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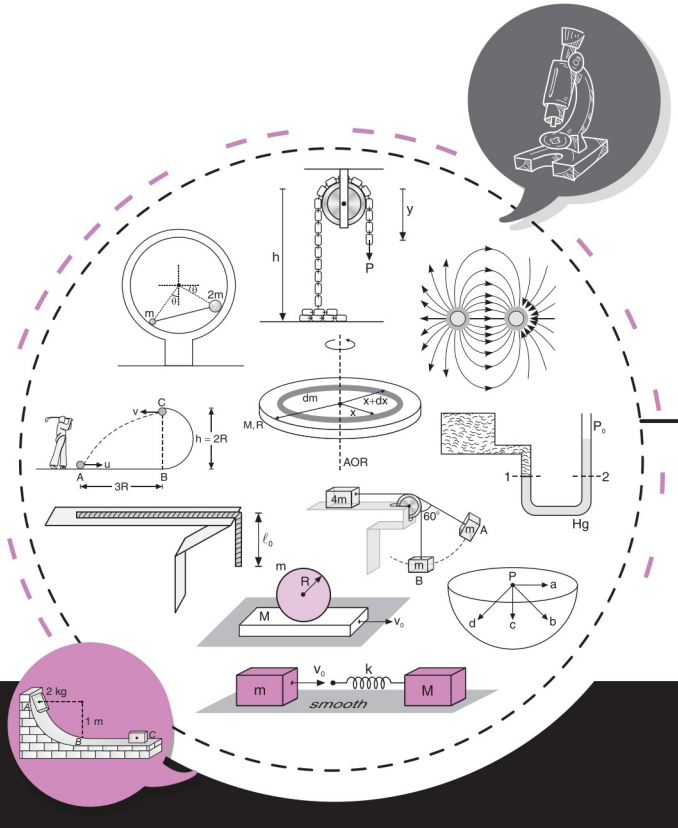

JEE ADVANCED PHYSICS

Mechanics – II

Pearson

Rahul Sardana

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CHAPTER INSIGHT

Learning Objectives

Help the students set an aim to achieve the major take-aways from a particular chapter.

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Explore the Concept of Work, Its Relation to Kinetic Energy
(c) Concept of Power
(d) Conservation of Energy

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Newton's Second Law and Concept of Centre of Mass
(d) Work Energy Theorem for System of Particles and its Applications from Centre Of Mass

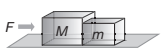
Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Moment of Inertia
(b) Parallel and Perpendicular Axis Theorem
(c) Rotational Kinematics
(d) Combined Effect of Rotation and Translation Motion
(e) Instantaneous Axis of Rotation
(f) Pure Rolling
(g) Conservation of Energy
(i) General Motion of a Rigid Body
(j) Concept of Toppling
(k) Shifting of Normal Reaction
(l) Work-Energy Principle
(m) Modified Newton's Second Law for Fixed Axis Rotation
(n) Uniform and Accelerated Pure Rolling
(o) Angular Momentum and its Conservation

ILLUSTRATION 5

Two blocks having masses M and m are placed on a smooth horizontal floor and a horizontal force F is applied on the system as shown. Calculate the work done by normal reaction (between the blocks) on two blocks as the system moves to the right through a distance d.



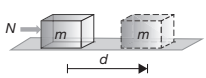
SOLUTION

Work done by normal rea
mass M is

$$W_M = Nd\cos(180°)$$

Work done by the normal
of mass m is

$$W_m = Nd\cos(0°) = Nd$$



Total work done by normal reaction (acting between two blocks) on the system of two blocks is

$$W_{net} = \Delta W_m + \Delta W_m$$

ILLUSTRATION 20

A small block of mass m is released from the top of a rough inclined plane as shown in Figure.

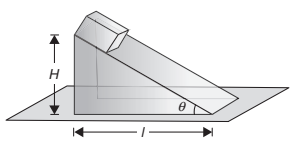


ILLUSTRATION 18

A planet of radius R equal to one-tenth the radius of earth has the same mass density as earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere calculate the force applied at the top of the wire by a person holding it in place.

Take the radius of earth to be 6×10^6 m and the accel- earth to be 10 ms^{-2}.

$$\Rightarrow \quad M_{planet} = \frac{M_{earth}}{10^3} = \frac{M_e}{1000}$$

Let the acceleration due to gravity at surface of planet and at the surface of earth be g_p and g_e respectively. Then

$$g_p = \frac{GM_{planet}}{R^2_{planet}} = \frac{GM_e(10)^2}{(10)^3 R_e^2} = \frac{GM_e}{10R_e^2}$$

Theory with Illustrations

Elaborative and simple theory helps the students to understand the illustrations supporting the theory. Please note that theory and problem solving techniques are based on simple learning program IF → THEN → ELSE. I would suggest you not to attempt the illustrations without going through the theory of that section.

Test Your Concepts-II

Based on Kinetic Energ

1. An engine working at a constant power P draws load of mass m against a resistance force f. Find th maximum speed of the load and the time taken attain half this speed.
2. An automobile of mass m accelerates, starting fro rest. The engine supplies constant power P.
   (a) Find the velocity of the automobile as a fun

Test Your Concepts-II

Based on Mot

1. A cubical block of ice of mass m_1 and edge L placed in a large tray of mass m_2. If the ice mel how far does the centre of mass of the system i plus tray come down?
2. Figure shows a fixed wedge on which two bloc of masses 2 kg and 3 kg are placed on its smoo

Test Your Concepts-III

Based on Instantaneous Axis of Rotati

1. A uniform ball of radius r rolls without slippi along the loop-the-loop track in Figure. It star from rest at height h above the bottom of th loop. If the ball is not to leave the track at the to of the loop, what is the least value h can ha (in terms of the radius $R \gg r$ of the loop)? Wh would h have been if the ball were to slide alor a frictionless track instead of rolling?

Test Your Concepts-IV

Based on Conservation Laws

1. If a body of mass m is projected vertically upwar from the surface of the earth (of radius R) so as t reach a height nR above the surface. Then find
   (a) the increase in its potential energy.
   (b) the velocity with which it must be projected.
2. If a body is released from a great distance from th centre of the earth, find its velocity when it strike the surface of the earth. Given, $R = 6400$ km an $g = 9.8\ \text{ms}^{-2}$.
3. Two particles of masses 20 kg and 10 kg are initiall at a distance of 1 m. Find the speeds of the particle

Test Your Concepts

These topic based exercise sets are based on simple, single concept classification technique. These are meant for students practice after they study a particular topic and want to practice more on that topic learnt. Finally, in case of any difficulty they can refer to the hints and solutions to these exercise sets given at the end of the book.

Problem Solving Techniques

These techniques ensure that students become capable enough to solve a variety of problems in an easy and quick manner.

Problem Solving Technique(s)

(a) The work done by a spring force is negative
(b) The work done by the spring force only depends on the initial and final points.
(c) The net work done by the spring force is zero for any path that returns to the initial point.
(d) The work done by the ment of its free end cl area of the trapezoid.

$$W_s = -\frac{1}{2}k\left(x_f^2 - x_i^2\right.$$

Problem Solving Technique(s)

In one such important situation when we have

$$\Sigma \vec{F}_{\text{external}} = \vec{0}$$

then, $\vec{v}_{cm} = \text{constant}$

Further if we have a two particle system initially at rest (or both its particle to be at rest), then as above result we have

$$\vec{}_{\text{nitial}} = \vec{0}$$

tem, this result can be inter-

Problem Solving Techn

(a) The gravitational force independent of the pr the properties of the in
(b) Gravitational force is co work done in displacing a body from one place to another is independent of path. It depends only on initial and final positions.
(c) The mass of air bubble in material medium is negative.
(d) The gravitational force obeys Newton's Third Law i.e.

$$\vec{F}_{12} = -\vec{F}_{21}$$

Problem Solving Technique(s)

WORD OF ADVICE

(a) Although the concept of IC is conveniently used to determine the velocity of any point in a body, **however generally, IC does not have zero acceleration and therefore, it should not be used for finding the acceleration of any point in the body.**
(b) When a body is subjected to general plane motion, the point determined as the instantaneous centre of zero velocity for the body can

hnique(s)

e rolling the velocity of the is zero but despite the relation $a = R\alpha$, the acceleration of the bottommost point is not zero. Because acceleration of any point P can be given as

$$\vec{a}_P = \vec{a}_C + \vec{a}_{PC}$$

Here, $\vec{a}_{PC}$ has two components:

Tangential acceleration $a_t = r\alpha$ (which is perpendicular to CP) and radial or normal acceleration $a_r = r\omega^2$ (which is along PC)

Conceptual Note(s)

Work done by a force is defined as the dot product of the force $(\vec{F})$ and the displacement $(\Delta\vec{r})$ the point of application of force with respect to the observer calculating the work.

Mathematically work done by a force is given by

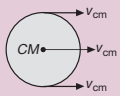
$$W = \vec{F}\cdot\Delta\vec{r} = \vec{F}\cdot(\vec{r}_2 - \vec{r}_1)$$

Conceptual Note(s)

Here the thrust force (F_t) applied by the chain on the table will be vertically downwards. Since $F_t = v_r\left(\frac{dm}{dt}\right)$, and in this expression v_r is downwards and $\frac{dm}{dt}$ is positive. Hence, F_t will act downwards.

Conceptual Note(s)

(a) Linear Motion or Rectilinear Motion in which a particles move with the same translational velocity that equals the velocity of centre of mass. Also, angular velocity $\omega = 0$ for the case of pure translational motion as shown in Figure.



(b) Rotational Motion in which the velocity of centre of mass of the particles is zero because the diametrically opposite points have velocities equal in magnitude but opposite in direction as show

Conceptual Note(s)

(a) If the particle is launched from the surface of the earth with a velocity $u > v_e$, then the body will reach infinity with a non zero velocity v which can be calculated by using the Law of Conservation of Energy, so

$$(U+K)_{\text{at surface}} = (U+K)_{\text{at }\infty}$$

$$\Rightarrow \quad -\frac{GmM}{R} + \frac{1}{2}mu^2 = 0 + \frac{1}{2}mv^2$$

(b) If the particle is launched from the surface of the earth with a velocity $u < v_e$, then the body will attain a maximum height h (say) and then return to the surface of the earth. The maximum height h can be calculated by using the Law of

Conceptual Notes

The Conceptual Notes, Remarks, Words of Advice, Misconception Removals provide warnings to the students about common errors and help them avoid falling for conceptual pitfalls.

Chapter End Solved Problems

These are based on multiple concept usage in a single problem approach so as to expose a student's brain to the ultimate throttle required to take the JEE examination.

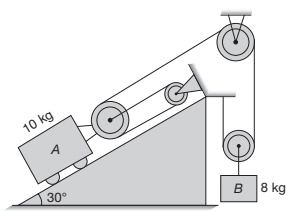
SOLVED PROBLEMS

PROBLEM 1

Two smooth spheres, A and B, of equal radius, lie on a horizontal table. A is of mass m and B is of mass $3m$. The spheres are projected towards each other with velocity vectors $5\hat{i}+2\hat{j}$ and $2\hat{i}-\hat{j}$ respectively and when they collide the line joining their centres is parallel to the vector $\hat{i}$. If the coefficient of restitution between A and B is $\frac{1}{2}$, find their velocities

PROBLEM 11

If the system is released from rest, determine the speeds of both masses after B has moved 1 m. Neglect friction and masses of the pulleys. $(g = 10\ \text{ms}^{-2})$.



SOLUTION

PROBLEM 8

A body is projected vertically upwards from the surface of earth with a velocity sufficient to carry it to infinity. Calculate the time taken by it to reach a height h.

SOLUTION

Let v be the velocity of the body at a distance r from the centre of earth. Applying Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mv_e^2 - \frac{GMm}{R} \qquad \ldots(1)$$

where, $v_e = \sqrt{2gR}$ and $g = \frac{GM}{R^2}$

Substituting in equation (1), we get

PROBLEM 3

A uniform circular disc of mass M and radius a is pivoted at a point O on its circumference so that it can rotate about the tangent at O, which is horizontal, the centre of the disc describing a vertical circle of centre O in a plane perpendicular to the tangent. The point diametrically opposite to O is A and the disc is just displaced from rest when A is vertically above O.

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Question: which ONLY ONE is correct.

1. Two equal masses are attached to the two ends of a spring of force constant k. The masses are pulled out symmetrically to stretch the spring by a length $2x_0$

MULTIPLE CORRECT CHOICE TYPE QUESTION

This section contains Multiple Correct Choice Type Question which ONE OR MORE is/are correct.

1. An object is revolving around the earth of radius R_e at height h from earth's surface. Select the correct statement(s).
   (A) Its time period is independent of h.
   (B) Its time period depends on h.
   (C) Orbital velocity of an object depends on h.

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions o followed by questions. Each question has four choices (A), (B), competitiveness there may be a few questions that may have r

Comprehension I

The mean radius of the earth's 1.5×10^{11} m and that of the orbit Based on above information, questions.

1. The mercury will revolve arou $\sqrt{2}$

INTEGER/NUMERICAL ANSWER TYPE QUESTI

In this section, the answer to each question is a numerical valu given in the question(s).

1. Calculate the ratio of the orbital radius of a communi- f earth so that it can cover rth during its revolution.

n newton, by the gravi- cle is moved along the on of gravitational field 1, 1) m to (−2, 3) m.

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having correct. Each question contains STATEMENT 1 and STATEMI

Bubble (A) If both statements are TRUE and STATEMENT 2
Bubble (B) If both statements are TRUE but STATEMENT 2
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 i

1. **Statement-1:** Kepler's Second Law can be understood by conservation of angular momentum principle.
   Statement-2: Kepler's Second Law is related with

Practice Exercise

Inclusion of all types of questions asked in JEE Advanced in adequate numbers helps you with enough practice

Archive JEE Main and Advanced

From this fully updated section, students get to know the actual pattern of the problems asked in the past examinations.

ARCHIVE: JEE MAIN

1. **[Online September 2020]**
   The mass density of a spherical galaxy varies over a large distance r from its center. In that r a small star is in a circular orbit of radius R. The period of revolution, T, depends on R as
   (A) $T \propto R$ (B) $T^2 \propto \frac{1}{R^3}$
   (C) $T^2 \propto R$ (D) $T^2 \propto R^3$

2. **[Online September 2020]**
   The height h at which the weight of a body will same as that at the same depth h from the surf the earth is (Radius of the earth is R and effect

Multiple Correct Choice Type Problems

In this section each question has four choices (A), (B), and (D), out of which ONE OR MORE is/are correct.

1. **[JEE (Advanced) 2020]**
   A rod of mass m and length L, pivoted at one of ends, is hanging vertically. A bullet of the same m moving at speed v strikes the rod horizontally a distance x from its pivoted end and gets embedd in it. The combined system now rotates with angu speed ω about the pivot. The maximum angular sp ω_M is achieved for $x = x_M$. Then

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. **[JEE (Advanced) 2019]**
   Consider a spherical gaseous cloud of mass density

Reasoning Based Questions

This section contains Reasoning type questions, each hav ing four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 an STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMEN 2 is the correct explanation of STATEMENT 1
Bubble (B) If both statements are TRUE but STATEMEN 2 is not the correct explanation of STATEMENT 1.

Linked Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension I

One twirls a circular ring (of mass M and radius R

Test Your Concepts-I (Based on Work Done by a Constant and Variable Force)

1. **Method I:**

Let a be the net downwar
Then

$$v^2 - u^2 = 2ah$$

$$\Rightarrow \quad k^2(gh) - 0^2 = 2ah$$

$$\Rightarrow \quad a = \frac{k^2 g}{2}$$

If f be the air drag, th
Newton's Second Law, we g

$$mg - f = ma$$

$$\Rightarrow \quad f = mg - ma$$

Multiple Correct Choice Type Questions

1. $I_1 = \frac{1}{2}MR^2 + \frac{1}{2}M\ell^2$

$4R^2)$

$= \frac{5}{6}MR^2$

$4R^2)$

$= \frac{11}{6}MR^2$

MR^2

orrect.

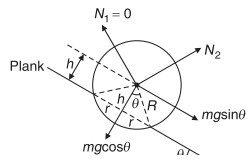
ation of Angular Momentum

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. At the verge of toppling, $N_1 = 0$

$$(mg\sin\theta)h = (mg\cos\theta)r$$



Single Correct Choice Type Questions

1. Horizontal and vertical components of initial velocity are

$$u_x = 20\sqrt{2}\cos 45° = 20$$

and $u_y = 20\sqrt{2}\sin 45° = 20$

After 1 s, horizontal comp
while the vertical compone

$$v_y = u_y - gt$$

$$\Rightarrow \quad v_y = 20 - (10)(1) = 10$$

Due to explosion one part
Conservation of Linear M
ponent of second part v
Therefore, maximum heig
part will be

$$H = h_1 + h_2$$

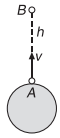
Where, h_1 be the height atta

Integer/Numerical Answer Type Questions

1. At height h, we have

$$g_h = \frac{g}{\quad}$$ ...(1)

(1) we get,



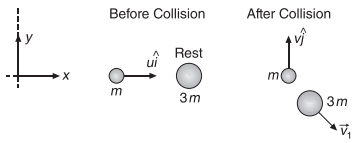
vation of Energy, from A to

$\frac{GMm}{R+h} + 0$

ARCHIVE: JEE MAIN

1. Applying Law of Conservation of Momentum, we get

$$\vec{p}_i = \vec{p}_f$$



$$\Rightarrow \quad (m)(u\hat{i}) + (3m)(\vec{0}) = (m)(v\hat{j}) + (3m)\vec{v}_1$$

$$\Rightarrow \quad mui - mvj = 3m\vec{v}_1$$

Hints and Explanations

Exhaustive solutions with shortcuts (where ever needed), help students enhance their problem-solving skills.

PREFACE

In the past few years, the IIT-JEE has evolved as an examination designed to check a candidate's true scientific skills. The examination pattern needs one to see those little details which others fail to see. These details tell us how much in-depth we should know to explain a concept in the right direction. Keeping the present-day scenario in mind, this series is written for students, to allow them not only to learn the tools but also to see why they work so nicely in explaining the beauty of ideas behind the subject. The central goal of this series is to help the students develop a thorough understanding of Physics as a subject. This series stresses on building a rock-solid technical knowledge based on firm foundation of the fundamental principles followed by a large collection of formulae. The primary philosophy of this book is to guide the aspirants towards detailed groundwork for strong conceptual understanding and development of problem-solving skills like mature and experienced physicists.

This updated Third Edition of the book will help the aspirants prepare for both Advanced and Main levels of JEE conducted for IITs and other elite engineering institutions in India. This book will also be equally useful for the students preparing for Physics Olympiads.

This book is enriched with detailed exhaustive theory that introduces the concepts of Physics in a clear, concise, thorough and easy-to-understand language. A large collection of relevant problems is provided in eight major categories (including updated archive for JEE Advanced and JEE Main), for which the solutions are demonstrated in a logical and stepwise manner.

We have carefully divided the series into seven parts to make the learning of different topics seamless for the students. These parts are

- Mechanics – I
- Mechanics – II
- Waves and Thermodynamics
- Electrostatics and Current Electricity
- Magnetic Effects of Current and Electromagnetic Induction
- Optics
- Modern Physics

Finally, I would like to thank all my teacher friends who had been a guiding source of light throughout the entire journey of writing this book.

To conclude, I apologise in advance for the errors (if any) that may have inadvertently crept in the text. I would be grateful to the readers who bring errors of any kind to my attention. I truly welcome all comments, critiques and suggestions. I hope this book will nourish you with the concepts involved so that you get a great rank at JEE.

PRAYING TO GOD FOR YOUR SUCCESS AT JEE. GOD BLESS YOU!

Rahul Sardana

ABOUT THE AUTHOR

Rahul Sardana, is a Physics instructor and mentor having rich and vast experience of about 20 years in the field of teaching Physics to JEE Advanced, JEE Main and NEET aspirants. Along with teaching, authoring books for engineering and medical aspirants has been his passion. He authored his first book '*MCQs in Physics*' in 2002 and since then he has authored many books exclusively for JEE Advanced, JEE Main and NEET examinations.

He is also a motivational speaker having skills to motivate students and ignite the spark in them for achieving success in all colours of life. Throughout this journey, by the Grace of God, under his guidance and mentorship, many of his students have become successful engineers and doctors.

You can reach him through email at sardanaeducation@gmail.com.

CHAPTER 1

Work, Energy, Power and Conservation of Energy

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Explore the Concept of Work, Its Relation to Kinetic Energy
(b) Energy Associated with Motion of Objects
(c) Concept of Power
(d) Conservation of Energy
(e) Motion of Body in a Vertical Circle

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

WORK, ENERGY, POWER AND LAW OF CONSERVATION OF ENERGY

INTRODUCTION

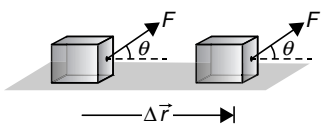
In Mechanics-I, we had analysed motion by using concepts such as position, velocity, acceleration, and force. However, after learning these concepts we may find difficulty in describing some types of motion by applying Newton's laws directly e.g. a block sliding down a curved slope may be difficult to describe using Newton's Laws. So, in this Chapter, we shall be analysing motion using the concepts of work and energy. Unlike force, which is a vector physical quantity, work and energy both are scalar physical quantities and they can be associated with particles as well as the systems of particles. These new concepts of work and energy will provide us with powerful methods to solve a wide variety of problems.

WORK DONE BY A CONSTANT FORCE

The work W done by a constant force F when the point of application of force undergoes a displacement Δr is defined as

$$W = F\Delta r\cos\theta \qquad \ldots(1)$$

where θ is the angle between $\vec{F}$ and $\Delta\vec{r}$ as indicated in Figure.



Only the component of $\vec{F}$ along $\Delta\vec{r}$, that is, $F\cos\theta$, contributes to the work done. Strictly speaking, the work is done by the source or agent that applies the force. Work is a scalar quantity and its SI unit is the joule (J). From equation (1), we see that

$$1\ \mathrm{J} = 1\ \mathrm{Nm}$$

From (1) we can also conclude that, work done is also defined as the dot product of force and its displacement as given by the following equation (2)

$$W = \vec{F}\cdot\Delta\vec{r} \qquad \ldots(2)$$

In terms of rectangular components, the two vectors are

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \text{ and } \Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

Hence, equation (2) may be written as

$$W = \vec{F}\cdot\Delta\vec{r} = F_x\Delta x + F_y\Delta y + F_z\Delta z \quad \ldots(3)$$

The work done by a given force on a body depends only on the force, the displacement, and the angle between them. It does not depend on the velocity or the acceleration of the body, or on the presence of other forces.

Since the work is a scalar, its value also does not depend on the orientation of the coordinate axes. Since the magnitude of a displacement in a given time interval depends on the velocity of the frame of reference used to measure the displacement, the calculated **work also depends on the reference frame.**

Conceptual Note(s)

Work done by a force is defined as the dot product of the force $(\vec{F})$ and the displacement $(\Delta\vec{r})$ of the point of application of force with respect to the observer calculating the work.

Mathematically work done by a force is given by

$$W = \vec{F}\cdot\Delta\vec{r} = \vec{F}\cdot(\vec{r}_2 - \vec{r}_1)$$

If a number of forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$,... $\vec{F}_n$ are acting on a body and it shifts from position vector $\vec{r}_1$ to position vector $\vec{r}_2$, then

$$W = (\vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n)\cdot(\vec{r}_2 - \vec{r}_1)$$

ILLUSTRATION 1

A box is moved over a horizontal path by applying force $F = 80$ N at an angle $\theta = 60°$ to the horizontal. What is the work done during the displacement of the box over a distance of 0.5 km.

SOLUTION

By definition, $W = F\Delta r\cos\theta$

Here $F = 80$ N, $\Delta r = 0.5$ km $= 500$ m, $\theta = 60°$

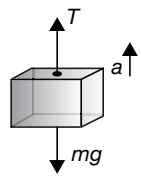
$\Rightarrow \quad W = (80)(500)\cos(60°) = 20$ kJ

ILLUSTRATION 2

A load of mass $m = 3000$ kg is lifted by a winch with an acceleration $a = 2$ ms^{-2}. Find the work done during the first one and a half seconds from the beginning of motion.

SOLUTION

The height to which the body is lifted during the first t second is $h = \frac{1}{2}at^2$



The tension in the rope is given by

$$T = mg + ma$$

Work done by tension is given by

$$W = Th\cos(0°) = m(g+a)\left(\frac{1}{2}at^2\right)$$

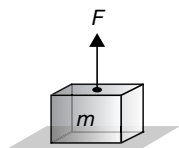
Here $m = 3000$ kg, $a = 2$ ms^{-2}, $g = 10$ ms^{-2}, $t = 1.5$ s

$\Rightarrow \quad W = (3000)(10+2)\left[\frac{1}{2}(2)(1.5)^2\right]$

$\Rightarrow \quad W = 81$ kJ

ILLUSTRATION 3

A block of mass $m = 4$ kg is pulled by a force $F = 20$ N upwards through a height $h = 2$ m as shown in Figure.



Calculate the work done on the block by the applied force F and its weight. Take $g = 10$ ms^{-2}.

SOLUTION

Weight of the block is $mg = (4)(10) = 40$ N

Work done by the applied force $W_F = Fh\cos(0°)$

The angle between force and displacement is $0°$, so

$$W_F = (20)(2)(1) = 40 \text{ J}$$

Similarly, work done by the weight of the block is

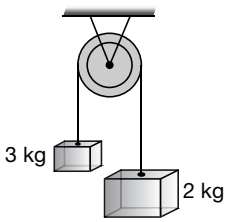
$$W_{mg} = (mg)(h)\cos(180°)$$

$$\Rightarrow \quad W_{mg} = (40)(2)(-1) = -80 \text{ J}$$

Negative sign indicates that work is done against the gravitational pull of the earth.

ILLUSTRATION 4

Two unequal masses of 3 kg and 2 kg are attached at the two ends of a light inextensible string passing over a smooth pulley as shown in figure.



If the system is released from rest, find the work done by string on both the blocks in 2 s. Take $g = 10 \text{ ms}^{-2}$.

SOLUTION

The acceleration of the blocks in the system is

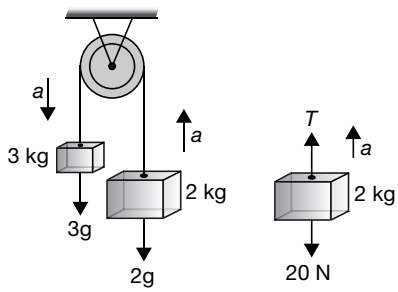
$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$$

$$\Rightarrow \quad a = \left(\frac{3-2}{3+2}\right)10 = 2 \text{ ms}^{-2}$$

Displacement of both the blocks in 1 s is

$$S = \frac{1}{2}at^2 = \frac{1}{2}(2)(2)^2 = 4 \text{ m}$$

Free body diagram of 2 kg block is shown in figure.



Using $\Sigma F = ma$, we get

$$T - 20 = 2a$$

$$\Rightarrow \quad T = 24 \text{ N}$$

Work done by string (tension) on 2 kg block in 2 s is

$$W_1 = (T)(S)\cos(0°)$$

$$\Rightarrow \quad W_1 = (24)(4)(1) = 96 \text{ J}$$

Similarly, work done by string on 3 kg block in 2 s will be

$$W_2 = (T)(S)(\cos 180°)$$

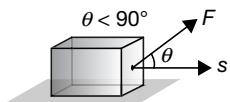
$$\Rightarrow \quad W_2 = (24)(4)(-1) = -96 \text{ J}$$

POSITIVE AND NEGATIVE WORK

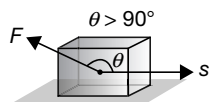
Work done by a force may be positive or negative depending on the angle θ between the force and displacement.

When the angle θ is acute $(\theta < 90°)$, then the component of force is parallel to the displacement and the work done is positive as shown in Figure (a).

When the angle θ is obtuse $(\theta > 90°)$, then the component of force is antiparallel to the displacement and the work done by force is negative as shown in Figure (b).



(a)
Positive work done by a force F



(b)
Negative work done by a force F

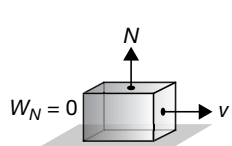
ZERO WORK DONE

Since we know that $W = F\Delta r \cos\theta$, so the work done by a force is zero when

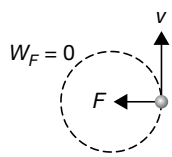
(a) EITHER $F = 0$

(b) OR $\Delta r = 0$

(c) OR $\cos\theta = 0$ i.e. $\theta = 90°$



(a)



(b)

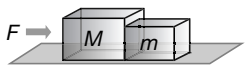
When the force and the displacement are perpendicular, the work done by the force is zero.

(a) the normal reaction N is perpendicular to displacement, therefore, $W_N = 0$.

(b) the centripetal force is perpendicular to displacement, thus $W_F = 0$.

ILLUSTRATION 5

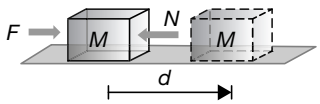
Two blocks having masses M and m are placed on a smooth horizontal floor and a horizontal force F is applied on the system as shown. Calculate the work done by normal reaction (between the blocks) on two blocks as the system moves to the right through a distance d.



SOLUTION

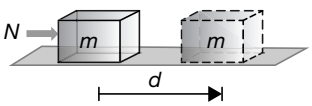
Work done by normal reaction acting on the block of mass M is

$$W_M = Nd\cos(180°) = -Nd$$



Work done by the normal reaction acting on the block of mass m is

$$W_m = Nd\cos(0°) = Nd$$



Total work done by normal reaction (acting between two blocks) on the system of two blocks is

$$W_{net} = \Delta W_m + \Delta W_m$$

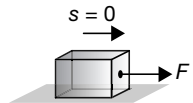
$$\Rightarrow \quad W_{net} = Nd + (-Nd)$$

$$\Rightarrow \quad W_{net} = 0$$

WORK DONE BY FRICTION

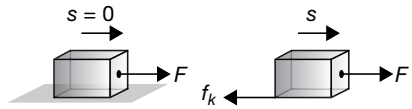
There is a misconception that the force of friction always does negative work. In reality, the work done by friction may be zero, positive or negative depending upon the situation.

In figure (a), when a block is pulled by a force F and the block does not move, the work done by friction is zero.



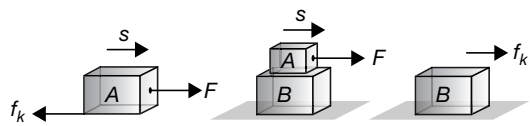
(a)

In figure (b), when a block is pulled by a force F on a stationary surface, the work done by the kinetic friction is negative.



(b)

In figure (c), block A is placed on the block B. When the block A is pulled with a force F, the friction force does negative work on block A and positive work on block B. The kinetic friction and displacement are oppositely directed in case of block A while in case of block B they are in the same direction.



(c)

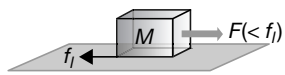
WORK DONE BY STATIC FRICTION

CASE-1:

Consider a block of mass M is placed on a rough horizontal surface. Let a pulling force F act horizontally on the block as shown in Figure.

When the applied force F is less than or equal to maximum possible frictional force between block and the surface i.e. limiting friction f_l, then the block will not slip on the surface and the work done by frictional force acting on the block will be zero because the displacement of block will be zero.

$$F \le f_l$$

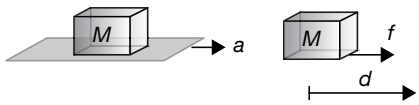


CASE-2:

When the surface is accelerating towards the right with an acceleration a and the block does not slip on the surface, then the frictional force f (which may be

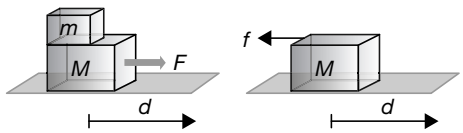
static or limiting) on the block will be in the direction of motion of the block and work done by frictional force to move the block through a distance d is

$$W = fd\cos(0°) = fd$$



CASE-3:

When another block of mass m is placed on the block of mass M placed on a smooth surface and the block of mass M is pulled by a force F through a distance d as shown in Figure.



In the case of no slipping between the two blocks, work done on the lower block by the frictional force f (which may be static or limiting) applied due to the upper block is

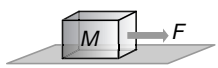
$$W = fd\cos(180°) = -fd$$

Similarly, work done on the upper block by the frictional force f (which may be static or limiting) applied due to the lower block is

$$W = fd\cos(0°) = fd$$

WORK DONE BY KINETIC FRICTION

Consider a block of mass M is placed on a rough horizontal surface. Let a pulling force F act horizontally on the block as shown in Figure.



When the applied force F is more than the limiting friction f_l, then the block will move on the surface and the friction between the block and surface will be kinetic given by

$$f_k = \mu_k N$$

This f_k will be directed opposite to the direction of motion of the block w.r.t. the surface.

When the block gets displaced by distance d, then the work done by kinetic friction acting on the block is given by

$$W = f_k N\cos(180°) = -\mu_k Nd$$

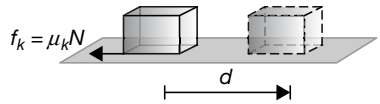
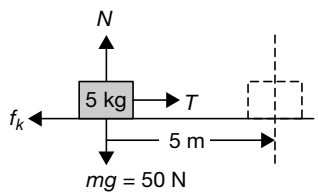


ILLUSTRATION 6

A block of mass 5 kg is placed on a rough horizontal ground with coefficient of friction 0.1. It is being pulled by means of a light horizontal rope, with a constant speed of $3\ \text{ms}^{-1}$. Calculate the work done on the block due to the tension in the rope, the force of friction and normal reaction, when the body moves through 5 m. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

The free body diagram (FBD) of the block is shown in the Figure.



Since the body moves with a constant speed, so its acceleration is zero.

$$\Rightarrow \quad N = mg = 50\ \text{N}$$

$$\Rightarrow \quad T = f_k = \mu N = 0.1 \times 50 = 5\ \text{N}$$

So, work done by tension T is

$$W_T = T \times 5 \times \cos(0°) = 25\ \text{J}$$

Work done by kinetic friction f_k is

$$W_{fk} = f_k \times 5 \times \cos(180°) = -25\ \text{J}$$

Work done by gravitational force mg is

$$W_{mg} = (mg)(5)\cos(90°) = 0$$

Work done by normal reaction N is

$$W_N = (N)(5)\cos(90°) = 0$$

Please note that the total work done is

$$W = 25 - 25 = 0$$

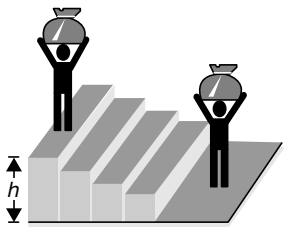
Since we observe that the block is moving with constant speed, therefore, there is no change in kinetic energy and hence in accordance with Work-Energy Theorem, work done by all forces acting on the block is zero.

DEPENDENCE OF WORK ON FRAME OF REFERENCE

Work as defined, depends on frame of reference also. When we change from one inertial reference frame to another inertial reference frame, the force does not change, while displacement may change, so the work done by a force will be different.

FOR EXAMPLE

(a) When a porter with a bag on his head moves up a staircase, work done by the upward lifting force relative to him will be zero.

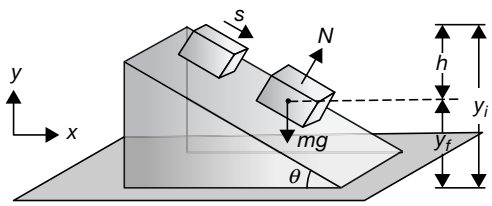


This is because, the displacement of the porter with respect to himself is zero. However, relative to a person on the ground, the work done by the porter will be *mgh*.

(b) Suppose a person is pushing a box in a moving train (in the same direction as the movement of train) by applying a force $\vec{F}$ on it. In the reference frame of the train, the work done by the force will be $\vec{F}\cdot\vec{r}$ where $\vec{r}$ is the displacement of the block with respect to the train. But in the reference frame of the earth the work will be $\vec{F}\cdot(\vec{r}+\vec{r}_0)$ where $\vec{r}_0$ is the displacement of the train with respect to the earth.

WORK DONE BY GRAVITY

Consider a block of mass m which slides down a smooth inclined plane of angle θ as shown in Figure.



Let us assume the coordinate axes as shown in the figure, to specify the components of the two vectors - although the value of work will not depend on the orientation of the axes.

Now, the force of gravity, $\vec{F}_g = -mg\hat{j}$

and the displacement is given by

$$\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

The work done by gravity is

$$W_g = \vec{F}_g\cdot\Delta\vec{r} = -mg\hat{j}\cdot\left(\Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}\right)$$

$$\Rightarrow \quad W_g = -mg\Delta y \qquad \left\{\because\ \hat{j}\cdot\hat{i} = 0, \hat{j}\cdot\hat{j} = 1, \hat{j}\cdot\hat{k} = 0\right\}$$

Since $\Delta y = y_f - y_i = -h$

The work done by gravity is

$$W_g = -mg\left(y_f - y_i\right) = +mgh$$

If the block moves in the upward direction, then the work done by gravity is negative i.e., work is done against the gravitational pull and is given by

$$W_g = -mgh$$

Conceptual Note(s)

(a) The work done by the force of gravity depends only on the initial and final vertical coordinates, not on the path taken.

(b) The work done by gravity is zero for any path that returns to its initial point.

When several forces act on a body one may calculate the work done by each force individually. The net work done on the body is the algebraic sum of individual contributions.

$$W_{\text{net}} = \vec{F}_1\cdot\Delta\vec{r}_1 + \vec{F}_2\cdot\Delta\vec{r}_2 + \ldots\ldots\ldots + F_n\cdot\Delta\vec{r}_n$$

$$\Rightarrow\ W_{\text{net}} = W_1 + W_2 + \ldots\ldots\ldots + W_n$$

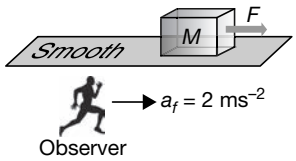
WORK DONE BY PSEUDO FORCE

When a body is being displaced under the action of a force and we are observing this from a non-inertial frame, then to calculate the total work done, we have to also consider the work done by pseudo force(s).

ILLUSTRATION 7

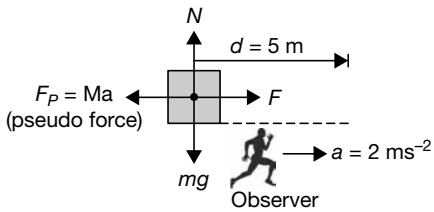
A block of mass $M = 5\text{ kg}$ is placed on a smooth horizontal surface and a force $F = 25\text{ N}$ starts acting on it parallel smooth to the plane. Calculate the net work done on the block as seen by an observer

who is accelerating in the direction of the force with an acceleration of $2\ \text{ms}^{-2}$ if the block gets displaced by $5\ \text{m}$ in the direction of the force with respect to the observer.



SOLUTION

FBD of the block w.r.t. the observer is shown in Figure.



Work done by weight of the body and the normal reaction force will be zero.

Since, work done by the applied force F is

$$W_F = Fd\cos(0°) = Fd = (25)(5) = 125\ \text{J}$$

and work done by the pseudo force F_{ps} is

$$W_{ps} = F_{ps}d\cos(180°) = -F_{ps}d$$

Since, $F_{ps} = ma = (5)(2) = 10\ \text{N}$

$$\Rightarrow \quad W_{ps} = -F_{ps}d = -(10)(5) = -50\ \text{J}$$

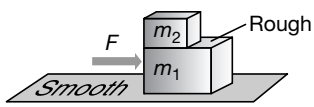
So, net work done W is given by

$$W = W_F + W_{ps}$$

$$\Rightarrow \quad W = 125 + (-50) = 75\ \text{J}$$

ILLUSTRATION 8

Two blocks of masses m_1 and m_2 are placed one above the other. There is no friction between the lower block and ground. The lower block is being pushed by a constant horizontal force F as shown in Figure.



There is sufficient friction between the blocks so that they do not slip over each other. Draw the free body diagram of the upper block with respect to the ground frame and with respect to the frame attached to the lower block. Find the work done by various forces on the upper block in the two frames, as the arrangement moves through l.

SOLUTION

Since the two blocks move together, so

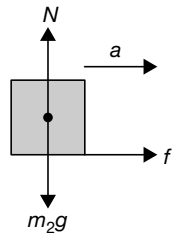
$$F = (m_1 + m_2)a$$

where a is their common acceleration.

$$\Rightarrow \quad a = \frac{F}{m_1 + m_2}$$

As long as the blocks move together, friction between the blocks will be static or limiting.

FBD of m_2 with respect to ground



$$N = m_2 g \ \text{(equilibrium along vertical)}$$

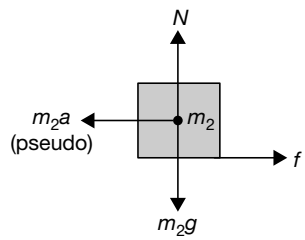
If f be the friction between the blocks, then

$$f = m_2 a \ \text{(by Newton's 2nd law).}$$

$$W_N = 0\,,\ W_{m_2 g} = 0 \ \text{and}$$

$$W_{fr} = (m_2 a)l\cos 0° = \frac{m_2 Fl}{m_1 + m_2}$$

FBD of m_2 with respect to m_1



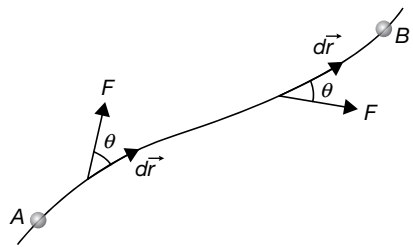
In the frame attached to m_1, the mass m_2 does not move. Therefore, work done by all the forces is zero.

WORK DONE BY A VARIABLE FORCE

When the magnitude and direction of a force vary in three dimensions, it can be expressed as a function of the position vector $\vec{F}(r)$, or in terms of the coordinates $\vec{F}(x,y,z)$. The work done by such a force in an infinitesimal displacement $d\vec{r}$ is

$$dW = \vec{F}\cdot d\vec{r}$$

The total work done in going from point A to point B as shown in the Figure is



A particle moves along a curved path subject to a variable force $\vec{F}$. The work done by the force in a displacement $d\vec{s}$ is $dW = \vec{F}\cdot d\vec{r}$.

$$W_{A\to B} = \int_A^B \vec{F}\cdot d\vec{r} = \int_A^B (F\cos\theta)\, dr$$

In terms of rectangular components,

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \text{ and } d\vec{r} = \hat{i}dx + \hat{j}dy + \hat{k}dz$$

$$\Rightarrow \quad W_{A\to B} = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

ILLUSTRATION 9

A force $F = (5+2x+3x^2)$ acts on a particle in x-direction where F is in newton and x in metre. Find the work done by this force during a displacement from $x = 1$ m to $x = 2$ m.

SOLUTION

As the force is variable, we shall find the work done in a small displacement from x to $x+dx$ and then integrate it to find the total work. The work done in this small displacement is

$$dW = Fdx = (5+2x+3x^2)dx$$

Thus, $W = \int_1^2 dW = \int_1^2 (5+2x+3x^2)dx$

$$\Rightarrow \quad W = (5x+x^2+x^3)\Big|_1^2 = 5(1)+(4-1)+(8-1)$$

$$\Rightarrow \quad W = 15 \text{ J}$$

ILLUSTRATION 10

A force varying with distance is given as $F = ae^{-bx}$ acts on a particle of mass m moving in a straight line. Find the work done on the particle in its displacement from origin to a distance d.

SOLUTION

Since the applied force varies with displacement, so the work done by the force is given by

$$W = \int_0^d Fdx = \int_0^d ae^{-bx}dx$$

$$\Rightarrow \quad W = -\frac{a}{b}\left(e^{-bx}\right)\Big|_0^d = \frac{a}{b}(1-e^{-bd})$$

ILLUSTRATION 11

A force $F = -\frac{k}{x^2}(x \neq 0)$ acts on a particle in x-direction. Find the work done by this force in displacing the particle from $x = a$ to $x = +2a$, where k is a positive constant.

SOLUTION

$$W = \int Fdx = \int_{+a}^{+2a}\left(\frac{-k}{x^2}\right)dx = \left(\frac{k}{x}\right)\Big|_{+a}^{+2a} = -\frac{k}{2a}$$

Problem Solving Technique(s)

It is important to note that work comes out to be negative which is quite obvious as the force acting on the particle is in negative x-direction $\left(F = -\frac{k}{x^2}\right)$ while displacement is along positive x-direction (from $x = a$ to $x = 2a$)

ILLUSTRATION 12

An object is displaced from point $A(2, 3, 4)$ m to a point $B(1, 2, 3)$ m under the influence of a constant force $\vec{F} = (2\hat{i} + 3\hat{j} + 4\hat{k})$ N. Calculate the work done by this force in the process.

SOLUTION

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$$

$$\Rightarrow \quad W = \int_{(2,3,4)\text{ m}}^{(1,2,3)\text{ m}} (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$\Rightarrow \quad W = (2x + 3y + 4z)\Big|_{(2,3,4)\text{ m}}^{(1,2,3)\text{ m}} = -9 \text{ J}$$

ALTERNATE SOLUTION:

Since, $\vec{F}$ = constant, we can also use $W = \vec{F} \cdot \Delta\vec{r}$

Here, $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$$\Delta\vec{r} = (i + 2\hat{j} + 3\hat{k}) - (2\hat{i} + 3\hat{j} + 4\hat{k}) = (-\hat{i} - \hat{j} - \hat{k})$$

ILLUSTRATION 13

An object is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j})$ m to position vector $\vec{r}_2 = (4\hat{i} + 6\hat{j})$ m under a force $\vec{F} = (3x^2\hat{i} + 2y\hat{j})$ N. Calculate the work done by this force.

SOLUTION

$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}, \text{ where } d\vec{r} = \hat{i}dx + \hat{j}dy + \hat{k}dz$$

$$\Rightarrow \quad W = \int_{\vec{r}_1}^{\vec{r}_2} (3x^2\hat{i} + 2y\hat{j}) \cdot (\hat{i}dx + \hat{j}dy + \hat{k}dz)$$

$$\Rightarrow \quad W = \int_{\vec{r}_1}^{\vec{r}_2} (3x^2dx + 2ydy)$$

$$\Rightarrow \quad W = \int_{(2,3)}^{(4,6)} d(x^3) + \int_{(2,3)}^{(4,6)} d(y^2) = (x^3 + y^2)\Big|_{(2,3)}^{(4,6)}$$

$$\Rightarrow \quad W = 83 \text{ J}$$

ILLUSTRATION 14

An object is displaced from a point $A(0, 0, 0)$ to $B(1\text{ m}, 1\text{ m}, 1\text{ m})$ under a force $\vec{F} = (y\hat{i} + x\hat{j})$N. Calculate the work done by this force in this process.

SOLUTION

$$W = \int_A^B \vec{F} \cdot d\vec{r}, \text{ where } d\vec{r} = \hat{i}dx + \hat{j}dy + \hat{k}dz$$

$$\Rightarrow \quad W = \int_A^B (y\hat{i} + x\hat{j}) \cdot (\hat{i}dx + \hat{j}dy + \hat{k}dz)$$

$$\Rightarrow \quad W = \int_A^B (ydx + xdy) = \int_A^B d(xy)$$

$$\Rightarrow \quad W = (xy)\Big|_{(0,0,0)}^{(1,1,1)}$$

$$\Rightarrow \quad W = 1 \text{ J}$$

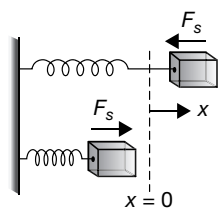
WORK DONE BY SPRING FORCE

If x be the displacement of the free end of the spring from its equilibrium position then, the restoring force (F_S) in the spring is given by Hooke's Law

$$F_S = -kx$$

where x is the extension or compression of the spring.

The negative sign signifies that the restoring force is always opposite to the extension $(x > 0)$ or the compression $(x < 0)$ in the spring. In simple words, the force tends to restore the system to its equilibrium position as shown in Figure.



The work done by the spring force for a displacement from x_i to x_f is given by

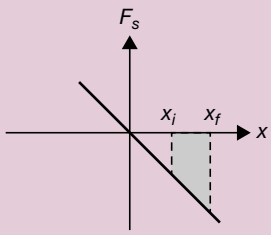
$$W_S = \int_{x_i}^{x_f} F_S dx \cos(180°) = -\int_{x_i}^{x_f} kx dx$$

$$\Rightarrow \quad W_S = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

Problem Solving Technique(s)

(a) The work done by a spring force is negative
(b) The work done by the spring force only depends on the initial and final points.
(c) The net work done by the spring force is zero for any path that returns to the initial point.
(d) The work done by the spring when the displacement of its free end changes from x_i to x_f is the area of the trapezoid.

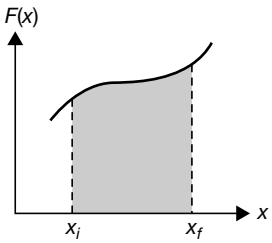
$$W_s = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$



Graphically, the work done by the spring force in a displacement from x_i to x_f is the shaded area (as shown in the figure) which is the difference in the areas of two triangles.

WORK DONE AS AREA UNDER F-X GRAPH

In general, the work done by a variable force $F(x)$ from an initial point x_i to final point x_f is given by the **area under the force - displacement curve** as shown in the figure.



(a) The work done by a non-constant force is approximately equal to sum of the areas of the rectangles.
(b) The area under the curve is given by the integral

$$W = \int F(x)dx$$

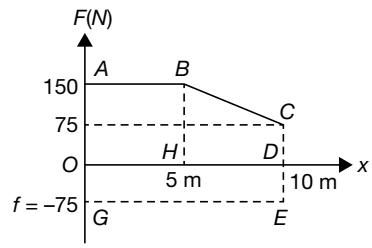
Area (work done) above the x-axis is taken as positive, and below the x-axis is taken as negative.

ILLUSTRATION 15

A gardener pushes a lawn roller on a rough surface. He applies a force of 150 N over a distance of 5 m. After some time, he gets tired and his applied force reduces to 75 N linearly with distance. The total distance moved by the gardener is 10 m. If the frictional force offered by the ground to the gardener is 75 N, then plot the graph of force applied by the gardener against the displacement. Also calculate the work done by the two forces over a distance of 10 m.

SOLUTION

The plot of force F applied by the gardener and the opposing frictional force $f = 75$ N vs displacement is shown in Figure.



At $x = 10$ m, $F = 75\ \text{N}(\neq 0)$ and the frictional force is also $f = |\vec{f}| = 75$ N. So, the frictional force opposes motion and acts in a direction opposite to F. Taking the direction of F as positive, the frictional force f is therefore shown on the negative side of the force axis.
The work done by the gardener is

$$W_F = \begin{pmatrix}\text{Area of}\\ \text{rectangle}\\ ABHO\end{pmatrix} + \begin{pmatrix}\text{Area of}\\ \text{trapezium}\\ BCDH\end{pmatrix}$$

$$\Rightarrow \quad W_F = (150)(5) + \frac{1}{2}(150+75)(5)$$

$$\Rightarrow \quad W_F = 750 + 562.5 = 1312.5\ \text{J}$$

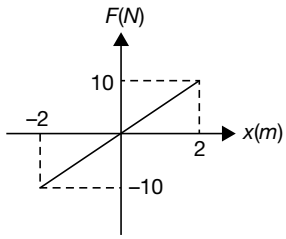
Work done by frictional force f is

$$W_f = \text{Area of rectangle } ODEG$$

$$\Rightarrow \quad W_f = (-75)\times 10 = -750 \text{ J}$$

ILLUSTRATION 16

A force F acting on a particle varies with the position x as shown in Figure.



Find the work done by this force in displacing the particle from

(a) $x = -2$ m to $x = 0$

(b) $x = 0$ to $x = 2$ m

SOLUTION

(a) From $x = -2$ m to $x = 0$, displacement of the particle is along positive x-direction while force acting on the particle is along negative x-direction. Therefore, work done is negative and given by the area under F-x graph.

$$\Rightarrow \quad W = -\frac{1}{2}(2)(10) = -10 \text{ J}$$

(b) From $x = 0$ to $x = 2$ m, displacement of particle and force acting on the particle both are along positive x-direction. Therefore, work done is positive and given by the area under F-x graph, or

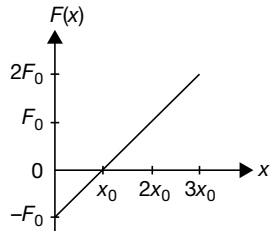
$$W = \frac{1}{2}(2)(10) = 10 \text{ J}$$

ILLUSTRATION 17

The force exerted on an object is $F = F_0\left(\frac{x}{x_0} - 1\right)$. Calculate the work done in moving the object from $x = 0$ to $x = 3x_0$ by plotting the $F(x)$ graph and then calculating the area under the graph. Also calculate the work by evaluating the line integral of force.

SOLUTION

The graph shows F as a function of x is shown in Figure.



Since, $F = F_0\left(\frac{x}{x_0} - 1\right)$, so the work done is negative as the object moves from $x = 0$ to $x = x_0$ and positive as it moves from $x = x_0$ to $x = 3x_0$. Since work done is the area under F-x graph so

$$W = -\frac{1}{2}F_0x_0 + \frac{1}{2}(2F_0)(2x_0) = \frac{3}{2}F_0x_0$$

The line integral of the force is the work done, so

$$W = \int_0^{3x_0} F_0\left(\frac{x}{x_0} - 1\right)dx$$

$$\Rightarrow \quad W = F_0\left(\frac{x^2}{2x_0} - x\right)\Bigg|_0^{3x_0} = \frac{3}{2}x_0F_0$$

Test Your Concepts-I

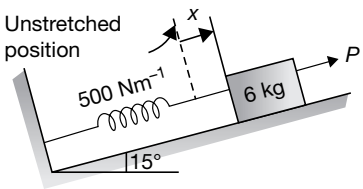
Based on Work Done by a Constant and Variable Force

(Solutions on page H.3)

1. A ball dropped from a height h strikes the ground with a speed of $k\sqrt{gh}$ where $k < \sqrt{2}$. Calculate the work done by air drag.
2. A block is pulled a distance ℓ_0 along a rough horizontal table by a horizontal string. If the tension in the string is T, the mass of the block is m, the normal reaction is N and frictional force is f. Find the work done by each of these forces.
3. A particle is pulled a distance ℓ up a rough plane inclined at an angle θ to the horizontal by a string

inclined at an angle ϕ to the plane $(\theta+\phi<90°)$. If the tension in the string is *T*, the normal reaction between the particle and the plane is *N*, the frictional force is *f* and the mass of the particle is *m*. Find the work done by each of these forces.

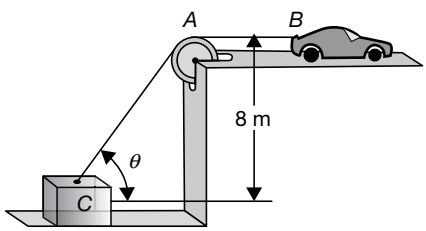
4. The spring is unstretched at the position $x = 0$. Under the action of a force *P*, the cart moves from the initial position $x_1 = -150$ mm to the final position $x_2 = 80$ mm. Calculate the work done on the cart by



(a) the spring and
(b) its weight.

5. In a research apparatus, one of the forces exerted on a proton is $\vec{F} = -\alpha x^2 \hat{i}$, where α has unit of Nm^{-2}. Calculate the work done by $\vec{F}$, when the proton undergoes a displacement along the straight line path
(a) from the point (0.1, 0)m to the point (0.1, 0.4)m
(b) along the straight-line path from the point (0.1, 0)m to the point (0.3, 0)m
(c) along the straight-line path from the point (0.3, 0)m to the point (0.1, 0)m

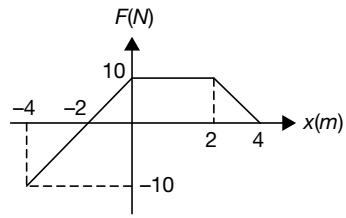
6. The 100 kg block is being dragged across the smooth surface by means of a car. If the towing cable passes over a small pulley at *A*, calculate the work done on the block when $\theta = 60°$. The block is at rest when $\theta = 30°$ and the car exerts a constant force $F = 500$ N on the cable.



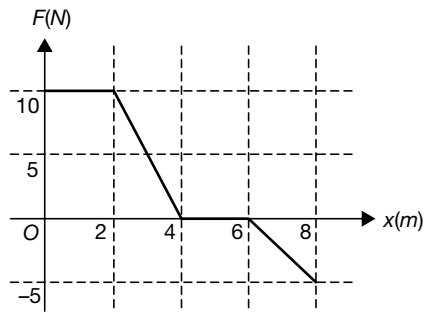
7. A force $\vec{F} = \left(3x^2\hat{i} + 2y\hat{j}\right)$N displaces an object from position vector $\vec{r}_1 = \left(2\hat{i} + 3\hat{j}\right)$m to $\vec{r}_2 = \left(4\hat{i} + 6\hat{j}\right)$ m. Calculate the work done by this force.

8. A cutting tool under microprocessor control has several forces acting on it. One force is $\vec{F} = -\alpha x y^2 \hat{j}$, a force in the negative *y*-direction whose magnitude depends on the position of the tool. The constant is $\alpha = 1.5\ Nm^{-3}$. Consider the displacement of the tool from the origin to the point $(2, 2)$m.
(a) Calculate the work done, in joule, on the tool by $\vec{F}$ if this displacement is along the straight line $y = x$ that connects these two points.
(b) Calculate the work done on the tool by $\vec{F}$ if the tool is first moved out along the *x*-axis to the point (2, 0)m and then moved parallel to the *y*-axis to (2, 2)m.
(c) Compare the work done by $\vec{F}$ along these two paths. Is $\vec{F}$ conservative or non-conservative?

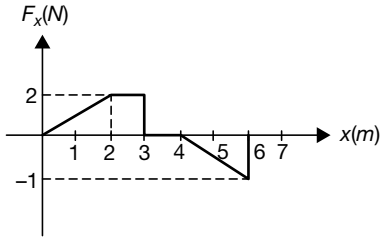
9. Force acting on a particle varies with displacement as shown in figure. Find the work done by this force on the particle from $x = -4$ m to $x = +4$ m.



10. A 5 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in the figure. Find the work done by this force as the block moves from the origin to $x = 8$ m.

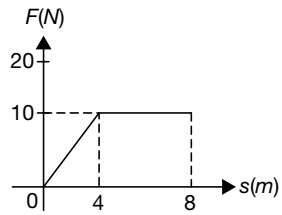


11. A child applies a force $\vec{F}$ parallel to the x-axis to a block moving on a horizontal surface. As the child controls the speed of the block, the *x*-component of the force varies with the *x*-coordinate of the block as shown in figure. Calculate the work done by the force $\vec{F}$ when the block moves



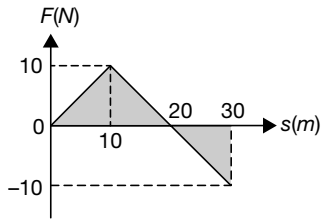
(a) from $x = 0$ to $x = 3$ m
(b) from $x = 3$ m to $x = 4$ m
(c) from $x = 4$ m to $x = 7$ m
(d) from $x = 0$ to $x = 7$ m

12. Force acting on a particle varies with displacement as shown in figure. Calculate the total work done by the force.



13. A force $\vec{F} = -k(y\hat{i} + x\hat{j})$, where k is a positive constant, acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a, 0)$ and then parallel to the y-axis to the point (a, a). Calculate the total work done by the force on the particle.

14. The force versus displacement graph for a body is shown in the Figure.



Calculate the work done by force to displace the body from zero to
(a) 10 m
(b) 20 m
(c) 30 m

15. A block of mass m is suspended from the ceiling of a lift by a light rope. The lift is going down with an acceleration $\frac{g}{2}$. Calculate the work done by various forces acting on the block as it goes down through h.

ENERGY

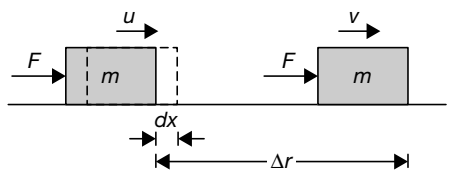
The capacity of the body to do work is called **Energy**. Energy can be of many types like kinetic energy, potential energy heat energy, light energy, sound energy, electrostatic energy, magnetic energy, atomic energy, nuclear energy etc. However, for this chapter we restrict ourselves to the study of **Mechanical Energy, which is the sum of kinetic energy and the potential energy.**

The Kinetic Energy is the energy possessed by a body by virtue of its motion.

The Potential Energy is the energy possessed by the body by virtue of its position or configuration.

CONCEPT OF KINETIC ENERGY AND WORK ENERGY THEOREM (OR THE CLASSICAL WORK ENERGY (CWE) THEOREM)

Consider a block of mass m undergoing a displacement Δr and a change in velocity under the action of a constant net force F, as shown in Figure.



Let the block the given an infinitesimal displacement dx under the influence of the force F. If dW is the work done, then

$$dW = Fdx\cos(0°) = Fdx$$

$$\Rightarrow \quad dW = m\left(\frac{dv}{dt}\right)dx \qquad \left\{\because F = m\frac{dv}{dt}\right\}$$

$$\Rightarrow \quad dW = m\left(\frac{dx}{dt}\right)dv = mvdv \qquad \left\{\because \frac{dx}{dt} = v\right\}$$

$$\Rightarrow \quad W = \int dW = m\int_u^v vdv$$

$$\Rightarrow \quad W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = F\Delta r \qquad \ldots(1)$$

where, the terms

$\frac{1}{2}mv^2$ = Final Kinetic Energy of the block and

$\frac{1}{2}mu^2$ = Initial Kinetic Energy of the block

Also, from (1), we observe that the work done by an external force acting on the block equals the change in kinetic energy of the block, i.e.

$$(W_{ext})_{total} = \Delta K \qquad \{\text{Work Energy Theorem}\}$$

The quantity $K = \frac{1}{2}mv^2$ is a scalar and is called the **Kinetic Energy** of the particle which is the energy possessed by a particle by virtue of its motion.
Thus, the equation (1) takes the form

$$W = K_f - K_i = \Delta K \qquad \ldots(2)$$

The work done by a force changes the kinetic energy of the particle. This is called the **Work-Energy Theorem.**

In general, **the net work done by the resultant of all the forces acting on the particle is equal to the change in kinetic energy of a particle.** So, we have

$$W_{net} = \Delta K \qquad \ldots(3)$$

Conceptual Note(s)

(a) The kinetic energy of an object is a measure of the amount of work needed to increase its speed from zero to a given value.
(b) The kinetic energy of a particle is the work it can do on its surroundings in coming to rest.
(c) Since the velocity and displacement of a particle depend on the frame of reference, the numerical values of the work and the kinetic energy also depend on the frame.

ILLUSTRATION 18

The velocity v of a particle of mass m moving along x-axis is given by $v = b\sqrt{x}$, where b is a constant. Calculate the work done by the force acting on the particle during its motion from $x = 0$ to $x = 4$ m.

SOLUTION

Initial velocity (u) of the particle is the velocity at $x = 0$ i.e.

$$u = v|_{x=0} = b\sqrt{0} = 0$$

Final velocity (v) of the particle is the velocity at $x = 4$ i.e.

$$v = v|_{x=4} = b\sqrt{4} = 2b$$

According to Work-Energy Theorem, we have

$$W = \Delta K = K_f - K_i = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\Rightarrow \quad W = \frac{1}{2}m(2b)^2 = 2mb^2$$

IMPORTANCE OF THE WORK ENERGY THEOREM

The work kinetic energy theorem does not represent a new independent law of mechanics. The theorem is useful for solving problems in which the work done can be easily computed and in which we are interested in finding the particle's speed at certain positions.

One method to do that is to apply Newton's Law of Motion and find out the acceleration of the particle from where we can get its speed. **However, this method is lengthier and particularly not useful when we are interested only in speed and not in acceleration of the particle.**

The other method is to use the Work-Energy Theorem because it has the advantage that it does away with all intermediate steps (i.e. what is happening during the motion). We have to simply calculate the sum of the works done by individual forces and equate that to the change in kinetic energy of the particle. This discussion also applies to the Conservation of Mechanical Energy principle which will follow the article on potential energy.

ILLUSTRATION 19

A particle of mass 0.01 kg travels along a curve with a velocity given by $4\hat{i} + 16\hat{k}$ ms^{-1}. After some time, its velocity becomes $8\hat{i} + 20\hat{j}$ ms^{-1} due to the action of

a conservative force. Calculate the work done on the particle during this interval of time.

SOLUTION

Since, work done equals the change in kinetic energy, so

$$W = \Delta K = K_f - K_i = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

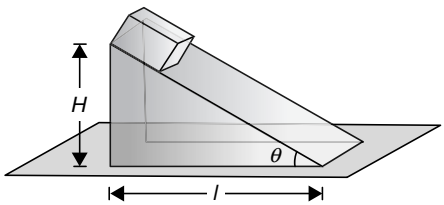
where, $u = \sqrt{4^2 + 16^2} = \sqrt{272}$ and

$$v = \sqrt{8^2 + 20^2} = \sqrt{464}$$

$$\Rightarrow \quad W = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2} \times 0.01(464 - 272) = 0.96 \text{ J}$$

ILLUSTRATION 20

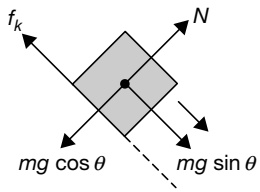
A small block of mass m is released from the top of a rough inclined plane as shown in Figure.



The coefficient of friction between the block and inclined plane is μ. Apply Work-Energy Theorem to calculate the speed of block as it passes the lowest point.

SOLUTION

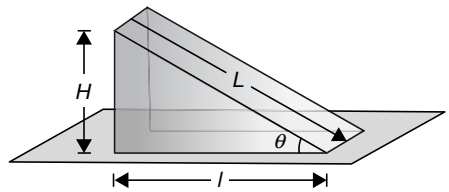
The FBD of the block is shown in Figure.



The initial speed is zero and let the final speed of the block as it passes the lowest point is v. According to Work-Energy Theorem, we have

$$W_{\text{Total}} = \Delta K = \frac{1}{2}mv^2 - 0$$

Let us now find W_{Total} by calculating the work done by each force.



Work done by normal reaction N is $W_N = 0$

Work done by $mg\cos\theta$ is $W_{mg\cos\theta} = 0$

Work done by $mg\sin\theta$ is

$$W_{mg\sin\theta} = (mg\sin\theta)(L)\cos(0°) = mgL\sin\theta$$

Since $\sin\theta = \frac{H}{L}$

$$\Rightarrow \quad W_{mg\sin\theta} = mg\left(\frac{H}{L}\right)(L) = mgH$$

Work done by kinetic friction f_k is

$$W_{f_k} = (\mu N)(L)\cos(180°) = -\mu mgL\cos\theta$$

Since $\cos\theta = \frac{l}{L}$

$$\Rightarrow \quad W_{f_k} = -\mu mg\left(\frac{l}{L}\right)(L) = -\mu mgl$$

So, the total work done W is given by

$$W = mgH - \mu mgl$$

Applying Work-Energy Theorem, we get

$$\frac{1}{2}mv^2 = mgH - \mu mgl$$

$$\Rightarrow \quad v = \sqrt{2g(H - \mu l)}$$

ILLUSTRATION 21

A running man has half the kinetic energy that a boy of half his mass has. The man speeds up by 1 ms^{-1} so as to have same kinetic energy as that of boy. Find the original speed of the man and the boy.

SOLUTION

Let mass of the man be M, then

$$(\text{K.E.})_{\text{man}} = \frac{1}{2}(\text{K.E.})_{\text{boy}}$$

$$\Rightarrow \quad \frac{1}{2}Mv_m^2 = \frac{1}{2}\left[\frac{1}{2}\left(\frac{M}{2}\right)v_b^2\right]$$

$$\Rightarrow \quad v_m = \frac{v_b}{2} \qquad \ldots(1)$$

Further, $\frac{1}{2}M(v_m+1)^2 = \frac{1}{2}\left(\frac{M}{2}\right)v_b^2$

$$\Rightarrow \quad v_m + 1 = \frac{v_b}{\sqrt{2}} \qquad \ldots(2)$$

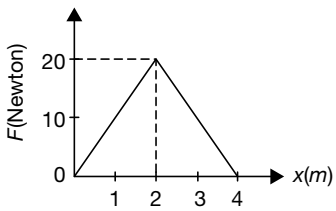
Solving (1) & (2), we get

$$v_b = 2(\sqrt{2}+1) = 4.82 \text{ ms}^{-1}$$

$$v_m = \sqrt{2}+1 = 2.41 \text{ ms}^{-1}$$

ILLUSTRATION 22

The graph between the resistive force F acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 kg and initial velocity is 2 ms^{-1}. Find the kinetic energy when the distance covered by the body is 5 m.



SOLUTION

Initial kinetic energy of the body is

$$K_i = \frac{1}{2}mu^2 = \frac{1}{2}\times 25\times(2)^2 = 50 \text{ J}$$

Since Work done equals the change in kinetic energy, so

$$W = \Delta K = K_f - K_i$$

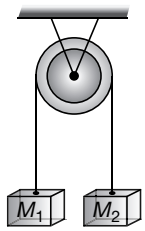
$$\Rightarrow \quad K_f = W + K_i$$

Now, work done against resistive force is equal to the negative of area under F-x graph, so

$$K_f = 50 - \frac{1}{2}\times 4\times 20 = 50 - 40 = 10 \text{ J}$$

ILLUSTRATION 23

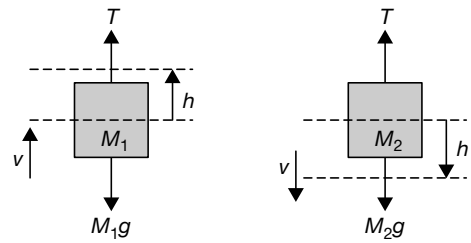
A system consists of two blocks having masses M_1 and M_2, (where $M_2 > M_1$) connected by a light inextensible string passing over a smooth pulley as shown in Figure.



Calculate the acceleration of the blocks by applying the Work-Energy Theorem.

SOLUTION

Since $M_2 > M_1$, so when the block M_2 moves down by a distance h, then the block M_1 will move up through the same distance h. The speed of M_1 and M_2 at this instant will be the same, say v as shown in figure.



Net work done on M_1 is

$$W_1 = Th\cos(0°) + M_1gh\cos(180°)$$

$$\Rightarrow \quad W_1 = (T - M_1g)h$$

Net work done on M_2 is

$$W_2 = M_2g\cos(0°) + Th\cos(180°)$$

$$\Rightarrow \quad W_2 = (M_2g - T)h$$

So, total work done is

$$W = W_1 + W_2 = (M_2 - M_1)gh$$

According to Work-Energy Theorem, we have

$$W = \Delta K$$

$$\Rightarrow \quad (M_2 - M_1)gh = \frac{1}{2}M_1v^2 + \frac{1}{2}M_2v^2$$

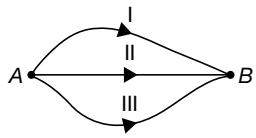
$$\Rightarrow \quad v^2 = 2\left(\frac{M_2 - M_1}{M_1 + M_2}\right)gh$$

Comparing with $v^2 = 2ah$, we get

$$a = \left(\frac{M_2 - M_1}{M_1 + M_2}\right)g$$

CONSERVATIVE AND NON-CONSERVATIVE FORCES

Statement I: Force is said to be conservative in nature when the work done by the force is independent of the path followed between any two points.

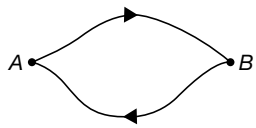


Mathematically, if for a force we have

$$W_{\substack{A\to B \\ \text{Path I}}} = W_{\substack{A\to B \\ \text{Path II}}} = W_{\substack{A\to B \\ \text{Path III}}}$$

then F (the force) must be conservative in nature.

Statement II: Force is said to be conservative in nature when the work done by a force in a closed loop is zero.



Mathematically, if for a force we have

$$W_{A\to B} + W_{B\to A} = 0,$$

then F (the force) must be conservative in nature.

Statement III: Force is said to be conservative in nature when the line integral of the force i.e. $\int \vec{F}\cdot d\vec{l}$ (which is also the work done) is independent of the path followed between any two points.

Mathematically, if for a force we have

$$\int_{\text{Path I}} \vec{F}\cdot d\vec{l} = \int_{\text{Path II}} \vec{F}\cdot d\vec{l} = \int_{\text{Path III}} \vec{F}\cdot d\vec{l}$$

then $\vec{F}$ is a conservative force

Statement IV: Force is said to be conservative in nature when the line integral of a force over a closed path/loop is zero.

Mathematically, if for a force we have

$$\oint \vec{F}\cdot d\vec{l} = 0$$

then $\vec{F}$ is a conservative force.

EXAMPLE

(a) Electrostatic forces, gravitational forces, elastic forces, all constant forces are conservative forces.

(b) All forces which do not obey the above specified conditions are called **non-conservative forces**.

(c) Frictional force, air drag, viscous drag i.e. all dissipative forces.

POTENTIAL ENERGY

When we throw a ball upwards with an initial velocity, it rises to a certain height and becomes stationary for a moment.

What happens to the lost kinetic energy?

We know with our experience that the ball returns to our hands with a speed equal to its initial value (assuming air drag to be absent). The initial kinetic energy is somehow stored and is later fully recovered in the form of kinetic energy.

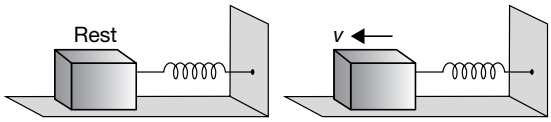
The ball must have some energy at the new height that it does not have at the previous level. That new energy by virtue of its position is called the **Potential Energy**.

Potential energy is the energy associated with the relative positions of two or more interacting particles.

Potential energy fits well to the idea of energy as the capacity to do work. The gravitational potential energy of an object raised off the ground can be used to compress or expand a spring or to lift another weight. As a coil spring unwinds, or a straight spring returns to its natural length, the stored elastic potential energy can be used to do work.

EXAMPLE

If a block is attached to a compressed spring, the elastic potential energy can be converted into kinetic energy of the block as shown in figure. The block gains kinetic energy when the compressed spring is released.

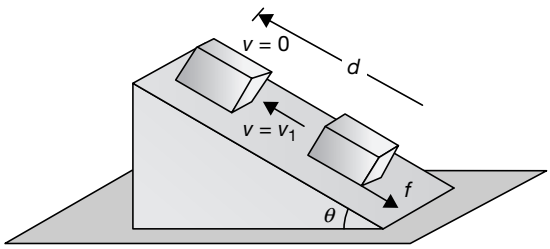


In the above discussion we have seen that in the case of gravity and elastic spring the kinetic energy imparted initially is stored as potential energy for a short time which is regained, later on. But this is not true in all cases.

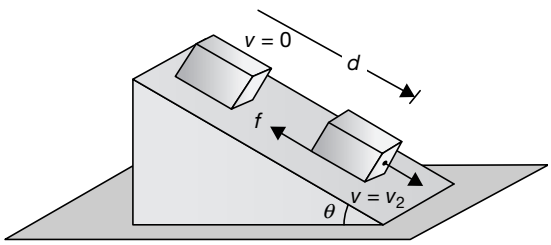
The forces, such as gravity and spring force, which do work in a reversible manner are called **Conservative Forces**. In contrast, the force, such as frictional force, which does work in an irreversible manner are called **Non-Conservative Forces**.

EXAMPLE

Consider block placed at rest on a rough horizontal surface. If we impart it some initial kinetic energy, it starts sliding on the surface, the frictional force does negative work on the block, decreasing its kinetic energy to zero. But it does not come back to our hand no matter how long we wait! The frictional force has used up the kinetic energy in a non - reversible way.



$W_g = -mgd \sin\theta$, $W_f = -fd$



$W_g = mgd \sin\theta$, $W_f = -fd$

A block slides up and down on a rough inclined plane. In the complete trip, the work done by gravity is zero whereas the work done by friction is negative.

THE POTENTIAL ENERGY IS DEFINED ONLY FOR CONSERVATIVE FORCES

The change in potential energy as a particle moves from point A to point B is equal to the negative of the work done by the associated conservative force

$$\Delta U = U_B - U_A = -W_C$$

Using definition of work

$$U_B - U_A = -\int_A^B \vec{F}_C \cdot d\vec{r}$$

From this equation, we see that, starting with potential energy U_A at point A, we obtain a unique value U_B at point B, because W_C has the same value for all paths. When a block slides along a rough floor, the work done by the force of friction on the block depends on the length of the path taken from point A to point B. **There is no unique value for the work done, so one cannot assign unique values for potential energy at each point. Hence, non-conservative force cannot have potential energy.**

When the forces within a system are conservative, external work done on the system is stored as potential energy and is fully recoverable.

Note that the potential energy is always defined with respect to a reference point.

CONSERVATIVE SYSTEM AND CONCEPT OF POTENTIAL ENERGY (U)

For a conservative system, since work done is independent of the path followed between any two points or it just depends upon the final state and initial state of the body. So, there must exist a certain scalar physical quantity such that the work done by this conservative force (W_c) is equal to the decrease in the value of this new scalar quantity. This scalar quantity is called the **Potential Energy,** U. So

$$W_c = -\Delta U = U_i - U_f$$

EXAMPLE

Just think of a body being dropped from a height h to the ground, then $U_{\text{initial}} = mgh$ and when it lands on the ground then $U_{\text{final}} = 0$. Also, the work done by the

gravitational force (a conservative force) is equal to *mgh* and this happens to be the decrease in potential energy of the falling body.

So, we can simply conclude that the work done by a conservative force (W_c) is equal to the decrease in potential energy. Hence, we have

$$W_c = \begin{pmatrix}\text{Decrease in}\\ \text{Potential Energy}\end{pmatrix} = U_{\text{initial}} - U_{\text{final}} = -\Delta U$$

GRAVITATIONAL POTENTIAL ENERGY (NEAR THE EARTH'S SURFACE)

The work done by gravity on a particle of mass m whose vertical coordinate changes from y_A to y_B is

$$W_g = -mg(y_B - y_A)$$

Since, we know that $W_g = -\Delta U = -(U_B - U_A)$

Thus, gravitational potential energy at the point B near the surface of the earth is given by

$$U_B = U_A + mgh$$

If we assume potential energy at the point A to be zero, then potential energy at the point B is given by

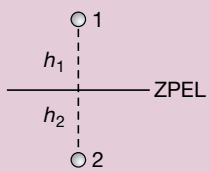
$$U_B = 0 + mgh = mgh$$

Conceptual Note(s)

(a) It is very important to **assign a Zero Potential Energy Level (ZPEL)** before we calculate the potential energy of a particle.

(b) Gravitational Potential Energy U depends upon the choice of **ZPEL** but Gravitational Potential Energy Difference ΔU is independent of the choice of **ZPEL.**

(c) If one point lies above **ZPEL** and other below **ZPEL**, then



$U_1 = +mgh_1$, $U_2 = -mgh_2$ with respect to the specified ZPEL. However,

$$U_1 - U_2 = \Delta U = mg(h_1 + h_2)$$

irrespective of the location of ZPEL.

SPRING POTENTIAL ENERGY OR ELASTIC POTENTIAL ENERGY

The work done by the spring force when the displacement of the free end changes from x_i to x_f is given by

$$W_S = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

By definition $W_S = -\Delta U_S = U_i - U_f$

$$\Rightarrow \quad U_f = U_i + \frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

If we assume the potential energy stored in the spring at equilibrium is zero and all the extensions to be measured from equilibrium, then $x_i = 0$ and $U_i = 0$. Thus, final energy stored in the spring, when $x_f = x$ is

$$U_f = U_S = \frac{1}{2}kx^2$$

The potential energy function for an ideal spring is a parabolic function of extension or compression x as shown in figure.

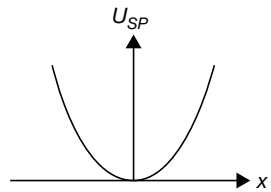
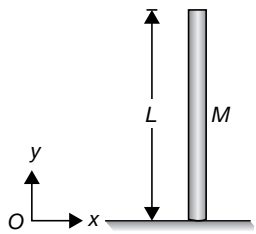


ILLUSTRATION 24

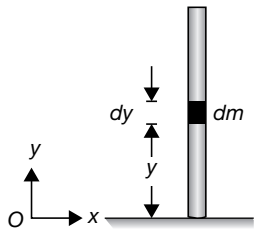
A uniform rod of mass M and length L is held vertically upright on a horizontal surface as shown in the figure.



Calculate the potential energy of the rod if the zero potential energy level is assumed at the horizontal surface.

SOLUTION

Since the parts of the rod are at different levels with respect to the horizontal surface, therefore, we have to use the integration to find its potential energy. Consider a small element of length dy at a height y from the horizontal as shown in Figure.



If dm is the mass of the element, then

$$dm = \frac{M}{L}dy$$

Its potential energy dU is given by

$$dU = (dm)gy = \frac{M}{L}gydy$$

Integrating, we get

$$U = \frac{Mg}{L}\int_0^L ydy = \frac{Mg}{L}\left(\frac{y^2}{2}\Big|_0^L\right)$$

$$\Rightarrow \quad U = \frac{1}{2}MgL$$

Conceptual Note(s)

Note that the potential energy of the rod is equal to the product of Mg and height of the center of mass $\left(\frac{L}{2}\right)$ from the surface.

POWER

Power is defined as the rate at which work is done. If an amount of work ΔW is done in a time interval Δt, then the **average power** is defined to be

$$P_{av} = \frac{W}{t} = \frac{\Delta W}{\Delta t}$$

The SI unit of power is Js^{-1} which is given the name watt (W) in the honour of James Watt.

Thus, $1\ \text{W} = 1\ \text{Js}^{-1}$

Instantaneous power is the limiting value of P_{av} when $\Delta t \to 0$, so we have

$$P_{\text{ins}} = \frac{dW}{dt}$$

The work done by force F on a object that has an infinitesimal displacement $d\vec{r}$ is $dW = \vec{F}\cdot d\vec{r}$. The instantaneous power may be written as

$$P = \frac{dW}{dt} = \vec{F}\cdot\frac{d\vec{r}}{dt}$$

$$\Rightarrow \quad P = \vec{F}\cdot\vec{v} \qquad \left\{\because\ \vec{v} = \frac{d\vec{r}}{dt}\right\}$$

Since the work and energy are closely related, a more general definition of power is the rate of energy transfer from one body to another, or the rate at which energy is transformed from one form to another.

$$P = \frac{dE}{dt}$$

ILLUSTRATION 25

A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_r is varying with time t as $a_r = k^2rt^2$, where k is a constant, Calculate the power delivered to the particle by the forces acting on it.

SOLUTION

Let the instantaneous speed of the particle be v, then centripetal acceleration is given by

$$a_r = \frac{v^2}{r}$$

According to the problem we have $a_r = k^2rt^2$, so we get

$$\Rightarrow \quad \frac{v^2}{r} = k^2rt^2$$

$$\Rightarrow \quad v = krt$$

The tangential acceleration is given by

$$a_t = \frac{dv}{dt} = kr$$

The tangential force is $F_t = ma_t = mkr$

Hence, power delivered is

$$P = F_t v = (mkr)(krt)$$

$$\Rightarrow \quad P = mk^2r^2t$$

ILLUSTRATION 26

An elevator that can carry a maximum load of $1500\ \text{kg}$ (elevator + passengers) is moving up with a constant speed of $2\ \text{ms}^{-1}$. The frictional force opposing the motion is $3000\ \text{N}$. Calculate the minimum power delivered by the motor to the elevator in kilo watt and in horse power. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

Since the elevator is moving up with a constant speed of $2\ \text{ms}^{-1}$, so the net downward force on the elevator is

$$F = mg + f$$

$$\Rightarrow \quad F = (1500 \times 10) + 3000$$

$$\Rightarrow \quad F = 18000\ \text{N}$$

The motor must supply enough power to balance this force. If P be the power required for this purpose, then

$$P = Fv = (18000)(2) = 36000\ \text{W}$$

$$\Rightarrow \quad P = 36\ \text{kW}$$

Since $1\ \text{hp} = 746\ \text{W}$

$$\Rightarrow \quad P = \frac{36000}{746}\ \text{hp}$$

$$\Rightarrow \quad P = 48.3\ \text{hp}$$

ILLUSTRATION 27

A block of mass m is pulled by a constant power P placed on a rough horizontal plane. The friction coefficient between the block and surface is μ. Find the maximum velocity of the block.

SOLUTION

Power $P = Fv = \text{constant}$

$$\Rightarrow \quad F = \frac{P}{v}$$

$$\Rightarrow \quad F \propto \frac{1}{v}$$

as v increases, F decreases.

when $F = \mu mg$, net force on block becomes zero, i.e., it has maximum or terminal velocity, so

$$P = (\mu mg)v_{\max}$$

$$\Rightarrow \quad v_{\max} = \frac{P}{\mu mg}$$

ILLUSTRATION 28

A particle is moving along x-axis under the action of a force F which varies with its position x as $F \propto \frac{1}{\sqrt[4]{x}}$.

Calculate the variation of power due to this force with x.

SOLUTION

Since, $F \propto x^{-1/4}$

$$\Rightarrow \quad a \propto x^{-1/4}$$

$$\Rightarrow \quad a = kx^{-1/4} \text{ (where } k \text{ is a proportionality constant)}$$

Since, $a = \frac{dv}{dt} = \frac{dx}{dt}\frac{dv}{dx} = v\frac{dv}{dx}$

$$\Rightarrow \quad v\frac{dv}{dx} = kx^{-1/4}$$

$$\Rightarrow \quad vdv = kx^{-1/4}dx$$

Integrating, we get

$$\int_0^v vdv = k\int_0^x x^{-1/4}dx$$

$$\Rightarrow \quad \frac{v^2}{2} = k\frac{x^{3/4}}{3/4}$$

$$\Rightarrow \quad v^2 \propto x^{3/4}$$

$$\Rightarrow \quad v \propto x^{3/8}$$

Since $P = Fv$

$$\Rightarrow \quad P \propto (x^{-1/4})(x^{3/8})$$

$$\Rightarrow \quad P \propto x^{1/8}$$

Test Your Concepts-II

Based on Kinetic Energy, Potential Energy and Power

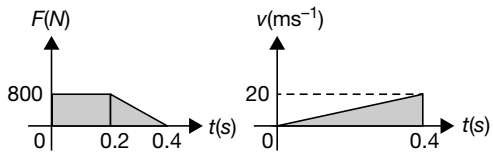
(Solutions on page H.5)

1. An engine working at a constant power P draws a load of mass m against a resistance force f. Find the maximum speed of the load and the time taken to attain half this speed.

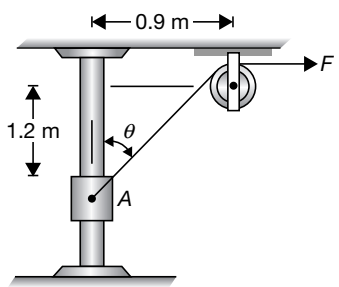
2. An automobile of mass m accelerates, starting from rest. The engine supplies constant power P.
(a) Find the velocity of the automobile as a function of time.
(b) Find the position of the automobile as function of time.

3. A particle moving in a straight line is acted upon by a force which works at a constant rate and changes its velocity from u and v over a distance x. Prove that the time taken in it is $\frac{3(u+v)x}{2(u^2+v^2+uv)}$.

4. A baseball having a mass of 0.4 kg is thrown such that the force acting on it varies with time as shown in the first graph. Also, the velocity of the ball acting in the direction same as that of force varies with time as shown in the second graph. Determine the power applied as a function of time and the work done in $t = 0.4$ s.



5. The 50 N collar starts from rest at A and is lifted with a constant speed of $0.6\ \text{ms}^{-1}$ along the smooth rod. Determine the power developed by the force F at the instant shown.



6. A train has a constant speed of $40\ \text{ms}^{-1}$ on a level road against resistive force of magnitude 3×10^4 N. Find the power of engine.

7. A particle is moving along x-axis under the action of a force F which varies with its position x as $F \propto \frac{1}{\sqrt{x}}$. If the power due to this force is proportional to x^n. Calculate n.

8. A body of mass 5 kg is placed at origin. A force starts acting on the body given by $\vec{F} = (2+3x)\hat{i}$, where x is the distance of body from origin in meters. Calculate the speed of the body as it passes through $x = 5$ m.

9. A particle of mass 0.5 kg travels in a straight line with velocity $v = ax\sqrt{x}$ where $a = 5\ \text{m}^{-1/2}\text{s}^{-1}$. Calculate the work done by the all forces when the particle is displaced from $x = 0$ to $x = 2$ m.

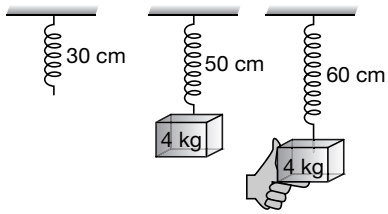
10. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of wind energy interception by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to v^n. Calculate n.

11. A pendulum bob has potential energy U_O when held taut in a horizontal position. The bob is allowed to fall until it is 30° away from the horizontal position, when it has potential energy U_A. It continues to fall until the string is vertical, when it has potential energy U_B. Calculate $\frac{U_O - U_A}{U_A - U_B}$

12. A spring acquires a potential energy of 30 J when stretched by a length of 20 cm. If the spring is now stretched further by 40 cm, calculate the additional amount of work required to do so.

13. Power of the engine of a motor boat is 50 HP. If resistance force F of water increases with speed v (in ms^{-1}) of the boat according to $F = 20v$ (in kN). Calculate the maximum speed of this motor boat in kmh^{-1}.

14. On attaching a 4 kg mass to a spring of length 30 cm, the spring stretches by 20 cm. Then the mass is pulled down until the length of the spring becomes 60 cm. Calculate the amount of elastic energy stored in the spring.



15. A brick of mass 5 kg and dimensions (in cm) $20\times10\times5$ lies on the ground on its largest face. If it is made to stand on its smallest face. Calculate the change in its gravitational potential energy.

16. A metre scale of mass 0.1 kg pivoted at the top end is turned from vertical position so that it is inclined at 30° with the horizontal in the vertical plane. Calculate the work done by gravity.

LAW OF CONSERVATION OF MECHANICAL ENERGY

Since, for a conservative system we have

$$W_c = \Delta K \qquad \ldots(1)$$

Also, we know that

$$W_c = -\Delta U \qquad \ldots(2)$$

Equating (1) and (2), we get

$$\Delta K = -\Delta U$$

$$\Rightarrow \quad \Delta(K+U) = 0$$

$$\Rightarrow \quad K+U = \text{constant}$$

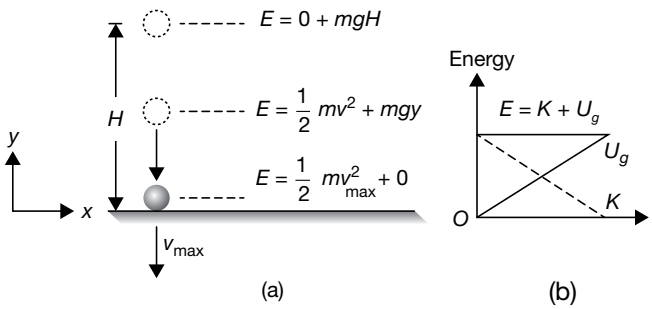
$$\Rightarrow \quad K_i + U_i = K_f + U_f$$

$$\Rightarrow \quad \left(\begin{array}{c}\text{Total Initial}\\ \text{Mechanical Energy}\end{array}\right) = \left(\begin{array}{c}\text{Total Final}\\ \text{Mechanical Energy}\end{array}\right)$$

Let us discuss few fundamental examples where we apply the Law of Conservation of Mechanical Energy to systems.

CASE-1: OBJECT FALLING FREELY

When an object falls from height H, its potential energy is converted to kinetic energy. At height H, the energy is $E = mgH$. Just as it lands, $E = \frac{1}{2}mv_{\max}^2$.



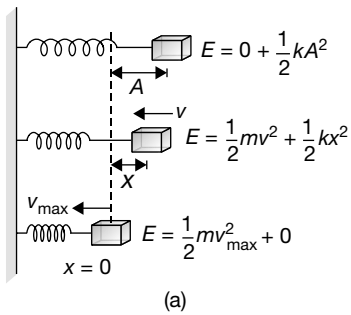
The potential energy and the kinetic energy vary linearly with vertical height y.

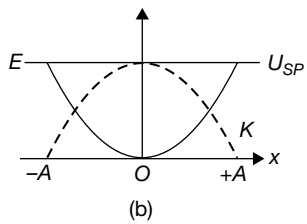
At any instant, the mechanical energy E remains constant. So,

$$E = K + U = \frac{1}{2}mv^2 + mgy = \text{constant}$$

CASE-2: MASS ATTACHED TO A SPRING

When a mass m is attached to a spring of force constant k, then at maximum extension in the spring (called the amplitude), $KE = 0$ and $PE = \frac{1}{2}kA^2$.





At extension x, we have

$$KE = \frac{1}{2}mv^2 \text{ and } PE = \frac{1}{2}kx^2$$

At mean position, we have

$$PE = 0 \text{ and}$$

$$KE = \frac{1}{2}mv_{max}^2$$

Here too, at any instant, the mechanical energy E remains constant. So, we have

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

Problem Solving Technique(s)

To solve problems using the concept of Conservation of Mechanical Energy we can follow the series of steps mentioned below.

(a) Draw diagrams of the system showing initial and final configuration and assume a coordinate system.

(b) Specify the reference level for potential energy. **In case of spring**, it is advisable to assume zero potential energy at the natural length of the spring. **In case of gravity**, any convenient level can be chosen as reference frame.

(c) Looking at the initial configuration, **ask yourself What forms of energy are present in the system?**

- **(i)** if the particle is moving, include $\frac{1}{2}mv_i^2$
- **(ii)** if the particle is not located at the reference level, include mgy_i
- **(iii)** if the spring is stretched or compressed, include $\frac{1}{2}kx_i^2$

(d) Looking at the final configuration, **ask yourself What forms of energy are present?**

- **(i)** if the particle is moving, include $\frac{1}{2}mv_f^2$
- **(ii)** if the particle is not located at the reference, include mgy_f
- **(iii)** if the spring is stretched or compressed, include $\frac{1}{2}kx_f^2$

(e) Equate the initial and final total energies

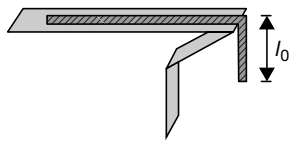
$$K_i + U_i = K_i + U_f$$

$$\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2$$

(f) Solve for the unknown.

ILLUSTRATION 29

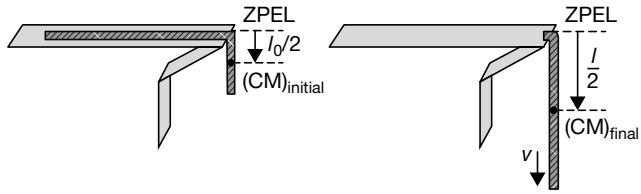
A rope of length $l = 80$ cm and mass $m = 2$ kg is hanging from the end of a plane so that the length l_0 of the vertical segment is 50 cm as shown in Figure.



The other end of the rope is fixed by a nail. At a certain instant, the nail is pushed out, what is the velocity of the rope at the moment it completely slides off the plane? Neglect the friction.

SOLUTION

We assume the Zero Potential Energy Level (ZPEL) at the horizontal plane. Assuming the rope to have a mass m and a length l. If λ is the mass per unit length of the rope then $\lambda = \frac{m}{l}$. The initial and final configuration of the rope are shown in Figure.



Initial kinetic energy of the rope is

$$K_i = 0$$

Initial potential energy of the hanging portion of the rope is obtained by locating the centre of mass (CM) of the hanging portion of the rope, which lies $\frac{l_0}{2}$ below the ZPEL.

$$U_i = 0 + \left(\frac{m}{l}l_0\right)g\left(-\frac{l_0}{2}\right) = -\frac{ml_0^2}{2l}g$$

Please note that, the part of rope lying over the table i.e. ZPEL, has zero potential energy.

Let v be the final velocity of the rope at the moment it completely slides off the plane. Then final kinetic energy of the rope is

$$K_f = \frac{1}{2}mv^2$$

Final potential energy of the rope is obtained by locating the centre of mass (CM) of the fully hanging rope, which lies $\frac{l}{2}$ below the ZPEL.

$$U_f = -mg\left(\frac{l}{2}\right)$$

Using the Law of Conservation of Energy, we get

$$K_f + U_f = K_i + U_i$$

$$\Rightarrow \quad \frac{1}{2}mv^2 - mg\left(\frac{l}{2}\right) = 0 - \frac{ml_0^2 g}{2l}$$

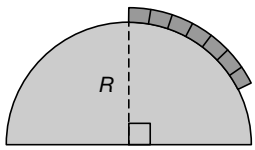
$$\Rightarrow \quad v = \sqrt{\frac{g}{l}\left(l^2 - l_o^2\right)}$$

Substituting $l = 0.8$ m, $l_0 = 0.5$ m and $g = 10\ \text{ms}^{-2}$, we get

$$v = \sqrt{5}\ \text{ms}^{-1} = 2.24\ \text{ms}^{-1}$$

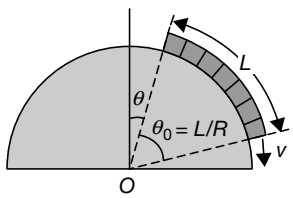
ILLUSTRATION 30

A uniform rope of mass m and length L is placed on the top of a smooth hemispherical surface. The rope is held at rest and released such that it slides down along the curvature as shown in Figure. Calculate the speed of the chain, as it moves through angle θ on the surface.

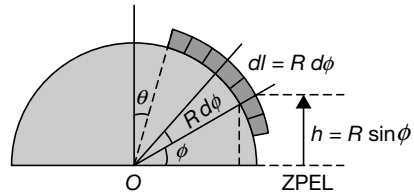


SOLUTION

Let θ_0 be the angle subtended by the rope at the centre of the sphere, then $\theta_0 = \frac{L}{R}$. As the rope slides down, it loses potential energy and gains kinetic energy. At any instant, the angular position of the rope is shown in the Figure.



Let us first calculate the gravitational potential energy of the rope. For that, consider a small element of length dl, having mass dm and subtending an angle $d\phi$ at the centre as shown in Figure.



Taking the Zero Potential Energy Level (ZPEL) at the bottom of the hemisphere, the gravitational potential energy for this infinitesimal element is given by

$$dU = (dm)gh$$

where, $h = R\sin\phi$ is the height of the element above ZPEL.

$$dm = \frac{m}{L}dl = \frac{m}{L}(Rd\phi)$$

$$\Rightarrow \quad dU = \frac{m}{L}gR^2\sin\phi d\phi\,.$$

At the instant the upper end of the rope makes an angle θ with the vertical, the lower end of the rope makes an angle $\phi = \frac{\pi}{2} - (\theta + \theta_0)$ with the horizontal and at the same instant the upper end of the rope makes and angle $\phi = \frac{\pi}{2} - \theta$ with the horizontal.

So, to get the potential energy of the rope (with respect to ZPEL) at the instant mentioned, integrating by applying limits on ϕ from the lower end of the rope to the upper end of the rope, we get

$$U = \frac{m}{L}gR^2 \int_{\frac{\pi}{2}-(\theta+\theta_0)}^{\frac{\pi}{2}-\theta} \sin\phi d\phi$$

$$\Rightarrow \quad U = \frac{mgR^2}{L}(-\cos\phi)\Big|_{\frac{\pi}{2}-(\theta+\theta_0)}^{\frac{\pi}{2}-\theta}$$

$$\Rightarrow \quad U = \frac{mgR^2}{L}\left\{-\cos\left(\frac{\pi}{2}-\theta\right) + \cos\left(\frac{\pi}{2}-(\theta+\theta_0)\right)\right\}$$

$$\Rightarrow \quad U = \frac{mgR^2}{L}\left[\sin(\theta+\theta_0) - \sin\theta\right] \qquad \ldots(1)$$

PE of the rope, when it is just released, can be calculated by taking $\theta = 0$ in the above expression i.e. equation (1). Thus, we have

$$U_i = \frac{mgR^2}{L}\sin\theta_0$$

By Law of Conservation of Mechanical Energy, we have

$$(\text{Loss in GPE of Rope}) = (\text{Gain in KE of Rope})$$

$$\Rightarrow \quad U_i - U_f = K_f - K_i$$

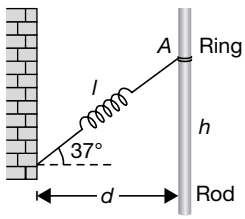
$$\Rightarrow \quad \frac{mgR^2}{L}\left[\sin\theta_0 - \left(\sin(\theta+\theta_0) - \sin\theta\right)\right] = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2gR^2}{L}\left[\sin\theta_0 + \sin\theta - \sin(\theta+\theta_0)\right]}$$

where $\theta_0 = \frac{L}{R}$

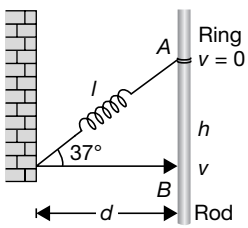
ILLUSTRATION 31

One end of a light spring of natural length d and spring constant k is fixed on a rigid wall and the other is attached to a smooth ring of mass m which can slide without friction on a vertical rod fixed at a distance d from the wall. Initially the spring makes an angle of 37° with the horizontal as shown in Figure.



When the system is released from rest, find the speed of the ring when the spring becomes horizontal. Take $\sin(37^\circ) = \frac{3}{5}$.

SOLUTION



If l is the stretched length of the spring, then from figure, we have

$$\frac{d}{l} = \cos(37^\circ) = \frac{4}{5}$$

$$\Rightarrow \quad l = \frac{5d}{4}$$

So, the extension in the spring is

$$x = l - d = \frac{5d}{4} - d = \frac{d}{4} \text{ and}$$

$$h = l\sin(37^\circ) = \left(\frac{5d}{4}\right)\left(\frac{3}{5}\right) = \frac{3d}{4}$$

Now taking point B as reference level and applying Law of Conservation of Mechanical Energy between A and B,

$$(U+K)_{at\ A} = (U+K)_{at\ B}$$

At A, $h = \frac{3d}{4}$ and $x = \frac{d}{4}$

At B, $h = 0$ and $x = 0$

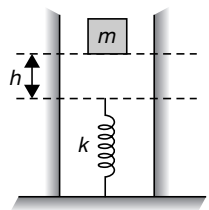
$$\Rightarrow \quad mgh + \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{3}{4}mgd + \frac{1}{2}k\left(\frac{d}{4}\right)^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = d\sqrt{\frac{3g}{2d} + \frac{k}{16m}}$$

ILLUSTRATION 32

A small block of mass m is released from a height h above the free end of a light spring of spring constant k as shown in Figure. Calculate the



(a) speed of the block at the instant when the compression in the spring is x.
(b) maximum compression in the spring.
(c) compression in the spring when the block is in equilibrium.
(d) acceleration of the block in situations (a) and (b).

SOLUTION

(a) When the spring has been compressed by x, the block has moved down by a distance $(h+x)$. Let v be the speed of the block at this instant, then by Law of Conservation of Energy, we have

$$mg(h+x)=\frac{1}{2}kx^2+\frac{1}{2}mv^2 \qquad \ldots(1)$$

$$\Rightarrow \quad v=\sqrt{2g(h+x)-\frac{kx^2}{m}}$$

(b) When compression is maximum, speed of the block is zero, so we have

$$mg(h+x)=\frac{1}{2}kx^2$$

$$\Rightarrow \quad x^2-\left(\frac{2mg}{k}\right)x-\frac{2mgh}{k}=0$$

Solving the quadratic equation, we get

$$x_1=\frac{mg}{k}+\sqrt{\frac{m^2g^2}{k^2}+\frac{2mgh}{k}} \text{ and}$$

$$x_2=\frac{mg}{k}-\sqrt{\frac{m^2g^2}{k^2}+\frac{2mgh}{k}}$$

Rejecting the negative value x_2 of the maximum compression $x_{\max}$, we get

$$x_{\max}=x_1=\frac{mg}{k}+\sqrt{\frac{m^2g^2}{k^2}+\frac{2mgh}{k}}$$

(c) When the block is in equilibrium, net force on it is zero. If x_{eq} be the compression in the spring at equilibrium, then

$$mg=kx_{eq}$$

$$\Rightarrow \quad x=x_{eq}=\frac{mg}{k}$$

(d) At any instant, the forces acting on the block are

(i) Weight mg, acting downwards and

(ii) Spring force kx, acting upwards

If a be the acceleration of the block, then

$$a=\frac{mg-kx}{m}, \text{ downwards}$$

For situation in (a), we have

$$a=g-\frac{k}{m}x, \text{ downwards}$$

For situation in (b), we have

$$a=g-\frac{k}{m}x_{\max}=g-\frac{k}{m}x_1$$

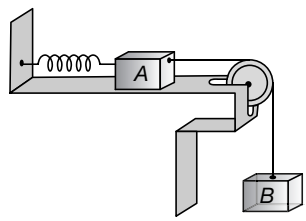
$$\Rightarrow \quad a=g-\frac{k}{m}\left(\frac{mg}{k}+\sqrt{\frac{m^2g^2}{k^2}+\frac{2mgh}{k}}\right)$$

$$\Rightarrow \quad a=-\sqrt{g^2+\frac{2kgh}{m}}$$

The negative sign shows that the acceleration is in upward direction.

ILLUSTRATION 33

Consider the situation shown in Figure, where the Mass of block A is m and that of block B is $2m$. The force constant of spring is K. Friction is absent everywhere.



The system is released from rest with the spring unstretched. Calculate

(a) the maximum extension of the spring x_m.

(b) the speed of block A when the extension in the spring is $x=\frac{x_m}{2}$.

(c) net acceleration of block B when extension in the spring is $x=\frac{x_m}{4}$.

SOLUTION

(a) At maximum extension in the spring, the blocks come momentarily to rest, so

$$v_A=v_B=0$$

Applying Conservation of Mechanical Energy, we have decrease in gravitational potential energy (GPE) of the block B must be equal to the increase in elastic potential energy (EPE) of the spring i.e.

$$\begin{pmatrix}\text{Decrease in GPE}\\ \text{of block } B\end{pmatrix}=\begin{pmatrix}\text{Increase in EPE}\\ \text{of spring}\end{pmatrix}$$

$$\Rightarrow \quad m_B g x_m = \frac{1}{2} K x_m^2$$

$$\Rightarrow \quad 2mgx_m = \frac{1}{2} K x_m^2$$

$$\Rightarrow \quad x_m = \frac{4mg}{K}$$

(b) At $x = \frac{x_m}{2} = \frac{2mg}{K}$, both the blocks move with the same speed, so $v_A = v_B = v$ (say).

Applying Law of Conservation of Mechanical Energy, we have

$$\begin{pmatrix}\text{Decrease}\\ \text{in GPE}\\ \text{of block}\end{pmatrix} = \begin{pmatrix}\text{Increase}\\ \text{in EPE}\\ \text{of spring}\end{pmatrix} + \begin{pmatrix}\text{Increase}\\ \text{in KE of}\\ \text{both blocks}\end{pmatrix}$$

$$\Rightarrow \quad m_B g x = \frac{1}{2} K x^2 + \frac{1}{2}(m_A + m_B) v^2$$

$$\Rightarrow \quad 2mg\left(\frac{2mg}{K}\right) = \frac{1}{2} K \left(\frac{2mg}{K}\right)^2 + \frac{1}{2}(3m) v^2$$

$$\Rightarrow \quad v = 2g\sqrt{\frac{m}{3K}}$$

(c) At $x = \frac{x_m}{4} = \frac{mg}{K}$, net upward force on block B is Kx or mg and net downward force on block B is $2mg$

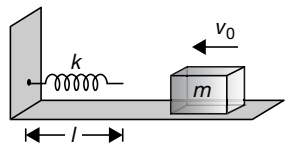
$$\Rightarrow \quad F_{\text{net}} = 2mg - mg = mg \qquad \text{(downwards)}$$

$$\Rightarrow \quad a = \frac{F_{\text{net}}}{m_A + m_B} = \frac{mg}{3m}$$

$$\Rightarrow \quad a = \frac{g}{3} \qquad \text{(downwards)}$$

ILLUSTRATION 34

A block of mass m is moving on a smooth horizontal surface with speed v_0 towards a vertical wall as shown. A light spring of spring constant k is attached to the wall such that the block compresses the spring as it moves closer to the wall. The natural length of the spring is l_0. Calculate the speed of the block when the spring has been compressed through x. Also calculate the maximum compression produced in the spring.



SOLUTION

By Law of Conservation of Energy, we have

$$\Delta U + \Delta K = 0$$

When the block collides with the spring then loss in kinetic energy of the block equals the gain in elastic potential energy of the spring. So, we have

$$\frac{1}{2} m v_0^2 - \frac{1}{2} m v^2 = \frac{1}{2} k x^2$$

$$\Rightarrow \quad v = \sqrt{v_0^2 - \frac{k}{m} x^2}$$

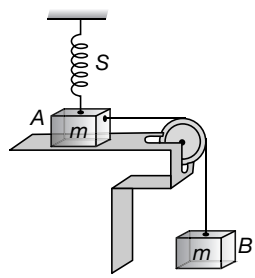
When compression in the spring is maximum i.e. $x = x_{\max}$, then the block comes momentarily to rest i.e. $v = 0$ and we can say that the total kinetic energy of the block is converted as the elastic potential energy of the spring. Hence

$$\frac{1}{2} m v_0^2 = \frac{1}{2} k x_{\max}^2$$

$$\Rightarrow \quad x_{\max} = \sqrt{\frac{m v_0^2}{k}} = v_0 \sqrt{\frac{m}{k}}$$

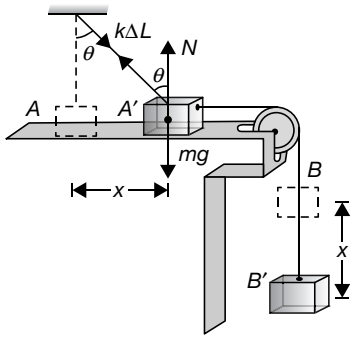
ILLUSTRATION 35

Two blocks A and B each having mass of 0.32 kg are connected by a light string passing over a smooth pulley as shown in Figure. The horizontal surface on which the block A slides is smooth. The block A is attached to a spring of force constant $40\ \text{Nm}^{-1}$ whose other end is fixed to a support 0.40 m above the horizontal surface. Initially, when the system is released to move, the spring is vertical and unstretched. Find the velocity of the block A at the instant it breaks off the surface below it. $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$.



SOLUTION

Let the block breaks off the surface when it travels a distance x as shown in figure.



So, the block A starts loosing contact with the surface below it at point A' after travelling a distance x. In this process the block B will shift from B to B' such that $BB' = AA' = x$ (as string is inextensible) and so there is a loss of gravitational potential energy of block B equal to mgx.

This energy is partly stored as elastic potential energy in the spring which is stretched by ΔL and partly appears as kinetic energy of blocks A and B. So, by Conservation of Mechanical Energy, we have

$$mgx = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}k(\Delta L)^2$$

$$\Rightarrow \quad v^2 = gx - \frac{k}{2m}(\Delta L)^2 \qquad \ldots(1)$$

Now, for vertical equilibrium of block A at A',

$$N + F\cos\theta = mg \qquad \ldots(2)$$

But for spring $F = k\Delta L$ and for breaking off, we have $N = 0$. So, the above equation reduces to

$$k\Delta L\cos\theta = mg \qquad \ldots(3)$$

where, $\Delta L = \frac{L}{\cos\theta} - L = L\left(\frac{1}{\cos\theta} - 1\right)$ $\ldots(4)$

So, substituting the value of ΔL from Equations (3) in (4) and solving for $\cos\theta$, we get

$$\cos\theta = 1 - \frac{mg}{kL} = 1 - \frac{0.32\times 10}{40\times 0.40} = \frac{4}{5}$$

$$\Rightarrow \quad \Delta L = \left(\frac{L}{\cos\theta} - L\right) = \frac{0.4\times 5}{4} - 0.4 = 0.1 \text{ m and}$$

$$x = L\tan\theta = (0.4)\left(\frac{3}{4}\right) = 0.3 \text{ m}$$

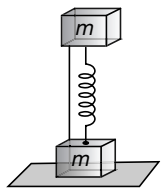
Substituting these values of ΔL and x in Equation (1), we get

$$v = \sqrt{10\times 0.3 - \frac{40\times(0.1)^2}{2\times 0.32}} = \sqrt{3 - 0.625} = \sqrt{2.375}$$

$$\Rightarrow \quad v = 1.54 \text{ ms}^{-1}$$

ILLUSTRATION 36

A system consists of two identical cubes, each of mass m, linked together by a compressed light spring of force constant k as shown in Figure.



The cubes are also connected by a thread which is burnt at a certain moment. At what values of initial compression x_0 of the spring, will the lower cube bounce up after the thread is burnt through?

SOLUTION

Let x be the elongation in the spring when it returns. Then by Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}kx_0^2 = \frac{1}{2}kx^2 + mg(x + x_0)$$

$$\Rightarrow \quad kx^2 + 2mgx - kx_0^2 + 2mgx_0 = 0$$

$$\Rightarrow \quad kx = -mg \pm (mg - kx_0)$$

$$\Rightarrow \quad kx = -kx_0 \text{ or } kx = kx_0 - 2mg$$

Since $kx = -kx_0$ gives $x = -x_0$ and this is not acceptable.

$$kx = kx_0 - 2mg$$

The lower cube will bounce up if $kx \geq mg$ (the weight of the lower cube)

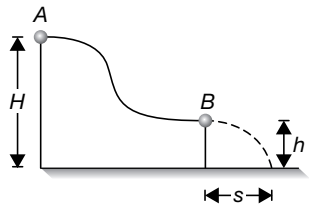
$$\Rightarrow \quad kx_0 - 2mg \geq mg$$

$$\Rightarrow \quad x_0 \geq \frac{3mg}{k}$$

Thus, for all values of x_0 greater than $\frac{3mg}{k}$ the lower cube will bounce up.

ILLUSTRATION 37

A small disc A slides down with initial velocity equal to zero from the top of a smooth hill of height H having a horizontal portion as shown in Figure.



What must be the height of the horizontal portion h to ensure the maximum distance s covered by the disc? What is it equal to?

SOLUTION

In get the velocity at point B, applying the Law of Conservation of Energy. So,

$$\begin{pmatrix}\text{Loss in Gravitational}\\ \text{Potential Energy}\end{pmatrix}=\begin{pmatrix}\text{Gain in}\\ \text{Kinetic Energy}\end{pmatrix}$$

$$\Rightarrow \quad mg(H-h)=\frac{1}{2}mv^2$$

$$\Rightarrow \quad v=\sqrt{2g(H-h)}$$

Since $y=u_yt+\frac{1}{2}a_yt^2$, where $u_y=0$, $a_y=g$ and $y=h$, so $h=\frac{1}{2}gt^2$

$$\Rightarrow \quad t=\sqrt{\frac{2h}{g}}$$

Since $x=u_xt+\frac{1}{2}a_xt^2$, where $u_x=v$, $a_x=0$ and $x=s$

$$\Rightarrow \quad s=v\times t=\sqrt{2g(H-h)}\times\sqrt{\frac{2h}{g}}$$

$$\Rightarrow \quad s=\sqrt{4h(H-h)} \qquad \ldots(1)$$

For maximum value of s or s^2, we have $\frac{d}{dh}(s^2)=0$

$$\frac{d}{dh}(4Hh-4h^2)=0$$

$$\Rightarrow \quad 4H-8h=0$$

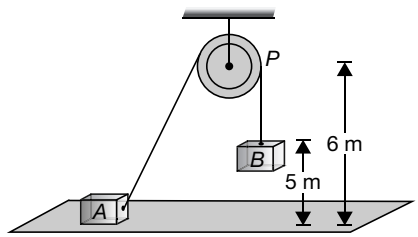
$$\Rightarrow \quad h=\frac{H}{2}$$

Substituting $h=\frac{H}{2}$, in Equation (1), we get

$$s=\sqrt{4\left(\frac{H}{2}\right)\left(H-\frac{H}{2}\right)}=\sqrt{H^2}=H$$

ILLUSTRATION 38

A block A of mass m is held at rest on a smooth horizontal floor. A light frictionless, small pulley is fixed at a height of 6 m from the floor. A light inextensible string of length 16 m, connected with A passes over the pulley and another identical block B is hung from the string. The initial height of B is 5 m from the floor as shown in Figure.



When the system is released from rest, B starts to move vertically downwards and A slides on the floor towards right.

(a) If at an instant, string makes an angle θ with the horizontal, calculate relation between velocity u of A and v of B.

(b) Calculate v when B strikes the floor.

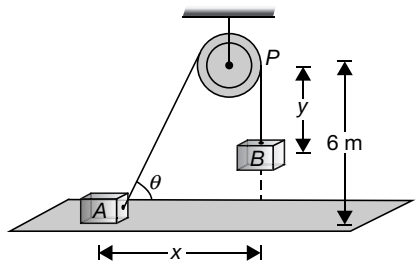
Take $g=10\ \text{ms}^{-2}$ and $\cos(37°)=0.8$

SOLUTION

(a) From the figure, we observe that

$$y+AP=16$$

$$\Rightarrow \quad y+\sqrt{x^2+36}=16$$



Differentiating with respect to time, we get

$$\frac{x}{\sqrt{(36+x^2)}}\frac{dx}{dt}+\frac{dy}{dt}=0$$

Here, $\frac{dy}{dt}=v=v_B$

As t increases, x decreases, hence

$$-\frac{dx}{dt}=u=v_A$$

$\Rightarrow \quad -u\cos\theta+v=0$

$\Rightarrow \quad u=v\sec\theta$

(b) When B strikes the floor, $y=6$ m, then

$$AP=16-6=10 \text{ m}$$

Now, $\sin\theta=\left(\frac{6}{10}\right)$

$\Rightarrow \quad \theta=37^\circ \qquad \{\because \sin^2(37^\circ)+\cos^2(37^\circ)=1\}$

Now, according to Law of Conservative of Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{PE of } B\end{pmatrix}=\begin{pmatrix}\text{Gain in}\\ \text{KE of } A\end{pmatrix}+\begin{pmatrix}\text{Gain in}\\ \text{KE of } B\end{pmatrix}$$

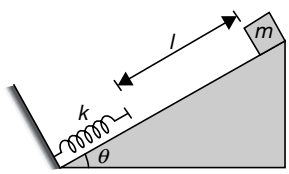
$\Rightarrow \quad mg(5)=\frac{1}{2}mu^2+\frac{1}{2}mv^2$

But $u=v\sec(37^\circ)=\frac{5v}{4}$

Solving, we get $v=\left(\frac{40}{\sqrt{41}}\right)$ ms^{-1}

ILLUSTRATION 39

An ideal light spring S can be compressed 1 m by a force of 100 N. This spring is placed at the bottom of a frictionless inclined plane which makes an angle of 30° with the horizontal as shown in Figure.



A 10 kg mass is released from rest from the top of the incline and is brought to rest momentarily after compressing the spring 2 m. Calculate the distance through which the mass slides before coming to rest. Also calculate the speed of the mass just before it hits the spring.

SOLUTION

For the spring S, we have $F=kx_0$

Since it is given that, for $F=100$ N, the spring can be compressed by $x_0=1$ m, so, we have

$$k=\frac{F}{x_0}=100 \text{ Nm}^{-1}$$

If spring compresses through $x=2$ m by the mass, then by applying Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in GPE}\\ \text{of the Body}\end{pmatrix}=\begin{pmatrix}\text{Gain in EPE}\\ \text{of the Spring}\end{pmatrix}$$

$\Rightarrow \quad mg\sin\theta(l+x)-\frac{1}{2}kx^2=0$

$\Rightarrow \quad (10)(10)(0.5)(l+2)-\frac{1}{2}(100)(2)^2=0$

$\Rightarrow \quad l+2-4=0$

$\Rightarrow \quad l=2$ m

So, the total distance through which the mass slides before coming to rest is

$$l+x=2+2=4 \text{ m}$$

When mass hits the spring, then by Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in GPE}\\ \text{of the Body}\end{pmatrix}=\begin{pmatrix}\text{Gain in KE}\\ \text{of the Body}\end{pmatrix}$$

$\Rightarrow \quad \frac{1}{2}mv^2=mgh=mg(l\sin\theta)$

$\Rightarrow \quad v=\sqrt{2gl\sin\theta}$

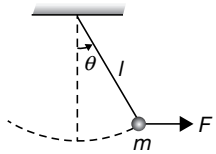
$\Rightarrow \quad v=\sqrt{20}$ ms^{-1}

Test Your Concepts-III

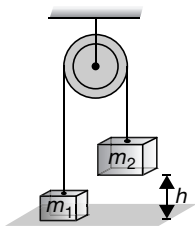
Based on Conservation of Energy and Work Energy Theorem

(Solutions on page H.8)

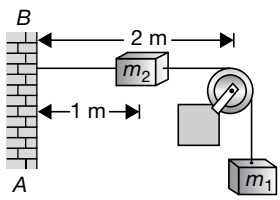
1. An object of mass m is tied to a string of length l and a variable force F is applied on it which brings the string gradually at angle θ with the vertical. Find the work done by the force F.



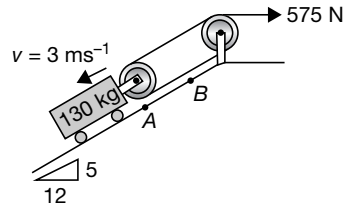
2. Two blocks with masses $m_1 = 3$ kg and $m_2 = 5$ kg are connected by a light string that slides over a frictionless pulley as shown in figure. Initially, m_2 is held 5 m off the floor while m_1 is on the floor. The system is then released. At what speed does m_2 hit the floor?



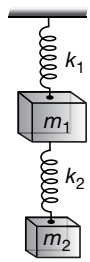
3. In the arrangement shown, calculate the speed of the bodies 1 and 2 having masses 0.5 kg and 2 kg respectively at the moment the block m_2 hits the wall AB. Assume that the bodies are released from rest. Take $g = 10\ \text{ms}^{-2}$.



4. The 130 kg carriage has an initial velocity of $3\ \text{ms}^{-1}$ down the incline at A, when a constant force of 575 N is applied to the hoisting cable as shown. Calculate the velocity of the carriage when it reaches B, a distance of 3 m above A on the incline. Show that in the absence of friction this velocity is independent of whether the initial velocity of the carriage at A was up or down the incline. Take $g = 10\ \text{ms}^{-2}$.



5. In the arrangement shown, we have $k_1 = 1500\ \text{Nm}^{-1}$, $k_2 = 500\ \text{Nm}^{-1}$, $m_1 = 2$ kg and $m_2 = 1$ kg. Calculate the

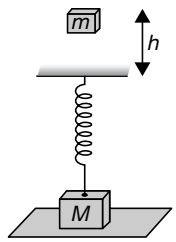


(a) potential energy stored in the springs in equilibrium.

(b) work done in slowly pulling down m_2 by 8 cm.

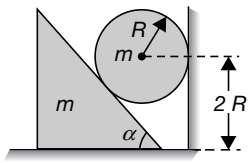
Take $g = 10\ \text{ms}^{-2}$

6. A block of mass m is dropped onto a spring of constant k from a height h. The second end of the spring is attached to another block of mass M as shown.

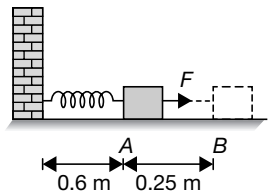


Find the minimum value of h so that the block M bounces off the ground if the block of mass m sticks to the spring immediately after it comes into contact with it.

7. A sphere of mass m held at a height $2R$ between a wedge of same mass m and a rigid wall, is released. Assuming that all the surfaces are frictionless, find the speed of both the bodies when the sphere hits the ground.



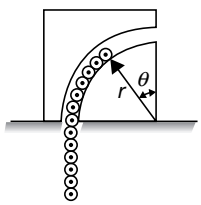
8. A 0.5 kg block attached to a spring with length 0.6 m and force constant $k = 40\ \text{Nm}^{-1}$ is at rest with the back of the block at point *A* on a frictionless, horizontal table. The mass of the spring is negligible. The block is pulled to the right along the surface with a constant horizontal force $F = 20$ N.



(a) What is the block's speed when the back of the block reaches point *B*, which is 0.25 m to the right of point *A*?

(b) When the back of the block reaches point *B*, the block is set free. In the subsequent motion, how close does the block get to the wall where the left end of the spring is attached?

9. The flexible bicycle type chain of length $\frac{\pi r}{2}$ and mass per unit length λ is released from rest with $\theta = 0°$ in the smooth circular channel and falls through the hole in the supporting surface. Determine the velocity *v* of the chain as the last link leaves the slot.



10. A helicopter lifts a 80 kg astronaut 15 m vertically from the ocean by means of a cable. The acceleration of the astronaut is $\frac{g}{10}$. Calculate the

(a) work done on the astronaut by the force from the helicopter

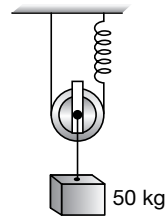
(b) work done on the astronaut by the gravitational force

(c) kinetic energy and

(d) speed of the astronaut just before reaching the helicopter.

Take $g = 10\ \text{ms}^{-2}$

11. The system is released from rest with the spring initially stretched 76 mm. Calculate the velocity *v* of the block of mass 50 kg after it has dropped 12 mm. The spring has a stiffness of $1000\ \text{Nm}^{-1}$. Neglect the mass of the small pulley.



12. A 1.5 kg block is initially at rest on a horizontal frictionless surface when a horizontal force in the positive direction of *x*-axis is applied to the block. The force is given by $\vec{F}(x) = (2.5 - x^2)\hat{i}$ N, where *x* is in metre and the initial position of the block is $x = 0$.

(a) What is the kinetic energy of the block as it passes through $x = 2$ m?

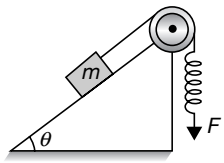
(b) What is the maximum kinetic energy of the block between $x = 0$ and $x = 2$ m?

13. A certain spring is found not to obey Hooke's Law. It exerts a restoring force $F_x(x) = -\alpha x - \beta x^2$ when stretched or compressed, where $\alpha = 80\ \text{Nm}^{-1}$ and $\beta = 24\ \text{Nm}^{-2}$. The mass of the spring is negligible.

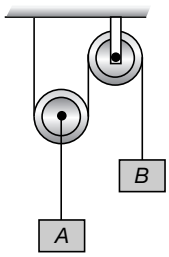
(a) Calculate the potential energy function *U*(*x*) for this spring. Let $U = 0$ when $x = 0$.

(b) An object with a mass of 2 kg on a frictionless horizontal surface is attached to this spring, pulled a distance 1 m to the right (the $+x$ direction) to stretch the spring and released. Calculate the approximate speed of the object when it is 0.5 m to the right of the $x = 0$ equilibrium position?

14. A block rests on an inclined plane as shown in figure. A spring of force constant *k* to which it is attached via a pulley is being pulled downward with gradually increasing force. The value of μ_s is known. Calculate the potential energy *U* of the spring at the moment when the block begins to move.



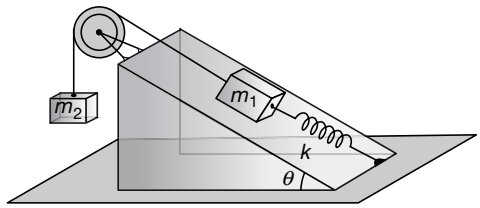
15. Block *A* has a mass of 30 kg and block *B* has a mass of D 5 kg. Calculate the distance *A* must descend from rest before it obtains a speed of $2\ \text{ms}^{-1}$. Neglect the mass of the cord and pulleys. Assume friction to be absent. Take $g = 10\ \text{ms}^{-2}$.



16. Two blocks with masses m_1 = 2 kg and m_2 = 3 kg hang on either side of a pulley as shown in figure. Block m_1 is on an incline $(\theta = 30°)$ and is attached to a spring whose stiffness constant is $40\ \text{Nm}^{-1}$. The system is released from rest with the spring in its natural length. Find

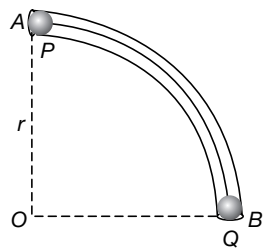
(a) the maximum extension of the spring

(b) the speed of m_1 when the extension is 0.5 m. Ignore friction and mass of the pulley.



17. A block of mass *m* is pushed against a spring of spring constant *k* fixed at one end to a wall. The block can slide on a frictionless table. The natural length of the spring is L_0 and it is compressed to half its natural length when the block is released. Find the velocity of the block as a function of its distance *x* from the wall.

18. A smooth narrow tube in the form of an arc *AB* of a circle of centre *O* and radius *r* is fixed so that *A* is vertically above *O* and *OB* is horizontal. Two particles *P* and *Q* of mass *m* and 2*m* respectively are connected to each other with a light inextensible string of length $\left(\frac{\pi r}{2}\right)$ are placed inside the tube with *P* at *A* and *Q* at *B* and released from rest. Assuming the string remains taut during motion, calculate the speed of particles when *P* reaches *B*.



MODIFIED WORK-ENERGY THEOREM (MWET)

If external forces, pseudo forces, non-conservative forces and internal forces are present in the system and the work done by the respective forces be W_{ext}, W_{ps}, W_{nc} and W_{int} then according to ***Modified Work-Energy Theorem (MWET)***, we have

$$W_{ext} + W_{ps} + W_{nc} + W_{int} = \Delta U + \Delta K$$

Conceptual Note(s)

(a) Work done by internal forces may or may not be zero.

(b) If pseudo forces, non-conservative forces are absent, external forces are present and if work done by internal forces is zero, then we have

$$W_{ext} = \Delta U + \Delta K$$

(c) If external forces, pseudo forces are absent, non-conservative forces are present and if work done by internal forces is zero, then we have

$$W_{nc} = \Delta U + \Delta K$$

LAW OF CONSERVATION OF MECHANICAL ENERGY

If external forces, pseudo forces and non-conservative forces all are absent, then we have

$$\Delta U + \Delta K = 0$$

which simply implies that total mechanical energy $(U+K)$ is constant i.e.

$$(U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

Since, for a conservative system we have

$$W_c = \Delta K \quad \ldots(1)$$

Also, we have

$$W_c = -\Delta U \quad \ldots(2)$$

Equating (1) and (2), we get

$$\Delta K = -\Delta U$$

$$\Rightarrow \quad \Delta(K+U) = 0$$

$$\Rightarrow \quad K + U = \text{constant}$$

$$\Rightarrow \quad K_i + U_i = K_f + U_f$$

$$\Rightarrow \quad \begin{pmatrix}\text{Total Initial}\\ \text{Mechanical Energy}\end{pmatrix} = \begin{pmatrix}\text{Total Final}\\ \text{Mechanical Energy}\end{pmatrix}$$

Since, any non-conservative system has got both conservative nature and a non-conservative nature, so using Work-Energy Theorem, we get

$$W = W_c + W_{nc} = \Delta K$$

and $W_c = -\Delta U$

$$\Rightarrow \quad -\Delta U + W_{nc} = \Delta K$$

$$\Rightarrow \quad W_{nc} = \Delta U + \Delta K$$

$$\Rightarrow \quad W_{nc} = (U_f - U_i) + (K_f - K_i)$$

$$\Rightarrow \quad W_{nc} = (U_f + K_f) - (U_i + K_i)$$

$$\Rightarrow \quad W_{nc} = \begin{pmatrix}\text{Total Final}\\ \text{Mechanical}\\ \text{Energy}\end{pmatrix} - \begin{pmatrix}\text{Total Initial}\\ \text{Mechanical}\\ \text{Energy}\end{pmatrix}$$

Conceptual Note(s)

So, to conclude we can say that the **Modified Work-Energy Theorem (MWET)**

$$W_{\text{ext}} + W_{\text{ps}} + W_{\text{nc}} + W_{\text{int}} = \Delta U + \Delta K$$

is a handy tool to solve problems.

WORK-ENERGY THEOREM FOR NON-CONSERVATIVE SYSTEM

For a non-conservative system, if W is the total work done (due to conservative and non-conservative nature of the system) then

$$W = W_c + W_{nc} = \Delta K = K_f - K_i$$

Conceptual Note(s)

A non-conservative system is bound to have a conservative part attached to it (It is impossible to have something that is purely impure like, purely impure milk or purely impure honey etc.). Hence for a non-conservative system we have

$$W_{\text{total}} = W_{\text{c}} + W_{\text{nc}} = \Delta K$$

Since, we know that the work done by a conservative force is given by

$$W_c = -\Delta U$$

$$\Rightarrow \quad -\Delta U + W_{nc} = \Delta K$$

$$\Rightarrow \quad W_{nc} = \Delta U + \Delta K$$

$$\Rightarrow \quad W_{nc} = (U_f - U_i) + (K_f - K_i)$$

$$\Rightarrow \quad W_{nc} = (U_f + K_f) - (U_i + K_i)$$

$$\Rightarrow \quad W_{nc} = \begin{pmatrix}\text{Total Final}\\ \text{Mechanical}\\ \text{Energy}\end{pmatrix} - \begin{pmatrix}\text{Total Initial}\\ \text{Mechanical}\\ \text{Energy}\end{pmatrix}$$

Conceptual Note(s)

(a) So, to conclude we can say that work done by a non-conservative force is the sum of change in potential energy and change in kinetic energy or work done by a non-conservative force equal the change in value of total mechanical energy.

(b) However, if we have a non-conservative system on which some external force is also acting, then, from Work Energy Theorem, we have

$$W_{\text{total}} = \Delta K$$

$$\Rightarrow \quad W_{\text{c}} + W_{\text{nc}} + W_{\text{ext}} = \Delta K \quad \ldots(1)$$

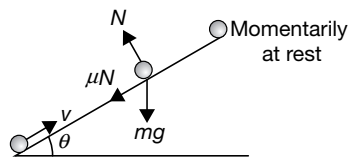
This formula happens to be the master key to be used for any kind of system.

ILLUSTRATION 40

A particle is launched straight up with a kinetic energy K up a rough inclined plane of inclination θ and coefficient of friction μ. Prove that the work done against friction before the particle first comes to rest is $\dfrac{\mu K\cos\theta}{\sin\theta+\mu\cos\theta}$.

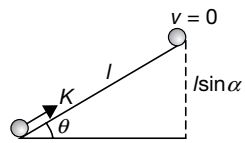
SOLUTION

Let us consider the mass of the particle to be m, the normal reaction on the particle exerted by the plane to be N, the initial velocity of the particle to be v and the distance travelled by the particle up the plane before it first comes to rest be l.



Since $W_{nc}=\Delta U+\Delta K$

$$\Rightarrow\quad -(\mu N)l=mgl\sin\theta+\left(\frac{1}{2}m(0)^2-\frac{1}{2}mu^2\right)$$



$$\Rightarrow\quad -\mu mgl\cos\theta=mgl\sin\theta-K$$

$$\Rightarrow\quad K=mgl\sin\theta+\mu mgl\cos\theta$$

$$\Rightarrow\quad K=mgl(\sin\theta+\mu\cos\theta)$$

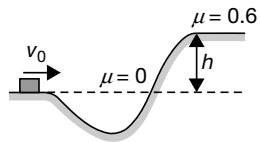
The work done W_f against friction force $f=\mu N=\mu mg\cos\theta$ is given by

$$W_f=\mu Nl=\mu mgl\cos\theta=\frac{\mu K\cos\theta}{\sin\theta+\mu\cos\theta}$$

Please note that when we write "*work done by friction force*" then we shall be applying the negative sign with the work done but when we write the "*work done against friction*", then we do not apply a negative sign with the work done.

ILLUSTRATION 41

A block slides along a track from one level to a higher level, by moving through an intermediate valley as shown in Figure. The track is frictionless until the block reaches the higher level, where a frictional force stops the block after it covers a distance d. If the coefficient of kinetic friction between the block and the rough surface is 0.6, the initial speed of the block is $v_0=6\ \text{ms}^{-1}$, the height difference $h=1.1$ m, then calculate d. Take $g=10\ \text{ms}^{-2}$.



SOLUTION

According to Modified Work-Energy Theorem, we have

$$W_{nc}=\Delta U+\Delta K$$

Applying this for the motion of block till it stops, we get

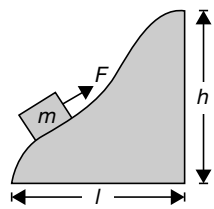
$$-\mu mgd=mgh-\frac{1}{2}mv_0^2$$

$$\Rightarrow\quad d=\frac{v_0^2-2gh}{2\mu g}$$

$$\Rightarrow\quad d=\frac{36-2\times10\times1.1}{2\times0.6\times10}=1.167\ \text{m}$$

ILLUSTRATION 42

A body of mass m was slowly hauled up the hill as shown in the figure by a force F which at each point was directed along a tangent to the trajectory as shown in Figure.



Calculate the work performed by this force, if the height of the hill is h, the length of its base is l and the coefficient of friction is μ.

SOLUTION

Four forces are acting on the body

1. the external applied force (F)
2. weight (mg)

3. normal reaction (N)
4. friction (f)

Using Work-Energy Theorem

$$W_{\text{total}} = \Delta K$$

$$\Rightarrow \quad W_c + W_{nc} + W_{ext} = \Delta K$$

$$\Rightarrow \quad W_{nc} + W_{ext} = \Delta U + \Delta K$$

Since the body is hauled up slowly, so

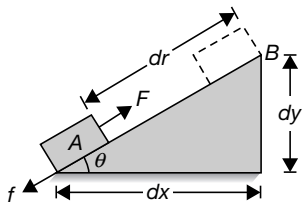
$$K_f = K_i = 0$$

$$\Rightarrow \quad \Delta K = 0$$

$$\Rightarrow \quad W_{nc} + W_{ext} = \Delta U \qquad \ldots(1)$$

Since, $\Delta U = -mgh$ and at all instants, the normal reaction is perpendicular to displacement at all points, so work done by the normal reaction is $W_N = 0$.

To calculate the work done by the non-conservative force i.e. friction, let us consider a very small displacement $d\vec{r}$ given to body. The **magnified view** of this is shown here in Figure.



Since, $f = \mu mg\cos\theta$

Also, $(dW_{AB})_f = -fds$

$$\Rightarrow \quad (dW_{AB})_f = -(\mu mg\cos\theta)ds$$

$$\Rightarrow \quad (dW_{AB})_f = -\mu mg(dl) \qquad \{\because\ ds\cos\theta = dl\}$$

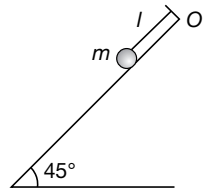
$$\Rightarrow \quad W_f = -\mu mg\int_0^l dl = -\mu mgl$$

Substituting these values in Equation (1), we get

$$W_F = mgh + \mu mgl$$

ILLUSTRATION 43

A particle of mass m is attached to one end of a light elastic string whose other end is fixed to a point O on an inclined plane having inclination angle $45°$ with the horizontal. The natural length of the string is l and its force constant is $\frac{mg}{l}$. The particle is held on the inclined plane so that the string lies just unstretched along a line of greatest slope and then released from rest as shown in Figure.



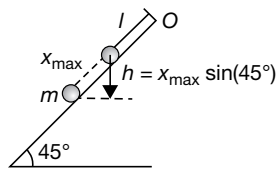
(a) If the inclined plane is smooth, then find the lowest position that can be reached by the particle. Also locate the equilibrium position.

(b) Assume that the inclined plane is rough enough with coefficient of friction between the particle and the plane to be μ. If the particle stops in first descend after covering a distance d along the greatest slope, then find d.

Please note that all the positions have to be measured with respect to the point O.

SOLUTION

(a) When the particle reaches the lowest position, then it momentarily comes to a stop. If $x_{\max}$ be the extension of the string when the particle is at its lowest position



By Law of Conservation of Energy, we have

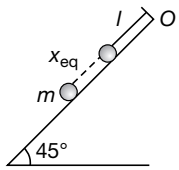
$$\begin{pmatrix}\text{Loss in GPE}\\ \text{of the particle}\end{pmatrix} = \begin{pmatrix}\text{Gain in EPE}\\ \text{of the string}\end{pmatrix}$$

$$\Rightarrow \quad mg(x_{\max}\sin 45°) = \frac{1}{2}\left(\frac{mg}{l}\right)x_{\max}^2$$

$$\Rightarrow \quad x_{\max} = \sqrt{2}l$$

So, the lowest position is located from O at a distance

$$l_1 = l + x_{\max} = l + \sqrt{2}l = (1+\sqrt{2})l$$



For equilibrium of the particle, the component of the weight acting down the incline is balanced by the elastic force.

$$\Rightarrow \quad mg\sin(45°) = \left(\frac{mg}{l}\right)x_{eq}$$

$$\Rightarrow \quad x_{eq} = \frac{l}{\sqrt{2}}$$

So, the equilibrium position is located from O at a distance

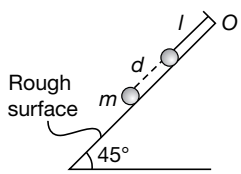
$$l_2 = l + x_{eq} = l + \frac{l}{\sqrt{2}} = l\left(1 + \frac{1}{\sqrt{2}}\right)$$

(b) According to Modified Work-Energy Theorem, we have

$$\Rightarrow \quad W_{nc} = \Delta U + \Delta K$$

$$\Rightarrow \quad W_{nc} = \left(\Delta U_g + \Delta U_e\right) + \Delta K$$

where ΔU_g is the change in gravitational potential energy of the mass and ΔU_e is the change in elastic potential energy of the string. Please note that the initial velocity and the final velocity of the mass both are zero and hence $\Delta K = 0$.



$$\Rightarrow \quad -\mu mgd\cos(45°) = -mgd\sin(45°) + \frac{1}{2}kd^2$$

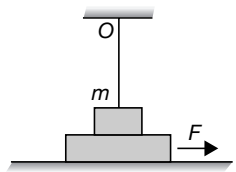
$$\Rightarrow \quad \frac{1}{2}\left(\frac{mg}{l}\right)d^2 + \mu mgd\cos(45°) = mgd\sin(45°)$$

$$\Rightarrow \quad \frac{d}{2l} + \frac{\mu}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad d = \sqrt{2}l(1-\mu)$$

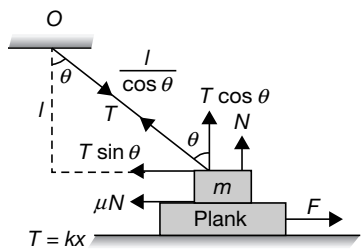
ILLUSTRATION 44

A horizontal plane supports a plank with a bar of mass $1\ \text{kg}$ placed on it and attached by a light elastic non deformed cord of length $40\ \text{cm}$ to a point O as shown in figure. The coefficient of friction between the bar and plank is 0.2. The plank is slowly shifted to the right until the bar starts sliding over it. It occurs at the moment when the cord deviates from the vertical by $30°$. Calculate the work done (in millijoule) that has been performed by that moment due to friction force acting on the bar in the reference frame attached to the plane.



SOLUTION

The intermediate force diagram of the system, when the cord makes an angle $\theta(<30°)$ is shown in Figure.



At this instant, the extension in cord is

$$x = \frac{l}{\cos\theta} - l = l(\sec\theta - 1) \qquad \ldots(1)$$

If the force constant of the cord is k, then the tension developed in the cord is given by $T = kx$
The bar will start sliding when

$$kx\sin\theta = f_l = \mu N \qquad \ldots(2)$$

where $N + kx\cos\theta = mg$

$$\Rightarrow \quad N = mg - kx\cos\theta$$

Substituting this value of N in equation (1), we get

$$kx\sin\theta = \mu\left(mg - kx\cos\theta\right)$$

$$\Rightarrow \quad kx\left(\sin\theta + \mu\cos\theta\right) = \mu mg$$

$$\Rightarrow \quad k = \frac{\mu mg}{x\left(\sin\theta + \mu\cos\theta\right)}$$

Substituting the value of x from equation (1), we get

$$\Rightarrow \quad k = \frac{\mu mg}{l(\sec\theta - 1)\left(\sin\theta + \mu\cos\theta\right)}$$

According to Modified Work-Energy Theorem (MWET), we have

$$W_{nc} = \Delta U + \Delta K$$

where, $\Delta U = \frac{1}{2}kx^2$ and $\Delta K = 0$

$$\Rightarrow \quad W_{nc} = \frac{1}{2}kx^2 = \frac{kx^2}{2}$$

Conceptual Note(s)

Alternatively, you can also think that in the process of shifting the plank slowly $(\Delta K = 0)$, the total work done against friction just increases the elastic potential energy of the cord by $\Delta U = \frac{1}{2}kx^2$. So, work done against friction is equal to the increment in potential energy of the spring

$$\Rightarrow \quad W_{nc} = \frac{\mu mgl^2(\sec\theta - 1)^2}{2l(\sec\theta - 1)(\sin\theta + \mu\cos\theta)}$$

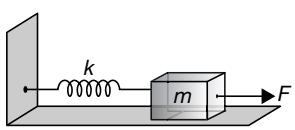
$$\Rightarrow \quad W = \frac{\mu mgl(1 - \cos\theta)}{2\cos\theta(\sin\theta + \mu\cos\theta)}$$

Substituting the numerical data, we get

$$W = 0.09 \text{ J} = 90 \text{ mJ}$$

ILLUSTRATION 45

a block of mass m is resting on a smooth horizontal surface. It is connected to a rigid wall by a light spring of spring constant k. The spring is in its natural length and a constant horizontal force F starts acting on the block towards right as shown in Figure.



Calculate the speed of the block when extension in the spring is x and when the block passes the equilibrium position. Also calculate the maximum extension produced in the spring.

SOLUTION

According to the Modified Work-Energy Theorem (MWET), we have

$$W_{ext} = \Delta U + \Delta K \qquad \ldots(1)$$

When the applied external force F displaces the block through x, then the spring also extends by x. If v is the velocity of the block at this instant then, we have

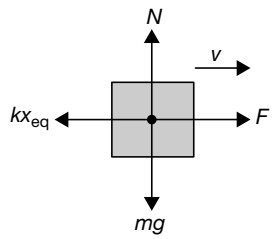
$$W_{ext} = Fx, \ \Delta U = \frac{1}{2}kx^2, \ \Delta K = \frac{1}{2}mv^2$$

Substituting these values in equation (1), we get

$$Fx = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2Fx - kx^2}{m}} \qquad \ldots(2)$$

When the block passes the equilibrium position, then net force on the block is zero as shown in Figure.



$$\Rightarrow \quad F = kx_{eq}$$

$$\Rightarrow \quad x_{eq} = \frac{F}{k}$$

Since from equation (2), we have

$$v = \sqrt{\frac{2Fx - kx^2}{m}}$$

Substituting $x_{eq} = \frac{F}{k}$ in this equation, we get

$$v = \sqrt{\frac{\frac{2F^2}{k} - \frac{F^2}{k}}{m}} = \frac{F}{\sqrt{mk}}$$

At the maximum extension, speed of the block becomes zero, so from equation (2), we get at $x = x_{max}$, $v = 0$.

$$\Rightarrow \quad \sqrt{\frac{2Fx_{max} - kx_{max}^2}{m}} = 0$$

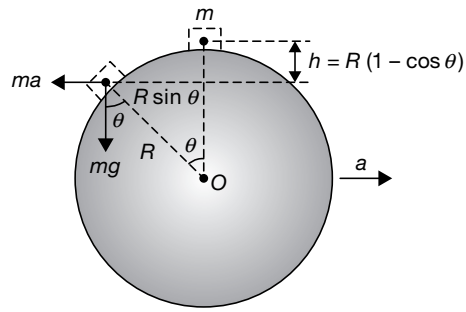
$$\Rightarrow \quad x_{max} = \frac{2F}{k}$$

ILLUSTRATION 46

A block is lying at the top of a smooth sphere of radius R. The sphere begins to accelerate with a constant horizontal acceleration a. Calculate the speed of block when it is at an angular position θ from the initial position relative to the sphere.

SOLUTION

First of all, we need to understand that since the block lies on the sphere, so let the speed of the particle be v with respect to the sphere (that happens to be the ground for the particle). However, the sphere is accelerating and hence it becomes a non-inertial frame for the particle due to which a pseudo force $F_{pseudo} = ma$ acts on the block opposite to the acceleration of the sphere. Now, due to this force the block is displaced to the left by $R\sin\theta$ as shown in Figure.



According to Modified Work-Energy Theorem (MWET), we have

$$W_{pseudo} = \Delta U + \Delta K$$

$$\Rightarrow \quad F_{ps}\Delta x = -mgh + \frac{1}{2}mv^2$$

$$\Rightarrow \quad maR\sin\theta = -mgR(1-\cos\theta) + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{R\left(a\sin\theta + g(1-\cos\theta)\right)}$$

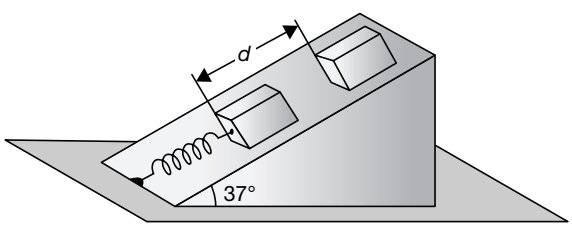
Test Your Concepts-IV

Based on Work Energy Theorem for Non-conservative Systems

(Solutions on page H.13)

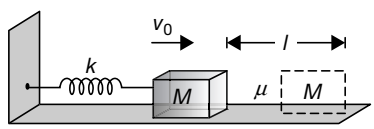
1. A box of mass m is gently placed on a conveyor belt that moves at a constant speed v. The coefficient of kinetic friction is μ_k.
- **(a)** What is the work done by friction?
- **(b)** How far does the box move before reaching its final speed?
- **(c)** When the box reaches its final speed, how far has the belt moved?

2. A block of mass $m = 0.2$ kg is held against, but not attached to a spring $(k = 50\ \text{Nm}^{-1})$ which is compressed by $x_0 = 20$ cm, as shown in figure. When the block is released, the block slides $d = 50$ cm up the rough incline before coming to rest. Find



- **(a)** the force of friction
- **(b)** the speed of the block as it leaves the spring.

3. The block of mass M shown in figure initially has a velocity v_0 to the right and its position is such that the spring exerts no force on it, i.e., the spring is neither stretched nor compressed. The block moves to

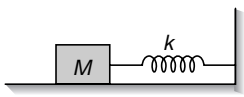


the right a distance l before stopping in the dotted position shown. The spring constant is k and the coefficient of kinetic friction between block and the table is μ.
- **(a)** What is the work done on it by the friction force?
- **(b)** What is the work done on it by the spring force?
- **(c)** Are there other forces acting on the block and if so, what work do they do?

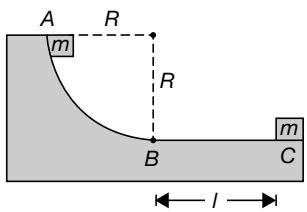
(d) What is the total work done on the block?

(e) Use the work-energy theorem to find the value of l in terms of M, v_0, μ, g and k

4. A block of mass M slides along a horizontal table with speed v_0. At $x = 0$ it hits a spring with spring constant k and begins to experience a friction force. The coefficient of friction is variable and is given by $\mu = bx$ where b is a constant. Find the loss in mechanical energy when the block has first come momentarily to rest.



5. In a truck-loading station at a post office a small package of mass $m = 200$ g is released from rest at point A on a track that is one quarter of a circle with radius $R = 2$ m. The size of the package is much less than 1.6 m, so the package can be treated as a particle. It slides down the track and reaches point B with a speed of $4\ \text{ms}^{-1}$. From point B it slides on a level surface a distance of $l = 4$ m to point C, where it comes to rest.

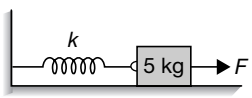


(a) What is the coefficient of kinetic friction on the horizontal surface?

(b) How much work is done on the package by friction as it slides down the circular arc from A to B?

Take $g = 10\ \text{ms}^{-2}$

6. A 5 kg block is attached to an unstretched spring of constant $k = 2000\ \text{Nm}^{-1}$. The coefficients of static and kinetic friction between the block and the plane are 0.6 and 0.36, respectively. If a force F is slowly applied to the block until the tension in the spring reaches 100 N and then suddenly removed, determine



(a) the velocity of the block as it returns to its initial position,

(b) the maximum velocity achieved by the block.

Take $g = 10\ \text{ms}^{-2}$

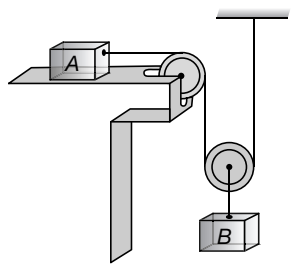
7. An object of mass 5 kg falls from rest through a vertical distance of 20 m and reaches a velocity of $10\ \text{ms}^{-1}$. Calculate the work done by push of the air on the object.

Take $g = 10\ \text{ms}^{-2}$

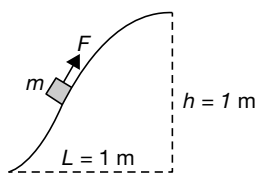
8. Two blocks of masses m_1 and m_2 connected by a light spring rest on a horizontal plane. The coefficient of friction between the blocks and the surface is equal to μ. What minimum constant force has to be applied in the horizontal direction to the block of mass m_1 in order to shift the other block?

9. A disc of mass 50 g slides with the zero initial velocity down an inclined plane set at an angle 30° to the horizontal. Having traversed a distance of 50 cm along the horizontal plane, the disc stops. Find the work performed by the friction forces over the whole distance, assuming the friction coefficient 0.15 for both inclined and horizontal planes. Take $g = 10\ \text{ms}^{-2}$.

10. In the arrangement shown in figure $m_A = 4$ kg and $m_B = 1$ kg. The system is released from rest and block B is found to have a speed $0.3\ \text{ms}^{-1}$ after it has descended through a distance of 1 m. Find the coefficient of friction between the block and the table is μ. Neglecting friction elsewhere and taking $g = 10\ \text{ms}^{-2}$.



11. A body of mass m = 1 kg was slowly hauled up a rough hill having friction coefficient 0.5 by a force F, which at each point was directed along a tangent to the trajectory. Calculate the work done against friction.



RELATION BETWEEN $\vec{F}_C$ AND U

Any conservative force $\vec{F}_C$ equals the negative gradient $(-\vec{\nabla})$ of the potential energy U. So,

$$\vec{F} = -\vec{\nabla}U = -\left(\hat{i}\frac{\partial U}{\partial x} + \hat{j}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right)$$

Problem Solving Technique(s)

(a) Gradient is represented by the symbol $\vec{\nabla}$. (**Read as Del Operator** or Nabla Operator).

(b) Geometrically it gives slope of a scalar function in 3-D space.

(c) Gradient can only be applied on scalar functions and never on vectors.

(d) Once gradient is applied on a scalar function the result thus obtained is a vector.

(e) Mathematically, $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

So, $\vec{F} = -\vec{\nabla}U = -\left(\hat{i}\frac{\partial U}{\partial x} + \hat{j}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right)$

where,

$$\frac{\partial U}{\partial x} = \begin{pmatrix}\text{Partial Derivative of } U \text{ w.r.t. } x\\ \text{i.e. Derivative of } U \text{ w.r.t. } x\\ \text{keeping } y \text{ and } z \text{ constant}\end{pmatrix}$$

$$\frac{\partial U}{\partial y} = \begin{pmatrix}\text{Partial Derivative of } U \text{ w.r.t. } y\\ \text{i.e. Derivative of } U \text{ w.r.t. } y\\ \text{keeping } x \text{ and } z \text{ constant}\end{pmatrix}$$

$$\frac{\partial U}{\partial z} = \begin{pmatrix}\text{Partial Derivative of } U \text{ w.r.t. } z\\ \text{i.e. Derivative of } U \text{ w.r.t. } z\\ \text{keeping } x \text{ and } y \text{ constant}\end{pmatrix}$$

EXAMPLE

If $U = kxy$, k is a constant, then

$$\vec{F} = -\left(\hat{i}\frac{\partial U}{\partial x} + \hat{j}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right)$$

Now, $\frac{\partial U}{\partial x} = ky$, $\frac{\partial U}{\partial y} = kx$, $\frac{\partial U}{\partial z} = 0$

So, $\vec{F} = -k\left(y\hat{i} + x\hat{j}\right)$ is the force corresponding to potential energy function $U = kxy$.

ILLUSTRATION 47

The potential energy function for a particle in a region of space is given by $U = 2x^2 + 3y^3 + 2z$, in joule. Calculate the force acting on the particle at the point $(1,2,3)$ m. Assume that x, y and z are expressed in metre.

SOLUTION

Since we know that,

$$\vec{F} = -\vec{\nabla}U = -\left(\hat{i}\frac{\partial U}{\partial x} + \hat{j}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right)$$

$$\Rightarrow \quad \vec{F} = -\left(4x\hat{i} + 9y^2\hat{j} + 2\hat{k}\right)$$

$$\Rightarrow \quad \vec{F}\big|_{\text{at } (1,2,3)\text{ m}} = -\left(4\hat{i} + 36\hat{j} + 2\hat{k}\right) \text{ N}$$

ILLUSTRATION 48

Force acting on a particle in a conservative force field is $\vec{F} = \left(2x\hat{i} + 3y^2\hat{j}\right)$. Find the potential energy function, if it is zero at origin.

SOLUTION

$$\int_{(0,0,0)}^{(x,y,z)} dU = -\int_{(0,0,0)}^{(x,y,z)} \vec{F}\cdot\vec{dr}$$

$$= -\int_{(0,0,0)}^{(x,y,z)} \left(2x\hat{i} + 3y^2\hat{j}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$\Rightarrow \quad U(x,y,z) - U(0,0,0) = -\int_{(0,0,0)}^{(x,y,z)} \left(2x\,dx + 3y^2\,dy\right)$$

Since, we are given that $U(0,0,0) = 0$

$$\Rightarrow \quad U(x,y,z) = -(x^3 + y^3)$$

ILLUSTRATION 49

The potential energy function for the force between two atoms in a diatomic molecule can be expressed approximately as $U(r) = \frac{a}{r^{12}} - \frac{b}{r^6}$, where a and b are constants and r is the separation between the atoms.

(a) Determine the force function $F(r)$.
(b) Find the value of for which the molecule will be in the stable equilibrium.

SOLUTION

(a) The force between the two atoms is given by

$$F(r) = -\frac{dU}{dr}$$

$$\Rightarrow \quad F(r) = \frac{12a}{r^{13}} - \frac{6b}{r^7}$$

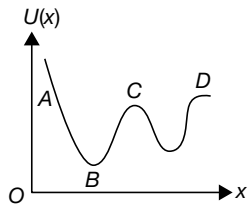
(b) For stable equilibrium $F(r) = 0$ and $\frac{d^2U}{dr^2} > 0$

Thus, $+\frac{12a}{r^{13}} - \frac{6b}{r^7} = 0$

$$\Rightarrow \quad r = \left(\frac{2a}{b}\right)^{1/6}$$

POTENTIAL ENERGY CURVE

A graph plotted between the potential energy of a particle and its displacement from the centre of force is called potential energy curve. Figure shows a graph of potential energy function $U(x)$ for one dimensional motion.



NATURE OF FORCE

Since the negative gradient of the potential energy gives force i.e.

$$F = -\frac{dU}{dx}$$

(a) **Attractive Force:** On increasing x, if U increases, then $\frac{dU}{dx} = \text{positive}$ i.e. F is negative and hence force is attractive in nature. In graph this is represented in region BC.

(b) **Repulsive Force:** On increasing x, if U decreases, then $\frac{dU}{dx} = \text{negative}$, i.e. F is positive in direction i.e. force is repulsive in nature. In graph this is represented in region AB.

(c) **Zero force (OR Equilibrium)**: On increasing x, if U does not change, then $\frac{dU}{dx} = 0$ i.e. F is zero

Points B, C and D represent the point of zero force or equilibrium is attained by the particle at these points.

TYPES OF EQUILIBRIUM

If net force acting on a particle is zero, it is said to be in equilibrium. Since the force acting on a particle is

$$F = -\frac{dU}{dx}$$

So, for equilibrium, we have

$$\frac{dU}{dx} = 0$$

However, equilibrium of particle can be of three types as explained.

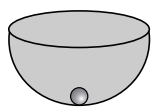
Stable

When a particle is displaced slightly from a position, then a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position. In stable equilibrium, the potential energy is minimum i.e.

$$F = -\frac{dU}{dx} = 0 \text{ and } \frac{d^2U}{dx^2} = \text{positive}$$

i.e. rate of change of $\frac{dU}{dx}$ is positive.

EXAMPLE:



A marble placed at the bottom of a hemispherical bowl.

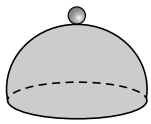
Unstable

When a particle is displaced slightly from a position, then a force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium in which potential energy is maximum, i.e. when

$$F = -\frac{dU}{dx} = 0 \text{ and } \frac{d^2U}{dx^2} = \text{negative}$$

i.e. rate of change of $\frac{dU}{dx}$ is negative.

EXAMPLE:



A marble balanced on top of a hemispherical bowl.

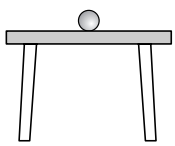
Neutral

When a particle is slightly displaced from a position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium in which potential energy is constant, i.e. when

$$F = -\frac{dU}{dx} = 0 \text{ and } \frac{d^2U}{dx^2} = 0$$

i.e. rate of change of $\frac{dU}{dx}$ is zero.

EXAMPLE:



A marble placed on horizontal table.

ILLUSTRATION 50

The potential energy of a conservative system is given by $U = ax^2 - bx$, where a and b are positive constants. Find the equilibrium position and discuss whether the equilibrium is stable, unstable or neutral.

SOLUTION

In a conservative field $F = -\frac{dU}{dx}$

$$\Rightarrow \quad F = -\frac{d}{dx}(ax^2 - bx) = b - 2ax$$

For equilibrium $F = 0$

$$\Rightarrow \quad b - 2ax = 0$$

$$\Rightarrow \quad x = \frac{b}{2a}$$

From the given equation we can see that $\frac{d^2U}{dx^2} = 2a$ (positive), i.e., U is minimum. Therefore, $x = \frac{b}{2a}$ is the stable equilibrium position.

ILLUSTRATION 51

The potential energy of a particle of mass 1 kg free to move along x-axis is given by $U(x) = \left(\frac{x^2}{2} - x\right)$ joule. If total mechanical energy of the particle is 2 J, then calculate the maximum speed of the particle. Ignore all other forces and assume that the conservative force corresponding to above potential energy is only acting on particle.

SOLUTION

The total mechanical energy of the particle at any instant is sum of kinetic and potential energy of the particle. So, we have

$$\text{KE} + \text{PE} = 2\text{ J}$$

$$\Rightarrow \quad \frac{1}{2}mv^2 + \left(\frac{x^2}{2} - x\right) = 2$$

$$\Rightarrow \quad v^2 = 2x - x^2 + 4 \qquad \ldots(1)$$

For maximum speed v, we have

$$\frac{d}{dx}(v^2) = 0$$

$$\Rightarrow \quad 2 - 2x = 0$$

$$\Rightarrow \quad x = 1\text{ m}$$

So, the speed is maximum at $x = 1\text{ m}$ and this maximum speed is obtained by substituting $x = 1\text{ m}$ in equation (1).

$$v_{\max}^2 = 2 - 1 + 4$$

$$\Rightarrow \quad v_{\max} = \sqrt{5}\text{ ms}^{-1}$$

Test Your Concepts-V

Based on Relation Between Conservative Force and Potential Energy and Types of Equilibrium

(Solutions on page H.15)

1. The potential energy function of a particle in a region of space is given by $U=(2xy+yz)$ J, where x, y and z are in metre. Calculate the force acting on the particle at a general point $P(x, y, z)$.

2. Suppose that the potential energy U in some region of space is given by

$$U(x, y, z)=U_0\exp(-az)\cos(ax),$$

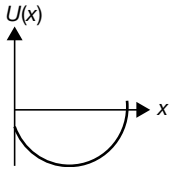
where a is a positive constant. Find the force corresponding to this potential energy.

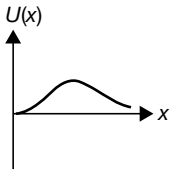
3. Find the potential energy function $U(x, y)$ corresponding to a force $\vec{F}=2axy\hat{i}+a(x^2-y^2)\hat{j}$ where a is a constant. Assume that the potential energy at the origin is U_0.

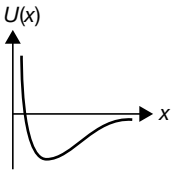
4. A force is given by $\vec{F}=-4(y\hat{i}+x\hat{j})$. Calculate the potential energy at (2, 3) m assuming that the potential energy at the origin is –4 J.

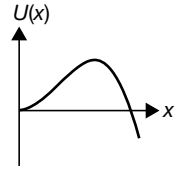
5. A particle is moving along x-axis has potential energy $U=(2-20x+5x^2)$ joule where x is in metre. The particle is released at $x=-3$ m. Calculate the maximum value of x.

6. A particle, which is constrained to move along x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x)=-kx+ax^3$. Here, k and a are positive constant. For $x\geq 0$, the functional form of the potential energy $U(x)$ of the particle is

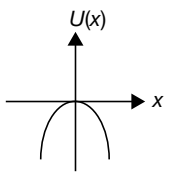
(A) 

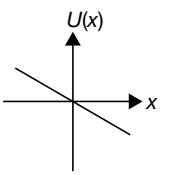
(B) 

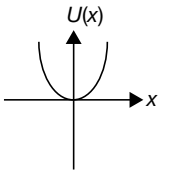
(C) 

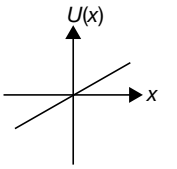
(D) 

7. A particle is placed at the origin and a force $F=kx$ is acting on it (where k is a positive constant). If $U(0)=0$, the graph of $U(x)$ versus x will be : (where U is the potential energy function)

(A) 

(B) 

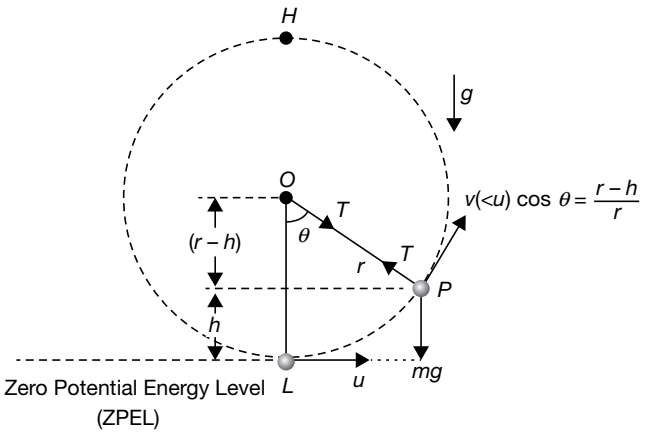
(C) 

(D) 

MOTION IN A VERTICAL CIRCLE

MOTION IN A VERTICAL CIRCLE

Consider a particle of mass m attached to a string of length r as shown in Figure.



Let the particle be whirled in a vertical circle (of radius r), then the speed of the particle is different at different points of the vertical circular path. Due to this, the centripetal force on the particle and hence the tension in the string changes continuously. Let the particle have speed u at the lowest point L, v at any point P and v_H at the highest point H. At the point P, the following forces act on the particle.

(i) the weight of the particle, mg, acting vertically downwards.
(ii) the tension T, in the string.

VELOCITY AT THE POINT P

Let us consider an intermediate point P (between L and H), such that it is at a vertical height h from the lowest point. If this point makes an angle θ with the initial position of the string, then we have

$$\cos\theta = \frac{r-h}{r}$$

Assuming the zero potential energy level (ZPEL) line to pass through L (as shown) and using the Law of Conservation of Mechanical Energy, we get

Version I	**Version II**
Loss in KE = Gain in PE	$(U+K)_L = (U+K)_P$
$\Rightarrow \quad \frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgh$	$0 + \frac{1}{2}mu^2 = mgh + \frac{1}{2}mv^2$

So, from both we get

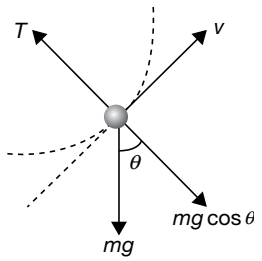
$$v^2 = u^2 - 2gh \qquad \ldots(1)$$

$$\Rightarrow \quad v = \sqrt{u^2 - 2gh} \qquad \ldots(2)$$

TENSION IN THE STRING AT ANY POINT P

The net force, $T - mg\cos\theta$, acting radially inwards is responsible for providing the centripetal force to the particle to move in a circle of radius r. So, we have

$$T - mg\cos\theta = \frac{mv^2}{r} \qquad \ldots(3)$$



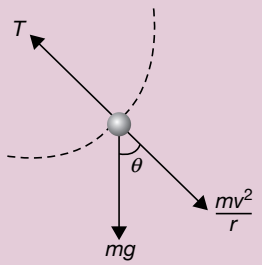
Inertial Frame

Since $\cos\theta = \frac{r-h}{r}$ and $v^2 = u^2 - 2gh$, so we get

$$T - mg\left(\frac{r-h}{r}\right) = \frac{m}{r}\left(u^2 - 2gh\right)$$

$$\Rightarrow \quad T = \frac{m}{r}\left(u^2 - 3gh + gr\right) \qquad \ldots(4)$$

We could have obtained the same result when observed from the frame attached to the particle i.e., a non-inertial frame as shown. From the frame attached to the particle it appears as if the particle is in equilibrium, so we have



$$T = mg\cos\theta + \frac{mv^2}{r} \qquad \{\text{same as (3)}\}$$

TENSION AT THE LOWEST POINT *L*

At the lowest point L, $h = 0$, so we get

$$T_L = \frac{m}{r}\left(u^2 + gr\right) \qquad \ldots(5)$$

TENSION AT THE HIGHEST POINT *H*

At the highest point H, $h = 2r$, so we get

$$T_H = \frac{m}{r}\left(u^2 - 5gr\right) \qquad \ldots(6)$$

So, from (5) and (6), we observe that

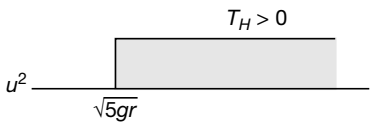
$$T_L - T_H = 6mg \qquad \ldots(7)$$

CONDITION FOR LOOPING THE LOOP

For "looping the loop" i.e., to just complete the loop, we must have

$$T_H \geq 0$$

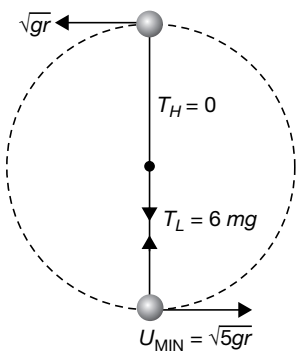
$$\Rightarrow \quad u^2 - 5gr \geq 0$$



$$\Rightarrow \quad u = u_{MIN} = \sqrt{5gr}$$

So, the minimum velocity (with which the particle should be launched from the lowest point) such that it just completes the circle is

$$u_{MIN} = \sqrt{5gr} \qquad \ldots(8)$$



Corresponding to this minimum velocity $u_{MIN} = \sqrt{5gr}$, velocity at maximum height H is obtained by putting $h = 2r$ in $v = \sqrt{u^2 - 2gh}$, so

$$v = \sqrt{5gr - 2g(2r)} = \sqrt{gr}$$

TENSION AND VELOCITY AT THE POINT *M* (MIDWAY BETWEEN *L* AND *H*)

At the point M, midway between L and H, we have $h = r$, so we get

$$v_M = \sqrt{u^2 - 2gr} \text{ and } T_M = \frac{m}{r}\left(u^2 - 2gr\right)$$

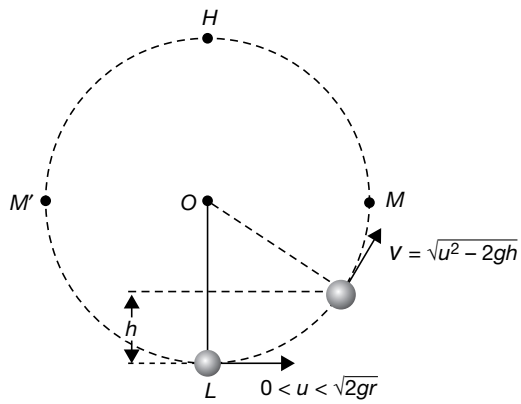
Now, from these equations, we **observe that the particle will reach** M only when

$$u^2 \geq 2gr$$

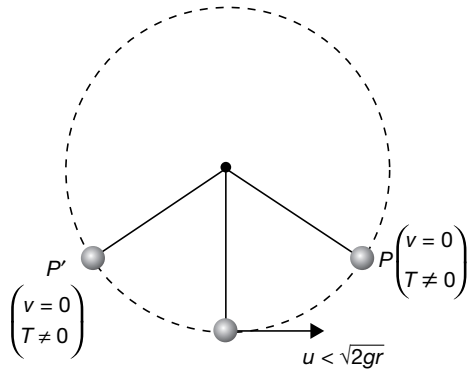
$$\Rightarrow \quad u \geq \sqrt{2gr}$$

FOR $0 < u < \sqrt{2gr}$ (BETWEEN L AND M)

For $u < \sqrt{2gr}$, the particle will not reach M and will oscillate between M and M'.



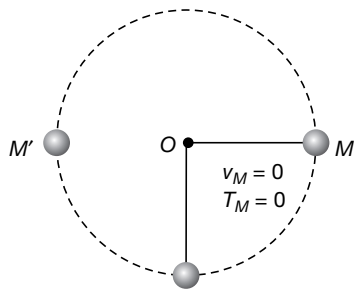
So, for $u < \sqrt{2gr}$, it will oscillate between P and P' with $v = 0$ at P (and P') and also everywhere between P and P' we observe $T \neq 0$.



i.e., for $u < \sqrt{2gr}$ we observe that v becomes zero earlier than T and hence the motion is reversed at P (or P') when $v = 0$ i.e., the point of reversal of motion.

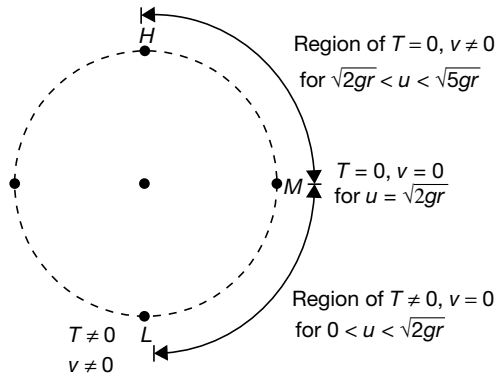
FOR $u = \sqrt{2gr}$

For $u = \sqrt{2gr}$, the particle will just be able to reach M (or M') i.e., it will still now oscillate between M and M'. Significantly, we observe that at M (or M') we have $v_M = 0$ as well as $T_M = 0$ i.e., $v_M = v_{M'} = 0$ and $T_M = T_{M'} = 0$

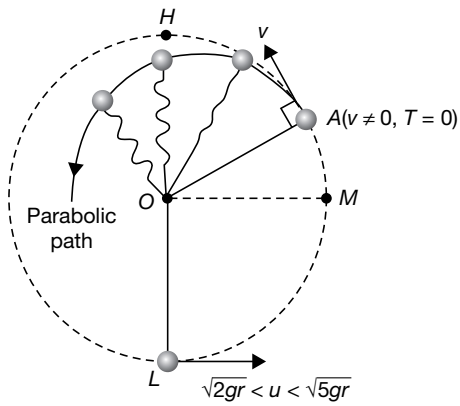


FOR $\sqrt{2gr} < u < \sqrt{5gr}$ (BETWEEN M AND H)

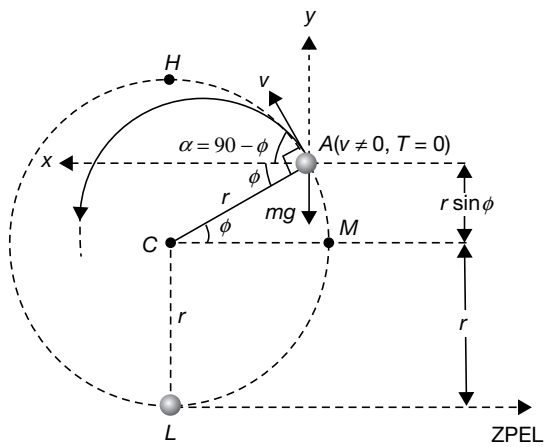
For this region we observe that T becomes zero before v (as discussed and concluded from the previous discussion). Hence for this region, we observe a point, say A, such that at this point A, tension in the string is zero (just as if it was absent) and the velocity of the particle at this point is non-zero.



Due to this non-zero velocity of the particle (and the string being absent, $T = 0$) at A, the particle behaves like a projectile, launched from A (with a non-zero initial velocity) and is then free to move under the influence of gravity.



Let the origin be now fixed at A as shown.

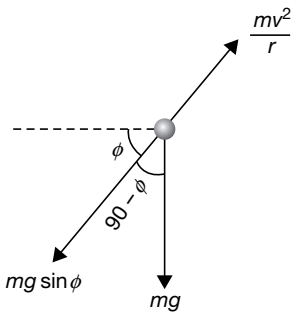


Applying Law of Conservation of Energy, we get

$$(U+K)_{\text{at } L} = (U+K)_{\text{at } A}$$

$$\Rightarrow \quad 0+\frac{1}{2}mu^2 = mg(r+r\sin\phi)+\frac{1}{2}mv^2 \qquad \ldots(1)$$

Now, at A, we have $T=0$, so we get



$$mg\sin\phi = \frac{mv^2}{r}$$

$$\Rightarrow \quad v^2 = gr\sin\phi \qquad \ldots(2)$$

Further for an oblique projectile launched from A, with initial velocity v at an angle of projection $\alpha(=90-\phi)$ we have

$$y = x\tan\alpha - \frac{gx^2}{2v^2\cos^2\alpha} \qquad \ldots(3)$$

$$\Rightarrow \quad y = x\tan(90-\phi) - \frac{gx^2}{2v^2\cos^2(90-\phi)}$$

$$\Rightarrow \quad y = x\cot\phi - \frac{gx^2}{2(gl\sin\phi)\sin^2\phi}$$

$$\Rightarrow \quad y = x\cot\phi - \frac{gx^2}{2gl\sin^3\phi} \qquad \ldots(4)$$

Now, if we asked to find the condition for the projectile to pass through (say),

(a) the point of suspension C, then we find the coordinates of $C(r\cos\phi,\ -r\sin\phi)$ and use equations (1), (2) and (3) (or (4)) to get the desired results.

(b) the lowest point L, then we find the coordinates of $L(r\cos\phi, -r(1+\sin\phi))$ and again make use of equations (1), (2) and (3) (or (4)) to get the desired conditions or results.

The following ILLUSTRATIONS make use of the fundamentals discussed above to have a smooth understanding of the concept.

ILLUSTRATION 52

A body weighing 0.4 kg is whirled in a vertical circle with a string making 2 revolutions per second. If the radius of the circle is 1.2 m. Find the tension

(a) at the top of the circle,
(b) at the bottom of the circle.

Given: $g = 10\ \text{ms}^{-2}$ and $\pi = 3.14$

SOLUTION

Angular velocity

$$\omega = 2 \text{ rev/s} = 4\pi \text{ rad s}^{-1} = 12.56 \text{ rad s}^{-1}$$

$$\{\because 1 \text{ rev} = 2\pi \text{ radian}\}$$

(a) At the top of the circle

$$T = \frac{mv^2}{r} - mg$$

$$\Rightarrow \quad T = mr\omega^2 - mg = m(r\omega^2 - g)$$

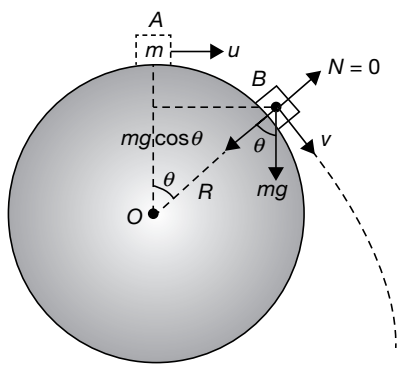
$$\Rightarrow \quad T = 0.4(1.2\times12.56\times12.56-9.8)\text{ N} = 71.2\text{ N}$$

(b) At the lowest point

$$T = m(r\omega^2+g) = 80\text{ N}$$

MOTION OF A BODY ON A SPHERICAL SURFACE

Consider a small body of mass m, placed at the top of a spherical fixed smooth surface of radius R. When the body is given a horizontal velocity u, then it moves some distance in circular path along the spherical surface and at some point B, it breaks off the surface i.e. loses contact with the surface $(N=0)$ and follows the projectile trajectory i.e. a parabolic path. As shown in Figure.



Let us first calculate the velocity of the body at point B, when it makes an angle θ with the vertical position. This can be done by applying the Law of Conservation of Mechanical Energy, according to which, we have

$$\begin{pmatrix}\text{Loss in GPE}\\ \text{of the Body}\end{pmatrix}=\begin{pmatrix}\text{Gain in KE}\\ \text{of the Body}\end{pmatrix}$$

$$\Rightarrow \quad mgR(1-\cos\theta)=\frac{1}{2}mv^2-\frac{1}{2}mu^2$$

$$\Rightarrow \quad \frac{1}{2}mv^2=\frac{1}{2}mu^2+mgR(1-\cos\theta)$$

$$\Rightarrow \quad v=\sqrt{u^2+2gR(1-\cos\theta)} \qquad \ldots(1)$$

When the body breaks off the surface, then the force applied on the block by the sphere is zero i.e. $N=0$. At this point, the component of weight $mg\cos\theta$ acting towards the centre of the circle provides the necessary centripetal force to the body to move in a circular path, so we have

$$mg\cos\theta=\frac{mv^2}{R}$$

Substituting the value of v from Equation (1), we get

$$gR\cos\theta=u^2+2gR(1-\cos\theta)$$

$$\Rightarrow \quad 3gR\cos\theta=u^2+2gR$$

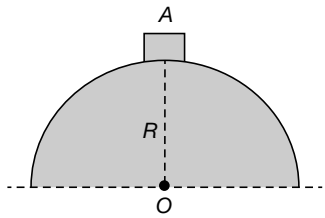
$$\Rightarrow \quad \cos\theta=\frac{u^2+2gR}{3gR}$$

If the horizontal velocity u i.e. the velocity of projection of the body is given, then we can get the angle θ with the vertical where the body leaves the spherical surface and starts moving as a projectile.

ILLUSTRATION 53

A block of mass m is released from the top of a frictionless fixed hemisphere as shown. Find

(a) the angle with the vertical where it breaks off.
(b) the velocity at the instant when it breaks off.
(c) the height where it breaks off.

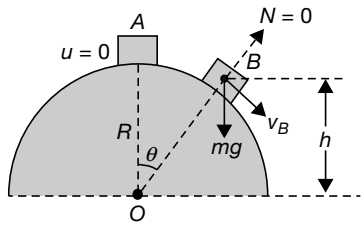


SOLUTION

At B, $N=0$

$$\Rightarrow \quad mg\cos\theta=\frac{mv_B^2}{R}$$

$$\Rightarrow \quad v_B=\sqrt{gR\cos\theta} \qquad \ldots(1)$$



Now by equation of energy between A and B we have,

$$0+mgR=\frac{1}{2}mv_B^2+mgh$$

Substituting v_B from (1) and $h=R\cos\theta$, we get

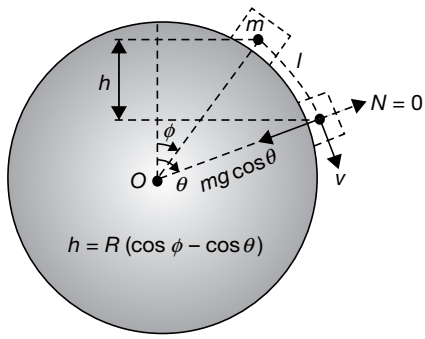
$$v_B=\sqrt{\frac{2}{3}gR} \text{ and } h=\frac{2R}{3} \text{ from the bottom}$$

ILLUSTRATION 54

A small block of mass m is placed on the outer surface of a smooth fixed sphere of radius R at a point where the radius makes an angle ϕ with the vertical. The block is given a small push such that it starts moving on the surface of the sphere. Calculate the distance travelled by the block before it leaves contact with the sphere.

SOLUTION

When the block starts falling along the circular path, then at the instant of breaking off the surface i.e. when the normal reaction force between the block and the sphere is zero. Let θ be the angle which radius makes with the vertical at the instant when block leaves the surface as shown in Figure.



The necessary centripetal force is provided to the block by the component of weight $mg\cos\theta$ i.e.

$$\frac{mv^2}{R} = mg\cos\theta$$

$$\Rightarrow \quad v = \sqrt{Rg\cos\theta} \qquad \ldots(1)$$

where v is the instantaneous velocity of the block at an angle θ from the vertical. As the block falls from rest through $h = R(\cos\phi - \cos\theta)$, then by Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in GPE}\\ \text{of the Body}\end{pmatrix} = \begin{pmatrix}\text{Gain in KE}\\ \text{of the Body}\end{pmatrix}$$

$$\Rightarrow \quad mgh = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{2gh}$$

$$\Rightarrow \quad v = \sqrt{2gR(\cos\phi - \cos\theta)} \qquad \ldots(2)$$

Equating (1) and (2), we get

$$\sqrt{Rg\cos\theta} = \sqrt{2gR(\cos\phi - \cos\theta)}$$

$$\Rightarrow \quad 3\cos\theta = 2\cos\phi$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{2}{3}\cos\phi\right)$$

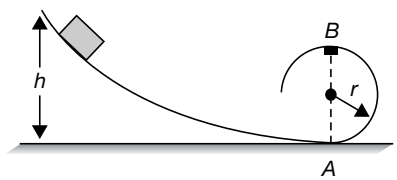
Thus distance l travelled by the box before leaving the contact with the sphere is

$$l = R(\theta - \phi)$$

$$\Rightarrow \quad l = R\left[\cos^{-1}\left(\frac{2}{3}\cos\phi\right) - \phi\right]$$

ILLUSTRATION 55

A block is released from the top of a smooth vertical track, which ends in a circle of radius r as shown in figure.



(a) Find the minimum value of h so that the block completes the circle.

(b) If $h = 3r$, find normal reaction when the block is at the points A and B.

(c) If $h = 2r$, find the velocity of the block when it loses the contact with the track.

SOLUTION

(a) For completing the circle, velocity at lowest point of circle (say A) is $\sqrt{5gr}$ from energy conservation

$$mgh = \frac{1}{2}m\left(\sqrt{5gr}\right)^2$$

$$\Rightarrow \quad h = \frac{5r}{2}$$

(b) $h = 3r$

From energy conservation velocity at point A and B are

$$mg(3r) = \frac{1}{2}mv_A^2$$

$$\Rightarrow \quad v_A = \sqrt{6gr}$$

$$\Rightarrow \quad mg(3r) = mg2r + \frac{1}{2}mv_B^2$$

$$\Rightarrow \quad v_B = \sqrt{2gr}$$

Therefore, normal reaction at A and B is

$$N_A - mg = \frac{mv_A^2}{r}$$

$$\Rightarrow \quad N_A = 7mg$$

$$\Rightarrow \quad N_B + mg = \frac{mv_B^2}{r}$$

$$\Rightarrow \quad N_B = mg$$

(c) $h = 2r$

It loses contact with the track when normal reaction is zero

$$\frac{mv^2}{r} = mg\cos\theta \qquad \ldots(1)$$

From energy conservation

$$mgh = mgr(1 + \cos\theta) + \frac{1}{2}mv^2 \qquad \ldots(2)$$

From (1) and (2)

$$v = \sqrt{\frac{2g(h-r)}{3}} = \sqrt{\frac{2gr}{3}}$$

ILLUSTRATION 56

A heavy particle is suspended by a string of length l from a fixed point O. The particle is given a horizontal velocity v_0. The string slacks at some angle and the particle proceeds on a parabola. Find the value v_0 if the particle passes through the point of suspension.

SOLUTION

Let the string slacks at point B as shown in figure.

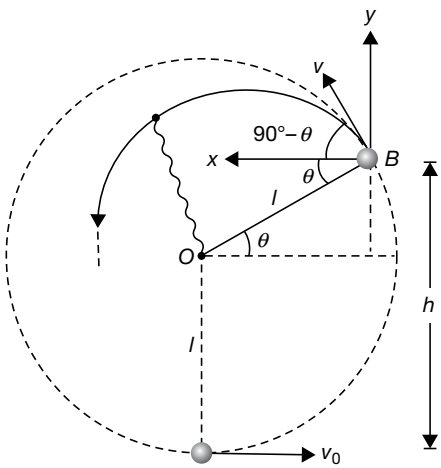
$$h = l + l\sin\theta = l(1+\sin\theta) \quad \ldots(1)$$

Applying Conservation of Mechanical Energy between points A and B, we get

$$v^2 = v_0^2 - 2gh \quad \ldots(2)$$

At point B, the string slacks, so we have

$$T = 0 \text{ and hence } mg\sin\theta = \frac{mv^2}{l}$$



$$\Rightarrow \quad v^2 = gl\sin\theta \quad \ldots(3)$$

After B, path of the particle is a projectile and it passes through O. Co-ordinate of point O with origin at B and x and y axes as shown in figure are:

$$(x, y) = (l\cos\theta, -l\sin\theta)$$

Angle of projection of the particle is $\alpha = 90° - \theta$ and the velocity of projection is v. So, substituting the above data in equation of an oblique projectile, i.e.,

$$y = x\tan\alpha - \frac{gx^2}{2v^2\cos^2\alpha}$$

we get,

$$-l\sin\theta = (l\cos\theta)\tan(90°-\theta) - \frac{g(l\cos\theta)^2}{2v^2\cos^2(90°-\theta)}$$

$$\Rightarrow \quad -l\sin\theta = \frac{l\cos^2\theta}{\sin\theta} - \frac{gl^2\cos^2\theta}{2v^2\sin^2\theta}$$

Substituting, $v^2 = gl\sin\theta$ from Equation (3), we get

$$l\sin\theta = \frac{gl^2\cos^2\theta}{2(gl\sin\theta)(\sin^2\theta)} - \frac{l\cos^2\theta}{\sin\theta}$$

$$\Rightarrow \quad 2\sin^4\theta = \cos^2\theta - 2\sin^2\theta\cos^2\theta$$

$$\Rightarrow \quad 2\sin^4\theta = (1-\sin^2\theta) - 2\sin^2\theta(1-\sin^2\theta)$$

$$\Rightarrow \quad 3\sin^2\theta = 1$$

$$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{3}}$$

From Equation (2), we get

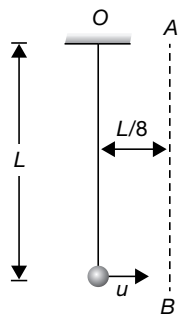
$$v_0^2 = v^2 + 2gh = (gl\sin\theta) + 2gl(1+\sin\theta)$$

$$\Rightarrow \quad v_0^2 = gl(2+3\sin\theta) = gl(2+\sqrt{3})$$

$$\Rightarrow \quad v_0 = \sqrt{gl(2+\sqrt{3})}$$

ILLUSTRATION 57

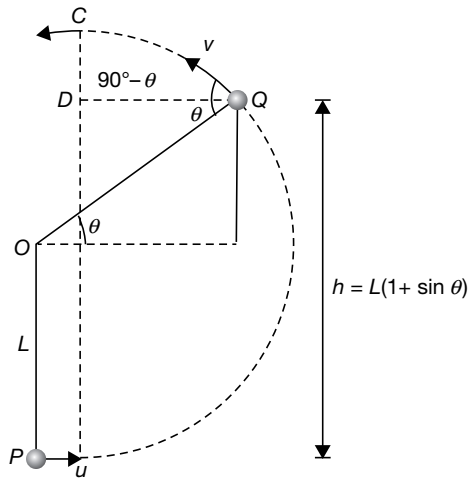
A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance of $\frac{L}{8}$ from O as shown.



The particle is given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find u.

SOLUTION

Let the string slacks at point Q as shown in figure. From P to Q path is circular and beyond Q, path is parabolic. At point C, velocity of particle becomes horizontal, therefore, QD is half of the range of the projectile.



Now we have following equations:

(a) $T_Q = 0$

$$\Rightarrow \quad mg\sin\theta = \frac{mv^2}{L} \qquad \ldots(1)$$

(b) $v^2 = u^2 - 2gh$

$$v^2 = u^2 - 2gL(1+\sin\theta) \qquad \ldots(2)$$

(c) $QD = \frac{1}{2}(\text{Range})$

$$\Rightarrow \quad \left(L\cos\theta - \frac{L}{8}\right) = \frac{v^2\sin 2(90^\circ-\theta)}{2g}$$

$$\Rightarrow \quad L\left(\cos\theta - \frac{1}{8}\right) = \frac{v^2\sin 2\theta}{2g} \qquad \ldots(3)$$

Equation (3) can be written as

$$\left(\cos\theta - \frac{1}{8}\right) = \left(\frac{v^2}{gL}\right)\sin\theta\cos\theta$$

From Equation (1), substituting value of $\left(\frac{v^2}{gL}\right) = \sin\theta$, we get

$$\left(\cos\theta - \frac{1}{8}\right) = \sin^2\theta\cos\theta = (1-\cos^2\theta)\cos\theta$$

$$\Rightarrow \quad \cos\theta - \frac{1}{8} = \cos\theta - \cos^3\theta$$

$$\Rightarrow \quad \cos^3\theta = \frac{1}{8}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60^\circ$$

From Equation (1), we get

$$v^2 = gL\sin\theta = gL\sin 60^\circ$$

$$\Rightarrow \quad v^2 = \frac{\sqrt{3}}{2}gL$$

Substituting this value of v^2 and $\theta = 60^\circ$ in Equation (2), we get

$$u^2 = v^2 + 2gL(1+\sin\theta)$$

$$\Rightarrow \quad u^2 = \frac{\sqrt{3}}{2}gL + 2gL\left(1+\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad u^2 = \frac{3\sqrt{3}}{2}gL + 2gL = gL\left(2+\frac{3\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad u = \sqrt{gL\left(2+\frac{3\sqrt{3}}{2}\right)} \approx 2.14\sqrt{gL}$$

Conceptual Note(s)

Since $u = 2.14\sqrt{gL} = \sqrt{4.6gL}$

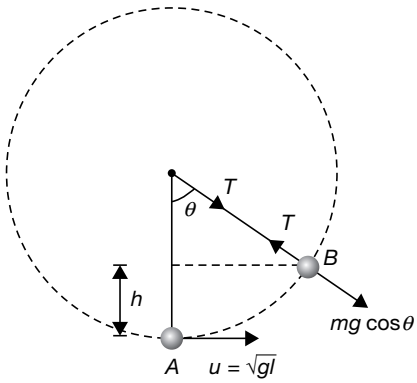
This value of u lies between $\sqrt{2gL}$ and $\sqrt{5gL}$

ILLUSTRATION 58

A heavy particle hanging from a fixed point by a light inextensible string of length l is projected horizontally with speed $\sqrt{gl}$. Find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle.

SOLUTION

Let $T = mg$ at angle θ as shown in figure



$$h = l(1-\cos\theta) \qquad \ldots(1)$$

Applying Conservation of Mechanical Energy between points A and B, we get

$$\frac{1}{2}m(u^2 - v^2) = mgh$$

Here, $u^2 = gl$ $\ldots(2)$

and $v =$ speed of particle in position B

$$\Rightarrow \quad v^2 = u^2 - 2gh \qquad \ldots(3)$$

Further, $T - mg\cos\theta = \frac{mv^2}{l}$

$$\Rightarrow \quad mg - mg\cos\theta = \frac{mv^2}{l} \qquad (T = mg)$$

$$\Rightarrow \quad v^2 = gl(1-\cos\theta) \qquad \ldots(4)$$

Substituting values of v^2, u^2 and h from Equations (4), (2) and (1) in Equation (3), we get

$$gl(1-\cos\theta) = gl - 2gl(1-\cos\theta)$$

$$\Rightarrow \quad \cos\theta = \frac{2}{3}$$

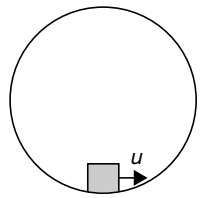
$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Substituting $\cos\theta = \frac{2}{3}$ in Equation (4), we get

$$v = \sqrt{\frac{gl}{3}}$$

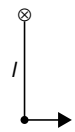
CONDITION FOR LOOPING THE LOOP IN SOME OTHER CASES

CASE-1: A mass moving on a smooth vertical circular track.



Mass moving along a smooth vertical circular loop. Condition for just looping the loop, normal at highest point $= 0$. By calculation similar to article (motion in vertical circle). Minimum horizontal velocity at lowest point is $u_{\min} = \sqrt{5gl}$.

CASE 2: A particle attached to a light rod rotated in vertical circle.

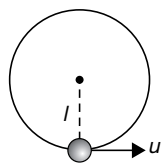


Condition for just looping the loop, velocity $v = 0$ at highest point (even if tension is zero, rod won't slack and a compressive force can appear in the rod).
By energy conservation, velocity at lowest point

$$\frac{1}{2}mu_{\min}^2 = mg(2l)$$

$$\Rightarrow \quad u_{\min} = \sqrt{4gl} \qquad \{\text{for completing the circle}\}$$

CASE 3: A bead attached to a ring and rotated.

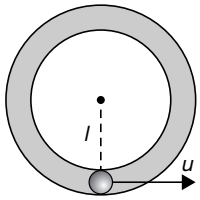


Condition for just looping the loop, velocity $v = 0$ at highest point (even if normal is zero, the bead will not lose contact with the track, normal can act radially outward).
By energy conservation, velocity at lowest point

$$\frac{1}{2}mu_{\min}^2 = mg(2l)$$

$$\Rightarrow \quad u_{\min} = \sqrt{4gl} \qquad \{\text{for completing the circle}\}$$

CASE 4: A block rotated between smooth surfaces of a pipe.



Condition for just looping the loop, velocity $v = 0$ at highest point (even if normal is zero, the bead will not lose contact with the track, normal can act radially outward).

By energy conservation, we have

$$\frac{1}{2}mu_{\min}^2 = mg(2l)$$

$$\Rightarrow \quad u_{\min} = \sqrt{4gl} \qquad \{\text{for completing the circle}\}$$

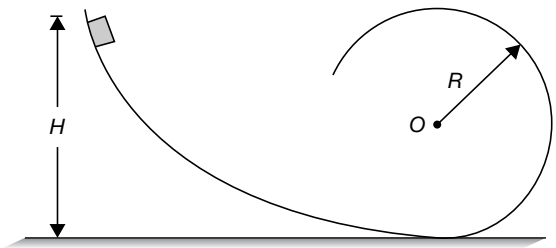
So, the minimum speed, at the top, required to perform a vertical loop is $15.65\ \text{ms}^{-1}$.

Test Your Concepts-VI

Based on Vertical Circle

(Solutions on page H.16)

1. A small body is allowed to slide on a frictionless track from rest position as shown in figure. What must be the minimum height in terms of *R*, so that body may successfully complete the loop.



2. A child revolves a stone of mass 0.5 kg tied to the end of a string of length 40 cm in a vertical circle. The speed of the stone at the lowest point of the circle is $3\ \text{ms}^{-1}$. Calculate the tension in the string at this point.

3. The bob of a simple pendulum of length $L = 2\text{m}$ has a mass $m = 2$ kg and a speed $u = 1\ \text{ms}^{-1}$ when the string is at 37° to the vertical. Find the tension in the string at

(a) the lowest point in its swing

(b) the highest point

Given $\cos(37°) = 0.8$

4. The bob of a 2 m pendulum describes an arc of circle in a vertical plane. If the tension in the cord is 2.5 times the weight of the bob for the position shown, calculate the velocity and the acceleration of the bob in that position.

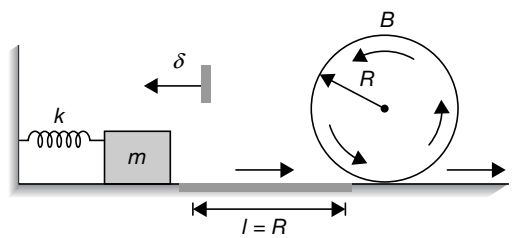
5. A small block with a mass of 0.8 kg is attached to a cord passing through a hole in a horizontal frictionless surface. The block is originally revolving at a distance of 0.4 m from the hole with a speed of $1\ \text{ms}^{-1}$. The cord is then pulled from below, shortening the radius of the circle in which the block revolves to 0.2 m. At this new distance the speed of the block is observed to be $4\ \text{ms}^{-1}$. Calculate the

(a) tension in the cord in the original situation when the block has speed $v = 1\ \text{ms}^{-1}$.

(b) tension in the cord in the final situation when the block has speed $v = 4\ \text{ms}^{-1}$.

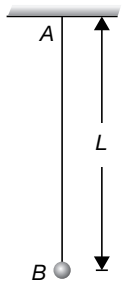
(c) work that had to be done by the person pulling the cord.

6. A spring of spring constant *k* is compressed and suddenly released, sending the particle of mass *m* sliding along the track shown. Determine the minimum spring compression δ for which the particle will not lose contact with looping the loop track. The sliding surface is smooth except for the rough portion of length *l* equal to radius of the loop, *R*, where the coefficient of kinetic friction is μ_k.

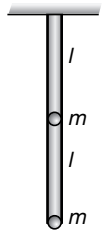


7. A particle is projected along the inside of a vertical hoop of radius r from its lowest point with such a velocity that it leaves the hoop and returns to the point of projection again. Find the velocity of projection and determine where the particle leaves the hoop.

8. A bob B of mass m of the pendulum AB is given an initial velocity $\sqrt{3gL}$ in the horizontal direction. Calculate the maximum height of the bob from the starting point if AB happens to be a very light
(a) rod.
(b) string.



9. Two point masses each of mass m are connected to the light rod of length l which is free to rotate in a vertical plane as shown. Calculate the minimum horizontal velocity is given to mass so that it completes the circular motion in vertical plane.



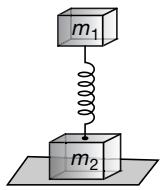
10. You may have seen in a circus a motorcyclist driving in vertical loops inside a death well (a hollow spherical chamber with holes, so that the cyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m?

11. A stone tied to a string of length 5 m is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at the lowest position and has a speed of 10 ms^{-1}. Calculate the magnitude of the change in velocity as it reaches a position where the string is horizontal. $\left(\text{Take } g = 10 \text{ ms}^{-2}\right)$

SOLVED PROBLEMS

PROBLEM 1

Two plates of masses m_1 and m_2 are connected by a spring of force constant k. What force should be applied to the upper plate such that when the force is removed, the lower plate is just raised? Disregard the mass of the spring.



SOLUTION

Let F be the required force applied on the upper plate in the downward direction. If x_1 be the depression produced in the spring. Then for equilibrium, under the influence of

(a) the downward applied force F,
(b) upward spring force kx and
(c) the weight mg always acting downwards

$$\Rightarrow \quad F + m_1 g = kx_1 \qquad \ldots(1)$$

When released, let the spring extend by x_2. The lower mass will be lifted if

$$kx_2 \geq m_2 g \qquad \ldots(2)$$

Applying the Law of Conservation of Mechanical Energy between the initial and final positions of the spring, we get

$$\begin{pmatrix}\text{Energy}\\ \text{Gained}\\ \text{by Spring}\\ \text{due to}\\ \text{compression}\end{pmatrix} + \begin{pmatrix}\text{Loss}\\ \text{in}\\ \text{GPE}\\ \text{of } m_1\end{pmatrix} = \begin{pmatrix}\text{Energy}\\ \text{Gained}\\ \text{by}\\ \text{Spring}\\ \text{due to}\\ \text{extension}\end{pmatrix} + \begin{pmatrix}\text{Gain}\\ \text{in}\\ \text{GPE}\\ \text{of } m_1\end{pmatrix}$$

Since m_2 stays on the ground, initially and finally, so we have not taken any term that involves the potential energy of the mass m_2.

$$\Rightarrow \quad \frac{1}{2}kx_1^2 - m_1 g x_1 = \frac{1}{2}kx_2^2 + m_1 g x_2$$

$$\Rightarrow \quad \frac{1}{2}kx_1^2 = \frac{1}{2}kx_2^2 + m_1 g(x_1 + x_2)$$

$$\Rightarrow \quad k^2 x_1^2 = k^2 x_2^2 + (2m_1 g x_1 + 2m_1 g x_2)k$$

$$\Rightarrow \quad (kx_1 - m_1 g)^2 = (kx_2 + m_1 g)^2$$

$$\Rightarrow \quad kx_1 - m_1 g = (kx_2 + m_1 g)$$

$$\Rightarrow \quad F + m_1 g - m_1 g = (kx_2 + m_1 g) \qquad \{\text{using (1)}\}$$

$$\Rightarrow \quad F = kx_2 + m_1 g$$

$$\Rightarrow \quad kx_2 = F - m_1 g$$

For lifting the lower mass, $kx_2 \geq m_2 g$

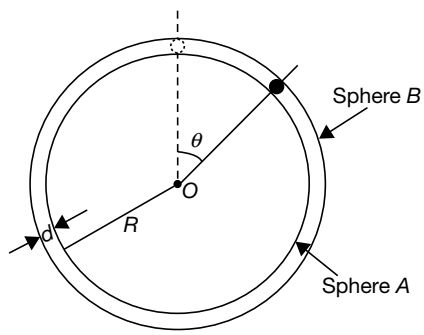
$$\Rightarrow \quad F - m_1 g \geq m_2 g$$

$$\Rightarrow \quad F \geq m_1 g + m_2 g$$

For just lifting, $F = (m_1 + m_2)g$

PROBLEM 2

A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less then d. All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by θ (shown in the figure).



(a) Express the total normal reaction force exerted by the spheres on the ball as a function of angle θ.
(b) Let N_A and N_B denote the magnitudes of the normal reaction forces on the ball exerted by the spheres A and B, respectively. Sketch the

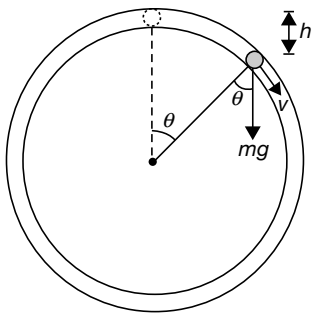
variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two separate graphs, taking $\cos\theta$ on the horizontal axes.

SOLUTION

(a) $h = \left(R + \frac{d}{2}\right)(1 - \cos\theta)$

Velocity of ball at angle θ is

$$v^2 = 2gh = 2\left(R + \frac{d}{2}\right)(1 - \cos\theta)g \qquad \ldots(1)$$



Let N be the total normal reaction (away from centre) at angle θ. Then

$$mg\cos\theta - N = \frac{mv^2}{\left(R + \frac{d}{2}\right)}$$

Substituting value of v^2 from Equation (1), we get

$$mg\cos\theta - N = 2mg(1 - \cos\theta)$$

$$\Rightarrow \quad N = mg(3\cos\theta - 2)$$

(b) The ball will lose contact with the inner sphere when

$$N = 0$$

$$\Rightarrow \quad 3\cos\theta - 2 = 0$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

After this it makes contact with outer sphere and normal reaction starts acting towards the centre.

Thus, for $\theta \le \cos^{-1}\left(\frac{2}{3}\right)$

$$N_B = 0$$

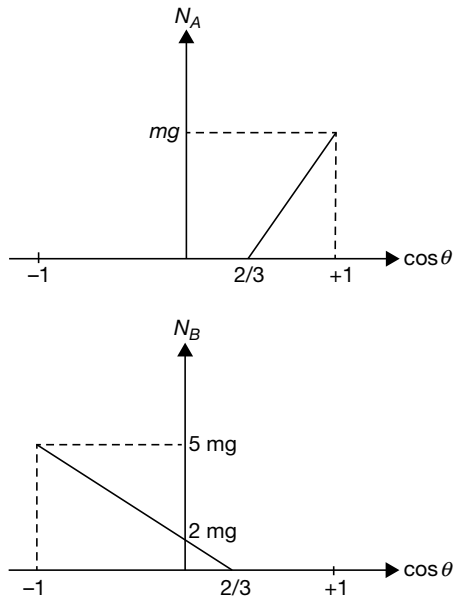
and $N_A = mg(3\cos\theta - 2)$

and for $\theta \ge \cos^{-1}\left(\frac{2}{3}\right)$

$$N_A = 0$$

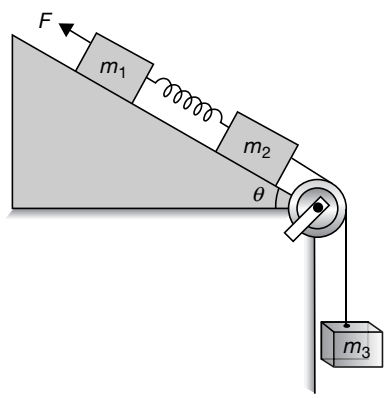
and $N_B = mg(2 - 3\cos\theta)$

The corresponding graphs are as follows:



PROBLEM 3

Two blocks of mass $m_1 = 2$ kg and $m_2 = 4$ kg are attached by light ideal spring of force constant $k = 1000\ \text{Nm}^{-1}$. The system is kept on a smooth inclined plane inclined $30°$ with horizontal. A force $F = 15$ N is applied on m_1 and system is released from rest. The block m_2 is attached with a light string whose another end is connected with a mass $m_3 = 1$ kg. Assume initially the spring is at relaxed position and the system is released from rest. Find the maximum extension of the spring.



SOLUTION

At some instant of time let m_1 be displaced downwards by x_1 and m_2 be displaced downwards by x_2, with $x_2 > x_1$. Elongation of spring at this instant is then given by

$$x = x_2 - x_1$$

Writing equation of motion for m_1, m_2 and m_3, we have,

For m_1, we have

$$m_1 g \sin\theta + kx - F = m_1 a_1$$

$$\Rightarrow \quad a_1 = \frac{m_1 g \sin\theta + kx - F}{m_1} \qquad \ldots(1)$$

For m_2, we have

$$T + m_2 g \sin\theta - kx = m_2 a_2 \qquad \ldots(2)$$

For m_3, we have

$$m_3 g - T = m_3 a_2 \qquad \ldots(3)$$

Adding (2) and (3), we get

$$a_2 = \frac{m_2 g \sin\theta - kx + m_3 g}{m_2 + m_3} \qquad \ldots(4)$$

Acceleration of m_2 relative to m_1 is given by

$$a = a_2 - a_1$$

$$\Rightarrow \quad a = \frac{m_2 g \sin\theta - kx + m_3 g}{(m_2 + m_3)} - \frac{m_1 g \sin\theta + kx - F}{m_1}$$

Substituting the given values, we get

$$a = 8.5 - 700x$$

$$\Rightarrow \quad v\frac{dv}{dx} = 8.5 - 700x$$

$$\Rightarrow \quad \int_0^0 v\,dv = \int_0^{x_{max}} (8.5 - 700x)\,dx$$

$$\Rightarrow \quad x_{max} = 0.024 \text{ m}$$

$$\Rightarrow \quad x_{max} = 2.4 \text{ cm}$$

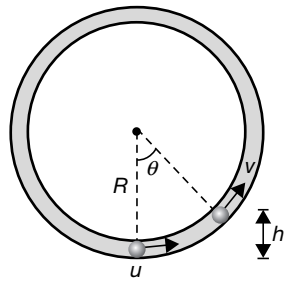
PROBLEM 4

A smooth circular tube of radius R is fixed in a vertical plane. A particle is projected from its lowest point with a velocity just sufficient to carry it to the highest point. Calculate the time taken by the particle to reach the end of the horizontal diameter.

SOLUTION

Minimum velocity of particle at the lowest position to complete the circle should be $\sqrt{4gR}$ inside a tube.

So, $u = \sqrt{4gR}$



At any instant, h has a function of θ is given by

Since, $h = R(1 - \cos\theta)$

$$\Rightarrow \quad v^2 = u^2 - 2gh$$

$$\Rightarrow \quad v^2 = 4gR - 2gR(1 - \cos\theta)$$

$$\Rightarrow \quad v^2 = 2gR(1 + \cos\theta)$$

$$\Rightarrow \quad v^2 = 2gR\left(2\cos^2\left(\frac{\theta}{2}\right)\right)$$

$$\Rightarrow \quad v = 2\sqrt{gR}\cos\left(\frac{\theta}{2}\right)$$

Since $v = \frac{ds}{dt}$

$$\Rightarrow \quad ds = v\,dt$$

Since, $ds = R\,d\theta$

$$\Rightarrow \quad R\,d\theta = 2\sqrt{gR}\cos\left(\frac{\theta}{2}\right)\cdot dt$$

$$\Rightarrow \quad \int_0^t dt = \frac{1}{2}\sqrt{\frac{R}{g}}\int_0^{\pi/2} \sec\left(\frac{\theta}{2}\right)d\theta$$

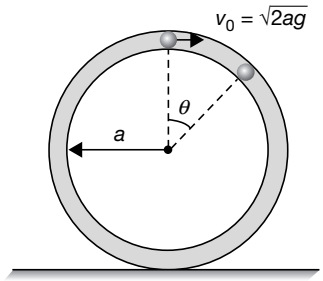
$$\Rightarrow \quad t = \sqrt{\frac{R}{g}}\left[\ln\left(\sec\frac{\theta}{2} + \tan\frac{\theta}{2}\right)\right]_0^{\pi/2}$$

$$\Rightarrow \quad t = \sqrt{\frac{R}{g}}\ln(1 + \sqrt{2})$$

PROBLEM 5

A heavy particle slides under gravity down the inside of a smooth vertical tube held in vertical plane. It starts from the highest point with velocity $\sqrt{2ag}$,

where a is the radius of the circle. Find the angular position θ (as shown in figure) at which the vertical acceleration of the particle is maximum.



SOLUTION

At position θ, we have

$$v^2 = v_0^2 + 2gh$$

where $h = a(1-\cos\theta)$

$$\Rightarrow \quad v^2 = \left(\sqrt{2ag}\right)^2 + 2ag(1-\cos\theta)$$

$$\Rightarrow \quad v^2 = 2ag(2-\cos\theta) \qquad \ldots(1)$$

Also, we have

$$N + mg\cos\theta = \frac{mv^2}{a}$$

$$\Rightarrow \quad N + mg\cos\theta = 2mg(2-\cos\theta)$$

$$\Rightarrow \quad N = mg(4-3\cos\theta)$$

Net vertical force acting on the particle is

$$F = N\cos\theta + mg = mg\left(4\cos\theta - 3\cos^2\theta + 1\right)$$

This force (or acceleration) will be maximum when $\frac{dF}{d\theta} = 0$

$$\Rightarrow \quad -4\sin\theta + 6\sin\theta\cos\theta = 0$$

$$\Rightarrow \quad \sin\theta(-4 + 6\cos\theta) = 0$$

OR

Either $\sin\theta = 0$	$\cos\theta = \frac{2}{3}$
$\Rightarrow \quad \theta = 0°$	$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{2}{3}\right)$
(Not Acceptable)	

Hence, the desired position is at $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

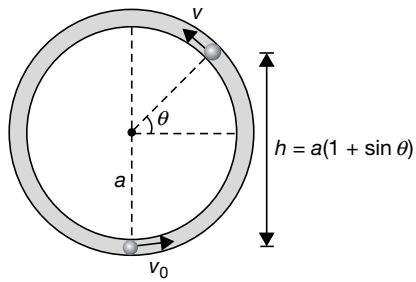
PROBLEM 6

A particle is constrained to move on a smooth vertical circular hoop of radius a. It is projected from the lowest point with velocity just sufficient to carry it to the highest point. Find the time after which the reaction between the particle and the hoop is zero.

SOLUTION

$$v_0 = \sqrt{4ga}$$

$$v^2 = v_0^2 - 2gh = \left(\sqrt{4ga}\right)^2 - 2ga(1+\sin\theta)$$



$$v^2 = 2ga(1-\sin\theta)$$

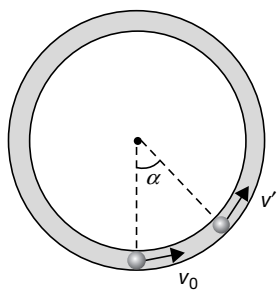
$N = 0$ at $mg\sin\theta = \frac{mv^2}{a}$

$$\Rightarrow \quad mg\sin\theta = 2mg(1-\sin\theta)$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{2}{3}\right)$$

$$\Rightarrow \quad \theta = 41.8° \text{ (with the horizontal)}$$

Now, at angle α, $(v')^2 = v_0^2 - 2ga(1-\cos\alpha)$



$$\Rightarrow \quad (v')^2 = \left(\sqrt{4ga}\right)^2 - 2ga(1-\cos\alpha)$$

$$\Rightarrow \quad (v')^2 = 2ga(1+\cos\alpha) = 4ga\cos^2\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \quad v' = 2\cos\left(\frac{\alpha}{2}\right)\sqrt{ga}$$

$$\Rightarrow \quad \omega = \frac{v'}{a} = 2\cos\left(\frac{\alpha}{2}\right)\sqrt{\frac{g}{a}}$$

$$\Rightarrow \quad \frac{d\alpha}{dt} = 2\cos\left(\frac{\alpha}{2}\right)\sqrt{\frac{g}{a}}$$

$$\Rightarrow \quad \int_{0^\circ}^{\frac{\pi}{2}+\theta} \sec\left(\frac{\alpha}{2}\right)d\alpha = 2\sqrt{\frac{g}{a}}\int_0^t dt$$

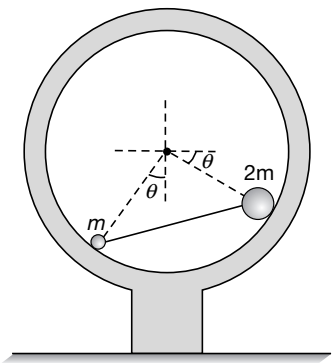
$$\Rightarrow \quad 2\sqrt{\frac{g}{a}}t = 2\ln\left[\tan\left(\frac{\pi}{4}+\frac{\alpha}{4}\right)\right]_{0^\circ}^{\pi/2+\theta}$$

$$\Rightarrow \quad \sqrt{\frac{g}{a}}t = \ln\left[\tan\left(\frac{\pi}{4}+\frac{\pi}{8}+\frac{\theta}{4}\right)\right] - \ln\left[\tan\left(\frac{\pi}{4}\right)\right]$$

$$\Rightarrow \quad t = 1.544\sqrt{\frac{g}{a}}$$

PROBLEM 7

The two particles of mass m and $2m$ respectively are connected by a rigid rod of negligible mass and slide with negligible friction in a circular path of radius r on the inside of the vertical circular ring. If the system is released from rest at $\theta = 0^\circ$ determine



(a) the velocity v of the particles when the rod passes the horizontal position,

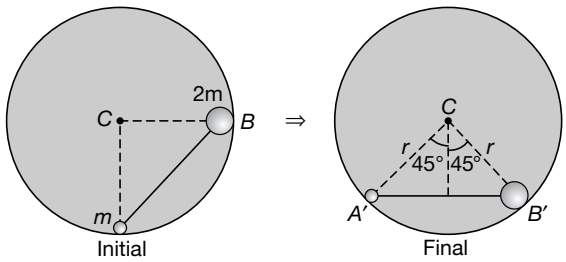
(b) the maximum velocity v_{max} of the particles and

(c) the maximum value of θ.

SOLUTION

(a) We observe that $\angle ACB = 90^\circ$

From constraint relations we conclude that speeds of both the particles will be same.



By Law of Conservation of Mechanical Energy, we get

$$\begin{pmatrix}\text{Decrease in}\\ \text{Potential}\\ \text{Energy of}\\ \text{mass } 2m\end{pmatrix} = \begin{pmatrix}\text{Increase in}\\ \text{Potential}\\ \text{Energy of}\\ \text{mass } m\end{pmatrix} + \begin{pmatrix}\text{Increase in}\\ \text{Kinetic}\\ \text{Energies}\\ \text{of both}\end{pmatrix}$$

$$\Rightarrow \quad 2mg(h_B) = mg(h_A) + \frac{1}{2}(3m)(v^2)$$

$$\Rightarrow \quad 2g(h_B) = g(h_A) + \frac{3}{2}v^2 \qquad \ldots(1)$$

where $h_B = r\cos 45^\circ = 0.707r$

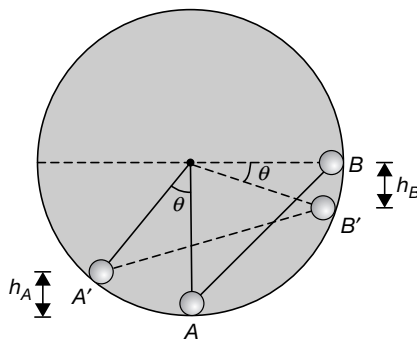
and $h_A = r(1-\cos 45^\circ) = 0.293r$

Substituting the values, we get

$$(2)(0.707)gr = 0.293gr + 1.5v^2$$

$$\Rightarrow \quad v = 0.864\sqrt{gr}$$

(b) At any angle θ, we have



$$h_B = r\sin\theta$$

and $h_A = r(1-\cos\theta)$

Hence equation (1) can be written as

$$2g(r\sin\theta) = gr(1-\cos\theta) + \frac{3}{2}v^2$$

$$\Rightarrow \quad v^2 = \frac{2}{3}gr(2\sin\theta + \cos\theta - 1) \qquad \ldots(2)$$

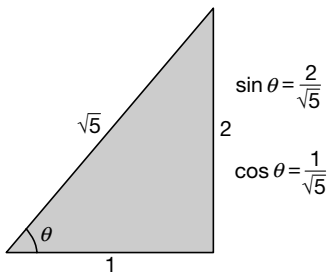
For v to be maximum, we have

$$\frac{d}{d\theta}(v^2)$$

$$\Rightarrow \quad 2\cos\theta - \sin\theta = 0$$

$$\Rightarrow \quad \tan\theta = 2$$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{2}{3}gr(2\sin\theta + \cos\theta - 1)}$$



Substituting the values of θ we get

$$v_{max} = 0.908\sqrt{gr}$$

(c) At $\theta = \theta_{max}$, velocity of both the particles will momentarily be zero. Hence substituting $v^2 = 0$ in equation (2) we get

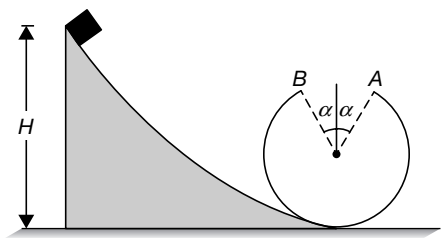
$$2\sin\theta + \cos\theta - 1 = 0$$

Solving this equation, we get

$$\theta_{max} \cong 127^\circ$$

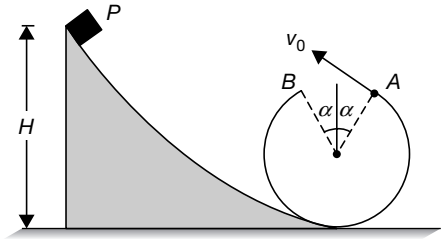
PROBLEM 8

A small object loops a vertical loop of radius R in which a symmetrical section of 2α has been removed. Find the maximum and minimum heights from which the object after loosing contact with the loop at point A and flying through the air, will reach point B. Find the corresponding angles of the section removed for which this is possible.



SOLUTION

Let v_0 be the speed of the object at point A. Between A and B path of the object is a parabola, with



AB = Range

$$\Rightarrow \quad 2R\sin\alpha = \frac{2v_0^2\sin\alpha\cos\alpha}{g}$$

$$\Rightarrow \quad v_0^2 = \frac{gR}{\cos\alpha} \qquad \ldots(1)$$

By Law of Conservation of Mechanical Energy applied at P and A, we get

$$mgH = mgR(1+\cos\alpha) + \frac{1}{2}mv_0^2$$

$$\Rightarrow \quad \frac{H}{R} = 1 + \cos\alpha + \frac{v_0^2}{2gR}$$

Since $\frac{v_0^2}{gR} = \frac{1}{\cos\alpha}$ {from equation (1)}

$$\Rightarrow \quad \frac{H}{R} = 1 + \cos\alpha + \frac{1}{2\cos\alpha} = k \qquad \{\text{say}\}$$

$$\Rightarrow \quad 2\cos^2\alpha - 2(k-1)\cos\alpha + 1 = 0$$

$$\Rightarrow \quad \cos^2\alpha - (k-1)\cos\alpha + \frac{1}{2} = 0$$

$$\Rightarrow \quad \cos\alpha = \frac{1}{2}\left(k - 1 \pm \sqrt{(k-1)^2 - 2}\right)$$

For roots to be real, the discriminant

$$(k-1)^2 - 2 \geq 0$$

$$\Rightarrow \quad k - 1 \geq \sqrt{2}$$

$$\Rightarrow \quad k \geq \left(1 + \sqrt{2}\right) \qquad \ldots(2)$$

Also, we know that

$$\cos\alpha \leq 1$$

$$\Rightarrow \quad \frac{1}{2}\left((k-1) + \sqrt{(k-1)^2 - 2}\right) \leq 1$$

$$\Rightarrow \quad (k-1)+\sqrt{(k-1)^2-2} \le 2$$

$$\Rightarrow \quad \sqrt{(k-1)^2-2} \le 2-(k-1)$$

$$\Rightarrow \quad (k-1)^2-2 \le (2-(k-1))^2$$

$$\Rightarrow \quad 4k \le 10$$

$$\Rightarrow \quad k \le 25 \qquad \ldots(3)$$

Hence from (2) and (3), we get

$$1+\sqrt{2} \le k \le 2.5$$

$$\Rightarrow \quad (1+\sqrt{2})R \le H \le 2.5R \qquad \left\{\text{as } k = \frac{H}{R}\right\}$$

$$\Rightarrow \quad 2.414R \le H \le 2.5R$$

For the limiting values of cosines we have for, $k = 1+\sqrt{2}$

$$\Rightarrow \quad k-1 = \sqrt{2}$$

$$\Rightarrow \quad \cos\alpha_1 = \frac{\sqrt{2}}{2}$$

i.e., $\alpha_1 = 45°$ and for, $k = 2.5$

$$\Rightarrow \quad k-1 = 1.5$$

$$\Rightarrow \quad \cos\alpha_2 = \frac{(1.5 \pm 0.5)}{2}$$

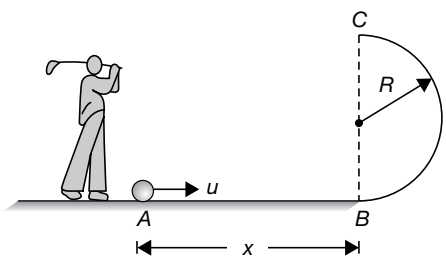
$$\Rightarrow \quad \cos\alpha_2 = 0.5 \text{ and } \cos\alpha_3 = 1$$

$$\Rightarrow \quad \alpha_2 = 60° \text{ and } \alpha_3 = 0°$$

So, $45° \le \alpha \le 60°$

PROBLEM 9

A person rolls a small ball with speed u along the floor from point A. If $x = 3R$, determine the required speed u so that the ball returns to A after rolling on the circular surface in the vertical plane from B to C and becoming a projectile at C. What is the minimum value of x for which the game could be played if contact must be maintained at point C. Neglect friction.

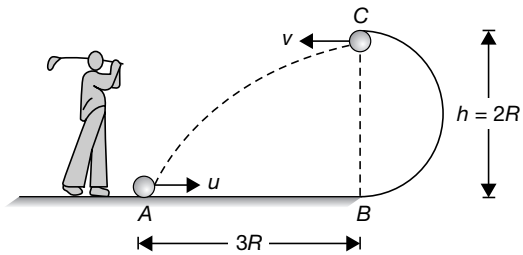


SOLUTION

(i) Let v be the velocity at the highest point, then

$$v^2 = u^2 - 2gh$$

$$\Rightarrow \quad v^2 = u^2 - 4gR \qquad \ldots(1)$$



After point C, path of the ball becomes projectile with initial velocity in horizontal direction. Hence substituting in

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta} \qquad \ldots(2)$$

So, we get

$$-2R = 3R\tan(0°) - \frac{g(3R)^2}{2(u^2-4gR)\cos^2(0°)}$$

$$\Rightarrow \quad 2R = \frac{9gR^2}{2(u^2-4gR)}$$

$$\Rightarrow \quad 4u^2 - 16gR = 9gR$$

$$\Rightarrow \quad u^2 = \frac{25}{4}gR$$

$$\Rightarrow \quad u = \frac{5}{2}\sqrt{gR} \qquad \{u > \sqrt{5gR}\}$$

(ii) Minimum value of v to maintain contact at C is $\sqrt{gR}$. Hence substituting $v = \sqrt{gR}$ in equation (2), we get

$$-2R = x\tan(0°) - \frac{gx^2}{2(gR)\cos^2(0°)}$$

$$\Rightarrow \quad 4gR^2 = gx^2$$

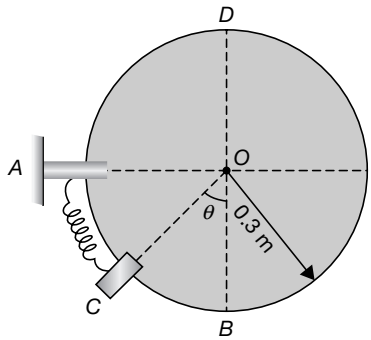
$$\Rightarrow \quad x = 2R$$

Hence minimum value of x is

$$x_{\min} = 2R$$

PROBLEM 10

A thin circular rod is supported in a vertical plane by a bracket at A. Attached to the bracket and loosely wound around the rod is a spring of constant $k = 40\ \text{Nm}^{-1}$ and underformed length equal to arc of the circle AB. A $0.2\ \text{kg}$ collar C, not attached to the spring, can slide without friction along the rod. The collar is released from rest at an angle θ with the vertical.



(a) Make the equation for minimum value of θ for which the collar will pass through D and reach point A.

(b) Determine the velocity of collar as it reaches point A for minimum value of θ. (Take $g = 10\ \text{ms}^{-2}$).

SOLUTION

(a) Compression of the spring in position C is

$$x = CB = R\theta = 0.3\theta \qquad \{R = 0.3\ \text{m}\}$$

The height difference between C and D is

$$h = R(1+\cos\theta) = 0.3(1+\cos\theta)$$

By Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}kx^2 = mgh$$

$$\Rightarrow \frac{1}{2}(40)(0.3\theta)^2 = (0.2)(10)(0.3)(1+\cos\theta)$$

$$\Rightarrow \theta^2 = \frac{1}{3}(1+\cos\theta)$$

$$\Rightarrow 3\theta^2 = 1+\cos\theta$$

(b) For the above angle, velocity of collar is zero at point D. The height difference between A and D is

$$h = R = 0.3\ \text{m}$$

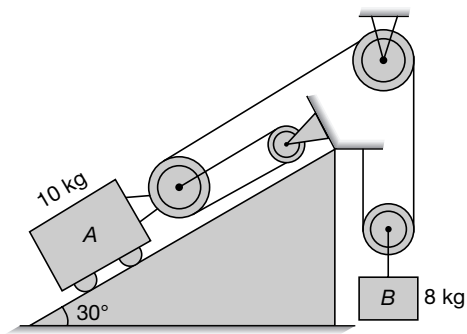
So, velocity of collar at point A is

$$v = \sqrt{2gh} = \sqrt{2(10)(0.3)}$$

$$\Rightarrow v = 2.45\ \text{ms}^{-1}$$

PROBLEM 11

If the system is released from rest, determine the speeds of both masses after B has moved 1 m. Neglect friction and masses of the pulleys. $\left(g = 10\ \text{ms}^{-2}\right)$.



SOLUTION

From constraint relations we observe that

$$3v_A = 2v_B$$

$$\Rightarrow v_B = 1.5v_A$$

$$\Rightarrow v_A = \frac{2}{3}v_B$$

$$\Rightarrow s_A = \frac{2}{3}s_B$$

Now as the block B moves $\frac{1}{2}$ m vertically downwards, block A will move $\frac{2}{3}$ m along the plane or $\frac{2}{3}\sin(30°)$ vertically upwards.

By Law of Conservation of Mechanical Energy, we get

$$\begin{pmatrix}\text{Decrease}\\ \text{in PE}\\ \text{of block } B\end{pmatrix} = \begin{pmatrix}\text{Increase}\\ \text{in PE}\\ \text{of block } A\end{pmatrix} + \begin{pmatrix}\text{Increase}\\ \text{in KE of both}\\ \text{the blocks}\end{pmatrix}$$

$$\Rightarrow (8)(g)(1) = (10)(g)\left(\frac{2}{3}\sin 30°\right) + \frac{1}{2}(8)\left(v_B^2\right) + \frac{1}{2}(10)\left(v_A\right)^2$$

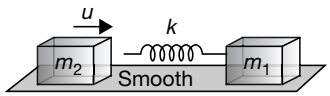
$$\Rightarrow 80 = \frac{100}{3} + \frac{1}{2}(8)\left(1.5v_A\right)^2 + \frac{1}{2}(10)v_A^2$$

$$\Rightarrow \quad \frac{140}{3} = 11v_A^2$$

$$\Rightarrow \quad v_A \cong 2 \text{ ms}^{-1} \text{ and } v_B \cong 3 \text{ ms}^{-1}$$

PROBLEM 12

A block of mass m_1 is resting on a smooth horizontal surface. A light spring of spring constant k is attached to the block. Another block of mass m_2 is moving with a speed u towards the stationary block as shown in Figure. Calculate the maximum compression in the spring.



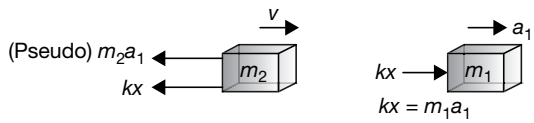
SOLUTION

Let us consider the motion of block m_2 as seen by m_1. As the block m_2 approaches m_1, it compresses the spring and the spring opposes the motion of m_2. When the speed of m_2 becomes zero relative to m_1, then compression in the spring is maximum.

At any instant, let the compression in the spring be x, then at this instant, the block m_1 has an acceleration a_1 given by

$$a_1 = \frac{kx}{m_1}, \text{ towards the right.}$$

The free body diagram of m_2 as seen from the reference frame attached to m_1 (i.e. a non-inertial frame) is shown in Figure.



Let speed of m_2 relative to m_1 be v. The force F acting on m_2 at this instant is given by

$$F = m_2 a_1 + kx = m_2\left(\frac{kx}{m_1}\right) + kx$$

$$\Rightarrow \quad F = \left(\frac{m_1 + m_2}{m_1}\right)kx$$

Work done by this force on m_2 is given by

$$W = \int_0^x F dx \cos(180°) = -\int_0^x \left(\frac{m_1 + m_2}{m_1}\right)kx dx$$

$$\Rightarrow \quad W = -\left(\frac{m_1 + m_2}{m_1}\right)\left(\frac{1}{2}kx^2\right)$$

According to Work-Energy Theorem, we have

$$W = \Delta K = \frac{1}{2}m_2 v^2 - \frac{1}{2}m_2 u^2$$

$$\Rightarrow \quad \frac{1}{2}m_2 v^2 - \frac{1}{2}m_2 u^2 = -\left(\frac{m_1 + m_2}{m_1}\right)\left(\frac{1}{2}kx^2\right)$$

When $x = x_{\max}$, then both the blocks move with the same velocity and hence $v = 0$.

$$\Rightarrow \quad -\frac{1}{2}m_2 u^2 = -\left(\frac{m_1 + m_2}{m_1}\right)\left(\frac{1}{2}kx_{\max}^2\right)$$

$$\Rightarrow \quad x_{\max} = u\sqrt{\left(\frac{m_2 m_1}{m_2 + m_1}\right)\frac{1}{k}}$$

where the term $\frac{m_2 m_1}{m_2 + m_1}$ is called as the reduced mass μ of the system.

PROBLEM 13

A bungee jumper of mass 50 kg attached to a light elastic cord (bungee cord) of unstretched length L jumps from a tall bridge of height 100 m. The cord first straightens and then extends as the jumper falls. This prevents the jumper from hitting the water. Assume that the bungee cord behaves like a spring with stiffness constant 100 Nm^{-1}.

(a) Calculate the maximum allowed length L of the unstretched bungee cord to keep the jumper alive? (Assume that the stiffness constant of the cord is constant throughout the fall of the bungee jumper)

(b) Before the bungee jumper jumps off the bridge, his instructor of same mass verified the stiffness constant of the bungee cord by lowering himself very slowly from the bridge to the full extent of the cord and measured his remaining distance d from the water surface. Calculate d.

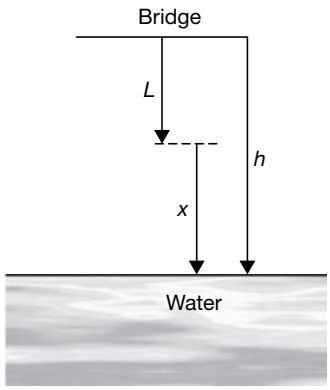
Assume air drag to be negligible, no heat is dissipated in the string, only gravitation and spring forces to be present.

SOLUTION

Given that, L is the unstretched length of the cord, $h = 100$ m, $k = 100 \text{ Nm}^{-1}$, $m = 50$ kg. Let x denote

the distance through which the bungee jumper has fallen from the bridge

(a) We want to choose L so that the jumper comes to a stop just before touching the water surface. At this moment, the extension in the cord is $x = (h - L)$ and the jumper has fallen through h as shown in Figure.



So, by Law of Conservation of Mechanical Energy, we have loss in gravitational potential energy of the jumper equals the gain in elastic potential energy of the bungee cord.

$$\Rightarrow \quad mgh = \frac{1}{2}kx^2$$

$$\Rightarrow \quad mgh = \frac{1}{2}k(h-L)^2$$

$$\Rightarrow \quad L = h - \sqrt{\frac{2mgh}{k}}$$

$$\Rightarrow \quad L \approx 69 \text{ m}$$

(b) The bungee jumper will hang freely at the distance d above the water when the weight of the instructor is balanced by the spring force developed in the cord i.e. at equilibrium. If x_{eq} be the extension in the bungee cord at equilibrium, then we have

$$kx_{eq} = mg$$

where $x_{eq} = h - L - d$

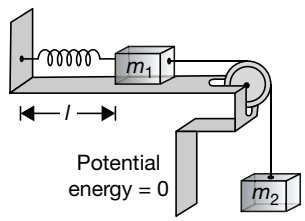
$$\Rightarrow \quad k(h-L-d) = mg$$

$$\Rightarrow \quad d = h - L - \frac{mg}{k}$$

$$\Rightarrow \quad d \approx 26 \text{ m}$$

PROBLEM 14

In the arrangement shown in Figure, two blocks of masses $m_1 = M$ and $m_2 = 2M$ are taken. If $M = 20$ kg, the force constant of the spring (of natural length l_0) is $k = 10$ kNm^{-1}, $g = 10$ ms^{-2}, the frictional force between M and table is 80 N, the system is held at rest with $l = (l_0 + 10)$ cm and then released. Calculate the



(a) kinetic energy of the sliding block, when it has moved 2 cm from its point of release.

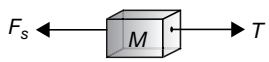
(b) kinetic energy of the sliding block m when it first slides back through the point where the spring is unstretched.

(c) maximum kinetic energy attained by the sliding block while it is sliding from its point of release to the point where the spring is unstretched.

SOLUTION

(a) *Before we arrive at any conclusion, we must first find whether the block of mass M moves to the right or to the left, when released.*

For that purpose, let us draw the FBD of the block M (imagining the friction to be absent) as shown in Figure.



As of now, we do not know the direction of the friction force too. So firstly, let us calculate the net force acting on M in the horizontal direction.

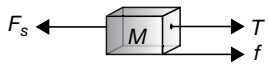
The spring force F_s acting initially on the block is

$$F_s = kx = 10000 \text{ Nm}^{-1} \times 0.1 \text{ m} = 1000 \text{ N}$$

The tension T acting on the block is

$$T = 2Mg = 2 \times 20 \text{ kg} \times 10 \text{ ms}^{-2} = 400 \text{ N}$$

Since, $F_s > T$, so we conclude that the block M goes to the left and hence the block $2M$ will go upwards. The friction on M will act to the right as shown in Figure.



So, the net force that moves the block of M to the right is $F_s - T - f = 1000 - 400 - 80 = 520$ N

According to the Modified Work-Energy Theorem (MWET) applied on the system, we have

$$W_{nc} = \Delta U + \Delta K \qquad \ldots(1)$$

where ΔU is the sum of $\Delta U = \Delta U_e + \Delta U_g$

where, $W_{nc} = -(80)\left(\frac{2}{100}\right) = -1.6$ J

For the spring, $k = 10$ kNm^{-1} $= 10000$ Nm^{-1}

Initial extension in the spring is

$$x_1 = 10 \text{ cm} = 0.1 \text{ m}$$

So, the initial elastic energy stored in the spring is

$$(U_e)_i = \frac{1}{2}kx_1^2 = 50 \text{ J}$$

Final extension in the spring is

$$x_2 = (10 - 2) \text{ cm} = 0.08 \text{ m}$$

So, the final elastic energy stored in the spring is

$$(U_e)_f = \frac{1}{2}kx_2^2 = 32 \text{ J}$$

$$\Rightarrow \Delta U_e = 32 - 50 = -18 \text{ J}$$

The initial gravitational potential energy (GPE) is assumed to be zero at the initial position of mass $m_2 = 2M$. When the mass $2M$ moves up by 2 cm its GPE increases by

$$(\Delta U_g)_{2M} = m_2 gh = (40)(10)\left(\frac{2}{100}\right) = 8 \text{ J}$$

However, there is no change in the GPE of the mass m_1.

So, $\Delta U_g = 8$ J

The initial kinetic energy of both the blocks is zero, so

$$K_i = 0$$

Finally, if v is the speed of each block, then

$$K_f = \frac{1}{2}(M + 2M)v^2 = \frac{3}{2}Mv^2$$

Since, we know that

$$W_{nc} = (\Delta U_e + \Delta U_g) + \Delta K$$

$$\Rightarrow -1.6 = (-18 + 8) + \frac{3}{2}Mv^2$$

$$\Rightarrow \frac{3}{2}Mv^2 = 8.4$$

$$\Rightarrow \frac{1}{2}Mv^2 = \frac{8.4}{3} = 2.8 \text{ J}$$

(b) When the spring attains its natural length after the block $m_2 = 2M$ moves up by $x_1 = 0.1$ m i.e. when $l = l_0$, then the elastic potential energy stored in the spring is zero. So, $U_f = 0$

Since, we know that

$$W_{nc} = \Delta U + \Delta K$$

where, $\Delta U = \Delta U_e + \Delta U_g$

$$\Delta U_g = (2M)gx_1 = 2(200)(0.1) = 40 \text{ J}$$

$$\Delta U_e = 0 - \frac{1}{2}kx_1^2 = -50 \text{ J}$$

$$\Delta K = \frac{1}{2}(M + 2M)v^2 = \frac{3}{2}Mv^2 \text{ and}$$

$$W_{\text{nc}} = -fx_1 = -(80)(0.1) = -8 \text{ J}$$

Substituting the values in above equation, we get

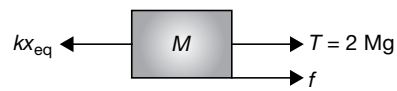
$$\frac{3}{2}Mv^2 + (40 - 50) = -8$$

$$\Rightarrow \frac{3}{2}Mv^2 = 2$$

$$\Rightarrow \frac{1}{2}Mv^2 = \frac{2}{3} \text{ J}$$

Hence, the kinetic energy of the sliding block when it first slides back through the point where the spring is unstretched is $\frac{2}{3}$ J

(c) Maximum kinetic energy is attained by the sliding block when it passes through the mean position (where net force on m_1 is zero), because at the mean position potential energy is minimum, so kinetic energy is maximum.



$$\Rightarrow kx_{eq} = 2Mg + f$$

$$\Rightarrow x_{eq} = \frac{2Mg + f}{k}$$

$$\Rightarrow \quad x_{eq} = \frac{2(20)(10)+80}{k}$$

$$\Rightarrow \quad x_{eq} = \frac{480}{10000}\ \text{m} = 0.048\ \text{m}$$

Again applying, $W_{nc} = \Delta U + \Delta K$

where, $\Delta U = \Delta U_e + \Delta U_g$ and

$$\Delta U_g = -2Mg\left(0.1 - x_{eq}\right) = -400(0.1 - 0.048)$$

$$\Rightarrow \quad \Delta U_g = -400(0.052) = -20.8\ \text{J}$$

$$\Delta U_e = \frac{1}{2}k\left(x_{eq}^2 - x_1^2\right)$$

$$\Rightarrow \quad \Delta U_e = \frac{1}{2}(10000)\left[(0.048)^2 - (0.1)^2\right]$$

$$\Rightarrow \quad \Delta U_e = -38.48\ \text{J}$$

Since $W_{nc} = -f\left(0.1 - x_{eq}\right) = -80(0.1 - 0.048)$

$$\Rightarrow \quad W_{nc} = -4.16\ \text{J}$$ and

$$\Delta K = \frac{1}{2}(M + 2M)v_{\max}^2 = \frac{3}{2}v_{\max}^2$$

Substituting these values in $W_{nc} = \Delta U + \Delta K$, we get

$$-4.16 = (-38.48 - 20.8) + 3\left(\frac{1}{2}Mv_{\max}^2\right)$$

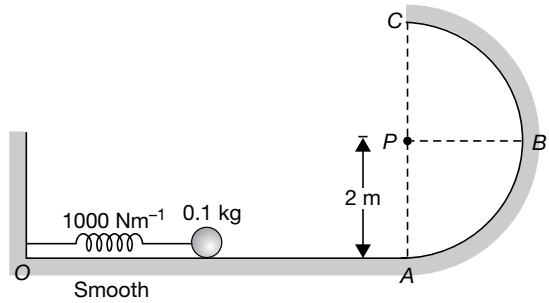
$$\Rightarrow \quad 3\left(\frac{1}{2}Mv_{\max}^2\right) = 55.12$$

$$\Rightarrow \quad \frac{1}{2}Mv_{\max}^2 = \frac{55.12}{3} \approx 18.4\ \text{J}$$

Hence, the maximum kinetic energy attained by the sliding block while it is sliding from its point of release to the point where the spring is unstretched is 18.4 J

PROBLEM 15

A small ball of mass 0.1 kg is placed on a smooth plane surface OA which acquires a semi-circular shape ABC of radius 2 m. The ball just touches a light spring of stiffness 1000 Nm^{-1} The ball is pushed to the left so as to compress the spring by a distance x and is then released. This ball then starts moving towards the circular track ABC as shown in Figure. Take $g = 10\ \text{ms}^{-2}$.



(a) Calculate the minimum work done by external agent to push the ball to the left through 50 cm.

(b) If the ball is pushed to the left by 5 cm and released, then calculate the normal force on the ball just after crossing A and the maximum angle made by the ball with PA on the circular track when it comes to rest.

(c) Find the minimum distance $x_{\min}$ through which the ball should be pushed to the left and released so that it can reach upto the point C.

(d) If the ball is pushed to left through $0.7x_{\min}$, then calculate the reaction force between ball and track at point B and the maximum height attained by the ball above horizontal surface OA.

SOLUTION

(a) When the work done by external force to push the ball against the spring is minimum, there must be no kinetic energy associated with the ball. The work done is solely responsible for increasing the potential energy of the spring. So, we have

$$W = \frac{1}{2}kx^2$$

$$\Rightarrow \quad W = \frac{1}{2} \times 1000 \times \left(\frac{5}{100}\right)^2 = 1.25\ \text{J}$$

(b) When the ball leaves the spring, it moves to the right with a speed v, such that the potential energy of the spring is converted to kinetic energy of the ball.

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad v = x\sqrt{\frac{k}{m}}$$

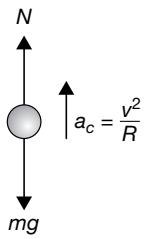
Since, $k = 1000\ \text{Nm}^{-1}$, $m = 0.1\ \text{kg}$, $x = 5\ \text{cm}$

$$\Rightarrow \quad v = 0.05 \times \sqrt{\frac{1000}{0.1}} = 5 \text{ ms}^{-1}$$

When the ball crosses the point A, then its path becomes circular and it experiences a centripetal acceleration

$$a_c = \frac{v^2}{r}$$

Applying Newton's Second Law, we get



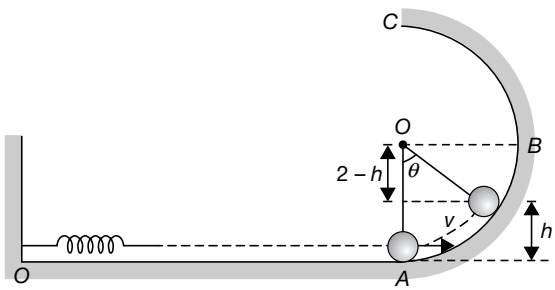
$$N - mg = ma_c$$

$$\Rightarrow \quad N = mg + \frac{mv^2}{R}$$

$$\Rightarrow \quad N = (0.1)(10) + \frac{(0.1)(5)^2}{2}$$

$$\Rightarrow \quad N = 1 + 1.25 = 2.25 \text{ N}$$

As the ball rises on the track, its gravitational potential energy increases and kinetic energy decreases. When the ball comes to rest, then its kinetic energy becomes zero. Let this happen at a height h as shown in Figure.



Then by Law of Conservation of energy, we get

$$mgh + 0 = 0 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad h = \frac{v^2}{2g} = \frac{5^2}{2 \times 10} = \frac{25}{20} = 1.25 \text{ m}$$

Since, $\cos\theta = \frac{2-h}{2} = \frac{2-1.25}{2} = \frac{0.75}{2} = \frac{3}{8}$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{3}{8}\right)$$

(c) Let the spring be compressed by $x_{\min}$ and the speed of the ball after leaving contact with spring be v, then by Law of Conservation of Energy, we have

$$\frac{1}{2}mv^2 = \frac{1}{2}kx_{\min}^2$$

$$\Rightarrow \quad v = x_{\min}\sqrt{\frac{k}{m}}$$

Now, to complete the circle, minimum speed required by the ball at A should be $\sqrt{5gR} = 10 \text{ ms}^{-1}$.

$$\Rightarrow \quad x_{\min}\sqrt{\frac{k}{m}} = \sqrt{5gR}$$

$$\Rightarrow \quad x_{\min} = 0.1 \text{ m}$$

(d) When the spring is compressed by $0.7x_{\min} = 0.07$ m, then the speed acquired by the ball while crossing the point A is

$$v = v_A = x\sqrt{\frac{k}{m}}$$

$$\Rightarrow \quad v_A = 0.07 \times \sqrt{\frac{1000}{0.1}} = 7 \text{ ms}^{-1}$$

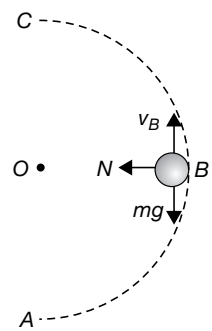
As the ball reaches B, its speed becomes v_B such that by Law of Conservation of Energy, we get

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_B^2 + mgR$$

$$\Rightarrow \quad v_B = \sqrt{v^2 - 2gR} = \sqrt{49 - 40}$$

$$\Rightarrow \quad v_B = 3 \text{ ms}^{-1}$$

The FBD of ball at B is shown in Figure.



The radial acceleration of the ball at this instant is

$$a_c = \frac{v_B^2}{R} = \frac{9}{2} \text{ ms}^{-2}$$

Applying Newton's Second Law, we get

$$N = \frac{mv_B^2}{R} = \frac{(0.1)9}{2} = 0.45 \text{ N}$$

Now as we see that the speed of the ball at the lowest point A is 7 ms^{-1} which is less than $\sqrt{5gR} = 10 \text{ ms}^{-1}$, so the ball will leave the circular track say at D and follow a parabolic path.

Let OD make an angle θ with OC and let v_D be the speed of the ball at this position. If h be the height of the ball at D, then

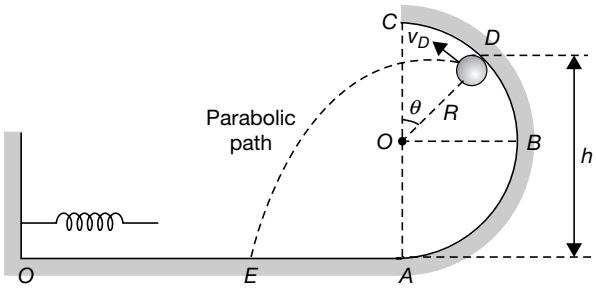
$$h = R(1+\cos\theta)$$

By Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}mv_A^2 - \frac{1}{2}mv_D^2 = mgR(1+\cos\theta)$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}mv_D^2 + mgR(1+\cos\theta) \quad \ldots(1)$$

FBD of ball at D, is shown in Figure.



As ball leaves the track at D, normal reaction is zero and the component of weight i.e. $mg\cos\theta$ acting towards the centre O provides the necessary centripetal force to the particle. So, applying Newton's Second Law in radial direction, we get

$$mg\cos\theta = \frac{mv_D^2}{R}$$

$$\Rightarrow v_D^2 = gR\cos\theta \quad \ldots(2)$$

Substituting equation (2) in (1), we get

$$\frac{1}{2}mv^2 = \frac{1}{2}mgR\cos\theta + mgR(1+\cos\theta)$$

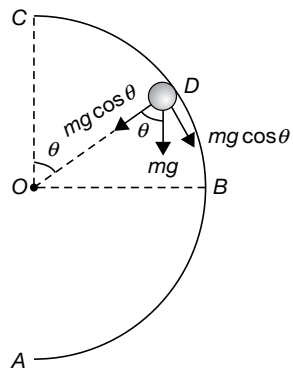
$$\Rightarrow v^2 - 2gR = 3gR\cos\theta$$

$$\Rightarrow \cos\theta = \frac{v^2-2gR}{3gR} = \frac{7^2-2(10)(2)}{3(10)(2)} = \frac{9}{60}$$

$$\Rightarrow \cos\theta = \frac{3}{20}$$

$$\Rightarrow h = R(1+\cos\theta) = R\left(1+\frac{3}{20}\right) = \frac{23R}{20}$$

$$\Rightarrow h = \frac{46}{20} = 2.3 \text{ m}$$



From point D, the ball moves in a parabolic path under the action of gravity alone. We can also think that the ball becomes an oblique projectile at the point D, where it is launched with a velocity $v_D = \sqrt{gR\cos\theta}$ making an angle of θ with the horizontal. The height H (from this point onwards) to which this oblique projectile rises further is given by

$$H = \frac{v_D^2 \sin^2\theta}{2g}$$

$$\Rightarrow H = \frac{gR\cos\theta\sin^2\theta}{2g} = \frac{R}{2}(\cos\theta\sin^2\theta)$$

Since $\cos\theta = \frac{3}{20}$

$$\Rightarrow \sin^2\theta = 1-\cos^2\theta = 1-\frac{9}{400} = \frac{391}{400}$$

$$\Rightarrow H = \left(\frac{2}{2}\right)\left(\frac{3}{20}\right)\left(\frac{391}{400}\right) = \frac{1173}{8000} \text{ m}$$

Maximum height h' above OA is

$$h' = 2.3 + \frac{1173}{8000} = 2.45 \text{ m}$$

PROBLEM 16

A small body of mass m is located on a rough horizontal plane at the point O. The body is given a horizontal velocity u. Calculate the power developed by the friction force during the whole time of motion, if the friction coefficient between the block and the surface is μ. Also calculate the maximum power developed by the friction force, if the friction coefficient varies with distance x from the point O as $\mu = \alpha x$, where α is a positive constant.

SOLUTION

The frictional force acting on the body

$$f_k = \mu mg$$

Retardation provided by this force is

$$a = \mu g$$

If t is the total time taken by the body to come to rest, then

$$0 = u + (-a)t$$

$$\Rightarrow \quad t = \frac{u}{\mu g}$$

Magnitude of change in kinetic energy due to friction is

$$|\Delta K| = \left|0 - \frac{1}{2}mu^2\right| = \frac{1}{2}mu^2$$

Power developed by friction during the whole time of motion, i.e. the average power is given by

$$\text{Average Power} = \frac{\text{Change in kinetic energy}}{\text{Total time of motion}}$$

$$\Rightarrow \quad P_{av} = \frac{|\Delta K|}{t}$$

$$\Rightarrow \quad P_{av} = \frac{\frac{1}{2}mu^2}{\left(\frac{u}{\mu g}\right)} = \frac{1}{2}\mu mug$$

When friction coefficient is $\mu = \alpha x$, the friction force on the body when it is at a distance x from the point O is

$$f_k = \alpha xmg$$

Retardation due to this force is

$$a = -\alpha gx$$

$$\Rightarrow \quad v\frac{dv}{dx} = -\alpha gx$$

$$\Rightarrow \quad vdv = -\alpha gxdx$$

Integrating the above expression to get the velocity at a distance x from point O, gives

$$\int_u^v vdv = -\int_0^x \alpha gxdx$$

$$\Rightarrow \quad \frac{v^2 - u^2}{2} = -\frac{\alpha gx^2}{2}$$

$$\Rightarrow \quad v = \sqrt{u^2 - \alpha gx^2}$$

Instantaneous power P due to friction force at a distance x from point O is

$$P = \vec{F} \cdot \vec{v}$$

$$\Rightarrow \quad P = P_{\text{ins}} = -\alpha mgx\sqrt{\left(u^2 - \alpha gx^2\right)} \qquad \ldots(1)$$

This power is maximum when $\frac{dP}{dx} = 0$

$$\Rightarrow \quad \frac{dP}{dx} = \frac{(\alpha mgx)(\alpha gx)}{\sqrt{u^2 - \alpha gx^2}} - \alpha mg\sqrt{u^2 - \alpha gx^2} = 0$$

$$\Rightarrow \quad x = \frac{u}{\sqrt{2\alpha g}} \qquad \ldots(2)$$

Equation (2) gives the value of x at which instantaneous power is maximum. Substituting the above value of x in Equation (1), we get maximum power to be

$$P_{\max} = -\frac{\alpha mug}{\sqrt{2\alpha g}}\sqrt{u^2 - \frac{u^2}{2}}$$

$$\Rightarrow \quad P_{\max} = -\frac{1}{2}mu^2\sqrt{\alpha g}$$

PROBLEM 17

A small box of mass m is kept on a fixed, smooth sphere of radius R at a position where the radius through the box makes an angle of $30°$ with the vertical. The box is released from this position. Calculate the force exerted by the sphere on the box just after the release. Also calculate the distance travelled by the box before it leaves contact with the sphere. Given that $\cos^{-1}(0.577) \approx 54°$

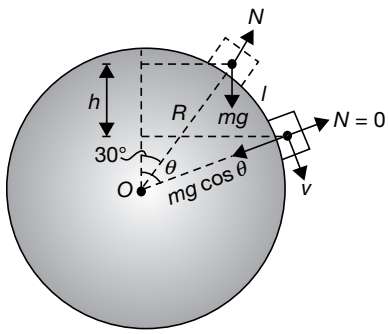
SOLUTION

At the instant of releasing the box, the speed of box is zero. Hence the box will push the sphere with a force that equals the component of its weight acing towards the centre of the sphere i.e. $mg\cos\theta$. So, we have

$$N = mg\cos(30°)$$

$$\Rightarrow \quad N = \frac{\sqrt{3}}{2}mg$$

Let the box loose contact with the sphere at an angle θ from the vertical as shown in Figure.



At this instant its normal reaction becomes zero, and hence at this point, we have

$$\frac{mv^2}{R} = mg\cos\theta$$

$$\Rightarrow \quad v = \sqrt{gR\cos\theta} \qquad \ldots(1)$$

Velocity of particle (v) at this point can be given by applying the Law of Conservation of Energy (as it has fallen a distance h), according to which we have

$$\begin{pmatrix}\text{Loss in GPE}\\ \text{of the Body}\end{pmatrix} = \begin{pmatrix}\text{Gain in KE}\\ \text{of the Body}\end{pmatrix}$$

$$\Rightarrow \quad mgh = \frac{1}{2}mv^2$$

where, $h = R(\cos 30° - \cos\theta) = R\left(\frac{\sqrt{3}}{2} - \cos\theta\right)$

$$\Rightarrow \quad v = \sqrt{2gh} = \sqrt{2gh\left(\frac{\sqrt{3}}{2} - \cos\theta\right)} \qquad \ldots(2)$$

Equating (1) and (2), we get

$$\cos\theta = \frac{\sqrt{3} - 2\cos\theta}{2}$$

$$\Rightarrow \quad 3\cos\theta = \sqrt{3}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \cos^{-1}(0.577)$$

$$\Rightarrow \quad \theta = 54°$$

Angular displacement of the box before leaving the sphere is

$$\Delta\theta = 54° - 30° = 24°$$

Since, $180° = \pi$ radian

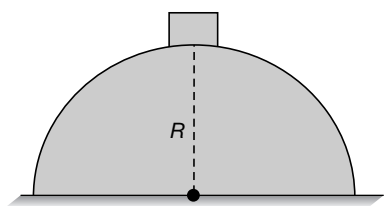
$$\Rightarrow \quad \Delta\theta = 24 \times \frac{\pi}{180} = \frac{2\pi}{15}$$

So, distance travelled l by the box is given by

$$l = R\Delta\theta = R\left(\frac{2\pi}{15}\right) = \frac{2\pi R}{15}$$

PROBLEM 18

A smooth surface hemisphere is fixed on a ground as shown in figure. From the topmost point of it, a small block starts sliding with no initial velocity. Calculate the distance s between the centre of base circle of hemisphere and the point where particle strikes the ground. Also find the time taken by the particle to strike the ground.



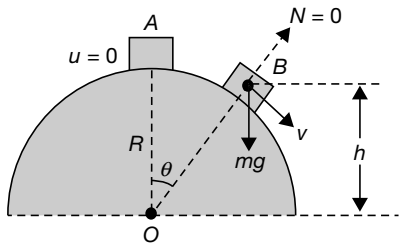
SOLUTION

Let the block leave the contact with hemisphere at point B, then at B, we have

$$N = 0$$

$$\Rightarrow \quad mg\cos\phi = \frac{mv_B^2}{R}$$

$$\Rightarrow \quad v_B = \sqrt{gR\cos\phi} \qquad \ldots(1)$$



Applying Law of Conservation of Energy between A and B we get,

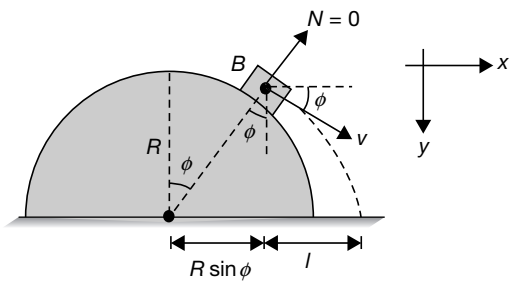
$$0 + mgR = \frac{1}{2}mv_B^2 + mgh$$

Substituting v_B from (1) and $h = R\cos\phi$, we get

$$v_B = \sqrt{\frac{2}{3}gR}$$

$h = \frac{2R}{3}$ from the bottom and

$$\cos\phi = \frac{2}{3}$$



After the block leaves contact with the hemisphere, it leaves the circular path and follows a parabolic path, because at the instant of leaving the hemisphere, the block has a velocity $v = \sqrt{\frac{2}{3}gR}$ and makes an angle ϕ below the horizontal.

If t is the time taken by the block to hit the ground, then considering the origin at the point B, we get

$$l = (v\cos\phi)t \qquad \ldots(1)$$

$$R\cos\phi = (v\sin\phi)t + \frac{1}{2}gt^2 \qquad \ldots(2)$$

$$\Rightarrow \quad t^2 + \left(\frac{2v\sin\phi}{g}\right)t - \frac{2R\cos\phi}{g} = 0 \qquad \ldots(3)$$

where, $v = \sqrt{\frac{2}{3}gR}$, $\cos\phi = \frac{2}{3}$ and $\sin\phi = \sqrt{1-\frac{4}{9}} = \frac{\sqrt{5}}{3}$

$$\Rightarrow \quad \frac{2v\sin\phi}{g} = \frac{2}{g}\sqrt{\frac{2gR}{3}}\frac{\sqrt{5}}{3} = \sqrt{\frac{40R}{27g}} = \frac{2}{3}\sqrt{\frac{10R}{3g}}$$

So, equation (3) becomes

$$t^2 + \left(\frac{2}{3}\sqrt{\frac{10R}{3g}}\right)t - \frac{4R}{3g} = 0$$

Solving this quadratic equation, we get

$$t = -\frac{1}{3}\sqrt{\frac{10R}{3g}} + \sqrt{\frac{10R}{27g} + \frac{4R}{3g}}$$

$$t = -\frac{1}{3}\sqrt{\frac{10R}{3g}} + \frac{1}{3}\sqrt{\frac{46R}{3g}}$$

$$\Rightarrow \quad t = \frac{1}{3}\sqrt{\frac{R}{3g}}\left(\sqrt{46} - \sqrt{10}\right)$$

So, the distance s is given by

$$s = R\sin\phi + l = R\sin\phi + (v\cos\phi)t$$

$$\Rightarrow \quad s = \frac{\sqrt{5}R}{3} + \left(\sqrt{\frac{2gR}{3}}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\sqrt{\frac{R}{3g}}\left(\sqrt{46} - \sqrt{10}\right)\right)$$

$$\Rightarrow \quad s = \frac{R}{3}\left(\sqrt{5} + \frac{4}{9}\left(\sqrt{23} - \sqrt{5}\right)\right)$$

$$\Rightarrow \quad s = \frac{R}{27}\left(4\sqrt{23} + 5\sqrt{5}\right)$$

Method-2 (to find s)

Speed of particle at point B, where it leaves the hemisphere is

$$v = \sqrt{\frac{2}{3}gR}$$

For an oblique projectile launched with a speed at an angle ϕ below the horizontal, the equation of trajectory is given by

$$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$$

Since the launch angle is below the horizontal, so $\theta = -\phi$

$$\Rightarrow \quad y = -x\tan\phi - \frac{gx^2}{2v^2\cos^2\phi} \qquad \ldots(4)$$

Since, at the point where the block leaves the hemisphere, we have

$$\cos\phi = \frac{2}{3}$$

$$\Rightarrow \quad \sin\phi = \frac{\sqrt{5}}{3}$$

$$\Rightarrow \quad \tan\phi = \frac{\sqrt{5}}{2}$$

Substituting these values in equation (4), we get

$$y = -\frac{\sqrt{5}}{2}x - \frac{gx^2}{2\left(\frac{2gR}{3}\right)\left(\frac{4}{9}\right)}$$

$$\Rightarrow \quad y = \left(-\frac{\sqrt{5}}{2}\right)x - \left(\frac{27}{16R}\right)x^2 \qquad \ldots(5)$$

where, $y = -R\cos\phi = -\frac{2R}{3}$ and $x = l$

Substituting these values in equation (5), we get

$$-\frac{2}{3}R = -\frac{\sqrt{5}}{2}l - \frac{27l^2}{16R}$$

$$\Rightarrow \quad l^2 + \frac{8\sqrt{5}}{27}lR - \frac{32}{81}R^2 = 0$$

$$\Rightarrow \quad l = -\frac{4\sqrt{5}}{27}R + \frac{R}{2}\sqrt{\frac{64\times5}{27\times27} + \frac{4\times32}{27\times3}\times\frac{9}{9}}$$

$$\Rightarrow \quad l = -\frac{4\sqrt{5}R}{27} + \frac{4R}{27}\sqrt{5+18}$$

$$\Rightarrow \quad l = \frac{4R}{27}\left(\sqrt{23} - \sqrt{5}\right)$$

Distance s will be given by

$$s = R\sin\phi + l$$

$$\Rightarrow \quad s = \frac{\sqrt{5}}{3}R + \frac{4R}{27}\left(\sqrt{23} - \sqrt{5}\right)$$

$$\Rightarrow \quad s = \frac{R}{27}\left(5\sqrt{5} + 4\sqrt{23}\right)$$

PROBLEM 19

A particle of mass m approaches a region of force starting from $r \to +\infty$. The potential energy function in terms of distance r from the origin is given by,

$$U(r) = \begin{cases} \frac{K}{2a^3}\left(3a^2 - r^2\right), & \text{for } 0 \le r \le a \\ \frac{K}{r}, & \text{for } r \ge a \end{cases}$$

Calculate the force $F(r)$ acting on the particle and find out whether the force is repulsive or attractive in nature. Also, calculate the minimum velocity with which a particle should be launched from infinity towards the origin, such that it just crosses the origin.

Assuming that the particle is launched from infinity towards the origin with a speed $\sqrt{\frac{2K}{ma}}$, then describe the motion of the particle.

SOLUTION

Since, we know that $F = -\frac{dU}{dr}$

For $0 \le r \le a$, we have

$$U = \frac{K}{2a^3}\left(3a^2 - r^2\right)$$

$$\Rightarrow \quad F = -\frac{Kr}{a^3}$$

For, $r \ge a$, we have

$$U = \frac{K}{r}$$

$$\Rightarrow \quad F = \frac{k}{r^2}$$

In both the regions i.e. $0 \le r \le a$ and $r \ge a$, we see that on increasing r, the potential energy decreases. Hence force is repulsive

Let the particle be launched from infinity towards the origin with a velocity $v_{\min}$. Then by Law of Conservation of Mechanical Energy, the total mechanical energy at infinity must be equal to the total mechanical energy at the origin O.

$$\Rightarrow \quad U_\infty + K_\infty = U_O + K_O$$

where $U_\infty = 0$, $K_\infty = \frac{1}{2}mv_{\min}^2$, $U_O = \frac{3K}{2a}$, $K_O = 0$

$$\Rightarrow \quad 0 + \frac{1}{2}mv_{\min}^2 = \frac{3K}{2a} + 0$$

$$\Rightarrow \quad v_{\min} = \sqrt{\frac{3K}{ma}}$$

So, for the particle to just cross O, the minimum velocity it should be given at infinity is $v_{\min} = \sqrt{\frac{3K}{ma}}$

However, if the particle is imparted a speed less than the minimum speed, then instead of reaching O, the particle will go closest to O and then starts travelling back towards infinity.

Since $v = \sqrt{\frac{2K}{ma}} < v_{\min}$

Hence the particle will not be able to cross the origin.

PROBLEM 20

A car starts from rest in a circular flat road of radius R with an acceleration a. The friction coefficient between the road and the tyres is μ. Find the distance car will travel before it start skidding.

SOLUTION

During the circular motion, the car will skid when net force acting on it exceeds the limiting friction. Since, tangential acceleration of car is a, so the speed of car after travelling a distance s is given by

$$v = \sqrt{2as}$$

Corresponding to this speed v of the car, the centripetal acceleration is

$$a_C = \frac{v^2}{R} = \frac{2as}{R}$$

Net acceleration (a_{net}) of the car is

$$a_{\text{net}} = \sqrt{a_T^2 + a_C^2} = \sqrt{a^2 + \left(\frac{2as}{R}\right)^2}$$

Net force acting on car is given by

$$F_{\text{net}} = ma_{\text{net}} = m\sqrt{a^2 + \left(\frac{2as}{R}\right)^2}$$

For the car to skid, this net force must be greater than the limiting friction $f_l = \mu N = \mu mg$. Hence, we have

$$m\sqrt{a^2 + \frac{4a^2s^2}{R^2}} \geq \mu mg$$

$$\Rightarrow \quad s = \sqrt{\left(\mu^2 g^2 - a^2\right)\frac{R^2}{4a^2}}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. Two equal masses are attached to the two ends of a spring of force constant k. The masses are pulled out symmetrically to stretch the spring by a length $2x_0$ over its natural length. The work done by the spring on each mass.

(A) kx_0^2 (B) $-\frac{1}{2}kx_0^2$

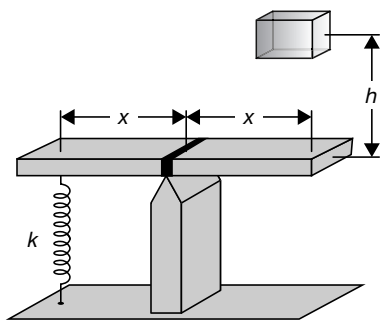
(C) $2kx_0^2$ (D) $-kx_0^2$

2. An ideal spring is hung vertically from the ceiling. Spring extends by 6 cm from its natural length when 2 kg mass is attached to it. A downward external force is now applied to the mass to extend the spring slowly by an additional 10 cm. Work done by the external force is

(A) –1.2 J (B) –1.8 J

(C) 2.4 J (D) 1.6 J

3. A crate of mass m falls from a height h onto the end of a platform, as shown in the figure. The spring is initially unstretched and the mass of the platform can be neglected. Assuming that there is no loss of energy, the maximum elongation of the spring is



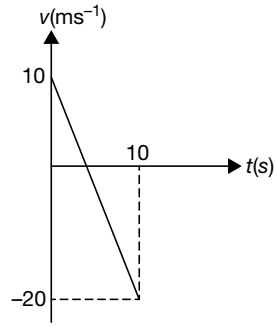
(A) $\frac{mg+\sqrt{m^2g^2+2mghk}}{k}$

(B) $\frac{mg-\sqrt{m^2g^2+2mghk}}{k}$

(C) $\frac{k}{\sqrt{m^2g^2+2mghk}}$

(D) $\frac{\sqrt{m^2g^2+2mghk}}{k}$

4. Velocity-time graph of a particle of mass 2 kg, moving in a straight line is as shown in figure. Work done by all the forces acting on the particle from $t=0$ to $t=10$ s is



(A) 300 J (B) –300 J

(C) 400 J (D) –400 J

5. A particle moves with a velocity $\vec{v}=5\hat{i}-3\hat{j}+6\hat{k}$ ms^{-1} under the influence of a constant force $\vec{F}=10\hat{i}+10\hat{j}+20\hat{k}$ N. The instantaneous power applied to the particle is

(A) 200 Js^{-1} (B) 40 Js^{-1}

(C) 140 Js^{-1} (D) 170 Js^{-1}

6. A ball suspended by a thread swings in a vertical plane so that its acceleration at the extreme position is equal to that at the lowest position. The thread deflection angle from the vertical in the extreme position is (Given: $\tan(26.5°)=0.5$)

(A) 45° (B) 37°

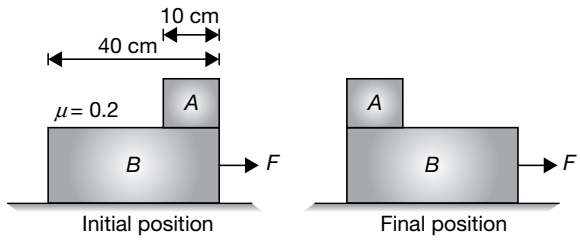
(C) 53° (D) 60°

7. A force acts on a 3 g particle in such a way that the position of the particle as a function of time is given by $x=3t-4t^2+t^3$, where x is in metre and t in second. The work done during the first 4 s is

(A) 578 mJ (B) 528 mJ

(C) 498 mJ (D) 458 mJ

8. Block A of mass 45 kg is placed on a block B of mass 160 kg. Now block B is displaced by a force 50 cm horizontally towards right. During the same time block A just reaches the left end of block B as shown in Figure.

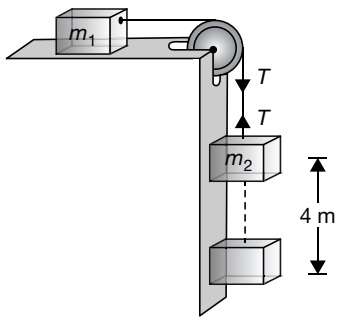


Work done by frictional force on block A in ground frame during this activity is

(A) –18 J (B) 18 J

(C) –45 J (D) 45 J

9. Two masses $m_1 = 10$ kg and $m_2 = 5$ kg are connected by an ideal string as shown in the figure. The coefficient of friction between m_1 and the surface is 0.2. Assuming that the system is released from rest, the velocity when m_2 has descended by 4 m is



(A) 17 ms^{-1} (B) 14 ms^{-1}

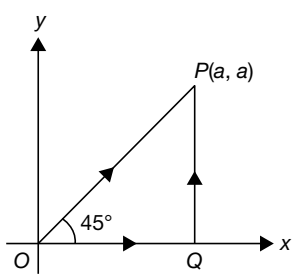
(C) 4 ms^{-1} (D) 3.5 ms^{-1}

10. A force $\vec{F} = \left(2\hat{i} + 5\hat{j} + \hat{k}\right)$ N is acting on a particle. The particle is first displaced from $(0, 0, 0)$ to $(2, 2, 0)$ m along the path $x = y$ and then from $(2, 2, 0)$ m to $(2, 2, 2)$ m along the path $x = 2$ m, $y = 2$ m. The total work done in the complete path is

(A) 8 J (B) 10 J

(C) 12 J (D) 16 J

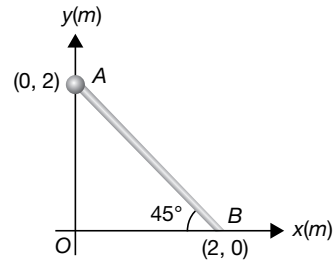
11. A particle is moved from $(0, 0)$ to (a, a) under a force $\vec{F} = \left(3\hat{i} + 4\hat{j}\right)$ along two paths, Path 1 along OP and Path 2 along OQP. Let W_1 and W_2 be the work done by this force along these two paths. Then



(A) $W_1 = W_2$ (B) $W_1 = 2W_2$

(C) $W_2 = 2W_1$ (D) $W_2 = 4W_1$

12. In an xy horizontal plane, a force field $\vec{F} = -(40 \text{ Nm}^{-1})\left(y\hat{i} + x\hat{j}\right)$ is present where x and y are the coordinates of any point lying in the plane. A smooth rod AB is fixed in the plane as shown in the figure. A particle of mass 5 kg is to be released with a velocity in this force field such that it reaches the point B. The minimum velocity that must be imparted to the particle along the rod at A such that it reaches B is



(A) 1 ms^{-1} (B) 2 ms^{-1}

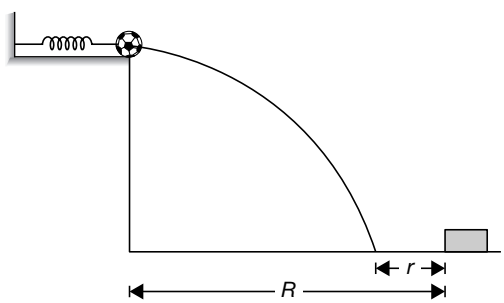
(C) 3 ms^{-1} (D) 4 ms^{-1}

13. Work done by a force $\vec{F} = \left(2\hat{i} + 3\hat{j}\right)$ N is zero, when a particle is moved along the line $3y + kx = 5$. Then k equals

(A) 2 (B) 4

(C) 6 (D) 8

14. A person wants to hit a small box on the floor with a marble as shown in Figure.



He compresses the spring by a distance d and releases but the marble falls short by a distance r from the box. How much should he compress the spring from natural length to land the ball in the box?

(A) $\frac{d^2}{R-r}$ (B) $\frac{d(R-r)}{R}$

(C) $\frac{d(R-r)}{R+r}$ (D) $\frac{Rd}{R-r}$

15. An object of mass m slides down a hill of height h of arbitrary shape and after travelling a certain horizontal path stops because of friction. The friction coefficient is different for different segments for the entire path but is independent of the velocity and direction of motion. The work that a force must perform to return the object to its initial position along the same path is

(A) mgh (B) $2mgh$

(C) $4mgh$ (D) $-mgh$

16. A horizontal force in the positive direction of x-axis is applied to a 1.5 kg block initially at rest on a horizontal frictionless surface. The force is given by $\vec{F}=(4-x^2)\hat{i}\text{ N}$, where x is in metre and the initial position of the block is $x=0$. The maximum kinetic energy of the block between $x=0$ and $x=2\text{ m}$ is

(A) 2.33 J (B) 8.67 J

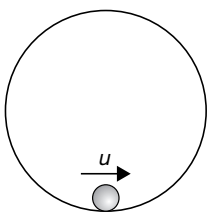
(C) 5.33 J (D) 6.67 J

17. In PROBLEM 24, the maximum positive displacement x is

(A) $2\sqrt{3}\text{ m}$ (B) 2 m

(C) 4 m (D) $\sqrt{2}\text{ m}$

18. A particle is given an initial speed u inside a smooth spherical shell so that it is just able to complete the circle. Acceleration of the particle, when its velocity is vertical will be



(A) $3g$ (B) g

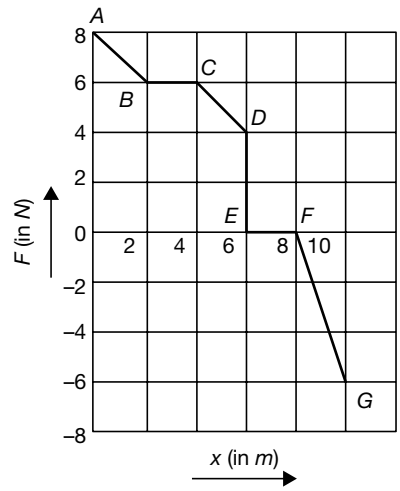
(C) $\sqrt{10}g$ (D) $\sqrt{6}g$

19. The work done in moving a particle from a point $(1, 1)$ to $(2, 3)$ in a plane and in a force field with potential $u=\lambda(x+y)$ is

(A) 3λ (B) -3λ

(C) 0 (D) λ

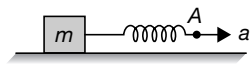
20. A 1.0 kg block moves in a straight line on a horizontal frictionless surface under the influence of a force that varies with position as shown in the figure. The work done by the force as it moves from the origin to a point $x=10\text{ m}$ is



(A) 5 J (B) 30 J

(C) −10 J (D) 22 J

21. A block of mass m is attached with a spring in its natural length, of spring constant k. The other end A of spring is moved with a constant acceleration a away from the block as shown in the figure. The maximum extension in the spring is (Assume that initially block and spring both are at rest w.r.t. ground frame)



(A) $\frac{ma}{k}$ (B) $\frac{1}{2}\frac{ma}{k}$

(C) $\frac{2ma}{k}$ (D) $\frac{4ma}{k}$

22. A particle moves in the x-y plane with velocity $\vec{v}=\alpha\hat{i}+\beta t\hat{j}$. The magnitude of tangential, normal and total accelerations at $t=\frac{\alpha\sqrt{3}}{\beta}$ are

(A) $\frac{\sqrt{3}}{2}\beta$, $\frac{\beta}{2}$ and β respectively

(B) $\frac{\beta}{2}$, $\frac{\beta}{2}$ and β respectively

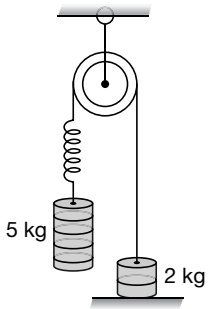
(C) $\sqrt{2}\beta$, $\sqrt{2}\beta$ and β respectively

(D) None of the above

23. A body of mass 0.5 kg is taken up an inclined plane of length 10 m and height 5 m and then allowed to slide down to the bottom again. The coefficient of friction between the body and the plane is 0.1. The work done by the frictional force over the round trip is $\left(g=10\text{ ms}^{-2}\right)$

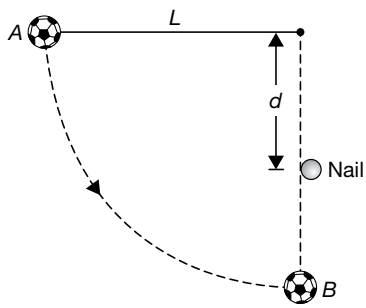
(A) 5 J (B) $5\sqrt{3}$ J
(C) –5 J (D) $-5\sqrt{3}$ J

24. In the system shown the 5 kg block is released from rest. The pulley and spring are massless and friction is absent everywhere. The force constant of spring is $40\ \text{Nm}^{-1}$. The speed of 5 kg block when 2 kg block leaves contact with ground is (Take $g = 10\ \text{ms}^{-2}$)



(A) $\sqrt{2}\,\text{ms}^{-1}$ (B) $2\sqrt{2}\ \text{ms}^{-1}$
(C) $2\ \text{ms}^{-1}$ (D) $4\sqrt{2}\ \text{ms}^{-1}$

25. In the given system, when the ball of mass m is released from position A, it swings down the dotted arc. A nail is located at a distance d below the point of suspension such that the ball just swings completely around a circle centered on the nail. Then minimum value of d possible is



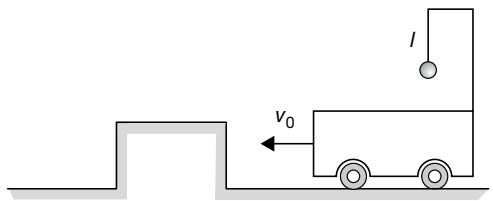
(A) 0.6 L (B) 0.4 L
(C) 0.5 L (D) 0.3 L

26. A 60 g bullet is fired through a stack of fibre board sheets 200 mm thick. If the bullet approaches the stack with a velocity of $600\ \text{ms}^{-1}$ and emerges out with a velocity of $300\ \text{ms}^{-1}$, the average resistance offered to the bullet is
(A) 40.5 kN (B) 2 kN
(C) 20.25 kN (D) 10 kN

27. A particle of mass m begins to slide down a fixed smooth sphere from the top. What is its tangential acceleration when it leaves contact with the sphere?

(A) $\frac{2g}{3}$ (B) $\frac{\sqrt{5}}{3}g$
(C) g (D) $\frac{g}{3}$

28. A bob is suspended from a crane by a cable of length $l = 5$ m. The crane and load are moving at a constant speed v_0. The crane is stopped by an obstacle such that the bob on the cable swings out an angle of $60°$. The initial speed v_0 is $\left(g = 9.8\ \text{ms}^{-2}\right)$



(A) $10\ \text{ms}^{-1}$ (B) $7\ \text{ms}^{-1}$
(C) $4\ \text{ms}^{-1}$ (D) $2\ \text{ms}^{-1}$

29. A chain of length l and mass m lies on the surface of a smooth sphere of radius $R(R > l)$ with one end tied on the top of the sphere. Then the gravitational potential energy of the chain with reference level at the centre of sphere is given by

(A) $\frac{mR^2g}{l}\sin\left(\frac{l}{R}\right)$ (B) $\frac{mR^2g}{l}\cos\left(\frac{l}{R}\right)$
(C) $\frac{mR^2g}{l}\cot\left(\frac{R}{l}\right)$ (D) $\frac{mR^2g}{l}\tan\left(\frac{R}{l}\right)$

30. A particle of mass m attached to a string of length l is describing circular motion on a smooth plane inclined that makes an angle α with the horizontal. For the particle to reach the highest point its minimum velocity at the lowest point should be

(A) $\sqrt{5gl}$ (B) $\sqrt{5gl(\cos\alpha + 1)}$
(C) $\sqrt{5gl\tan\alpha}$ (D) $\sqrt{5gl\sin\alpha}$

31. A block is attached to a spring of stiffness k. The other end of the spring is attached to a fixed wall. The entire system lie on a horizontal surface and the spring is in natural state. The natural length of the spring is l_0. If the block is slowly lifted up vertically to a height $\frac{5}{12}l_0$ from its initial position. The work done by the

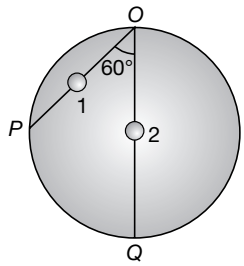
(A) lifting force is $\frac{kl_0^2}{288} + \frac{5}{12}mgl_0$

(B) lifting force is $-\frac{5}{12}mgl_0 + \frac{kl_0^2}{288}$

(C) gravity is $\frac{5}{12}mgl_0$

(D) spring force is $\frac{kl_0^2}{288}$

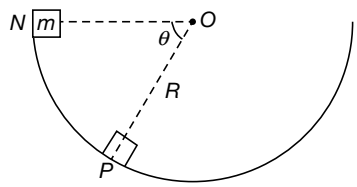
32. Two particles 1 and 2 are descending on two frictionless chords OP and OQ. The ratio of the speeds of the particles 1 and 2 respectively, when they reach the circumference is



(A) $\frac{1}{4}$ (B) $\frac{1}{2\sqrt{2}}$

(C) $\frac{1}{2}$ (D) 1

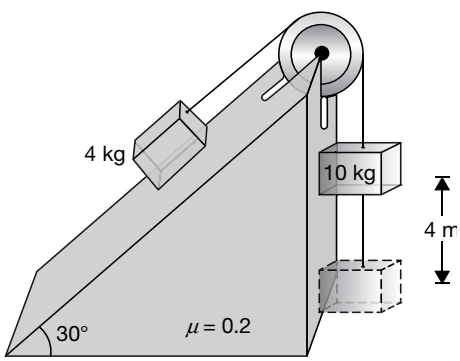
33. An object of mass m is released from rest at N to move along the fixed smooth circular track of radius R as shown in figure. The ratio of magnitudes of centripetal force and normal reaction of the track on the object at the point P is



(A) $\frac{2}{3}\sin\theta$ (B) $\frac{2}{3}\cos\theta$

(C) $\frac{2}{3}$ (D) $\frac{3}{2}$

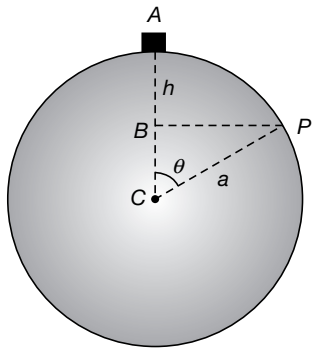
34. Two blocks of masses 10 kg and 4 kg are connected by a flexible inextensible string as shown in the figure. The pulley is assumed to be frictionless. The coefficient of friction between the inclined surface and the 4 kg block is 0.2 and angle of inclination of the plane is 30°. Assuming that the system is released from rest, the velocity of 10 kg block when it has moved 4 m down is



(A) 5.6 ms^{-1} (B) 6.5 ms^{-1}

(C) 41.8 ms^{-1} (D) 13 ms^{-1}

35. A small body A starts sliding off the top of a smooth sphere of radius a. The angle θ corresponding to the point P at which the body breaks off the sphere is



(A) $\cos^{-1}\left(\frac{h}{a}\right)$ (B) $\cos^{-1}\left(\frac{2h}{a}\right)$

(C) $\cos^{-1}\left(\frac{a}{h}\right)$ (D) $\cos^{-1}\left(\frac{a}{2h}\right)$

36. A body is displaced from $(0, 0)$ to $(1, 1)$ m along the path $x = y$ by a force $\vec{F} = \left(x^2\hat{j} + y\hat{i}\right)$ N. The work done by this force is

(A) $\frac{4}{3}$ J (B) $\frac{5}{6}$ J

(C) $\frac{3}{2}$ J (D) $\frac{7}{5}$ J

37. A dam is situated at a height of 550 metre above sea level and supplies water to a power house which is at a height of 50 metre above sea level. 2000 kg of water passes through the turbines per second. The maximum electrical power output of the power house if the whole system were 80% efficient is

(A) 8 MW (B) 10 MW

(C) 12.5 MW (D) 16 MW

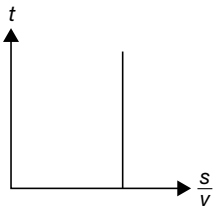
38. A particle of mass m attached to an inextensible light string is moving in a vertical circle of radius r. Velocity at the highest point is v_0 and is just sufficient to complete the vertical circle. Tension in the string when it becomes horizontal is

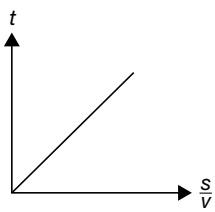
a. $\frac{3mv_0^2}{r}$ b. $\frac{9mv_0^2}{r}$ c. $3mg$

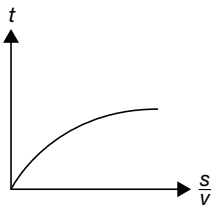
(A) a only (B) b only

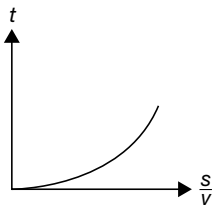
(C) Both b and c (D) Both a and c

39. A body is moved from rest along a straight line by a machine delivering constant power. The ratio of displacement and velocity, $\left(\frac{s}{v}\right)$ varies with time t as

(A) 

(B) 

(C) 

(D) 

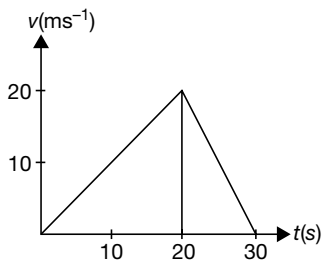
40. A particle suspended from a fixed point, by a light inextensible thread of length L is projected horizontally from its lowest position with velocity $\sqrt{\frac{7gL}{2}}$. The string will slack after swinging through an angle θ, such that θ equals

(A) $30°$ (B) $135°$
(C) $120°$ (D) $150°$

41. A man pulls a bucket of water from a depth of h from a well. If the mass of the rope and that of bucket full of water are m and M respectively, the work done by the man is

(A) $(M+m)gh$ (B) $\left(M+\frac{m}{2}\right)gh$
(C) $\left(\frac{M+m}{2}\right)gh$ (D) $\left(\frac{M}{2}+m\right)gh$

42. v-t graph of an object of mass 2 kg is shown. Select the incorrect statement



(A) The average acceleration of the object is zero
(B) The average velocity of the object is zero
(C) The average force on the object is zero
(D) Work done on the object in 30 s is zero

43. A body of mass 2 kg, initially at rest, is acted upon simultaneously by two forces, one of 4 N and the other of 3 N, acting at right angles to each other. The kinetic energy of the body after 20 s is

(A) 500 J (B) 1250 J
(C) 2500 J (D) 5000 J

44. Power supplied to a particle of mass 2 kg varies with time t (in second) as $P=\frac{3t^2}{2}$ W. If velocity of particle at $t=0$ is $v=0$. The velocity of particle at time $t=2$ s will be

(A) $1\ \text{ms}^{-1}$ (B) $2\ \text{ms}^{-1}$
(C) $2\sqrt{2}\ \text{ms}^{-1}$ (D) $4\ \text{ms}^{-1}$

45. A particle of mass m is projected with velocity u at an angle α with horizontal. During the period when the particle descends from highest point to the position where its velocity vector makes an angle $\frac{\alpha}{2}$ with horizontal. Work done by gravity force is

(A) $\frac{1}{2}mu^2\tan^2\alpha$ (B) $\frac{1}{2}m^2\tan^2\frac{\alpha}{2}$
(C) $\frac{1}{2}mu^2\cos^2\alpha\tan^2\frac{\alpha}{2}$ (D) $\frac{1}{2}mu^2\cos^2\frac{\alpha}{2}\sin^2\alpha$

46. A uniform rod of length 1 m and mass 100 g is pivoted at one end and is hanging vertically. It is displaced through $60°$ from the vertical. The increase in its potential energy is $\left(g=10\ \text{ms}^{-2}\right)$

(A) 0.25 J (B) 0.5 J
(C) 0.75 J (D) 1.0 J

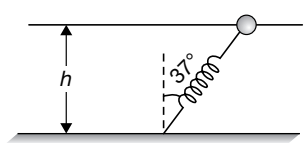
47. A running man has half the kinetic energy of a boy of half his mass. The man speeds up by $1.0\ \text{ms}^{-1}$ and then has the same kinetic energy as the boy. The original speed of the boy was

(A) $2.4\ \text{ms}^{-1}$ (B) $9.6\ \text{ms}^{-1}$
(C) $4.8\ \text{ms}^{-1}$ (D) $7.2\ \text{ms}^{-1}$

48. A body of mass m accelerates uniformly from rest to a velocity v_0 in time t_0. The instantaneous power delivered to the body at any time t is

(A) $\frac{mv_0t}{t_0}$ (B) $\frac{mv_0{}^2t}{t_0}$
(C) $\frac{mv_0t^2}{t_0}$ (D) $\frac{mv_0^2}{t_0^2}t$

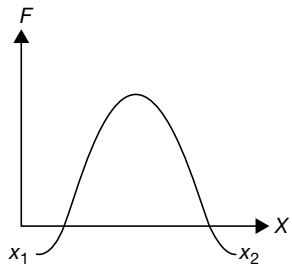
49. One end of a spring of natural length h and spring constant k is fixed at the ground and the other is fitted with a smooth ring of mass m which is allowed to slide on a horizontal rod fixed at a height h. Initially the spring makes an angle of $37°$ with the vertical when the system is released from rest. The speed of the ring when the spring becomes vertical is



(A) $\sqrt{\frac{k}{m}}$ (B) $\frac{h}{2}\sqrt{\frac{k}{m}}$

(C) $\frac{h}{3}\sqrt{\frac{k}{m}}$ (D) $\frac{h}{4}\sqrt{\frac{k}{m}}$

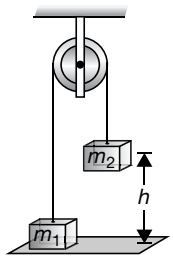
50. The force acting on a particle moving along x-axis varies with the position of the particle as shown in the figure. The particle is in stable equilibrium at



(A) $x = x_1$ (B) $x = x_2$

(C) both x_1 and x_2 (D) neither x_1 nor x_2

51. Two masses m_1 and $m_2 (m_2 > m_1)$ are positioned as shown in figure, m_1 being on the ground and m_2 at a height h above the ground. When m_2 is released, the speed at which it hits the ground will be



(A) $\sqrt{\frac{2ghm_1}{m_2}}$ (B) $\sqrt{\frac{2gh(m_1-m_2)}{(m_1+m_2)}}$

(C) $\sqrt{\frac{2gh(m_1+m_2)}{(m_1-m_2)}}$ (D) $\sqrt{\frac{2gh(m_2-m_1)}{(m_1+m_2)}}$

52. A block of mass m slides down a rough inclined plane of inclination θ with horizontal with zero initial velocity. The coefficient of friction between the block and the plane is μ with $\theta > \tan^{-1}(\mu)$. The rate of work done by the force of friction at time t is

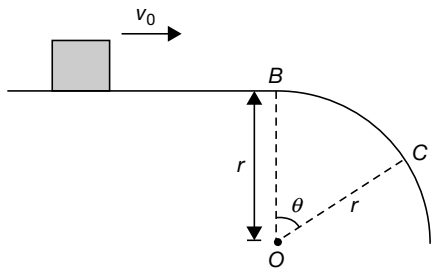
(A) $\mu mg^2 t \sin\theta$

(B) $mg^2 t(\sin\theta - \mu\cos\theta)$

(C) $\mu mg^2 t\cos\theta(\sin\theta - \mu\cos\theta)$

(D) $\mu mg^2 t\cos\theta$

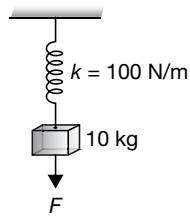
53. A small block slides with velocity $v_0 = \frac{1}{2}\sqrt{gr}$ on the horizontal frictionless surface as shown in the figure. The block leaves the surface at point C. The angle θ in the figure is



(A) $\cos^{-1}\left(\frac{4}{9}\right)$ (B) $\cos^{-1}\left(\frac{3}{4}\right)$

(C) $\cos^{-1}\left(\frac{1}{2}\right)$ (D) None of the above

54. A vertical spring of force constant $100\ \text{Nm}^{-1}$ is attached with a hanging mass of $10\ \text{kg}$. Now an external force is applied slowly on the mass so that the spring is stretched by additional $2\ \text{m}$. The work done by the force F is $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$



(A) 200 J (B) 400 J

(C) 450 J (D) 600 J

55. A child on a swing is 1 m above the ground at the lowest point and 6 m above the ground at the highest point. The horizontal speed of the child at the lowest point of the swing is approximately

(A) $8\ \text{ms}^{-1}$ (B) $10\ \text{ms}^{-1}$

(C) $12\ \text{ms}^{-1}$ (D) $14\ \text{ms}^{-1}$

56. A force F acting on a body depends on its displacement x as $F \propto x^{-\frac{1}{3}}$. The power delivered by F will depend on displacement as

(A) $x^{\frac{2}{3}}$ (B) $x^{-\frac{5}{3}}$

(C) $x^{\frac{1}{2}}$ (D) x^0

57. Two conservative forces $\vec{F}_1$ and $\vec{F}_2$ act on an object. The relation between W_1 and W_2, where $W_1 = \oint(\vec{F}_1 + \vec{F}_2)\cdot d\vec{l}$ and $W_2 = \oint(\vec{F}_1 - \vec{F}_2)\cdot d\vec{l}$

(A) $W_1 > W_2$ (B) $W_1 = W_2 \neq 0$

(C) $W_1 = W_2 = 0$ (D) $W_1 < W_2$

58. A smooth sphere of radius R is made to translate in a straight line with a constant acceleration whose magnitude is equal to the value of acceleration due to gravity. A particle kept on the top of the sphere is released from there with zero velocity with respect to the sphere. The speed of particle with respect to the sphere as a function of angle θ with the vertical, as it slides down is

(A) $\frac{\sqrt{Rg(\sin\theta+\cos\theta)}}{2}$

(B) $\sqrt{Rg(1+\cos\theta-\sin\theta)}$

(C) $\sqrt{4Rg\sin\theta}$

(D) $\sqrt{2Rg(1+\sin\theta-\cos\theta)}$

59. A 100 kg drop hammer H is operated by a motor. It falls from a height of 9 m on thick nail N which is to be driven into the hard block. The hammer has to be lifted to height of 9 m in 12 s with constant velocity. If the efficiency of the motor is assumed to be 80%, the power required by the motor is approximately

(A) 0.49 kW (B) 4.9 kW

(C) 0.94 kW (D) 6 kW

60. A block of mass 1 kg is attached to one end of a spring of force constant $k = 20\ \text{Nm}^{-1}$. The other end of the spring is attached to a fixed rigid support. This spring block system is made to oscillate on a rough horizontal surface having $\mu = 0.04$. The initial displacement of the block from the equilibrium position is $a = 30$ cm. The number of times the block passes from the mean position before coming to rest is $(g = 10\ \text{ms}^{-2})$

(A) 15 (B) 11

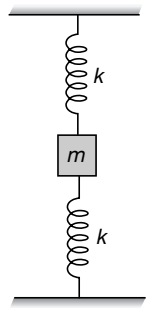
(C) 7 (D) 6

61. A motor has an electrical input of 30 kJ and is used to raise a 100 kg load to a height of 25 m when fitted to a crane winch. What is the efficiency of winch? $(g = 10\ \text{ms}^{-2})$

(A) 7.5% (B) 75%

(C) 83.3% (D) 0.75%

62. A block of mass m is attached to the two springs in vertical plane as shown in the figure. If initially both the springs are at their natural lengths. Then velocity of the block is maximum at displacement x given as

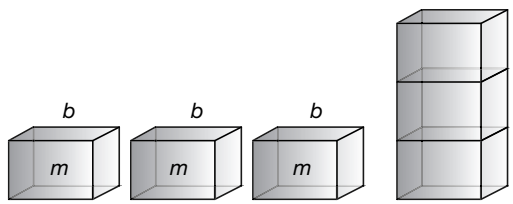


(A) $x = \frac{mg}{2k}$ (B) $x = \frac{mg}{k}$

(C) $x = \frac{mg}{4k}$ (D) $x = \frac{3mg}{2k}$

x is the displacement of the block from its initial position.

63. Three similar cubical slabs each of mass m and edge b are lying on ground. Work done to arrange them one over the other is



(A) $1.5mgb$ (B) $2mgb$

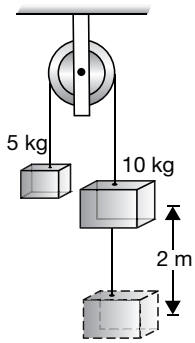
(C) $3mgb$ (D) $4mgb$

64. The potential energy function for the force between two atoms in a diatomic molecule can be expressed approximately as $U = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are positive constants and x is the distance between atoms. The dissociation energy (the energy needed to break up the molecule into separated atoms to infinity) is

(A) $\frac{b^2}{2a}$ (B) $\frac{b^2}{a}$

(C) $\frac{b^2}{4a}$ (D) $\frac{4b^2}{a}$

65. Two blocks of masses 10 kg and 5 kg are connected by a flexible but inextensible string over a shaft as shown in the figure. The system starts from rest with 10 kg mass moving downward. If a constant frictional force of 25 N acts at the shaft, the velocity when the 10 kg block has moved 2 m down is



(A) $2.6\ \text{ms}^{-1}$ (B) $6.2\ \text{ms}^{-1}$
(C) $3.2\ \text{ms}^{-1}$ (D) $4\ \text{ms}^{-1}$

66. Force acting on a particle moving in a straight line varies with the velocity of the particle as $F = \frac{K}{v}$, where K is a positive constant. The work done by this force in time t is

(A) $\frac{K}{v^2}t$ (B) $2Kt$
(C) Kt (D) $\frac{2Kt}{v^2}$

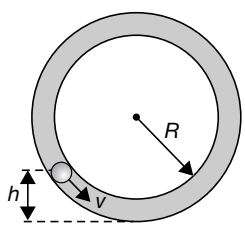
67. A 2 HP pump is used to lift water to the overhead tank on the roof of a building. It has to be operated 8 hours per day and the cost of 1 unit of electricity is ₹6. The electricity bill for month of June will be

(A) ₹2148 (B) ₹2450
(C) ₹1580 (D) ₹840

68. A pendulum of mass 1 kg and length 1 m is released from rest when it makes an angle of $60°$ with the vertical. The power delivered by all the forces acting on the bob when it makes an angle of $30°$ with the vertical is $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$

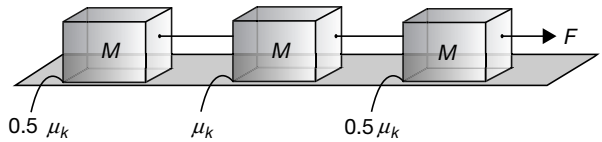
(A) Zero (B) 13.4 watt
(C) 20.4 watt (D) 24.6 watt

69. The minimum speed v with which a small ball should be pushed inside a smooth vertical tube of radius R from a height h such that it may reach the top of the tube is



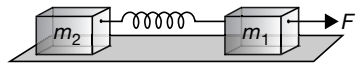
(A) $\sqrt{2g(h+2R)}$ (B) $\frac{5}{2}R$
(C) $\sqrt{g(5R-2h)}$ (D) $\sqrt{2g(2R-h)}$

70. Let a system of three blocks each of mass M, attached by light cords, placed on a horizontal surface move through a distance x with constant velocity. Then work done by the external force is



(A) $3\mu_k Mgx$ (B) $2\mu_k Mgx$
(C) Zero (D) $\frac{2}{3}\mu_k Mgx$

71. Two blocks of masses $m_1 = 1\ \text{kg}$ and $m_2 = 2\ \text{kg}$ are connected by a non-deformed light spring. They are lying on a rough horizontal surface. The coefficient of friction between the blocks and the surface is 0.4. The minimum constant force F that has to be applied in horizontal direction to the block of mass m_1 in order to just shift the other block is $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$

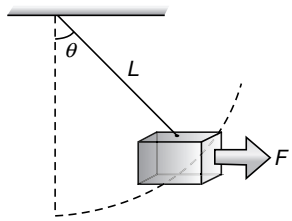


(A) 8 N (B) 15 N
(C) 10 N (D) 25 N

72. A man throws a brick to a height of 5 m where it reaches with a speed of $10\ \text{ms}^{-1}$. If he throws it such that it just reaches that required height, then the percentage of energy saved is $\left(g = 10\ \text{ms}^{-2}\right)$

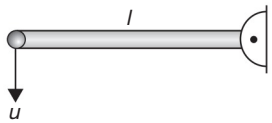
(A) 29% (B) 46%
(C) 38% (D) 50%

73. An object of mass m is tied to string of length L and a variable horizontal force is applied on it which starts at zero and gradually increases (it is pulled extremely slowly so that equilibrium exists at all times) until the string makes an angle θ with the vertical. Work done by the force F is



(A) $mgL(1-\cos\theta)$
(B) $mgL(1-\sin\theta)$
(C) mgL
(D) $mgL(1+\tan\theta)$

74. A ball of mass m is attached to one end of a light rod of length l, the other end of which is hinged. The minimum velocity v that should be imparted to the ball downwards, so that it can complete the circle is



(A) $\sqrt{gl}$ (B) $\sqrt{2gl}$

(C) $\sqrt{3gl}$ (D) $\sqrt{5gl}$

75. A simple pendulum swings with angular amplitude θ. When string is vertical, tension in it is twice the tension in extreme position. Then, $\cos\theta$ is equal to

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

76. A particle is released from a height H has at a certain instant kinetic energy twice its potential energy. The height and speed of particle at that instant are

(A) $\frac{H}{3}, \sqrt{\frac{2gH}{3}}$ (B) $\frac{2H}{3}, \sqrt{\frac{2gH}{3}}$

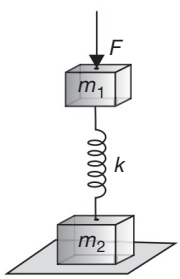
(C) $\frac{H}{3}, 2\sqrt{\frac{gH}{3}}$ (D) $\frac{H}{3}, \sqrt{2gH}$

77. A uniform flexible chain of mass m and length $2l$ hangs in equilibrium over a smooth horizontal pin of negligible diameter. One end of the chain is given a small vertical displacement so that the chain slips over the pin. The speed of chain when it leaves pin is

(A) $\sqrt{2gl}$ (B) $\sqrt{gl}$

(C) $\sqrt{4gl}$ (D) $\sqrt{3gl}$

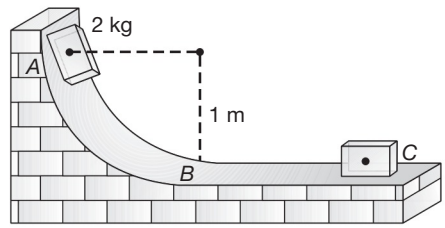
78. A system consists of two blocks of masses m_1 and m_2 respectively connected by a spring of force constant k. The force (F) that should be applied to the upper block for which the lower one just lifts after the force is removed is



(A) $m_1 g$ (B) $\frac{m_1 m_2}{m_1 + m_2} g$

(C) $(m_1 + m_2)g$ (D) $m_2 g$

79. A block of mass 2 kg is released from A on the track that is one quadrant of a circle of radius 1 m. It slides down the track and reaches B with a speed of $4\ \text{ms}^{-1}$ and finally stops at C at a distance of 3 m from B. The work done against the force of friction is



(A) 10 J (B) 20 J

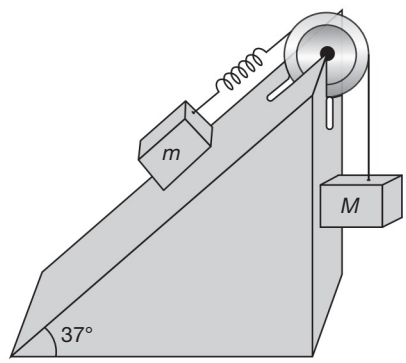
(C) 2 J (D) 6 J

80. A particle moves on a rough horizontal ground with initial velocity v_0 such that $\frac{3}{4}$th of its kinetic energy is lost against friction in time t_0. The coefficient of friction between the particle and the ground is

(A) $\frac{v_0}{gt_0}$ (B) $\frac{v_0}{2gt_0}$

(C) $\frac{3v_0}{4gt_0}$ (D) $\frac{v_0}{4gt_0}$

81. A block of mass m is attached to a light spring of force constant k. The block is placed over a rough inclined surface for which the coefficient of friction is $\mu = \frac{3}{4}$. The minimum value of M required to move the block up the plane is (Neglect mass of string and pulley and friction in pulley)



(A) $\frac{3}{5}$ m (B) $\frac{4}{5}$ m

(C) 2 m (D) $\frac{3}{2}$ m

82. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to

(A) x (B) x^2

(C) $\ln x$ (D) e^x

83. The potential energy of a particle of mass m is given by $U = \begin{cases} \frac{1}{2}kx^2 & \text{for } x < 0 \\ 0 & \text{for } x \geq 0 \end{cases}$. If total mechanical energy of the particle is E. Then its speed at $x = \sqrt{\frac{2E}{k}}$ is

(A) Zero (B) $\sqrt{\frac{2E}{m}}$

(C) $\sqrt{\frac{E}{m}}$ (D) $\sqrt{\frac{E}{2m}}$

84. A car of mass 1000 kg moves on a rough road where road friction is 10 percent of its weight and air friction 2 percent of its weight. To keep the car at a uniform speed of $36\ \text{kmhr}^{-1}$, the power required will be

(A) 112 kW (B) 56 kW

(C) 12 kW (D) 6 kW

85. A particle of mass 0.5 kg is displaced from position $\vec{r}_1(2, 3, 1)$ m to $\vec{r}_2(4, 3, 2)$ m by applying a force of magnitude 30 N which is acting along $(\hat{i} + \hat{j} + \hat{k})$. The work done by the force is

(A) $10\sqrt{3}$ J (B) $30\sqrt{3}$ J

(C) 30 J (D) $\sqrt{15}$ J

86. A 2 kg block is dropped from a height of 0.4 m on a spring of force constant $k = 1960\ \text{Nm}^{-1}$. The maximum compression of the spring is

(A) 0.1 m (B) 0.2 m

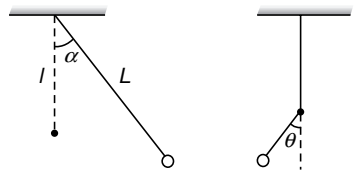
(C) 0.3 m (D) 0.4 m

87. A stone of mass 1 kg tied to a light inextensible string of length $L = \frac{10}{3}$ m is whirling in a circular path of radius L in a vertical plane. If the ratio of the maximum tension in the string to the minimum tension in the string is 4 and if g is taken to be $10\ \text{ms}^{-2}$, the speed of the stone at the highest point of the circle is

(A) $20\ \text{ms}^{-1}$ (B) $10\sqrt{3}\ \text{ms}^{-1}$

(C) $5\sqrt{2}\ \text{ms}^{-1}$ (D) $10\ \text{ms}^{-1}$

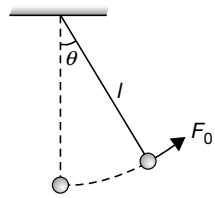
88. A simple pendulum consisting of a mass M attached in a string of length L is released from rest at an angle α. A pin is located at a distance l below the pivot point. When the pendulum swings down, the string hits the pin as shown in the figure. Assuming that after hitting the pin, the pendulum just oscillates, then the maximum angle θ which string makes with the vertical after hitting the pin is



(A) $\cos^{-1}\left(\frac{L\cos\alpha + l}{L + l}\right)$ (B) $\cos^{-1}\left(\frac{L\cos\alpha + l}{L - l}\right)$

(C) $\cos^{-1}\left(\frac{L\cos\alpha - l}{L - l}\right)$ (D) $\cos^{-1}\left(\frac{L\cos\alpha - l}{L + l}\right)$

89. An agent applies force of constant magnitude F_0 always in the tangential direction as shown in the figure. Find the speed of the bob when string becomes horizontal, assuming that it is at rest at its lowest point



(A) $\sqrt{\frac{l}{m}(\pi F_0 - 2mg)}$ (B) $\sqrt{lg}$

(C) $\sqrt{\frac{\pi F_0 l}{m} - 4mg}$ (D) $\sqrt{\frac{F_0 l}{m}}$

90. The tangential acceleration of a particle in a circular motion of radius 2 m is $a_T = \alpha t\ \text{ms}^{-2}$ (where α is a constant) Initially the particle is at rest. At 2 second from the start, total acceleration of the particle is equally inclined to both the accelerations. The value of constant α is

(A) $\frac{1}{2}\ \text{ms}^{-3}$ (B) $1\ \text{ms}^{-3}$

(C) $2\ \text{ms}^{-3}$ (D) $4\ \text{ms}^{-3}$

91. A car weighing 1000 kg is travelling on a level road with a uniform velocity of $60\ \text{kmh}^{-1}$. The resistance offered to the motion due to air and friction is 50 N. The work done by the car in travelling a distance of 1 km is

(A) 5×10^4 J (B) 60×10^3 J

(C) 60×10^6 J (D) 3000 J

92. A body is attached to the lower end of a vertical spiral spring and it is gradually lowered to its equilibrium position. This stretches the spring by a length d. If the same body attached to the same spring is allowed to fall suddenly, what would be the maximum stretching in this case?

(A) d (B) $2d$

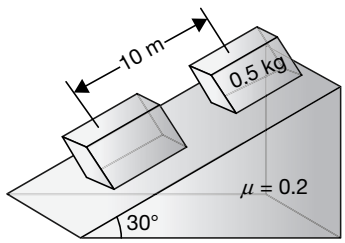
(C) $3d$ (D) $\frac{1}{2}d$

93. A block of mass 4 kg slides on a horizontal smooth surface of a table with a speed $1.6\ \text{ms}^{-1}$. It comes to rest in compressing a spring lying in its path. If the spring has a force constant of $2\ \text{Nm}^{-1}$, the spring should be compressed by

(A) 0.22 m (B) 1.4 m

(C) 2.2 m (D) 0.4 m

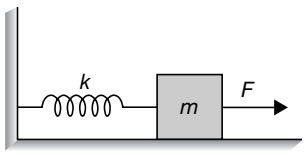
94. A block of mass 0.5 kg has an initial velocity of $10\ \text{ms}^{-1}$ down an inclined rough plane of angle 30°, the coefficient of friction between the block and inclined surface being 0.2. The velocity of the block after it travels a distance of 10 m is



(A) $24\ \text{ms}^{-1}$ (B) $13\ \text{ms}^{-1}$

(C) $8\ \text{ms}^{-1}$ (D) $17\ \text{ms}^{-1}$

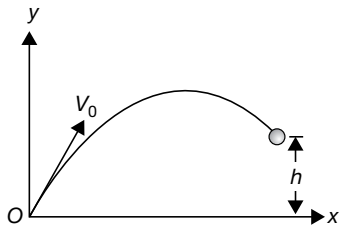
95. If the block in the shown arrangement is acted upon by a constant force F for $t \geq 0$, its maximum speed will be



(A) $\frac{F}{\sqrt{mk}}$ (B) $\frac{2F}{\sqrt{mk}}$

(C) $\frac{F}{\sqrt{2mk}}$ (D) $\frac{\sqrt{2}F}{\sqrt{mk}}$

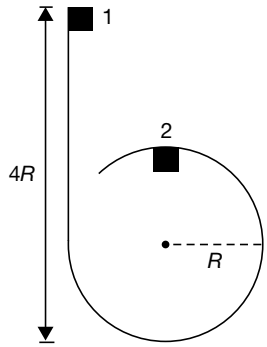
96. A projectile is fired from point O with an initial speed V_0. The speed at an altitude h from the ground is



(A) $\sqrt{V_0^2 + 2gh}$ (B) $\sqrt{2gh}$

(C) $\sqrt{V_0^2 - 2gh}$ (D) $\sqrt{2gh - V_0^2}$

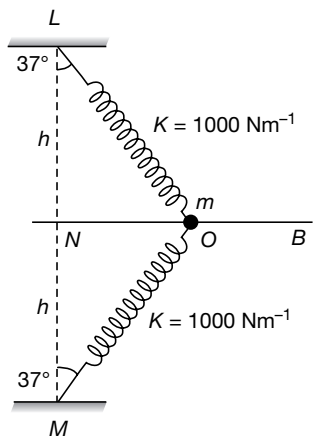
97. A cube of mass M starts at rest from point 1 at a height $4R$, where R is the radius of the circular track. The cube slides down the frictionless track and around the loop. The force which the track exerts on the cube at point 2 is



(A) $3Mg$ (B) Mg

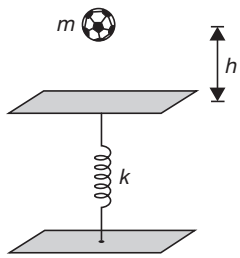
(C) $2Mg$ (D) Cube will not reach the point 2

98. A bead of mass $5\ \text{kg}$ is free to slide on the horizontal rod AB. They are connected to two identical springs of natural length h ms as shown. If initially bead was at O and M is vertically below L then, velocity of bead (in ms^{-1}) at point N will be



(A) $5h$ (B) $13.3h$
(C) $8h$ (D) $4h$

99. A ball of mass m is dropped from height h on a light platform fixed at the top of a vertical spring as shown in Figure.



Speed of the ball will be maximum at the instant when compression x in the spring is

(A) $\frac{mg}{k}$
(B) $\frac{mg}{2k}$
(C) $\frac{2mg}{k}$
(D) Zero

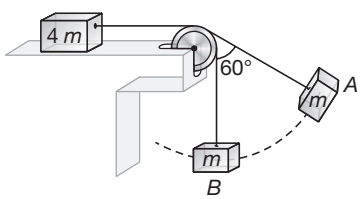
MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. A particle is taken from point A to point B under the influence of a force field. Now it is taken back from B to A and it is observed that the work done in taking the particle from A to B is not equal to the work done in taking it from B to A. If W_{nc} and W_c is the work done by non-conservative forces and conservative forces present in the system respectively, ΔU is the change in potential energy, Δk is the change in kinetic energy, then

(A) $W_{nc} - \Delta U = \Delta k$ (B) $W_c = -\Delta U$
(C) $W_{nc} + W_c = \Delta k$ (D) $W_{nc} - \Delta U = -\Delta k$

2. In the system shown in the figure the mass m moves in a circular arc of angular amplitude $60°$. Mass $4m$ is stationary. Then

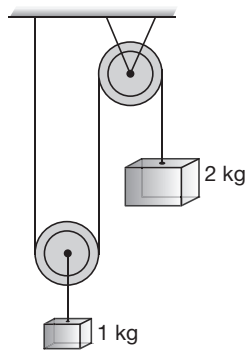


(A) the minimum value of coefficient of friction between the mass $4m$ and the surface of the table is 0.5
(B) the work done by gravitational force on the block m is positive when it moves from A to B
(C) the power delivered by the tension when m moves from A to B is zero
(D) the kinetic energy of m in position B equals the work done by gravitational force on the block when it moves from position A to B

3. An engine is pulling a train of mass m on a level track at a uniform speed v. The resistive force offered per unit mass is f.

(A) Power produced by the engine is mfv.
(B) The extra power developed by the engine to maintain a speed v up a gradient of h in s is $\frac{mghv}{s}$.
(C) The frictional force exerting on the train is mf on the level track.
(D) None of above is correct.

4. In the pulley-block system shown in figure strings are light, the pulleys are light and smooth. The system is released from rest. At $t = 0.3$ s



(A) work done on 2 kg block by gravity is 6 J
(B) work done on 2 kg block by string is –2 J
(C) work done on 1 kg block by gravity is –1.5 J
(D) work done on 1 kg block by string is 2 J

5. A block of mass 2 kg is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force $F = 40$ N. The kinetic energy of the particle increases 40 J in a given interval of time. Then (Take $g = 10\ \text{ms}^{-2}$)

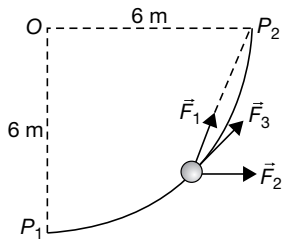
(A) Tension in the string is 40 N

(B) Displacement of the block in the given interval of time is 2 m.

(C) Power developed by this force varies linearly with time

(D) Power developed by this force varies parabolically with displacement.

6. A smooth track in the form of a quarter circle of radius 6 m lies in the vertical plane. A particle moves from P_1 to P_2 under the action of forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$. Force $\vec{F}_1$ is always toward P_2 and is always 20 N in magnitude. Force $\vec{F}_2$ always acts horizontally and is always 30 N in magnitude. Force $\vec{F}_3$ always acts tangentially to the track and is of magnitude 15 N. Select the correct alternative(s)



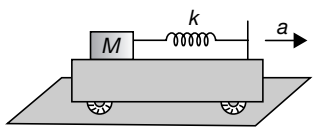
(A) $\vec{F}_1$ is conservative in nature

(B) work done by $\vec{F}_1$ is 120 J

(C) work done by $\vec{F}_2$ is 180 J

(D) work done by $\vec{F}_3$ is 45π J

7. A block of mass M is attached with a spring of spring constant k. The whole arrangement is placed on the frictionless surface of a vehicle as shown in figure. If the vehicle starts moving towards right with an acceleration a (there is no friction anywhere), then



(A) maximum elongation in the spring is $\frac{Ma}{k}$

(B) maximum elongation in the spring is $\frac{2Ma}{k}$

(C) maximum compression in the spring is $\frac{2ma}{k}$

(D) maximum compression in the spring is zero

8. A particle is acted upon by a conservative force $\vec{F} = (7\hat{i} - 6\hat{j})$ N (no other force is acting on the particle). Under the influence of this force particle moves from $(0, 0)$ to $(-3\text{ m}, 4\text{ m})$ then

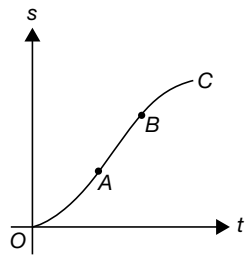
(A) work done by the force is 3 J

(B) work done by the force is -45 J

(C) at $(0, 0)$ speed of the particle must be zero

(D) at $(0, 0)$ speed of the particle must not be zero

9. Displacement time graph of a particle moving in a straight line is as shown in figure. Select the correct alternative(s)



(A) Work done by all the forces in region OA and BC is positive.

(B) Work done by all the forces in region AB is zero.

(C) Work done by all the forces in region BC is negative

(D) Work done by all the forces in region OA is negative.

10. A block initially suspended by an ideal spring of force constant k is pulled down by applying a constant force F and if maximum displacement of block from its initial mean position of rest is x_0 then

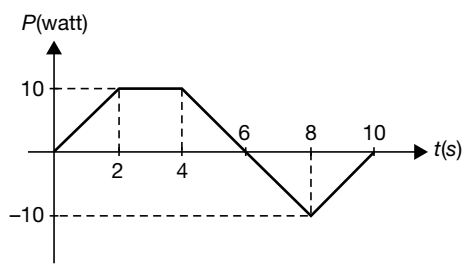
(A) increase in energy stored in spring is kx_0^2

(B) $x_0 = \frac{3F}{2k}$

(C) work done by applied force F is Fx_0

(D) $x_0 = \frac{2F}{k}$

11. Power due to a force acting on a block varies with time t as shown in figure. Then angle between force acting on the block and its velocity is



(A) acute at $t = 1$ s
(B) $90°$ at $t = 3$ s
(C) obtuse at $t = 7$ s
(D) change in kinetic energy from $t = 0$ to $t = 10$ s is 20 J

12. A particle is moving in a conservative force field from point A to point B. U_A and U_B are the potential energies of the particle at points A and B and W_c is the work done in the process of taking the particle from A to B.

(A) $W_c = U_B - U_A$ (B) $W_c = U_A - U_B$
(C) $U_A > U_B$ (D) $U_B > U_A$

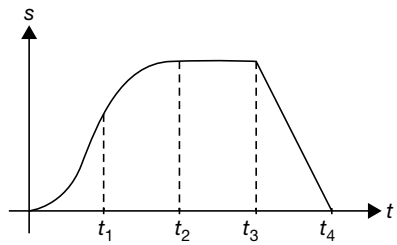
13. Out of the following statements, select the correct alternative(s)
(A) Work done by static friction is always zero.
(B) Work done by kinetic friction can also be positive.
(C) Kinetic energy of a system cannot be increased without applying any external force on the system.
(D) Work energy theorem is valid for non-inertial frames also.

14. Kinetic energy of a particle is continuously increasing with time. It means
(A) resultant force is always along the direction of motion
(B) resultant force is always normal to the direction of motion
(C) resultant force is at an angle less than $90°$ to the direction of motion
(D) its height above the ground level may be decreasing
(E) power associated with resultant force is not equal to zero

15. The potential energy of a particle of mass 1 kg in a conservative field is given as $U = (3x^2y^2 + 6x)$ J, where x and y are measured in meter. Initially particle is at (1, 1) and at rest, then
(A) initial acceleration of particle is $6\sqrt{5}$ ms^2
(B) work done to slowly bring the particle to origin is 9 J
(C) work done to slowly bring the particle to origin is –9 J
(D) if particle is left free it moves in straight line

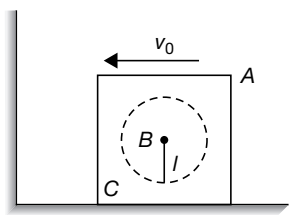
16. A block of mass 2 kg is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force $F = 40$ N. The kinetic energy of the particle is increased by 40 J in a given interval of time. Then $(g = 10 \text{ ms}^{-2})$
(A) tension in the string is 40 N
(B) displacement of the block in the given interval of time is 2 m
(C) work done by gravity is –20 J
(D) work done by tension is 80 J

17. Displacement time graph of a particle moving in a straight line is as shown in figure. From the graph we observe that the work done on the block is



(A) positive from 0 to t_1
(B) negative from t_1 to t_2
(C) zero from t_2 to t_3
(D) negative from t_3 to t_4

18. A Block A is placed on a smooth horizontal surface and a particle C is suspended with the help of light rod from point B of the block as shown. Now both the block A and the particle C are given velocity v_0 towards left. The block A strikes a fixed wall and suddenly stops. Then, (The rod BC is free to rotate about B)



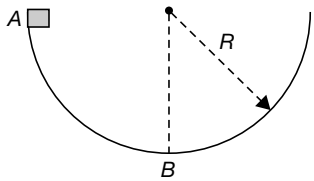
(A) The smallest velocity v_0 for which the particle C will swing in a full circle about the point B is $\sqrt{4gl}$.
(B) The smallest velocity v_0 for which the particle C will swing in a full circle about the point B is $\sqrt{gl}$.
(C) Velocity of point C at the highest point of the circle (for the smallest value of v_0) is zero.
(D) Velocity of point C at the highest point of the circle (for the smallest value of v_0) is $\sqrt{gl}$.

19. A force acting on an object does zero work, if
(A) the force is always perpendicular to its acceleration
(B) the object is stationary but the point of application of the force moves on the object

(C) the force is always perpendicular to its velocity
(D) the object moves in such a way that the point of application of the force remains fixed

20. If the kinetic energy of a body is directly proportional to time t, the magnitude of the force acting on the body is
(A) directly proportional to $\sqrt{t}$
(B) inversely proportional to $\sqrt{t}$
(C) directly proportional to the speed of the body
(D) inversely proportional to the speed of the body

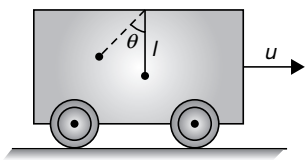
21. A small block of mass m is released from rest from position A inside a smooth hemispherical bowl of radius R as shown in figure. The incorrect statement(s) is/are



(A) Acceleration of block is constant throughout
(B) Acceleration of block at A is g
(C) Acceleration of block at B is $3g$
(D) Acceleration of block at B is $2g$

22. A particle of mass 5 kg moving in the x-y plane has its potential energy given by $U=(-7x+24y)\text{ J}$, where x and y are in metre. The particle is initially at origin and has a velocity $\vec{u}=(14.4\hat{i}+4.2\hat{j})\text{ ms}^{-1}$.
(A) The particle has a speed of 25 ms^{-1} at $t=4\text{ s}$.
(B) The particle has an acceleration of 5 ms^{-2}.
(C) The acceleration of the particle is perpendicular to its initial velocity.
(D) None of the above is correct.

23. A pendulum of length l is suspended on a flat car that is moving with a velocity u on the horizontal road. If the car is suddenly stopped, then, (assume bob of pendulum does not collide anywhere)



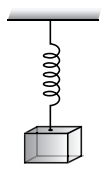
(A) the maximum angle θ with the initial vertical line through which the pendulum swing is $\sin^{-1}\left[\frac{u}{2\sqrt{gl}}\right]$
(B) the maximum angle θ with the initial vertical line through which the pendulum swing is $2\sin^{-1}\left[\frac{u}{2\sqrt{gl}}\right]$
(C) if maximum angle is $60°$, $l=5\text{ m}$ and $g=9.8\text{ ms}^{-2}$ then the initial speed of car u is 7 ms^{-1}
(D) if maximum angle θ is $30°$, $l=5\text{ m}$ and $g=9.8\text{ ms}^{-2}$, then the initial speed of car u is 6 ms^{-1}

24. A small spherical ball is suspended through a string of length l. The whole arrangement is placed in a vehicle which is moving with velocity v. Now suddenly the vehicle stops and ball starts moving along a circular path. If tension in the string at the highest point is twice the weight of the ball and v' be the velocity of ball at the highest point, then
(A) $v=\sqrt{5gl}$ (B) $v=\sqrt{7gl}$
(C) $v'=\sqrt{gl}$ (D) $v'=\sqrt{3gl}$

25. One end of a light spring of force constant k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. In a displacement, the work done by the spring is $\frac{1}{2}kx^2$. The possible case(s) may be
(A) The spring was initially stretched by a distance x and finally was in its natural length.
(B) The spring was initially in its natural length and finally it was compressed by a distance x.
(C) The spring was initially compressed by a distance x and finally was in its natural length.
(D) The spring was initially in its natural length and finally stretched by a distance x.

26. If a particle under the action of a force F has potential energy U then in equilibrium
(A) $F=0$ and $U=0$
(B) $F\neq 0$ and $U=0$
(C) $F=0$ and $U\neq 0$
(D) $F\neq 0$ and $U\neq 0$

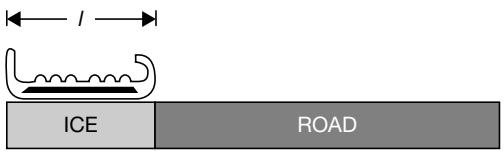
27. A block suspended from a spring is released when the spring is unstretched. Then, which of the following OPTIONS is/are correct?



(A) Block starts oscillating simple harmonically.
(B) Throughout the motion block is accelerated.
(C) Maximum acceleration of block is g.
(D) In the upward motion of block, spring is detached from the block at its mean position, then block will rise to a height greater than from where it was released.

28. The potential energy U, in joule, of a particle of mass 1 kg moving in x-y plane varies as $U = 3x + 4y$, where (x, y) are the co-ordinates of the particle in metre. If the particle starts from rest from $(6, 4)$ at time $t = 0$ then
(A) the particle has constant acceleration
(B) the particle has zero acceleration
(C) the speed of particle when it crosses the y-axis is $10\ \text{ms}^{-1}$
(D) co-ordinates of particle at $t = 1\ \text{s}$ are $(4.5, 2)$

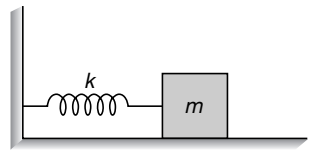
29. A sledge moving over a smooth horizontal surface of ice at a velocity v_0 drives out on a horizontal road and comes to a halt as shown. The sledge has a length l, mass m and friction between runners and road is μ.



(A) No work is done by the friction to switch the sledge from ice to the road.
(B) A work of $\frac{1}{2}\mu mgl$ is done against friction while sledge switches completely on to road.
(C) The distance covered by the sledge on the road is $\left(\frac{v_0^2}{2\mu g} - \frac{l}{2}\right)$.
(D) Total distance moved by the sledge before stopping is $\left(\frac{v_0^2}{2\mu g} + \frac{l}{2}\right)$.

30. A particle moves in a straight line with constant acceleration under a constant force F. Select the incorrect alternative(s)
(A) power developed by this force varies linearly with time
(B) power developed by this force varies parabolically with time
(C) power developed by this force varies linearly with displacement
(D) power developed by this force varies parabolically with displacement

31. The spring is given a compression a and released. The block of mass m again comes to rest when the spring extends by b. During this process, the work done by the spring on the block is W. If μ be the coefficient of friction between the block and the surface, then



(A) $W = \frac{1}{2}k(a+b)^2$ (B) $W = \frac{1}{2}k(a^2 - b^2)$
(C) $\mu = \frac{k(a-b)}{2mg}$ (D) $\mu = \frac{k(a+b)}{2mg}$

32. One end of a light spring of force constant k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. For a possible displacement, the work done by the spring is $\frac{1}{2}kx^2$. The possible case(s) may be
(A) the spring was initially stretched by a distance x and finally was in its natural length
(B) the spring was initially in its natural length and finally it was compressed by a distance x
(C) the spring was initially compressed by a distance x and finally was in its natural length
(D) the spring was initially in its natural length and finally stretched by a distance x

33. A ball of mass m is attached to the lower end of a light vertical spring of force constant k. The upper end of the spring is fixed. The ball is released from rest with the spring at its normal length and comes to rest again after descending through a distance x. Then select the correct statement(s).
(A) $x = \frac{mg}{k}$
(B) $x = \frac{2mg}{k}$
(C) The ball will have no acceleration at the position where it has descended through $\frac{x}{2}$.
(D) The ball will have an upward acceleration equal to g at its lowermost position.

34. The potential energy U for a force field $\vec{F}$ is such that $U = -kxy$, where k is a constant.
(A) $\vec{F} = ky\hat{i} + kx\hat{j}$
(B) $\vec{F} = kx\hat{i} + ky\hat{j}$
(C) The force $\vec{F}$ is a conservative force
(D) The force $\vec{F}$ is a non-conservative force.

35. During projectile motion, the power due to gravitational force
(A) is constant throughout
(B) varies linearly with time
(C) is positive for complete path
(D) is negative for first half, zero at topmost point and positive for rest half

36. Two blocks A and B having different kinetic energies K_A and $K_B (> K_A)$ are released on rough horizontal ground having same coefficient of friction for both the blocks. Then
(A) momentum of B is greater than momentum of A
(B) more work has to be done by friction to stop B
(C) B will travel more distance before stopping
(D) from the given data we cannot compare the distances travelled by them before stopping

37. Which of the following can be negative
(A) kinetic energy (B) potential energy
(C) mechanical energy (D) energy

38. A force $F = -kx^3$ is acting on a block moving along x-axis. Here, k is a positive constant. Work done by this force is
(A) positive in displacing the block from $x = 3$ to $x = 1$
(B) positive in displacing the block from $x = -1$ to $x = -3$
(C) negative in displacing the block from $x = 3$ to $x = 1$
(D) negative in displacing the block from $x = -1$ to $x = -3$

39. Suppose a car is modeled as a cylinder moving with a speed v. If A is the area of cross-section of the car and ρ is the density of air then
(A) power loss due to air resistance is $\frac{1}{2}A\rho v^3$
(B) power loss due to air resistance is $A\rho v^3$
(C) drag force is $\frac{1}{2}A\rho v^2$
(D) drag force is $A\rho v^2$

40. A block is suspended by an ideal spring of force constant k. If the block is pulled down by applying constant force F and if maximum displacement of block from its initial position of rest is x_0 then
(A) Increase in energy stored in spring is kx_0^2
(B) $x_0 = \frac{3F}{2k}$
(C) $x_0 = \frac{2F}{k}$
(D) Work done by applied force F is Fx_0

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

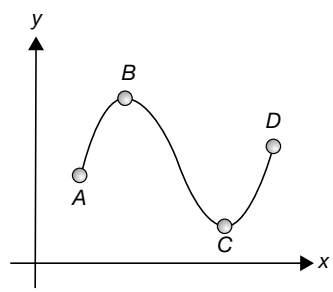
Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** Under the action of a force of constant magnitude, work done is path independent.
Statement-2: Work done by force of gravity is path independent.

2. **Statement-1:** Two cars of unequal mass are moving with equal velocity in straight line. On applying equal resistive force on both the cars, work done to stop them are equal.
Statement-2: Work done is equal to change in its kinetic energy.

3. **Statement-1:** A car and heavy truck have the same speed moving on a road. Same force is applied on both and both stop in same distance.
Statement-2: Same force will cause the different retardation in the two vehicles.

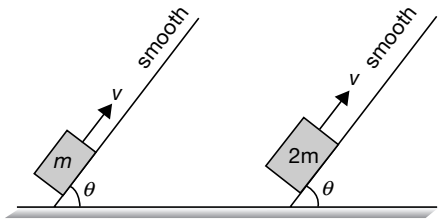
4. **Statement-1:** The potential energy of a particle varies with distance x as shown in the graph.



The force acting on the particle is zero at point B and C.
Statement-2: The slope of y-x curve is zero at point B and C.

5. **Statement-1:** A body is connected to a string and if it just completes a circle, it must have zero velocity at the top.
Statement-2: A body is projected in vertically upward direction, at the highest point the acceleration of particle is non-zero.

6. **Statement-1:** Consider the two situation shown in the figure. In first case height attained by particle above ground is same as the height attained by particle in case II.
Statement-2: As planes are smooth so initial kinetic energy is converted into potential energy. Initial kinetic energy in two cases is different.



7. **Statement-1:** Power developed in a uniform circular motion is always zero.
Statement-2: Work done in case of a uniform circular motion is zero.

8. **Statement-1:** A heavy weight is suspended from the spring. A person raises the weight slowly till the spring become slack. The work done by person is W. The energy stored on the stretched spring was E. The gain in gravitational potential energy is $(W+E)$.
Statement-2: Work done by the spring force is always negative.

9. **Statement-1:** A bomb at rest explodes into two fragments of different masses. The kinetic energies of the two fragments will be in inverse ratio of their masses.
Statement-2: Kinetic energy of a body is inversely proportional to its momentum.

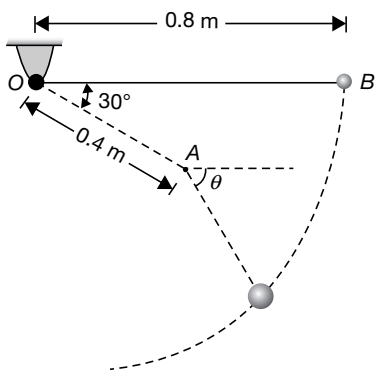
10. **Statement-1:** When a man raises himself up by pulling down on a horizontal bar as in a gymnasium, it is work done by the normal reaction from the bar that goes on to increase his potential energy.
Statement-2: Normal force applied by the bar on the man is responsible for lifting the centre of mass of the man.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options).

Comprehension 1

A small sphere B of mass m is released from rest in the position shown and swings freely in a vertical plane, first about O and then about the peg A after the cord comes in contact with the peg.



Based on above information, answer the following questions.

1. The tension in the cord just before it comes in contact with the peg is

(A) $\frac{mg}{2}$ (B) mg

(C) $\frac{3mg}{2}$ (D) $2mg$

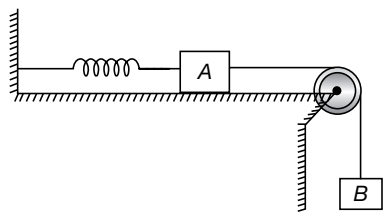
2. The tension in the cord just after it comes in contact with the peg is

(A) $\frac{5mg}{2}$ (B) $\frac{3mg}{2}$

(C) $\frac{mg}{2}$ (D) $\frac{mg}{4}$

Comprehension 2

Consider the set up shown below in figure. Block A has a mass m and the block B has a mass $2m$. The block A is attached to a rigid wall through a spring of spring constant k. The system is released from rest with the spring unstretched. Assume friction to be absent everywhere.



Based on the above facts, answer the following questions.

3. The maximum extension x_{max} produced in spring is

(A) $\frac{mg}{k}$ (B) $\frac{2mg}{k}$

(C) $\frac{3mg}{k}$ (D) $\frac{4mg}{k}$

4. The speed of the block A, when the extension in the spring is half the maximum extension is

(A) $g\sqrt{\frac{m}{k}}$ (B) $2g\sqrt{\frac{2m}{3k}}$

(C) $g\sqrt{\frac{2m}{3k}}$ (D) $2g\sqrt{\frac{m}{3k}}$

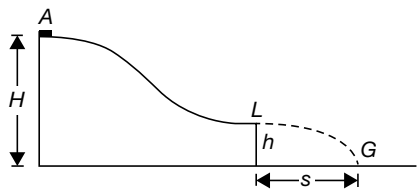
5. Net acceleration of the block B, when extension in the spring is one fourth the maximum extension is

(A) g (B) $\frac{g}{2}$

(C) $\frac{g}{3}$ (D) $\frac{g}{4}$

Comprehension 3

A small disc A slides down with zero initial velocity from the top of a smooth hill of height H which also has a horizontal portion as shown in figure.



Based on the above facts and figures, answer the following questions.

6. The speed of launch of the particle as a projectile at lowest point of the hill L is

(A) $\sqrt{2gH}$ (B) $\sqrt{2gh}$

(C) $\sqrt{2g(H+h)}$ (D) $\sqrt{2g(H-h)}$

7. Time taken by the particle to go from L to G is

(A) $\sqrt{\frac{2H}{g}}$ (B) $\sqrt{\frac{2h}{g}}$

(C) $\sqrt{\frac{2(H+h)}{g}}$ (D) $\sqrt{\frac{2(H-h)}{g}}$

8. The height of the horizontal portion h to ensure the maximum distance s covered by the disc is

(A) H (B) $\frac{H}{2}$

(C) $\frac{H}{3}$ (D) $\frac{H}{4}$

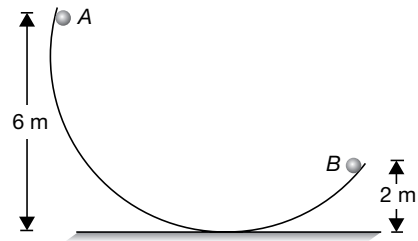
9. The maximum distance then covered by the disc is s_{max}. Then s_{max} equals

(A) $\frac{H}{4}$ (B) $\frac{H}{3}$

(C) $\frac{H}{2}$ (D) H

Comprehension 4

A ball is released from point A as shown in figure. The ball leaves the track at B. All surfaces are smooth.



Based on above information, answer the following questions.

10. Let h be the maximum height from ground reached by ball after leaving track at B. Then

(A) $h = 6$ m

(B) $h < 6$ m

(C) $h > 6$ m

(D) speed of ball at B will change if shape of track is changed keeping h_A and h_B constant

11. If track makes an angle $30°$ with horizontal at B then maximum height attained by ball will be

(A) 3 m (B) 4 m

(C) 4.5 m (D) 5 m

Comprehension 5

The work done by a variable force $\vec{F}$ is measured by taking an infinitesimal length element $d\vec{l} = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz$ such that $W = \int \vec{F} \cdot \vec{dl}$. A particle of mass 2 kg is being acted upon by a variable force $\vec{F} = k\left(x^2y^3\hat{i} + x^3y^2\hat{j}\right)$. The particle is taken from $(0, 0)$ to (a, a) along different paths. Based on the above facts, answer the following questions.

12. The work done to take the particle from $(0, 0)$ to $(a, 0)$ (for $k = 24$) is

(A) Zero (B) $4a^6$

(C) $6a^6$ (D) $8a^6$

13. The work done to take the particle from $(a, 0)$ to (a, a) (for $k = 9$) is

(A) Zero (B) a^6

(C) $3a^6$ (D) $5a^6$

14. The work done to take the particle from $(0, 0)$ to $(a, 0)$ to (a, a) is

(A) ka^6 (B) $\frac{ka^6}{2}$

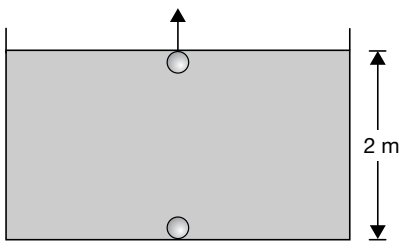
(C) $\frac{ka^6}{3}$ (D) $\frac{ka^6}{4}$

15. The force $\vec{F} = k\left(x^3y^2\hat{i} + x^2y^3\hat{j}\right)$ is a

(A) conservative force
(B) non-conservative force
(C) force which is neither conservative nor non-conservative
(D) None of above facts seems to be true

Comprehension 6

A ball of mass 2 kg is released from the bottom of a tank filled with water upto 2 m. On reaching the top its kinetic energy is found to be 16 J. Ignoring the viscosity and taking $g = 10\ \text{ms}^{-2}$, answer the following questions.



16. The work done by upthrust is W_1 and that done by gravity is W_2, then

(A) $W_1 = 32$ J, $W_2 = -16$ J (B) $W_1 = -16$ J, $W_2 = 32$ J

(C) $W_1 = 40$ J, $W_2 = -24$ J (D) $W_1 = 56$ J, $W_2 = -40$ J

17. Density of the ball in $\frac{\text{kg}}{\text{m}^3}$ is

(A) $\frac{5}{7}\times10^3$ (B) $\frac{3}{4}\times10^3$

(C) $\frac{2}{3}\times10^3$ (D) $\frac{1}{3}\times10^3$

Comprehension 7

The radial component of a conservative force (like electrostatic or gravitational force) can be calculated from the potential energy function U, by using the formula $F = -\frac{dU}{dr}$. Consider a particle bound to a certain point at a distance r from the centre of the force such that the potential energy of this particle varies with r as $U(r) = \frac{A}{r^2} - \frac{B}{r}$ where A and B are positive constants. Based on the above facts, answer the following questions.

18. For the particle, the equilibrium distance is r_0. Then

(A) $r_0 = \frac{A}{B}$ (B) $r_0 = \frac{2A}{B}$

(C) $r_0 = \frac{3A}{B}$ (D) $r_0 = \frac{B}{4A}$

19. The nature of the equilibrium

(A) is stable
(B) is unstable
(C) is neutral
(D) non-predictable, as data provided is insufficient

20. The particle is moved from the equilibrium position to infinity. A work W is done in the process. Then

(A) $W = \frac{B^2}{2A}$ (B) $W = \frac{B^2}{4A}$

(C) $W = \frac{2B^2}{A}$ (D) $W = \frac{4B^2}{A}$

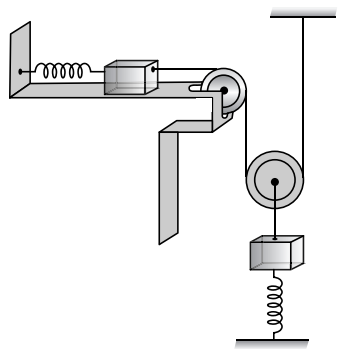
21. It is found that at a particular radial position, the total energy possessed by the particle is $E = -\frac{3B^2}{16A}$. Assuming the motion to be purely radial, the velocity will be zero at position given by (in terms of equilibrium distance r_0)

(A) r_0 (B) $\frac{r_0}{3}$

(C) $\frac{2r_0}{3}$ (D) $\frac{r_0}{5}$

Comprehension 8

The system is released from rest with both the springs in unstretched positions. The mass of each block is 1 kg and force constant of each spring is $10\ \text{Nm}^{-1}$.



Based on above information, answer the following questions.

22. Extension of horizontal spring, in equilibrium, is

(A) 0.2 m (B) 0.4 m

(C) 0.6 m (D) 0.8 m

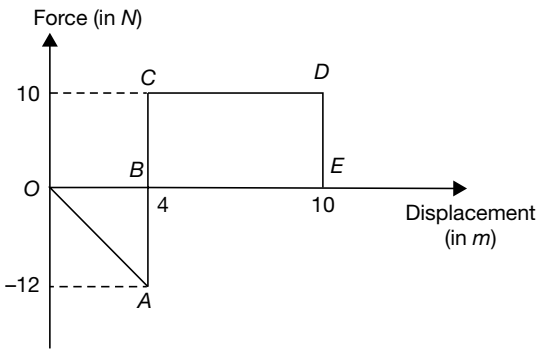
23. Maximum speed of the block placed horizontally is

(A) $1.26\ \text{ms}^{-1}$ (B) $1.93\ \text{ms}^{-1}$

(C) $2.21\ \text{ms}^{-1}$ (D) $3.21\ \text{ms}^{-1}$

Comprehension 9

The kinetic energy is the capacity of the body to do work by virtue of its motion. For a body of mass m moving with a velocity v, it is mathematically defined as $\text{KE} = \frac{1}{2}mv^2$.



For conservative forces doing work, the kinetic energy always increases (or remains constant) and for non-conservative forces doing work, the kinetic energy decreases. A plot of force vs displacement is shown here. The conservative force is shown as positive and that due to non-conservative force is shown as negative. Consider a body of mass 4 kg having a velocity of $20\ \text{ms}^{-1}$ at the origin of the coordinates. Based on the above facts and figures, answer the following questions.

24. The kinetic energy of the particle at the origin of the coordinates is

(A) 400 J (B) 600 J

(C) 800 J (D) 1000 J

25. The kinetic energy of the body at B, $x = 4$ m is

(A) 376 J (B) 576 J

(C) 776 J (D) 824 J

26. The kinetic energy of the body at E, $x = 10$ m is

(A) 1036 J (B) 836 J

(C) 636 J (D) 436 J

27. Let us assume the body to return to O under the action of these forces, then the kinetic energy of the body at the origin of the coordinates will

(A) again be 800 J

(B) be more than 800 J

(C) be less than 800 J

(D) be greater or less than 800 J

Comprehension 10

A bob of mass 1 kg is suspended from an inextensible string of length 1 m. When the string makes an angle of $60°$ with the vertical, speed of the bob is $4\ \text{ms}^{-1}$. Based on above information, answer the following questions.

28. Net acceleration of the bob at this instant is

(A) $10.4\ \text{ms}^{-2}$ (B) $16\ \text{ms}^{-2}$

(C) $18.2\ \text{ms}^{-2}$ (D) $20.4\ \text{ms}^{-2}$

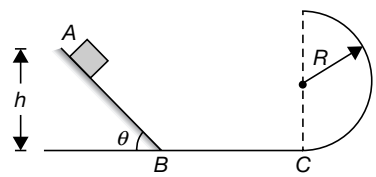
29. The maximum height upto which the bob will rise with respect to the bottom most point is

(A) 1.3 m (B) > 1.3 m

(C) < 1.3 m (D) > 2 m

Comprehension 11

A block is released from height $h(\neq 0)$ on a rough track AB as shown in figure. The coefficient of friction between the block and the surface is $\mu = 0.5$ and the track BC is smooth. From C onwards there is a circular smooth track of radius $R = 50$ cm. Given $\tan\theta = \frac{3}{4}$ and $g = 10\ \text{ms}^{-2}$. Based on above information, answer the following questions.



30. For what value of h block does not leave contact with any surface.

(A) $h = 2.5$ m (B) $h \le 2$ m

(C) $h = 2$ m (D) $h \le 1.5$ m

31. If the block is placed on the above calculated height, how many times will the block cross point C?

(A) Infinite number of times

(B) 2 times

(C) 4 times

(D) 3 times

Comprehension 12

A projectile of mass $m = 1$ kg is projected with velocity v_0 at angle θ. If $\vec{a}$ be the acceleration of the projectile $\vec{v}$ be its velocity, h be its height, K be its kinetic energy and U be its potential energy at any instant, then based on above information, answer the following questions.

32. The magnitude of rate of change of potential energy of the projectile is numerically equal to the magnitude of

(A) $\vec{a}\times\vec{v}$ (B) $\vec{a}\cdot\vec{v}$

(C) $|\vec{a}|\times|\vec{v}|$ (D) $\dfrac{|\vec{a}|}{|\vec{v}|}$

33. A graph is plotted between $\dfrac{K}{U}$ (on y-axis), and $\dfrac{1}{h}$ (on x-axis) then the graph will be

(A) a straight line passing through origin
(B) a straight line not passing through origin
(C) a parabola passing through origin
(D) a parabola not passing through origin

Comprehension 13

A particle of mass 2 kg moving along x axis is being acted upon by a single conservative force denoted by $F(x)$. The potential energy of the particle is $U(x)=20+(x-2)^2$, where U is in joule and x is in metre. The velocity that the particle possesses at $x=5$ m is $4\ \text{ms}^{-1}$. Based on the above facts, answer the following questions.

34. The system possesses a mechanical energy equal to

(A) 25 J (B) 35 J
(C) 45 J (D) 55 J

35. The maximum kinetic energy possessed by the particle is

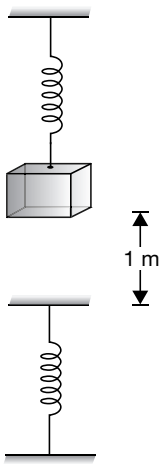
(A) 15 J (B) 25 J
(C) 30 J (D) 35 J

36. For what value of x (say x_0) will the particle be in equilibrium?

(A) $x_0=1$ m (B) $x_0=2$ m
(C) $x_0=3$ m (D) $x_0=4$ m

Comprehension 14

A block of mass $m=2$ kg is suspended by a spring of force constant $k=10\ \text{Nm}^{-1}$. An another spring of same value of force constant is 1 m below it. Initially both the springs were unstretched. The block is released from rest.



Based on above information, answer the following questions.

37. The maximum extension in the upper spring is

(A) 3.26 m (B) 1.67 m
(C) 2.82 m (D) None of these

38. The equilibrium position of the block from where it was released is

(A) 1 m (B) 2 m
(C) 1.2 m (D) 1.5 m

Comprehension 15

The velocity versus displacement of a particle of mass 2 kg moving in a straight line is given by

$$v=\sqrt{9+4s}$$

where v is in ms^{-1} and s in metre. Based on above information, answer the following questions.

39. Work done by all the forces acting on the particle in a time interval from $t=0$ to $t=2$ s is

(A) 10 J (B) 20 J
(C) 30 J (D) 40 J

40. The average power is $\dfrac{3}{4}^{\text{th}}$ the instantaneous power at time t_0, then

(A) $t_0=1.5$ s (B) $t_0=2$ s
(C) $t_0=2.5$ s (D) $t_0=3$ s

Comprehension 16

A pendulum of mass m, length l has angular amplitude θ. Tension in the string at extreme position is T_1 and at bottom is T_2. Based on above information, answer the following questions.

41. If $T_2=T_1$, then θ is equal to

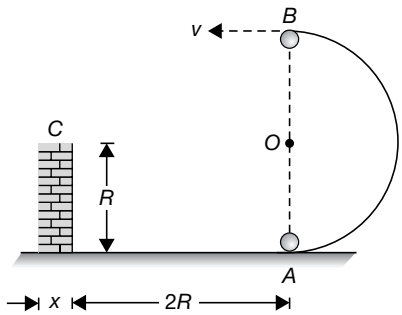
(A) $\cos^{-1}\left(\dfrac{1}{2}\right)$ (B) $\cos^{-1}\left(\dfrac{1}{3}\right)$
(C) $\cos^{-1}\left(\dfrac{2}{3}\right)$ (D) $\cos^{-1}\left(\dfrac{3}{4}\right)$

42. If maximum tension which the string can bear is twice the weight of bob, then the maximum value of θ is

(A) $\cos^{-1}\left(\dfrac{1}{3}\right)$ (B) $\cos^{-1}\left(\dfrac{2}{3}\right)$
(C) $\cos^{-1}\left(\dfrac{3}{4}\right)$ (D) $\dfrac{\pi}{3}$

Comprehension 17

A small ball is given some velocity at point A towards right so that it moves on the semicircular track and does not leave contact upto the highest point B. After leaving the highest point B, it falls at the top of a building of height R and width $x(x\ll 2R)$. (All the surfaces are frictionless).



Based on above information, answer the following questions.

43. The velocity given to the ball at point A so that it may hit the top of the building is

(A) $\sqrt{gR}$ (B) $\sqrt{2gR}$

(C) $\sqrt{4gR}$ (D) $\sqrt{6gR}$

44. If the collision of ball with the building is elastic, then the angle with the horizontal at which the ball will rebound from the top of the building is

(A) $30°$ (B) $45°$

(C) $60°$ (D) $90°$

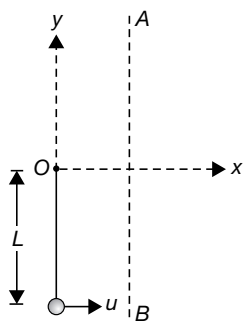
45. The horizontal distance of the ball from the foot of building where the ball strikes the horizontal ground is

(A) $\sqrt{2}R$ (B) $(1+\sqrt{2})R$

(C) $2(1+\sqrt{2})R$ (D) $12R$

Comprehension 18

A particle of mass m is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance $\frac{3\sqrt{3}}{8}L$ from O as shown in figure. The object is given a horizontal velocity $u=\sqrt{\frac{7gL}{2}}$. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Based on above information, answer the following questions.



46. The magnitude of the change in its velocity as it reaches a position, where the string is horizontal

(A) $\sqrt{\frac{3gL}{2}}$ (B) $\sqrt{\frac{5gL}{2}}$

(C) $\sqrt{3gL}$ (D) $\sqrt{5gL}$

47. When the particle's motion ceases to be circular, the angle made by the string with the horizontal is

(A) $30°$ (B) $60°$

(C) $37°$ (D) $\cos^{-1}\left(\frac{2}{3}\right)$

48. When the particle leaves the circular path its velocity vector is

(A) $\sqrt{\frac{gL}{8}}\left(-\hat{i}+\sqrt{3}\hat{j}\right)$ (B) $\sqrt{\frac{gL}{2}}\left(-\hat{i}+\sqrt{3}\hat{j}\right)$

(C) $\sqrt{\frac{gL}{2}}\left(-\sqrt{3}\hat{i}+\hat{j}\right)$ (D) $\sqrt{\frac{gL}{8}}\left(-\sqrt{3}\hat{i}+\hat{j}\right)$

Comprehension 19

The Work Energy Theorem for a particle says, that the total work done on a particle equals the change in its kinetic energy. It means, to change the K.E. of a particle, we have to apply some force on it and the force must do work on it. On a system of particles, the work done by external and internal forces on the system is equal to the change in K.E. of the system. Based on above information, answer the following questions.

49. When you lift a stone from the floor and put it on top of the table slowly, there is no change in its K.E. This implies.

(A) You have not done any work on the stone because had you done any work, then according to W-E theorem the K.E. of the stone would have changed.

(B) Work done by the force of gravity on the stone is also zero as that has also not lead to any change in K.E. of stone.

(C) Gravity has done a positive work and you have done a negative work. Therefore, the total work done on the stone equals zero and hence change in K.E. equals zero

(D) The net work done on the stone is zero and hence its change in K.E. is zero

50. Suppose a heavy box is kept on a smooth inclined plane and is pushed up by a force F parallel to the plane.

(A) The work done by the force F as well as work done by the force of gravity both depend on how fast was the box moving at A and B as the box goes from A to B

(B) The work done by the force of gravity alone depends on how fast was the box moving at A and B as the box goes from A to B

(C) The work done by the force F alone will depend on how fast was the box moving at A and B as the box goes from A to B

(D) None of them depends on how fast the box was moving at A and B as the box goes from A to B

51. A balls is given a speed V_0 on a rough horizontal surface. The ball travels through a distance l on the surface and stops.

(A) The work done by K-friction can be found by W.E. theorem as $-\mu mgl = 0 - \frac{1}{2}mv_0^2$ and hence work done by kinetic friction is positive

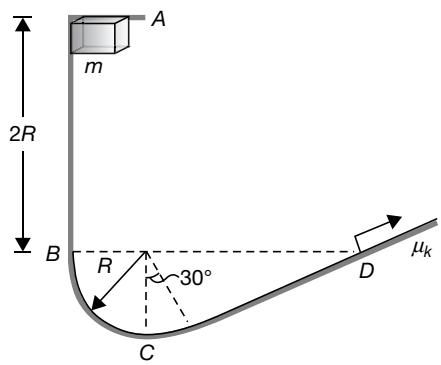
(B) The work done by the kinetic friction can never be positive. Therefore, work done is either $-\mu mgl$ or $-\frac{1}{2}mv_0^2$, only expressed in two different ways

(C) In this case the work done by kinetic friction is negative and it can be expressed by either, $-\mu mgl$ or $-\frac{1}{2}mv_0^2$

(D) The work done by kinetic friction is negative and is μmgl and not $-\frac{1}{2}mv_0^2$ because $\frac{1}{2}mv_0^2$ is the change in K.E. and that is a result of work done by all the forces like gravity and normal reaction as well

Comprehension 20

A block of mass m is released on a track shown in figure. The track is smooth from A to D and rough from point D onwards. Based on above information, answer the following questions.



52. The normal force N_B exerted by the track just after crossing point B is

(A) $2mg$ (B) $3mg$

(C) $4mg$ (D) $5mg$

53. The normal force N_C at C is

(A) $2mg$ (B) $3mg$

(C) $4mg$ (D) $7mg$

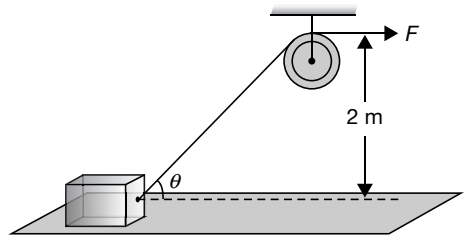
54. The distance travelled along the incline beyond point D where the block stops is

(A) $\frac{2R}{\sqrt{3}\mu_k + 1}$ (B) $\frac{4R}{\mu_k + 1}$

(C) $\frac{4R}{\sqrt{3}\mu_k + 1}$ (D) $\frac{4R}{\sqrt{3}\mu_k + 3}$

Comprehension 21

A force $F = 50$ N is applied at one end of a string, the other end of which is tied to a block of mass 10 kg. The block is free to move on a frictionless horizontal surface. Take initial instant as $\theta = 30°$ and final instant as $\theta = 37°$. For the time between these two instants, answer the following questions? Take $\sin(37°) = \frac{3}{5}$.



55. Net work done by the force F on the block is

(A) $\frac{20}{3}$ J (B) $\frac{50}{3}$ J

(C) $\frac{100}{3}$ J (D) 75 J

56. What is the final velocity of the block if initially it was at rest?

(A) $\sqrt{5}$ ms^{-1} (B) $\sqrt{\frac{25}{3}}$ ms^{-1}

(C) $\sqrt{\frac{20}{3}}$ ms^{-1} (D) 5 ms^{-1}

57. Find the ratio of initial acceleration to final acceleration of the block.

(A) $\frac{3\sqrt{5}}{8}$ (B) $\frac{5\sqrt{3}}{8}$

(C) $\frac{8\sqrt{3}}{5}$ (D) $\frac{8\sqrt{5}}{3}$

Comprehension 22

A particle moves in x-y plane under the action of a path dependent force $F = y\hat{i} + x\hat{j}$. The work done by the force as it moves in x-y plane can be evaluated by solving the

line integral of force i.e. $\int \vec{F} \cdot d\vec{l}$, where $d\vec{l} = dx\hat{i} + dy\hat{j}$. The position coordinates x and y may vary according to some constraint determined by the path followed by the particle. Based on the information shared, answer the following questions.

58. When the particle moves along a straight-line path from origin to the point (a, a), the work done by the force on the particle is

(A) a^2 (B) $2a$

(C) $\frac{a^2}{2}$ (D) Zero

59. When the particle moves from $(0, 0)$ to $(a, 0)$ and then from $(a, 0)$ to (a, a) in straight line paths, then work done by the force on the particle is

(A) a^2 (B) $2a$

(C) $\frac{a^2}{2}$ (D) Zero

60. The infinitesimal work done (dW) by the given force when displaced through $d\vec{l}$ is given by

$$dW = \vec{F} \cdot d\vec{l} = ydx + xdy$$

From this, it can be inferred that the work done by the force

(A) depends only on initial and final values of x and y

(B) depends on initial and final values and on the path followed

(C) is zero for any values of x and y.

(D) is zero when object returns to original position after following any path.

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

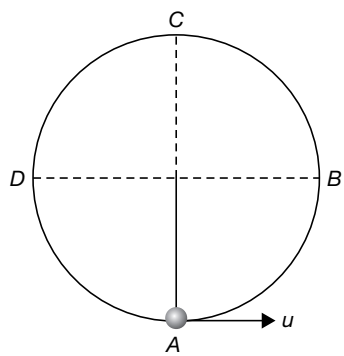
If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	(p)	(q)	(r)	(s)	(t)
B	(p)	(q)	(r)	(s)	(t)
C	(p)	(q)	(r)	(s)	(t)
D	(p)	(q)	(r)	(s)	(t)

1. Match the following

COLUMN-I	COLUMN-II
(A) Work done by all the forces	(p) Change in potential energy
(B) Work done by conservative forces	(q) Change in kinetic energy
(C) Work done by external forces	(r) Change in mechanical energy
(D) Work done by non-conservative forces	(s) Decrease in potential energy

2. A particle is suspended from a string of length l and is given a velocity $u = 3\sqrt{gl}$ at the bottom. If v_B, T_B and v_C, T_C be the velocity, tension at points B and C, then match the following.

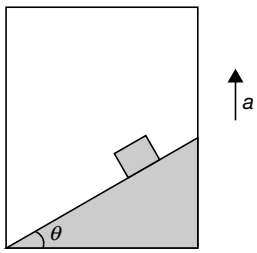


COLUMN-I	COLUMN-II
(A) $\frac{v_B^2}{gl}$	(p) $\frac{7}{2}$
(B) $\frac{v_C^2}{gl}$	(q) 4

(*Continued*)

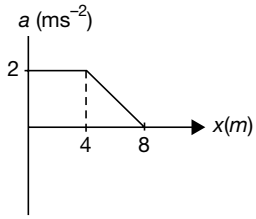
COLUMN-I	COLUMN-II
(C) $\frac{T_B}{2mg}$	(r) 5
(D) $\frac{T_C}{mg}$	(s) 7

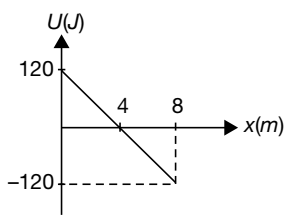
3. A block of mass $1\ \text{kg}$ is stationary with respect to a rough wedge making an angle of $30°$ with horizontal as shown in figure. Starting from rest with an acceleration of $2\ \text{ms}^{-2}$ the work done on the block at $t = 4\ \text{s}$.



COLUMN-I	COLUMN-II
(A) by the gravity	(p) 144 J
(B) by the normal reaction	(q) 32 J
(C) by the friction	(r) 56 J
(D) by all the forces	(s) 48 J

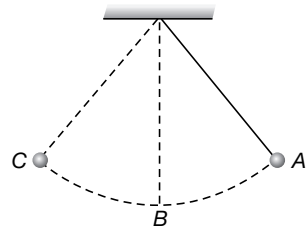
4. Acceleration versus x and potential energy versus x graph of a particle of mass $1\ \text{kg}$ moving along x-axis are shown in figure. Assume that at $x = 0$, the velocity of the particle is $4\ \text{ms}^{-1}$, match the following at $x = 8\ \text{m}$.





COLUMN-I	COLUMN-II
(A) Kinetic energy	(p) 120 J
(B) Work done by conservative forces	(q) 240 J
(C) Total work done	(r) 128 J
(D) Work done by external forces	(s) 112 J
	(t) –112 J

5. A pendulum is released from point A as shown in figure. At any instant, the net force on the bob is making an angle θ with the string. Then match the following.



COLUMN-I	COLUMN-II
(A) For $\theta = 30°$	(p) Particle may be moving along BA
(B) For $\theta = 120°$	(q) Particle may be moving along CB
(C) For $\theta = 90°$	(r) Particle is at A
(D) For $\theta = 0°$	(s) Particle is at B
	(t) None of these

6. Match the following.

COLUMN-I	COLUMN-II
(A) Magnetic potential energy	(p) Positive
(B) Electrostatic potential energy	(q) Negative
(C) Gravitational potential energy	(r) Zero
(D) Elastic potential energy	(s) Not defined

7. Match the following.

COLUMN-I	COLUMN-II
(A) If the work done by force in cyclic path is zero, the force is	(p) Non conservative
(B) If the work done by a force in cyclic path is not zero, the force is	(q) Negative
(C) Work done by friction force can be	(r) Conservative
(D) Work done by spring force can be	(s) Positive

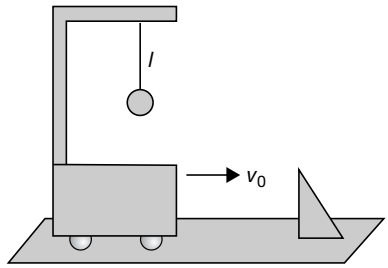
8. Match the following.

COLUMN-I	COLUMN-II
(A) Conservative force	(p) Potential can be defined
(B) Non conservative force	(q) Potential can't be defined
(C) Gravitational force	(r) Path dependent work
(D) Frictional force	(s) Position dependent work

9. A particle of mass $500\ \text{g}$ moves along a horizontal circle of radius $16\ \text{m}$ such that normal acceleration of particle varies with time as $a_n = 9t^2$.

COLUMN-I	COLUMN-II
(A) Tangential force on particle at $t = 1\ \text{s}$ (in Newton)	(p) 72
(B) Total force on particle at $t = 1\ \text{s}$ (in Newton)	(q) 36
(C) Power delivered by total force at $t = 1\ \text{s}$ (in watt)	(r) 7.5
(D) Average power developed by total force over first one second (in watt)	(s) 6

10. A bob of mass $2\ \text{kg}$ is suspended from a vehicle by a rope of length $l = 5\ \text{m}$. The vehicle and the bob are moving at a constant speed v_0. The vehicle is suddenly stopped by a bumper and the bob on the rope swings out a maximum angle of $60°$. Taking $g = 10\ \text{ms}^{-2}$, match the following.



COLUMN-I	COLUMN-II
(A) Net force, in N, acting on the bob at lowest point just after the vehicle is stopped	(p) $5\sqrt{3}$
(B) Acceleration, in ms^{-2}, of the bob at lowest point	(q) $10\sqrt{3}$
(C) Net force, in N, acting on the bob at its highest point	(r) 10
(D) Acceleration, in ms^{-2}, of the bob at its highest point	(s) 20

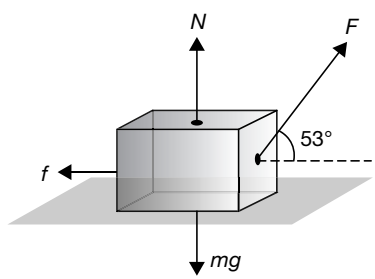
INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. A small block of mass 1 kg is kept on a rough inclined wedge of inclination $45°$ fixed in an elevator. The elevator goes up with a uniform velocity $v = 2\ \text{ms}^{-1}$ and the block does not slide on the wedge. Calculate the work done, in joule by the force of friction on the block in $1\ \text{s}$. Take $g = 10\ \text{ms}^{-2}$.

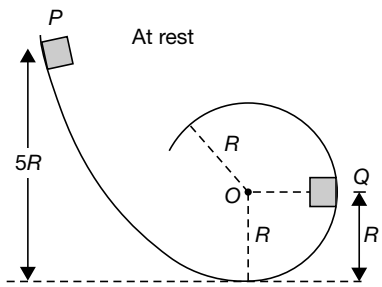
2. A train of mass $2 \times 10^5\ \text{kg}$ has a constant speed of $20\ \text{ms}^{-1}$ up a hill inclined at $\theta = \sin^{-1}\left(\frac{1}{50}\right)$ to the horizontal when the engine is working at 8×10^5 watt. Calculate the resistance to the motion of the train, in newton. Take $g = 9.8\ \text{ms}^{-2}$.

3. A block of mass $m = 4\ \text{kg}$ is dragged $2\ \text{m}$ along a horizontal surface by a force $F = 30\ \text{N}$ acting at $53°$ to the horizontal. The initial speed is $3\ \text{ms}^{-1}$ and $\mu_k = \frac{1}{8}$. Given that $\cos(37°) = \frac{3}{5}$.



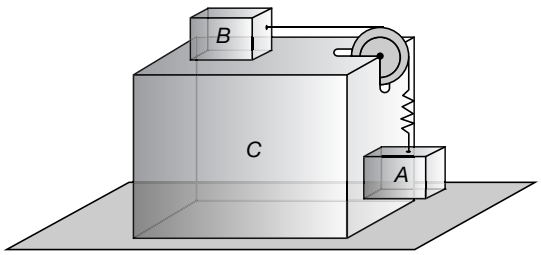
(a) Find the change in kinetic energy of the block, in joule.
(b) Find its final speed, in cms^{-1}.

4. A small block of mass m slides along a smooth frictionless track as shown in the figure.



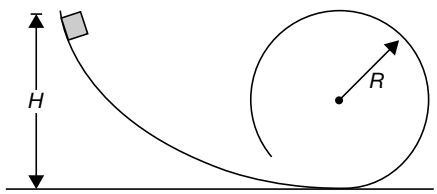
(a) If it starts from rest at P, the resultant force acting on it at Q is $\sqrt{x}$ mg. Calculate x
(b) At height $h = *R$ above the bottom of the loop should the block be released so that the force it exerts against the track at the top of the loop equals its weight. Find $*$.

5. Two blocks A and B are connected to each other by a string and a spring. The string passes over a frictionless pulley as shown in figure.



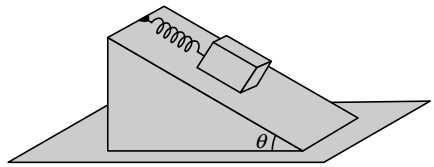
Block B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C, both with the same uniform speed. The coefficient of friction between the surfaces of the blocks is 0.2. The force constant of the spring is $1960\ \text{Nm}^{-1}$. If the mass of block A is 2 kg, calculate the mass of block B, in kg, and the energy stored in the spring in mJ. Take $g = 9.8\ \text{ms}^{-2}$.

6. A body slides down an inclined surface which ends into a vertical loop of radius $R = 40$ cm as shown in figure.



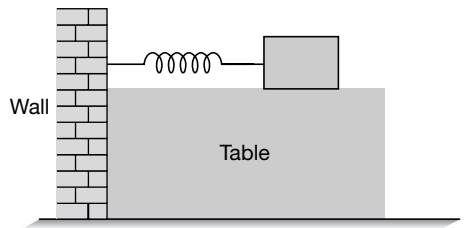
What must be the height H, in cm, of the inclined surface for the body not to fall at the uppermost point of the loop? Assume friction to be absent.

7. A block of mass $m = 2$ kg is attached to a spring whose spring constant is $k = 8\ \text{Nm}^{-1}$. The block slides on an incline for which $\mu_k = \frac{1}{8}$ and $\theta = 37°$ as shown in figure.



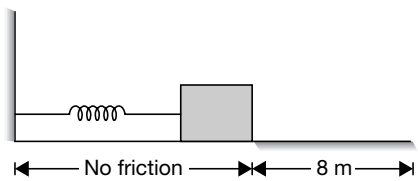
If the block starts at rest with the spring unextended, what is its speed, in ms^{-1}, when it has moved a distance $d = 0.5$ m down the incline?

8. A small block of ice with mass 120 g is placed against a horizontal compressed spring mounted on a horizontal table top that is 2.5 m above the floor as shown in figure.



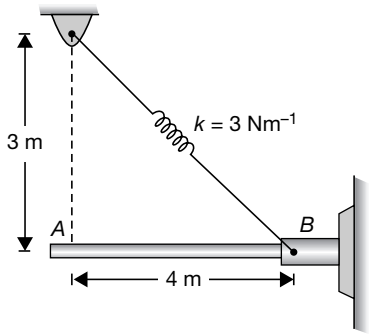
The spring has a force constant $k = 2400\ \text{Nm}^{-1}$ and is initially compressed 5 cm. The mass of the spring is negligible. The spring is released and the block slides along the table, goes off the edge and travels to the floor. If there is negligible, friction between the ice and the table and ice does not melt in the process, what is the speed of the block of ice, in ms^{-1}, when it reaches the floor. Take $g = 10\ \text{ms}^{-2}$

9. Figure shows, a 4 kg block accelerated by a compressed spring whose spring constant is $640\ \text{Nm}^{-1}$. After leaving the spring at the spring's relaxed length, the block travels over a horizontal surface, with a coefficient of kinetic friction of 0.25, for a distance of 8 m before stopping.



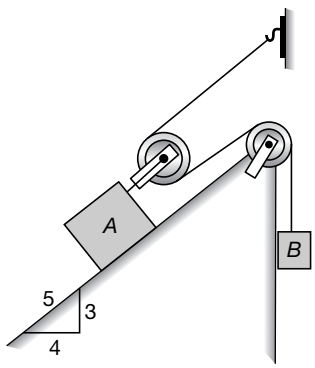
(a) Calculate the increase in the thermal energy, in joule, of the block-floor system.
(b) Calculate the maximum kinetic energy of the block, in joule.
(c) Through what distance is the spring compressed, in cm, before the block begins to move?

10. It is observed that when the 2 kg smooth collar is slided back to point B and released from rest, the collar attains a speed of $2\ \text{ms}^{-1}$ when it arrives at point A as shown in figure.



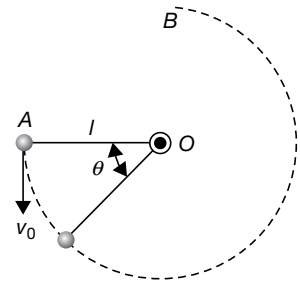
Calculate the necessary unstretched length of the spring, in metre, if it has a stiffness $k = 2\ \text{Nm}^{-1}$.

11. Block A has a weight of 300 N and block B has a weight of 50 N. If the coefficient of kinetic friction between the incline and block A is $\mu_k = \frac{1}{5}$ as shown in figure.



The speed, in ms^{-1}, of block A after it moves 1 m down the plane, starting from rest is $\frac{4}{5}\sqrt{*}$, where $*$ is not readable. Find $*$. Neglect the mass of the cord and pulleys. Take $g = 10\ \text{ms}^{-2}$.

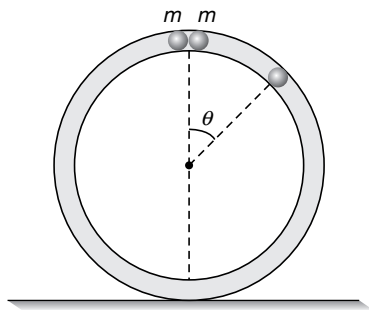
12. The sphere at A is given a downward velocity v_0 of magnitude $5\ \text{ms}^{-1}$ and swings in a vertical plane at the end of a rope of length $l = 2$ m attached to a support at O as shown in figure.



The angle at which the rope will break is $\theta = \sin^{-1}\left(\frac{1}{*}\right)$, where $*$ is not readable knowing that the maximum tension that the rope can withstand is twice the weight of the sphere and taking $g = 10\ \text{ms}^{-2}$, find $*$.

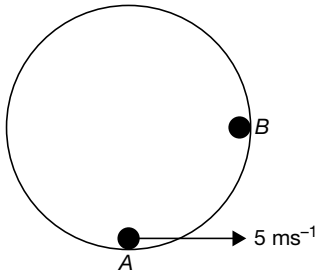
13. A particle is suspended from a fixed point by a string of length 5 m. It is projected from the equilibrium position with such a velocity that the string slackens after the particle has reached a height 8 m above the lowest point.
(a) The velocity of the particle (just before the string slackens), in ms^{-1}, is found to be $\sqrt{*}$, where $*$ is not readable, find $*$.
(b) The height to which the particle can rise further, in cm, is $24x$. Find x.

14. A circular tube of mass M is placed vertically on a horizontal surface as shown in the figure.



Two small spheres, each of mass m, just fit in the tube are released from the top. If θ gives the angle between radius vector of either ball with the vertical, obtain the value of the ratio $\frac{m}{M}$, if the tube just breaks its contact with ground when $\theta = 60°$. Neglect any friction.

15. A ball of mass $1\ \text{kg}$ moves inside a smooth fixed spherical shell of radius $1\ \text{m}$ with an initial velocity $v = 5\ \text{ms}^{-1}$ from the bottom as shown in figure.



The total force acting on the particle at point B is $x\sqrt{x}$ N. Calculate x

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1. [Online September 2020]

A person pushes a box on a rough horizontal platform surface. He applies a force of 200 N over a distance of 15 m. Thereafter, he gets progressively tired and his applied force reduces linearly with distance to 100 N. The total distance through which the box has been moved is 30 m. What is the work done by the person during the total movement of the box?

(A) 5690 J (B) 5250 J

(C) 3280 J (D) 2780 J

2. [Online September 2020]

If the potential energy between two molecules is given by $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation between molecules, and the potential energy are

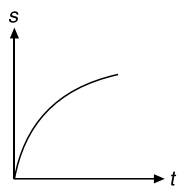
(A) $\left(\frac{B}{A}\right)^{\frac{1}{6}}, 0$ (B) $\left(\frac{B}{2A}\right)^{\frac{1}{6}}, -\frac{A^2}{2B}$

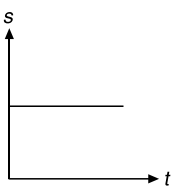
(C) $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, -\frac{A^2}{4B}$ (D) $\left(\frac{2B}{A}\right)^{\frac{1}{6}}, -\frac{A^2}{2B}$

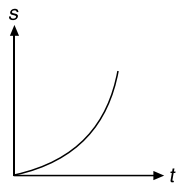
3. [Online September 2020]

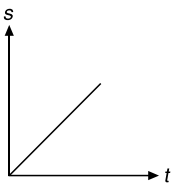
A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) time (t) graph that describes the motion of the particle is

(Graphs are drawn schematically and are not to scale)

(A) 

(B) 

(C) 

(D) 

4. [Online September 2020]

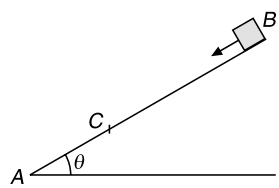
A body of mass 2 kg is driven by an engine delivering a constant power 1 Js^{-1}. The body starts from rest and moves in a straight line. After 9 second, the body has moved a distance …… (in metre).

5. [Online September 2020]

A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F (in N) is …… $\left(g = 10\ \text{ms}^{-2}\right)$

6. [Online September 2020]

A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If $BC = 2AC$, the coefficient of friction is given by $\mu = k\tan\theta$. The value of k is ……



7. [Online January 2020]

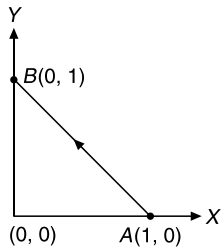
An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed 3 ms^{-1}. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator $\left(g = 10\ \text{ms}^{-2}\right)$ must be at least

(A) 56300 W (B) 48000 W

(C) 66000 W (D) 62360 W

8. [Online January 2020]

Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point $A(1,0)$ to $B(0,1)$ along the line segment shown is ……
(All quantities are in SI units)



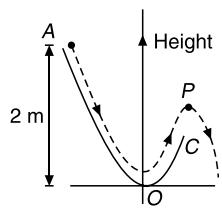
(A) $\frac{3}{2}$ (B) 1
(C) 2 (D) $\frac{1}{2}$

9. [Online January 2020]

A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to …… ms^{-1}. Take 1 HP = 746 W and $g = 10\ ms^{-2}$.

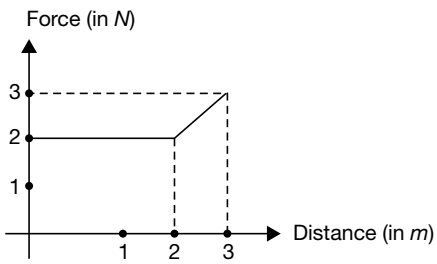
10. [Online January 2020]

A particle $(m = 1\ kg)$ slides down a frictionless track (AOC) starting from rest at a point A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaches its highest point P (height 1 m), the kinetic energy of the particle (in J) is …… (Figure drawn is schematic and not to scale, take $g = 10\ ms^{-2}$)



11. [Online April 2019]

A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the Figure. The kinetic energy of the particle after it has travelled 3 m is



(A) 4 J (B) 2.5 J
(C) 5 J (D) 6.5 J

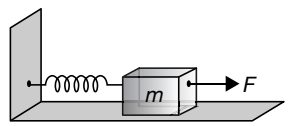
12. [Online April 2019]

A uniform cable of mass M and length L is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be

(A) $\frac{MgL}{n^2}$ (B) $nMgL$
(C) $\frac{MgL}{2n^2}$ (D) $\frac{2MgL}{n^2}$

13. [Online January 2019]

A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is



(A) $\frac{F}{\pi\sqrt{mk}}$ (B) $\frac{\pi F}{\sqrt{mk}}$
(C) $\frac{2F}{\sqrt{mk}}$ (D) $\frac{F}{\sqrt{mk}}$

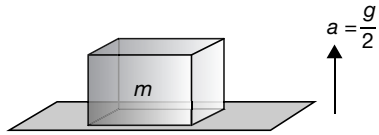
14. [Online January 2019]

A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?

(A) 950 J (B) 900 J
(C) 850 J (D) 875 J

15. [Online January 2019]

A block of mass m is kept on a platform which starts from rest with constant acceleration $\frac{g}{2}$ upward, as shown in figure. Work done by normal reaction on block in time t is



(A) $\frac{3mg^2t^2}{8}$ (B) $-\frac{mg^2t^2}{8}$
(C) 0 (D) $\frac{mg^2t^2}{8}$

16. [Online January 2019]

A particle which is experiencing a force, given by $\vec{F} = 3\hat{i} - 12\hat{j}$, undergoes a displacement of $\vec{d} = 4\hat{i}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy of the end of the displacement?

(A) 15 J (B) 9 J

(C) 12 J (D) 10 J

17. [Online January 2019]

A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then

(A) $K_2 = 2K_1$ (B) $K_2 = \frac{K_1}{4}$

(C) $K_2 = K_1$ (D) $K_2 = \frac{K_1}{2}$

18. [2018]

A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is

(A) $-\frac{k}{4a^2}$ (B) $\frac{k}{2a^2}$

(C) Zero (D) $-\frac{3}{2}\frac{k}{a^2}$

19. [Online 2018]

Two particles of the same mass m are moving in circular orbits because of force, given by $F(r) = -\frac{16}{r} - r^3$. The first particle is at a distance $r = 1$ and the second, at $r = 4$. The best estimate for the ratio of kinetic energies of the first and the second particle is closest to

(A) 10^{-1} (B) 6×10^{-2}

(C) 6×10^{2} (D) 3×10^{-3}

20. [Online 2018]

A body of mass m starts moving from rest along x-axis so that its velocity varies as $v = a\sqrt{s}$ where a is a constant and s is the distance covered by the body. The total work done by all the forces acting on the body in the first t seconds after the start of the motion is

(A) $\frac{1}{4}ma^4t^2$ (B) $4ma^4t^2$

(C) $\frac{1}{8}ma^4t^2$ (D) $8ma^4t^2$

21. [2017]

A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10$ ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be

(A) 10^{-3} kgm^{-1} (B) 10^{-3} kgs^{-1}

(C) 10^{-4} kgm^{-1} (D) 10^{-1} kgm^{-1}s^{-1}

22. [2017]

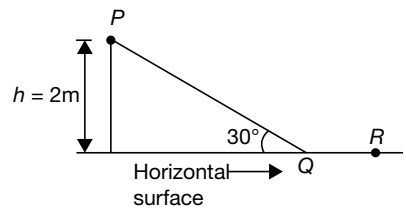
A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 second will be

(A) 4.5 J (B) 22 J

(C) 9 J (D) 18 J

23. [2016]

A point particle having some mass, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance $x(=QR)$, are respectively close to



(A) 0.29 and 6.5 m

(B) 0.2 and 6.5 m

(C) 0.2 and 3.5 m

(D) 0.29 and 3.5 m

24. [2016]

A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8$ ms^{-2}

(A) 12.89×10^{-3} kg (B) 2.45×10^{-3} kg

(C) 6.45×10^{-3} kg (D) 9.89×10^{-3} kg

25. [Online 2016]

Concrete mixture is made by mixing cement, stone and sand in a rotating cylinder drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute (rpm) to ensure proper mixing is close to [Take the radius of the drum to be 1.25 m and its axle to be horizontal]

(A) 27.0 (B) 0.4

(C) 1.3 (D) 8.0

26. [Online 2016]

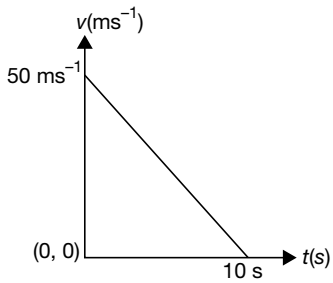
A car of weight W is on an inclined road that rises by 100 m over a distance of 1 km and applies a constant frictional force $\frac{W}{20}$ on the car. While moving uphill on the road at a speed of 10 ms^{-1}, the car needs power P. If it needs power $\frac{P}{2}$ while moving downhill at speed v then value of v is

(A) 20 ms^{-1} (B) 5 ms^{-1}

(C) 15 ms^{-1} (D) 10 ms^{-1}

27. [Online 2016]

Velocity-time graph for a body of mass 10 kg is shown in figure. Work-done on the body in first two seconds of the motion is



(A) –9300 J (B) 12000 J

(C) –4500 J (D) –12000 J

28. [Online 2016]

A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by n^2Rt^2 where n is a constant. The power delivered to the particle by the force acting on it, is

(A) $\frac{1}{2}Mn^2R^2t^2$ (B) Mn^2R^2t

(C) MnR^2t^2 (D) MnR^2t

29. [Online 2015]

A block of mass $m = 0.1$ kg is connected to a spring of unknown spring constant k. It is compressed to a distance x from its equilibrium position and released from rest. After approaching half the distance $\left(\frac{x}{2}\right)$ from equilibrium position, it hits another block and comes to rest momentarily, while the other block moves with a velocity 3 ms^{-1}. The total initial energy of the spring is

(A) 1.5 J (B) 0.6 J

(C) 0.3 J (D) 0.8 J

30. [Online 2015]

A particle is moving in a circle of radius r under the action of a force $F = \alpha r^2$ which is directed towards centre of the circle. Total mechanical energy (kinetic energy + potential energy) of the particle is (take potential energy $= 0$ for $r = 0$)

(A) αr^3 (B) $\frac{1}{2}\alpha r^3$

(C) $\frac{4}{3}\alpha r^3$ (D) $\frac{5}{6}\alpha r^3$

31. [2014]

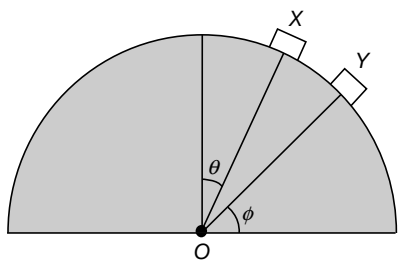
When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is

(A) $aL^2 + bL^3$ (B) $\frac{1}{2}(aL^2 + bL^3)$

(C) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (D) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

32. [Online 2014]

A particle is released on a vertical smooth semi-circular track from point X so that OX makes angle θ from the vertical (see figure). The normal reaction of the track on the particle vanishes at point Y where OY makes angle ϕ with the horizontal. Then



(A) $\sin\phi = \cos\phi$ (B) $\sin\phi = \frac{1}{2}\cos\theta$

(C) $\sin\phi = \frac{2}{3}\cos\theta$ (D) $\sin\phi = \frac{3}{4}\cos\theta$

33. [Online 2014]

A spring of unstretched length 1 has a mass m with one end fixed to a rigid support. Assuming spring to be made of a uniform wire, the kinetic energy possessed by it if its free end is pulled with uniform velocity v is

(A) $\frac{1}{2}mv^2$ (B) mv^2

(C) $\frac{1}{3}mv^2$ (D) $\frac{1}{6}mv^2$

34. [Online 2014]

A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be most likely proportional to

(A) v^4 (B) v^2

(C) v (D) v^3

35. [Online 2014]

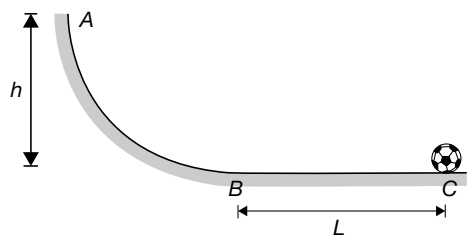
A bullet looses $\left(\frac{1}{n}\right)^{\text{th}}$ of its velocity passing through one plank. The number of such planks that are required to stop the bullet can be

(A) $\frac{n^2}{2n-1}$ (B) $\frac{2n^2}{n-1}$

(C) Infinite (D) n

36. [Online 2014]

A small ball of mass m starts at a point A with speed v_0 and moves along a frictionless track AB as shown. The track BC has coefficient of friction μ. The ball comes to stop at C after travelling a distance L which is



(A) $\frac{2h}{\mu}+\frac{v_0^2}{2\mu g}$ (B) $\frac{h}{\mu}+\frac{v_0^2}{2\mu g}$

(C) $\frac{h}{2\mu}+\frac{v_0^2}{\mu g}$ (D) $\frac{h}{2\mu}+\frac{v_0^2}{2\mu g}$

37. [2013]

This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement 1: A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f=\left(\frac{m}{M+m}\right)$

Statement 2: Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(A) Statement 1 is true, Statement 2 is true, Statement-II is **correct** explanation of Statement-I.

(B) Statement-I is true, Statement-II is true, Statement-II is **not** a correct explanation of Statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

38. [Online 2013]

A 70 kg man leaps vertically into the air from a crouching position. To take the leap the man pushes the ground with a constant force F to raise himself. The centre of gravity rises by 0.5 m before he leaps. After the leap the c.g. rises by another 1 m. The maximum power delivered by the muscles is [Take $g=10\text{ ms}^{-2}$]

(A) 6.26×10^3 Watts at the start

(B) 6.26×10^3 Watts at take off

(C) 6.26×10^4 Watts at the start

(D) 6.26×10^4 Watts at take off

39. [2012]

This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements. If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement 1: If stretched by the same amount, work done on S_1, will be more than that on S_2.

Statement 2: $k_1<k_2$.

(A) Statement 1 is true, Statement 2 is false.

(B) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.

(C) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

(D) Statement 1 is false, Statement 2 is true.

40. [Online 2012]

A particle gets displaced by $\Delta\vec{r}=\left(2\hat{i}+3\hat{j}+4\hat{k}\right)\text{ m}$ under the action of a force $\vec{F}\left(7\hat{i}+4\hat{j}+3\hat{k}\right)$. The change in its kinetic energy is

(A) 38 J (B) 70 J
(C) 52.5 J (D) 126 J

41. [2010]
The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x)=\frac{a}{x^{12}}-\frac{b}{x^{6}}$, where a and b are constant and x is the distance between the atoms. If the dissociation energy of the molecule is $D=\left[U(x=\infty)-U_{\text{at equilibrium}}\right]$, D is

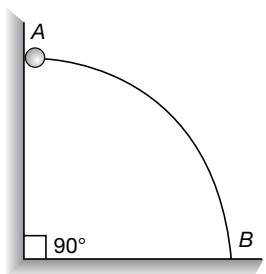
(A) $\frac{b^2}{6a}$ (B) $\frac{b^2}{2a}$
(C) $\frac{b^2}{12a}$ (D) $\frac{b^2}{4a}$

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct)

1. [JEE (Advanced) 2014]
A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is

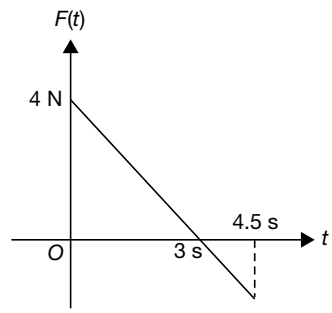


(A) always radially outwards
(B) always radially inwards
(C) radially outwards initially and radially inwards later
(D) radially inwards initially and radially outwards later

2. [JEE (Advanced) 2013]
The work done on a particle of mass m by a force, $K\left[\frac{x}{\left(x^2+y^2\right)^{\frac{3}{2}}}\hat{i}+\frac{y}{\left(x^2+y^2\right)^{\frac{3}{2}}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the x-y plane is

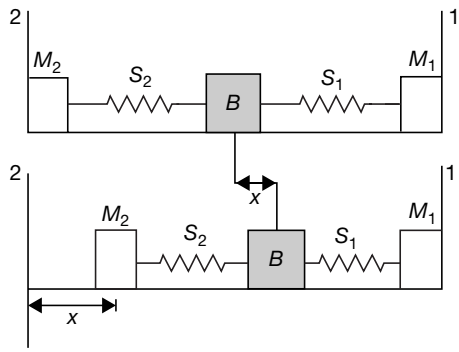
(A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$
(C) $\frac{K\pi}{2a}$ (D) 0

3. [IIT-JEE 2010]
A block of mass 2 kg is free to move along the x axis. It is at rest and from $t=0$ onwards it is subjected to a time-dependent force $F(t)$ in the x direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 s is



(A) 4.50 J (B) 7.50 J
(C) 5.06 J (D) 14.06 J

4. [IIT-JEE 2008]
A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$, respectively. The other ends are attached to two supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ is

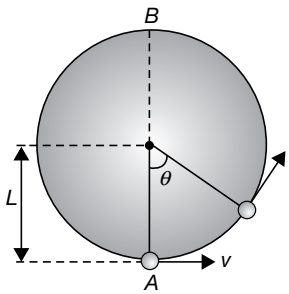


(A) 4 (B) 2

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

5. [IIT-JEE 2008]

A bob of mass M is suspended by a massless string of length L. The horizontal velocity v at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies



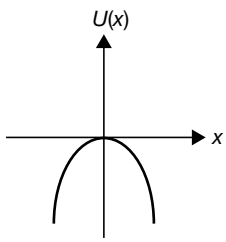
(A) $\theta = \frac{\pi}{4}$ (B) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

(C) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (D) $\frac{3\pi}{4} < \theta < \pi$

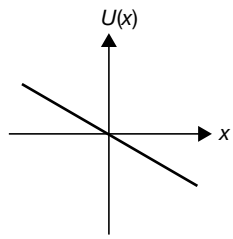
6. [IIT-JEE 2004]

A particle is placed at the origin and a force $F = kx$ is acting on it (where k is a positive constant). If $U(0) = 0$, the graph of $U(x)$ versus x will be: (where U is the potential energy function)

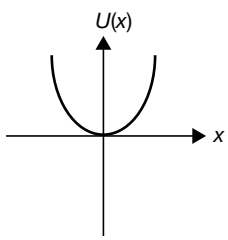
(A)



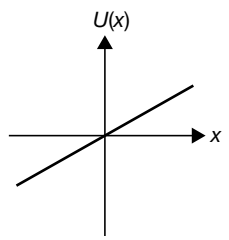
(B)



(C)

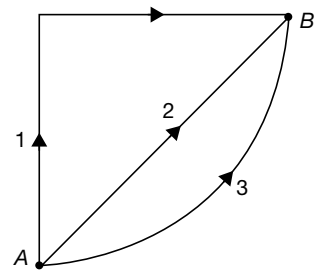


(D)



7. [IIT-JEE 2003]

If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m. Find the correct relation between W_1, W_2 and W_3



(A) $W_1 > W_2 > W_3$ (B) $W_1 = W_2 = W_3$

(C) $W_1 < W_2 < W_3$ (D) $W_2 > W_1 > W_3$

8. [IIT-JEE 2002]

An ideal spring with spring constant k is hung from the ceiling and a block of mass M is attached to its lower end. The block is released with the spring initially unstretched. Then the maximum extension in the spring is

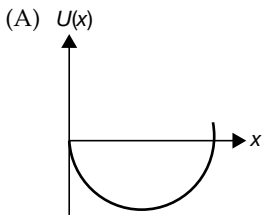
(A) $\frac{4Mg}{k}$ (B) $\frac{2Mg}{k}$

(C) $\frac{Mg}{k}$ (D) $\frac{Mg}{2k}$

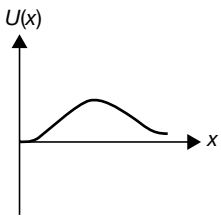
9. [IIT-JEE 2002]

A particle, which is constrained to move along x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here, k and a are positive constant. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is

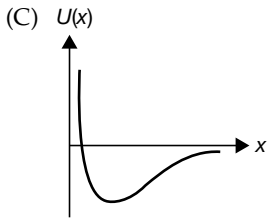
(A)



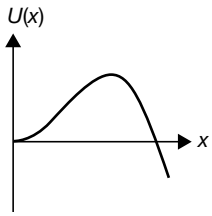
(B)



(C)



(D)



10. [IIT-JEE 2000]

A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of wind energy interception by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to

(A) v (B) v^2

(C) v^3 (D) v^4

11. [IIT-JEE 1999]

A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of

(A) $\left(\frac{2}{3}\right)k$ (B) $\left(\frac{3}{2}\right)k$

(C) $3k$ (D) $6k$

12. [IIT-JEE 1998]

A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at the lowest position and has a speed u. The magnitude of the change in velocity as it reaches a position where the string is horizontal is

(A) $\sqrt{u^2-2gL}$ (B) $\sqrt{2gL}$

(C) $\sqrt{u^2-gL}$ (D) $\sqrt{2\left(u^2-gL\right)}$

13. [IIT-JEE 1998]

A force $\vec{F}=-k(y\hat{i}+x\hat{j})$, where k is a positive constant, acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive x-axis to the point $(a,0)$, and then parallel to the y-axis to the point (a,a). The total work done by the force on the particle is

(A) $-2ka^2$ (B) $2ka^2$

(C) $-ka^2$ (D) ka^2

14. [IIT-JEE 1994]

A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c=k^2rt^2$, where k is a constant. The power delivered to the particle by the force acting on it is

(A) $2\pi mk^2r^2$ (B) mk^2r^2t

(C) $\frac{\left(mk^4r^2t^5\right)}{3}$ (D) Zero

15. [IIT-JEE 1985]

A uniform chain of length L and mass M is lying on a smooth table and one-third of its length is hanging vertically down over the edge of the table. The work required to pull the hanging part on the table is

(A) MgL (B) $\frac{MgL}{3}$

(C) $\frac{MgL}{9}$ (D) $\frac{MgL}{18}$

16. [IIT-JEE 1984]

A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to

(A) $t^{\frac{1}{2}}$ (B) $t^{\frac{3}{4}}$

(C) $t^{\frac{3}{2}}$ (D) t^2

17. [IIT-JEE 1980]

Two masses of 1 g and 4 g are moving with equal kinetic energies. The ratio of the magnitudes of their momenta is

(A) 4 : 1 (B) $\sqrt{2}$: 1

(C) 1 : 2 (D) 1 : 16

18. [IIT-JEE 1980]

If a machine is lubricated with oil,

(A) the mechanical advantage of the machine increases

(B) the mechanical efficiency of the machine increases

(C) both its mechanical advantage and efficiency increases

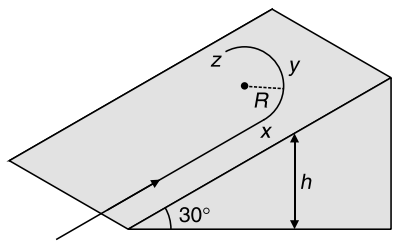
(D) its efficiency increases, but its mechanical advantage decreases

Multiple Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct)

1. JEE (Advanced) 2020

A student skates up a ramp makes an angle 30° with the horizontal. He/she starts (as shown in Figure) at the bottom of the ramp with speed v_0 and wants to turn around over a semicircular path xyz of radius R during which he/she reaches a maximum height h (at point y) from the ground as shown in Figure. Assume that the energy loss is negligible and the force required for this turn at the highest point is provided by his/her weight only. Then (g is the acceleration due to gravity)



(A) $v_0^2-2gh=\frac{1}{2}gR$

(B) $v_0^2-2gh=\frac{\sqrt{3}}{2}gR$

(C) the centripetal force required at points x and z is zero
(D) the centripetal force required is maximum at points x and z

2. [IIT-JEE 1987]
A particle is acted upon by a force of constant magnitude which is always directed perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. From the information given, we conclude that the particle
(A) has constant velocity.
(B) has constant acceleration.
(C) has constant kinetic energy.
(D) moves in a circular path.

Reasoning Based Questions

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. [IIT-JEE 2007]
Statement-1: A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of $30°$ with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.
Statement-2: The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

Matrix Match/Column Match Type Questions

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. [JEE (Advanced) 2015]
A particle of unit mass is moving along the x axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in **COLUMN-I** (a and U_0 are constants). Match the potential energies in **COLUMN-I** to the corresponding statements in **COLUMN-II**.

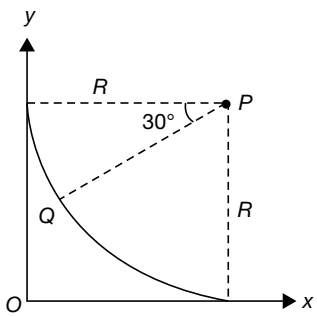
COLUMN-I	COLUMN-II
(A) $U_1(x)=\frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(p) The force acting on the particle is zero at $x=a$
(B) $U_2(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(q) The force acting on the particle is zero at $x=0$
(C) $U_3(x)=\frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(r) The force acting on the particle is zero at $x=-a$
(D) $U_4(x)=\frac{U_0}{2}\left[\frac{x}{a}-\frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(s) The particle experiences an attractive force towards $x=0$ in the region $\lvert x\rvert < a$
	(t) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x=-a$

Linked Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure, is 150 J. (Take the acceleration due to gravity, $g = 10\text{ ms}^{-2}$)



1. [JEE (Advanced) 2013]
The speed of the block when it reaches the point Q is

(A) 5 ms^{-1} (B) 10 ms^{-1}

(C) $10\sqrt{3}\text{ ms}^{-1}$ (D) 20 ms^{-1}

2. [JEE (Advanced) 2013]
The magnitude of the normal reaction that acts on the block at the point Q is

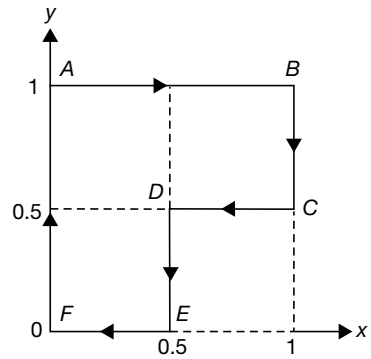
(A) 7.5 N (B) 8.6 N

(C) 11.5 N (D) 22.5 N

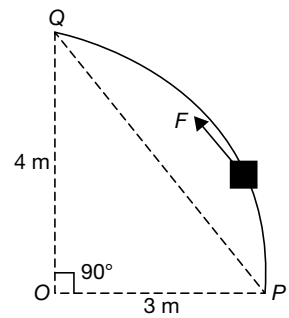
Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. [JEE (Advanced) 2019]
A particle is moved along a path *AB-BC-CD-DE-EF-FA*, as shown in figure, in presence of a force $\vec{F} = \left(\alpha y\hat{i} + 2\alpha x\hat{j}\right)$ N, where x and y are in meter and $\alpha = -1\text{ Nm}^{-1}$. The work done on the particle by this force $\vec{F}$ will be_____ joule.



2. [JEE (Advanced) 2014]
Consider an elliptically shaped rail PQ in the vertical plane with $OP = 3\text{ m}$ and $OQ = 4\text{ m}$. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see figure). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is $(n \times 10)$ J. The value of n is (Take acceleration due to gravity to be 10 ms^{-2})

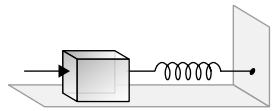


3. [JEE (Advanced) 2013]
A bob of mass m, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are massless and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{l_1}{l_2}$ is

4. [JEE (Advanced) 2013]
A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is

5. [IIT-JEE 2011]

A block of mass $0.18\ \text{kg}$ is attached to a spring of force constant $2\ \text{Nm}^{-1}$. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure.



The block slides a distance of $0.06\ \text{m}$ and comes to rest for the first time. The initial velocity of the block in ms^{-1} is $V = \frac{N}{10}$. Find the value of N.

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Work Done by a Constant and Variable Force)

1. $-mgh\left(1-\frac{k^2}{2}\right)$
2. $-f\ell_0$
3. $-mg\ell\sin\theta$
4. (a) 4.03 J (b) –3.5 J
5. (a) 0 (b) $-\frac{\alpha}{3}(0.026)$ (c) $\frac{\alpha}{3}(0.026)$
6. –3380 J
7. 83 J
8. (a) –6 J (b) –8 J
   (c) Force is non-conservative in nature.
9. 30 J
10. 25 J
11. (a) 4 J (b) 0 (c) –1 J (d) 3 J
12. 60 J
13. $-ka^2$
14. (a) 50 J (b) 100 J (c) 50 J
15. Work done by the gravitational force is mgh

    Work done by the tension force is $-\frac{mgh}{2}$

Test Your Concepts-II (Based on Kinetic Energy, Potential Energy and Power)

1. $\frac{P}{f}, \frac{Pm}{8r^2}$
2. (a) $\sqrt{\frac{2Pt}{m}}$ (b) $\left(\frac{8P}{9m}\right)^{1/2}t^{3/2}$
4. $P=\begin{cases}40t & \text{for } t\le 0.2\text{ s}\\ -100t^2+60t & \text{for } t>0.2\text{ s}\end{cases}$, 2533 J
5. 30 W
6. 1.2×10^6 watt
7. $-\frac{1}{4}$
8. $\sqrt{19}$ ms^{-1}
9. 50 J
10. 3
11. 1
12. 240 J
13. 4.9 kmh^{-1}
14. 9 J
15. 3.75 J
16. –0.25 J

Test Your Concepts-III (Based on Conservation of Energy and Work Energy Theorem)

1. $mg\ell(1-\cos\theta)$
2. 5 ms^{-1}
3. 3.54 ms^{-1}, 3.96 ms^{-1}
4. 6.25 ms^{-1}
5. (a) 0.4 J (b) 1.2 J
6. $\frac{M^2g}{2km}+\frac{Mg}{k}$
7. $\sqrt{2gR}\cos\alpha$, $\sqrt{2gR}\sin\alpha$
8. (a) 3.87 ms^{-1} (b) = 0.1 m
9. $\sqrt{rg\left(\frac{4}{\pi}+\frac{\pi}{2}\right)}$
10. (a) 13.2 kJ (b) –12 kJ (c) 1.2 kJ (d) 5.47 ms^{-1}
11. 0.374 ms^{-1}
12. (a) 2.33 J (b) 2.635 J
13. (a) $\frac{\alpha x^2}{2}+\frac{\beta x^3}{3}$ (b) 6 ms^{-1}
14. $\frac{\left[mg(\sin\theta+\mu_s\cos\theta)\right]^2}{2k}$
15. 0.5 m
16. (a) 0.98 m (b) 1.39 ms^{-1}
17. $\sqrt{\frac{k}{m}\left[\frac{L_0^2}{4}-(L_0-x)^2\right]}$
18. $\sqrt{\frac{2}{3}(1+\pi)gr}$

Test Your Concepts-IV (Based on Work Energy Theorem for Non-conservative Systems)

1. (a) $+\frac{1}{2}mv^2$ (b) $\frac{v^2}{2\mu_k g}$ (c) $2d$

2. (a) 0.82 N (b) 2.45 ms^{-1}
3. (a) $-\mu Mg\ell$ (b) $-\frac{1}{2}k\ell^2$
   (c) Gravitational force, normal reaction, zero
   (d) $-\left(\mu Mgl+\frac{1}{2}kl^2\right)$
   (e) $\frac{1}{k}\left[\sqrt{\mu^2M^2g^2+Mkv_0^2}-\mu Mg\right]$
4. $\left(\frac{M^2v_0^2}{2}\right)\left(\frac{gb}{k+Mgb}\right)$
5. (a) 0.2 (b) –2.4 J
6. (a) 0.8 ms^{-1} (b) 0.82 ms^{-1}
7. –750 J
8. $\left(m_1+\frac{m_2}{2}\right)\mu g$
9. –0.044 J
10. 0.115
11. 5 J

Test Your Concepts-V (Based on Relation Between Conservative Force and Potential Energy and Types of Equilibrium)

1. $-\left(2y\hat{i}+(2x+z)\hat{j}+y\hat{k}\right)$
2. $aU_0\exp(-az)\left[\sin(ax)\hat{i}+\cos(ax)\hat{k}\right]$
3. $U_0-ax^2y+\frac{ay^3}{3}$
4. 20 J
5. 7 m
6. D
7. A

Test Your Concepts-VI (Based on Vertical Circle)

1. $\frac{5}{2}R$
2. 16.15 N
3. (a) 29 N (b) 15.5 N
4. 5.66 ms^{-1}, 16.75 ms^{-2}
5. (a) 2 N (b) 64 N (c) 1.2 J
6. $\sqrt{\frac{mgR}{k}}\sqrt{5+2\mu_k}$
7. $\sqrt{\frac{7rg}{2}}$, 30°
8. (a) $\frac{3L}{2}$ (b) $\frac{40}{27}L$
9. $\sqrt{\frac{48g\ell}{5}}$
10. 15.65 ms^{-1}
11. 10 ms^{-1}

Single Correct Choice Type Questions

1. D	**2.** D	**3.** A	**4.** A	**5.** C	**6.** C	**7.** B	**8.** B	**9.** D	**10.** D
11. A	**12.** D	**13.** A	**14.** D	**15.** B	**16.** C	**17.** A	**18.** C	**19.** B	**20.** B
21. C	**22.** A	**23.** D	**24.** B	**25.** A	**26.** A	**27.** B	**28.** B	**29.** A	**30.** D
31. A	**32.** C	**33.** C	**34.** B	**35.** B	**36.** B	**37.** A	**38.** D	**39.** B	**40.** C
41. B	**42.** B	**43.** C	**44.** B	**45.** C	**46.** A	**47.** C	**48.** D	**49.** D	**50.** B
51. D	**52.** C	**53.** B	**54.** A	**55.** B	**56.** D	**57.** C	**58.** D	**59.** C	**60.** C
61. C	**62.** A	**63.** C	**64.** C	**65.** A	**66.** C	**67.** A	**68.** B	**69.** D	**70.** B
71. A	**72.** D	**73.** A	**74.** B	**75.** D	**76.** C	**77.** B	**78.** C	**79.** B	**80.** B
81. A	**82.** B	**83.** B	**84.** A	**85.** B	**86.** A	**87.** D	**88.** C	**89.** A	**90.** B
91. A	**92.** B	**93.** C	**94.** B	**95.** A	**96.** C	**97.** A	**98.** A	**99.** A	

Multiple Correct Choice Type Questions

1. A, B, C	**2.** A, B, C, D	**3.** A, B, C	**4.** A, B, C, D	**5.** A, B, C, D
6. A, C, D	**7.** B, D	**8.** B, D	**9.** B, C	**10.** C, D
11. A, C, D	**12.** B, C	**13.** B, D	**14.** A, C, D, E	**15.** A, C
16. A, B, D	**17.** A, B, C	**18.** A, C	**19.** B, C, D	**20.** B, D
21. A, C	**22.** A, B, C	**23.** B, C	**24.** B, D	**25.** A, C
26. A, C	**27.** A, C, D	**28.** A, C, D	**29.** B, C, D	**30.** B, C
31. B, C	**32.** A, C	**33.** B, C, D	**34.** A, C	**35.** B, D
36. B, D	**37.** B, C	**38.** A, D	**39.** A, C	**40.** C, D

Reasoning Based Questions

1. D	**2.** D	**3.** D	**4.** A	**5.** D	**6.** A	**7.** A	**8.** C	**9.** C	**10.** D

Linked Comprehension Type Questions

1. C	**2.** A	**3.** D	**4.** D	**5.** C	**6.** D	**7.** B	**8.** B	**9.** D	**10.** B
11. A	**12.** A	**13.** C	**14.** C	**15.** A	**16.** D	**17.** A	**18.** B	**19.** A	**20.** B
21. C	**22.** B	**23.** B	**24.** C	**25.** C	**26.** B	**27.** C	**28.** C	**29.** C	**30.** D
31. A	**32.** B	**33.** B	**34.** C	**35.** B	**36.** B	**37.** C	**38.** D	**39.** D	**40.** A
41. D	**42.** D	**43.** D	**44.** B	**45.** C	**46.** D	**47.** A	**48.** C	**49.** D	**50.** C
51. C	**52.** C	**53.** D	**54.** C	**55.** C	**56.** C	**57.** B	**58.** A	**59.** A	**60.** A, D

Matrix Match/Column Match Type Questions

1. A → (q)	B → (s)	C → (r)	D → (r)
2. A → (s)	B → (r)	C → (p)	D → (q)
3. A → (t)	B → (p)	C → (s)	D → (q)
4. A → (r)	B → (q)	C → (p)	D → (t)
5. A → (p, q)	B → (t)	C → (r)	D → (s)
6. A → (p, q, r)	B → (p, q, r)	C → (q, r)	D → (p, r)
7. A → (r)	B → (p)	C → (q, s)	D → (q, s)
8. A → (p, s)	B → (q, r)	C → (p, s)	D → (q, r)
9. A → (s)	B → (r)	C → (p)	D → (q)
10. A → (s)	B → (r)	C → (q)	D → (p)

Integer/Numerical Answer Type Questions

1. 10	**2.** 800	**3.** (a) 32, (b) 500	**4.** (a) 65, (b) 3	**5.** 0.098
6. 100	**7.** 2	**8.** 10	**9.** (a) 80, (b) 80, (c) 50	**10.** 3
11. 2	**12.** 4	**13.** 4	**14.** 2	**15.** **5**

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1. B	**2.** C	**3.** D	**4.** 18	**5.** 150	**6.** 3	**7.** C	**8.** B	**9.** 1.9	**10.** 10
11. D	**12.** C	**13.** D	**14.** B	**15.** A	**16.** A	**17.** A	**18.** C	**19.** B	**20.** C
21. C	**22.** A	**23.** D	**24.** A	**25.** A	**26.** C	**27.** C	**28.** B	**29.** B	**30.** D
31. C	**32.** C	**33.** D	**34.** D	**35.** A	**36.** B	**37.** D	**38.** B	**39.** D	**40.** A
41. D									

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. C	**2.** D	**3.** C	**4.** C	**5.** D	**6.** A	**7.** B	**8.** B	**9.** D	**10.** C
11. B	**12.** D	**13.** C	**14.** B	**15.** D	**16.** C	**17.** C	**18.** B		

Multiple Correct Choice Type Problems

1. A, D **2.** C, D

Reasoning Based Questions

1. C

Matrix Match/Column Match Type Questions

1. (A) → (p, q, r, t) (B) → (q, s) (C) → (p, q, r, s) (D) → (p, r, t)

Linked Comprehension Type Questions

1. B **2.** A

Integer/Numerical Answer Type Questions

1. 0.75	**2.** 5	**3.** 5	**4.** 5	**5.** 4

CHAPTER 2

Centre of Mass, Conservation of Linear Momentum and Collisions

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Newton's Second Law and Concept of Centre of Mass
(b) Centre of Mass for Various Bodies and Configurations
(c) Concept of Motion of Centre of Mass
(d) Work Energy Theorem for System of Particles and its Applications from Centre Of Mass Reference Frame (C-frame)
(e) Conservation of Linear Momentum, Variable Mass Systems
(f) Collisions: Head on and Oblique

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

CENTRE OF MASS

INTRODUCTION

In the previous chapters, we had been discussing motion for the point objects. However, we come across mechanical systems which are actually not point objects. A mechanical system is either a group of particles (such as atoms of a gas enclosed in a container) or an extended object (such as an acrobat following a projectile motion after being fired from the human canon). The motion of a mechanical system can be easily described in terms of a special point called the **Centre of Mass** of the system. We shall also see that the centre of mass of the system moves as if the entire mass of the system were concentrated at that point. Furthermore, if a resultant force $\vec{F}_{ext}$ acts on the system and the total mass of the system is M, then acceleration of the centre of mass of system is $\vec{a}_{CM} = \frac{\vec{F}_{ext}}{M}$. **So, we observe that the system moves as if the resultant force was being applied to a single particle of mass M, placed at the centre of mass. In simple words the centre of mass is the point from a chosen origin where the entire mass of an extended system can be assumed to be concentrated and instead of discussing about the motion of the particles of the system individually (to study the motion of the system) we can directly discuss about the motion of the centre of mass to get the same results.**

CENTRE OF MASS AND NEWTON'S SECOND LAW

Consider a mechanical system formed by a collection of particles m_1, m_2, m_3,...., m_n. Let a force $\vec{f}_1$ be applied on particle 1, $\vec{f}_2$ on particle 2 and so on such that their respective accelerations are $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$,....,

$\vec{a}_n$. Now, according to Newton's Second Law, the net force $\vec{F}$ acting on the mechanical system is given by

$$\vec{F} = \vec{f}_1 + \vec{f}_2 + ... + \vec{f}_n$$

$$\Rightarrow \quad \vec{F} = m_1\vec{a}_1 + m_2\vec{a}_2 + ... + m_n\vec{a}_n \qquad ...(1)$$

If M be the total mass of the system and $\vec{a}$ be the acceleration of the system, then

$$\vec{F} = M\vec{a} = m_1\vec{a}_1 + m_2\vec{a}_2 + ... + m_n\vec{a}_n \qquad ...(2)$$

where, $M = m_1 + m_2 + ... + m_n$

Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ be the respective velocities of the particles at an instant and $\vec{v}$ be the velocity of the system at that instant. Then

$$\vec{a}_1 = \frac{d\vec{v}_1}{dt},\ \vec{a}_2 = \frac{d\vec{v}_2}{dt}, \ldots,\ \vec{a}_n = \frac{d\vec{v}_n}{dt} \text{ and } \vec{a} = \frac{d\vec{v}}{dt}$$

So, from equation (2), we get

$$M\left(\frac{d\vec{v}}{dt}\right) = m_1\left(\frac{d\vec{v}_1}{dt}\right) + m_2\left(\frac{d\vec{v}_2}{dt}\right) + ... + m_n\left(\frac{d\vec{v}_n}{dt}\right)$$

$$\Rightarrow \quad \frac{d}{dt}(M\vec{v}) = \frac{d}{dt}\left(m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n\right)$$

Integrating both sides, we get

$$M\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n \qquad ...(3)$$

$$\Rightarrow \quad \vec{v} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n}{m_1 + m_2 + ... + m_n} \qquad ...(4)$$

Further, if $\vec{r}_1, \vec{r}_2, \ldots., \vec{r}_n$ be the respective position vectors of the particles **(from the selected convenient origin)** at any instant t and $\vec{r}$ be the position vector of M, then

$$\vec{v}_1 = \frac{d\vec{r}}{dt},\ \vec{v}_2 = \frac{d\vec{r}_2}{dt}, \ldots.,\ \vec{v}_n = \frac{d\vec{r}_n}{dt},\ \vec{v} = \frac{d\vec{r}}{dt}$$

So, from (3), we get

$$M\left(\frac{d\vec{r}}{dt}\right) = m_1\left(\frac{d\vec{r}_1}{dt}\right) + m_2\left(\frac{d\vec{r}_2}{dt}\right) + ... + m_n\frac{d\vec{r}_n}{dt}$$

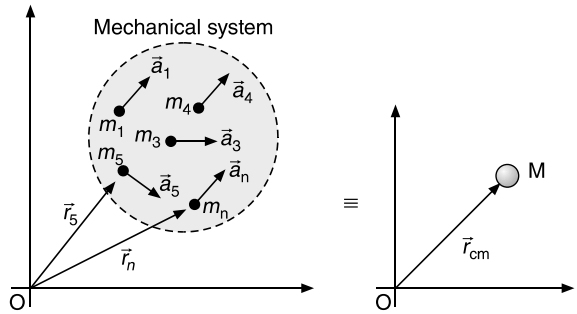
$$\Rightarrow \quad \frac{d}{dt}(M\vec{r}) = \frac{d}{dt}\left(m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n\right)$$

Integrating, we get

$$M\vec{r} = m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n$$

$$\Rightarrow \quad \vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n}$$

This position vector of the combined mass M, is called as the centre of mass of the system of particles, denoted by $\vec{r}_{cm}$.



Furthermore, $\vec{v}_{cm}$ is actually the velocity of centre of mass and $\vec{a}_{cm}$ is the acceleration of the centre of mass. Hence, we have

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ... + m_n\vec{a}_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_i m_i\vec{a}_i}{\sum_i m_i}$$

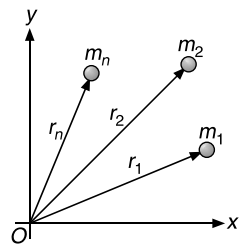
$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_i m_i\vec{v}_i}{\sum_i m_i}$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r} + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$$

These equations help us to reduce a many particle mechanical system to a single particle system (of mass M) placed at a position vector $\vec{r}_{cm}$.

DEFINITION OF CENTRE OF MASS

Consider a system of n discrete particles having position vectors $\vec{r}_1, \vec{r}_2, \ldots\ldots., \vec{r}_n$ as shown in Figure.



The position vector for the centre of mass $\vec{r}_{cm}$ is

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n}$$

$$\Rightarrow \quad \vec{r}_{cm} = \frac{\sum m_i\vec{r}_i}{M} \qquad ...(1)$$

where, $M = \Sigma m_i$ is the total mass of the system.

The components of equation (1) are defined as

$$x_{cm} = \frac{\Sigma m_i x_i}{M},\ y_{cm} = \frac{\Sigma m_i y_i}{M} \text{ and } z_{cm} = \frac{\Sigma m_i z_i}{M}$$

(a) The location of the centre of mass is independent of the reference frame used to locate it.

(b) The centre of mass of the system of particles depends only on the masses of the particles and the positions of the particles relative to one another.

(c) The centre of mass has a tendency to shift towards the heavier side of the system.

(d) The above formulae are equally applicable for collinear and non collinear systems.

(e) However, it must be noted that to locate the position of the centre of mass an Origin of convenience has to be selected first.

ANY SYSTEM CAN BE THOUGHT OF AS A TWO PARTICLE SYSTEM

Dear Students, please remember that any particle system can be reduced to a two-particle system. As an example, let us consider a four particle system, then its CM is given by

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4\vec{r}_4}{m_1 + m_2 + m_3 + m_4} \quad \ldots(1)$$

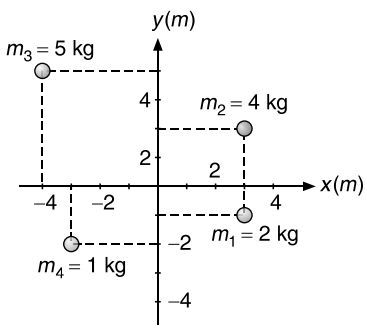
Now, if we want that particles 1, 3 should be a single particle and 2, 4 should be another single particle, then equation (1) can be re-written as

$$\vec{r}_{cm} = \frac{\left(\frac{m_1\vec{r}_1 + m_3\vec{r}_3}{m_1 + m_3}\right)(m_1 + m_3) + \left(\frac{m_2\vec{r}_2 + m_4\vec{r}_4}{m_2 + m_4}\right)(m_2 + m_4)}{(m_1 + m_3) + (m_2 + m_4)}$$

$$\Rightarrow \quad \vec{r}_{cm} = \frac{(\vec{r}_{cm \text{ of } 1\,\&\,3})(m_1 + m_3) + (\vec{r}_{cm \text{ of } 2\,\&\,4})(m_2 + m_4)}{m_1 + m_2 + m_3 + m_4}$$

ILLUSTRATION 1

Find the center of mass of the four point masses as shown in figure.



SOLUTION

The total mass $M = 12$ kg, from equation, we have

$$x_{cm} = \frac{(2)(3)+(4)(3)+(5)(-4)+(1)(-3)}{2+4+5+1} = -\frac{5}{12} \text{ m}$$

$$y_{cm} = \frac{(2)(-1)+(4)(3)+(5)(4)+(1)(-2)}{2+4+5+1} = \frac{28}{12} \text{ m}$$

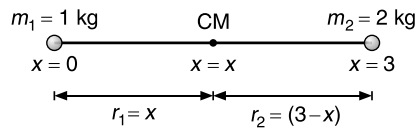
The position of the cm is $\vec{r}_{cm} = \left(-0.42\hat{i} + 2.3\hat{j}\right)$ m

ILLUSTRATION 2

Two particles of mass 1 kg and 2 kg are located at $x = 0$ and $x = 3$ m. Find the position of their centre of mass.

SOLUTION

Since, both the particles lie on x-axis, the cm will also lie on x-axis. Let the cm is located at $x = x$, then



r_1 = distance of cm from the particle of mass 1 kg

$\Rightarrow \quad r_1 = x$

and r_2 = distance of cm from the particle of mass 2 kg

$\Rightarrow \quad r_2 = (3 - x)$

Using $\frac{r_1}{r_2} = \frac{m_2}{m_1}$

$\Rightarrow \quad \frac{x}{3-x} = \frac{2}{1}$

$\Rightarrow \quad x = 2$ m

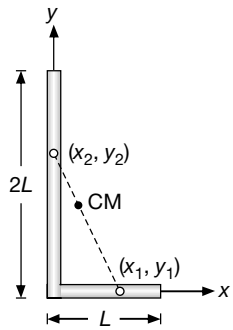
Thus, the cm of the two particles is located at $x = 2$ m

ILLUSTRATION 3

A thin rod of length $3L$ is bent at right angles at a distance L from one end. Locate the centre of mass of the arrangement with respect to the corner. Take $L = 1.2$ m.

SOLUTION

The centre of mass of each arm lies at its midpoint. The centre of mass of the two arms can be found by treating each arm as a point particle placed at its centre of mass. From the diagram we see that $x_1 = \frac{L}{2}$, $y_1 = 0$ and $x_2 = 0$, $y_2 = L$.



Since the distribution of the mass of the rod is uniform so the masses of the parts that are bent are m and $2m$ which have the respective lengths as L and $2L$. By definition, we get

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{M} = \frac{L}{6}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2}{M} = \frac{2L}{3}$$

The position of the centre of mass is given by

$$\vec{r}_{cm} = \left(-0.2\hat{i} + 0.8\hat{j}\right) \text{ m}$$

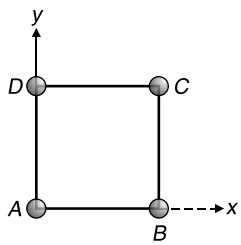
Note that the centre of mass may not necessarily have a physical mass present at it and may lie within or outside the body.

ILLUSTRATION 4

Four particles of mass 1 kg, 2 kg, 3 kg and 4 kg are placed at the four vertices A, B, C and D of a square of side 1 m. Find the position of centre of mass of the particles.

SOLUTION

Assuming A as the origin, AB as x-axis and AD as y-axis, we have



$$m_1 = 1 \text{ kg at } (x_1, y_1) = (0, 0)$$

$$m_2 = 2 \text{ kg at } (x_2, y_2) = (1, 0)$$

$$m_3 = 3 \text{ kg at } (x_3, y_3) = (1, 1)$$

and $$m_4 = 4 \text{ kg at } (x_4, y_4) = (0, 1)$$

Co-ordinates of their cm are

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$\Rightarrow \quad x_{cm} = \frac{(1)(0)+2(1)+3(1)+4(0)}{1+2+3+4}$$

$$\Rightarrow \quad x_{cm} = \frac{5}{10} = \frac{1}{2} \text{ m} = 0.5 \text{ m}$$

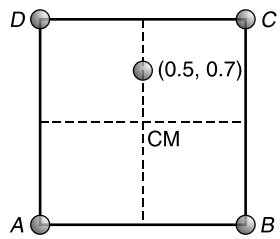
Similarly, $y_{cm} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$

$$y_{cm} = \frac{(1)(0)+2(0)+3(1)+4(1)}{1+2+3+4}$$

$$y_{cm} = \frac{7}{10} \text{ m} = 0.7 \text{ m}$$

$$\Rightarrow \quad (x_{cm}, y_{cm}) = (0.5, 0.7) \text{ m}$$

Thus, position of cm of the four particles is as shown in Figure.



So, we see that the centre of mass of a system has a tendency to shift towards the heavier side of the system.

CENTRE OF MASS FOR THIN SHEETS AND LAMINAR BODIES

For thin sheets or laminar type (2-dimensional) bodies, mass is directly proportional to area A, so the formulae for finding the position of centre of mass is modified and is given by

$$\vec{r}_{cm} = \frac{A_1\vec{r}_1 + A_2\vec{r}_2 + \ldots + A_n\vec{r}_n}{A_1 + A_2 + \ldots + A_n}$$

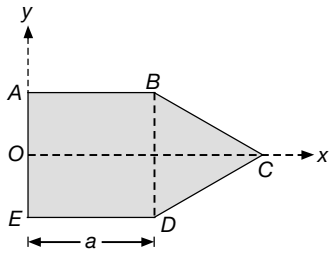
$$\Rightarrow \quad x_{cm} = \frac{A_1x_1 + A_2x_2 + \ldots + A_nx_n}{A_1 + A_2 + \ldots + A_n}$$

$$\Rightarrow \quad y_{cm} = \frac{A_1y_1 + A_2y_2 + \ldots + A_ny_n}{A_1 + A_2 + \ldots + A_n} \text{ and}$$

$$\Rightarrow \quad z_{cm} = \frac{A_1z_1 + A_2z_2 + \ldots + A_nZ_n}{A_1 + A_2 + \ldots + A_n}$$

ILLUSTRATION 5

A uniform lamina $ABCDE$ is made from a square $ABDE$ and an equilateral triangle BCD. Find the centre of mass of the lamina.



SOLUTION

Here, A_1 = area of square $ABDE = (a)(a) = a^2$

$$A_2 = \text{area of triangle} = \frac{1}{2}(a)\left(\frac{\sqrt{3}a}{2}\right) = \frac{\sqrt{3}}{4}a^2$$

Coordinates of centre of mass of square is $(x_1, y_1) = \left(\frac{a}{2}, 0\right)$ and coordinates of centre of mass of the triangle is $(x_2, y_2) = \left(a + \frac{a}{2\sqrt{3}}, 0\right)$

Now, $x_{cm} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$

$$\Rightarrow \quad x_{cm} = \frac{(a^2)\left(\frac{a}{2}\right) + \left(\frac{\sqrt{3}a^2}{4}\right)\left(a + \frac{a}{2\sqrt{3}}\right)}{a^2 + \frac{\sqrt{3}}{4}a^2} = 0.74a$$

$y_{cm} = 0$ as y_1 and y_2 both are zero.

Therefore, coordinates of cm of the given lamina are $(0.74a, 0)$.

CENTRE OF MASS FOR A SYSTEM HAVING A CAVITY

Consider a body of mass m_1 having its centre of mass at $\vec{r}_1$ from a fixed origin O (say). If a portion of mass m_2 having its centre of mass at $\vec{r}_2$ is removed from this body, then the centre of mass of the remaining body is given by

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 - m_2} \quad \textbf{OR} \quad \vec{r}_{cm} = \frac{A_1\vec{r}_1 - A_2\vec{r}_2}{A_1 - A_2}$$

$$\Rightarrow \quad x_{cm} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2} \quad \textbf{OR} \quad x_{cm} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

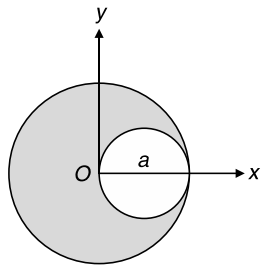
$$\Rightarrow \quad y_{cm} = \frac{m_1y_1 - m_2y_2}{m_1 - m_2} \quad \textbf{OR} \quad y_{cm} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$$

$$\Rightarrow \quad z_{cm} = \frac{m_1z_1 - m_2z_2}{m_1 - m_2} \quad \textbf{OR} \quad z_{cm} = \frac{A_1z_1 - A_2z_2}{A_1 - A_2}$$

Please note that here A is the area of the corresponding portion if the body is in the shape of a lamina.

ILLUSTRATION 6

A disc of diameter a is removed from a uniform disc of radius a. Locate the position of centre of mass of the left out lamina shown in figure.



SOLUTION

Coordinates of centre of mass of large circle are $(x_1, y_1) = (0, 0)$ and coordinates of centre of mass of small circle are $(x_2, y_2) = \left(\frac{a}{2}, 0\right)$. Since

$$x_{cm} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

where, A_1 is area of complete circle, i.e. $A_1 = \pi a^2$ and A_2 is the area of cavity, i.e. $A_2 = \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$

$$\Rightarrow \quad x_{cm} = \frac{-\left(\pi a^2/4\right)(a/2)}{\pi a^2 - \left(\pi a^2/4\right)} = -\frac{a/8}{3/4} = -\frac{a}{6}$$

and $y_{cm} = 0$ as y_1 and y_2 both are zero.

Therefore, coordinates of cm of the lamina shown in figure are $\left(-\frac{a}{6}, 0\right)$. Please note that Centre of Mass has a tendency to shift towards heavier side of system.

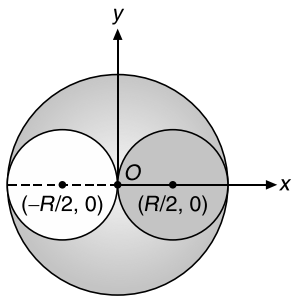
ILLUSTRATION 7

From a uniform disc of radius R, a disc of diameter R has been cut out from the left part and this removed part is then placed opposite to the cavity such that centres of the full disc, cavity and the removed disc

all lie in the same line and the removed disc just touches it cavity. Locate the centre of mass of the resulting disc assuming the origin to be at the centre of the full disc.

SOLUTION

Since the arrangement is symmetric about x-axis, so centre of mass of the system lies on the x-axis. Also, due to uniform thickness, mass is proportional to area.



The x-coordinate of centre of mass of the system is

$$x_{cm} = \frac{A_1x_1 - A_2x_2 + A_3x_3}{A_1 - A_2 + A_3}$$

where, $A_1 = \pi R^2$ is the area of the full disc,

$$A_2 = A_3 = \pi\left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4} \text{ is the area of cavity}$$

x_1, x_2 and x_3 are the x coordinates of the centre of mass of full disc at origin, the removed disc at $\left(-\frac{R}{2}, 0\right)$ and the removed disc when placed opposite to the cavity tangentially at $\left(\frac{R}{2}, 0\right)$.

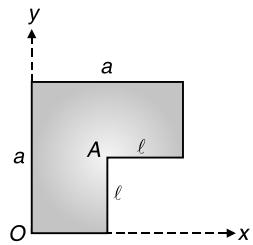
$$\Rightarrow \quad x_{cm} = \frac{(\pi R^2)(0) - \left(\frac{\pi R^2}{4}\right)\left(-\frac{R}{2}\right) + \left(\frac{\pi R^2}{4}\right)\left(\frac{R}{2}\right)}{\pi R^2 - \frac{\pi R^2}{4} + \frac{\pi R^2}{4}}$$

$$\Rightarrow \quad x_{cm} = \frac{R}{4}$$

Hence the coordinate of centre of mass of system will be $\left(\frac{R}{4}, 0\right)$.

ILLUSTRATION 8

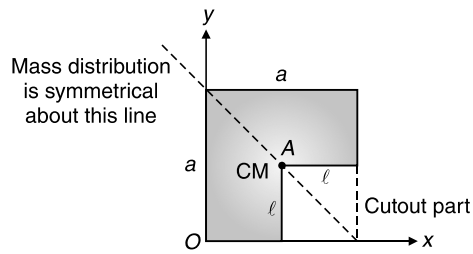
A square metal plate of side a from which a square plate of side $l(<a)$ has been cut as shown in the figure. Calculate the ratio $\left(\frac{a}{l}\right)$ so that the centre of mass of the remaining L-shaped plate coincides with the point A.



SOLUTION

For centre of mass to lie at point A, we have

$$r_{cm} = (x_{cm}, y_{cm}) = (l, l)$$



$$\Rightarrow \quad y_{cm} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2} = \frac{a^2\left(\frac{a}{2}\right) - l^2\left(\frac{l}{2}\right)}{a^2 - l^2} = l$$

$$\Rightarrow \quad a^3 - l^3 = 2l(a^2 - l^2)$$

$$\Rightarrow \quad (a-l)(a^2 + l^2 + al) = 2l(a+l)(a-l)$$

$$\Rightarrow \quad a^2 - al - l^2 = 0 \qquad \ldots(1)$$

Solving equation (1), we get

$$\frac{a}{l} = \frac{\sqrt{5}-1}{2}$$

CENTRE OF MASS OF EXTENDED BODIES

A rigid body, such as a meter stick, can be thought of as a system of closely packed particles and hence it also has a centre of mass. *Since the number of particles in the body is very large and their spacing is extremely small, so we can treat the body as a continuous distribution of mass*. For continuous distribution of mass, the centre of mass is defined as

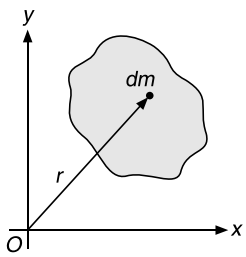
$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm$$

where $\vec{r}$ is the position vector of the centre of mass of an infinitesimal mass element dm. The components of this equation are

$$x_{cm} = \frac{1}{M}\int x\,dm\ ,\ y_{cm} = \frac{1}{M}\int y\,dm \text{ and}$$

$$z_{cm} = \frac{1}{M}\int z\,dm$$

Often, we deal with homogeneous objects having a point, a line, or a plane of symmetry. Then the centre of mass will lie at the point, on the line, or in the plane of symmetry. **For example,** the centre of mass of a homogeneous rod will lie at the centre of the rod, the centre of mass of a homogenous sphere will be at the centre of the sphere, the centre of mass of a cone will be at the axis of the cone, etc.



In the above formulae we have taken an infinitesimal point element of mass dm placed at a distance dx or dy or dz from the selected convenient origin O at respective distance x or y or z from O.

Please note that, the same set of formulae cannot be extended to systems where we have to find the centre of mass of the extended system with the help of the elements that themselves are extended systems and are not point elements, like a disc where the infinitesimal elements are taken in the form of concentric rings which are not point like. In that case we modify the above formula as

$$\vec{r}_{cm} = \begin{bmatrix} \frac{1}{M}\int \vec{r}\,dm & \text{for infinitesimal elements} \\ \frac{1}{M}\int \left(\vec{r}_{\text{cm of the infinitesimal element}}\right) dm & \text{for extended elements} \end{bmatrix}$$

ILLUSTRATION 9

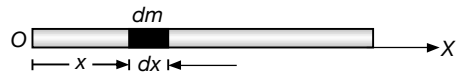
If a homogeneous thin rod of mass m and l is given, from symmetry, we know that its centre of mass coincides with the geometrical centre (mid-point) of the rod. Prove this by calculation for the centre of mass of the rod.

SOLUTION

Let us choose the x-axis along the length of the rod and origin at the left end of the rod, as shown in figure. A differential mass element of the rod having mass dm and length dx is also shown in the figure at a distance x from the origin.

Mass of the element, dm, equals to the linear mass density (mass per unit length $\lambda = \frac{m}{l}$) times the length of the element, dx. Therefore,

$$dm = \lambda dx = \frac{m}{l}dx$$



If x_{cm} be the coordinate of the centre of mass of the rod, then

$$\Rightarrow \quad x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^l x\left(\frac{m}{l}\right)dx}{m} \qquad \left\{\because \int dm = m\right\}$$

$$\Rightarrow \quad x_{cm} = \frac{1}{l}\int_0^l x\,dx = \frac{1}{l}\left(\frac{l^2}{2}\right)$$

$$\Rightarrow \quad x_{cm} = \frac{l}{2}$$

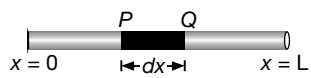
Hence, the centre of mass of the rod lies at the mid-point of the rod.

ILLUSTRATION 10

A rod of length L is placed along the x-axis between $x = 0$ and $x = L$. The linear mass density, λ, of the rod varies with the distance x from the origin as $\lambda = \lambda_0 + ax$. Here, λ_0 and a are constants. Find the position of centre of mass of this rod.

SOLUTION

Mass of element PQ of length dx situated at $x = x$ as shown in figure is



$$dm = \lambda dx = (\lambda_0 + ax)dx$$

The cm of the element has coordinates $(x, 0, 0)$. Therefore, x-coordinate of cm of the rod will be

$$x_{cm} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(\lambda dx)}{\int_0^L \lambda dx} = \frac{\int_0^L (x)(\lambda_0 + ax)dx}{\int_0^L (\lambda_0 + ax)dx}$$

$$\Rightarrow \quad x_{cm} = \frac{\left(\frac{\lambda_0 x^2}{2} + \frac{ax^3}{3}\right)\Big|_0^L}{\left(\lambda_0 x + \frac{ax^2}{2}\right)\Big|_0^L} = \frac{3\lambda_0 L + 2aL^2}{6\lambda_0 + 3aL}$$

The result obtained can be rearranged to obtain

$$x_{cm} = \frac{L}{2}\left(\frac{3\lambda_0 + 2aL}{3\lambda_0 + \frac{3}{2}aL}\right)$$

In the above expression numerator of the term within square bracket is greater than the denominator. Hence, $x_{cm} > \frac{L}{2}$

The y-coordinate of cm of the rod is

$$y_{cm} = \frac{\int y\,dm}{\int dm} = 0 \qquad \{\text{as } y = 0\}$$

Similarly, $z_{cm} = 0$

Hence, the centre of mass of the rod lies at $\left(\frac{3aL + 2bL^2}{6a + 3bL}, 0, 0\right)$

ILLUSTRATION 11

Find the centre of mass of a thin semi-circular ring of radius R.

SOLUTION

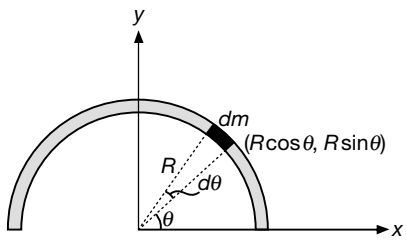
From symmetric distribution of the body we see that the centre of mass must lie along the y-axis, so $x_{cm} = 0$. In this case it is convenient to express the mass element in terms of the angle θ, measured in radian. The element, which subtends an angle $d\theta$ at the origin, has a length $Rd\theta$ and a mass $dm = Rd\theta$.

Coordinates of the infinitesimal element dm are

$$x = R\cos\theta \text{ and } y = R\sin\theta$$

So, after finding these coordinates for the infinitesimal element we use the fundamental formula to calculate the centre of mass according to which we have

$$x_{cm} = \frac{1}{M}\int x\,dm \text{ and } y_{cm} = \frac{1}{M}\int y\,dm$$



So, we get $x_{cm} = 0$ and

$$y_{cm} = \frac{1}{M}\int_0^{\pi} \lambda R^2 \sin\theta\, d\theta = \frac{\lambda R^2}{M}(-\cos\theta)\Big|_0^{\pi} = \frac{2\lambda R^2}{M}$$

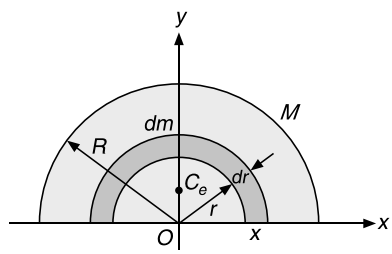
The total mass of the ring is $M = \pi R\lambda$ and hence $y_{cm} = \frac{2R}{\pi}$.

ILLUSTRATION 12

Locate the centre of mass of a homogeneous semi-circular disc of radius R and mass M.

SOLUTION

A semi-circular disc can be subdivided into large number of semi-circular ring elements, as shown in Figure.



According to result obtained in the last Illustration, all these elemental rings have centre of mass on the symmetrical axis of the disc (i.e., on the y-axis for chosen reference frame).

Now, if we replace all these elemental rings by points masses placed at their centre of mass position, then,

given disc reduces to a system of large number of particles distributed on the y-axis. So, to locate the centre of mass of the disc, centre of mass of these particles can be found and used equivalently.
Consider an infinitesimal semi-circular ring of radius r having thickness dr and mass dm, then

$$dm = \frac{M}{(\pi R^2/2)}(\pi r dr)$$

$$dm = \frac{2M}{R^2} r dr \qquad \ldots(1)$$

The centre of mass of the chosen extended element of radius r, thickness dr is at the point C_e, as shown in the same figure, then, coordinates of C_e can be given as

$$x = 0 \text{ and} \qquad \ldots(2)$$

$$y = \frac{2r}{\pi} \qquad \ldots(3)$$

Now, the chosen element can be treated as a point mass placed at the point C_e. If we consider all elemental rings shown in figure in a similar way, then, the coordinates of the centre of mass of the given semi-circular disc, X and Y, can be obtained by using equation. Therefore,

$$x_{cm} = \frac{\int x dm}{\int dm} = 0 \qquad \{\text{using equation (2)}\}$$

and $$y_{cm} = \frac{\int y dm}{\int dm} = \frac{\int \left(\frac{2r}{\pi}\right) dm}{\int dm} \qquad \{\text{using equation (3)}\}$$

$$\Rightarrow \quad y_{cm} = \frac{\frac{2}{\pi}\int_0^R r\left(\frac{2M}{R^2}\right)(r dr)}{M} = \frac{4}{\pi R^2}\int_0^R r^2 dr$$

$$\Rightarrow \quad y_{cm} = \left(\frac{4}{\pi R^2}\right)\left(\frac{R^3}{3}\right) = \frac{4R}{3\pi}$$

Hence, the centre of mass of the given semi-circular disc lies on the symmetrical axis of the disc at a distance of $\frac{4R}{3\pi}$ from its centre.

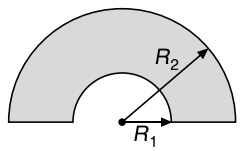
Centre of mass of a complete circular disc (uniform) lies at the centre of the disc.
While choosing elements, you must take care the following:

(a) the centre of mass of the element itself must be known. (If your chosen element is a point mass then its centre of mass coincides with its position.)
(b) by using simple integration techniques your element must be able to cover the entire body.

$$\vec{r}_{cm} = \begin{bmatrix} \frac{1}{M}\int \vec{r}\, dm, & \text{for infinitesimal elements} \\ \frac{1}{M}\int \left(\vec{r}_{\substack{\text{cm of the} \\ \text{infinitesimal} \\ \text{element}}}\right) dm, & \text{for extended elements} \end{bmatrix}$$

ILLUSTRATION 13

Locate the centre of mass of an annular semi-circular disc of inner radius R_1 and outer radius R_2 as shown in Figure.

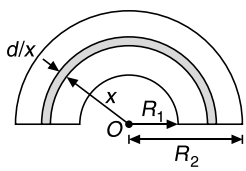


SOLUTION

Let σ be the mass per unit area of the disc. To locate the centre of mass of the annular disc, we consider a half ring of radius x width dx having mass dm, then we have

$$dm = \sigma dA = \sigma(\pi x dx)$$

Centre of mass of this infinitesimal half ring is at a distance $\frac{2x}{\pi}$ from O as shown in Figure.



$$\Rightarrow \quad y_{cm} = \frac{1}{M}\int_{R_1}^{R_2}(\sigma\pi x dx)\frac{2x}{\pi} = \frac{2\sigma}{M}\int_{R_1}^{R_2}\sigma x^2 dx$$

$$\Rightarrow \quad y_{cm} = \frac{2\sigma}{\sigma\pi\left(R_2^2 - R_1^2\right)/2}\left(\frac{x^3}{3}\Big|_{R_1}^{R_2}\right)$$

$$\Rightarrow \quad y_{cm} = \frac{4\left(R_2^3 - R_1^3\right)}{3\pi\left(R_2^2 - R_1^2\right)}$$

We can also find the centre of mass of this object by considering it to be complete half disc of radius R_2 and a then removing a smaller half disc of radius R_1 from it. If y_{cm} be the centre of mass of this disc, then we have

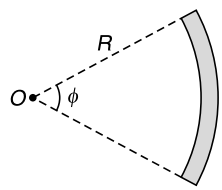
$$y_{cm} = \frac{m_{full}\left(y_{cm}\right)_{full} - m_{removed}\left(y_{cm}\right)_{removed}}{m_{full} - m_{removed}}$$

$$\Rightarrow \quad y_{cm} = \frac{\left(\sigma\pi R_2^2\right)\left(4R_2/3\pi\right) - \left(\sigma\pi R_1^2\right)\left(4R_1/3\pi\right)}{\sigma\left(\pi R_2^2 - \pi R_1^2\right)/2}$$

$$\Rightarrow \quad y_{cm} = \frac{4}{3\pi}\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

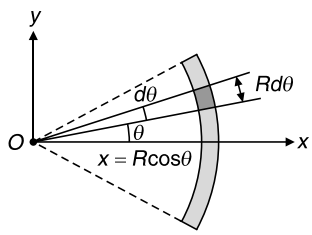
ILLUSTRATION 14

Calculate the distance of centre of mass (from O) of the uniform circular arc subtending an angle ϕ at the centre O as shown in Figure.



SOLUTION

Let the mass of the arc be M. Consider an infinitesimal element of mass dm subtending an angle $d\theta$ at the centre of the arc as shown in Figure.



Its x coordinate is $R\cos\theta$ and its mass is

$$dm = \left(\frac{M}{\phi}\right)d\theta$$

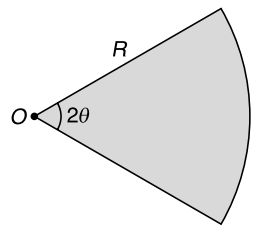
So, centre of mass of arc is given by

$$x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_{-\phi/2}^{+\phi/2}(R\cos\theta)\left(\frac{M}{\phi}d\theta\right)$$

$$\Rightarrow \quad x_{cm} = \frac{2R}{\phi}\sin\left(\frac{\phi}{2}\right)$$

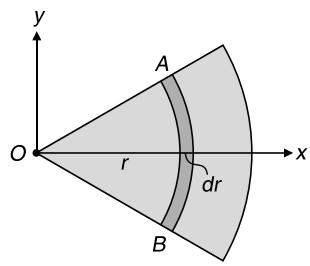
ILLUSTRATION 15

Find the distance of centre of mass from point O of a thin uniform plate having the shape of a sector of a circular disc of radius R subtending an angle 2θ at the centre O as shown in Figure.



SOLUTION

Let M be the mass of the sector of the disc. Consider an elemental arc AB of radius r and width dr as shown in Figure.



If σ be the surface mass density of the disc, then the mass m of the arc AB is given by

$$dm = \sigma(2r\theta dr) = 2\sigma r\theta dr$$

As derived in the earlier ILLUSTRATION, due to symmetry the centre of mass of this arc must be on lie on the angle bisector i.e. on x-axis at distance $x = \frac{r\sin\theta}{\theta}$ from O. The centre of mass of sector is

$$x_{cm} = \frac{1}{M}\int x\,dm, \text{ where } M = \sigma\left[R^2(2\theta)\right]$$

$$\Rightarrow \quad x_{cm} = \frac{1}{M}\int_0^R\left(\frac{r\sin\theta}{\theta}\right)(2\sigma r\theta dr)$$

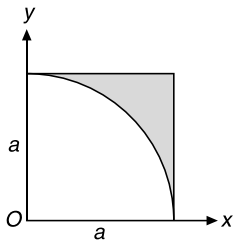
$$\Rightarrow \quad x_{cm} = \frac{1}{2\sigma R^2\theta}\left[\int_0^R \left(\frac{r\sin\theta}{\theta}\right)(2\sigma r\theta dr)\right]$$

$$\Rightarrow \quad x_{cm} = \frac{\sin\theta}{R^2\theta}\left(\int_0^R r^2 dr\right) = \left(\frac{\sin\theta}{R^2\theta}\right)\left(\frac{r^3}{3}\Big|_0^R\right)$$

$$\Rightarrow \quad x_{cm} == \left(\frac{\sin\theta}{R^2\theta}\right)\left(\frac{R^3}{3}\right) = \frac{R\sin\theta}{3\theta}$$

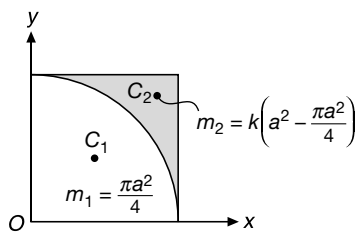
ILLUSTRATION 16

A quarter circular disc of radius a is removed from a square plate of side a as shown in Figure. Find the coordinates of centre of mass of the remaining part.



SOLUTION

The centre of mass of quarter disc taken out from the square plate lies at the point $C_1\left(\frac{4a}{3\pi}, \frac{4a}{3\pi}\right)$. Let the centre of mass of the remaining part be at $C_2(x, y)$ such that the combined centre of mass of the quarter disc and the remaining part must be located at centre of square, i.e. at $\left(\frac{a}{2}, \frac{a}{2}\right)$.



Since, we know that for the full square plate (made up of quarter circular disc and the remainder plate), the centre of mass is given by $\left(\frac{a}{2}, \frac{a}{2}\right)$, so we get

$$\frac{a}{2} = \frac{m_{\text{quarter}}\left(x_{cm}\right)_{\text{quarter}} + m_{\text{remainder}}\left(x_{cm}\right)_{\text{remainder}}}{M}$$

$$\Rightarrow \quad \frac{a}{2} = \frac{1}{\sigma a^2}\left[\sigma\left(\frac{\pi a^2}{4}\right)\left(\frac{4a}{3\pi}\right) + (\sigma a^2)\left(\frac{4-\pi}{4}\right)x\right]$$

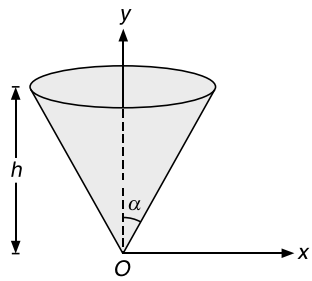
$$\Rightarrow \quad \frac{a}{2} = \frac{a}{3} + \left(\frac{4-\pi}{4}\right)x$$

$$\Rightarrow \quad x = \frac{2a}{3(4-\pi)}$$

Similarly, we get $y = \frac{2a}{3(4-\pi)}$

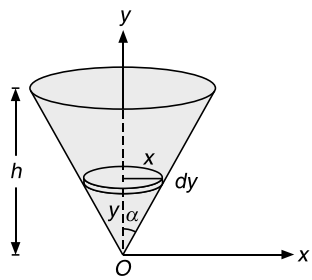
ILLUSTRATION 17

Find the centre of mass of a uniform solid cone of height h and semi-angle α, as shown in the figure.



SOLUTION

We assume the apex of the cone at the origin. It is clear that the centre of mass will lie along the y-axis. We divide the cone into disc of radius x and thickness dy as shown in Figure.



The volume of such a disc is

$$dV = \pi x^2 dy = \pi\left(y\tan\alpha\right)^2 dy$$

The mass of the disc is $dm = \rho dV$

Let us first find the total mass M of the cone

$$M = \int dm = \pi\rho\tan^2\alpha\int_0^h y^2 dy$$

$$\Rightarrow \quad M = \left(\pi\rho\tan^2\alpha\right)\left(\frac{h^3}{3}\right) \qquad \ldots(1)$$

The position of the centre of mass is given by

$$y_{cm} = \frac{1}{M}\int y dm = \frac{1}{M}\pi\rho\tan^2\alpha\int_0^h y^3 dy$$

$$\Rightarrow \quad y_{cm} = \frac{1}{M}\pi\rho\tan^2\alpha\int_0^h y^3 dy$$

$$\Rightarrow \quad y_{cm} = \left(\frac{1}{M}\pi\rho\tan^2\alpha\right)\left(\frac{h^4}{4}\right) \qquad \ldots(2)$$

Equating equations (1) and (2), we get

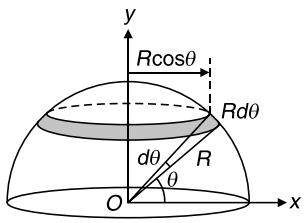
$$y_{cm} = \frac{3h}{4}$$

ILLUSTRATION 18

Locate the centre of mass of a uniform thin hemispherical shell of radius R from its centre O.

SOLUTION

Consider a hollow hemispherical shell of mass M and radius R. Let us consider an elemental circular strip of angular width $d\theta$ at an angular distance θ from the base of the hemisphere as shown in Figure.



This strip will have an area $dA = (2\pi R\cos\theta)(Rd\theta)$
The mass dm of this strip is

$$dm = \sigma dA = \left(\frac{M}{2\pi R^2}\right)(2\pi R\cos\theta)(Rd\theta)$$

Due to symmetry, we observe that centre of mass lies on y axis. So, $x_{cm} = 0$. Also,

$$y_{cm} = \frac{1}{M}\int\left(\begin{array}{c}\text{Mass of}\\ \text{Strip}\end{array}\right)\left(\begin{array}{c}y \text{ coordinate of}\\ \text{CM of strip}\end{array}\right)$$

Here y coordinate of this strip of mass dm can be taken as $R\sin\theta$, so we have

$$y_{cm} = \frac{1}{M}\int_0^{\pi/2}(R\sin\theta)dm$$

$$\Rightarrow \quad y_{cm} = \frac{1}{M}\int_0^{\pi/2}\left(\frac{M}{2\pi R^2}2\pi R^2\cos\theta d\theta\right)R\sin\theta$$

$$\Rightarrow \quad y_{cm} = R\int_0^{\pi/2}\sin\theta\cos\theta d\theta$$

$$\Rightarrow \quad y_{cm} = \frac{R}{2}\int_0^{\pi/2}(2\sin\theta\cos\theta)d\theta = \frac{R}{2}\int_0^{\pi/2}\sin(2\theta)d\theta$$

$$\Rightarrow \quad y_{cm} = \frac{R}{2}$$

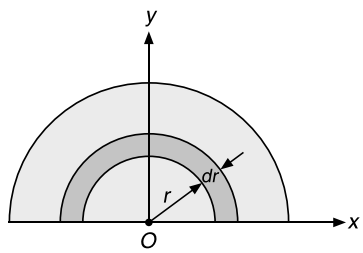
So, centre of mass of hemispherical shell lies at y axis at a distance $\frac{R}{2}$ from O i.e. centre of shell.

ILLUSTRATION 19

Locate the centre of mass of a uniform solid hemisphere of radius R.

SOLUTION

For a solid hemisphere, the mass is distributed uniformly over its volume. Consider an infinitesimal hemispherical shell of radius r thickness dr having mass dm as shown in Figure.



Also, we note that from symmetry, $x_{CM} = 0$
Now, centre of mass of a hemispherical shell is at $\frac{r}{2}$ from centre, so we have

$$y_{cm} = \frac{1}{M}\left[\int_0^R\left(\begin{array}{c}\text{Mass of}\\ \text{Shell}\end{array}\right)\left(\begin{array}{c}y \text{ coordinate}\\ \text{of CM of Shell}\end{array}\right)\right]$$

$$\Rightarrow \quad y_{cm} = \frac{1}{M}\left[\int_0^R(y_{\text{cm shell}})dm\right]$$

where, $dm = \rho dV = \left(\frac{M}{2\pi R^3/3}\right)(2\pi x^2 dx)$

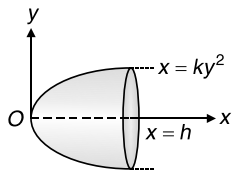
$$\Rightarrow \quad dm = \left(\frac{3M}{R^3}\right)x^2 dx$$

$$\Rightarrow \quad y_{cm} = \frac{1}{M}\left[\int_0^R\left(\frac{3M}{R^3}x^2 dx\right)\frac{x}{2}\right] = \frac{3}{2R^3}\left(\frac{x^4}{4}\Big|_0^R\right)$$

$$\Rightarrow \quad y_{cm} = \frac{3R}{8}$$

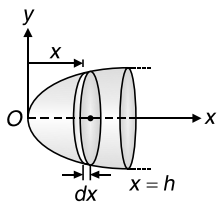
ILLUSTRATION 20

A solid generated by the rotation of a parabola is called a paraboloid. Find the location of centre of mass of such a paraboloid (from O) of uniform density formed by rotating a parabola $x = ky^2$ about x-axis. Assume that the height of object is h as shown in Figure.



SOLUTION

To find centre of mass of a paraboloid, let us consider an infinitesimal disc of radius y, width dx, mass dm at a distance x from origin as shown in Figure.



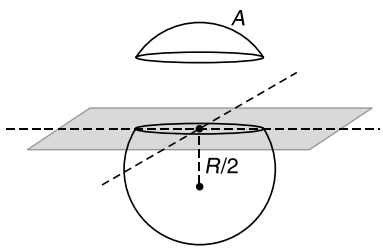
$$\Rightarrow \quad dm = \rho\left(\pi y^2\right)dx = \pi\rho\left(\frac{x}{k}\right)dx = \frac{\pi\rho x dx}{k}$$

$$\Rightarrow \quad x_{cm} = \frac{\int x dm}{M} = \frac{\int_0^h \pi\rho x^2 dx}{\int_0^h \pi\rho x dx} = \frac{x^3/3\big|_0^h}{x^2/2\big|_0^h}$$

$$\Rightarrow \quad x_{cm} = \frac{h^3/3}{h^2/2} = \frac{2h}{3}$$

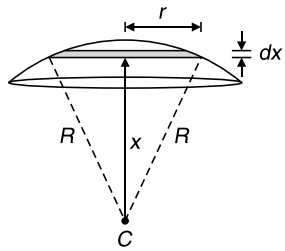
ILLUSTRATION 21

A small part A is cut from a solid sphere by a plane at a distance $\frac{R}{2}$ from its centre as shown in Figure. Find the location of centre of mass of object A from the centre of the sphere.



SOLUTION

To find centre of mass of object A, we consider a small elemental disc of width dx at a distance x from the centre C as shown in Figure.



Radius r of this elemental disc is

$$r = \sqrt{R^2 - x^2}$$

If ρ be the density of the sphere, then mass of the elemental disc will be

$$dm = \rho dV = \rho\left(\pi r^2 dx\right) = \pi\rho\left(R^2 - x^2\right)dx$$

Mass of object A can be obtained as

$$M = \int_{R/2}^{R} dm = \pi\rho \int_{R/2}^{R} \left(R^2 - x^2\right)dx$$

$$\Rightarrow \quad M = \pi\rho\left(R^2 x - \frac{x^3}{3}\right)\Bigg|_{R/2}^{R}$$

$$\Rightarrow \quad M = \pi\rho\left[\left(R^3 - \frac{R^3}{3}\right) - \left(\frac{R^3}{2} - \frac{R^3}{24}\right)\right] = \frac{5}{24}\pi\rho R^3$$

Since, $y_{cm} = \frac{1}{M}\int_{R/2}^{R} x dm$

$$\Rightarrow \quad y_{cm} = \frac{24}{5\rho\pi R^3}\int_{R/2}^{R} \rho\pi\left(R^2 - x^2\right)x dx$$

$$\Rightarrow \quad y_{cm} = \frac{24}{5R^3}\left(\frac{R^2 x^2}{2} - \frac{x^4}{4}\right)\Bigg|_{R/2}^{R}$$

$$\Rightarrow \quad y_{cm} = \frac{12}{5R^3}\left[\left(R^4 - \frac{R^4}{2}\right) - \left(\frac{R^4}{4} - \frac{R^4}{32}\right)\right]$$

$$\Rightarrow \quad y_{cm} = \frac{27}{40}R$$

Test Your Concepts-I

Based on Centre of Mass

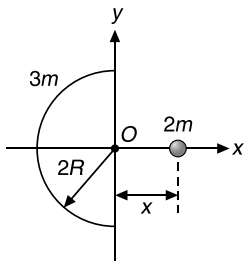
(Solutions on page H.69)

1. There are two masses m_1 and m_2 placed at a distance ℓ apart, let the centre of mass of this system is at a point named *C*. If m_1 is displaced by ℓ_1 towards *C* and m_2 is displaced by ℓ_2 away from *C*. Find the distance, from *C* where new centre of mass will be located.

2. Two particles having masses m_1 and m_2 are placed at a separation r between them. Let m_2 be displaced away from m_1 by a distance x. By what distance, m_1 should be displaced in order to keep the position of centre of mass unchanged.

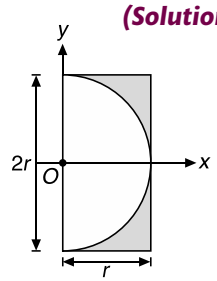
3. The position vector of three particles of mass $m_1 = 1\,\text{kg}$, $m_2 = 2\,\text{kg}$ and $m_3 = 3\,\text{kg}$ are $\vec{r}_1 = (\hat{i} + 4\hat{j} + \hat{k})m$, $\vec{r}_2 = (\hat{i} + \hat{j} + \hat{k})m$ and $\vec{r}_3 = (2\hat{i} - \hat{j} - 2\hat{k})m$ respectively. Find the position vector of their centre of mass.

4. A ring of mass $3m$, radius $2R$ and a particle of mass $2m$ are kept as shown in Figure such that the centre of mass of the combination lies at the origin. Calculate x.



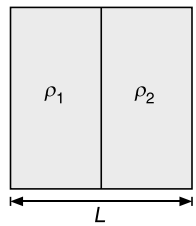
5. Consider a rectangular plate of dimensions $a \times b$. If this plate is considered to be made up of four rectangles of dimensions $\frac{a}{2} \times \frac{b}{2}$ and we now remove one out of four rectangles. Find the position where the centre of mass of the remaining system will be.

6. Determine the coordinate of centre of mass of a thin homogeneous plate having the form of a rectangle with sides r and $2r$ from which a semicircle with a radius r is cut out.

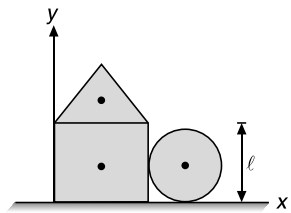


7. The coordinates of a triangle *ABC* are *A*(1, 2), *B*(4, 6) and *C*(−3, −2). Three particles of masses 1 kg, 2 kg and *m* kg are placed the vertices of the triangle. If the coordinates of the centre of mass are $\left(\frac{3}{5}, 2\right)$, calculate the mass *m*.

8. Half of the square plate shown in figure is made of a material of density ρ_1 and the other half of density ρ_2. The length of the plate is *L*. Locate the centre of mass of the plate.

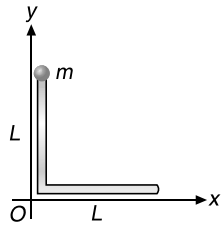


9. A square sheet, a disc and an equilateral triangle having same surface mass densities are placed as shown in Figure. Find the coordinates of centre of mass of the three bodies.

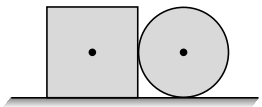


10. The linear mass density of a thin rod of length ℓ varies with the distance x from one end as $\lambda = \lambda_0 \frac{x^2}{l^2}$. Find the position of centre of mass of rod.

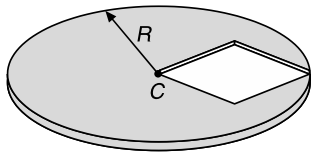
11. Find the centre of mass of the system of a L-shaped rod of mass m and a particle of mass m attached at the end of the rod as shown in Figure.



12. A uniform square plate of side a and a disc of diameter $2a$ having same mass per unit area are kept in contact as shown in Figure. Locate the position of the centre of mass of the system from the centre of square.



13. A square portion is removed from the disc as shown in Figure. Locate centre of mass of the remaining portion from the centre of the full disc.



14. A rod of length L, having negligible thickness, has a linear mass density that increases linearly with distance from one end of the rod from λ_1 to λ_2 at the other end. Locate the centre of mass of the rod from the lighter end.

VELOCITY AND ACCELERATION OF CENTER OF MASS

The instantaneous velocity of the centre of mass is given by $\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt}$. From definition, we have

$$\vec{v}_{cm} = \frac{1}{M}\Sigma\left(m_i \frac{d\vec{r}_i}{dt}\right) = \frac{1}{M}\left(\Sigma m_i \vec{v}_i\right)$$

The total momentum of a system of particles is equivalent to that of a single (imaginary) particle of mass $M = \Sigma m_i$ moving at the velocity of the centre of mass $\vec{v}_{cm}$. So, we have

$$\vec{p}_{system} = \vec{p}_1 + \vec{p}_2 + \ldots\ldots\ldots\ldots + \vec{p}_n$$

$$\Rightarrow \quad \vec{p}_{system} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots\ldots + m_n\vec{v}_n$$

$$\Rightarrow \quad \vec{p}_{system} = M\vec{v}_{cm}$$

Using the concept of centre of mass, we may deal with the translational motion of extended objects or systems of particles, as if they were point particle with all the mass concentrated at the centre of mass. Differentiating equation, with respect to time, we get

$$a_{cm} = \frac{\Sigma m_i \vec{a}_i}{M} = \frac{\Sigma \vec{F}_i}{M}$$

Since the internal forces cancel in pairs, therefore,

$$\Sigma \vec{F}_i = \vec{F}_{ext}$$

$$\Rightarrow \quad \vec{F}_{ext} = M\vec{a}_{cm}$$

The centre of mass accelerates as if it were a point particle of mass $M = \Sigma m_i$ and the net external force were applied at this point.

QUICK RECAP

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_n\vec{r}_n}{m_1 + m_2 + \ldots + m_n}$$

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + \ldots + m_nx_n}{m_1 + m_2 + \ldots + m_n}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + \ldots + m_ny_n}{m_1 + m_2 + \ldots + m_n}$$

$$z_{cm} = \frac{m_1z_1 + m_2z_2 + \ldots + m_nz_n}{m_1 + m_2 + \ldots + m_n}$$

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_n\vec{v}_n}{m_1 + m_2 + \ldots + m_n}$$

$$\vec{p}_{cm} = \vec{p}_1 + \vec{p}_2 + \ldots + \vec{p}_n$$

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots + m_n\vec{a}_n}{m_1 + m_2 + \ldots + m_n} \text{ and}$$

$$\vec{F}_{cm} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n$$

ILLUSTRATION 22

Two particles one of mass 1 kg and the other of 2 kg are projected simultaneously with the same speed from the roof of a tower, the one of mass 1 kg vertically upwards and the other vertically downwards. What is the acceleration of centre of mass of these two particles?

SOLUTION

In this Problem, students generally get three answers, i.e. g, $\frac{g}{3}$ *and zero*. However, the correct answer is g.

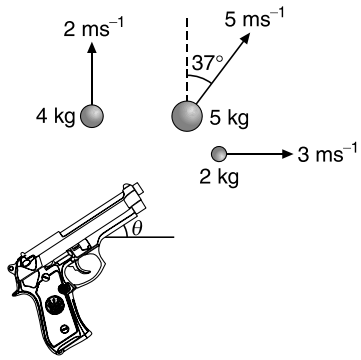
Since, $\vec{a}_{cm} = \dfrac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

where, $\vec{a}_1 = \vec{a}_2 = g$ {downwards}

$$\Rightarrow \quad \vec{a}_{cm} = \frac{(1)(g)+(2)(g)}{1+2} = g \qquad \{\text{downwards}\}$$

ILLUSTRATION 23

A gun whose barrel is inclined at some angle to the horizontal fires three shells of mass 2 kg, 5 kg and 4 kg which move in different directions as shown in Figure.



If the centre of mass of the three shells moves along the direction in which the barrel points, then calculate the angle made by the gun's barrel with the horizontal. Take $\cos(53^\circ) = 3/5$.

SOLUTION

The x and y components of velocity of centre of mass of the shells are given by

$$(v_{cm})_x = \frac{(2)(3)+(5)(5\sin 37^\circ)+(4)(0)}{4+5+2}$$

$$\Rightarrow \quad (v_{cm})_x = \frac{6+(25)(3/5)}{4+5+2} = \frac{21}{11}\ \text{ms}^{-1}$$

$$(v_{cm})_y = \frac{(2)(0)+(5)(5\cos 37^\circ)+(4)(2)}{4+5+2}$$

$$\Rightarrow \quad (v_{cm})_y = \frac{(25)(4/5)+8}{4+5+2} = \frac{28}{11}\ \text{ms}^{-1}$$

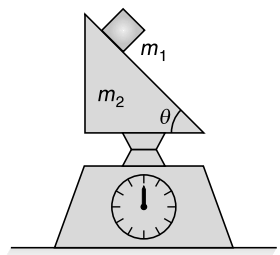
If θ be the angle which the gun's barrel makes with the horizontal, then we have

$$\tan\theta = \tan^{-1}\left[\frac{(v_{cm})_y}{(v_{cm})_x}\right] = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\Rightarrow \quad \theta = 53^\circ$$

ILLUSTRATION 24

A wedge of mass m_2 is placed at rest on a scale as shown in figure. A small block of mass m_1 slides down the frictionless incline of the wedge. Find the scale reading while the block slides.



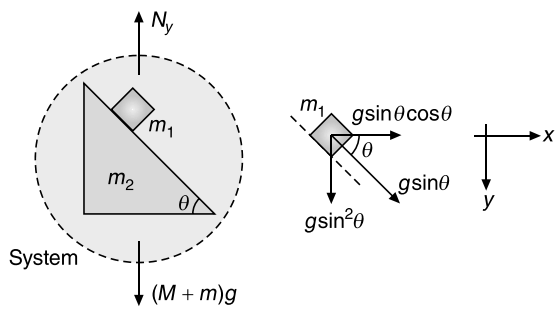
SOLUTION

Acceleration of m_1 in the vertical downward direction, i.e. along y direction will be

$$a_y = g\sin\theta(\sin\theta) = g\sin^2\theta$$

$$\Rightarrow \quad (a_{cm})_y = \frac{m_1 g\sin^2\theta + 0}{m_1 + m_2} = \frac{m_1 g\sin^2\theta}{m_1 + m_2}$$

The free body diagram of the system is shown in Figure.



The scale reading will be the force exerted by the wedge on the measuring scale or vice versa. Applying

Newton's Second Law to the centre of mass of the system in the vertically downward direction, i.e. along the y direction, we get

$$(m_1+m_2)g-N_y=(m_1+m_2)(a_{cm})_y$$

$$\Rightarrow \quad N_y=(m_1+m_2)g-m_1g\sin^2\theta$$

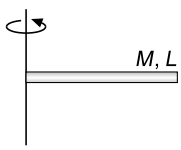
$$\Rightarrow \quad N_y=m_1g(1-\sin^2\theta)+m_2g$$

$$\Rightarrow \quad N_y=m_1g\cos^2\theta+m_2g$$

$$\Rightarrow \quad N_y=(m_1\cos^2\theta+m_2)g$$

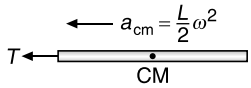
ILLUSTRATION 25

A uniform rod of mass M and length L is tied to a vertical shaft. It rotates in horizontal plane about the vertical axis at angular velocity ω. How much horizontal force does the shaft exert on the rod?



SOLUTION

Let T be the force applied horizontally on the rod. The acceleration of centre of mass is $\left(\frac{L}{2}\right)\omega^2$ because it is moving in a circle of radius $\frac{L}{2}$.



Using $\vec{F}_{ext}=M\vec{a}_{cm}$, we get

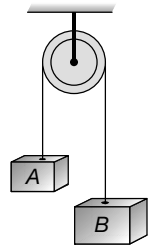
$$T=M\left(\frac{L}{2}\right)\omega^2$$

So, the horizontal force exerted is

$$T=Mr_{cm}\omega^2=\frac{1}{2}ML\omega^2$$

ILLUSTRATION 26

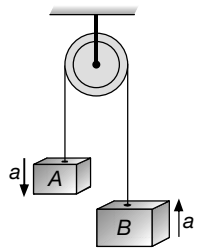
In the arrangement shown in figure, $m_A=2$ kg and $m_B=1$ kg. String is light and inextensible. Find the acceleration of centre of mass of both the blocks. Neglect friction everywhere.



SOLUTION

Net pulling force on the system is

$$(m_A-m_B)g=(2-1)g=g$$



Total mass being pulled is $m_A+m_B=3$ kg

$$\Rightarrow \quad a=\frac{\text{Net pulling force}}{\text{Total mass}}=\frac{g}{3}$$

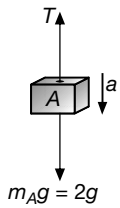
Now, $\vec{a}_{cm}=\dfrac{m_A\vec{a}_A+m_B\vec{a}_B}{m_A+m_B}$

$$\Rightarrow \quad |\vec{a}_{cm}|=\frac{(2)(a)-(1)(a)}{1+2}=\frac{a}{3}$$

$$\Rightarrow \quad |\vec{a}_{cm}|=\frac{g}{9}\text{, downwards}$$

Alternate Method:

Free body diagram of block A is shown in figure.

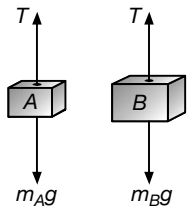


$$2g-T=m_A(a)$$

$$\Rightarrow \quad T=2g-m_Aa$$

$$\Rightarrow \quad T=2g-(2)\left(\frac{g}{3}\right)=\frac{4g}{3}$$

Free body diagrams of A and B both are as shown in figure



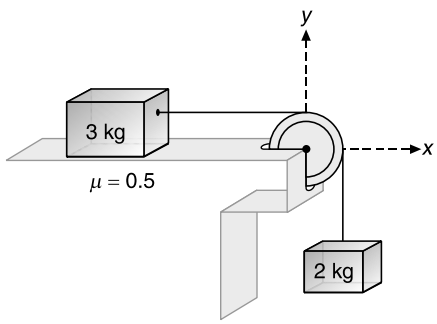
Since, $|\vec{a}_{cm}| = \dfrac{\text{Net force on both the blocks}}{m_A + m_B}$

$$\Rightarrow \quad |\vec{a}_{cm}| = \frac{(m_A + m_B)g - 2T}{2+1} = \frac{3g - (8g/3)}{3}$$

$$\Rightarrow \quad |\vec{a}_{cm}| = \frac{g}{9}, \text{ downwards}$$

ILLUSTRATION 27

In the arrangement shown in Figure, calculate the acceleration of centre of mass of the system. Take $g = 10\ \text{ms}^{-2}$.



SOLUTION

The limiting friction between the 3 kg block and the surface is

$$f_l = \mu N = (0.5)(3)(10) = 15\ \text{N}$$

The weight of the 2 kg block, i.e. 20 N provides the driving force to the system, due to which magnitude of acceleration of both masses is given by

$$a = \frac{20-15}{2+3} = \frac{5}{5} = 1\ \text{ms}^{-2}$$

So, for $m_1 = 3$ kg, we have $\vec{a}_1 = 1\hat{i}\ \text{ms}^{-2}$ and that for $m_2 = 2$ kg, we have $\vec{a}_2 = -1\hat{j}\ \text{ms}^{-2}$. Since

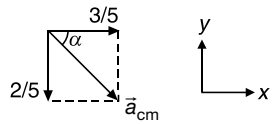
$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$$

$$\Rightarrow \quad \vec{a}_{cm} = \frac{3\hat{i} - 2\hat{j}}{3+2} = \frac{3}{5}\hat{i} - \frac{2}{5}\hat{j}$$

Magnitude of acceleration of CM is given by

$$a_{cm} = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = \frac{\sqrt{13}}{5}\ \text{ms}^{-2}$$

The direction of motion of centre of mass is calculated from the Figure shown.

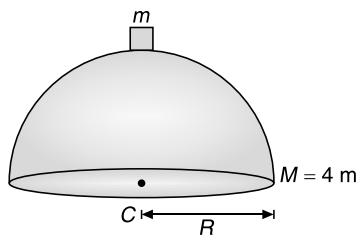


Since, $\tan\alpha = \dfrac{2/5}{3/5} = \dfrac{2}{3}$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{2}{3}\right), \text{ below the horizontal}$$

ILLUSTRATION 28

A small block of mass m starts sliding down from rest along the smooth surface of a fixed hollow hemisphere of same mass m. Calculate the distance of centre of mass of block and hemisphere from centre of hemisphere C when block m separates from the surface of hemisphere.



SOLUTION

When m starts from rest, it breaks off at an angular displacement of $\theta = \cos^{-1}\left(\dfrac{2}{3}\right)$ with the vertical. At this point, if the centre of mass of block and hemisphere is located at a point $P(x_{cm}, y_{cm})$ with origin at C, then we have

$$x_{cm} = \frac{m\left(\sqrt{5}R/3\right) + 4m(0)}{5m} = \frac{R}{3\sqrt{5}}$$

$$y_{cm} = \frac{m(2R/3) + (4m)(R/2)}{5m} = \frac{8}{15}R$$

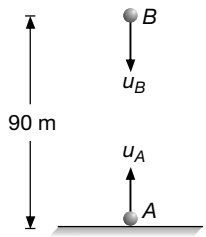
So, the distance CP of centre of mass is

$$r_{cm} = \sqrt{x_{cm}^2 + y_{cm}^2}$$

$$\Rightarrow \quad r_{cm} = R\sqrt{\frac{1}{45} + \frac{64}{225}} = R\sqrt{\frac{69}{225}} = R\sqrt{\frac{23}{75}}$$

ILLUSTRATION 29

Two particles A and B of mass 1 kg and 2 kg respectively are projected in the directions shown in figure with speeds $u_A = 200\ \text{ms}^{-1}$ and $u_B = 55\ \text{ms}^{-1}$. Initially they were 90 m apart. Find the maximum height attained by the centre of mass of the particles. Assume acceleration due to gravity to be constant during entire motion and take $g = 10\ \text{ms}^{-2}$.



SOLUTION

Let us first find location of centre of mass of system. Assume origin at A, we get

$$r_{cm} = \frac{(1)(0)+2(90)}{3}$$

$$\Rightarrow \quad r_{cm} = 60\ \text{cm from } A$$

i.e., cm is at height 60 m from the ground at time $t = 0$

Since $\vec{a}_A = \vec{a}_B = g$

$$\Rightarrow \quad \vec{a}_{cm} = \frac{m_A\vec{a}_A + m_B\vec{a}_B}{m_A + m_B} = g = 10\ \text{ms}^{-2}, \text{ downwards}$$

Also, $\vec{u}_{cm} = \dfrac{m_A\vec{u}_A + m_B\vec{u}_B}{m_A + m_B}$

$$\Rightarrow \quad \vec{u}_{cm} = \frac{(1)(200)-(2)(55)}{1+2} = 30\ \text{ms}^{-1}, \text{ upwards}$$

If h be the height attained by centre of mass beyond 60 m, then since

$$v_{cm}^2 = u_{cm}^2 + 2a_{cm}h$$

$$\Rightarrow \quad 0 = (30)^2 - (2)(10)h$$

$$\Rightarrow \quad h = \frac{900}{20} = 45\ \text{m}$$

Therefore, maximum height attained by the centre of mass is

$$H = 60 + 45 = 105\ \text{m}$$

MOTION OF THE CENTRE OF MASS

Now we can discuss the physical importance of the centre of mass concept. Consider the motion of a group of particles m_1, m_2, ……, m_n and whose total mass is $M = \Sigma m_i$ which is a constant. From the definition of the centre of mass, we have

$$M\vec{r}_{cm} = m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots\ldots\ldots\ldots + m_n\vec{r}_n$$

Differentiating this equation wrt time, we obtain

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\ldots\ldots + m_n\vec{v}_n$$

where $v_1 = \dfrac{d\vec{r}_1}{dt}$ is the velocity of the first particle, etc.,

and $v_{cm} = \dfrac{d\vec{r}_{cm}}{dt}$ is the velocity of the centre of mass.

Differentiating the above equation wrt time, we obtain

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots\ldots\ldots + m_n\vec{a}_n$$

where $\vec{a}_1$ is the acceleration of the first particle, etc., and $\vec{a}_{cm}$ is the acceleration of the centre of mass. Now, from Newton's Second Law the force $\vec{F}_1$ acting on the first particle is given by $\vec{F}_1 = m_1\vec{a}_1$. Likewise, $\vec{F}_2 = m_2\vec{a}_2$, etc. We can then write above equation as

$$M\vec{a}_{cm} = \vec{F}_1 + \vec{F}_2 + \ldots\ldots\ldots + \vec{F}_n$$

Among all these forces the internal forces are exerted by the particles on each other. However, from Newton's Third Law, these internal forces will occur in equal and opposite pairs, so that they contribute nothing to the sum. The right hand sum in the above equation then represents the sum of only the external forces acting on all the particles (system). We can then rewrite the above equation as

$$Ma_{cm} = \Sigma\vec{F}_{external}$$

This states that the centre of mass of a system of particles moves as though all the mass of the system is concentrated at the centre of mass and all the external forces were applied at that point.

Problem Solving Technique(s)

In one such important situation when we have

$$\Sigma \vec{F}_{\text{external}} = \vec{0}$$

then, $\vec{v}_{cm} = \text{constant}$

Further if we have a two particle system initially at rest (or both its particle to be at rest), then as a consequence of the above result we have

$$(\vec{v}_{cm})_{\text{final}} = (\vec{v}_{cm})_{\text{initial}} = \vec{0}$$

$$\Rightarrow \quad \vec{r}_{cm} = \text{constant}$$

$$\Rightarrow \quad \Delta\vec{r}_{cm} = \vec{0}$$

For a two particle system, this result can be interpreted as

$$\Delta\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}\right) = \vec{0}$$

$$\Rightarrow \quad m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2 = \vec{0}$$

where, $\Delta\vec{r}_1$ and $\Delta\vec{r}_2$ are the displacements of the particles m_1 and m_2 with respect to the ground. The displacements $\Delta\vec{r}_1$ and $\Delta\vec{r}_2$ are in opposite directions.

DISPLACEMENT OF CENTRE OF MASS OF A SYSTEM OF PARTICLES

Consider a system of n point masses m_1, m_2, m_3,…., m_n whose position vectors from origin O are given by $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_n$, respectively. Then, the position vector of the centre of mass of the system is given by

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_n\vec{r}_n}{m_1 + m_2 + \ldots + m_n} \qquad \ldots(1)$$

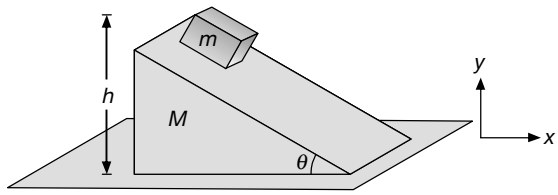
If $\Delta\vec{r}_1$, $\Delta\vec{r}_2$, $\Delta\vec{r}_3$ …, $\Delta\vec{r}_n$ be the displacement vectors of the particles of system, then the displacement vector of centre of mass of this system of particles can be directly given by

$$\Delta\vec{r}_{cm} = \frac{m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2 + m_3\Delta\vec{r}_3 + \ldots + m_n\Delta\vec{r}_n}{m_1 + m_2 + m_3 + \ldots + m_n} \qquad \ldots(2)$$

If net external force on the system in a particular direction is zero and initially the centre of mass of the system is at rest, then the centre of mass will not move along that particular direction even though some particles (or bodies) of the system may move along that direction.

ILLUSTRATION 30

A block of mass m is released from the top of a wedge of mass M as shown in figure. Find the displacement of wedge on the horizontal ground when the block reaches the bottom of the wedge. Neglect friction everywhere.



SOLUTION

Here the system is wedge + block. Net force on the system in horizontal direction (x-direction) is zero, therefore, the centre of mass of the system will not move in x-direction. Hence when block moves to the right the wedge moves to the left. So, we can apply,

$$m_R\Delta x_R = m_L\Delta x_L = x_L m_L \qquad \ldots(1)$$

Let x be the displacement of wedge to the left. Then

Δx_L = displacement of wedge towards left = x

m_L = mass of wedge = M

Δx_R = displacement of block with respect to ground towards right = $h\cot\theta - x$

and m_R = mass of block = m

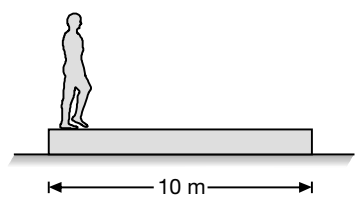
Substituting in equation (1), we get

$$m(h\cot\theta - x) = xM$$

$$\Rightarrow \quad x = \frac{mh\cot\theta}{M + m}$$

ILLUSTRATION 31

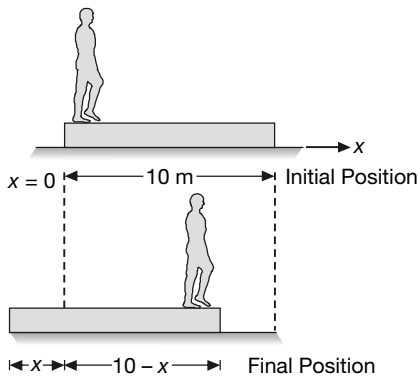
A wooden plank of mass $20\ \text{kg}$ is resting on a smooth horizontal floor. A man of mass $60\ \text{kg}$ starts moving from one end of the plank to the other end. The length of the plank is $10\ \text{m}$. Find the displacement of the plank over the floor when the man reaches the other end of the plank.



SOLUTION

Method I:

Here the system is man + plank. Net force on this system in horizontal direction is zero and initially the centre of mass of the system is at rest. Therefore, the centre of mass does not move in horizontal direction. Let x be the displacement of the Plank. Assuming the origin, i.e., $x = 0$ at the position shown in Figure



As we said earlier also, the centre of mass will not move in horizontal direction (x-axis). Therefore, for centre of mass to remain stationary.

$$x_i = x_f$$

$$\frac{(60)(0)+20\left(\frac{10}{2}\right)}{60+20} = \frac{(60)(10-x)+20\left(\frac{10}{2}-x\right)}{60+20}$$

Please note that the centre of mass of the plank lies at its centre.

$$\Rightarrow \quad \frac{5}{4} = \frac{6(10-x)+2\left(\frac{10}{2}-x\right)}{8} = \frac{60-6x+10-2x}{8}$$

$$\Rightarrow \quad 5 = 30-3x+5-x$$

$$\Rightarrow \quad 4x = 30$$

$$\Rightarrow \quad x = \frac{30}{4} \text{ m}$$

$$\Rightarrow \quad x = 7.5 \text{ m}$$

Method II:

In this situation discussed above we can also apply

$$\Sigma m_R x_R = \Sigma m_L x_L \qquad \ldots(1)$$

where, $\Sigma m_R x_R$ is the summation of product of x and m of the particles (or bodies) which are moving towards right (R) and $\Sigma m_L x_L$ is the summation of product of x and m of the particles (or bodies) which are moving towards left (L).

However, remember the following two conditions while using the above equation.

(i) This equation can be applied when centre of mass does not move in x-direction and no external force is acting along x-direction.

(ii) In the above equation x is the displacement of particle relative to ground.

Here, displacement of plank towards left is $x_L = x$, mass of plank is $m_L = 20$ kg, displacement of man relative to ground towards right is $x_R = 10 - x$ and mass of man is $m_R = 60$ kg. From (1), we get

$$(10-x)(60) = 20x$$

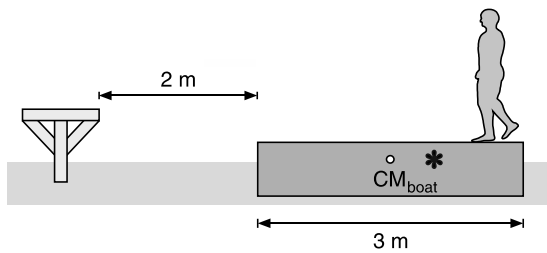
$$\Rightarrow \quad x = 30-3x$$

$$\Rightarrow \quad 4x = 30$$

$$\Rightarrow \quad x = \frac{30}{4} = 7.5 \text{ m}$$

ILLUSTRATION 32

A man of mass $m_1 = 60$ kg is at the rear of a stationary boat of mass $m_2 = 40$ kg and length 3 m, which can move freely on the water, as shown in the Figure.



The front of the boat is 2 m from the dock. What happens when the man walks to the front? Treat the boat as a uniform object.

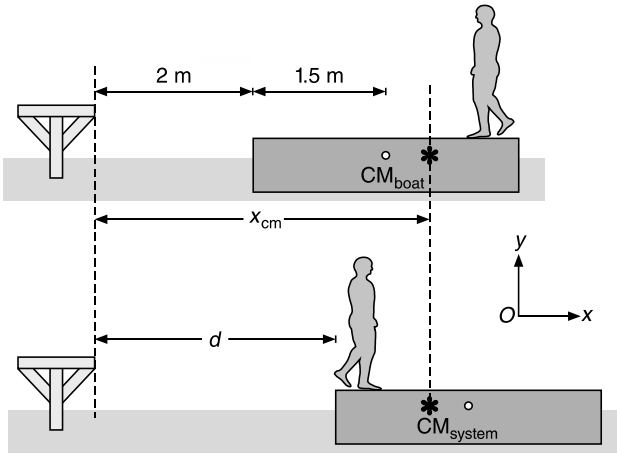
SOLUTION

Since there is no net external force, so x_{cm} is fixed. We treat the boat as a point particle whose mass is concentrated at its centre. In terms of the initial positions, the centre of mass is

$$(x_{cm})_{initial} = \frac{m_1 x_1 + m_2 x_2}{M} = \frac{(60)(5)+(40)(3.5)}{100}$$

$$\Rightarrow \quad (x_{cm})_{initial} = 4.4 \text{ m} \qquad \ldots(1)$$

The initial position of the centre of mass is indicated by a star as shown in Figure.



When the person walks relative to the boat, the position of the center of mass of the system remains unchanged.

After the man walks to the front end or the opposite end of the boat, then let the front end be at a distance d from the dock.

In terms of the new positions,

$$(x_{cm})_{final} = \frac{m_1 d + m_2 (d + 1.5)}{100} \qquad \ldots(2)$$

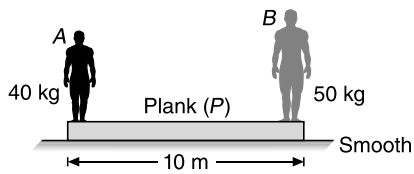
On equating (1) and (2), we get

$$d = 3.8 \text{ m}$$

The negative displacement of the man is accompanied by a positive displacement of the boat such that the centre of mass stays fixed. The total momentum of the system remains zero at all times.

ILLUSTRATION 33

Two men A and B are standing at the ends of a 10 kg plank of length 10 m. The plank is resting on a smooth horizontal surface. The system is initially at rest and masses are shown in Figure.



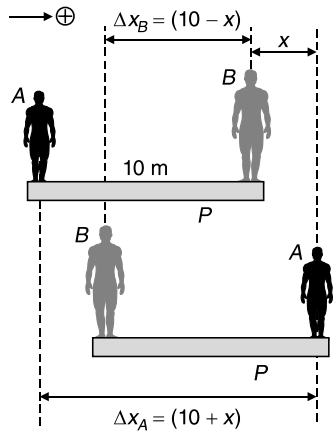
Find the displacement of plank when both A and B interchange their positions. Also find the displacement of the plank if both A and B move towards each other to finally meet at the middle of the plank.

SOLUTION

Let us consider man A, man B and plank as a system, then there is no external force acting on system in horizontal direction. Since the system is initially at rest, hence the displacement of centre of mass should also be zero.

$$\Delta x_{cm} = 0 = \frac{m_A \Delta x_A + m_B \Delta x_B + m_P \Delta x_P}{m_A + m_B + m_P} \qquad \ldots(1)$$

When both A and B exchange their position, then let the plank be displaced rightwards through x as shown in Figure.



So, displacement of A is

$$\Delta x_A = 10 + x, \text{ rightwards}$$

and displacement of B is

$$\Delta x_B = -(10 - x), \text{ leftwards}$$

From Equation (1), we get

$$\frac{40(10 + x) + 50(-10 + x) + 10(x)}{40 + 50 + 10} = 0$$

$$\Rightarrow \quad -100 + 100x = 0$$

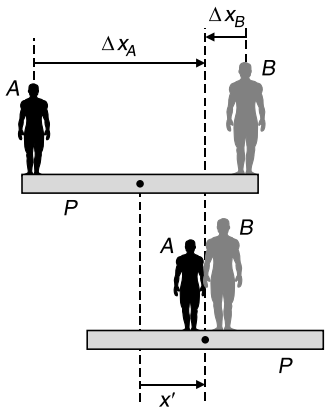
$$\Rightarrow \quad x = 1 \text{ m}$$

Hence the plank will move 1 m towards right.

When both A and B move towards each other to finally meet at the middle of the plank, then too the displacement of the centre of mass will be zero, i.e.

$$\Delta x_{cm} = 0$$

Let the displacement of the plank in this case be x' rightwards as shown in Figure.



So, displacement of A is

$$\Delta x_A = 5 + x', \text{ rightwards}$$

and displacement of B is

$$\Delta x_B = -(5 - x'), \text{ leftwards}$$

$$\Rightarrow \quad \Delta x_{cm} = \frac{40(5+x') + 50(-5+x') + 10x'}{40+50+10} = 0$$

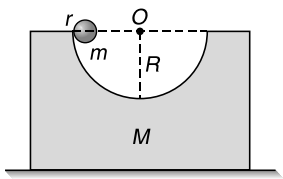
$$\Rightarrow \quad -50 + 100x' = 0$$

$$\Rightarrow \quad x' = \frac{1}{2} \text{ m}$$

Hence the displacement of the plank will be $\frac{1}{2}$ m towards right.

ILLUSTRATION 34

Calculate the displacement of M when the sphere reaches the bottom of the groove shown in Figure. Assume all surfaces to be frictionless.



SOLUTION

Since no force acts on the system in the horizontal direction, so the x position of the centre of mass of the system remains fixed. When the sphere reaches the bottom of the groove, let the mass M move towards the left by x, then displacement of the sphere will be $(R - r - x)$. Since, for centre of mass not to move, we have

$$m_1 \Delta x_1 = m_2 \Delta x_2$$

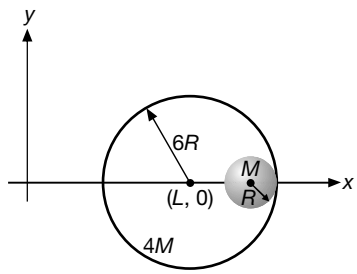
where Δx_1 and Δx_2 are the displacements of M and m with respect to ground and both displacements are in opposite directions.

$$\Rightarrow \quad Mx = m(R - r - x)$$

$$\Rightarrow \quad x = \frac{m(R-r)}{M+m}$$

ILLUSTRATION 35

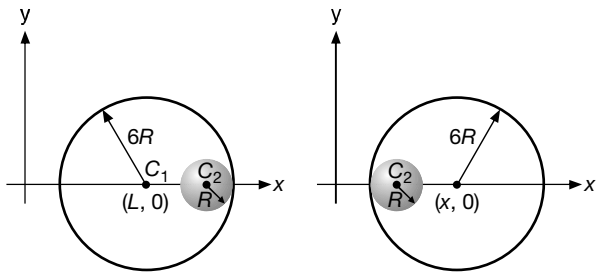
A small sphere of radius R is held against the inner surface of larger sphere of radius $6R$ (as shown in figure). The masses of large and small spheres are $4M$ and M respectively. This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the coordinates of the centre of the large sphere, when the smaller sphere reaches the other extreme position.



SOLUTION

Method I:

Since all the surfaces are smooth, no external force is acting on the system in horizontal direction. Therefore, the centre of mass of the system in horizontal direction will remain stationary.



$$x_i = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(4M)(L) + M(L+5R)}{4M+M}$$

$$\Rightarrow \quad x_i = L + R \qquad \ldots(1)$$

Let, $(x, 0)$ be the coordinates of the centre of large sphere in final position. Then x-coordinates of cm in final position will be

$$x_f = \frac{(4M)(x) + M(x-5R)}{4M+M} = (x-R) \qquad \ldots(2)$$

Equating equations (1) and (2), we get

$$x = L + 2R$$

Therefore, coordinates of large sphere, when the smaller sphere reaches the other extreme position, are $(L+2R, 0)$.

Method II:

Since no force is acting along the x direction and initially the system is at rest, so the displacement of the centre of mass of the system along the x direction is zero due to which we get

$$m_1 \Delta x_1 = m_2 \Delta x_2$$

So, when M moves to left, i.e. to the other extreme, then $4M$ moves to right, say by x. Consequently M moves effectively to the left by $(10R - x)$ such that

$$M(10R - x) = (4M)x$$

$$\Rightarrow \quad x = 2R$$

So, new coordinates of centre of big sphere are $(L+2R,\ 0)$.

ILLUSTRATION 36

A projectile is fired at a speed of $100\ \text{ms}^{-1}$ at an angle of $37°$ above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio $1:3$, the smaller coming to rest. Find the distance from the launching point to the point where the heavier piece lands.

SOLUTION

At the highest point, the projectile has horizontal velocity. The lighter part comes to rest. Hence, the heavier part will move with increased horizontal velocity. In vertical direction, both parts have zero velocity and undergo same acceleration. Thus, both will hit the ground together. As internal forces do not affect the motion of the centre of mass, the centre of mass hits the ground at the position where the original projectile would have landed. The range of the original projectile is,

$$x_{cm} = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{2(10^4)(3/5)(4/5)}{10}\ \text{m}$$

$$\Rightarrow \quad x_{cm} = 960\ \text{m}$$

The centre of mass will hit the ground at this position. As the smaller block comes to rest after breaking, it falls down vertically and hits the ground at half of the range, i.e., at $x = 480$ m. If the heavier block hits the ground at x_2, then

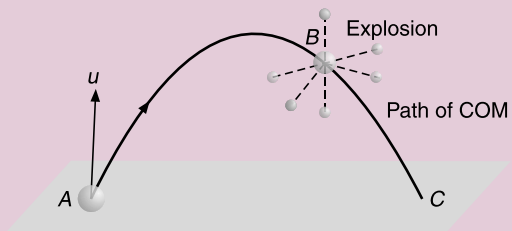
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = 960\ \text{m}$$

$$\Rightarrow \quad 960 = \frac{(m)(480) + (3m)(x_2)}{(m+3m)}$$

$$\Rightarrow \quad x_2 = 1120\ \text{m}$$

Conceptual Note(s)

If a projectile explodes in mid-air, the path of the centre of mass remains unchanged. This is because, during explosion no external force (except gravity) acts on the centre of mass. The situation is shown in figure.



Path of centre of mass (cm) is ABC, even though the different parts travel in different directions after explosion.

ILLUSTRATION 37

A rope thrown over a pulley has a ladder with a man A on one of its ends and a counter balancing mass M on its other end. The man whose mass is m, climbs upwards by $\Delta\vec{r}$ relative to the ladder and then stops. Assuming the masses of the pulley and the rope to be negligible, pulley to be frictionless at the axis, calculate the displacement of the centre of mass of this system.

SOLUTION

Counter balancing mass M applied at the other end means that

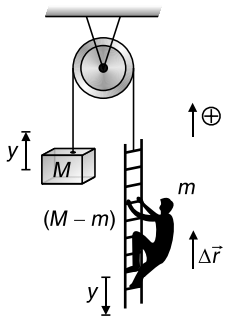
$$M = m + m_{\text{ladder}}$$

$\Rightarrow \quad m_{\text{ladder}} = M - m$

$m_{\text{pulley}} \approx m_{\text{rope}} \approx 0$

Displacement of man with respect to ladder is

$y_{m/l} = \Delta r$



When man moves up by Δr, then let ladder move down by y and hence counter weight moves up by y. So, effective displacement of man is $(\Delta r - y)$.
The displacement of CM is given by

$$\Delta r_{\text{cm}} = \frac{m_l \Delta r_l + m_m \Delta r_m + m_l \Delta r_l}{m_\ell + m_m + m_c}$$

$$\Rightarrow \quad \Delta r_{\text{cm}} = \frac{(M-m)y + m(\Delta r - y) + M(-y)}{M - m + m + M}$$

$$\Rightarrow \quad \Delta r_{\text{cm}} = \frac{m\Delta r}{2M}, \text{ upwards}$$

ILLUSTRATION 38

A raft of mass M with a man of mass m aboard stays motionless on the surface of a lake. The man moves a distance l_0 relative to the raft with velocity v_0 and then stops. Assuming the water resistance to be negligible, find the displacement of the raft l relative to the shore. Also calculate the horizontal component of the force which the man acted on the raft during the motion.

SOLUTION

Since, centre of mass will remain stationary, so when man moves ℓ_0 relative to the raft (say forward), then raft moves backwards, say by x. Hence effective displacement of man with respect to ground is $(l_0 - x)$.

$$\Rightarrow \quad m(l_0 - x) = Mx$$

$$\Rightarrow \quad x = \frac{ml_0}{m+M}, \text{ opposite to motion of man.}$$

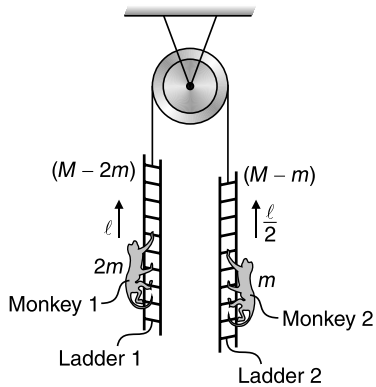
Also, $v = \frac{dx}{dt} = \frac{d}{dt}\left(\frac{ml_0}{m+M}\right) = \left(\frac{m}{m+M}\right)\frac{dl_0}{dt}$

$$\Rightarrow \quad v = \left(\frac{m}{m+M}\right)v_0, \text{ opposite to velocity of man}$$

$$\Rightarrow \quad F = M\frac{dv}{dt} = \left(\frac{Mm}{M+m}\right)\left(\frac{dv_0}{dt}\right)$$

ILLUSTRATION 39

Two ladders are hanging from ends of a light rope that passes over a light and smooth pulley. A monkey of mass $2m$ hangs near the bottom of one ladder whose mass is $M - 2m$. Another monkey of mass m hangs near the bottom of the other ladder whose mass is $M - m$ as shown in Figure.



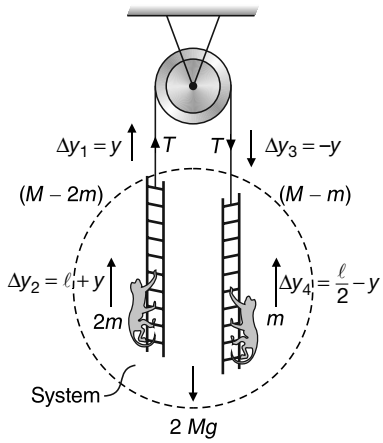
The monkey of mass $2m$ moves up a distance l with respect to the ladder. The monkey of mass m moves up a distance $l/2$ with respect to the ladder. Does the centre of mass of the system change? If so, then calculate the displacement of centre of mass of the system. Also calculate the displacement of ladder.

SOLUTION

In this problem let us consider both the monkeys and the ladders as a system. When the monkeys are at rest, the tension force in the string is balanced by the weight of the system. However, when a monkey starts moving up, then he pulls the respective ladder in downward direction due to which the tension in string increases. Hence, there will be a net upward external force acting on the system and the displacement of centre of mass of the system will be in upward direction.

Let the displacement of Ladder 1 in this process be y upwards, then the displacement of Ladder 2 will

be y downwards (because both the ladders are connected together by a single string). During this time the displacement of Monkey 1 will be $(l+y)$ and that of Monkey 2 will be $\left(\frac{l}{2}-y\right)$ as shown in Figure.



The displacement of the centre of mass of system is given by

$$\Delta y_{cm} = \frac{m_1\Delta y_1 + m_2\Delta y_2 + m_3\Delta y_3 + m_4\Delta y_4}{m_1 + m_2 + m_3 + m_4} \quad \ldots(1)$$

where, $m_1 = M - 2m$ is the mass of the Ladder 1

$\Delta y_1 = y$ is the displacement of the Ladder 1

$m_2 = 2m$ is the mass of the Monkey 2

$\Delta y_2 = l + y$ is the displacement of the Monkey 2

$m_3 = M - m$ is the mass of Ladder 2

$\Delta y_3 = -y$ is the displacement of Ladder 2

$m_4 = m$ is the mass of Monkey 2

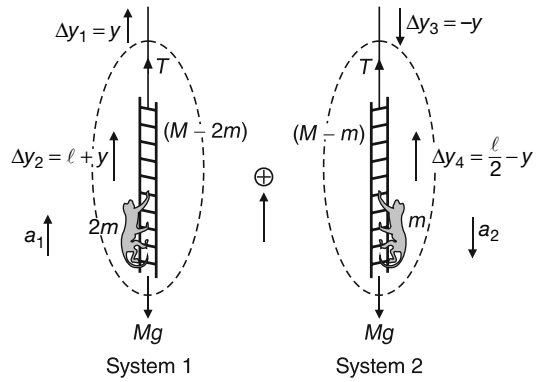
$\Delta y_4 = \frac{l}{2} - y$ is the displacement of Monkey 2

On substituting these values in equation (1), we get

$$\Delta y_{cm} = \frac{(M-2m)y + 2m(l+y) - (M-m)y + m\left(\frac{l}{2}-y\right)}{(M-2m) + 2m + (M-m) + m}$$

$$\Rightarrow \quad \Delta y_{cm} = \frac{5ml}{4M}$$

To calculate the displacement of ladder, let us consider 'Ladder 1 and Monkey 1' as System 1 and 'Ladder 2 and Monkey 2' as System 2.



Since both the systems have equal mass and equal net external forces, and initially both are at rest, so the displacements of centre of mass of both the systems should be equal.

We can also think of this situation by using Newton's Second Law applied on Systems 1 and 2 separately.

For System 1, we have

$$T - Mg = Ma_1 \quad \ldots(2)$$

For System 2, we have

$$Mg - T = Ma_1 \quad \ldots(3)$$

Also, both systems are connected by inextensible string, so we have $a_1 = a_2 = a$ (say)

From equations (2) and (3), we get

$$a_1 = a_2 = a = 0$$

$$\Rightarrow \quad a_{cm} = 0$$

Since initially the system is at rest, so the position of the centre of mass of the system remains fixed along the vertical direction.

$$\Rightarrow \quad (\Delta y_{cm})_1 = (\Delta y_{cm})_2$$

$$\Rightarrow \quad \frac{m_1\Delta y_1 + m_2\Delta y_2}{m_1 + m_2} = \frac{m_3\Delta y_3 + m_4\Delta y_4}{m_3 + m_4}$$

Taking the upward direction as positive, we get

$$\frac{y(M-2m) + 2m(l+y)}{(M-2m) + 2m} = \frac{-y(M-m) + m\left(\frac{l}{2}-y\right)}{(M-m) + m}$$

$$\Rightarrow \quad (M-2m)y + 2m(l+y) = -(M-m)y + m\left(\frac{l}{2}-y\right)$$

$$\Rightarrow \quad My - 2my + 2ml + 2my = -My + my + \frac{ml}{2} - my$$

$$\Rightarrow \quad My + 2ml = -My + \frac{ml}{2}$$

$$\Rightarrow \quad 2My = -\frac{3ml}{2}$$

$$\Rightarrow \quad y = -\frac{3ml}{4M}$$

The negative sign indicates that displacement of the ladder is in the downward direction.

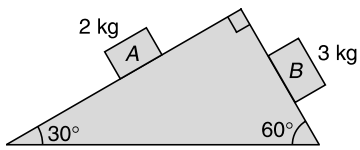
Test Your Concepts-II

Based on Motion of Centre of Mass

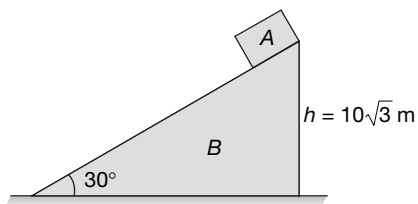
(Solutions on page H.72)

1. A cubical block of ice of mass m_1 and edge L is placed in a large tray of mass m_2. If the ice melts, how far does the centre of mass of the system ice plus tray come down?

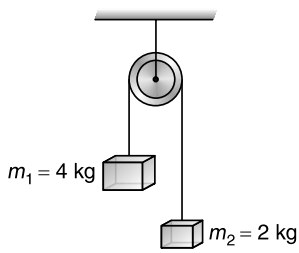
2. Figure shows a fixed wedge on which two blocks of masses 2 kg and 3 kg are placed on its smooth inclined surfaces. When the two blocks are released from rest, find the acceleration of centre of mass of the two blocks.



3. Block A has a mass of 5 kg and is placed on the top of smooth triangular block B having a mass of 25 kg. If the system is released from rest, determine the distance B moves, in metre, when A reaches the bottom. Neglect the size of block A.

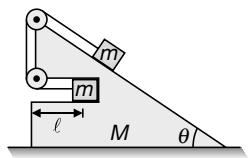


4. Two masses $m_1 = 4$ kg and $m_2 = 2$ kg are released from rest at $t = 0$ as shown in Figure. Calculate the distance travelled by centre of mass and its speed when $t = 9$ s.



5. A dog of mass 10 kg stands on a stationary boat of mass 40 kg so that he is 20 m from the shore. He then walks 8 m on the boat towards the shore and halts. How far is he from the shore now? Assume that there is no friction between the boat and water.

6. Find the displacement of the wedge when m comes out of the wedge of mass $4m$. Assume friction to be absent everywhere and the base angle of the wedge to be 60°.



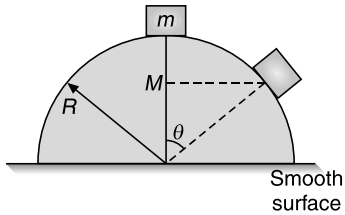
7. Two men A (mass M) and B (mass $2M$) are standing on the two opposite ends in a boat of mass $4M$ and length l on a frictionless water surface. Man A travels a distance of $\frac{l}{4}$ relative to the boat towards its centre and man B moves a distance $\frac{3l}{4}$ relative to boat and meet A. Find the distance travelled by the boat on water till A and B meet.

8. Two balls with masses $m_1 = 3$ kg and $m_2 = 5$ kg have initial velocities $v_1 = v_2 = 5\ \text{ms}^{-1}$ the directions as shown in the figure. They collide at the origin.

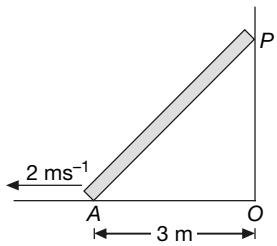
(a) Find the velocity of the centre of mass 3 s before the collision

(b) Find the position of the centre of mass 2 s after the collision

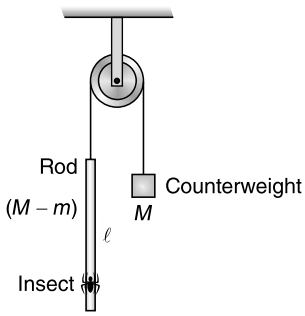
9. A block is released on the convex surface of a hemispherical wedge as shown in figure. Calculate the displacement of the wedge when the block reaches the angular position θ.



10. A ladder AP of length 5 m inclined to a vertical wall is slipping over a horizontal surface with velocity of $2\ \text{ms}^{-1}$, when A is at distance 3 m from ground. Calculate the velocity of centre of mass at this moment.



11. A rod of mass $M-m$ and length l carries an insect of mass m at its bottom end. The top end of the rod is connected with a string which passes over a smooth pulley and other end of the string is connected to a counter weight of mass M as shown in Figure.



Initially the insect is at rest. Consider insect, rod and counter weight as a system.

(a) Calculate the net external force acting on the system initially.

(b) If the insect starts crawling up on the rod with constant velocity, will the centre of mass of the system accelerate or not?

(c) If the insect moves up with acceleration, does the centre of mass of the system accelerate?

(d) Calculate the displacement of centre of mass of the system and that of the rod when insect reaches the other end of the rod.

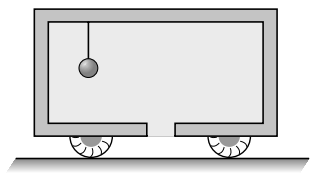
12. At one instant, the centre of mass of a system of two particles is located on the x-axis at $x=3$ m and has a velocity of $\left(6\ \text{ms}^{-1}\right)\hat{j}$. One of the particles is at the origin, the other particle has a mass of 0.1 kg and is at rest on the x-axis at $x=12$ m.

(a) Calculate the mass of the particle at the origin.

(b) Calculate the total momentum of this system.

(c) Calculate the velocity of the particle at the origin.

13. A cart of mass M is at rest on a frictionless horizontal surface and a pendulum bob of mass m hangs from the roof of the cart. The string breaks, the bob falls on the floor, makes several collisions on the floor and finally lands up in a small slot made on the floor. The horizontal distance between the string and the slot is L. Find the displacement of the cart during this process.



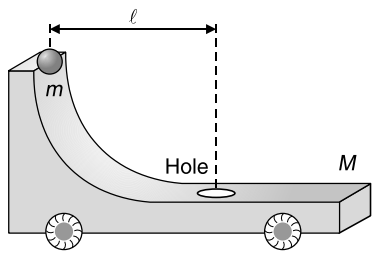
14. A stone is dropped at $t=0$. A second stone, with twice the mass of the first, is dropped from the same point at $t=200$ ms.

(a) How far below the release point is the centre of mass of the two stones at $t=400$ ms? (Neither stone has yet reached the ground).

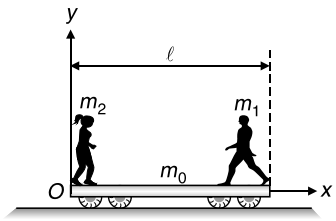
(b) How fast is the centre of mass of the two-stone system moving at that time?

Take $g=10\ \text{ms}^{-2}$

15. Figure shows a flat car of mass M on a frictionless road. A small massless wedge is fitted on it as shown. A small ball of mass m is released from the top of the wedge, it slides over it and falls in the hole at distance l from the initial position of the ball. Calculate the distance that the flat car moves till the ball gets into the hole.

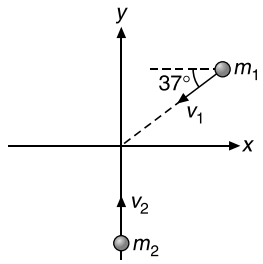


16. Initially a platform at rest at $x = 0$ has a man of mass m_1 and a woman of mass m_2 standing at the extreme corners of the platform of mass m_0 and length l as shown in Figure.



The man and the woman start to move towards each other. If x_1 be the displacement of the man relative to the platform, then obtain an expression for the displacement s of the platform as function of x_1 when the man and woman meet.

17. Two balls with masses $m_1 = 3$ kg and $m_2 = 5$ kg have initial velocities $v_1 = v_2 = 5$ ms^{-1} in the directions shown in figure. They collide at the origin. Calculate the velocity of the centre of mass 3 s before the collision. Also calculate the position of the centre of mass 2 s after the collision.



18. Two particles having masses m and nm start simultaneously from the intersection of two straight line with velocities nv and v respectively. It is observed that the path of their centre of mass is a straight line bisecting the angle θ between the given straight lines. Calculate the magnitude of the velocity of centre of mass.

CENTRE OF MASS REFERENCE FRAME OR CENTROIDAL FRAME OR C-FRAME OR ZERO MOMENTUM FRAME

A reference frame carried by the centre of mass of an isolated system of particles (i.e., a system not subjected to any external forces) is called as the **Centre of Mass Reference Frame or Centroidal Frame or C-frame or zero momentum frame**.

In the centre of mass reference frame or the centroidal frame,

(a) the position vector of centre of mass is zero, i.e. the origin is taken at the centre of mass.
(b) the sum of mass moments is zero, i.e. $\Sigma m_i \vec{r}_{ic} = \vec{0}$
(b) the velocity of centre of mass of system is zero.
(c) the linear momentum of centre of mass of system is also zero, i.e. $\Sigma m_i \vec{v}_{ic} = 0$. *This means that neither the internal forces nor the external forces can change the momentum of the system relative to centre of mass.*
(d) the acceleration of centre of mass is also zero.
(e) for a system of two particles, the momentum of the particles are equal in magnitude but opposite in direction.

KINETIC ENERGY OF A SYSTEM OF PARTICLES

Consider an n particle system having masses m_1, m_2, m_3, ..., m_n moving with respective velocities $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, ..., $\vec{v}_n$. If $\vec{v}_{cm} = \vec{v}_c$ be the velocity of the centre of mass, then with respect to the centre of mass the velocity of each particle is

$$\vec{v}_{1c} = \vec{v}_1 - \vec{v}_c$$

$$\vec{v}_{2c} = \vec{v}_2 - \vec{v}_c$$

$$\vec{v}_{3c} = \vec{v}_3 - \vec{v}_c \text{ and so on.}$$

For some i^{th} particle, we have

$$\vec{v}_{ic} = \vec{v}_i - \vec{v}_c$$

$$\Rightarrow \quad \vec{v}_i = \vec{v}_{ic} + \vec{v}_c \qquad \ldots(1)$$

So, velocity of each particle can be written as sum of velocity of centre of mass $\vec{v}_{cm}$ and the velocity of particle relative to centre of mass $\vec{v}_i$. The kinetic energy of this i^{th} particle is given by

$$K_i = \frac{1}{2} m_i \vec{v}_i^2 = \frac{1}{2} m_i \left(\vec{v}_{ic} + \vec{v}_c\right)^2$$

The total kinetic energy of the system is obtained by adding (or summating) these energies and hence, we get

$$K = \Sigma K_i = \frac{1}{2}\Sigma m_i v_i^2 = \frac{1}{2}\Sigma m_i (\vec{v}_i \cdot \vec{v}_i)$$

$$\Rightarrow \quad K = \frac{1}{2}\Sigma m_i (\vec{v}_{ic} + \vec{v}_c)\cdot(\vec{v}_{ic} + \vec{v}_c)$$

$$\Rightarrow \quad K = \frac{1}{2}\left(\Sigma m_i v_c^2 + \Sigma m_i v_{ic}^2\right) + \left(\vec{v}_c \cdot \Sigma m_i \vec{v}_{ic}\right)$$

In the last term we have removed $\vec{v}_c = \vec{v}_{\text{cm}}$ from the summation because it is same for each particle. The quantity $\Sigma m_i \vec{v}_{ic} = \vec{0}$, because the total momentum of the system relative to the centre of mass is zero.

$$K = \frac{1}{2}(\Sigma m_i) v_{\text{cm}}^2 + \frac{1}{2}\Sigma m_i v_{ic}^2 = \frac{1}{2}Mv_{\text{cm}}^2 + K_{\text{rel}}$$

$$\Rightarrow \quad K = K_{\text{cm}} + K_{\text{rel}}$$

where, M is the total mass, K_{cm} is the kinetic energy of the centre of mass of the system, K_{rel} is the kinetic energy of the particles relative to the centre of mass of the system *which is also called as the Internal Kinetic Energy of the system. The term* K_{rel} *may involve translation, rotation or vibration relative to the centre of mass.*

So, we conclude that the kinetic energy of a system of particles can be written as the sum of two terms,

(a) the kinetic energy associated with the motion of centre of mass, $K_{\text{cm}} = \frac{1}{2}Mv_{\text{cm}}^2$, where M is the total mass of the system and

(b) the kinetic energy associated with the motion of the particles of the system relative to the centre of mass, $K_{\text{rel}} = \frac{1}{2}\Sigma m_i v_{ic}^2$ where, $\vec{v}_{ic}$ is the velocity of the ith particle relative to the centre of mass.

When there is no external force, $\vec{v}_{\text{cm}}$ is constant and the kinetic energy associated with bulk motion, i.e. $K_{\text{cm}} = \frac{1}{2}Mv_{\text{cm}}^2$ does not change. Only the relative kinetic energy (K_{rel}) can change in isolated system.

If a frame is attached to the centre of mass, then kinetic energy of the system is minimum, because $K_{\text{cm}} = 0$ and hence

$$K_{\text{system}} = K_{\text{min}} = K_{\text{rel}} = K_{\text{internal}}$$

This makes us conclude that the internal kinetic energy of the system does not depend upon the reference frame and is the internal property of the system.

KINETIC ENERGY AND MOMENTUM OF TWO PARTICLE SYSTEM WITH RESPECT TO CENTRE OF MASS

Consider a system of two masses m_1 and m_2 having velocities $\vec{v}_1$ and $\vec{v}_2$. If $\vec{v}_{\text{cm}}$ be the velocity of the centre of mass, then we have

$$\vec{v}_{\text{cm}} = \vec{v}_c = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} \qquad \ldots(1)$$

Velocity of m_1 with respect to centre of mass is

$$\vec{v}_{1c} = \vec{v}_1 - \vec{v}_c = \left(\frac{m_2}{m_1 + m_2}\right)(\vec{v}_1 - \vec{v}_2) \qquad \ldots(2)$$

Momentum of m_1 with respect to centre of mass is

$$\vec{p}_{1c} = m_1\vec{v}_{1c} = \left(\frac{m_1 m_2}{m_1 + m_2}\right)(\vec{v}_1 - \vec{v}_2) = \mu(\vec{v}_1 - \vec{v}_2)$$

$$\Rightarrow \quad \vec{p}_{1c} = \mu(\vec{v}_1 - \vec{v}_2) \qquad \ldots(3)$$

where, $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is called as the reduced mass of a two particle system.

Velocity of m_2 with respect to the centre of mass is

$$\vec{v}_{2c} = \vec{v}_2 - \vec{v}_c = \left(\frac{m_1}{m_1 + m_2}\right)(\vec{v}_2 - \vec{v}_1) \qquad \ldots(4)$$

Momentum of m_2 with respect to centre of mass is

$$\vec{p}_{2c} = m_2\vec{v}_{2c} = \left(\frac{m_1 m_2}{m_1 + m_2}\right)(\vec{v}_2 - \vec{v}_1) = \mu(\vec{v}_2 - \vec{v}_1)$$

$$\Rightarrow \quad \vec{p}_{2c} = \mu(\vec{v}_2 - \vec{v}_1) \qquad \ldots(5)$$

So, from equations (3) and (5), we get

$$\vec{p}_{1c} + \vec{p}_{2c} = \vec{0}$$

Hence, we see that the total linear momentum of a two particle system with respect to the centre of mass is zero or we can also say that in the centre of mass reference frame, the two particles have momentum equal in magnitude but opposite in direction.

Similarly, we have the kinetic energy of the system with respect to the centre of mass given by

$$K_{\text{cm}} = \frac{1}{2}m_1 v_{1c}^2 + \frac{1}{2}m_2 v_{2c}^2 \qquad \ldots(6)$$

Substituting the values of $\vec{v}_{1c}$ and $\vec{v}_{2c}$ from equations (2) and (4) in equation (6), we get

$$K_{\text{cm}} = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(\vec{v}_1 - \vec{v}_2)^2 = \frac{1}{2}\mu|\vec{v}_{\text{rel}}|^2 \qquad \ldots(7)$$

where, $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ is the reduced mass of the system and $\vec{v}_{rel} = \vec{v}_1 - \vec{v}_2$ is the relative velocity of the masses with respect to each other.

If velocities are in same direction, then equation (7) can be re-written as

$$K_{cm} = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(v_1 - v_2)^2 = \frac{1}{2}\mu |v_{rel}|^2$$

If velocities are in the opposite direction, then equation (7) can be re-written as

$$K_{cm} = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(v_1 + v_2)^2 = \frac{1}{2}\mu |v_{rel}|^2$$

ILLUSTRATION 40

A particle of mass $m_1 = 4$ kg moves at $5\hat{i}$ ms^{-1}, while $m_2 = 2$ kg moves at $2\hat{i}$ ms^{-1}. Find

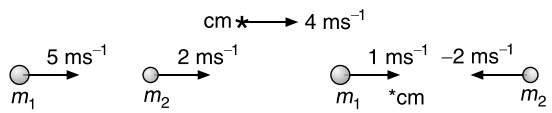
(a) the kinetic energy of centre of mass and
(b) the kinetic energy with respect to centre of mass.

SOLUTION

The velocity of the centre of mass is

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(4)(5)+(2)(2)}{4+2} = 4 \text{ ms}^{-1}$$

The velocities relative to centre of mass are shown in Figure.



(a) Laboratory Frame (b) Centre of mass Frame

The velocities of two particles and of their center of mass (a) relative to the laboratory and (b) relative to the frame in which the center of mass is at rest.

$$v_{1/cm} = v_1 - v_{cm} = 1 \text{ ms}^{-1}$$
$$v_{2/cm} = v_2 - v_{cm} = -2 \text{ ms}^{-1}$$

The two terms for the total kinetic energy are

(a) $K_{cm} = \dfrac{1}{2}(m_1 + m_2)v_{cm}^2 = 48$ J

(b) $K_{wrt\ cm} = \dfrac{1}{2}m_1 (v_{1/cm})^2 + \dfrac{1}{2}m_2 (v_{2/cm})^2 = 6$ J

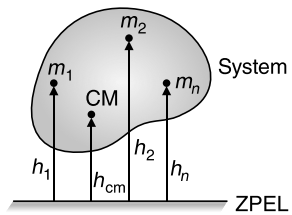
Please note that the kinetic energy with respect to the centre of mass is equal to

$$K_{wrt\ cm} = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$

$$\Rightarrow \quad K_{wrt\ cm} = \frac{1}{2}\left(\frac{(4)(2)}{4+2}\right)(5-3)^2 = 6 \text{ J}$$

GRAVITATIONAL POTENTIAL ENERGY OF A SYSTEM OF PARTICLES

Consider a system of particles having masses m_1, m_2, m_3, ……, m_n at heights or distances h_1, h_2, h_3, ……, h_n from the Zero Potential Energy Level (ZPEL) as shown in Figure.



For this system of discrete distribution of particles, let us take i^{th} particle of mass m_i situated at a height h_i from ZPEL. Then the gravitational potential energy of m_i is given by

$$U_i = m_i g h_i$$

The total potential energy of the system is obtained by summing up the potential energy of all particles. So, total potential energy of the system is

$$U = \Sigma U_i = \Sigma m_i g h_i = g \Sigma m_i h_i \qquad \ldots(1)$$

$$\Rightarrow \quad U = Mg\left(\frac{\Sigma m_i h_i}{M}\right) = Mgh_{cm}$$

where $h_{cm} = \dfrac{\Sigma m_i h_i}{M}$ is the height of centre of mass of the system from ZPEL.

WORK-ENERGY THEOREM FOR A SYSTEM OF PARTICLES

The work-energy theorem for a system of particles must include the works done by external as well as internal forces. So, we have

$$W_{ext} + W_{int} = \Delta K_{cm} + \Delta K_{rel}$$

Since all the basic interactions are conservative, one can always express internal work by the change of internal potential energy.

$$W_{int} = -\Delta U_{int}$$
$$W_{ext} = \Delta K_{cm} + (\Delta K_{rel} + \Delta U_{int}) = \Delta K_{cm} + \Delta E_{int}$$

So, from this equation, we conclude that external work done on a system of particles can change translational kinetic energy of centre of mass (CM) of system and can change the internal energy of the system (in whatever form possessed by it).

Please note that the internal energy may include the elastic potential energy, gravitational potential energy, thermal energy, electrostatic potential energy, electromagnetic energy, nuclear energy, chemical energy and so on.

WORK ENERGY THEOREM FROM THE CENTRE OF MASS REFERENCE FRAME

Since we know that most important tool for solving mechanics problems is the Work Energy Theorem, which can be applied from any frame of reference. However, when Work-Energy Theorem is applied from the centre of mass reference frame, then we have *"the work done by all forces (internal or external) is equal to change in kinetic energy of the system calculated with respect to centre of mass"*. Mathematically, we write

$$W_{\text{total}} = \Delta K_{\text{cm}}$$

$$\Rightarrow \quad W_{\text{internal}} + W_{\text{external}} = \Delta K_{\text{cm}}$$

$$\Rightarrow \quad W_{\text{internal}} + W_{\text{external}} = (K_{\text{cm}})_{\text{final}} - (K_{\text{cm}})_{\text{initial}}$$

For a two particle system, we can re-write

$$W_{\text{internal}} + W_{\text{external}} = \frac{1}{2}\mu\left(v_{\text{rel}}^2 - u_{\text{rel}}^2\right)$$

where, v_{rel} is the final relative velocity, u_{rel} is the initial relative velocity of the particles and μ is the reduced mass of the two particle system.

TOTAL WORK DONE BY PSEUDO FORCES IN CENTRE OF MASS REFERENCE FRAME OR CENTROIDAL FRAME

Let acceleration of centre of mass relative to an inertial frame is $\vec{a}_c$. Pseudo force on ith particle of mass m_I in centroidal frame is

$$\vec{F}_{\text{pseudo}} = m_i(-\vec{a}_c) = -m_i\vec{a}_c$$

Let displacement of ith particle in a time interval be $\Delta\vec{r}_i$ relative to the centroidal frame, then total work done by pseudo forces on all the particles in centroidal frame can now be expressed as

$$W = \vec{F}_{\text{pseudo}} \cdot \Delta\vec{r} = (-\Sigma m_i\vec{a}_c)\cdot\Delta\vec{r}_i = -\vec{a}_c \cdot \Sigma(m_i\Delta\vec{r}_i)$$

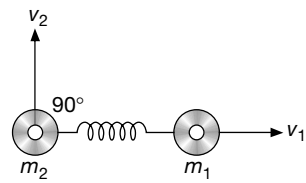
The displacement of the centre of mass in the centroidal frame is zero, so we get

$$W = -\vec{a}_c \cdot \vec{0} = 0$$

So, the total work done in centroidal frame on all the particles of a system by pseudo forces due to acceleration of centre of mass is zero.

ILLUSTRATION 41

Two small discs of masses m_1 and m_2 are connected by a weightless spring resting on a smooth horizontal plane. The discs are set in motion with initial velocities v_1 and v_2 whose directions are mutually perpendicular and in the same horizontal plane. Find the total initial energy E of this system with reference to the frame fixed to the centre of mass.



SOLUTION

Let $\vec{v}_{\text{cm}}$ be the velocity of the centre of mass. Then the velocity of m_1 relative to the centre of mass is $\vec{v}_{1/\text{cm}} = \vec{v}_1 - \vec{v}_{\text{cm}}$ and that of m_2 is $\vec{v}_{2/\text{cm}} = \vec{v}_2 - \vec{v}_{\text{cm}}$.

$$E = \frac{1}{2}m_1|\vec{v}_1 - \vec{v}_{\text{cm}}|^2 + \frac{1}{2}m_2|\vec{v}_2 - \vec{v}_{\text{cm}}|^2 \qquad \ldots(1)$$

The velocity of the cm is given by

$$(m_1 + m_2)\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\Rightarrow \quad v_{\text{cm}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

$$\Rightarrow \quad \vec{v}_1 - \vec{v}_{\text{cm}} = \frac{m_2(\vec{v}_1 - \vec{v}_2)}{m_1 + m_2}$$

$$\Rightarrow \quad |\vec{v}_1 - \vec{v}_{\text{cm}}| = \frac{m_2}{m_1 + m_2}|\vec{v}_1 - \vec{v}_2|$$

Since $\vec{v}_1$ and $\vec{v}_2$ are perpendicular to each other, so

$$|\vec{v}_1 - \vec{v}_2| = \sqrt{v_1^2 + v_2^2}$$

$$\Rightarrow \quad |\vec{v}_1 - \vec{v}_{\text{cm}}| = \frac{m_2}{m_1 + m_2}|\vec{v}_1 - \vec{v}_2| = \frac{m_2}{m_1 + m_2}\sqrt{v_1^2 + v_2^2}$$

Similarly, $|\vec{v}_2 - \vec{v}_{\text{cm}}| = \dfrac{m_1}{m_1 + m_2}\sqrt{v_2^2 + v_1^2}$

So, from equation (1), we get

$$E = \frac{1}{2}\left[\frac{m_1m_2^2 + m_2m_1^2}{(m_1 + m_2)^2}\right](v_1^2 + v_2^2)$$

$$\Rightarrow \quad E=\frac{1}{2}\left[\frac{m_1m_2\left(m_2+m_1\right)}{\left(m_1+m_2\right)^2}\right]\left(v_1^2+v_2^2\right)$$

$$\Rightarrow \quad E=\frac{1}{2}\mu\left(v_1^2+v_2^2\right), \text{ where } \mu=\frac{m_1m_2}{m_1+m_2}$$

The quantity μ is called the reduced mass of the system. In general, for an n particle system the reduced mass is defined as

$$\frac{1}{\mu}=\frac{1}{m_1}+\frac{1}{m_2}+\frac{1}{m_3}+\ldots\ldots\ldots\ldots+\frac{1}{m_n}$$

This character called as the REDUCED MASS has some unique properties.

(a) The concept of reduced mass is to be introduced for bonded systems where the particles forming the system share a bond (may be physical or chemical) between them and the motion of the system has to be studied with ease. Like the two masses connected with a spring, like CH_4 molecule or H_2O molecule, like a dumbbell like situation, like the double star system (to be studied in Gravitation and Satellites) and many more situations that will be encountered as we keep on studying further.

(b) This quantity has a value smaller than the lightest mass of the system and hence we have used the name reduced mass.
So, in bonded systems we can replace the entire mass of the system by a single mass called the reduced mass μ to make the study of such a system easier.

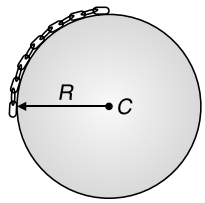
(c) In the event of a two particle system, if we have a massive and a light body bonded together, then the reduced mass of the system is approximately equal to the mass of the light body. This can be explained from the following argument.
Since for a two particle system we have

$$\mu=\frac{m_1m_2}{m_1+m_2}=\frac{mM}{m+M}$$

where, m is the mass of the light body and M is the mass of the massive body. Then in that case we have $m+M\approx M$ and hence the above expression for a two particle system reduces to $\mu\approx m$.

ILLUSTRATION 42

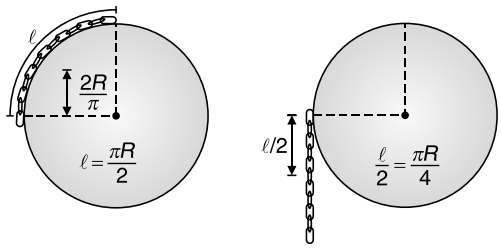
A chain AB of length equal to the quarter of a circle of radius R is placed on a smooth hemisphere as shown in figure. When it is released, it starts falling. Find the velocity of the chain when it falls of the end B from the hemisphere.



SOLUTION

As chain slips off the sphere, fall in its centre of mass height is

$$h=\frac{2R}{\pi}+\frac{\pi R}{4}$$



By work energy theorem, we use

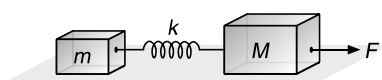
$$mgh=\frac{1}{2}mv^2$$

$$\Rightarrow \quad v=\sqrt{2gh}=\sqrt{2g\left(\frac{2R}{\pi}+\frac{\pi R}{4}\right)}$$

$$\Rightarrow \quad v=\sqrt{gR\left(\frac{4}{\pi}+\frac{\pi}{2}\right)}$$

ILLUSTRATION 43

A block of mass m is connected to another block of mass M by a massless spring of spring constant k. The blocks are kept on a smooth horizontal plane. Initially, the blocks are at rest and the spring is unstretched when a constant force F starts acting on the block of mass M to pull it. Find the maximum extension of the spring.



SOLUTION

Method I:

Let us solve the Problem in a frame of reference fixed to the centre of mass of the system. The centre of mass of the system (two blocks + spring) moves with an acceleration $a = \frac{F}{m+M}$. As this frame is accelerated with respect to the ground, we have to apply a pseudo force ma towards left on the block of mass m and Ma towards left on the block of mass M. The net external force on m is

$$F_1 = ma = \frac{mF}{m+M}, \text{ towards left}$$

and the net external force on M is

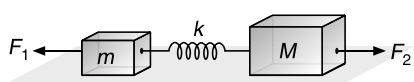
$$F_2 = F - Ma = F - \frac{MF}{m+M}$$

$$\Rightarrow \quad F_2 = \frac{mF}{m+M}, \text{ towards right}$$

Please note that the centre of mass is at rest in this frame (i.e. centroidal frame) and when the blocks move in opposite directions then at maximum extension, the blocks come to instantaneous rest and hence their final relative velocity is zero.

Assuming the left block to be displaced through a distance x_1 and the right block through a distance x_2 from the initial positions, then total work done by the external forces F_1 and F_2 in this period are

$$W_{ext} = F_1x_1 + F_2x_2 = \frac{mF}{m+M}(x_1 + x_2)$$



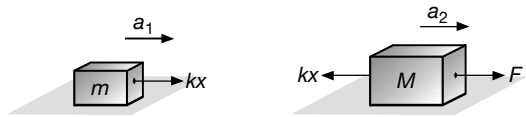
Since work done by pseudo forces in the centroidal frame is zero, so this work done should be equal to the increase in the potential energy of the spring as there is no change in the kinetic energy.

$$\Rightarrow \quad \frac{mF}{m+M}(x_1 + x_2) = \frac{1}{2}k(x_1 + x_2)^2$$

$$\Rightarrow \quad x_1 + x_2 = x_{max} = \frac{2mF}{k(m+M)}$$

Method II:

As a force F starts pulling M, it accelerates. This increases the length of the spring, developing a restoring force in the spring. Due to this restoring force, m also accelerates towards M. The length of the spring will increase till the velocity of M with respect to m is directed away from m. When this velocity becomes zero, the elongation acquires a maximum value.



Let at any instant, x be the elongation in the spring. The forces and accelerations are shown here.

$$a_1 = \frac{kx}{m} \text{ and } a_2 = \frac{F - kx}{M}$$

The acceleration of M w.r.t. m is

$$a_r = \frac{F - kx}{M} - \frac{kx}{m}$$

If v_r be the relative velocity of M w.r.t. m, then

$$a_r = \frac{v_r dv_r}{dx}$$

where, x is elongation in the spring.

$$\Rightarrow \quad \frac{v_r dv_r}{dx} = \frac{F}{M} - \left(\frac{k}{m} + \frac{k}{M}\right)x$$

$$\Rightarrow \quad v_r dv_r = \frac{F}{M}dx - k\left(\frac{M+m}{mM}\right)dx$$

$$\Rightarrow \quad \int_0^v v_r dv_r = \int_0^x \frac{F}{M}dx - \int_0^x \frac{k(m+M)}{mM}dx$$

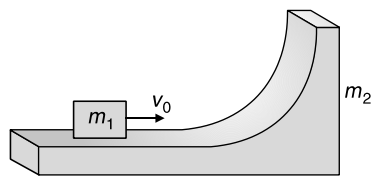
Initially, $v_r = 0$ and when elongation is maximum, then too $v_r = 0$. So, we have

$$\left(\frac{v_r^2}{2}\right)\Bigg|_0^0 = \frac{F}{M}(x)\Bigg|_0^{x_m} - \frac{k(m+M)}{Mm}\left(\frac{x^2}{2}\right)\Bigg|_0^{x_m}$$

$$\Rightarrow \quad x_m = \frac{2mF}{m+M}$$

ILLUSTRATION 44

A block of mass m_1 is projected with a velocity v_0 so that it climbs onto the smooth wedge of mass m_2. If the block does not leave the wedge, find the maximum height attained by the block.



SOLUTION

According to Work-Energy Theorem applied from CM frame, we have

$$W_{\text{internal}} + W_{\text{external}} = (K_{\text{cm}})_{\text{final}} - (K_{\text{cm}})_{\text{initial}} \quad \ldots(1)$$

Initial kinetic energy of system wrt CM is

$$(K_{\text{cm}})_{\text{initial}} = \frac{1}{2}\mu u_{\text{rel}}^2 = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(v_0 - 0)^2$$

$$\Rightarrow \quad (K_{\text{cm}})_{\text{initial}} = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)v_0^2$$

When the block attains a maximum height, then the final relative velocity v_{rel} (between block and wedge) becomes zero, i.e. $v_{\text{rel}} = 0$, so the final kinetic energy of system wrt CM is also zero.

$$\Rightarrow \quad (K_{\text{cm}})_{\text{final}} = \frac{1}{2}\mu v_{\text{rel}}^2 = 0$$

In this case, $W_{\text{int}} = 0$ but $W_{\text{ext}} = -m_1 g h$

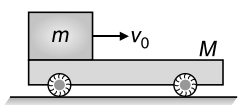
From equation (1), we get

$$-m_1 g h + 0 = -\frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)v_0^2$$

$$\Rightarrow \quad h = \frac{v_0^2}{2\left(1 + \frac{m_1}{m_2}\right)g}$$

ILLUSTRATION 45

A block of mass m is pushed with a velocity v_0 along the surface of a trolley car of mass M. If the horizontal ground is smooth and the coefficient of kinetic friction between the block and plank is k, then calculate the minimum distance of relative sliding between the block and plank.



SOLUTION

According to Work-Energy Theorem applied from CM frame, we have

$$W_{\text{internal}} + W_{\text{external}} = (K_{\text{cm}})_{\text{final}} - (K_{\text{cm}})_{\text{initial}} \quad \ldots(1)$$

Initial kinetic energy of system wrt CM is

$$(K_{\text{cm}})_{\text{initial}} = \frac{1}{2}\mu u_{\text{rel}}^2 = \frac{1}{2}\left(\frac{mM}{m+M}\right)(v_0 - 0)^2$$

$$\Rightarrow \quad (K_{\text{cm}})_{\text{initial}} = \frac{1}{2}\left(\frac{mM}{m+M}\right)v_0^2$$

Finally, when both move together, then the final relative velocity, i.e. $v_{\text{rel}} = 0$, so the final kinetic energy of system wrt CM is zero.

$$\Rightarrow \quad (K_{\text{cm}})_{\text{final}} = \frac{1}{2}\mu v_{\text{rel}}^2 = 0$$

Due to friction, work done is

$$W_{\text{int}} = f_k x \cos(180^\circ) = -kmgx \qquad \{\because \mu_k = k\}$$

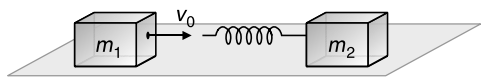
So, from equation (1), we get

$$0 + (-kmgx) = 0 - \frac{1}{2}\left(\frac{mM}{m+M}\right)v_0^2$$

$$\Rightarrow \quad x = \frac{v_0^2}{2\left(1 + \frac{m}{M}\right)kg}$$

ILLUSTRATION 46

Two smooth blocks of masses m_1 and m_2 attached with an ideal spring of stiffness k and kept on a horizontal surface. If m_1 is projected with a horizontal velocity v_0, find the maximum compression of the spring by applying the Work-Energy Theorem with respect to the centre of mass frame.



SOLUTION

When seen from the frame attached to the centre of mass, we observe that the linear momentum of system from CM frame is zero and no external force is acting on the system. According to Work-Energy Theorem applied from CM frame, we have

$$W_{\text{internal}} + W_{\text{external}} = (K_{\text{cm}})_{\text{final}} - (K_{\text{cm}})_{\text{initial}} \quad \ldots(1)$$

Initial kinetic energy of system wrt centre of mass is

$$(K_{\text{cm}})_{\text{initial}} = \frac{1}{2}\mu u_{\text{rel}}^2 = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(v_0 - 0)^2$$

$$\Rightarrow \quad (K_{\text{cm}})_{\text{initial}} = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)v_0^2$$

Finally, both the blocks move with a common velocity, so the final relative velocity

$$v_{\text{rel}} = 0$$

Hence, the final kinetic energy of system wrt centre of mass is zero.

$$\Rightarrow \quad (K_{cm})_{final} = \frac{1}{2}\mu v_{rel}^2 = 0$$

Since, $W_{int} = W_{spring} = -\Delta U = -\frac{1}{2}kx_{max}^2$

Using equation (1), we get

$$0 + \left(-\frac{1}{2}kx^2\right) = 0 - \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)v_0^2$$

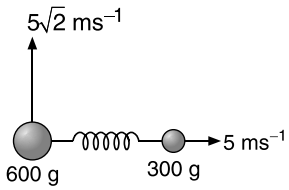
$$\Rightarrow \quad x = v_0\sqrt{\frac{1}{k}\left(\frac{m_1 m_2}{m_1 + m_2}\right)} = v_0\sqrt{\frac{\mu}{k}}$$

Test Your Concepts-III

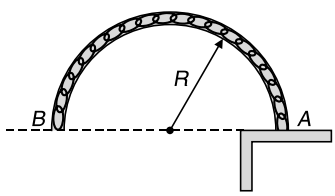
Based on Work Energy Theorem for System of Particles and its Applications from Centre of Mass Reference Frame

(Solutions on page H.76)

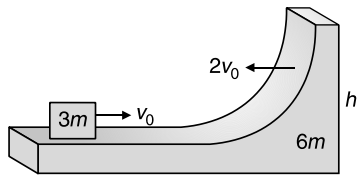
1. Two small balls of masses 600 g and 300 g are connected by a light spring resting on a smooth horizontal plane. The balls are set in motion with initial velocities $5\sqrt{3}$ ms^{-1} and 5 ms^{-1} whose initial directions are mutually perpendicular and in the same horizontal plane. Calculate the total initial kinetic energy of this system with reference to the centroidal frame.



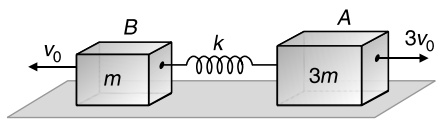
2. A heavy, flexible, uniform chain lies in a smooth semi-circular tube AB of radius R. The cross-sectional radius of the tube is much smaller than the radius of the tube (R). Assuming a slight disturbance to start the chain in motion, find the velocity with which it will emerge from the end B of the tube.



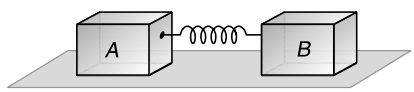
3. A block of mass $3m$ is projected to the right with a speed v_0 onto a smooth wedge of mass $6m$ which is simultaneously projected due to the left with a speed $2v_0$. If the particle attains the highest point of the wedge, calculate h.



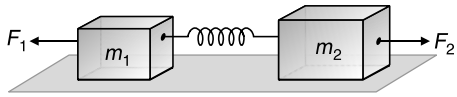
4. The block A and B of masses m and $3m$ are connected by an ideal spring of force constant k. Now both the blocks are given impulse in opposite direction so that they started moving with velocities v_0 and $3v_0$ as shown in Figure. Find the maximum stretch of spring.



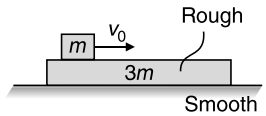
5. Two blocks A and B having masses $6m$ and $3m$ respectively lying on a smooth horizontal surface are connected by light spring of spring constant k. If the spring is stretched through x and then released, calculate the relative velocity of the blocks when the spring comes to its natural length.



6. Two constant forces F_1 and F_2 act on two smooth blocks having masses m_1 and m_2 interconnected by a light spring of stiffness k placed on a smooth horizontal surface as shown in Figure. Calculate the maximum elongation in the spring using work energy theorem from the centre of mass reference frame.

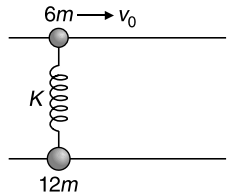


7. A small block of mass m is projected with velocity v_0 on a very long rough plank of mass $3m$ placed on a smooth horizontal surface as shown in Figure. Calculate the work done by friction when slipping stops.



8. A uniform chain is held on a frictionless table with one third of its length hanging over the edge. If the chain has a length l and a mass m, how much work is required to pull the hanging part back slowly on the table?

9. Two beads having masses $6m$ and $12m$ are connected with a light spring. The beads can slide over two frictionless parallel horizontal rails as shown in Figure.



The lighter bead is given an initial velocity v_0 in horizontal direction. Calculate the maximum extension produced in the spring.

CONSERVATION OF LINEAR MOMENTUM AND COLLISIONS

CONSERVATION OF LINEAR MOMENTUM

According to Newton's Second Law, we have

$$\vec{F}_{\text{ext}} = \vec{F} = \frac{d\vec{p}}{dt}$$

If we observe that $\vec{F}_{\text{ext}} = \vec{0}$

$$\Rightarrow \quad \frac{d\vec{p}}{dt} = \vec{0}$$

$$\Rightarrow \quad \vec{p} = \text{constant (both in magnitude and direction)}$$

$$\Rightarrow \quad \sum \vec{p}_{\text{initial}} = \sum \vec{p}_{\text{final}}$$

So, total initial momentum = final momentum (if $\vec{F}_{\text{ext}} = \vec{0}$)

LAW OF CONSERVATION OF LINEAR MOMENTUM

So, in the absence of an external force, the linear momentum of a particle (or the body) remains constant. This is called the **Law of Conservation of Linear Momentum.** The law can be extended to a system of particles or to the centre of mass of a system of particles.

If net force (or the vector sum of all the forces) on a system of particles is zero, the vector sum of linear momentum of all the particles remain conserved. So, when

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... + \vec{F}_n = \vec{0}$$

$$\Rightarrow \quad \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + ... + \vec{P}_n = \text{constant}$$

The same is the case for the centre of mass of a system of particles, i.e., when

$$\vec{F}_{\text{cm}} = \vec{0}, \ \vec{P}_{\text{cm}} = \text{constant}$$

Thus, the law of conservation of linear momentum can be applied to a single particle, to a system of particles or even to the centre of mass of the particles.

Please note that the law of conservation of linear momentum enables us to solve a number of problems which cannot be solved by a straight application of the relation $\vec{F} = m\vec{a}$.

EXAMPLE:

Suppose a particle of mass m initially at rest, suddenly explodes into two fragments of masses m_1 and m_2 which fly apart with velocities $\vec{v}_1$ and $\vec{v}_2$ respectively. Obviously, the forces resulting in the explosion of the particle must be internal forces, as no external force has been applied. In the absence of the external forces, therefore, the momentum must remain conserved and we should have

$$m\vec{v} = m_1\vec{v}_1 + m_2\vec{v}_2$$

Since, the particle was initially at rest, so $\vec{v} = \vec{0}$ and therefore,

$$m_1\vec{v}_1 + m_2\vec{v}_2 = 0$$

$$\Rightarrow \quad \vec{v}_1 = -\frac{m_2}{m_1}\vec{v}_2$$

$$\Rightarrow \quad \frac{|\vec{v}_1|}{|\vec{v}_2|} = \frac{m_2}{m_1}$$

Showing at once that the velocities of the two fragments must be inversely proportional to their masses and in opposite directions along the same line. This result could not possibly be arrived at from the relation $\vec{F} = m\vec{a}$, since we know nothing about the forces that were acting during the explosion. Nor, could we derive it by using the Law of Conservation of Energy.

Also, note that partial conservation of momentum along either of x, y or z direction does not guarantee total conservation. However, total conservation guarantees partial conservation.

FORCE			MOMENTUM			
F_x	F_y	F_z	p_x	p_y	p_z	$\vec{p}$
0	0	0	C	C	C	C
0	$\neq 0$	$\neq 0$	C	NC	NC	NC
$\neq 0$	0	$\neq 0$	NC	C	NC	NC
$\neq 0$	$\neq 0$	0	NC	NC	C	NC
$= 0$	$= 0$	$\neq 0$	C	C	C	NC

Conserved (C), Not Conserved (NC)

Problem Solving Technique(s)

While solving problems, we will come across situations where a force is acting on a particle/system. However, if on careful observation, we find that along a particular direction, the component of the external force is zero, then along this direction, momentum is conserved. However, total momentum is not conserved.

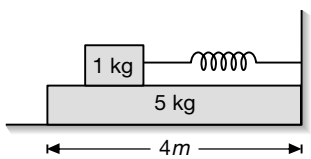
EXAMPLE: Take the case of oblique projectile where the only force acting on the particle is its own weight which has no component along the horizontal. So, momentum of the projectile is conserved along horizontal direction. However, total momentum is not conserved.

However, in situations where some external force is acting on the system, then momentum is not conserved and instead we can also make use of the Impulse-Momentum Theorem to get the desired results.

$$m\vec{v} - m\vec{u} = \int_0^t \vec{F}dt$$

ILLUSTRATION 47

A plank of mass 5 kg is placed on a frictionless horizontal plane. Further a block of mass 1 kg is placed over the plank. A massless spring of natural length 2 m is fixed to the plank by its one end. The other end of spring is compressed by the block by half of spring's natural length. The system is now released from the rest. What is the velocity of the plank when block leaves the plank? (The stiffness constant of spring is 100 Nm^{-1}).



SOLUTION

Let the velocity of the block and the plank, when the block leaves the spring be u and v respectively. If M is mass of plank, m is mass of block, then by Law of Conservation of Energy, we get

$$\frac{1}{2}kx^2 = \frac{1}{2}mu^2 + \frac{1}{2}Mv^2$$

$$\Rightarrow \quad 100 = u^2 + 5v^2 \qquad \ldots(1)$$

By Law of Conservation of Momentum, we get

$$mu + Mv = 0$$

$$\Rightarrow \quad u = -5v \qquad \ldots(2)$$

Solving equations (1) and (2), we have

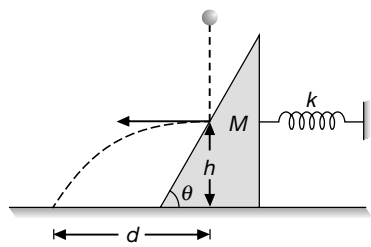
$$30v^2 = 100$$

$$\Rightarrow \quad v = \sqrt{\frac{10}{3}}\ \text{ms}^{-1}$$

From this moment until block falls, both plank and block keep their velocity constant. So, when block falls, velocity of plank is $\sqrt{\frac{10}{3}}\ \text{ms}^{-1}$.

ILLUSTRATION 48

A ball of mass m when dropped from certain height strikes a wedge kept on smooth horizontal surface and move horizontally just after impact as shown in diagram. If the ball strikes the ground at a distance d from its initial line of fall, determine the amplitude of oscillation of wedge after being hit by the ball.



SOLUTION

Since, $h = \frac{1}{2}gt^2$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}}$$

Let v_1 be the horizontal velocity of ball after collision. Then

$$d = v_1 t$$

$$\Rightarrow \quad v_1 = \frac{d}{t} = d\sqrt{\frac{g}{2h}}$$

Suppose, v_2 be the velocity of wedge just after collision. Then from conservation of linear momentum, we have,

$$Mv_2 = mv_1$$

$$\Rightarrow \quad v_2 = \frac{m}{M}v_1 = \frac{md}{M}\sqrt{\frac{g}{2h}}$$

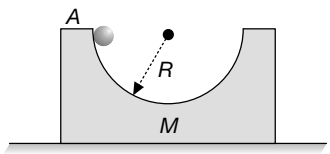
By Law of Conservation of Energy, we have

$$\frac{1}{2}kA^2 = \frac{1}{2}Mv_2^2$$

$$\Rightarrow \quad \text{Amplitude, } A = v_2\sqrt{\frac{M}{k}} = \frac{md}{M}\sqrt{\frac{gM}{2hk}}$$

ILLUSTRATION 49

A block of mass M with a semi-circular track of radius R rests on a horizontal frictionless surface shown in figure. A uniform cylinder of radius r and mass m is released from rest at the point A. The cylinder slips on the semi-circular frictionless track. How far has the block moved when the cylinder reaches the bottom of the track? How fast is the block moving when the cylinder reaches the bottom of the track?



SOLUTION

Applying $x_R m_R = x_L m_L$ we have

$$m(R-r-x) = Mx$$

Here $x =$ displacement of block towards left

$$\Rightarrow \quad x = \frac{m(R-r)}{M+m}$$

By Law of Conservation of Energy, we get

$$\begin{pmatrix}\text{Loss in Gravitation}\\ \text{Potential Energy}\\ \text{of Cylinder}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{K.E. of}\\ \text{Cylinder}\end{pmatrix} + \begin{pmatrix}\text{Gain in}\\ \text{K.E. of}\\ \text{Block}\end{pmatrix}$$

$$\Rightarrow \quad mg(R-r) = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 \qquad \ldots(1)$$

Also, by Law of Conservation of Linear Momentum, along the horizontal we have

$$mv = MV \qquad \ldots(2)$$

So, from (1) and (2), we get

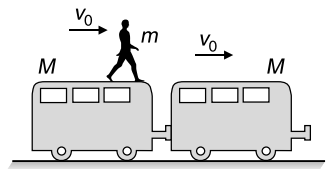
$$V = \frac{m}{M}v = m\sqrt{\frac{2g(R-r)}{M(M+m)}}$$

ILLUSTRATION 50

Two identical buggies move one after the other due to inertia (without friction) with the same velocity v_0. A man of mass m jumps into the front buggy from the rear buggy with a velocity u relative to his buggy. Knowing that the mass of each buggy is equal to M, find the velocities with which the buggies will move after that.

SOLUTION

If we take rear 'buggy + man' as system. There is no external force acting on system in horizontal direction. Hence linear momentum of the system should be conserved. Let v_R be the velocity of rear buggy just after man of mass m jumps to the front buggy, then velocity of man just after jumping will be $(v_R + u)$.



Initial momentum of rear buggy is

$$p_i = (M+m)v_0$$

When the man jumped into the front buggy with velocity u (relative to rear buggy), then momentum of rear buggy along with the man (in air) is

$$p_f = m(u+v_R) + Mv_R$$

By Law of Conservation of Momentum, we have

$$m(u+v_R) + Mv_R = (M+m)v_0$$

$$\Rightarrow \quad v_R = v_0 - \frac{mu}{(M+m)}$$

Similarly, initial momentum of front buggy along with the man (in air) is

$$p_i = Mv_0 + m(u+v_R)$$

Final momentum of front buggy is

$$p_f = (M+m)v_F$$

By Law of Conservation of Momentum, we have

$$\Rightarrow \quad Mv_0 + m(u+v_R) = (M+m)v_F$$

$$\Rightarrow \quad v_F = \left(\frac{M}{M+m}\right)v_0 + \left(\frac{m}{M+m}\right)(u+v_R)$$

Substituting the value of v_R, we get

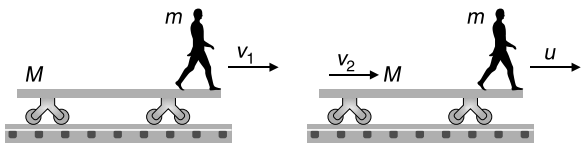
$$v_F = v_0 + \frac{Mmu}{(M+m)^2}$$

ILLUSTRATION 51

A railroad car of mass M with a man of mass m is moving with a velocity v_1. The man jumps in the direction of motion of car with a velocity u with respect to the car. Find the final velocities of the man and that of the car after jump.

SOLUTION

Taking 'man and car' as a system, we observe that there is no external force acting on this system along the horizontal direction. Due to this, the linear momentum of the system is conserved along the horizontal direction. After the man jumps, the car attains a velocity v_2 in the same direction, which is less than v_1, due to backward push of the man for jumping. After the jump, the man attains a velocity $(u+v_2)$ in the direction of motion of car with respect to ground.



Applying the Law of Conservation of Linear Momentum, we get

$$(M+m)v_1 = Mv_2 + m(u+v_2)$$

So, velocity of car after jump is

$$v_2 = \frac{(M+m)v_1 - mu}{M+m}$$

and velocity of man after jump is

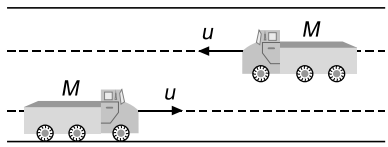
$$u+v_2 = \frac{(M+m)v_1 + Mu}{M+m}$$

ILLUSTRATION 52

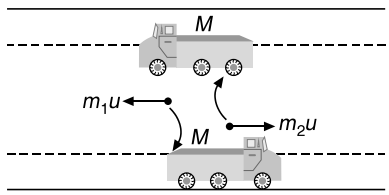
Two trucks each of mass M each (including the content it carries) are moving in opposite direction on adjacent parallel tracks with same velocity u. One truck is carrying potatoes and other is carrying onions. Each bag of potatoes has a mass m_1 and that of onions has a mass m_2. When trucks get close to each other while passing, the drivers exchange a bag each from their truck by throwing it towards the other one. Find the final velocity of each truck after the exchange of bags.

SOLUTION

At the time of exchange of bags, momentum in the direction of individual motion remains conserved. The situation is shown in Figure.



When a bag of potatoes having mass m_1 is thrown from the first truck in a direction perpendicular to its motion, towards the second truck, then during the throw, the bag has a velocity u in the direction of motion of the first truck as shown in figure.



When a bag of onions having mass m_2 is thrown from the second truck in a direction perpendicular to its motion, towards the first truck, then during the throw, the bag has a velocity u in the direction of motion of the second truck.

Similarly, when the second truck driver throws the onion bag of mass m_2, towards the first truck, it brings a momentum m_2u in the direction of the second truck.

Applying Conservation of Momentum on first truck carrying potatoes, we get

$$(M-m_1)u - m_2u = (M-m_1+m_2)v_1$$

So, velocity of first truck after exchange is

$$v_1 = \frac{Mu - m_1u - m_2u}{M-m_1+m_2}$$

Applying Conservation of Momentum on second truck carrying onions, we get

$$(M-m_2)u - m_1u = (M-m_2+m_1)v_2$$

So, velocity of second truck after exchange is

$$v_2 = \frac{Mu - m_1u - m_2u}{M-m_2+m_1}$$

INSTANTANEOUS IMPULSE

There are many occasions when a force acts for such a short time that the effect is instantaneous, e.g., a bat striking a ball. In such cases, although the magnitude of the force and the time for which it acts may each be unknown but the value of their product (i.e., impulse) can be known by measuring the initial and final momenta. Thus, we can write

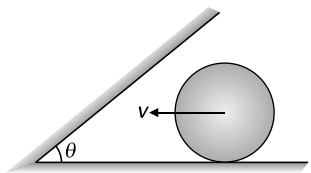
$$\vec{J} = \int \vec{F} dt = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

Regarding the impulse it is important to note that impulse applied to an object in a given time interval can also be calculated from the area under force-time $(F\text{-}t)$ graph in the same time interval.

ILLUSTRATION 53

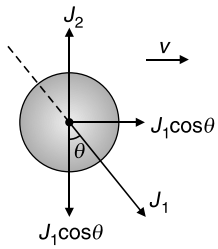
A sphere of mass m slides with velocity v on a frictionless surface towards a smooth inclined wall as shown in Figure. If the collision with the wall is perfectly elastic, find

(a) the impulse imparted by the wall on the sphere,
(b) the impulse imparted by the floor on the sphere.



SOLUTION

As the collision is elastic, so the sphere will rebound with the same speed after collision. The impulses acting on the sphere are shown in Figure.



$$J_1 \cos\theta = J_2 \quad \ldots(1)$$

$$-mv + J_1 \sin\theta = mv \quad \ldots(2)$$

Solving equation (1) and (2), we get

$$J_2 = 2mv\cot\theta$$

$$J_1 = 2mv\,\text{cosec}\,\theta$$

IMPULSIVE TENSIONS

When a string jerks, equal and opposite tensions act suddenly at each end. Consequently, equal and opposite impulses act on the objects to which the two ends of the string are attached. There are two cases to be considered.

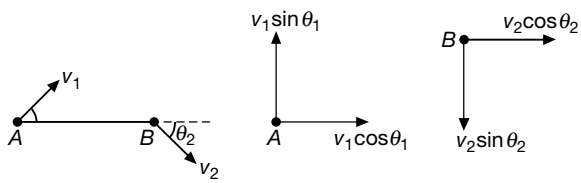
CASE-1: One end of the string is fixed

The impulse which acts at the fixed end of the string cannot change the momentum of the fixed object there. The object attached to the free end however will undergo a change in momentum in the direction of the string. The momentum remains unchanged in a direction perpendicular to the string where no impulsive forces act.

CASE-2: Both ends of string attached to movable objects

In this case equal and opposite impulses act on the two objects, producing equal and opposite changes in momentum. The total momentum of the system therefore remains constant, although the momentum of each individual object is changed in the direction of the string. Perpendicular to the string however, no impulse acts and the momentum of each particle in this direction is unchanged.

The velocities of two objects moving at the ends of a taut string are not independent. The relationship between them is understood by taking a taut string AB and particles A, B which are moving with velocities as shown in the Figure. The important components of velocity are those along the string AB.



If $v_1\cos\theta_1 > v_2\cos\theta_2$ **the string is not taut and**

If $v_2\cos\theta_2 > v_1\cos\theta_1$ **the string is snapped** i.e., it breaks

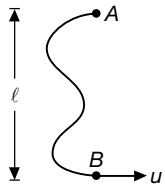
Hence, for the string to remain taut and unbroken, we must have

$$v_1\cos\theta_1 = v_2\cos\theta_2$$

Hence, we conclude that the two ends of a taut string have equal velocity components in the direction of the string.

ILLUSTRATION 54

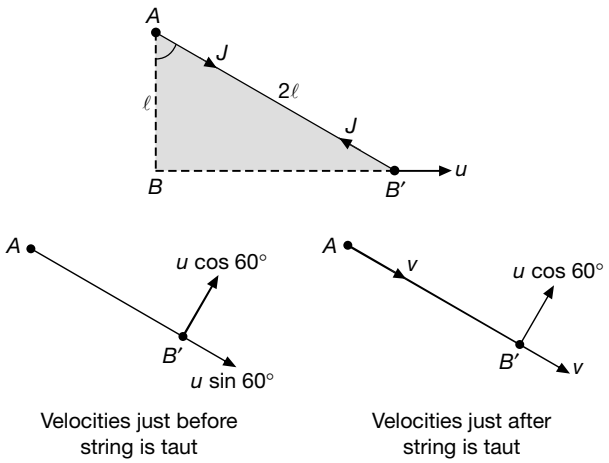
Two particles A and B of equal mass m each are attached by a string of length $2l$ and initially placed over a smooth horizontal table in the position shown in figure. Particle B is projected across the table with speed u perpendicular to AB as shown in figure.



Find the velocities of each particle after the string becomes taut and the magnitude of the impulsive tension.

SOLUTION

When the string becomes taut, both particles begin to move with equal velocity component v along the direction AB'



Velocities just before string is taut

Velocities just after string is taut

Perpendicular to AB' there is no impulse on either particle, velocity components in this direction are therefore unchanged.

Applying the Law of Conservation of Momentum in the direction AB', we get

$$0 + mu\sin(60°) = mv + mv$$

$$\Rightarrow \quad v = \frac{\sqrt{3}}{4}u$$

So, just after the string becomes taut, we have

(a) Velocity of mass A is $v_A = \frac{\sqrt{3}}{4}u$ along AB'

(b) Velocity of mass B is

$$v_B = \sqrt{(u\cos 60°)^2 + \left(\frac{\sqrt{3}}{4}u\right)^2} = \frac{\sqrt{7}}{4}u$$

In a direction inclined to AB' at $\tan^{-1}\left(\frac{u\cos 60°}{v}\right)$, i.e., at $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

The magnitude of impulsive tension (J) is found by calculating the change in momentum of one of the particles.

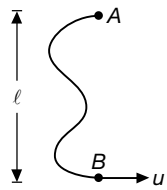
For the mass A, in the direction AB', we have

$$J = \Delta p_A = mv - 0$$

$$\Rightarrow \quad J = \frac{\sqrt{3}}{4}mu$$

ILLUSTRATION 55

A string AB of length $2l$ is fixed at A to a point on a smooth horizontal table. A particle of mass m attached to B is initially at a distance l from A as shown in figure.

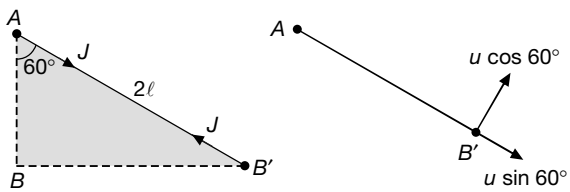


The particle is projected horizontally with speed u at right angles to AB. Find the impulsive tension J in the string when it becomes taut and the velocity of the particle immediately afterwards.

SOLUTION

When the string becomes taut $AB' = 2l$

$$\Rightarrow \quad \angle B'AB = 60°$$



Just before the string becomes taut, the components of the velocity of particle parallel and perpendicular

to AB' are $u\sin(60°)$ and $u\cos(60°)$ respectively. When the string becomes taut the length of AB is fixed and particle cannot travel further along the direction AB'. So, after the string becomes taut, the velocity of the particle is, therefore, perpendicular to AB' (as it cannot move any further along AB').
Since, we know that Impulse = Change in Momentum, so

(a) Along $B'A$, we have

$$J = 0 - (-mu\sin(60°))$$

$$\Rightarrow \quad J = \frac{\sqrt{3}}{2}mu$$

(b) Perpendicular to $B'A$ (no impulsive component)

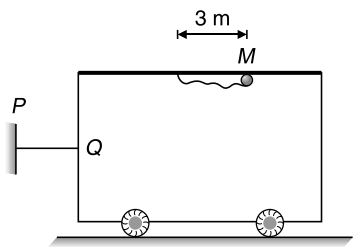
$$\Rightarrow \quad 0 = mv - mu\cos(60°)$$

$$\Rightarrow \quad v = u\cos(60°) = \frac{u}{2}$$

Hence, the velocity of the particle just after the string becomes taut is $\frac{u}{2}$, perpendicular to the string.

ILLUSTRATION 56

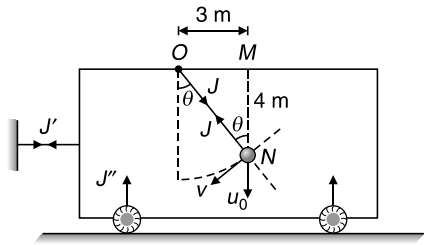
A particle of mass $\sqrt{5}$ kg is attached to a string of length 5 m attached to a fixed point O. The particle is released from the position shown in Figure.



Calculate the impulse developed in the string when it becomes taut, the velocity of the particle just after the string becomes taut and the impulse developed in this string PQ at this instant.

SOLUTION

When ball is released from rest from M then it falls to N, a distance 4 m below M, due to which an impulse will be developed in the string ON as shown in Figure.



Velocity of ball just before string becomes taut is

$$u = \sqrt{2\times10\times4} = \sqrt{80}\ \text{ms}^{-1} = 4\sqrt{5}\ \text{ms}^{-1}$$

Velocity of ball in direction perpendicular to the impulse will remain the same just before and after and the impulse is developed.

$$v = u\sin\theta = (4\sqrt{5})\left(\frac{3}{5}\right) = \frac{12}{\sqrt{5}}\ \text{ms}^{-1}$$

$$\Rightarrow \quad J = mu\cos\theta = \sqrt{5}(4\sqrt{5})\left(\frac{4}{5}\right) = 16\ \text{kgms}^{-1}$$

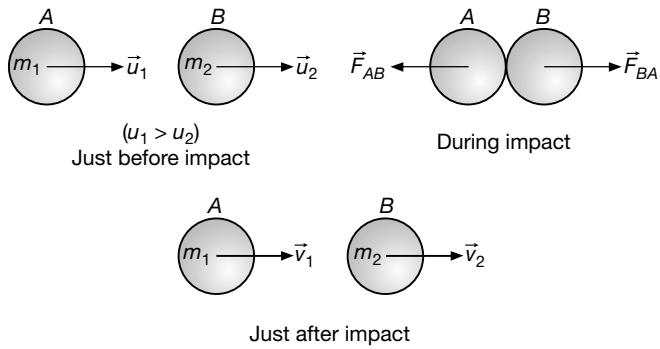
$$\Rightarrow \quad J' = J\sin\theta = (16)\left(\frac{3}{5}\right) = \frac{48}{5}\ \text{kgms}^{-1}$$

and impulse imparted by earth is

$$J'' = J\cos\theta = (16)\left(\frac{4}{5}\right) = \frac{64}{5}\ \text{kgms}^{-1}$$

CONSERVATION OF LINEAR MOMENTUM FOR A TWO PARTICLE SYSTEM

Consider two bodies A and B having masses m_1 and m_2 moving with initial velocities $\vec{u}_1$ and $\vec{u}_2$ (as shown).



Let the bodies collide such that during impact a force $\vec{F}_{AB}$ acts on A due to B and a force $\vec{F}_{BA}$ acts on B due to A. Then by Newton's Third Law

$$\vec{F}_{AB} = -\vec{F}_{BA} \qquad \ldots(1)$$

Let $\vec{v}_1$ and $\vec{v}_2$ be the velocities of the two bodies just after impact. Let us now calculate the change in momentum of A and B i.e., $\Delta\vec{p}_A$ and $\Delta\vec{p}_B$.
For A, we have

$$\Delta\vec{p}_A = m_1\vec{v}_1 - m_1\vec{u}_1 = \int_0^t \vec{F}_{AB}dt \qquad \ldots(2)$$

For B, we have

$$\Delta\vec{p}_B = m_2\vec{v}_2 - m_2\vec{u}_2 = \int_0^t \vec{F}_{BA}dt \qquad \ldots(3)$$

For $A+B$ system, by adding (2) and (3), we get

$$m_1\vec{v}_1 - m_1\vec{v}_1 + m_2\vec{v}_2 - m_2\vec{v}_2 = \int_0^t \vec{F}_{AB}dt + \int_0^t \vec{F}_{BA}dt$$

$$\Rightarrow \quad m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

Conceptual Note(s)

CHOICE OF A SYSTEM

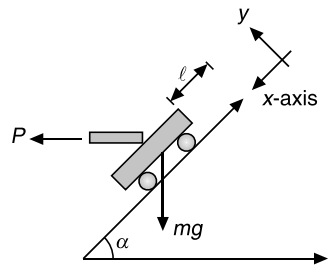
We observe that momentum is not conserved for A (because of force $\vec{F}_{AB}$ acting on it) and also not conserved for B (because of force $\vec{F}_{BA}$ acting on it). However, for $A+B$ **system net external force acting is zero**, hence momentum of $(A+B)$ system is conserved.

ILLUSTRATION 57

A cannon of mass m starts sliding freely down a smooth inclined plane at an angle α to the horizontal. After the cannon covered the distance l, a shot was fired, the shell leaving the cannon in the horizontal direction with a momentum P. As a consequence, the cannon is stopped. Assuming the mass of the shell to be negligible, as compared to that of the cannon, determine the duration of the shot.

SOLUTION

To start with, let us calculate the velocity of the cannon when the shot was fired.



$$v^2 - 0^2 = 2(g\sin\alpha)l \qquad \{\because\ a = g\sin\alpha\}$$

$$\Rightarrow \quad v = \sqrt{2gl\sin\alpha}$$

Initial momentum along x-axis is $(p_i)_x = mv$

Final momentum along x-axis is $(p_f)_x = P\cos\alpha$

Change in momentum along x-axis i.e., $\Delta p_x = (P\cos\alpha - mv)$

Since, Force = Rate of Change of Momentum

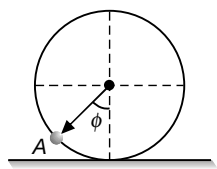
$$\Rightarrow \quad mg\sin\alpha = \frac{(P\cos\alpha - mv)}{t}$$

$$\Rightarrow \quad t = \frac{P\cos\alpha - mv}{mg\sin\alpha} = \frac{P\cos\alpha - m\sqrt{2gl\sin\alpha}}{mg\sin\alpha}$$

Please note that the normal reaction will have no component along x-axis.

ILLUSTRATION 58

A thin hoop of mass M and radius r is placed on a horizontal plane. At the initial instant, the hoop is at rest. A small washer of mass m with zero initial velocity slides from the upper point of the hoop along a smooth grove in the inner surface of the hoop. Determine the velocity u of the centre of the hoop at the moment when the washer is at a certain point A of the hoop, whose radius vector forms an angle ϕ with the vertical (figure). The friction between the hoop and the plane should be neglected.



SOLUTION

Let v_r be the velocity of washer relative to centre of hoop and u be the velocity of centre of hoop. Applying Law of Conservation of Linear Momentum along the horizontal, we get

$$m(v_r \cos\phi - u) = Mu \quad \ldots(1)$$

Applying Law of Conservation of Mechanical Energy, we get

$$\begin{pmatrix}\text{Loss in}\\ \text{GPE of}\\ \text{Washer}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{KE of}\\ \text{Hoop}\end{pmatrix} + \begin{pmatrix}\text{Gain in}\\ \text{KE of}\\ \text{Washer}\end{pmatrix}$$

$$\Rightarrow \quad mgr(1+\cos\phi) = \frac{M}{2}u^2 + \frac{m}{2}\left(v_r^2 + u^2 - 2uv_r\cos\phi\right) \quad \ldots(2)$$

Solving equations (1) and (2), we get

$$u = m\cos\phi\sqrt{\frac{2gr(1+\cos\phi)}{(M+m)(M+m\sin^2\phi)}}$$

Test Your Concepts-IV

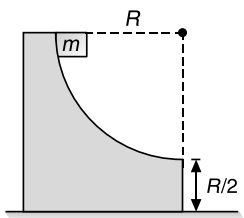
Based on Conservation of Linear Momentum

(Solutions on page H.78)

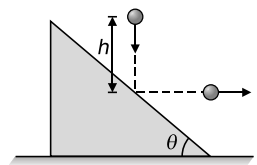
1. A man of mass m_1 is standing on a platform of mass m_2 kept on a smooth horizontal surface. The man starts moving on the platform with a velocity v_r relative to the platform. Find the recoil velocity of platform.

2. A railroad car of mass M is at rest on a frictionless track with a man of mass m standing at its edge. If the man jumps off from the car towards right with an initial velocity u, with respect to the car, find the velocity of the car after its jump.

3. A small cube of mass m slides down a circular path of radius R cut into a large block of mass M, as shown in figure. M rests on a table and both blocks move without friction. The blocks are initially at rest and m starts from the top of the path. Find the horizontal distance from the bottom of block when cube hits the table.



4. A particle of mass m is released from height h on an inclined plane. If after collision with the plane, the particle starts moving horizontally as shown in Figure.



Calculate the velocity of particle just after collision, impulse provided by the plane to the particle and loss in kinetic energy of the particle due to impact.

5. A 50 N projectile is fired from ground level with an initial velocity of 24 ms^{-1} making an angle of 60° with the horizontal. When it reaches its highest point, it explodes into two 25 N fragments. If one fragment is seen to travel vertically upward, and after they fall, they are 45 m apart, calculate the speed of each fragment just after the explosion. Neglect the size of the gun.

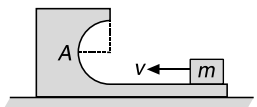
6. A projectile is launched at an angle θ_0 with velocity v_0. At the highest point of its path, it explodes in two parts having mass ratio 1 : 3. If the lighter mass retraces its path, then calculate velocity of the heavier part just after the explosion.

7. A wagon of mass M can move without friction along horizontal rails. A simple pendulum consisting of a sphere of mass m is suspended from the ceiling of the wagon by a string of length ℓ. At the initial moment the wagon and the pendulum are at rest and the string is deflected

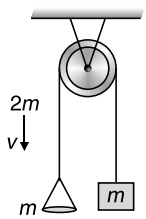
through an angle α from the vertical. Find the velocity of the wagon when the pendulum passes through its mean position.

8. Two particles, each of mass m, are connected by a light inextensible string of length $2l$. Initially they lie on a smooth horizontal table at points A and B distant l apart. The particle at A is projected across the table with velocity u. Find the speed with which the second particle begins to move, if the direction of u is
(a) along the line BA,
(b) at an angle of $120°$ with AB
(c) perpendicular to AB.
In each case also calculate (in terms of m and u) the impulsive tension in the sting.

9. Figure shows a small block of mass m which is started with a speed v on the horizontal part of the bigger block of mass M placed on a horizontal floor. The curved part of the surface shown is semi-circular. All the surfaces are frictionless. Find the speed of the bigger block when the smaller block reaches the point A of the surface.



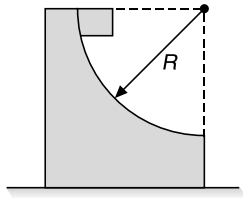
10. A block of mass m and a pan of equal mass are connected by a string going over a smooth light pulley. Initially, the system is at rest, then a particle of mass $2m$ falls on the pan and sticks to it. If the particle strikes the pan with a speed v, calculate the speed with which the system moves just after the collision.



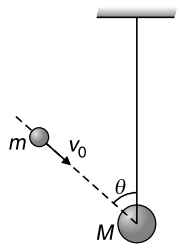
11. Two men, each of mass m stand on the edge of a stationary buggy of mass M. Assuming the friction to be negligible, calculate the velocity of the buggy after both men jump off with the same horizontal velocity u relative to the buggy,
(a) simultaneously
(b) one after the other

12. A shell of mass m is fired from a gun of mass km which can recoil freely on a horizontal plane, the elevation of the gun is $45°$. Find the ratio of the energy of the shell to that of the gun.

13. A small cube of mass m slides down a circular path of radius R cut into a large block of mass M as shown in figure. M rests on a table and both blocks move without friction. The blocks are initially at rest and m starts from top of the path. Find the velocity v of the cube as it leaves the block.



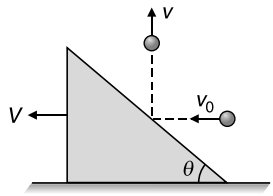
14. A bob of mass M is suspended by an inextensible string. A particle of mass m, moving with velocity v_0 strikes the bob as shown in Figure.



If the particle comes to rest just after collision, then calculate impulse due to tension in the string, velocity of the bob just after collision.

15. A heavy mass M resting on the ground is attached to a small mass m via massless inextensible string passing over a pulley. The string connected to M is loose. The smaller mass falls freely through a height h and the string becomes taut. Find the time from this instant when the heavier mass again makes contact with the ground. Also obtain the loss in kinetic energy when M is jerked into motion.

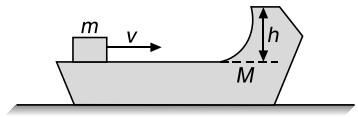
16. A right-angled wedge of mass M and base angle θ is lying on a smooth horizontal surface. A small ball of mass m, moving horizontally with velocity v_0 collides with wedge on inclined surface as shown in Figure.



If after collision the ball starts moving in vertical, upward direction, then calculate the impulse due to normal reaction applied by wedge on the ball, impulse due to normal reaction applied by ground on wedge, velocity of the ball and wedge just after collision. If coefficient of friction between ground and wedge is μ, then calculate impulse due to frictional force.

17. A bullet of mass 0.25 kg is fired with velocity 302 ms^{-1} into a block of wood of mass $m_1 = 37.5$ kg. It gets embedded into it. The block m_1 is resting on a long block m_2 of mass $m_2 = 12.5$ kg and the horizontal surface on which it is placed is smooth. The coefficient of friction between m_1 and m_2 of 0.5. Find the displacement of m_1 on m_2 and the common velocity of m_1 and m_2.

18. A body of mass M with a small box of mass m placed on it rests on a smooth horizontal surface. The box is set in motion in the horizontal direction with a velocity u as shown in Figure. Calculate the height relative to the initial level to which the box will rise after breaking off from the body M. Assume all surfaces to be smooth.



VARIABLE MASS

In our discussion of the conservation of linear momentum, we have so far dealt with systems whose mass remains constant. We now consider those systems whose mass is variable, i.e., those in which mass enters or leaves the system. A typical case is that of the rocket from which hot gases keep on escaping, thereby continuously decreasing its mass.

In such problems we apply a thrust force $\left(\vec{F}_t\right)$ to the main mass in addition to the all other forces acting on it. This thrust force is given by,

$$\vec{F}_t = \vec{v}_{\text{rel}}\left(\frac{dm}{dt}\right)$$

where, $\vec{v}_{\text{rel}}$ is the velocity of the mass gained or **mass ejected relative to the main mass.** In case of rocket this is also called the **exhaust velocity** of the gases, also $\frac{dm}{dt}$ is the rate at which mass is increasing or decreasing.

The expression for the thrust force can be derived from the Law of Conservation of Linear Momentum in the absence of any external forces on a system.

Let at some time t, the mass of the body be m and its velocity be $\vec{v}$. After some infinitesimal time say $t + dt$ its mass becomes $(m - dm)$ and velocity becomes $\vec{v} + d\vec{v}$. Let the mass dm be ejected with relative velocity $\vec{v}_r$. Absolute velocity of mass dm is therefore $\left(\vec{v}_r + \vec{v} + d\vec{v}\right)$. If no external forces are acting on the system, then linear momentum of the system will remain conserved, i.e.,

$$\Rightarrow \quad \vec{p}_i = \vec{p}_f$$

$$\Rightarrow \quad m\vec{v} = (m - dm)(\vec{v} + d\vec{v}) + dm\left(\vec{v}_r + \vec{v} + d\vec{v}\right)$$

$$\Rightarrow \quad m\vec{v} = m\vec{v} + md\vec{v} - \vec{v}dm - dmd\vec{v} + \vec{v}dm + \vec{v}_r dm + dmd\vec{v}$$

$$\Rightarrow \quad md\vec{v} = -\vec{v}_r dm$$

$$\Rightarrow \quad m\left(\frac{d\vec{v}}{dt}\right) = \vec{v}_r\left(-\frac{dm}{dt}\right)$$

where, $m\left(\frac{d\vec{v}}{dt}\right)$ = Thrust Force $\left(\vec{F}_t\right)$

and $-\frac{dm}{dt}$ = Rate at which mass is being ejected from system

Problem Solving Technique(s)

(a) Make a list of all the forces acting on the main mass and apply them on it.

(b) Apply an additional thrust force $\vec{F}_t$ on the mass, the magnitude of which is $\left|\vec{v}_r\left(\pm\frac{dm}{dt}\right)\right|$.

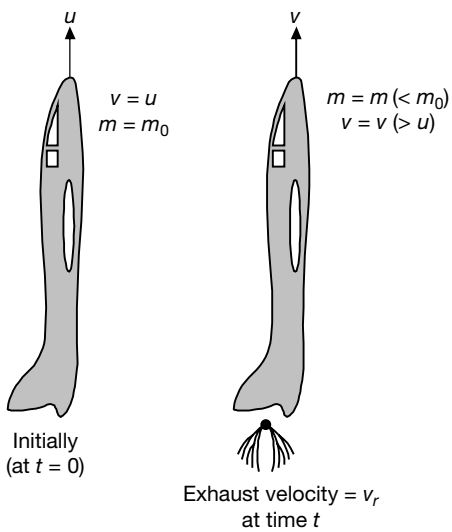
(c) The direction of this thrust force $\vec{F}_t$ is given by the direction of $\vec{v}_r$ **in case the mass is increasing** else it is the direction of $-\vec{v}_r$ if the mass is decreasing.

(d) Calculate the net force on the mass and apply $\vec{F}_{net} = m\frac{d\vec{v}}{dt}$ (m = mass at that particular instant).

(e) Integrate it within proper limits to find velocity at any time t.

ROCKET PROPULSION

Let m_0 be the mass of the rocket initially (i.e., at time $t = 0$) and m be its mass at any later time t. Let v be its velocity at time t let the velocity of the rocket be u at $t = 0$.



Further, let $-\frac{dm}{dt}$ be the mass of the gas ejected per unit time and v_r be the exhaust velocity of the gases. **Usually** $\left(-\frac{dm}{dt}\right)$ **and** v_r **are kept constant throughout the journey of the rocket**. Few equations which can be used in the problems of rocket propulsion. At time $t = t$. The forces that act on the rocket, at time t (say) are

(a) Thrust force on the rocket due to ejection of gases is

$$F_t = v_r\left(-\frac{dm}{dt}\right) \quad \{\text{upwards}\}$$

(b) Weight of the rocket is

$$W = mg \quad \{\text{downwards}\}$$

(c) So, net force acting on the rocket is

$$F_{\text{net}} = F_t - W \quad \{\text{upwards}\}$$

$$\Rightarrow F_{\text{net}} = v_r\left(-\frac{dm}{dt}\right) - mg$$

(d) Net acceleration of the rocket

$$a = \frac{F}{m}$$

Since, $F_{\text{net}} = m\frac{dv}{dt}$

$$\Rightarrow m\frac{dv}{dt} = -v_r\frac{dm}{dt} - mg$$

$$\Rightarrow \frac{dv}{dt} = \frac{v_r}{m}\left(-\frac{dm}{dt}\right) - g$$

$$\Rightarrow dv = v_r\left(-\frac{dm}{m}\right) - g\,dt$$

$$\Rightarrow \int_u^v dv = v_r\int_{m_0}^{m} -\frac{dm}{m} - g\int_0^t dt$$

$$\Rightarrow v - u = v_r \ln\left(\frac{m_0}{m}\right) - gt$$

$$\Rightarrow v = (u - gt) + v_r \ln\left(\frac{m_0}{m}\right)$$

Conceptual Note(s)

(a) $F_t = v_r\left(-\frac{dm}{dt}\right)$ is upwards, as v_r is downwards and $\frac{dm}{dt}$ is negative.

(b) If gravity is ignored and initial velocity of the rocket $u = 0$, then we get $v = v_r \ln\left(\frac{m_0}{m}\right)$.

(c) There can be many cases in which the mass of system varies with time. In all the cases of mass variation, we use momentum conservation at an intermediate instant from time t to $t + dt$.

ILLUSTRATION 59

A rocket, with an initial mass of 1000 kg , is launched vertically upwards from rest under gravity. The rocket burns fuel at the rate of 10 kgs^{-1}. The burnt matter is ejected vertically downwards with a speed of 2000 ms^{-1} relative to the rocket. If burning ceases after one minute, find the maximum velocity of the rocket. (Assume g to be constant and have a value 10 ms^{-2}).

SOLUTION

Since, we know that

$$v = (u - gt) + v_r \ln\left(\frac{m_0}{m}\right)$$

According to the problem, we have $u = 0$, $t = 60$ s, $g = 10\ ms^{-2}$, $v_r = 2000\ ms^{-1}$, $m_0 = 1000$ kg

and $m = 1000 - 10 \times 60 = 400$ kg

$$\Rightarrow \quad v = 0 - 600 + 2000 \ln\left(\frac{1000}{400}\right)$$

$$\Rightarrow \quad v = 2000 \ln(2.5) - 600$$

The maximum velocity of the rocket is

$$v = 200(10 \ln 2.5 - 3) = 1232.6\ ms^{-1}$$

ILLUSTRATION 60

A double stage rocket has an initial mass M_1. Gas is exhausted from the rocket at a constant rate of $\frac{dm}{dt}$ and with an exhaust velocity u relative to the rocket. When the mass of the rocket reaches the value M, the first stage of mass m of which fuel is exhausted is disengaged from the rocket. Then rocket continues to the second stage at the same rate and exhaust velocity as in the first stage, until it reaches a mass M_2. Calculate the rocket velocity at the end of the first and second stage (if the rocket had started from rest). Also calculate the final velocity of a single stage rocket of same initial mass M_1 and having same amount of fuel. Is this velocity greater or less than final velocity of the double stage rocket?

SOLUTION

The rocket has constant exhaust velocity given by $\frac{dm}{dt}$ during its motion. Since, we know that

$$m\frac{dv}{dt} = -v_r \frac{dm}{dt}$$

$$\Rightarrow \quad \int_0^{v_1} dv = -u \int_{M_1}^{M} \frac{dm}{m}$$

$$\Rightarrow \quad v_1 = u \ln\left(\frac{M_1}{M}\right)$$

After a mass m is disengaged, the rocket mass changes to $(M - m)$ at the start of second stage and hence we have

$$\int_{v_1}^{v_2} dv = -u \int_{M-m}^{M_2} \frac{dm}{m}$$

$$\Rightarrow \quad v_2 - v_1 = u \ln\left(\frac{M-m}{M_2}\right)$$

$$\Rightarrow \quad v_2 = u\left[\ln\left(\frac{M_1}{M}\right) + \ln\left(\frac{M-m}{M_2}\right)\right]$$

$$\Rightarrow \quad v_2 = u \ln\left[\frac{M_1(M-m)}{MM_2}\right]$$

However, if there is only single stage of mass M_1, then the final mass after exhaust of fuel will be $(M_2 + m)$. Hence, we have

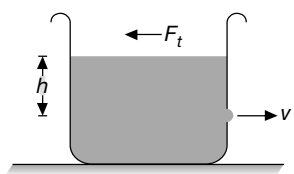
$$\int_0^{v_f} dv = -u \int_{M_1}^{M_2+m} \frac{dm}{m}$$

$$\Rightarrow \quad v_f = u \ln\left(\frac{M_1}{M_2 + m}\right)$$

So, we observe that $v_2 > v_f$.

LIQUID COMING OUT OF ORIFICE IN A BEAKER

A liquid of density ρ is filled in a container as shown in Figure.



The liquid comes out from the container through a orifice of area a at a depth h below the free surface of the liquid with a velocity v. This exerts a thrust force in the container in the backward direction. This thrust force is given by

$$F_t = v_r\left(-\frac{dm}{dt}\right)$$

Here, $v_r = v$ {in forward direction}

and $\left(-\frac{dm}{dt}\right) = \rho av$

Since, $\left(\frac{dV}{dt}\right)$ = Volume of liquid flowing per second

$\Rightarrow \quad \left(\frac{dV}{dt}\right) = av$

$\Rightarrow \quad \left(-\frac{dm}{dt}\right) = \rho\left(\frac{dV}{dt}\right) = \rho av$

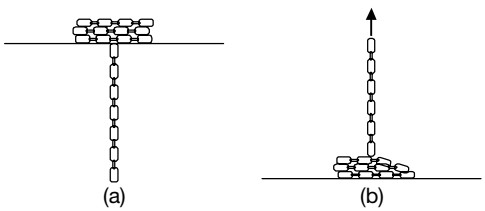
$\Rightarrow \quad F_t = v(\rho av)$

$\Rightarrow \quad F_t = \rho av^2$ {in backward direction}

Further, we will see in the chapter of fluid mechanics that $v = \sqrt{2gh}$.

CHAIN FALLING THROUGH A HOLE OR CHAIN BEING LIFTED VERTICALLY UP

Suppose, a chain of mass per unit length λ begins to fall through a hole in the ceiling as shown in figure (a) or the end of the chain piled on the platform is lifted vertically as in figure (b).



In both the cases, due to increase of mass in the portion of the chain which is moving with a velocity v at certain moment of time a thrust force acts on this part of the chain which is given by

$$F_t = v_r\left(\frac{dm}{dt}\right)$$

Here, $v_r = v$ and $\frac{dm}{dt} = \lambda v$

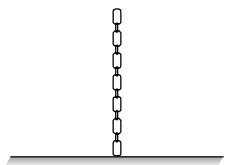
Here, v_r is upwards in case (a) and downwards in case (b). Thus,

$$F_t = \lambda v^2$$

The direction of F_t is upwards in case (a) and downwards in case (b).

ILLUSTRATION 61

A uniform chain of mass m and length l hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the table on the chain when half of its length has fallen on the table. The fallen part does not form heap.

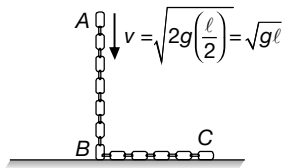


SOLUTION

The force exerted by the chain on the table consists of two parts,

(a) Weight of the portion BC of the chain lying on the table,

$$W = \frac{mg}{2} \quad \text{{downwards}}$$



(b) Thrust force exerted on table by the falling part of the chain, $F_t = \lambda v^2$

where, λ = mass per unit length of chain $= \frac{m}{l}$

and $v^2 = \left(\sqrt{gl}\right)^2 = gl$

$\Rightarrow \quad F_t = \left(\frac{m}{l}\right)(gl) = mg$ {downwards}

So, net force exerted by the chain on the table is

$$F = W + F_t = \frac{mg}{2} + mg = \frac{3}{2}mg \quad \text{{downwards}}$$

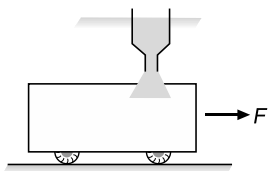
So, from Newton's Third Law the force exerted by the table on the chain will be $\frac{3}{2}mg$ (vertically upwards).

Conceptual Note(s)

Here the thrust force (F_t) applied by the chain on the table will be vertically downwards. Since $F_t = v_r\left(\frac{dm}{dt}\right)$, and in this expression v_r is downwards and $\frac{dm}{dt}$ is positive. Hence, F_t will act downwards.

ILLUSTRATION 62

A flat car of mass m_0 starts moving to the right due to a constant horizontal force F. Sand spills on the flat car from a stationary hopper. The rate of loading is constant and equals to μ kgs^{-1}. Find the time dependence of the velocity and the acceleration of the flat car in the process of loading. The friction is negligibly small.

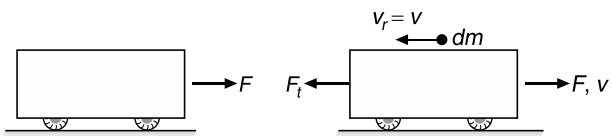


SOLUTION

Initial velocity of the flat car is zero

i.e., at $t = 0$, $v = 0$

Let v be its velocity at time t and m its mass at that instant i.e.,



at time t, $v = v$ and $m = m_0 + \mu t$ {backwards}

Here, $v_r = v$ and $\frac{dm}{dt} = \mu$

Thrust force, $F_t = v_r\frac{dm}{dt} = \mu v$ {backwards}

Net force on the flat car at time t is $F_{net} = F - F_t$

$$\Rightarrow \quad m\frac{dv}{dt} = F - \mu v \qquad \ldots(1)$$

$$\Rightarrow \quad (m_0 + \mu t)\frac{dv}{dt} = F - \mu v$$

$$\Rightarrow \quad \int_0^v \frac{dv}{F - \mu v} = \int_0^t \frac{dt}{m_0 + \mu t}$$

$$\Rightarrow \quad -\frac{1}{\mu}\left(\ln(F - \mu v)\right)\Big|_0^v = \frac{1}{\mu}\left(\ln(m_0 + \mu t)\right)\Big|_0^t$$

$$\Rightarrow \quad \ln\left(\frac{F}{F - \mu v}\right) = \ln\left(\frac{m_0 + \mu t}{m_0}\right)$$

Taking antilog both sides, we get

$$\frac{F}{F - \mu v} = \frac{m_0 + \mu t}{m_0}$$

$$\Rightarrow \quad v = \frac{Ft}{m_0 + \mu t}$$

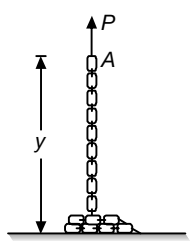
Since $\frac{dv}{dt} = a$, i.e. acceleration of flat car at time t, so from equation (1), we get

$$\frac{dv}{dt} = \frac{F - \mu v}{m} = \frac{F - \frac{F\mu t}{m_0 + \mu t}}{(m_0 + \mu t)}$$

$$\Rightarrow \quad a = \frac{Fm_0}{(m_0 + \mu t)^2}$$

ILLUSTRATION 63

A chain of length l and mass m lies in a pile on the floor. If its end A is raised vertically at a constant speed v_0, express in terms of the length y of chain which is off the floor at any given instant.



(a) the magnitude of the force P applied to end A.
(b) energy lost during the lifting of the chain.

SOLUTION

(a) When the chain is raised with a constant upward speed, then net force on it is zero. So,

P = Weight of length y of chain + Thrust force

$$\Rightarrow \quad P = \left(\frac{m}{l}\right)gy + \rho v_0^2 \qquad \left\{\text{here } \rho = \frac{m}{l}\right\}$$

$$\Rightarrow \quad P = \frac{m}{l}\left(gy + v_0^2\right)$$

(b) $\begin{pmatrix}\text{Energy Lost}\\ \text{During}\\ \text{Lifting}\end{pmatrix} = \begin{pmatrix}\text{Work done}\\ \text{by Applied}\\ \text{Force}\end{pmatrix} - \begin{pmatrix}\text{Increase in}\\ \text{Mechanical}\\ \text{Energy of}\\ \text{Chain}\end{pmatrix}$

$$\Rightarrow \quad -\Delta E = Py - \left(\frac{m}{l}y\right)g\left(\frac{y}{2}\right) - \frac{1}{2}\left(\frac{m}{l}y\right)v_0^2$$

$$\Rightarrow \quad -\Delta E = \frac{my}{l}\left(gy + v_0^2\right) - \frac{mgy^2}{2l} - \frac{myv_0^2}{2l}$$

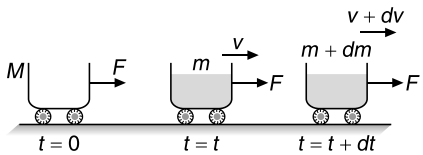
$$\Rightarrow \quad -\Delta E = \frac{1}{2}\left(\frac{mgy^2}{l} + \frac{myv_0^2}{l}\right) = \frac{my}{2l}\left(gy + v_0^2\right)$$

ILLUSTRATION 64

A tank car of mass M is at rest on a road. At $t = 0$, a force F starts acting on the tank car and also the rain fall starts, in vertical direction, as shown in figure. The rain is falling with a velocity v_r with respect to earth and the rate of collection of water in the tank is r kgs^{-1}. Find the velocity of the tank-car as a function of time t.

SOLUTION

The situation at time t and also at time $t + dt$ is shown in Figure.



At time t let mass of the car be m and its velocity be v. In the duration dt, a further mass dm is added and in this duration car gains a speed $v + dv$.

By Impulse Momentum Theorem, we have

$$Fdt = (m + dm)(v + dv) - mv$$

$$\Rightarrow \quad Fdt = vdm + mdv$$

where, $m = M + rt$ and $dm = rdt$

$$\Rightarrow \quad Fdt = v(rdt) + (M + rt)dv$$

$$\Rightarrow \quad (M + rt)dv = (F - rv)dt$$

$$\Rightarrow \quad \frac{dv}{F - rv} = \frac{dt}{M + rt}$$

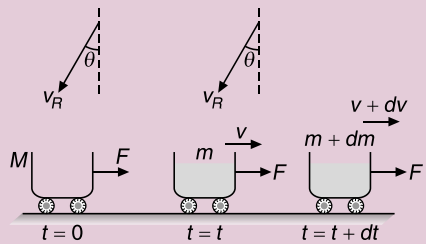
Integrating, we get

$$\int_0^v \frac{dv}{F - rv} = \int_0^t \frac{dt}{M + rt}$$

$$\Rightarrow \quad \ln\left(\frac{F}{F - rv}\right) = \ln\left(\frac{M + rt}{M}\right)$$

$$\Rightarrow \quad v = \frac{Frt}{Mr + r^2t}$$

Dear Students, this example can be modified if we consider rain to be falling at an angle θ with the vertical in the direction against the velocity of the car as shown in figure.



So, in this case we will add the momentum imparted by the rain to the car in the opposite direction in time dt. So, we get

$$Fdt = (m + dm)(v + dv) - \left[mv - (v_R \sin\theta)dm\right]$$

Now we can rearrange the terms of dv and dt according to the conditions given in the problem and then integrate it to get the velocity of car as a function of time.

ILLUSTRATION 65

A particle of mass M_0 initially at rest starts moving under action of a constant force $F\hat{i}$. It encounters the resistance of a stream of fine dust moving with velocity $-v_0\hat{i}$ which deposits on it at a constant rate k. Calculate the distance travelled by the particle when its mass becomes M.

SOLUTION

Applying Impulse Momentum Theorem to the particle, we get

$$Fdt = (m + dm)(v + dv) - (M_0v - v_0kdt)$$

$$\Rightarrow \quad (F - kv_0)dt = kvdt + (M_0 + kt)dv$$

$$\Rightarrow \quad (F - kv_0 - kv)dt = (M_0 + kt)dv$$

$$\Rightarrow \quad \int_0^v \frac{dv}{F - kv_0 - kv} = \int_0^t \frac{dt}{M_0 + kt}$$

$$\Rightarrow \quad \ln\left(\frac{F-kv_0-kv}{F-kv_0}\right)=\ln\left(\frac{M_0}{M_0+kt}\right)$$

$$\Rightarrow \quad \frac{F-kv_0-kv}{F-kv_0}=\frac{M_0}{M_0+kt}$$

$$\Rightarrow \quad F-kv_0-kv=\frac{(F-kv_0)M_0}{M_0+kt}$$

Since, $M=M_0+kt$ …(1)

$$\Rightarrow \quad kv=(F-kv_0)-\frac{(F-kv_0)M_0}{M}$$

$$\Rightarrow \quad v=\frac{1}{k}(F-kv_0)\left(1-\frac{M_0}{M}\right)$$

$$\Rightarrow \quad \frac{dx}{dt}=\left(\frac{F-kv_0}{k}\right)\left(1-\frac{M_0}{M}\right) \qquad \ldots(2)$$

From equation (1), we have $M=M_0+kt$

$$\Rightarrow \quad dM=kdt$$

$$\Rightarrow \quad dt=\frac{dM}{k}$$

So, equation (2) becomes

$$\frac{kdx}{dM}=\left(\frac{F-kv_0}{k}\right)\left(1-\frac{M_0}{M}\right)$$

$$\Rightarrow \quad dx=\left(\frac{F-kv_0}{k^2}\right)\left(1-\frac{M_0}{M}\right)dM$$

Integrating, we get

$$l=\int_0^l dx=\left(\frac{F-kv_0}{k^2}\right)\int_{M_0}^{M}\left(1-\frac{M_0}{M}\right)dM$$

$$\Rightarrow \quad l=\frac{F-kv}{k^2}\left[(M-M_0)-M_0\ln\left(\frac{M}{M_0}\right)\right]$$

$$\Rightarrow \quad l=\frac{F-kv}{k^2}\left[M-M_0\left(1+\ln\left(\frac{M}{M_0}\right)\right)\right]$$

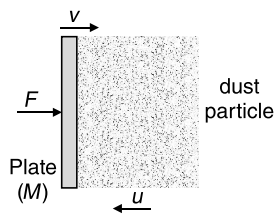
Test Your Concepts-V

Based on Variable Mass Systems

(Solutions on page H.83)

1. A cart loaded with sand moves along a horizontal floor due to a constant force F coinciding in direction with the cart's velocity. However, in this process the sand spills through a hole in the bottom with a constant rate μ kgs^{-1}. Find the acceleration and velocity of the cart at the moment t, if at the initial moment $t=0$ the cart with loaded sand had the mass m_0 and its velocity was equal to zero. Friction is to be neglected.

2. A plate of mass M is moved with constant velocity v against dust particles moving with velocity u in opposite direction as shown. The density of the dust is ρ kgm^{-3} and plate area is A. Calculate the force F required to keep the plate moving uniformly.



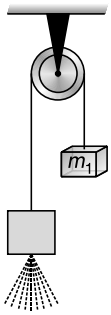
3. **(a)** A rocket set for vertical firing weighs 50 kg and contains 450 kg of fuel. It can have a maximum exhaust velocity of 2 kms^{-1}. What should be its minimum rate of fuel consumption

(i) to just lift it off the launching pad.

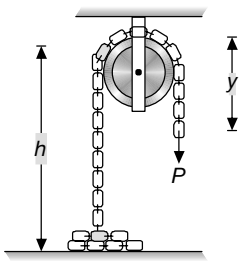
(ii) to give it an acceleration of 20 ms^{-2}.

(b) What will be the speed of the rocket when the rate of consumption of fuel is 10 kgs^{-1} after the entire fuel is consumed? Take $g=10$ ms^{-2}.

4. A mass m_1 is connected by a very light cable passing over a frictionless pulley to a container of water, whose mass is m_0 at $t=0$. If the container ejects water in downward direction at a constant rate of b kgs^{-1} with a velocity v_0 relative to the container, then calculate the acceleration of m_1 as a function of time.



5. Determine the force P required to give the open link chain a constant velocity $v = \frac{dy}{dt}$. The chain has a total length L and mass per unit length λ. Neglect the small size and mass of the pulley and any friction in the pulley.

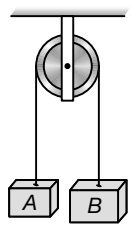


6. A rocket with initial mass M is launched by emitting gas with velocity v_0 relative to the rocket downwards. The mass of the gas emitted per second is k, where k is a constant such that $kv_0 < Mg$. Calculate the time when rocket starts to lift.

7. Find the mass of the rocket as a function of time, if it moves with a constant acceleration a, in absence of external forces. The gas escapes with a constant velocity u relative to the rocket and its mass initially was m_0.

8. A cart loaded with sand moves along a horizontal floor due to a constant force F directed along the cart's velocity. During this process, sand spills through a hole in the bottom at a constant rate of μ kgs^{-1}. Calculate acceleration and velocity of the cart at time t, if initially the cart loaded with sand had mass m_0 and zero velocity. Neglect friction.

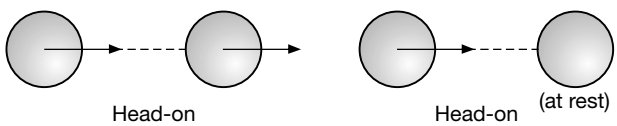
9. Two blocks A and B each masses of mass M_0 are hanging with the help of a light string, which passes over a massless pulley. Mass of A starts leaking out at a rate of μ kgs^{-1} with velocity v_0 with respect to the mass A. Find the velocity of block B as a function of time.



TYPES OF COLLISIONS

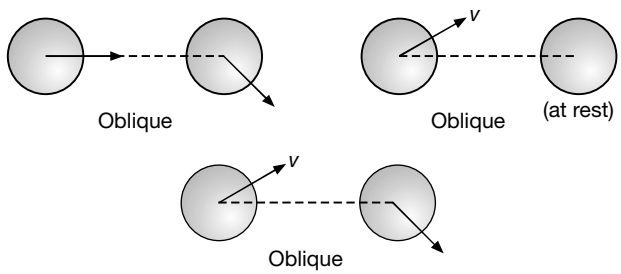
Head on Collision

Whenever the colliding bodies have their velocities along the line joining their centres, then the collision is said to be head-on.



Oblique Collision

Whenever the colliding bodies do not have their velocities along the line joining their centres, then the collision is said to be oblique.



WORD OF ADVICE

(a) These diagrams are drawn **just before** or **just after** the collision.

(b) Based on energy, we divide collisions into three categories

(i) Elastic collision: In such type of collisions, momentum and kinetic energy, both are conserved.

(ii) Inelastic collision: In such type of collisions, momentum is conserved. However, kinetic energy is not conserved $(k_i < k_i)$.

(iii) Super-elastic collision: In such types of collisions, momentum is conserved. However, the final kinetic energy is greater than initial kinetic energy $(k_f > k_i)$. As an example, a bomb initially at rest $(KE = 0)$ on exploding, has its different fragments moving in different directions with some finite kinetic energy such that total final momentum of the fragments is also zero (as a system). However, final kinetic energy is greater than initial kinetic energy (which was zero).

COEFFICIENT OF RESTITUTION (*e*)

The coefficient of restitution is only defined for elastic and inelastic collisions. It is the measure of the degree of elasticity of the collision and is defined as the ratio of the impulse due to reformation and impulse due to deformation of either body.

$$e = \frac{\text{Impulse due to Reformation}}{\text{Impulse due to Deformation}} = \frac{\int F_r dt}{\int F_d dt}$$

It can also be defined as the ratio of the relative velocity of separation just after the collision to the relative velocity of approach just before collision.

$$e = \frac{\left(\begin{array}{c}\text{Relative velocity of separation}\\ \text{just after collision}\end{array}\right)}{\left(\begin{array}{c}\text{Relative velocity of approach}\\ \text{just after collision}\end{array}\right)}$$

$$\Rightarrow \quad e = \left(\frac{v_2 - v_1}{u_1 - u_2}\right)$$

$$\Rightarrow \quad e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = -\left(\frac{v_1 - v_2}{u_1 - u_2}\right)$$

In general, we have, $0 \le e \le 1$

Conceptual Note(s)

(a) For elastic (or perfectly elastic) collisions, $e = 1$.

(b) For inelastic collisions, $0 \le e < 1$.

(c) For perfectly inelastic collisions, $e = 0$.
In this collision, bodies move independently before collision, but after collision stick to each other and move as are single body.
$\Rightarrow \left([e] = M^0L^0T^0\right)$

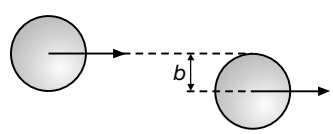
WORD OF ADVICE

The definition of '*e*' is applicable only for elastic and inelastic collisions.
However, if we could have applied this definition of '*e*' to a super elastic collision, then $e > 1$.
So, can '*e*' be greater than 1?
If the definition of '*e*' can be applied to a super elastic collision, then '*e*' can be greater than 1.

IMPACT PARAMETER (*b*)

It is the perpendicular distance between the velocities of the two colliding bodies.

$$[b] = M^0L^1T^0$$



For, $b = 0$, then collision is head-on and for

$0 < b < (r_1 + r_2)$, the collision is oblique

TYPES OF COLLISIONS

(a) Elastic Head-On: $e = 1$ and $b = 0$

(b) Inelastic Head-On: $0 < e < 1$ and $b = 0$

(c) Perfectly Inelastic Head-On: $e = 0$ and $b = 0$

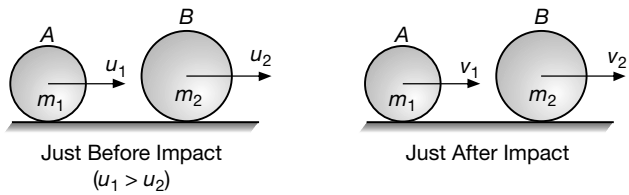
(d) Inelastic Oblique: $0 < e < 1$ and $0 < b < (r_1 + r_2)$

(e) Elastic Oblique: $e = 1$ and $0 < b < (r_1 + r_2)$

(f) Perfectly Inelastic: $e = 0$ and $0 < b < (r_1 + r_2)$

(g) Super-Elastic Collisions: $(K_{\text{final}} > K_{\text{initial}})$

HEAD-ON ELASTIC COLLISION



By Law of Conservation of Linear Momentum, we have

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \ldots(1)$$

($\vec{u}_1$, $\vec{u}_2$, $\vec{v}_1$ and $\vec{v}_2$ all are in same direction)

By Law of Conservation of Kinetic Energy, we have

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \ldots(2)$$

From (1), $m_1(u_1 - v_1) = m_2(v_2 - u_2)$ …(3)

From (2), $m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$ …(4)

Divide (4) by (3), we get

$$u_1 + v_1 = u_2 + v_2 \quad \ldots(5)$$

Rearranging (5), we get

$$u_1 - u_2 = v_2 - v_1 \quad \ldots(6)$$

$$\begin{pmatrix}\text{Relative velocity}\\ \text{of approach just}\\ \text{before collision}\end{pmatrix} = \begin{pmatrix}\text{Relative velocity}\\ \text{of approach just}\\ \text{after collision}\end{pmatrix}$$

From (6) and by definition of e, we get

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) = 1$$

From (5), we get

$$v_2 = u_1 + v_1 - u_2 \quad \ldots(7)$$

Put (7) in (1), we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(u_1 + v_1 - u_2)$$

Rearranging, we get

$$(m_1 + m_2)v_1 = (m_1 - m_2)u_1 + 2m_2u_2$$

$$\Rightarrow \quad v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

Simply replace subscript 2 by 1 and 1 by 2,

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

SPECIAL CASES

CASE-1: Colliding Bodies of equal masses

When colliding bodies are of equal masses, then

$m_1 = m_2 = m$, then $v_1 = u_2$ and $v_2 = u_1$

$m_1 = m_2$	Velocity Just		Momentum Just		Kinetic Energy Just	
	Before Impact	After Impact	Before Impact	After Impact	Before Impact	After Impact
Ball 1	u_1	u_2	mu_1	mu_2	$\frac{1}{2}mu_1^2$	$\frac{1}{2}mu_2^2$
Ball 2	u_2	u_1	mu_2	mu_1	$\frac{1}{2}mu_2^2$	$\frac{1}{2}mu_1^2$
Conclusion	Velocity Exchange		Momentum Exchange		Kinetic Energy Exchange	

i.e. for a perfectly elastic head on collision, the bodies exchange their velocity momentum and kinetic energy just after impact.

CASE-2: Target body at rest

If $u_2 = 0$, i.e. second body at rest, then

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 \text{ and } v_2 = \left(\frac{2m_1}{m_2 + m_2}\right)v_2$$

CASE-3: Colliding Bodies of equal masses and target body at rest

If $m_1 = m_2 = m$ (say) and $u_2 = 0$

i.e., a body collides with an identical body at rest, then, we get

$$v_1 = 0 \text{ and } v_2 = u_1$$

$m_1 = m_2$ and $u_2 = 0$	Velocity Just		Momentum Just		Kinetic Energy Just	
	Before Impact	After Impact	Before Impact	After Impact	Before Impact	After Impact
Ball 1	u_1	0	mu_1	0	$\frac{1}{2}mu_1^2$	0
Ball 2	0	u_1	0	mu_1	0	$\frac{1}{2}mu_1^2$
Conclusion	100% Transfer of Velocity		100% Transfer of Momentum		100% Transfer of Kinetic Energy	

So, we observe that in an elastic head-on collision if one body is at rest and the two bodies have equal mass then after collision hundred percent (100%)

transfer of velocity/momentum/kinetic energy takes place.

CASE-4: Massive body collides with a light body at rest

When a massive body collides with a light body at rest i.e., $m_1 \gg m_2$ and $u_2 = 0$

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$ and $v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$

So, for $m_1 \gg m_2$, we have $m_1 - m_2 \approx m_1$

and $m_1 + m_2 \approx m_1$

$\Rightarrow \quad v_1 \approx u_1$ and $v_2 \approx 2u_1$

So, when a massive body collides with a light body at rest (elastic head-on) then, massive body continues to move with the same velocity and the light body moves with twice the velocity of massive body.

CASE-5: Light body collides with the massive body at rest

When a light body collides with the massive body at rest, then $m_2 \gg m_1$ and $u_2 = 0$

Since $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$ and $v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$

So, for $m_2 \gg m_1$, we have

$m_1 - m_2 \approx -m_2$ and $m_1 + m_2 \approx m_2$

$\Rightarrow \quad v_1 \approx -u_1$ (rebound) and $v_2 \approx 2\left(\frac{m_1}{m_2}\right)v_1$

Since $m_1 \ll m_2$, i.e. $\frac{m_1}{m_2}$ is negligible

$\Rightarrow \quad v_2 \to 0$

So, when a light body collides with massive body at rest, then the light body rebounds and massive body continues to be at rest.

CASE-6: Massive body collides with a light body

When massive body collides with a light body, then $m_1 \gg m_2$ and $u_2 \neq 0$. Since, we have

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

For $m_1 \gg m_2$, $m_1 - m_2 \approx m_1$ and $m_1 + m_2 \approx m_1$

$\Rightarrow \quad v_1 \approx 2u_2 - u_1$ and $v_2 \approx u_2$

ILLUSTRATION 66

In an elastic head-on collision, if the second body is at rest, then calculate

(a) fraction of kinetic energy lost by m_1
(b) fraction of kinetic energy gained by m_2
(c) fraction of kinetic energy retained by m_1
(d) the condition for which the fraction of kinetic energy lost by m_1 is the maximum.

SOLUTION

$$u_2 = 0, v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 \text{ and } v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$

(a) Fraction of K.E. lost by m_1 is

$$f_{\text{lost by } m_1} = \frac{(k_1)_i - (k_1)_f}{(k_r)_i} = \frac{\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2}{\frac{1}{2}m_1u_1^2}$$

$$\Rightarrow \quad f_{\text{lost by } m_1} = 1 - \left(\frac{v_1}{u_1}\right)^2$$

$$\Rightarrow \quad f_{\text{lost by } m_1} = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

$$\Rightarrow \quad f_{\text{lost by } m_1} = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

(b) Since the collision is ideal,
So, fraction of K.E. gained by m_2 is equal to the fraction of K.E. lost by m_1
Hence,

$$f_{\text{gained by } m_2} = f_{\text{lost of } m_1} = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

(c) Fraction retained by m_1 is $(1 - f_{\text{lost of } m_1})$

$$\Rightarrow \quad f_{\text{retained by } m_1} = 1 - \frac{4m_1m_2}{(m_1 + m_2)^2} = \frac{(m_1 - m_2)^2}{(m_1 + m_2)^2}$$

(d) $f_{\text{lost by } m_1} = \text{MAX} = 1$

$$\Rightarrow \quad \frac{4m_1m_2}{(m_1 + m_2)^2} = 1$$

$$\Rightarrow \quad (m_1 - m_2)^2 = 0$$

$$\Rightarrow \quad m_1 = m_2$$

ILLUSTRATION 67

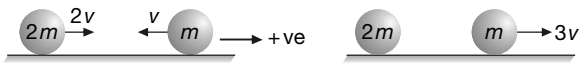
Two particles of mass m and $2m$ moving in opposite directions collide elastically with velocities v and $2v$. Find their velocities after collision.

SOLUTION

Since, $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$

and $v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$

Here, $v_1 = -v$, $v_2 = 2v$, $m_1 = m$ and $m_2 = 2m$



Substituting these values, we get

$$v_1' = \left(\frac{m - 2m}{m + 2m}\right)(-v) + \left(\frac{4m}{m + 2m}\right)(2v)$$

$$\Rightarrow \quad v_1' = \frac{v}{3} + \frac{8v}{3} = 3v$$

and $v_2' = \left(\frac{2m - m}{m + 2m}\right)(2v) + \left(\frac{2m}{m + 2m}\right)(-v)$

$$\Rightarrow \quad v_2' = \frac{2}{3}v - \frac{2}{3}v = 0$$

i.e., the second particle (of mass $2m$) comes to a rest while the first (of mass m) moves with velocity $3v$ in the direction shown in figure.

ILLUSTRATION 68

Two pendulum bobs of mass m and $2m$ collide elastically at the lowest point in their motion. If both the balls are released from a height H above the lowest point, to what heights do they rise for the first time after collision?

SOLUTION

Given, $m_1 = m$, $m_2 = 2m$, $v_1 = -\sqrt{2gH}$ and $v_2 = \sqrt{2gH}$

Since, the collision is elastic.

$$\Rightarrow \quad v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

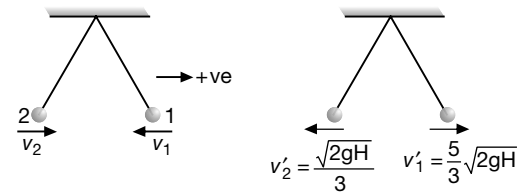
$$v_1' = \left(\frac{m - 2m}{m + 2m}\right)\left(-\sqrt{2gH}\right) + \left(\frac{4m}{m + 2m}\right)\sqrt{2gH}$$

$$v_1' = \frac{\sqrt{2gH}}{3} + \frac{4\sqrt{2gH}}{3} = \frac{5}{3}\sqrt{2gH}$$

and $v_2' = \left(\frac{2m - m}{m + 2m}\right)\left(\sqrt{2gH}\right) + \left(\frac{2m}{m + 2m}\right)\left(-\sqrt{2gH}\right)$

$$v_2' = \frac{\sqrt{2gH}}{3} - \frac{2\sqrt{2gH}}{3} = -\frac{\sqrt{2gH}}{3}$$

i.e., the velocities of the balls after the collision are as shown in figure.



Therefore, the heights to which the balls rise after the collision are

$$h_1 = \frac{(v_1')^2}{2g} \qquad \{\text{using } v^2 = u^2 - 2gh\}$$

$$\Rightarrow \quad h_1 = \frac{\left(\frac{5}{3}\sqrt{2gH}\right)^2}{2g}$$

$$\Rightarrow \quad h_1 = \frac{25}{9}H \text{ and } h_2 = \frac{(v_2')^2}{2g}$$

$$\Rightarrow \quad h_2 = \frac{\left(\frac{\sqrt{2gH}}{3}\right)^2}{2g} = \frac{H}{9}$$

Since the collision is elastic, mechanical energy of both the balls will remain conserved, or

$$E_i = E_f$$

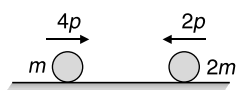
$$\Rightarrow \quad (m + 2m)gH = mgh_1 + 2mgh_2$$

$$\Rightarrow \quad 3mgH = (mg)\left(\frac{25}{9}H\right) + (2mg)\left(\frac{H}{9}\right)$$

$$\Rightarrow \quad 3mgH = 3mgH$$

ILLUSTRATION 69

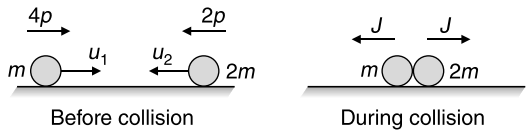
Two balls of masses m and $2m$ having momenta $4p$ and $2p$ respectively collide as shown in Figure.



During collision, the value of linear impulse between them is J. Calculate the coefficient of restitution e in terms of J and p. Find the condition for which collision is elastic and the condition for which it is perfectly inelastic.

SOLUTION

Before the collision, the directions of linear momenta and during the collision, the directions of linear impulses on the two colliding bodies are shown in Figure.



Linear impulse is equal to the change in linear momentum, so for ball m, we have

$$-J = p_f - p_i = p_f - 4p$$

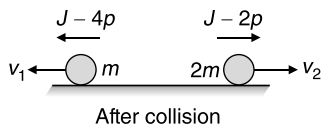
$$\Rightarrow \quad p_f = -(J - 4p), \text{ i.e. leftwards}$$

for ball $2m$, we have

$$J = p_f - (-2p) = p_f + 2p$$

$$\Rightarrow \quad p_f = J - 2p, \text{ i.e. rightwards}$$

Hence, momenta of the balls after collision are shown in Figure.



Let u_1, u_2 be the velocities of balls before collision and v_1, v_2 be the velocities of balls after collision.

Since, $v = \dfrac{\text{linear momentum}}{\text{mass}} = \dfrac{p}{m}$

Also, by the definition of the coefficient of restitution, we have

$$e = \frac{\text{Relative Velocity of Separation}}{\text{Relative Velocity of Aapproach}}$$

$$\Rightarrow \quad e = \frac{v_1 + v_2}{u_1 + u_2} = \frac{\dfrac{J - 4p}{m} + \dfrac{J - 2p}{2m}}{\dfrac{4p}{m} + \dfrac{2p}{2m}}$$

$$\Rightarrow \quad e = \frac{3J - 10p}{10p} = \frac{3J}{10p} - 1$$

For elastic collision, $e = 1$

$$\Rightarrow \quad \frac{3J}{10p} = 2$$

$$\Rightarrow \quad J = \frac{20}{3}p$$

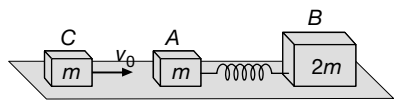
For perfectly inelastic collision, $e = 0$

$$\Rightarrow \quad \frac{3J}{10p} = 1$$

$$\Rightarrow \quad J = \frac{10}{3}p$$

ILLUSTRATION 70

Two blocks A and B of masses m and $2m$ respectively are placed on a smooth floor. They are connected by a spring. A third block C of mass m moves with a velocity v_0 along the line joining A and B and collides elastically with A, as shown in figure. At a certain instant of time t_0 after collision, it is found that the instantaneous velocities of A and B are the same. Further, at this instant the compression of the spring is found to be x_0. Determine (i) the common velocity of A and B at time t_0 and (ii) the spring constant.



SOLUTION

Initially, the blocks A and B are at rest and C is moving with velocity v_0 to the right.

As masses of C and A are same and the collision is elastic the body C transfers its whole momentum mv_0 to body A and as a result the body C stops and A starts moving with velocity v_0 to the right. At this instant the spring is uncompressed and the body B is still at rest.

The momentum of the system at this instant = mv_0

Now, the spring is compressed and the body B comes in motion. After time t_0, the compression of the spring is x_0 and common velocity of A and B is v (say).

As external force on the system is zero, the Law of Conservation of Linear Momentum gives

$$mv_0 = mv + (2m)v$$

$$\Rightarrow \quad v = \frac{v_0}{3}$$

The Law of Conservation of Energy gives

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}(2m)v^2 + \frac{1}{2}kx_0^2$$

$$\Rightarrow \quad \frac{1}{2}mv_0^2 = \frac{3}{2}mv^2 + \frac{1}{2}kx_0^2$$

$$\frac{1}{2}mv_0^2 = \frac{3}{2}m\left(\frac{v_0}{3}\right)^2 + \frac{1}{2}kx_0^2$$

$$\Rightarrow \quad \frac{1}{2}kx_0^2 = \frac{1}{2}mv_0^2 - \frac{1}{6}mv_0^2$$

$$\Rightarrow \quad \frac{1}{2}kx_0^2 = \frac{1}{3}mv_0^2$$

$$\Rightarrow \quad k = \frac{2}{3}\frac{mv_0^2}{x_0^2}$$

INELASTIC HEAD-ON

$$m_1u_2 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \ldots(1)$$

Since collision is inelastic, so we use

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right)$$

$$v_2 - v_1 = eu_1 - eu_2$$

$$v_2 = eu_1 - eu_2 + v_1 \qquad \ldots(2)$$

Putting (2) in (1), we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2\left(ev_1 - ev_2 + v_1\right)$$

$$m_1v_1 + m_2v_2 = m_1v_1 + m_2ev_1 - m_2ev_2 + m_2v_1$$

$$m_1u_1 + m_2u_2 + m_2ev_2 - m_2ev_1 = m_1v_1 + m_2v_1$$

$$v_1 = \frac{u_1\left(m_1 - m_2e\right)}{m_1 + m_2} + \left[\frac{(1+e)m_2}{m_1 + m_2}\right]v_2$$

$$v_2 = \left[\frac{(1+e)m_1}{m_1 + m_2}\right]u_1 + \left[\frac{m_2 - em_1}{m_1 + m_2}\right]u_2$$

CHECK POINT

Put e = 1 to get the results matching with those of perfectly elastic collision.

$$(\text{Loss in KE}) = (\text{Total Initial KE}) - (\text{Total Final KE})$$

$$\text{Loss} = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

$$\text{Loss} = \frac{1}{2}\underbrace{\left(\frac{m_1m_2}{m_1 + m_2}\right)}_{\substack{\text{Reduced Mass}\\(\text{discussed in CM})}}\left(1 - e^2\right)\left(u_1 - u_2\right)^2$$

CHECK POINT

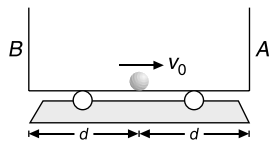
If $e = 1$, we observe

$$\text{Loss} = 0$$

which is the case of a perfectly elastic collision.

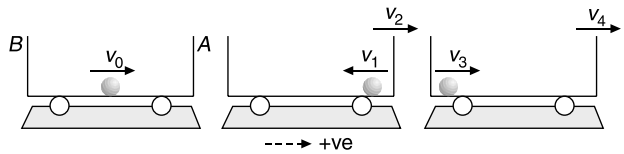
ILLUSTRATION 71

A ball of negligible size and mass m is given a velocity v_0 on the centre of the cart which has a mass M and is originally at rest. If the coefficient of restitution between the ball and walls A and B is e, determine the velocity of the ball and the cart just after the ball strikes A. Also, determine the total time needed for the ball to strike A, rebound, then strike B and rebound and then return to the centre of the cart. Neglect friction.



SOLUTION

Applying conservation of linear momentum and



$$e = \frac{\text{relative speed of separation}}{\text{relative speed of approach}}, \text{ we get}$$

$$\Rightarrow \quad mv_0 = Mv_2 - mv_1 \qquad \ldots(1)$$

$$\Rightarrow \quad v_1 + v_2 = ev_0 \qquad \ldots(2)$$

Solving these two equations, we get,

$$v_1 = \left(\frac{eM - m}{M + m}\right)v_0, \; v_2 = m\left(\frac{e+1}{M+m}\right)v_0$$

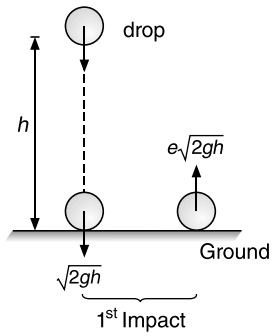
The desired time is

$$t = \frac{d}{v_0} + \frac{2d}{ev_0} + \frac{d}{e^2v_0}$$

$$\Rightarrow \quad t = \frac{d}{v_0}\left(1 + \frac{2}{e} + \frac{1}{e^2}\right)$$

BALL GROUND INELASTIC HEAD-ON COLLISION

For ball wall collision or ball ground inelastic head-on collision, we have



$$e=-\left(\frac{v_2-v_1}{u_2-u_1}\right)=-\left(\frac{0-v_1}{0-\sqrt{2gh}}\right)=-\frac{v_1}{\sqrt{2gh}}$$

$$\Rightarrow \quad v_1=-e\sqrt{2gh}$$

So, for a ball colliding inelastically head-on with the ground we observe that for the ball

$$\begin{pmatrix}\text{Speed just}\\ \text{after Collision}\end{pmatrix}_{n\text{-line}}=e\begin{pmatrix}\text{Speed just}\\ \text{before Collision}\end{pmatrix}_{n\text{-line}}$$

ILLUSTRATION 72

A ball of mass m is dropped from a height h to hit the ground and then rebound to a certain height. The process carries on and he ball continues to hit the ground time and again. The coefficient of restitution between the ground and the ball is e. Find the average speed of the ball, the total momentum imparted to the floor by the ball, the total distance travelled by the ball before coming the rest and the total momentum imparted to the floor by the ball.

SOLUTION

Velocity after 1st impact is $v_1=e\sqrt{2gh}$

Velocity just before second impact is $v_1=e\sqrt{2gh}$

Velocity just after second impact is

$$v_2=ev_1=e^2\sqrt{2gh}$$

Height after first impact is $h_1=\frac{v_1^2}{2g}=e^2h$

Total distance just before second impact is $2h_1$

Time taken between 1st and 2nd impact is

$$2t_1=2\left(\frac{v_1}{g}\right)=\frac{2e\sqrt{2gh}}{g}=2e\sqrt{\frac{2h}{g}}$$

Time taken between 2nd and 3rd impact is

$$2t_2=2\left(\frac{2v_1}{g}\right)=\frac{2e^2\sqrt{2gh}}{g}=2e^2\sqrt{\frac{2h}{g}}$$

After every impact, the ball loses some momentum which is gained by the grand. If Δp_1 is the change in momentum after first impact.

$$|\Delta p_1|=\left|m\left(-e\sqrt{2gh}-\sqrt{2gh}\right)\right|$$

$$\Rightarrow \quad |\Delta p_1|=m\sqrt{2gh}\,(1+e)$$

If Δp_2 is the change of momentum of ball after second impact, then

$$|\Delta p_2|=\left|m\left(-e^2\sqrt{2gh}-e\sqrt{2gh}\right)\right|$$

$$\Rightarrow \quad |\Delta p_2|=e\Delta p_1=em\sqrt{2gh}\,(1+e)$$

Similarly, $|\Delta p_3|=e^2m\sqrt{2gh}\,(1+e)$

Total momentum imparted by the ball to the floor is

$$\Delta p=\Delta p_1+\Delta p_2+\Delta p_3\ldots$$

$$\Rightarrow \quad \Delta p=m\sqrt{2gh}\,(1+e)(1+e+e^2+e^3+\ldots)$$

$$\Rightarrow \quad \Delta p=m\sqrt{2gh}\left(\frac{1+e}{1-e}\right)$$

Total time taken by the ball to come to rest is

$$t=\sqrt{\frac{2h}{g}}+2e\sqrt{\frac{2h}{g}}+2e^2\sqrt{\frac{2h}{g}}+2e^3\sqrt{\frac{2h}{g}}+\ldots.$$

$$\Rightarrow \quad t=\sqrt{\frac{2h}{g}}+2e\sqrt{\frac{2h}{g}}(1+e+e^2+\ldots)$$

$$\Rightarrow \quad t=\sqrt{\frac{2h}{g}}+2e\sqrt{\frac{2h}{g}}\left(\frac{1}{1-e}\right)$$

$$\Rightarrow \quad t=\sqrt{\frac{2h}{g}}\left[1+\frac{2e}{1-e}\right]=\left(\frac{1+e}{1-e}\right)\sqrt{\frac{2h}{g}}$$

Total distance travelled by the ball before coming to rest is

$$s=h+2e^2h+2e^4h+2e^6h\ldots.$$

$$\Rightarrow \quad s=h+2e^2h(1+e^2+e^4+\ldots)$$

$$\Rightarrow \quad s = h + \frac{2e^2h}{1-e^2} = h\left(1 + \frac{2e^2}{1-e^2}\right)$$

$$\Rightarrow \quad s = h\left(\frac{1+e^2}{1-e^2}\right)$$

The average speed v_{av} of the ball is given by

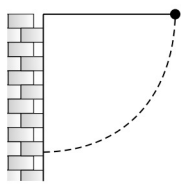
$$v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$\Rightarrow \quad v_{av} = \frac{h\left(\frac{1+e^2}{1-e^2}\right)}{\sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)}$$

$$\Rightarrow \quad v_{av} = \sqrt{\frac{gh}{2}}\left(\frac{1+e^2}{1-e^2}\right)$$

ILLUSTRATION 73

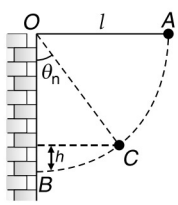
A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position (see figure) and released.



The ball hits the wall, the coefficient of restitution being $\frac{2}{\sqrt{5}}$. What is the minimum number of collisions after which the amplitude of oscillation becomes less than 60 degrees?

SOLUTION

As shown in figure initially when the bob is at A, its potential energy is mgl.



When the bob is released and it strikes the wall at B, its potential energy mgl is converted into its kinetic energy. If v be the velocity with which the bob strikes the wall, then

$$mgl = \frac{1}{2}mv^2 \text{ or } v = \sqrt{2gl} \qquad \ldots(1)$$

Speed of the bob after rebounding (first time)

$$v_1 = e\sqrt{2gl} \qquad \ldots(2)$$

The speed after second rebound is

$$v_2 = e^2\sqrt{2gl}$$

In general, after n rebounds, the speed of the bob is

$$v_n = e^n\sqrt{2gl} \qquad \ldots(3)$$

Let the bob rises to a height h after n rebounds. Applying the Law of Conservation of Energy, we have

$$\frac{1}{2}mv_n^2 = mgh$$

$$\Rightarrow \quad h = \frac{v_n^2}{2g} = \frac{(e^{2n})(2gl)}{2g} = le^{2n}$$

$$\Rightarrow \quad h = l\left(\frac{2}{\sqrt{5}}\right)^{2n} = l\left(\frac{4}{5}\right)^n \qquad \ldots(4)$$

If θ_n be the angle after n collisions, then

$$h = l - l\cos\theta_n = l(1-\cos\theta_n) \qquad \ldots(5)$$

From equations (4) and (5), we have

$$\left(\frac{4}{5}\right)^n l = l(1-\cos\theta_n)$$

$$\Rightarrow \quad \left(\frac{4}{5}\right)^n = (1-\cos\theta_n)$$

For θ_n to be less than $60°$, i.e., $\cos\theta_n$ is greater than $\frac{1}{2}$, i.e., $(1-\cos\theta_n)$ is less than $\frac{1}{2}$, we have

$$\left(\frac{4}{5}\right)^n < \left(\frac{1}{2}\right)$$

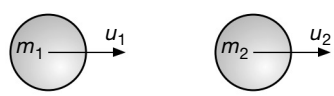
This condition is satisfied for $n = 4$

So, required number of collisions is 4.

PERFECTLY INELASTIC HEAD-ON COLLISION

CASE-1: Bodies moving in same direction

When the bodies are moving in the same direction as shown in Figure, then by conservation of linear momentum, we have



$$m_1u_1 + m_2u_2 = m_1v_{comb} + m_2v_{comb}$$

$$\Rightarrow \quad m_1u_1 + m_2u_2 = v_{comb}(m_1 + m_2)$$

$$\Rightarrow \quad v_{comb} = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

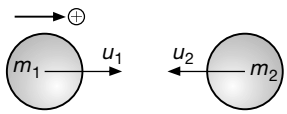
Also, loss in kinetic energy is

$$-\Delta K = K_i - K_f$$

$$\Rightarrow \quad -\Delta K = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$$

CASE-2: Bodies moving in opposite direction

When the bodies are moving in the opposite direction as shown in Figure, then by conservation of linear momentum, we have



$$m_1u_1 + m_2(-u_2) = m_1v_{comb} + m_2v_{comb}$$

$$\Rightarrow \quad m_1u_1 - m_2u_2 = v_{comb}(m_1 + m_2)$$

$$\Rightarrow \quad v_{comb} = \frac{m_1u_1 - m_2u_2}{m_1 + m_2}$$

OPTION-I	**OPTION-II**
If $v_{comb} = \oplus$, then	If $v_{comb} = \ominus$, then
v_{comb} is along u_1	v_{comb} is along u_2

Also, loss in kinetic energy is

$$-\Delta K = K_i - K_f$$

$$\Rightarrow \quad -\Delta K = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)(u_1 + u_2)^2$$

Test Your Concepts-VI

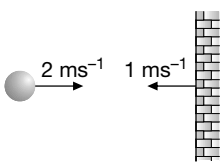
Based on Head-On Collisions

(Solutions on page H.85)

1. A ball of mass m moving at a speed u makes a head on collision with an identical ball at rest. The kinetic energy of the balls after the collision is 3/4th of the original. Find the coefficient of restitution.

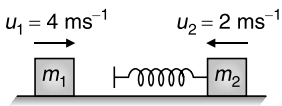
2. A ball of mass 4 kg moving with a velocity of 12 ms^{-1} impinges directly on another ball of mass 8 kg moving with velocity of 4 ms^{-1} in the same direction. Find their velocities after impact and calculate the loss of KE due to impact if $e = 0.5$.

3. A ball is moving with velocity 2 ms^{-1} towards a heavy wall moving towards the ball with speed 1 ms^{-1} as shown in figure. Assuming collision to be elastic, find the velocity of ball immediately after the collision.



4. A particle A of mass m moving on a smooth horizontal surface collides with a stationary particle B of mass $2m$ directly, situated at a distance d from a wall. The coefficient of restitution both between A and B, between B and the wall is $e = \frac{1}{4}$. Calculate the distance from the wall where they collide again. Assume that the entire motion takes place along a straight line perpendicular to the wall.

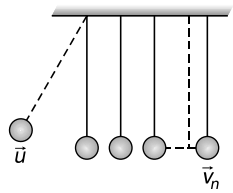
5. A block of mass $m_1 = 2$ kg initially moving to right with a speed of $u_1 = 4$ ms^{-1} on a frictionless, horizontal track, collides with a light spring attached to a second block of mass $m_2 = 1$ kg initially moving to left with a speed of $u_2 = 2$ ms^{-1} as shown in Figure.



Calculate the velocities of the blocks when spring again comes to original length.

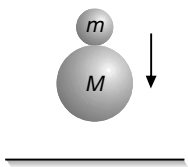
6. A small block of mass m moving with a speed v_0 collides elastically with an identical block kept at rest. Now the second block after travelling a distance d collides elastically with a heavy wall moving towards it with velocity u. The coefficient of friction between the blocks and the ground is μ. Find the maximum value of u for which no further collision takes place between the blocks.

7. n identical spheres of mass m are suspended with wires of equal length. The spheres are almost in contact with each other. Sphere 1 is pulled aside and released as shown in Figure.



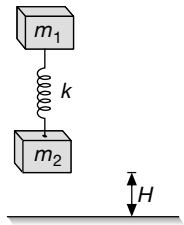
If sphere 1 strikes sphere 2 with velocity u, find an expression for velocity v_n of the nth sphere immediately after being struck by the one adjacent to it. Assume that the coefficient of restitution for each impact is e.

8. A small ball of mass m is placed on top of a super ball of mass M and the two balls are dropped to the floor from height h. How high does the small ball rise after the collision? Assume that the collisions with the superball are elastic and that $m \ll M$.



9. Three balls having masses m_1, m_2 and m_3 are lying in a straight line. The first ball moving with a certain velocity strikes the second ball directly and itself comes to rest. The second ball collides with the third and also comes to rest. If e is the coefficient of restitution for each ball, find the mass of ball m_2 in terms of m_1 and m_3.

10. A spring mass system is held at rest with the spring relaxed at a height H above the ground. Determine the minimum value of H so that the system has a tendency to rebound after hitting the ground. Given that the coefficient of restitution between m_2 and ground is zero.



11. Two particles of masses m and $2m$ moving in opposite directions collide elastically with velocity $2v$ and v, respectively. Find their velocities after collision.

12. A light spring of spring constant k is kept compressed between two blocks of masses m and M on a smooth horizontal surface. When released, the blocks acquire velocities in opposite directions. The spring loses contact with the block when it acquires natural length. If the spring was initially compressed through a distance x, find the final speeds of the two blocks.

OBLIQUE COLLISION

Physics of Oblique Collision

Before dealing with oblique collisions, we must keep in mind the following steps so that the problems can be solved with ease:

STEP-1: Draw a **common tangent line (t-line)** to the colliding surfaces at the point of impact. This line is called t-line.

STEP-2: Perpendicular to the common tangent line (t-line), draw the line of impact (also called n-line). The impact forces will be acting only along n-line (and have no component along t-line. **In other words, the impact forces are acting along the line joining the centres of the two bodies.**

STEP-3: Since impact forces have no components along t-line, hence the **individual momentum** or **individual velocity** of the particle must be conserved along t-line.

For *A*:

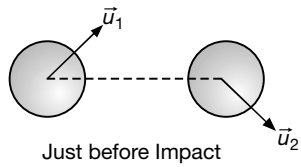
$$\left[(p_A)_i\right]_t = \left[(p_A)_f\right]_t$$

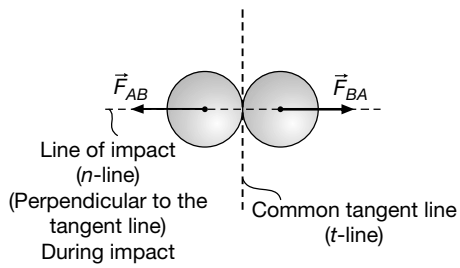
$$\Rightarrow \quad (u_1)_t = (v_1)_t$$

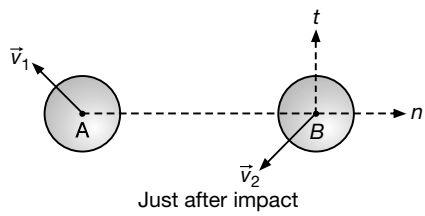
For *B*:

$$\left[(p_B)_i\right]_t = \left[(p_B)_f\right]_t$$

$$\Rightarrow \quad (u_2)_t = (v_2)_t$$







STEP-4: The **total momentum** of the system $(A+B$ system$)$ remains conserved along n-line.

For *A* (along *n*-line)

$$\Delta\vec{p}_A = \int \vec{F}_{AB}\,dt$$

$$\Rightarrow \quad m_1\left[(\vec{v}_1)_n - (\vec{u}_1)_n\right] = \int \vec{F}_{AB} \qquad \ldots(1)$$

For *B* (along *n*-line)

$$\Delta\vec{p}_B = \int \vec{F}_{BA}\,dt$$

$$\Rightarrow \quad m_2\left[(\vec{v}_2)_n - (\vec{v}_2)_n\right] = \int \vec{F}_{BA}\,dt \qquad \ldots(2)$$

Since $\vec{F}_{AB} = -\vec{F}_{BA}$ {from Newton's Third Law}

$$\Rightarrow \quad m_1(\vec{v}_1)_n - m_1(\vec{v}_1)_n = -m_2(\vec{v}_2)_n + m_2(\vec{v}_2)_n$$

$$\Rightarrow \quad m_1(\vec{v}_1)_n + m_2(\vec{v}_2)_n = m_1(\vec{v}_1)_n + m_2(\vec{v}_2)_n$$

$$\begin{pmatrix}\text{Total initial}\\ \text{momentum of}\\ A+B \text{ system}\end{pmatrix}_{\substack{\text{along}\\ n\text{-line}}} = \begin{pmatrix}\text{Total final}\\ \text{momentum of}\\ A+B \text{ system}\end{pmatrix}_{\substack{\text{along}\\ n\text{-line}}}$$

STEP-5: For elastic oblique collisions,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

STEP-6: For inelastic oblique collision (instead of STEP-5), if e be the coefficient of restitution at the point of impact, then

$$e = -\left[\frac{(v_2)_n - (v_1)_n}{(u_2)_n - (u_1)_n}\right]$$

Conceptual Note(s)

To summarise, we observe that we select two axis very smartly, the *t*-axis and the *n*-axis. Along *t*-axis (or *t*-line) no component of force acts, so individual momentum of particles is constant along *t*-axis. Along *n*-axis (or *n*-line) the net force on collective system is zero, so total momentum of system is conserved along *n*-line.

ILLUSTRATION 74

A ball of mass m hits a floor with a speed v_0 making an angle of incidence θ with the normal. The coefficient of restitution is e. Find the speed of the reflected ball and the angle of reflection ϕ of the ball.

SOLUTION

Since impact force has got no component along *t*-line, because it always acts along *n*-line, so the component of velocity v_0 along common tangent direction $v_0\sin\alpha$ will remain unchanged. Let v be the velocity of ball just after impact such that angle of reflection is ϕ with the vertical. So,

$$v\sin\phi = v_0\sin\theta \qquad \ldots(1)$$

Also, we know that

$$e = -\left[\frac{(v_2)_n - (v_1)_n}{(u_2)_n - (u_1)_n}\right]$$

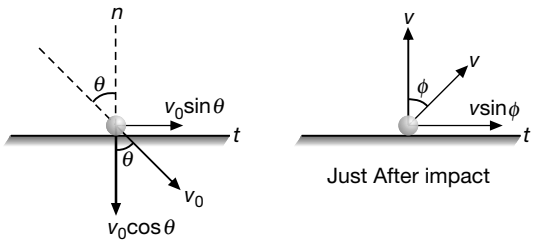
Now think of **1 as Ball** and **2 as Ground**

Since ground is at rest finally as well as initially, so $(v_2)_n = (u_2)_n = 0$

$$\Rightarrow \quad e = -\left(\frac{0 - v\cos\phi}{0 - (-v_0\cos\theta)}\right)$$

$$\Rightarrow \quad e = \frac{v\cos\phi}{v_0\cos\theta}$$

$$\Rightarrow \quad v\cos\phi = e(v_0\cos\theta) \qquad \ldots(2)$$



To calculate the speed of ball after impact, squaring (1) and (2) and adding, to get

$$v^2 = v_0^2\sin^2\theta + e^2(v_0\cos\theta)^2$$

$$\Rightarrow \quad v = v_0\sqrt{\sin^2\theta + e^2\cos^2\theta}$$

To calculate the angle of reflection, divide (1) by (2), to get

$$\tan\phi = \frac{\tan\theta}{e}$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{\tan\theta}{e}\right)$$

ILLUSTRATION 75

A sphere, of mass m, impinges obliquely on a sphere, of mass M, which is at rest. Show that, if $m = eM$, the directions of motion of the spheres after impact are at right angles.

SOLUTION

Let v_1 and v_2 be the components of velocity of m before collision, then we have

	Mass *m*		Mass *M*	
	***t*-line**	***n*-line**	***t*-line**	***n*-line**
Before collision	v_1	v_2	0	0
After collision	v_1	$v_3 = 0$	0	v_4

$$v_3 = \left(\frac{M - eM}{m + eM}\right)v_2 = 0 \qquad \{\because\ m = 2m\}$$

and $\quad v_4 = \left(\dfrac{m + eM}{m + M}\right)v_2 \neq 0$

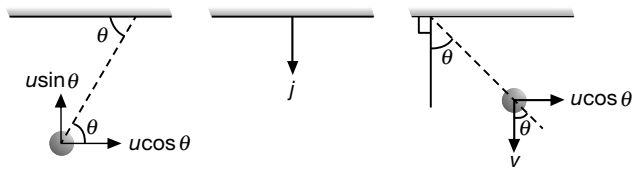
So, after collision m is along t-line while M along n-line and since t-line is perpendicular to n-line, so after impact both spheres move at right angles to each other.

ILLUSTRATION 76

The coefficient of restitution between a snooker ball and the side cushion is $\frac{1}{3}$. If the ball hits the cushion and then rebounds at right angles to its original direction, show that the angles made with the side cushion by the direction of motion before and after impact are $60°$ and $30°$ respectively.

SOLUTION

Let the original speed be u, in a direction making an angle θ with the side cushion.



Using the law of restitution

$$v = \frac{1}{3}(u\sin\theta)$$

After impact, $\tan\theta = \dfrac{u\cos\theta}{v} = \dfrac{3\cos\theta}{\sin\theta}$

$$\Rightarrow \quad \tan^2\theta = 3$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \theta = 60°$$

Therefore, the directions of motion before and after impact are at $60°$ and $30°$ to the cushion.

ILLUSTRATION 77

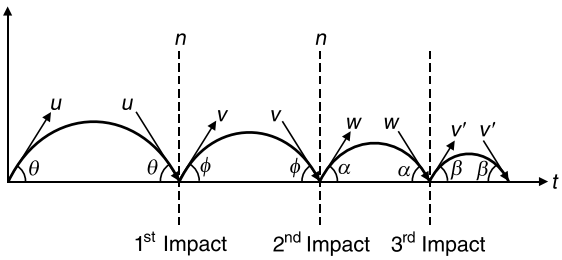
A projectile is launched with initial velocity u making angle θ with the horizontal. Assuming the coefficient of restitution between the projectile and the ground to be e, find the following:

(a) the total horizontal distance travelled before it comes to rest.

(b) the total time lapsed before it comes to rest.
(c) the average horizontal velocity during the tenure.
(d) the launch angle of the projectile just after n^{th} impact.

SOLUTION

About *t*-line, individual velocity of particle is conserved, so we have



$$v\cos\phi = u\cos\theta \quad \ldots(1)$$

$$w\cos\alpha = v\cos\phi = u\cos\theta \quad \ldots(2)$$

About *n*-line

$$\begin{pmatrix}\text{Speed After}\\ \text{Collision}\end{pmatrix}_{\substack{\text{along}\\ n\text{-line}}} = e\begin{pmatrix}\text{Speed Before}\\ \text{Collision}\end{pmatrix}_{\substack{\text{along}\\ n\text{-line}}}$$

After 1st Impact

$$v\sin\phi = e(u\sin\theta) \quad \ldots(3)$$

After 2nd Impact

$$w\sin\alpha = e(v\sin\phi) = e^2(u\sin\theta) \quad \ldots(4)$$

After 3rd Impact

$$v'\sin\beta = e(w\sin\alpha) = e^2(v\sin\phi) = e^3(u\sin\theta) \ldots(5)$$

(a) Now, $R_1 = \frac{2}{g}(u\cos\theta)(u\sin\theta) = R$

and $R_2 = \frac{2}{g}(v\cos\phi)(v\sin\phi)$

$$\Rightarrow R_2 = \frac{2}{g}(u\cos\theta)(eu\sin\theta) = eR$$

Similarly, $R_3 = e^2R$

Total horizontal distance travelled by the projectile before it comes to rest is

$$x = R + eR + e^2R + e^3R + \ldots$$

$$\Rightarrow x = R(1 + e + e^2 + \ldots)$$

$$\Rightarrow x = \frac{R}{1-e}$$

$$\Rightarrow x = \frac{2u^2\sin(2\theta)}{g(1-e)}$$

Total time lapsed is

$$T = \frac{2u\sin\theta}{g} + \frac{2v\sin\phi}{g} + \frac{2w\sin\alpha}{g} + \ldots$$

$$\Rightarrow T = \frac{2u\sin\theta}{g}(1 + e + e^2 + \ldots)$$

$$\Rightarrow T = \frac{2u\sin\theta}{g}\left(\frac{1}{1-e}\right)$$

(c) Average Horizontal Velocities

$$(v_{av})_x = \frac{\text{Total Horizontal Displacement}}{\text{Total Time Taken}}$$

$$\Rightarrow (v_{av})_x = \frac{x}{t} = \frac{\left(\frac{2u^2\sin(2\theta)}{g(1-e)}\right)}{\left(\frac{2u\sin\theta}{g(1-e)}\right)}$$

Since $\sin(2\theta) = 2\sin\theta\cos\theta$

$$\Rightarrow (v_{av})_x = u\cos\theta$$

(d) From (3) and (1), we get

$$\tan\phi = e\tan\theta$$

From (4) and (2), we get

$$\tan\alpha = e\tan\phi = e^2\tan\theta$$

So, after *n*th impact, we get

$$\tan(\theta_n) = e^n\tan\theta$$

$$\Rightarrow \theta_n = \tan^{-1}(e^n\tan\theta)$$

Conceptual Note(s)

In an oblique collision between two bodies *A* and *B* if body *B* is at rest, then after impact, the body at rest i.e., *B* will move along the *n*-line (or the line of impact). Because, for the body *B*, at rest, we have, along *t*-line

$$(v_B)_t = (u_B)_t = 0$$

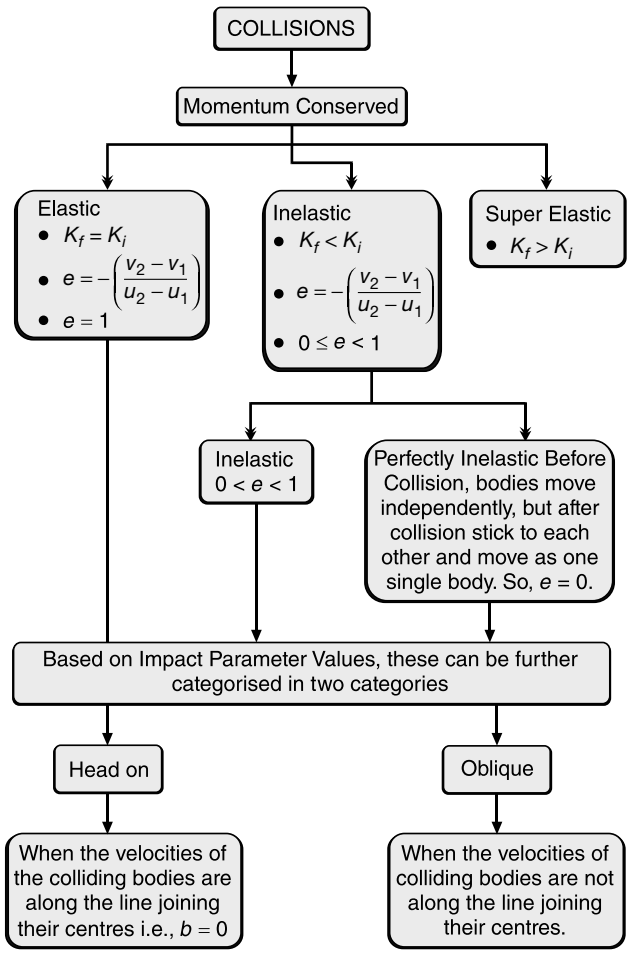
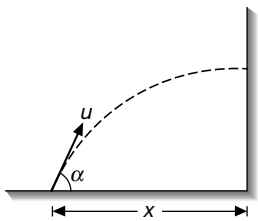

COLLISIONS
Momentum Conserved
Elastic
• $K_f = K_i$
• $e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right)$
• $e = 1$
Inelastic
• $K_f < K_i$
• $e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right)$
• $0 \le e < 1$
Super Elastic
• $K_f > K_i$
Inelastic
$0 < e < 1$
Perfectly Inelastic Before Collision, bodies move independently, but after collision stick to each other and move as one single body. So, $e = 0$.
Based on Impact Parameter Values, these can be further categorised in two categories
Head on
Oblique
When the velocities of the colliding bodies are along the line joining their centres i.e., $b = 0$
When the velocities of colliding bodies are not along the line joining their centres.

ILLUSTRATION 78

A ball is projected from a given point with velocity u at some angle with the horizontal and after hitting a vertical wall returns to the same point. If the distance of the point from the wall is x, then prove that $x < \frac{eu^2}{(1+e)g}$, where e is the coefficient of restitution.

SOLUTION

The situation discussed in the problem is shown in Figure.



Since, $\frac{x}{u\cos\alpha} + \frac{x}{eu\cos\alpha} = T = \frac{2u\sin\alpha}{g}$

$$\Rightarrow \quad x = \frac{eu^2 \sin 2\alpha}{(1+e)g}$$

At $\alpha = 45°$, $\sin(2\alpha) = 1$, so the value of x is maximum and is given by

$$x_{\max} = \frac{eu^2}{(1+e)g}$$

Hence, all other values of α will yield a value of x that happens to be less than $x_{\max} = \frac{eu^2}{(1+e)g}$.

$$\Rightarrow \quad x < x_{\max}$$

$$\Rightarrow \quad x < \frac{eu^2}{(1+e)g}$$

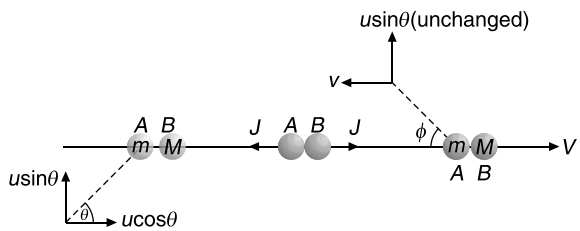
$$\Rightarrow \quad x_{\max} = \frac{eu^2}{(1+e)g} \text{ at } 2\alpha = 90°$$

ILLUSTRATION 79

Two smooth spheres A and B, of equal radius but masses m and M, are free to move on a horizontal table. A is projected with speed u towards B which is at rest. On impact, the line joining their centres is inclined at an angle θ to the velocity of A before impact. If e is the coefficient of restitution between the spheres, find the speed with which B begins to move. If A's path after impact is perpendicular to its path before impact, show that $\tan^2\theta = \frac{eM - m}{M + m}$.

SOLUTION

When B is struck by the impulse J, it begins to move in the direction of J as shown in the diagram.



Along the line of centres, we apply

(a) Conservation of Linear Momentum,
i.e., $mu\cos\theta = MV - mv$ …(1)

(b) Law of restitution,
i.e., $eu\cos\theta = V + v$ …(2)

Solving equations (1) and (2), we get

$$v = \frac{(eM - m)u\cos\theta}{(M+m)}$$

and $V = \dfrac{(1+e)mu\cos\theta}{M+m}$

Hence, $\tan\phi = \dfrac{u\sin\theta}{v} = \dfrac{(M+m)\tan\theta}{(eM-m)}$

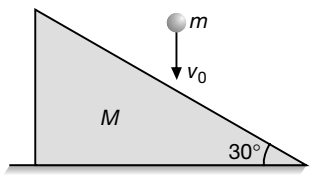
But the paths of A before and after impact are at right angles, therefore, $\cot\phi = \tan\theta$

$$\Rightarrow \quad \frac{(eM-m)}{(M+m)\tan\theta} = \tan\theta$$

$$\Rightarrow \quad \tan^2\theta = \frac{eM-m}{(M+m)}$$

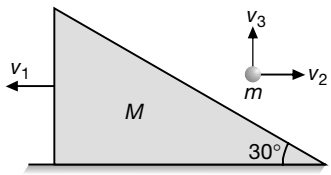
ILLUSTRATION 80

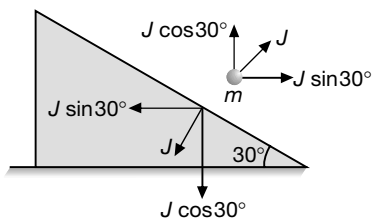
A ball of mass $m = 1\text{ kg}$ falling vertically with a velocity $v_0 = 2\text{ ms}^{-1}$ strikes a wedge of mass $M = 2\text{ kg}$ kept on a smooth, horizontal surface as shown in figure. The coefficient of restitution between the ball and the wedge is $e = \dfrac{1}{2}$. Find the velocity of the wedge and the ball immediately after collision.



SOLUTION

Let, J be the impulse between ball and wedge during collision and v_1, v_2 and v_3 be the **components of velocity** of the wedge and the ball in horizontal and vertical directions respectively as shown in Figure.





Since, Impulse = Change in Momentum

$$\Rightarrow \quad J\sin(30°) = Mv_1 = mv_2, \text{ along horizontal}$$

$$\Rightarrow \quad \frac{J}{2} = 2v_1 = v_2 \qquad \ldots(1)$$

Also, $J\cos(30°) = m(v_3 + v_0)$, along vertical

$$\Rightarrow \quad \frac{\sqrt{3}}{2}J = (v_3 + 2) \qquad \ldots(2)$$

Since $e = -\dfrac{(v_2)_n - (v_1)_n}{(u_2)_n - (u_1)_n}$

$$\begin{pmatrix}\text{Relative Velocity}\\ \text{of Separation just}\\ \text{after Impact}\end{pmatrix}_{n\text{ line}} = e\begin{pmatrix}\text{Relative Velocity}\\ \text{of Approach just}\\ \text{before Impact}\end{pmatrix}_{n\text{ line}}$$

$$\Rightarrow \quad (v_1 + v_2)\sin(30°) + v_3\cos(30°) = \frac{1}{2}(v_0\cos(30°))$$

$$\Rightarrow \quad v_1 + v_2 + \sqrt{3}v_3 = \sqrt{3} \qquad \ldots(3)$$

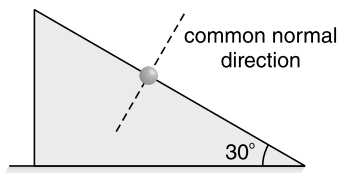
Solving equations (1), (2) and (3), we get

$$v_1 = \frac{1}{\sqrt{3}}\text{ ms}^{-1},\ v_2 = \frac{2}{\sqrt{3}}\text{ ms}^{-1} \text{ and } v_3 = 0$$

Thus, velocities of wedge and ball are

$$v_1 = \frac{1}{\sqrt{3}}\text{ ms}^{-1} \text{ and } v_2 = \frac{2}{\sqrt{3}}\text{ ms}^{-1} \text{ in horizontal}$$

direction as shown in Figure.



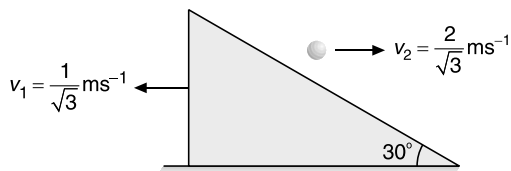


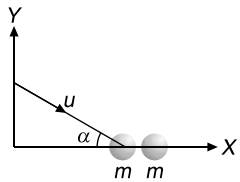
ILLUSTRATION 81

A ball moving translationally collides elastically with another stationary ball of the same mass. At the moment of impact the angle between the straight line passing through the centres of the balls and the direction of the initial motion of the striking ball is equal to $\alpha = 45°$. Assuming the balls to be smooth, find the fraction η of the kinetic energy of the striking

ball that turned into potential energy at the moment of the maximum deformation.

SOLUTION

Let a ball of mass m with velocity u collides with stationary ball as shown in figure.



Applying the Law of Conservation of Momentum along x-axis, we have

$$mu\cos\alpha = mv_{1x} + mv_{2x}$$

At the moment of maximum deformation

$$v_{1x} = v_{2x}$$

$$\Rightarrow \quad u\cos\alpha = v_{1x} + v_{1x} = 2v_{1x}$$

or $$v_{1x} = \frac{u\cos\alpha}{2}$$

Initial kinetic energy, $K_{\text{initial}} = \frac{1}{2}mu^2$

Final kinetic energy, $K_{\text{final}} = \frac{1}{2}mv_{1x}^2 + \frac{1}{2}mv_{2x}^2 + \frac{1}{2}mv_{1y}^2$

$$\Rightarrow \quad K_{\text{final}} = 2\left(\frac{1}{2}mv_{1x}^2\right) + \frac{1}{2}mv_{1y}^2$$

$$\Rightarrow \quad K_{\text{final}} = \frac{mu^2\cos^2\alpha}{4} + \frac{mu^2\sin^2\alpha}{2}$$

$$\Rightarrow \quad K_{\text{final}} = \frac{mu^2}{4}\cos^2 45^\circ + \frac{mu^2}{2}\sin^2 45^\circ$$

$$\Rightarrow \quad K_{\text{final}} = \frac{mu^2}{8} + \frac{mu^2}{4} = \frac{3mu^2}{8}$$

Therefore, potential energy at the moment of maximum deformation is

$$U = \frac{mu^2}{2} - \frac{3mu^2}{8} = \frac{mu^2}{8}$$

So, the desired fraction η is given by

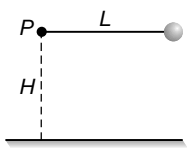
$$\eta = \frac{mu^2/8}{mu^2/2} = \frac{1}{4} = 0.25$$

Test Your Concepts-VII

Based on Oblique Collisions

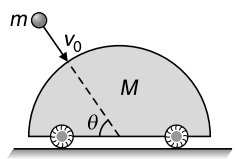
(Solutions on page H.88)

1. A point P is fixed at a height H above a perfectly inelastic smooth horizontal plane. A light inextensible string of length $L(>H)$ has one end attached to P and other is attached to a heavy particle. The particle is held at the level of P with string just taut and released from rest. Find the height of the particle above the plane when it is next instantaneously at rest.



2. A smooth sphere of mass m is moving on a horizontal plane with a velocity $3\hat{i} + \hat{j}$ when it collides with a vertical wall which is parallel to the vector $\hat{j}$. If the coefficient of restitution between the sphere and the wall is $\frac{1}{2}$, find the velocity of the sphere after impact, the loss in kinetic energy caused by the impact and the impulse $\vec{J}$ that acts on the sphere.

3. A putty ball of mass m strikes a smooth stationary wedge of mass M with a velocity v_0 making at an angle θ with the horizontal as shown in Figure.

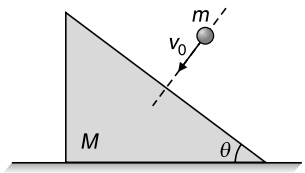


If the collision is perfectly inelastic, calculate the velocity of the wedge just after collision and the loss in kinetic energy of the system.

4. A sphere of mass m is moving with a velocity $4\hat{i} - \hat{j}$ when it hits a wall and rebounds with velocity $\hat{i} + 3\hat{j}$. Calculate the impulse it receives. Also calculate the coefficient of restitution between the sphere and the wall.

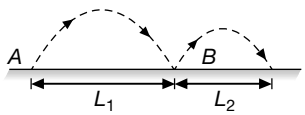
5. A ball of mass m, moving with a velocity u along x-axis, strikes another ball of mass $2m$ kept at rest. The first ball comes to rest after collision and the other breaks into two equal pieces. One of the pieces starts moving along y-axis with a speed v. What will be the speed of the other piece?

6. A ball of mass m normally hits a wedge of mass M lying at rest on a smooth horizontal surface as shown in Figure.

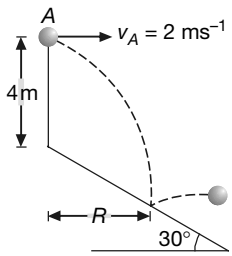


If v_0 be the velocity with which the ball hits the wedge, e be the coefficient of restitution between ball and the wedge, then calculate the velocity of wedge and ball just after collision.

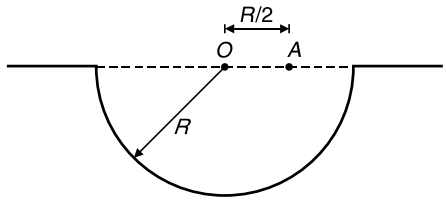
7. A projectile is launched from point A has a horizontal range L_1 as shown. If the coefficient of restitution at B is e, determine the distance L_2.



8. A 0.5 kg ball is thrown horizontally from point A with velocity $v_A = 2\ \text{ms}^{-1}$. Determine the horizontal distance R where the ball strikes the smooth inclined plane. If the coefficient of restitution is $e = 0.6$, determine the speed at which it bounces from the plane. Take $g = 9.8\ \text{ms}^{-2}$.



9. A semi cylindrical frictionless track of radius R is centred at the point O. A particle is released from point A such that $OA = R/2$ as shown in Figure.



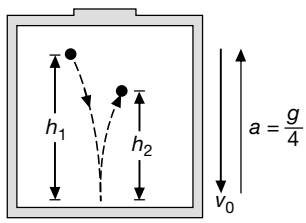
Calculate the coefficient of restitution e, if after collision with the track, the particle moves along the track. Also calculate e if the velocity of particle becomes horizontal just after collision with the track.

10. A ball is released from rest relative to the elevator at a distance h_1 above the floor. The speed of the elevator at the instant ball is released is v_0. Determine the bounce height h_2 of the ball

(a) if v_0 is constant and

(b) if an upward elevator acceleration $a = \frac{g}{4}$ begins at the instant the ball is released.

The coefficient of restitution for the impact is e.



11. A ball is projected from the ground with speed u at an angle α with horizontal. It collides with a wall at a distance a from the point of projection and returns to its original position. Find the coefficient of restitution between the ball and the wall.

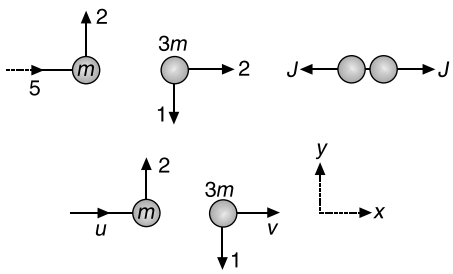
SOLVED PROBLEMS

PROBLEM 1

Two smooth spheres, A and B, of equal radius, lie on a horizontal table. A is of mass m and B is of mass $3m$. The spheres are projected towards each other with velocity vectors $5\hat{i}+2\hat{j}$ and $2\hat{i}-\hat{j}$ respectively and when they collide the line joining their centres is parallel to the vector $\hat{i}$. If the coefficient of restitution between A and B is $\frac{1}{3}$, find their velocities after impact and the loss in kinetic energy caused by the collision. Find also the magnitude of the impulses that act at the instant of impact.

SOLUTION

Since, during impact, the line joining the centres is parallel to the vector $\hat{i}$ so, the velocity components of A and B perpendicular to $\hat{i}$ stay unchanged by the impact.



Applying Law of Conservation of Linear Momentum and the Law of Restitution, we get

$$(m)(5)+(3m)(2)=mu+3mv \qquad \ldots(1)$$

and $$\frac{1}{3}(5-2)=v-u \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$u=2 \text{ and } v=3$$

The velocities of A and B after impact are therefore, $2\hat{i}+2\hat{j}$ and $3\hat{i}-\hat{j}$ respectively

Before impact, the kinetic energy of A and B is

$$(K_A)_i=\frac{1}{2}m(5^2+2^2)=\frac{29}{2}m$$

$$(K_B)_i=\frac{1}{2}(3m)(2^2+1^2)=\frac{15}{2}m$$

After impact, the kinetic energy of A and B is

$$(K_A)_f=\frac{1}{2}m(2^2+2^2)=4m$$

$$(K_B)_f=\frac{1}{2}(3m)(3^2+1^2)=15m$$

Therefore, the loss in K.E. at impact is

$$-\Delta K=(K_A+K_B)_i-(K_A+K_B)_f$$

$$\Rightarrow \quad -\Delta K=\left(\frac{29}{2}m+\frac{15}{2}m\right)-(4m+15m)=3m$$

To find the value of J, we consider the change in momentum along $\hat{i}$ for any one sphere.

For sphere B, we have

$$J=3m(3-2)$$

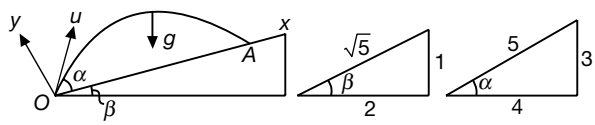
$$\Rightarrow \quad J=3m$$

PROBLEM 2

A particle P is projected from a point O on a smooth plane inclined to the horizontal at $\tan^{-1}\left(\frac{1}{2}\right)$. The particle is projected at $\tan^{-1}\left(\frac{3}{4}\right)$ to the plane. It hits the plane at higher point A and rebounds. Find the possible range of the coefficient of restitution e if the particle P continues to move up the plane after the impact.

SOLUTION

Time of flight of the particle is



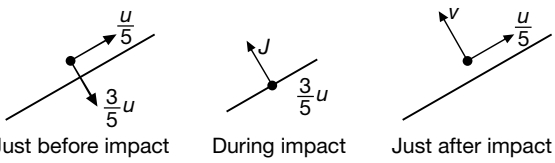
$$T=\frac{2u_y}{g\cos\beta}=\frac{2u\sin\alpha}{g\cos\beta}=\frac{(2u)\left(\frac{3}{5}\right)}{(g)\left(\frac{2}{\sqrt{5}}\right)}=\frac{3u}{\sqrt{5}g}$$

At A, $v_x=u_x+a_xT$ and $v_y=u_y+a_yT$

Substituting values, we get

$$v_x=\frac{u}{5}$$

$$v_y=-\frac{3}{5}u$$

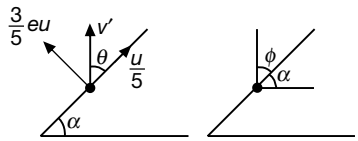


Just before impact During impact Just after impact

From the Figure, we get $u = e\left(\frac{3}{5}u\right)$

Just after impact, the resultant velocity v' of P is inclined to the plane at an angle θ, where

$$\tan\theta = \frac{\left(\frac{3}{5}\right)eu}{\left(\frac{u}{5}\right)} = 3e$$



P continues to move up the plane if,

$\theta < \phi$, where $\phi = \frac{\pi}{2} - \beta$

$\Rightarrow \quad \tan\theta < \tan\left(\frac{\pi}{2} - \beta\right)$

$\Rightarrow \quad \tan\theta < \cot\beta$

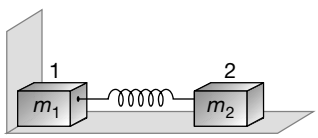
$\Rightarrow \quad 3e < 2$

$\Rightarrow \quad e < \frac{2}{3}$

Since, we also know that $e \geq 0$, so the possible range of e is $0 \leq e < \frac{2}{3}$

PROBLEM 3

Two bars of masses m_1 and m_2 connected by a light spring of force constant k at rest on a smooth horizontal plane. Bar 2 is shifted a small distance x to the left and then released. Find the velocity of the centre of mass of the system after bar 1 breaks off the wall.



SOLUTION

When bar 1 breaks off the wall, its velocity is zero. Let v_2 be the velocity of bar 2. Then the velocity of the cm is given by

$$(m_1 + m_2)v_{cm} = m_1(0) + m_2 v_2$$

$$\Rightarrow \quad v_{cm} = \frac{m_2 v_2}{m_1 + m_2}$$

Using the Law of Conservation of Energy

$$\frac{1}{2}kx^2 = \frac{1}{2}m_2 v_2^2$$

$$\Rightarrow \quad v_2 = x\sqrt{\frac{k}{m_2}}$$

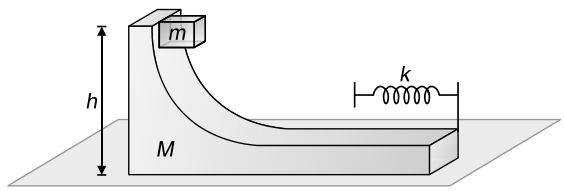
$$\Rightarrow \quad v_{cm} = \frac{m_2 x}{m_1 + m_2}\sqrt{\frac{k}{m_2}} = \frac{x}{m_1 + m_2}\sqrt{km_2}$$

Since the system is free from external force, the acceleration of its cm is zero. So, the velocity of the cm is always

$$v_{cm} = \frac{x}{m_1 + m_2}\sqrt{km_2}$$

PROBLEM 4

A block of mass m is released from rest from a height h onto a smooth sledge of mass M fitted with an ideal spring of stiffness k. Calculate the velocity of the block and sledge just before the block touches the spring. Also calculate the maximum compression in the spring.



SOLUTION

Since no external force is acting along the horizontal direction, so the linear momentum is conserved along the horizontal direction. Let the velocity of block and wedge be v and V respectively up to the case when the block reaches on horizontal surface on the wedge.

$$mv = MV \qquad \ldots(1)$$

Mechanical energy is also conserved in this case

$$\Delta K + \Delta U = 0$$

$$\Rightarrow \quad \left(\frac{1}{2}mv^2 + \frac{1}{2}MV^2 - 0\right) + (-mgh) = 0 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{1}{2}mv^2+\frac{1}{2}M\left(\frac{m}{M}v\right)^2=mgh$$

$$\Rightarrow \quad v=\sqrt{\frac{2Mgh}{M+m}} \text{ and } V=\sqrt{\frac{2m^2gh}{M(M+m)}}$$

We can also get the same result by applying the work energy theorem from centre of mass reference frame

$$W_{\text{gravity}}=\Delta K_{\text{cm}}$$

$$\Rightarrow \quad mgh=\frac{1}{2}\left(\frac{mM}{m+M}\right)(v+V)^2$$

Also, by conservation of linear momentum, we have

$$mv=MV$$

$$\Rightarrow \quad v=\frac{MV}{m}$$

$$\Rightarrow \quad mgh=\frac{1}{2}\left(\frac{mM}{m+M}\right)V^2\left(\frac{M}{m}+1\right)^2$$

$$\Rightarrow \quad V=\sqrt{\frac{2m^2gh}{M(m+M)}}$$

Since initial linear momentum of the system is zero and there is no external force on the system, hence at the instant of maximum compression of spring, the linear momentum of the system should also be zero. So, the final kinetic energy of system is also zero. By conservation of mechanical energy, we have

$$\Delta U+\Delta K=0 \qquad \ldots(3)$$

where, $\Delta K=K_f-K_i=0$

So, equation (3), becomes

$$\Delta U_{\text{gravity}}+\Delta U_{\text{spring}}=0$$

$$\Rightarrow \quad -mgh+\frac{1}{2}kx^2=0$$

$$\Rightarrow \quad x=\sqrt{\frac{2mgh}{k}}$$

PROBLEM 5

A uniform thin rod of mass M and length L is standing vertically along the y-axis on a smooth horizontal surface, with its lower end at the origin $(0, 0)$. A slight disturbance at time $t=0$ causes the lower end to slip on the smooth surface along the positive x-axis, and the rod starts falling.

(a) What is the path followed by the centre of mass of the rod during its fall?

(b) Find the equation of trajectory of a point P on the rod located at a distance r from the lower end. What is the shape of the path of this point?

SOLUTION

(a) As the floor is smooth, no horizontal forces act on the rod.

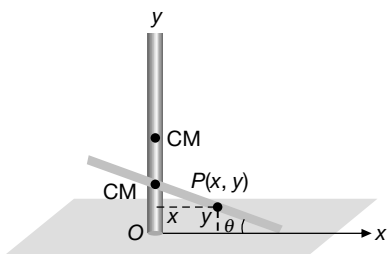
So, centre of mass moves vertically downwards or the path followed by centre of mass is a vertical line along y-axis, i.e., $x=0$

(b) At any instant for a point P on the rod.

$$x=\left[\left(\frac{L}{2}\right)-r\right]\cos\theta \text{ and } y=r\sin\theta$$

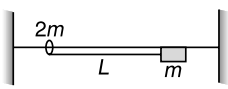
So, eliminating θ between these, we get trajectory of the point P as

$$\frac{x^2}{\left[\left(\frac{L}{2}\right)-r\right]^2}+\frac{y^2}{r^2}=1 \text{ which is an ellipse.}$$



PROBLEM 6

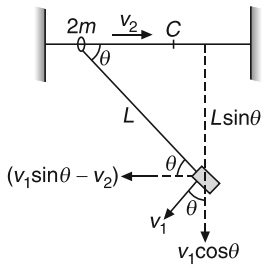
A bead can slide on a smooth straight wire and a particle of mass m attached to the bead by a light string of length l. The particle is held in contact with the wire with string taut and as shown in figure and then let fall. If the bead has a mass $2m$. Then when the string makes an angle θ with the wire, find the speed of the bead and the distance it slides up to this instant.



SOLUTION

When the system is released from rest, m starts falling and due to tension in string bead starts sliding to right. Since no external force is acting on system

along the horizontal direction, so centre of mass of the system will not displace in horizontal direction. Consider the situation when string makes an angle θ with the wire as shown in Figure.



Let centre of mass of system be at position C, then initially the distance of mass m from C is $\frac{2L}{3}$ and when it is at angle θ from the wire its horizontal distance from C is $\frac{2}{3}L\cos\theta$ and similarly that for bead are $\frac{L}{3}$ and $\frac{L}{3}\cos\theta$ respectively. So, the distance it slides is given by

$$d_{\text{bead}} = \frac{L}{3} - \frac{L}{3}\cos\theta = \frac{L}{3}(1-\cos\theta)$$

If at this instant the speed of block and bead are v_1 (relative to bead) and v_2 then using work energy theorem we get

$$mgL\sin\theta = \frac{1}{2}m\left(v_1^2 + v_2^2 - 2v_1v_2\sin\theta\right) + \frac{1}{2}(2m)v_2^2 \quad \ldots(1)$$

Using momentum conservation in horizontal direction as

$$\Sigma F_{\text{horizontal}} = 0$$

$$m(v_1\sin\theta - v_2) = (2m)v_2 \quad \ldots(2)$$

Solving equations (1) and (2), we get

$$\Rightarrow \quad v_1 = \frac{3v_2}{\sin\theta}$$

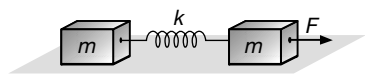
From equation (1), we get

$$2gL\sin\theta = v_2^2\left(1 + \frac{9}{\sin^2\theta} - 6\right)$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{2gL\sin^3\theta}{9 - 5\sin^2\theta}}$$

PROBLEM 7

Two blocks of equal mass m are connected by an unstretched spring and the system is kept at rest on a frictionless horizontal surface. A constant force F is applied on one of the blocks pulling it away from the other as shown in figure.



(a) Find the displacement of the centre of mass at time t.

(b) If the extension of the spring is x_0 at time t, find the displacement of the two blocks at this instant.

SOLUTION

(a) The acceleration of the centre of mass is

$$a_{\text{cm}} = \frac{F}{2m}$$

The displacement of the centre of mass at time t will be

$$x = \frac{1}{2}a_{\text{cm}}t^2 = \frac{Ft^2}{4m}$$

(b) Suppose the displacement of the first block is x_1 and that of the second is x_2. Then,

$$x = \frac{mx_1 + mx_2}{2m}$$

$$\Rightarrow \quad \frac{Ft^2}{4m} = \frac{x_1 + x_2}{2}$$

$$\Rightarrow \quad x_1 + x_2 = \frac{Ft^2}{2m} \quad \ldots(1)$$

Further, the extension of the spring is $x_1 - x_2$.

$$\Rightarrow \quad x_1 - x_2 = x_0 \quad \ldots(2)$$

From equations (1) and (2), we get

$$x_1 = \frac{1}{2}\left(\frac{Ft^2}{2m} + x_0\right) \text{ and } x_2 = \frac{1}{2}\left(\frac{Ft^2}{2m} - x_0\right)$$

PROBLEM 8

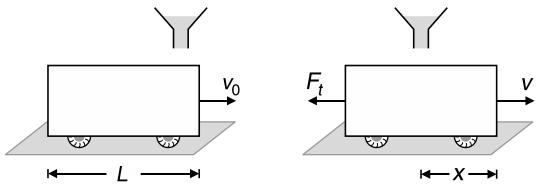
A railroad car of length L and mass m_0 when empty is moving freely on a horizontal track while being loaded with sand from a stationary chute at a rate $\frac{dm}{dt} = \lambda$. Assuming that the car was approaching the chute at a speed v_0, determine

(a) the mass of the car and its load after the car has cleared the chute,

(b) the speed of the car at that time.

SOLUTION

(a) At time t, $m = (m_0 + \lambda t)$



Thrust force $F_t = v_r \dfrac{dm}{dt} = \lambda v$ {backwards}

$$\Rightarrow \quad a_t = \frac{F_t}{m} = \frac{\lambda v}{(m_0 + \lambda t)}$$

$$\Rightarrow \quad \left(-\frac{dv}{dt}\right) = \left(\frac{\lambda v}{m_0 + \lambda t}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda}\int_{v_0}^{v}\frac{dv}{v} = -\int_0^t \frac{dt}{m_0 + \lambda t}$$

$$\Rightarrow \quad \frac{1}{\lambda}\ln\left(\frac{v}{v_0}\right) = \frac{-1}{\lambda}\ln\left(\frac{m_0 + \lambda t}{m_0}\right)$$

$$\Rightarrow \quad \frac{v}{v_0} = \left(\frac{m_0}{m_0 + \lambda t}\right)$$

$$\Rightarrow \quad v = v_0\left(\frac{m_0}{m_0 + \lambda t}\right) \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{dx}{dt} = v_0\left(\frac{m_0}{m_0 + \lambda t}\right)$$

$$\Rightarrow \quad \int_0^L dx = \int_0^t v_0\left(\frac{m_0}{m_0 + \lambda t}\right)dt$$

$$\Rightarrow \quad L = \frac{m_0 v_0}{\lambda}\left(\ln(m_0 + \lambda t)\right)\Big|_0^t$$

$$\Rightarrow \quad L = \frac{m_0 v_0}{\lambda}\ln\left(\frac{m_0 + \lambda t}{m_0}\right)$$

So, mass of car and its load after the car has cleared the chute is

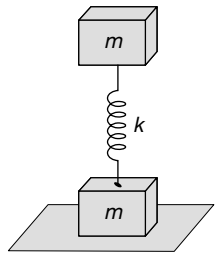
$$(m_0 + \lambda t) = m_0 e^{\frac{\lambda L}{m_0 v_0}}$$

(b) From equation (1) speed of car at that moment is

$$v = \frac{m_0 v_0}{m_0 e^{\frac{\lambda L}{m_0 v_0}}} = v_0 e^{-\frac{\lambda L}{m_0 v_0}}$$

PROBLEM 9

A system consists of two identical cubes, each of mass m, linked together by the compressed light spring of spring constant k. The cubes are connected by a thread which is burnt at a certain moment. Find



(a) at what values of x_0, the initial compression of the spring, the lower cube will bounce up after the thread has been burned through;

(b) to what maximum height will centre of mass of this system rise if the initial compression of the spring $\Delta l = \dfrac{7mg}{k}$.

SOLUTION

(a) Let x be the elongation in the spring when it returns. Then by Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}kx_0^2 = \frac{1}{2}kx^2 + mg(x + x_0)$$

$$\Rightarrow \quad kx^2 + 2mgx - kx_0^2 + 2mgx_0 = 0$$

$$\Rightarrow \quad kx = -mg \pm (mg - kx_0)$$

$$\Rightarrow \quad kx = -kx_0 \text{ or } kx = kx_0 - 2mg$$

Since $kx = -kx_0$ gives $x = -x_0$ and this is not acceptable.

$$kx = kx_0 - 2mg$$

The lower cube will bounce up if $kx \geq mg$ (the weight of the lower cube)

$$\Rightarrow \quad kx_0 - 2mg \geq mg$$

$$\Rightarrow \quad x_0 \geq \frac{3mg}{k}$$

Thus, for all values of x_0 greater than $\dfrac{3mg}{k}$ the lower cube will bounce up.

(b) If the initial compression, $\Delta l = \dfrac{7mg}{k}$ is greater than $\dfrac{3mg}{k}$, the lower mass m also moves up.

Let v be the velocity of the upper block at the position when the lower block just bounces off the floor, then by Law of Conservation of Energy, we get

$$\frac{1}{2}mv^2 = \frac{1}{2}k\left[(\Delta l)^2 - x^2\right] - mg(\Delta l + x)$$

$$\Rightarrow \quad v^2 = 32\frac{mg^2}{k} \qquad \left\{\because\ x = \frac{mg}{k}\right\}$$

Hence, the velocity of the centre of mass, at the moment lower block just bounces off the floor is

$$v_c = \frac{mv + 0}{2m} = \frac{v}{2}$$

If the centre of mass further rises to x', then by Law of Conservation of Energy

$$\frac{1}{2}(2m)v_c^2 = (2m)gx'$$

$$\Rightarrow \quad x' = \frac{v_c^2}{2g} = \frac{v^2}{8g} = \frac{1}{8g}\left(\frac{32mg^2}{k}\right) = \frac{4mg}{k}$$

$$\{a_{cm} = g\}$$

Before the break off of the lower mass, centre of mass has already moved up by a distance

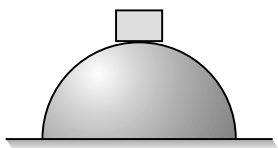
$$\frac{(\Delta l + x)m + 0}{2m} = \frac{4mg}{k} \qquad \left\{\Delta l = \frac{7mg}{k}\right\}$$

Hence, the maximum height attained by centre of mass of the system

$$h = \frac{4mg}{k} + \frac{4mg}{k} = \frac{8mg}{k}$$

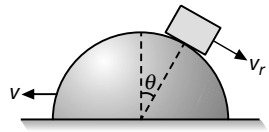
PROBLEM 10

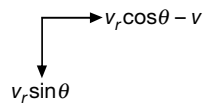
A particle of mass m is placed on top of a smooth hemisphere also of mass m which is placed on a smooth horizontal surface. If the particle begins to slide down due to a negligible small impulse, show that it will lose contact with the hemisphere when the radial line through it makes an angle θ given by the equation $\cos^3\theta - 6\cos\theta + 4 = 0$.



SOLUTION

Let v_r be the velocity of particle as it leaves contact with the sphere $(N = 0)$ and v the horizontal velocity of sphere at this instant.





Absolute components of velocity of particle

Applying Conservation of Linear Momentum in horizontal direction we get,

$$mv = m(v_r\cos\theta - v)$$

$$\Rightarrow \quad 2v = v_r\cos\theta \qquad \ldots(1)$$

Conservation of Mechanical Energy gives,

$$mgr(1 - \cos\theta) = \frac{1}{2}mv^2 + \frac{1}{2}m\left(v_r^2 + v^2 - 2vv_r\cos\theta\right)$$

$$\Rightarrow \quad gr(1 - \cos\theta) = v^2 + \frac{v_r^2}{2} - vv_r\cos\theta \qquad \ldots(2)$$

Equation of Law of Motion gives,

$$mg\cos\theta = \frac{mv_r^2}{r}$$

$$\Rightarrow \quad gr = \frac{v_r^2}{\cos\theta} \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$\cos^3\theta - 6\cos\theta + 4 = 0$$

PROBLEM 11

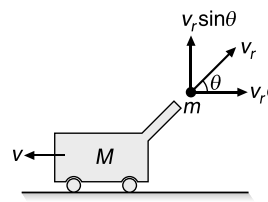
A gun of mass M (including the carriage) fires a shot of mass m. The gun along with the carriage is kept on a smooth horizontal surface. The muzzle speed of the bullet v_r is constant. Find

(a) the elevation of the gun with horizontal at which maximum range of bullet with respect to the ground is obtained.

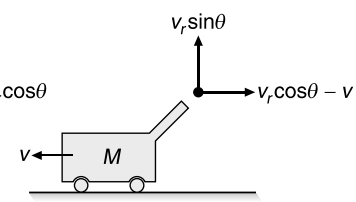
(b) the maximum range of the bullet.

SOLUTION

Muzzle velocity v_r is given to be constant



Component of velocity of bullet with respect to gun



Components of velocity of bullet with respect to ground

From Conservation of Linear Momentum in horizontal direction we have,

$$Mv = m(v_r \cos\theta - v)$$

$$\Rightarrow \quad v = \frac{mv_r \cos\theta}{M+m} \qquad \ldots(1)$$

Further, range of bullet on horizontal ground

$$R = \frac{2v_r \sin\theta}{g}(v_r \cos\theta - v)$$

$$R = \frac{2v_r \sin\theta}{g}\left(v_r \cos\theta - \frac{mv_r \cos\theta}{M+m}\right)$$

$$R = \frac{2Mv_r^2 \sin\theta\cos\theta}{(M+m)g}$$

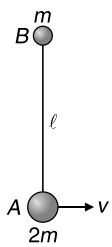
$$\Rightarrow \quad R = \left(\frac{M}{M+m}\right)\frac{v_r^2 \sin 2\theta}{g} \qquad \ldots(2)$$

(a) From equation (2) we see that maximum range is at $\theta = 45°$

(b) At $\theta = 45°$, $R_{max} = \left(\frac{M}{M+m}\right)\frac{v_r^2}{g}$

PROBLEM 12

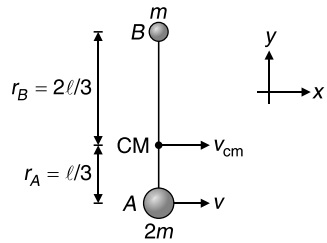
Two small spheres A and B of masses $2m$ and m respectively, are connected with an inextensible string of length l lie on a smooth horizontal plane. The sphere A is given a velocity of v along the horizontal plane perpendicular to line AB as shown in Figure. Calculate the tension in string during their subsequent motion by analysing the problem from the centre of mass reference frame.



SOLUTION

Since no external force is acting on the system of particles (i.e. the two spheres), so velocity of centre of mass of system remains unchanged and hence centre of mass of system will move along the x-direction and both spheres will move in circular path about centre of mass of the system.

In the centre of mass reference frame, the particles A and B move along circular path with same angular velocity, say ω.



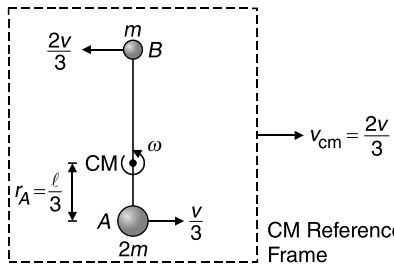
Velocity of centre of mass is given by

$$v_{cm} = \frac{m(0)+(2m)v}{3m} = \frac{2v}{3}$$

Distance of centre of mass from the sphere A is equal to the radius of the circle in which the sphere A revolves, so we have

$$r_A = \frac{(2m)(0)+ml}{2m+m} = \frac{l}{3}$$

In the centre of mass reference frame, the system is shown in the Figure.



Velocity of mass $2m$ with respect to centre of mass is given by

$$v_2 = v - \frac{2v}{3} = \frac{v}{3}$$

Hence, the angular velocity of particle A is given by

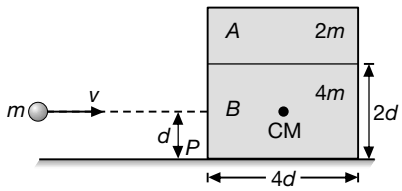
$$\omega = \frac{v/3}{l/3} = \frac{v}{l}$$

$$\Rightarrow \quad T = (2m)r\omega^2 = (2m)\left(\frac{l}{3}\right)\left(\frac{v^2}{l^2}\right)$$

$$\Rightarrow \quad T = \frac{2mv^2}{3l}$$

PROBLEM 13

A block A of mass $2m$ is placed on another block B of mass $4m$ which in turn is placed on a fixed table. The two blocks have the same length $4d$ and they are placed as shown in figure.



The coefficient of friction (both static and kinetic) between the block B and the table is μ. There is no friction between the two blocks. A small object of mass m moving horizontally along a line passing through the centre of mass of the block B and perpendicular to its face with a speed v collides elastically with the block B at a height d above the table.

(a) What is the minimum value of v (call it v_0), required to make the block A topple.

(b) If $v = 2v_0$, find the distance (from the point P) at which the mass m falls on the table after collision.

SOLUTION

If v_1 and v_2 are the velocities of object of mass m and block of mass $4m$, just after collision then by conservation of momentum,

$$mv = mv_1 + 4mv_2,$$

i.e., $$v = v_1 + 4v_2 \qquad \ldots(1)$$

Further, as collision is elastic

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}4mv_2^2,$$

i.e., $$v^2 = v_1^2 + 4v_2^2 \qquad \ldots(2)$$

Solving, these two equations we get either

$$v_2 = 0 \text{ or } v_2 = \frac{2}{5}v$$

Therefore, $v_2 = \frac{2}{5}v$

Substituting in equation (1)

$$v_1 = -\frac{3}{5}v$$

when $v_2 = 0$, $v_1 = v_2$, but it is physically unacceptable.

(a) Now, after collision the block B will start moving with velocity v_2 to the right. Since, there is no friction between blocks A and B the upper block A will stay at its position and will topple if B moves a distance s such that

$$s > 2d \qquad \ldots(3)$$

However, the motion of B is retarded by frictional force $f = \mu(4m+2m)g$ between table and its lower surface. So, the distance moved by B till it stops

$$0 = v_2^2 - 2\left(\frac{6\mu mg}{4m}\right)s,$$

i.e., $$s = \frac{v_2^2}{3\mu g}$$

Substituting this value of s in equation (3), we find that for toppling of A

$$v_2^2 > 6\mu gd$$

$$\Rightarrow \frac{2}{5}v > \sqrt{6\mu gd} \qquad \left\{\text{as } v_2 = \frac{2v}{5}\right\}$$

i.e., $$v > \frac{5}{2}\sqrt{6\mu gd}$$

$$\Rightarrow v_{\min} = v_0 = \frac{5}{2}\sqrt{6\mu gd}$$

(b) If $v = 2v_0 = 5\sqrt{6\mu gd}$, the object will rebound with speed

$$v_1 = \frac{3}{5}v = 3\sqrt{6\mu gd}$$

and as time taken by it to fall down

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2d}{g}} \qquad \{\text{as } h = d\}$$

The horizontal distance moved by it to the left of P in this time

$$x = v_1 t = 6d\sqrt{3\mu}$$

(a) Toppling will take place if line of action of weight does not pass through the base area in contact.

(b) v_1 and v_2 can also be obtained by using the equations of head on elastic collision

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

and $$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2 + \left(\frac{2m_1}{m_1 + m_2}\right)v_1$$

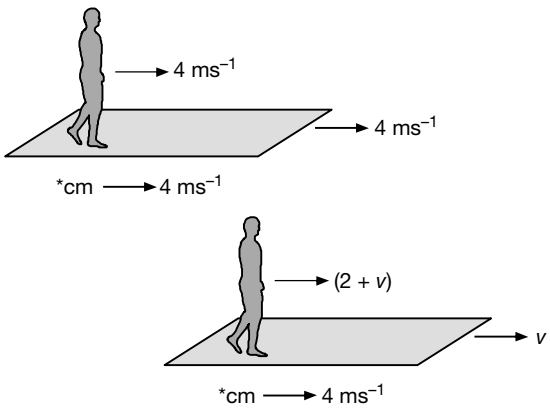
PROBLEM 14

A 75 kg man stands at the rear end of a platform of mass 25 kg and length 4 m, which moves initially at $4\hat{i}$ ms^{-1} over a frictionless surface. At $t = 0$, he walks

at $2\ \text{ms}^{-1}$ relative to the platform and then stops at the front end. During the period of walking, find the displacement of the platform, the man and the centre of mass

SOLUTION

Initially the man, the platform, and the centre of mass have the same velocity, $4\hat{i}\ \text{ms}^{-1}$, as shown in the Figure.



(a) A person at the rear of a platform moving at 4 ms^{-1}.
(b) The person walks at 2 ms^{-1} relative to the platform. The velocity of the center of mass does not change.

When he begins to walk forward, his increase in momentum must be compensated for by a decrease in the platform's momentum. Let us say that the velocity of the platform relative to the ground while he is walking i.e.

$$\vec{v}_{PG} = v_p\hat{i}$$

The velocity of man relative to the ground is then

$$\vec{v}_{MG} = \vec{v}_{MP} + \vec{v}_{PG} = (2 + v_P)(4\hat{i})$$

From the conservation of momentum, we have

$$(75+25)4 = 75(2+v_P) + 25v_P$$

Thus, the velocity of the platform is $v_P = 2.5\ \text{ms}^{-1}$ and the velocity of the man is $v_M = 4.5\ \text{ms}^{-1}$.

Since the velocity of man relative to platform is $2\ \text{ms}^{-1}$, therefore, it takes $t = \dfrac{4\ \text{m}}{2\ \text{ms}^{-1}} = 2\ \text{s}$ to walk from the rear to the front.

Displacement of platform is

$$x_P = (v_P)t = (2.5)(2) = 5\ \text{ms}^{-1}$$

Displacement of the man is

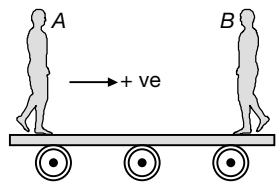
$$x_M = (v_M)t = (4.5)(2) = 9\ \text{m}$$

Displacement of the centre of mass

$$x_{\text{cm}} = (v_{\text{cm}})t = (4)(2) = 8\ \text{m}$$

PROBLEM 15

Two persons A and B, each of mass m are standing at the two extremes of rail-road car of mass M. Person A jumps to the left with a horizontal speed u with respect to the car. Thereafter, the person B jumps to the right, again with the same horizontal speed u with respect to the car. Find the velocity of the car after both the persons have jumped off.



SOLUTION

Initial velocity of car = 0

Let v_1 be the velocity of car (with person B) to the right, when person A jumps off the car. Then by Law of Conservation of Linear Momentum applied relative to earth, we get

$$0 = (m+M)v_1 + m(v_1 - u)$$

$$\Rightarrow \quad v_1 = \frac{mu}{M+2m} \qquad \ldots(1)$$

Let v_2 be the velocity of the car to the right when the second person B jumps off the car to the right with velocity u relative to car. Then again applying Law of Conservation of Linear Momentum, we get

$$(M+m)v_1 = Mv_2 + m(u+v_2)$$

$$\Rightarrow \quad v_2 = v_1 - \frac{mu}{(M+m)}$$

Substituting the value of v_1 from equation (1), we get,

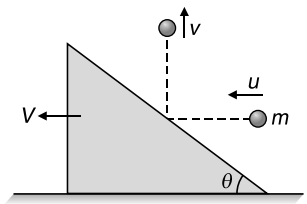
$$v_2 = \frac{mu}{M+2m} - \frac{mu}{(M+m)}$$

$$\Rightarrow \quad v_2 = \frac{-m^2u}{(M+2m)(M+m)} \qquad \{\text{towards right}\}$$

$$\text{OR} \quad v_2 = \frac{m^2u}{(M+2m)(M+m)} \qquad \{\text{towards left}\}$$

PROBLEM 16

A ball of mass m moving horizontally which velocity u hits a wedge of mass M. The wedge is situated on a smooth horizontal source. If after striking with wedge the ball starts moving in vertical direction and the wedge starts moving in horizontal plane as shown in Figure.



Calculate the velocity V of wedge, the velocity (v) with which the ball moves in vertical direction, the impulse imparted by the ball to the wedge and the coefficient of restitution e.

SOLUTION

Since no external force is acting on the system along the horizontal direction, so the linear momentum is conserved along the horizontal direction. So, we have

$$mu = MV$$

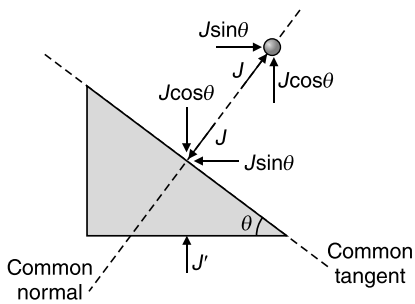
$$\Rightarrow \quad V = \frac{mu}{M} \qquad \ldots(1)$$

There is no impulse on the ball in the direction parallel to incline, so the velocity of ball along the incline, i.e. along the common tangent or the t-line is constant

$$\Rightarrow \quad u\cos\theta = v\sin\theta$$

$$\Rightarrow \quad v = u\cot\theta \qquad \ldots(2)$$

We can also find the values of v and V by using the impulse-momentum theorem. When ball hits the wedge, an impulse is generated (between ball and wedge) along the direction perpendicular to the incline, i.e. along the line of impact or the n-line.



For ball, we have

$$mu - J\sin\theta = 0$$

$$\Rightarrow \quad J\sin\theta = mu \qquad \ldots(3)$$

Also, $J\cos\theta = mv \qquad \ldots(4)$

From equation (3) and (4), we get

$$v = u\cot\theta$$

From equation (3), we have

For the wedge, we have

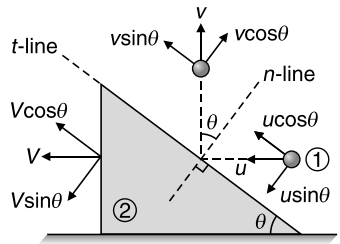
$$J\sin\theta = MV$$

$$\Rightarrow \quad V = \frac{J\sin\theta}{M}$$

Using equation (3), we get

$$V = \frac{mu}{M}$$

The velocities of ball, wedge before and after the collision along the t-line and the n-line are shown in Figure.



The coefficient of restitution is given by

$$e = \frac{V\sin\theta - (-v\cos\theta)}{u\sin\theta - 0}$$

$$\Rightarrow \quad e = \frac{m\sin^2\theta + M\cos^2\theta}{M\sin^2\theta}$$

$$\Rightarrow \quad e = \frac{m}{M} + \cot^2\theta$$

PROBLEM 17

A gun of mass M fires a shell of mass m and recoils horizontally. If the shell travels along the barrel with speed v. Find the speed with which the barrel begins to recoil when

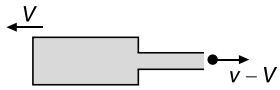
(a) the barrel is horizontal

(b) the barrel is inclined at an angle $30°$ to the horizontal.

In each case find the constant force required to bring the gun to rest in $t = 2$ s.

SOLUTION

(a) The speed of the shell is $v - V$ as it leaves the barrel, because the barrel is recoiling with speed V.
Before firing the shell, the gun is at rest and the total momentum is zero.



Using conservation of linear momentum (in the direction of the shell's motion)

$$0 = m(v - V) - MV$$

$$\Rightarrow \quad (M + m)V = mv$$

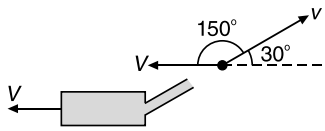
Therefore, the initial velocity of recoil is $\dfrac{mv}{M+m}$

If a constant force F_1 brings the gun to rest, it must exert in 2 second, an impulse equal to the initial momentum of the gun, so

$$2F_1 = M\left(\frac{mv}{M+m}\right)$$

So, the force required is $F_1 = \dfrac{Mmv}{2(M+m)}$

(b) This time the shell leaves the barrel with a velocity which is the resultant of two components inclined at $150°$.
That momentum is not conserved in the vertical direction because the impulse exerted by the ground on the gun is an external impulse which does change the total momentum.



Applying Law of Conservation of Linear Momentum in the direction of recoil, we get

$$0 = MV + m(V - v\cos 30°)$$

$$\Rightarrow \quad \frac{1}{2}mv\sqrt{3} = (M+m)V$$

$$\Rightarrow \quad V = \frac{mv\sqrt{3}}{2(M+m)}$$

and the force, F_2, required to stop the gun in two second is found by using Impulse - Momentum Theorem

$$2F_2 = M\left(\frac{mv\sqrt{3}}{2(M+m)}\right)$$

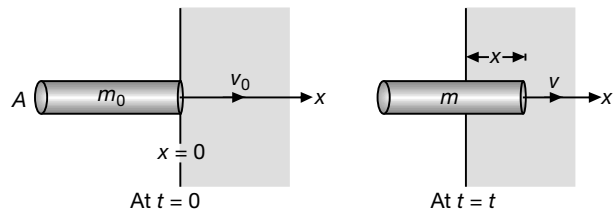
So, the required force is $F_2 = \dfrac{Mmv\sqrt{3}}{4(M+m)}$

PROBLEM 18

A cylindrical solid of mass 10^{-2} kg and cross-sectional area $10^{-4}\,\text{m}^2$ is moving parallel to its axis (the X-axis) with a uniform speed of $10^3\ \text{ms}^{-1}$ in the positive direction. At $t = 0$, its front face passes the plane $x = 0$. The region to the right of this plane is filled with stationary dust particles of uniform density $10^{-3}\ \text{kgm}^{-3}$. When a dust particle collides with the face of the cylinder, it sticks to its surface. Assuming that the dimensions of the cylinder remains practically unchanged, and that the dust sticks only to the front face of the cylinder, find the x-coordinate of the front of the cylinder at $t = 150$ s.

SOLUTION

Given, $m_0 = 10^{-2}$ kg, $A = 10^{-4}\ \text{m}^2$, $v_0 = 10^3\ \text{ms}^{-1}$ and $\rho_{\text{dust}} = \rho = 10^{-3}\ \text{kgm}^{-3}$



At any instant mass of cylinder is $m = m_0 +$ mass of dust collected so far $= m_0 + Ax\rho_{\text{dust}}$

$$\Rightarrow \quad m = m_0 + Ax\rho$$

The linear momentum at $t = 0$ is

$$p_0 = m_0 v_0$$

and momentum at t is

$$p_t = mv = (m_0 + Ax\rho)v$$

By Law of Conservation of Linear Momentum, we have

$$p_0 = p_t$$

$$\Rightarrow \quad m_0 v_0 = (m_0 + Ax\rho)v$$

$$\Rightarrow \quad m_0 v_0 = (m_0 + Ax\rho)\frac{dx}{dt}$$

$$\Rightarrow \quad (m_0 + Ax\rho)dx = m_0 v_0 dt$$

$$\Rightarrow \quad \int_0^x (m_0 + Ax\rho)dx = m_0 v_0 \int_0^{150} dt$$

$$\Rightarrow \quad \left(m_0 x + A\rho\frac{x^2}{2}\right)_0^x = (m_0 v_0 t)\Big|_0^{150}$$

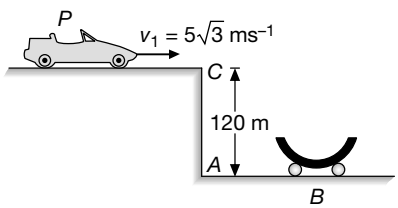
$$\Rightarrow \quad m_0 x + A\rho\frac{x^2}{2} = 150 m_0 v_0$$

Solving this quadratic equation and Substituting the values of m_0, A, ρ and v_0, we get a quadratic in x, which on solving yields a positive value of x as 10^5 m.

$$\Rightarrow \quad x = 10^5 \text{ m}$$

PROBLEM 19

A car P is moving with a uniform speed of $5\sqrt{3}$ ms^{-1} towards a carriage of mass 9 kg at rest kept on the rails at a point B as shown in figure.



The height AC is 120 m. Cannon balls of 1 kg are fired from the car with an initial velocity 100 ms^{-1} at an angle $30°$ with the horizontal. The first cannon ball hits the stationary carriage after a time t_0 and sticks to it. Determine t_0. At t_0, the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout. If the second cannon ball also hits and sticks to the carriage. What will be the horizontal velocity of the carriage just after the second impact? Take $g = 10$ ms^{-2}.

SOLUTION

Unless and until it is mentioned in the question, the velocity is taken relative to ground, so velocity of the cannon ball is relative to ground is 100 ms^{-1}.

Horizontal component of its velocity,

$$u_x = u\cos 30° = 100\left(\frac{\sqrt{3}}{2}\right) \text{ ms}^{-1} = 50\sqrt{3} \text{ ms}^{-1}$$

and vertical component of its velocity,

$$u_y = u\sin 30° = 100\left(\frac{1}{2}\right) \text{ ms}^{-1} = 50 \text{ ms}^{-1}$$

Vertical displacement of the ball when it strikes the carriage is -120 m or

$$s_y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad -120 = (50t) + \left(\frac{1}{2}\right)(-10t^2)$$

$$\Rightarrow \quad t^2 - 10t - 24 = 0$$

$$\Rightarrow \quad t = 12 \text{ s or } -2 \text{ s}$$

Ignoring the negative time, we get

$$t_0 = 12 \text{ s}$$

When it strikes the carriage, its horizontal component of velocity is still $50\sqrt{3}$ ms^{-1}. It sticks to the carriage. Let v_2 be the velocity of (carriage-cannon ball) system after collision. Then applying Conservation of Linear Momentum in horizontal direction

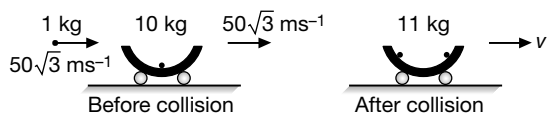
$$m_{\text{ball}} u_x = (m_{\text{ball}} + m_{\text{carriage}})v_2$$

$$\Rightarrow \quad (1 \text{ kg})(50\sqrt{3} \text{ ms}^{-1}) = (10 \text{ kg})(v_2)$$

$$\Rightarrow \quad v_2 = v_{\text{carriage}} = 5\sqrt{3} \text{ ms}^{-1}$$

The second cannon ball is fired when the first cannon ball strikes the carriage, i.e., after 12 second. In these 12 second the car will move forward a distance of $12v_1$ or $60\sqrt{3}$ m. This ball will strike the carriage only when the carriage also covers the same distance of $60\sqrt{3}$ m in next 12 s. This is possible only when the resistive force is zero, because then only

$$v_{\text{car}} = v_{\text{carriage}} = 5\sqrt{3} \text{ ms}^{-1}$$



Hence at the time of second collision, the horizontal component of velocity of cannon ball is $u_x = 50\sqrt{3}$ ms^{-1} and horizontal velocity of carriage and first cannon ball is $5\sqrt{3}$ ms^{-1}.

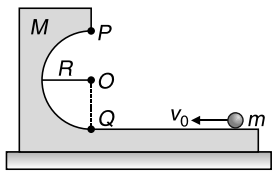
If v be the desired velocity of carriage after second collision. Then Conservation of Linear Momentum in horizontal direction gives

$$11v = (1)(50\sqrt{3}) + (10)(5\sqrt{3}) = 100\sqrt{3}$$

$$\Rightarrow \quad v = \frac{100\sqrt{3}}{11} \text{ ms}^{-1} \approx 15.75 \text{ ms}^{-1}$$

PROBLEM 20

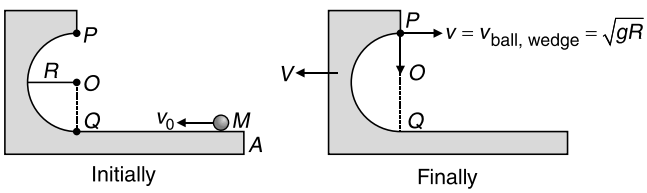
A small ball of mass m is projected with a minimum horizontal velocity v_0 on a smooth wedge of mass M so that it will reach the highest point P of the wedge as shown in Figure. Calculate v_0.



SOLUTION

For ball to reach the point P, its velocity with respect to wedge should be

$$v = \sqrt{gR}$$



Applying work energy theorem from the centre of mass reference frame at A and the highest point P, we get

$$W_{ext} + W_{int} = W_{ext} + 0 = \Delta K_{cm}$$

$$\Rightarrow \quad W_{gravity} = \Delta K_{cm}$$

$$\Rightarrow \quad -mg(2R) = \frac{1}{2}\mu\left(v_{rel}^2 - u_{rel}^2\right), \text{ where } \mu = \frac{mM}{m+M}$$

$$\Rightarrow \quad -2mgR = \frac{1}{2}\mu(V+v)^2 - \frac{1}{2}\mu v_0^2 \qquad \ldots(1)$$

Since, no external forces are acting on the system in horizontal direction, so linear momentum of the system is be conserved. Hence, we have

$$mv_0 = m(V-v) + MV$$

$$\Rightarrow \quad V = \frac{m(v_0+v)}{m+M} \qquad \ldots(2)$$

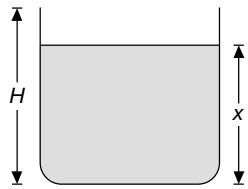
From equations (1) and (2), we get

$$v_0 = \sqrt{\left(5 + \frac{4m}{M}\right)gR}$$

PROBLEM 21

A liquid has been poured into a cylindrical vessel of mass M (the mass of the vessel bottom can be ignored) and height H. The linear density of the liquid, which is, the ratio of the mass of the liquid column to its height is denoted by α. Calculate the height x of the column of liquid at which the common centre of mass of the liquid plus the vessel is in the lowest position.

Assume the centre of mass of vessel to be at $\frac{H}{2}$ from ground.



SOLUTION

Distance of centre of mass of system (vessel plus liquid) from the bottom.

$$x_{cm} = \frac{M\left(\frac{H}{2}\right) + (\alpha x)\frac{x}{2}}{M + (\alpha x)}$$

For x_{cm} to be minimum,

$$\frac{dx_{cm}}{dx} = 0$$

$$\Rightarrow \quad (M+\alpha x)(\alpha x) - \left(M\frac{H}{2} + \frac{\alpha x^2}{2}\right)(\alpha) = 0$$

$$\Rightarrow \quad M\alpha x + \alpha^2 x^2 - \frac{MH\alpha}{2} - \frac{\alpha^2 x^2}{2} = 0$$

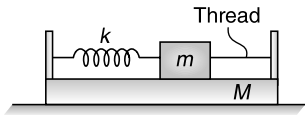
$$\Rightarrow \quad \alpha x^2 + 2Mx - MH = 0$$

$$\Rightarrow \quad x = \frac{-2M \pm \sqrt{4M^2 + 4MH\alpha}}{2\alpha}$$

$$\Rightarrow \quad x = \frac{\sqrt{M(M+H\alpha)} - M}{\alpha}$$

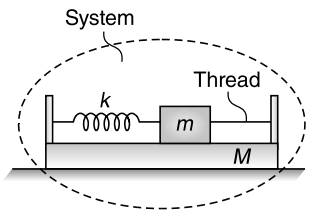
PROBLEM 22

A spring is connected with plank and other end of spring is connected with a block of mass m. Initially spring is stretched by a distance x_0 and block is connected with a thread which is connected to another end of the plank as shown in Figure. If thread is cut, calculate the maximum speed of the plank. Assume friction to be absent everywhere.



SOLUTION

Assume the system to be made of spring, plank and bloc as shown in Figure.



No external force acts on the system in horizontal direction.

Since no external force is acting on the system along the horizontal direction. Both linear momentum and mechanical energy will be conserved for the system. Let speed of block be v and maximum speed of the plank be V. Applying conservation of linear momentum at this situation, we get

$$mv = MV \qquad \ldots(1)$$

Applying conservation of mechanical energy, we get

$$\Delta K + \Delta U = 0$$

$$\left[\left(\frac{1}{2}MV^2 + \frac{1}{2}mv^2\right) - 0\right] + \left[0 - \frac{1}{2}kx_0^2\right] = 0 \qquad \ldots(2)$$

From equation (1), we have

$$v = \frac{MV}{m}$$

Substituting this value in equation (2), we get

$$\left[\frac{1}{2}MV^2 + \frac{1}{2}m\left(\frac{MV}{m}\right)^2\right] = \frac{1}{2}kx_0^2$$

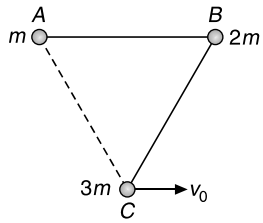
$$\Rightarrow \quad \frac{1}{2}\left(M + \frac{M^2}{m}\right)V^2 = \frac{1}{2}kx_0^2$$

$$\Rightarrow \quad V^2 = \frac{mkx_0^2}{Mm + M^2}$$

$$\Rightarrow \quad V = x_0\sqrt{\frac{mk}{M(m+M)}}$$

PROBLEM 23

Three particles A, B and C of mass m, $2m$ and $3m$ respectively lie on a smooth horizontal table at the vertices of an equilateral triangle. A and B as well as B and C are connected by light inextensible strings. C is given a velocity v_0 parallel to AB as shown in Figure.



Show that A eventually begins to move with a velocity $\frac{2v_0}{19}$.

SOLUTION

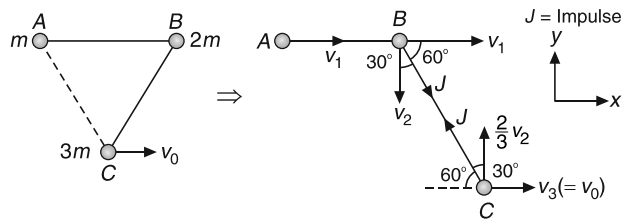
For B-C, system, by Law of Conservation of Momentum along y-direction, we have

$$0 = 3mv_{Cy} + 2mv_{By}$$

$$\Rightarrow \quad v_{Cy} = -\frac{2}{3}v_{By}$$

If v_1 and $-v_2$ be the x and y components of final velocity of B, then $v_{By} = -v_2$

$$\Rightarrow \quad v_{Cy} = \frac{2}{3}v_2$$



Now, applying Law of Conservation of Momentum along x-direction, we get

$$3mv_0 = mv_1 + 2mv_1 + 3mv_3$$

$$\Rightarrow \quad v_0 = v_1 + v_3 \qquad \ldots(1)$$

Components of velocities along BC for particles B and C will be equal. Hence,

$$v_1 \cos 60° + v_2 \cos 30° = v_3 \cos 60° - \frac{2}{3} v_2 \cos 30°$$

$$\Rightarrow \quad \frac{v_1}{2} + \frac{\sqrt{3} v_2}{2} = \frac{v_3}{2} - \frac{2\sqrt{3}}{6} v_2$$

$$\Rightarrow \quad v_1 - v_3 + \frac{5}{\sqrt{3}} v_2 = 0 \qquad \ldots(2)$$

Now, since Impulse = Change in Momentum, so for C, we have

Along x-axis, $J \cos(60°) = 3m(v_0 - v_3)$

$$\Rightarrow \quad \frac{J}{2} = 3m(v_0 - v_3) \qquad \ldots(3)$$

Along y-axis, $J \cos(30°) = 3m\left(\frac{2}{3} v_2\right)$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} J = 2mv_2 \qquad \ldots(4)$$

Solving equations (1), (2), (3) and (4) we get

$$v_1 = \frac{2}{19} v_0$$

PRACTICE EXERCISES

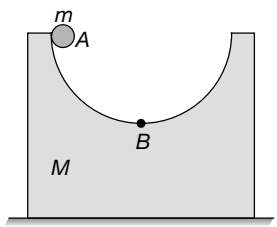
SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. A particle of mass $2m$ is projected at an angle of $45°$ with horizontal with a velocity of $20\sqrt{2}$ ms^{-1}. After 1 s explosion takes place and the particle is broken into two equal pieces. As a result of this explosion, one part comes to rest. The maximum height from the ground attained by the other part is $(g = 10 \text{ ms}^{-2})$

(A) 25 m (B) 35 m
(C) 40 m (D) 50 m

2. A block of mass $M = 2$ kg with a semi-circular track of radius $R = 1.1$ m rests on a horizontal frictionless surface. A uniform cylinder of radius $r = 10$ cm and mass $m = \frac{M}{2}$ is released from rest from the top point A. The cylinder slips on the semi-circular frictionless track. The speed of the block when the cylinder reaches the bottom of the track at B is $(\text{Take } g = 10 \text{ ms}^{-2})$

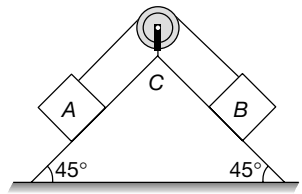


(A) $\sqrt{\frac{10}{3}}$ ms^{-1} (B) $\sqrt{\frac{4}{3}}$ ms^{-1}
(C) $\sqrt{\frac{5}{2}}$ ms^{-1} (D) $\sqrt{10}$ ms^{-1}

3. Three point masses of 1 g, 2 g and 3 g have their centre of mass at $(2, 2, 2)$. A fourth mass of 4 g is placed at position vector $\vec{r}$ such that the centre of mass of new system is now at $(0, 0, 0)$.

(A) $\vec{r} = (-1, -1, -1)$ (B) $\vec{r} = (-2, -2, -2)$
(C) $\vec{r} = (-3, -3, -3)$ (D) $\vec{r} = (-4, -4, -4)$

4. A system of two blocks A and B (having masses m and $2m$) and a wedge C (having mass $2m$) is released from rest as shown in figure. The displacement of wedge C when block B slides down the plane, a distance 10 cm is (neglect friction)



(A) $2\sqrt{2}$ cm (B) $3\sqrt{2}$ cm
(C) $4\sqrt{2}$ cm (D) $5\sqrt{2}$ cm

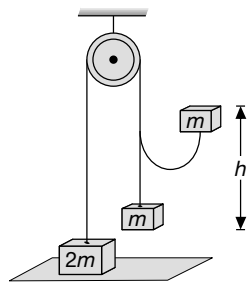
5. A body of mass m_0 is placed on a smooth horizontal surface. The mass of the body is decreasing exponentially with disintegration constant λ. Assuming that the mass is ejected backward with a relative velocity u. Assuming the body to be at rest initially, the speed of body at time t is

(A) $ue^{\lambda t}$ (B) $u\lambda t$
(C) $ue^{-\lambda t}$ (D) $u(1 - e^{-\lambda t})$

6. A bullet, moving with a speed of 150 ms^{-1}, strikes a wooden plank. After passing through the plank, its speed becomes 125 ms^{-1}. Another bullet of the same mass and size strikes the plank with a speed of 90 ms^{-1}. Its speed after passing through the plank would be

(A) 25 ms^{-1} (B) 35 ms^{-1}
(C) 50 ms^{-1} (D) 70 ms^{-1}

7. A mass $2m$ lying on a horizontal table is attached to a light inextensible string which passes over a smooth pulley and carries a mass m at the other end. If the mass m is raised vertically through a distance h and is then dropped, then the speed with which the mass $2m$ begins to rise is



(A) $\frac{\sqrt{gh}}{2}$ (B) $\sqrt{gh}$

(C) $\sqrt{2gh}$ (D) $\frac{\sqrt{2gh}}{3}$

8. An isolated particle of mass m is moving in horizontal plane $(x\text{-}y)$, along the x-axis, at a certain height above the ground. It suddenly explodes into two fragments of masses $\frac{m}{4}$ and $\frac{3m}{4}$. An instant later, the smaller fragment is at $y = +15$ cm. The larger fragment at this instant is at

(A) $y = -5$ cm (B) $y = +20$ cm

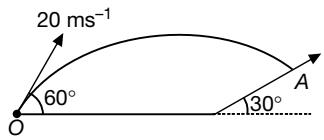
(C) $y = +5$ cm (D) $y = -20$ cm

9. A non-zero external force acts on a system of particles. The velocity and the acceleration of the centre of mass are found to be v_0 and a_0 at an instant t. It is possible that

(A) $v_0 = 0$, $a_0 = 0$ (B) $v_0 = 0$, $a \neq 0$

(C) $v_0 \neq 0$, $a_0 = 0$ (D) $v_0 \neq 0$, $a_0 \neq 0$

10. A ball is projected from the point O with velocity $20\ \text{ms}^{-1}$ at an angle of 60° with horizontal as shown in figure.



At highest point of its trajectory it strikes a smooth plane of inclination 30° at point A. The collision is perfectly inelastic. The maximum height from the ground attained by the ball is $(\text{Take } g = 10\ \text{ms}^{-2})$

(A) 15 m (B) 18.75 m

(C) 20.25 m (D) 22.5 m

11. A small block of mass M moves with a velocity of $5\ \text{ms}^{-1}$ towards another block of same mass M placed at a distance of 2 m on a rough horizontal surface. The coefficient of friction between the blocks and ground is 0.25 and the collision between the two blocks is elastic. The separation between the blocks, when both of them come to rest, is $(\text{Take } g = 10\ \text{ms}^{-2})$

(A) 1.5 m (B) 2 m

(C) 3 m (D) 4 m

12. After perfectly inelastic collision between two identical particles moving with same speed in different directions, the speed of the particles become half the initial speed. The angle between the velocities of the two before collision is

(A) 30° (B) 45°

(C) 60° (D) 120°

13. In one dimensional collision of two particles, the velocities are interchanged when

(i) collision is elastic and masses are equal

(ii) collision is inelastic but masses are unequal

Select the correct alternative

(A) only (i) is correct

(B) only (ii) is correct

(C) both (i) and (ii) are correct

(D) both (i) and (ii) are wrong

14. A hemisphere and a solid cone have a common base. The centre of mass of the common structure coincides with the centre of the common base. If R is the radius of hemisphere and h is height of the cone, then

(A) $\frac{h}{R} = \sqrt{3}$ (B) $\frac{h}{R} = \frac{1}{\sqrt{3}}$

(C) $\frac{h}{R} = 3$ (D) $\frac{h}{R} = \frac{1}{3}$

15. Velocity of centre of mass of two particles is v and the sum of the masses of two particles is m. Kinetic energy of the system is K. Then

(A) $K = \frac{1}{2}mv^2$ (B) $K < \frac{1}{2}mv^2$

(C) $K \geq \frac{1}{2}mv^2$ (D) $K > \frac{1}{2}mv^2$

16. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from the right edge of the plate with centres for both lying on the same line. The position of the centre of mass of the remaining portion is

(A) 9 cm to the right of the centre of full plate (thereafter referred to as "Origin").

(B) 9 cm to the left of the "Origin".

(C) $\frac{49}{9}$ cm to the right of the "Origin".

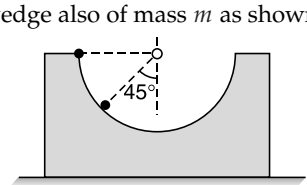
(D) $\frac{49}{9}$ cm to the left of the "Origin".

17. A neutron collides head-on and elastically with an atom of mass number A, which is initially at rest. The fraction of kinetic energy retained by the neutron is

(A) $\left(\frac{A}{A+1}\right)^2$ (B) $\left(\frac{A-1}{A+1}\right)^2$

(C) $\left(\frac{A-1}{A}\right)^2$ (D) $\left(\frac{A-1}{A+1}\right)$

18. A ball of mass m is released from point A inside a smooth wedge also of mass m as shown in Figure.



The speed of the wedge when the ball reaches point B is

(A) $\sqrt{\frac{gR}{3\sqrt{2}}}$ (B) $\sqrt{2gR}$

(C) $\sqrt{\frac{5gR}{2\sqrt{3}}}$ (D) $\sqrt{\frac{3gR}{2}}$

19. A ball moving with a speed v towards north, collides with an identical ball, moving with a speed v towards east. After collision the two balls stick together and move towards north-east. The speed of the combination is

(A) v (B) $v\sqrt{2}$

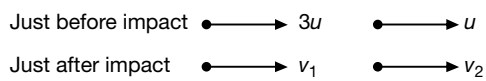
(C) $\frac{v}{\sqrt{2}}$ (D) $\frac{v}{2}$

20. In a one dimensional collision between two identical particles A and B, B is stationary and A has momentum P before impact. During impact B gives an impulse J to A. Then coefficient of restitution between the two is

(A) $\frac{2J}{P}+1$ (B) $\frac{2J}{P}-1$

(C) $\frac{J}{P}+1$ (D) $\frac{J}{P}-1$

21. Two smooth objects with a coefficient of restitution e, collide directly and bounce as shown, then Newton's Law of Restitution gives



(A) $4eu = v_2 + v_1$ (B) $2eu = v_1 - v_2$

(C) $2eu = v_2 - v_1$ (D) $2eu = v_2 + v_1$

22. Two particles of equal masses have velocities $\vec{v}_1 = 2\hat{i}\ \text{ms}^{-1}$ and $\vec{v}_2 = 2\hat{j}\ \text{ms}^{-1}$. First particle has an acceleration $\vec{a}_1 = \left(3\hat{i}+3\hat{j}\right)\ \text{ms}^{-2}$ while the acceleration of the other particle is zero. The centre of mass of the two particles moves in a

(A) circle (B) parabola

(C) straight line (D) ellipse

23. A rope thrown over a pulley has a ladder with a man of mass m on one of its ends and a counterbalancing mass M on its other end. The man climbs with a velocity v_r relative to ladder. Ignoring the masses of the pulley and the rope as well as the friction on the pulley axis, the velocity of the centre of mass of this system is

(A) $\frac{m}{M}v_r$ (B) $\frac{m}{2M}v_r$

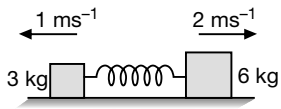
(C) $\frac{M}{m}v_r$ (D) $\frac{2M}{m}v_r$

24. Consider a system of two identical particles. One of the particle is at rest and the other has an acceleration a. The centre of mass has an acceleration

(A) ZERO (B) $\frac{1}{2}\vec{a}$

(C) $\vec{a}$ (D) $2\vec{a}$

25. Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant $k = 200\ \text{Nm}^{-1}$. Initially the spring is unstretched and velocities of $1\ \text{ms}^{-1}$ and $2\ \text{ms}^{-2}$ are imparted in opposite directions to the respective blocks as shown in figure.



The maximum extension of the spring will be

(A) 15 cm (B) 20 cm

(C) 25 cm (D) 30 cm

26. A ball moving horizontally with speed v strikes the bob of a simple pendulum at rest. The mass of the bob is equal to that of the ball. It the collision is elastic the bob will rise to a height

(A) $\frac{v^2}{g}$ (B) $\frac{v^2}{2g}$

(C) $\frac{v^2}{4g}$ (D) $\frac{v^2}{8g}$

27. In PROBLEM 26, if the collision is completely inelastic, the height to which the ball-bob system will rise is

(A) $\frac{v^2}{g}$ (B) $\frac{v^2}{2g}$

(C) $\frac{v^2}{4g}$ (D) $\frac{v^2}{8g}$

28. A particle of mass m collides elastically with a stationary particle and continues to move at an angle of 45° with respect to the original direction. The second particle also recoils at an angle of 45° to this direction. The mass of the second particle is

(A) m (B) $\sqrt{2}m$

(C) $\frac{m}{\sqrt{2}}$ (D) $\frac{m}{2}$

29. Two blocks m_1 and m_2, having masses 10 kg and 5 kg respectively, are placed on a frictionless horizontal surface and are connected by a light spring of force constant $5\ \text{Nm}^{-1}$. m_1 is in contact with a rigid wall. m_2 is pushed through a distance of 4 cm towards m_1 and then released. The velocity of the centre of mass (in cms^{-1}) of the system when m_1 breaks off the wall is

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$

(C) 2 (D) 4

30. A steel ball of radius 2 cm is initially at rest on a horizontal frictionless surface. It is struck head-on and elastically by another steel ball of radius 4 cm moving with a speed of 81 cms^{-1}. The speeds of the two balls after the collision are, respectively

(A) 63 cms^{-1}, 144 cms^{-1} (B) 144 cms^{-1}, 63 cms^{-1}

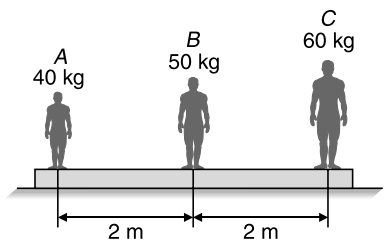
(C) 126 cms^{-1}, 72 cms^{-1} (D) 72 cms^{-1}, 126 cms^{-1}

31. A 1 kg ball, moving at 12 ms^{-1}, collides head-on with a 2 kg ball moving in the opposite direction at 24 ms^{-1}. If the coefficient of restitution is $\frac{2}{3}$, then the energy lost in the collision is

(A) 60 J (B) 120 J

(C) 240 J (D) 480 J

32. Three men A, B and C of mass 40 kg, 50 kg and 60 kg are standing on a plank of mass 90 kg, which is kept on a smooth horizontal plane. If A and C exchange their positions then B



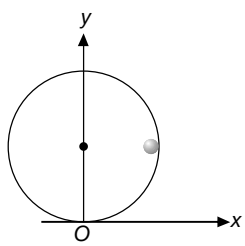
(A) shifts $\frac{1}{3}$ m, towards left

(B) shifts $\frac{1}{3}$ m, towards right

(C) shifts $\frac{5}{3}$ m, towards left

(D) will not shift at all

33. A small sphere of radius R held against the inner surface of a smooth spherical shell of radius $6R$ as shown in figure.



The masses of the shell and small spheres are $4M$ and M respectively. This arrangement is placed on a smooth horizontal table. The small sphere is now released. The x-coordinate of the centre of the shell when the smaller sphere reaches the other extreme position is

(A) R (B) $2R$

(C) $3R$ (D) $4R$

34. A ball hits the floor and rebounds after an inelastic collision. In this case

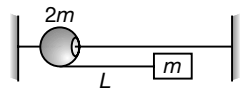
(A) the momentum of the ball just after the collision is the same as that just before the collision

(B) the mechanical energy of the ball remains the same in the collision

(C) the total momentum of the ball and the earth is conserved

(D) the total energy of the ball and the earth is conserved

35. A bead can slide on a smooth straight wire and a particle of mass m is attached to the bead by a light string of length L. The particle is held in contact with the wire with the string taut and is then let fall. If the bead has mass $2m$. Then, when the string makes an angle θ with the wire the bead will have slipped a distance



(A) $\frac{L}{2}(1-\cos\theta)$ (B) $L(1-\cos\theta)$

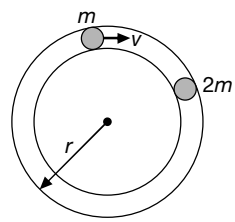
(C) $\frac{L}{6}(1-\cos\theta)$ (D) $\frac{L}{3}(1-\cos\theta)$

36. A particle of mass m moving with a velocity $\vec{v}$ makes a head-on elastic collision with another identical particle which is initially at rest. The velocity of the first particle after the collision is

(A) $-\vec{v}$ (B) $\vec{v}$

(C) $\frac{\vec{v}}{2}$ (D) ZERO

37. A particle of mass m moving with a speed v hits elastically another stationary particle of mass $2m$ on a smooth horizontal circular tube of radius r. The time in which the next collision will take place is equal to



(A) $\frac{\pi r}{v}$ (B) $\frac{2\pi r}{v}$

(C) $\frac{3\pi r}{2v}$ (D) $\frac{4\pi r}{v}$

38. A body of mass 2.9 kg is suspended from a string of length 2.5 m and is at rest. A bullet of mass 100 g, moving horizontally with a speed of 150 ms^{-1}, strikes and sticks to it. What is the maximum angle made by the string with the vertical after the impact? $(g = 10\ ms^{-2})$

(A) 30° (B) 45°

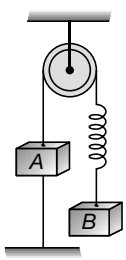
(C) 60° (D) 90°

39. A ball of mass m collides head-on and elastically with a ball of mass nm, initially at rest. The fraction of the incident energy transferred to the heavier ball is

(A) $\frac{n}{n+1}$ (B) $\frac{n}{(n+1)^2}$

(C) $\frac{2n}{(n+1)^2}$ (D) $\frac{4n}{(n+1)^2}$

40. In the figure shown, the blocks A and B have equal masses m. The system is released from rest with the spring unstretched. The string between A and ground is cut, when there is maximum extension in the spring. The acceleration of centre of mass of the two blocks at this instant is



(A) g (B) $\frac{g}{2}$

(C) $2g$ (D) ZERO

41. A particle of mass $3m$ is projected from the ground at some angle with horizontal such that its horizontal range is R. At the highest point of its path it breaks into two pieces of masses m and $2m$ respectively. The smaller mass comes to rest. The larger mass finally falls at a distance x from the point of projection where x equals

(A) $\frac{3}{4}R$ (B) $\frac{3}{2}R$

(C) $\frac{5}{4}R$ (D) $3R$

42. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 ms^{-1} to the heavier block in the direction of the lighter block. The velocity of the centre of mass is

(A) 30 ms^{-1} (B) 20 ms^{-1}

(C) 10 ms^{-1} (D) 5 ms^{-1}

43. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 ms^{-1} to the heavier block in the direction of the lighter block. The velocity of the center of mass is

(A) 30 ms^{-1} (B) 20 ms^{-1}

(C) 10 ms^{-1} (D) 5 ms^{-1}

44. A man hangs from a rope attached to a hot-air balloon. The mass of the man is greater than the mass of the balloon and its contents. The system is stationary in air. If the man now climbs up to the balloon using the rope, the centre of mass of the 'man plus balloon' system will

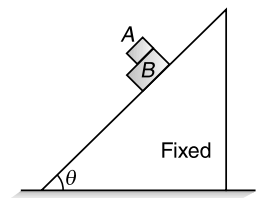
(A) remain stationary

(B) move up

(C) move down

(D) first move up and then returns to its initial position

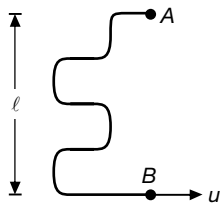
45. A block A slides over another block B which is placed over a smooth inclined plane as shown in figure. The coefficient of friction between the two blocks A and B is μ. Mass of block B is twice the mass of block A. The acceleration of the centre of mass of two blocks is



(A) $\frac{g\sin\theta - \mu g\cos\theta}{3}$ (B) $\frac{2g\sin\theta - \mu g\cos\theta}{3}$

(C) $\frac{g\sin\theta}{3}$ (D) $g\sin\theta$

46. Two particles A and B each of mass m are attached by a light inextensible string of length $2l$. The whole system lies on a smooth horizontal table with B initially at a distance l from A. The particle at end B is projected across the table with speed u perpendicular to AB. Velocity of ball A just after the string is taut is



(A) $\frac{u\sqrt{3}}{4}$ (B) $u\sqrt{3}$

(C) $\frac{u\sqrt{3}}{2}$ (D) $\frac{1}{2}u$

47. When a ball collides head-on and elastically with an identical ball on a horizontal frictionless surface, the first one comes to rest while the second one moves with the same velocity as that of the first ball before collision. This result

(A) can be derived by using momentum conservation alone

(B) can be derived by using energy conservation alone

(C) cannot be derived by using any of the two conservation principles

(D) can be derived by using both conservation of energy and momentum

48. In gravity-free space, a man of mass M standing at a height h above the floor throws a ball of mass m straight down with a speed u. When the ball reaches the floor, the distance of the man above the floor will be

(A) $h\left(1+\frac{m}{M}\right)$

(B) $h\left(2-\frac{m}{M}\right)$

(C) $2h$

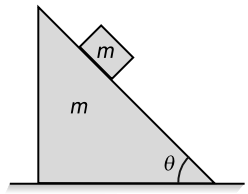
(D) a function of m, M, h and u

49. A body of mass m_1 moving with velocity u_1 collides with another body of mass m_2 at rest. After the collision the velocity of mass m_1 is $\frac{2}{3}u_1$, what is the ratio of $\frac{m_1}{m_2}$?

(A) 1 : 25 (B) 1 : 5

(C) 5 : 1 (D) 25 : 1

50. A block of mass m slides down an inclined wedge of same mass m shown in figure. Friction is absent everywhere. Acceleration of centre of mass of the block and wedge is



(A) ZERO (B) $\frac{g\cos^2\theta}{(1+\sin^2\theta)}$

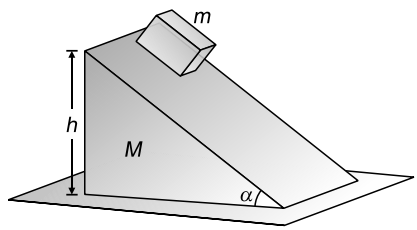
(C) $\frac{g\sin^2\theta}{(1+\sin^2\theta)}$ (D) $\frac{g\sin\theta}{(1+\cos\theta)}$

51. All the particles of a body are situated at a distance R from the origin. The distance of the centre of mass of the body from the origin is

(A) $=R$ (B) $\leq R$

(C) $>R$ (D) $\geq R$

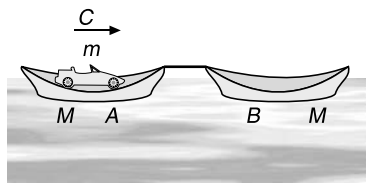
52. In the arrangement shown, the mass m starts from the top of the wedge of mass M. The system $(m+M)$ was initially at rest. Assume all surfaces in contact to be frictionless. The distance moved by M when m just reaches the floor is



(A) $\frac{mh}{M+m}$ (B) $\frac{mh\tan\alpha}{M+m}$

(C) $\frac{mh\cot\alpha}{M+m}$ (D) $\frac{mh\sin\alpha}{M+m}$

53. A car C of mass m is initially at rest on the boat A of mass M tied to the identical boat B of same, mass m through a massless inextensible string as shown in Figure.



The car accelerates from rest to velocity v_0 with respect to boat A in time t_0 sec. At time t_0 the car applies brake and comes to rest relative to boat in negligible time. Neglect friction between boat and water, the velocity of boat A just after applying brake is

(A) $\frac{Mmv_0}{(2M+m)(M+m)}$ (B) $\frac{Mmv_0}{(M+2m)(M+m)}$

(C) $\frac{2Mmv_0}{(M+2m)(M+m)}$ (D) ZERO

54. A gardener waters the plants by a pipe of diameter 1 cm. The water comes out at the rate of 20 ccs^{-1}. The reactionary force exerted on the hand of the gardener is

(A) ZERO (B) $1.62\times10^{-3}\text{ N}$

(C) $2.54\times10^{-5}\text{ N}$ (D) $5.1\times10^{-3}\text{ N}$

55. A particle of mass m is made to move with uniform speed v along the perimeter of a regular polygon of $2n$ sides. The magnitude of impulse applied at each corner of the polygon is

(A) $mv\sin\left(\frac{\pi}{2n}\right)$ (B) $mv\cos\left(\frac{\pi}{2n}\right)$

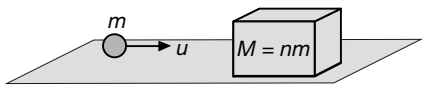
(C) $2mv\cos\left(\frac{\pi}{2n}\right)$ (D) $2mv\sin\left(\frac{\pi}{2n}\right)$

56. Two particles, each of mass m, moving in opposite directions with equal speeds along the same straight line strike elastically. If the velocities of the first and the second particle before collision are denoted by $+\vec{v}$, and $-\vec{v}$, respectively, then if there is no change in the line of motion of the two particles, their velocities after collision are

(A) $-\vec{v}$ and $+\vec{v}$ (B) $+\vec{v}$ and $-\vec{v}$

(C) 0 and $2\vec{v}$ (D) $2\vec{v}$ and 0

57. A bullet of mass m moving with velocity u passes through a wooden block having mass $M=nm$. The block is resting on a smooth horizontal floor. After passing through the block, the velocity of bullet is v. Its velocity relative to block is



(A) $\frac{nv-u}{n+1}$ (B) $\frac{nu-v}{n+1}$

(C) $\frac{(1+n)v-u}{n}$ (D) $\frac{(n+1)u+v}{2n+1}$

58. A girl throws a ball with initial velocity v at an inclination of $45°$. The ball strikes a smooth vertical wall at a horizontal distance d from the girl and after rebounding returns to her hand. The coefficient of restitution between the wall and the ball is

(A) $\frac{gd}{v^2}$ (B) $\frac{v^2}{gd}$

(C) $\frac{gd}{v^2-gd}$ (D) $\frac{v^2-gd}{v^2}$

59. A uniform sphere is placed on a smooth horizontal surface and a horizontal force F is applied on it at a distance h above the surface. The acceleration of the centre

(A) is maximum at $h=0$

(B) is maximum when $h=R$

(C) is maximum when $h=2R$

(D) is independent of h

60. A ball of mass m approaches a wall of mass $M(\gg m)$ with speed 4 ms^{-1} along the normal to the wall. The speed of wall is 1 ms^{-1} towards the ball. The speed of the ball after an elastic collision with the wall is

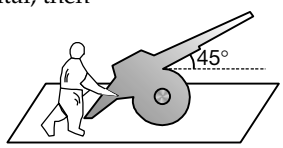
(A) 5 ms^{-1} away from the wall

(B) 9 ms^{-1} away from the wall

(C) 3 ms^{-1} away from the wall

(D) 6 ms^{-1} away from the wall

61. A bullet is fired from a gun whose barrel is inclined at an angle of $45°$ with horizontal. When the bullet leaves the barrel, it will be travelling at an angle ϕ to the horizontal, then



(A) $\phi=45°$ (B) $\phi<45°$

(C) $\phi>45°$ (D) $\phi=0$

62. Mass is non-uniformly distributed on the circumference of a ring of radius a and centre at origin. Let b be the distance of centre of mass of the ring from origin. Then

(A) $b=a$ (B) $b<a$

(C) $b>a$ (D) $0\le b\le a$

63. A bomb of mass 1 kg, initially at rest, explodes into three fragments of masses in the ratio $1:1:3$. The two pieces of equal mass fly off perpendicular to each other, each with a speed of 30 ms^{-1}. What is the velocity of the heavier fragment?

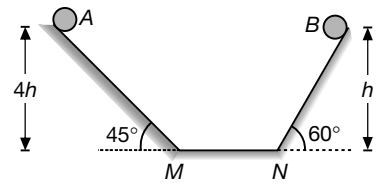
(A) $10\sqrt{2}\text{ ms}^{-1}$ at $45°$ with each of the other two fragments

(B) $10\sqrt{2}\text{ ms}^{-1}$ at $135°$ with each of the other two fragments

(C) 20 ms^{-1} at $45°$ with each of the other two fragments

(D) 20 ms^{-1} at $135°$ with each of the other two fragments

64. Two identical balls A and B are released from the positions shown in figure. They collide elastically on horizontal portion MN. The ratio of heights attained by A and B after collision will be (neglect friction)



(A) 1 : 4 (B) 2 : 1
(C) 4 : 13 (D) 2 : 5

65. From a circular disc of radius R, a square is cut out with the radius as its diagonal. The distance of the centre of mass of the remainder from the centre of the disc is

(A) $\frac{R}{\pi-2}$ (B) $\frac{R}{\pi}$
(C) $\frac{R}{2(2\pi-1)}$ (D) $\frac{R}{2(\pi-1)}$

66. A balloon has 2 g of air. If a small hole is pierced into it, the air comes out with a speed of $2\ \text{ms}^{-1}$. If the balloon shrinks completely in 5 s, the average thrust experienced by the balloon is

(A) 8×10^{-1} N (B) 8×10^{-2} N
(C) 8×10^{-3} N (D) 8×10^{-4} N

67. A ball A, moving with a speed u, collides directly with another similar ball B moving with a speed v in the opposite direction. A comes to rest after the collision. If the coefficient of restitution is e then $\frac{u}{v}$ is

(A) $\frac{1+e}{1-e}$ (B) $\frac{1-e}{1+e}$
(C) $\frac{e}{1-e}$ (D) $\frac{e}{1+e}$

68. A canon ball is fired with a velocity of $200\ \text{ms}^{-1}$ at an angle of $60°$ with the horizontal. At the highest point it explodes into three equal fragments. One goes vertically upwards with a velocity of $100\ \text{ms}^{-1}$ and other goes vertically downwards with $100\ \text{ms}^{-1}$. The third one moves with a velocity of

(A) $100\ \text{ms}^{-1}$ horizontally
(B) $300\ \text{ms}^{-1}$ horizontally
(C) $200\ \text{ms}^{-1}$ at $60°$ with horizontal
(D) $300\ \text{ms}^{-1}$ at $60°$ with horizontal

69. The maximum offset that can be obtained by piling up three identical bricks of length l is

(A) $\frac{2}{3}l$ (B) $\frac{4}{3}l$
(C) $\frac{5}{6}l$ (D) $\frac{11}{12}l$

70. A ball A of mass 1 kg, moving with a speed of $12\ \text{ms}^{-1}$, collides obliquely and elastically with another ball B which was initially at rest. Ball A then moves off at right angles to its initial direction with a speed of $5\ \text{ms}^{-1}$. The momentum of ball B after collision is

(A) $5\ \text{kgms}^{-1}$ (B) $11\ \text{kgms}^{-1}$
(C) $13\ \text{kgms}^{-1}$ (D) $17\ \text{kgms}^{-1}$

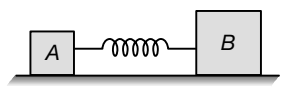
71. The displacement of a particle of mass 2 kg moving in a straight line varies with time as $x=(2t^3+2)m$. Impulse of the force acting on the particle over a time interval between $t=0$ and $t=1$ s is

(A) 10 Ns (B) 12 Ns
(C) 8 Ns (D) 6 Ns

72. A man of mass m moves with a constant speed on a plank of mass M and length L kept initially at rest on a frictionless horizontal surface, from one end to the other in time t. The speed of the plank relative to ground while man is moving, is

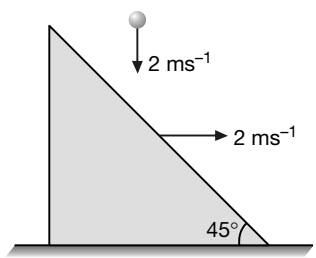
(A) $\frac{L}{t}\left(\frac{M}{m}\right)$ (B) $\frac{L}{t}\left(\frac{m}{M+m}\right)$
(C) $\frac{L}{t}\left(\frac{M}{M-m}\right)$ (D) None of these

73. Two blocks A and B of mass m and $2m$ are connected by a massless spring of force constant k and are placed on a smooth horizontal plane. The spring is stretched by an amount x and then released. The relative velocity of the blocks when the spring comes to its natural length is



(A) $\left(\sqrt{\frac{3k}{2m}}\right)x$ (B) $\left(\sqrt{\frac{2k}{3m}}\right)x$
(C) $\sqrt{\frac{2kx}{m}}$ (D) $\sqrt{\frac{3km}{2x}}$

74. A small ball falling vertically downwards with constant velocity $2\ \text{ms}^{-1}$ strikes elastically an inclined plane moving with velocity $2\ \text{ms}^{-1}$ as shown in figure. The velocity of rebound of the ball with respect to ground is



(A) $2\ \text{ms}^{-1}$ (B) $2\sqrt{2}\ \text{ms}^{-1}$

(C) $4\ \text{ms}^{-1}$ (D) $2\sqrt{5}\ \text{ms}^{-1}$

75. A $6000\ \text{kg}$ rocket is set for vertical firing. If the exhaust speed is $1000\ \text{ms}^{-1}$, the amount of gas that must be ejected per second to supply the thrust needed to overcome the weight of the rocket is $(g = 10\ \text{ms}^{-2})$

(A) 30 kg (B) 60 kg

(C) 75 kg (D) 90 kg

76. In PROBLEM 75, the amount of gas that must be ejected to give the rocket an initial upward acceleration of $20\ \text{ms}^{-2}$ is

(A) $90\ \text{kgs}^{-1}$ (B) $120\ \text{kgs}^{-1}$

(C) $150\ \text{kgs}^{-1}$ (D) $180\ \text{kgs}^{-1}$

77. Two particles A and B, initially at rest, move towards each other under a mutual force of attraction. At the instant when the speed of A is V and the speed of B is $2V$, the speed of the centre of mass of the system is

(A) ZERO (B) V

(C) $1.5V$ (D) $3V$

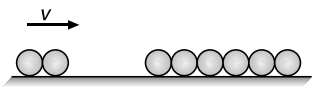
78. Ball A collides with an another identical ball B at rest as shown in Figure.

The value of coefficient of restitution e, for which the velocity of ball B becomes two times that of A after collision is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) $\frac{1}{6}$

79. Six identical steel balls are lined up in a straight frictionless groove made on a horizontal surface. Two similar balls moving with speed v collide elastically with the row of 6 balls from the left. Then



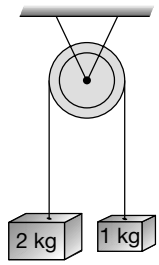
(A) all the 8 balls will move with a speed $\frac{v}{8}$ each

(B) all the 6 balls in the row will move with a speed $\frac{v}{6}$ each and the two incident balls will come to rest

(C) two balls from the row will move with a speed v other balls remaining at rest

(D) one ball from the row will move with a speed $2v$, other balls remaining at rest

80. Two blocks of masses $2\ \text{kg}$ and $1\ \text{kg}$ respectively are tied to the ends of a string which passes over a light frictionless pulley. The masses are held at rest at the same horizontal level and then released. The distance traversed by centre of mass in 2 second is $(\text{Take } g = 10\ \text{ms}^{-2})$



(A) 1.42 m (B) 2.22 m

(C) 3.12 m (D) 3.33 m

81. A shell is fired from a canon with a velocity v at an angle θ with the horizontal. At the highest point it explodes into two pieces of equal masses. One of the pieces retraces its path to the canon. The speed of the other piece immediately after the explosion is

(A) $3v\cos\theta$ (B) $2v\cos\theta$

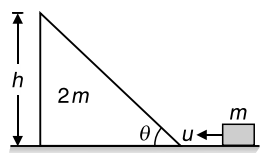
(C) $\frac{3}{2}v\cos\theta$ (D) $v\cos\theta$

82. A force $\vec{F} = (2\hat{i} + \hat{j} + 3\hat{k})N$ acts on a particle of mass $1\ \text{kg}$ for $t = 2\ \text{s}$. If initial velocity of particle is $\vec{u} = (2\hat{i} + \hat{j})\ \text{ms}^{-1}$, then the speed of particle at the $t = 2\ \text{s}$ will be

(A) $4\ \text{ms}^{-1}$ (B) $6\ \text{ms}^{-1}$

(C) $9\ \text{ms}^{-1}$ (D) $12\ \text{ms}^{-1}$

83. A block of mass m is pushed towards a movable wedge of mass $2m$ and height h with a velocity u. All surfaces are smooth. The minimum value of u for which the block will reach the top of the wedge is



(A) $\sqrt{\frac{3}{2}gh}$ (B) $\sqrt{3gh}$

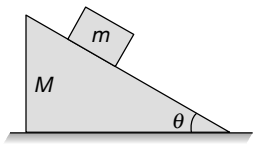
(C) $\sqrt{6gh}$ (D) $2\sqrt{gh}$

84. A moving particle of mass m makes a head-on elastic collision with a particle of mass $2m$ which is initially at rest. The fraction of the initial kinetic energy lost by the colliding particle is

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$

(C) $\frac{4}{9}$ (D) $\frac{8}{9}$

85. A block of mass m slides over a smooth wedge of mass M which is placed over a rough horizontal surface. If μ be the coefficient of friction between the wedge and the ground, then the centre of mass of the system will move towards left



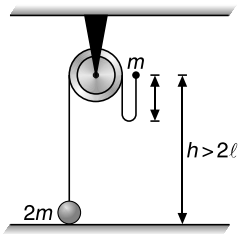
(A) if $mg\cos\theta\sin\theta > \mu(M+m)g$

(B) if $mg\sin\theta > \mu Mg$

(C) if $mg\cos\theta\sin\theta > \mu Mg$

(D) None of these

86. A heavy ball of mass $2m$ rests on the horizontal surface and the lighter balls of mass m is dropped from a height $h > 2l$ as shown in Figure. The upward velocity of the heavy ball at the instant when string gets taut is



(A) $\frac{4\sqrt{gl}}{3}$ (B) $\frac{2\sqrt{gl}}{3}$

(C) $\frac{\sqrt{gl}}{3}$ (D) $\frac{\sqrt{gl}}{2}$

87. A large tray of mass M has in it a cubical block of ice of mass m and edge length L. The ice starts melting. The shift in the height of centre of mass of system (ice + tray) is

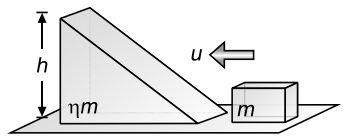
(A) $\frac{1}{2}\frac{mL}{(m+M)}$; upward

(B) $\frac{1}{2}\frac{ML}{(m+M)}$; upward

(C) $\frac{1}{2}\frac{mL}{(m+M)}$; downward

(D) $\frac{1}{2}\frac{ML}{(m+M)}$; downward

88. A block of mass m is pushed towards a movable wedge of mass ηm and height h, with a velocity u. Assume all surfaces to be smooth. The minimum value of u for which the block will reach the top of the wedge is



(A) $\sqrt{2gh}$ (B) $\eta\sqrt{2gh}$

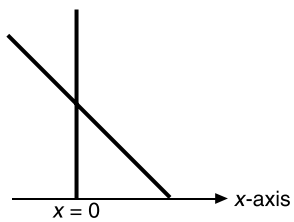
(C) $\sqrt{2gh\left(1+\frac{1}{\eta}\right)}$ (D) $\sqrt{2gh\left(1-\frac{1}{\eta}\right)}$

89. A particle of mass m_1 moves with speed v and collides head on with a stationary particle of mass m_2. If e be the coefficient of restitution and the first particle continues to move in the same direction, then $\frac{m_1}{m_2}$ is

(A) e (B) $> e$

(C) $< e$ (D) $> e^2$

90. A uniform rod of length l is kept vertically on a rough horizontal surface at $x = 0$. It is rotated slightly and released. When the rod finally falls on the horizontal surface the lower end will remain at



(A) $x = 0$ (B) $x = \frac{l}{2}$

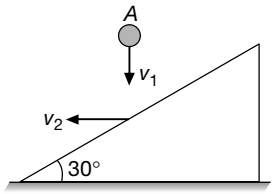
(C) $x < \frac{l}{2}$ (D) $x > \frac{l}{2}$

91. Two spheres of masses M and $2M$ are initially at rest at a distance R apart. Due to mutual force of attraction they approach each other. When they are at separation $\frac{R}{2}$, the acceleration of their centre of mass would be

(A) 0 (B) g ms^{-2}

(C) $3g$ ms^{-2} (D) $12g$ ms^{-2}

92. A ball A is falling vertically downwards with velocity v_1. It strikes elastically a wedge moving horizontally with velocity v_2 as shown in figure.



The ratio $\frac{v_2}{v_1}$, so that the ball bounces back vertically upwards relative to the wedge is

(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$

(C) 2 (D) $\frac{1}{2}$

93. The mass per unit length of a non-uniform rod of length L varies as $m = m_0\frac{x^2}{L}$, where m_0 is a constant and x is the distance of any point on the rod measured from one end. The centre of mass of the rod is at

(A) $\frac{L}{2}$ (B) $\frac{L}{3}$

(C) $\frac{3L}{2}$ (D) $\frac{3L}{4}$

94. A gun fires a shell and recoils horizontally. If the shell travels along the barrel with speed v, the ratio of speed with which the gun recoils if
CASE-1 the barrel is horizontal
CASE-2 inclined at an angle of $30°$ with the horizontal is

(A) 1 (B) $\frac{2}{\sqrt{3}}$

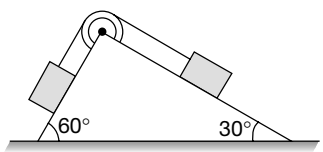
(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

95. A radioactive nucleus of mass number A, initially at rest, emits an α particle with speed v. The recoil speed of the daughter nucleus is

(A) $\frac{4v}{A-4}$ (B) $\frac{4v}{A}$

(C) $\frac{(A-4)v}{A}$ (D) $\frac{(A-4)v}{4}$

96. Two blocks of equal masses are tied to a light string, which passes over a massless pulley as shown in figure.



The magnitude of acceleration of centre of mass of both the blocks is (neglect friction everywhere)

(A) $\left(\frac{\sqrt{3}-1}{4\sqrt{2}}g\right)$ (B) $(\sqrt{3}-1)g$

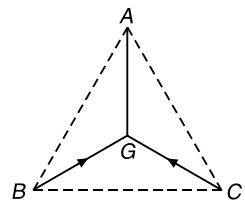
(C) $\frac{g}{2}$ (D) $\left(\frac{\sqrt{3}-1}{\sqrt{2}}\right)g$

97. A tennis ball bounces down a flight of stairs striking each step in turn and rebounding to the height of the step above. The coefficient of restitution is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $+\frac{1}{4}$ (D) 1

98. Three particles A, B and C of equal masses, moving with the same speed v along the medians of an equilateral triangle, collide at the centroid G of the triangle. After collision, A comes to rest and B retraces its path with speed v. The speed of C after the collision is



(A) ZERO (B) $\frac{v}{2}$ along GB

(C) v along BG (D) v along CG

99. A bomb of mass 12 kg, initially at rest, explodes into two pieces of masses 4 kg and 8 kg. The speed of the 8 kg mass is 6 ms^{-1}. The kinetic energy of the 4 kg mass is

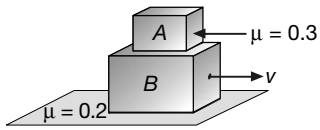
(A) 32 J (B) 48 J
(C) 144 J (D) 288 J

100. The momentum of a particle is $\vec{P} = \vec{A} + \vec{B}t^2$, where $\vec{A}$ and $\vec{B}$ are constant perpendicular vectors. The force acting on the particle when its acceleration makes an angle of $45°$ with the velocity is

(A) ZERO (B) $\left(2\sqrt{\frac{B}{A}}\right)\vec{A}$

(C) $\left(2\sqrt{\frac{A}{B}}\right)\vec{B}$ (D) $2\vec{B}$

101. The blocks A (of mass m) and B (of mass $2m$), in the given arrangement shown, are given together a horizontal velocity towards right. If a_{cm} be the subsequent acceleration of the centre of mass of the system of blocks then a_{cm} equals



(A) $0\ \text{ms}^{-2}$ (B) $2\ \text{ms}^{-2}$

(C) $\frac{5}{3}\ \text{ms}^{-2}$ (D) $\frac{7}{3}\ \text{ms}^{-2}$

102. A moving mass of $8\ \text{kg}$ collides elastically with a stationary mass of $2\ \text{kg}$. If E be the initial kinetic energy of the mass, the kinetic energy left with it after collision will be

(A) $0.80E$ (B) $0.64E$

(C) $0.36E$ (D) $0.08E$

103. A particle of mass 1 kg is projected with velocity $u = 40\ \text{ms}^{-1}$ at an angle of $30°$ with horizontal. The change in linear momentum of the particle after time $t = 1\ \text{s}$ is $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$

(A) $7.5\ \text{kgms}^{-1}$ (B) $15\ \text{kgms}^{-1}$

(C) $10\ \text{kgms}^{-1}$ (D) $20\ \text{kgms}^{-1}$

104. A projectile of mass $3\ \text{m}$ explodes at the highest point of its path and breaks into three equal parts such that one part retraces its path and the second one comes to rest. The range of the projectile was $100\ \text{m}$ if no explosion would have taken place. The distance of the third part from the point of projection when it finally lands on the ground is

(A) 100 m (B) 150 m

(C) 250 m (D) 300 m

105. A $6\ \text{kg}$ box sled is travelling on ice at a speed of $9\ \text{ms}^{-1}$ when a $12\ \text{kg}$ packet is dropped into it vertically. The velocity of the sled will now be

(A) $3\ \text{ms}^{-1}$ (B) $4\ \text{ms}^{-1}$

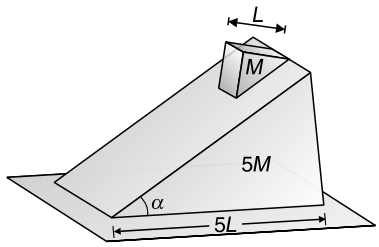
(C) $6\ \text{ms}^{-1}$ (D) $8\ \text{ms}^{-1}$

106. A smooth sphere is moving on a horizontal surface with velocity vector $2\hat{i} + 2\hat{j}$ immediately before it hits a vertical wall. The wall is parallel to $\hat{j}$ vector and the coefficient of restitution between the sphere and the wall is $e = \frac{1}{2}$. The velocity vector of the sphere after it hits the wall is

(A) $\hat{i} - \hat{j}$ (B) $-\hat{i} + 2\hat{j}$

(C) $-\hat{i} - \hat{j}$ (D) $2\hat{i} - \hat{j}$

107. The figure shows a small prism of mass M sliding on the bigger prism of mass $5M$ from the top to the bottom. Assuming all surfaces to be frictionless and the system (big prism + small prism) to be at rest initially, the distance moved by the prism combination is



(A) $\frac{3}{2}L$ to left (B) $\frac{3}{2}L$ to right

(C) $\frac{2}{3}L$ to right (D) $\frac{2}{3}L$ to left

108. A drum major's baton consists of two masses $2M$ and M separated by a thin light rod of length L. The baton is thrown into the air. Neglecting air drag, if $\vec{R}$ is the position vector of the centre of mass of the baton then the equation of motion for the centre of mass is

(A) $\frac{d^2\vec{R}}{dt^2} = \frac{2\vec{g}}{3}$ (B) $\frac{d^2\vec{R}}{dt^2} = \frac{\vec{g}}{3}$

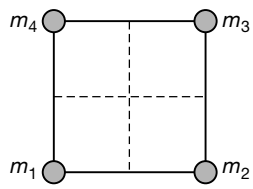
(C) $\frac{d^2\vec{R}}{dt^2} = \vec{g}$ (D) $\frac{d^2\vec{R}}{dt^2} = \frac{4\vec{g}}{3}$

109. Two billiard balls of the same size and mass are in contact on a billiard table. A third ball of the same size and mass strikes them symmetrically and comes to rest after the impact. The coefficient of restitution between the balls is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

110. Four particles of masses $m_1 = 2m$, $m_2 = 4m$, $m_3 = m$ and m_4 are placed at four corners of a square. What should be the value of m_4, so that the centre of mass of all the four particles are exactly at the centre of the square?



(A) $2m$ (B) $6m$

(C) $8m$ (D) None of these

111. A ball strikes a horizontal floor at an angle $\theta = 45°$. If the coefficient of restitution between the ball and the floor is $e = \frac{1}{2}$, then the fraction of its kinetic energy lost in collision is

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$

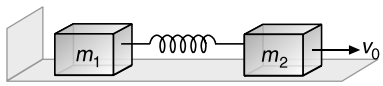
(C) $\frac{3}{8}$ (D) $\frac{5}{8}$

112. Two particles of masses m_1 and m_2 in projectile motion have velocities v_1 and v_2 respectively at time $t = 0$. They collide at time t_0. Their velocities become $\vec{v}_1'$ and $\vec{v}_2'$ at time $2t_0$, while still moving in air. The value of $\left|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)\right|$ is

(A) ZERO (B) $(m_1 + m_2)gt_0$

(C) $2(m_1 + m_2)gt_0$ (D) $\frac{1}{2}(m_1 + m_2)gt_0$

113. Two block m_1 and m_2 are pulled on a smooth horizontal surface, and are joined together with a spring of stiffness k as shown in Figure. Suddenly, the block m_2 receives a horizontal velocity v_0, then the maximum extension x_{max} in the spring is



(A) $v_0\sqrt{\frac{m_1}{k}}$ (B) $v_0\sqrt{\frac{2m_1m_2}{k(m_1 + m_2)}}$

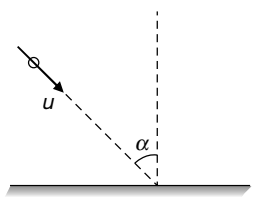
(C) $v_0\sqrt{\frac{m_1m_2}{2k(m_1 + m_2)}}$ (D) $v_0\sqrt{\frac{m_1m_2}{k(m_1 + m_2)}}$

114. A man of mass $80\ \text{kg}$ is riding on a small cart of mass $40\ \text{kg}$ which is rolling along a level floor at a speed of $2\ \text{ms}^{-1}$. He is running on the cart, so that his velocity relative to the cart is $3\ \text{ms}^{-1}$ in the direction opposite to the motion of cart. The speed of the centre of mass of the system is

(A) ZERO (B) $1\ \text{ms}^{-1}$

(C) $1.5\ \text{ms}^{-1}$ (D) $3\ \text{ms}^{-1}$

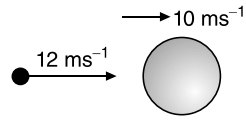
115. A ball of mass m collides with the ground at an angle α with the vertical. If the collision lasts for time t and e is coefficient of restitution between the ball and ground, then the average force exerted by the ground on the ball is



(A) $\frac{emu}{t}$ (B) $\frac{emu\cos\alpha}{t}$

(C) $\frac{2(1+e)mu\cos\alpha}{t}$ (D) $\frac{(1+e)mu\cos\alpha}{t}$

116. A light particle moving horizontally with a speed of $12\ \text{ms}^{-1}$ strikes head-on a very heavy block moving in the same direction at $10\ \text{ms}^{-1}$. The collision is one-dimensional and elastic. After the collision, the particle will



(A) move at $2\ \text{ms}^{-1}$ in its original direction

(B) move at $8\ \text{ms}^{-1}$ in its original direction

(C) move at $8\ \text{ms}^{-1}$ opposite to its original direction

(D) move at $12\ \text{ms}^{-1}$ opposite to its original direction

117. A small block of superdense material has a mass $\frac{M}{3}$, where M is the mass of earth. It is released from rest from a height h ($\ll$ radius of earth) above the surface of earth. The speed of the block at a height $\frac{h}{2}$ is

(A) $\sqrt{\frac{2gh}{3}}$ (B) $\sqrt{gh}$

(C) $\frac{\sqrt{3gh}}{2}$ (D) $\sqrt{2gh}$

118. An object, initially at rest, explodes into three fragments. The momenta of two parts are $2p\hat{i}$ and $p\hat{j}$ where p is a positive number. The momentum of the third part

(A) will be of magnitude $3p$ inclined at $\tan^{-1}\left(\frac{1}{2}\right)$ with x-axis

(B) will be of magnitude $\sqrt{5}p$ inclined at $\tan^{-1}(2)$ with x-axis

(C) will be of magnitude $3p$ inclined at $\pi - \tan^{-1}(2)$ with x-axis

(D) will be of magnitude $\sqrt{5}p$ inclined at $\pi - \tan^{-1}\left(\frac{1}{2}\right)$ with x-axis

119. There are some passengers inside a stationary railway compartment. The centre of mass of the compartment itself (without the passengers) is C_1, while the centre of mass of the 'compartment plus passengers' system is C_2. If the passengers move about inside the compartment,

(A) both C_1 and C_2 will move with respect to the ground

(B) neither C_1 nor C_2 will move with respect to the ground

(C) C_1 will move but C_2 will be stationary with respect to the ground

(D) C_2 will move but C_1 will be stationary with respect to the ground

120. A ball, moving with a speed of $9\ \text{ms}^{-1}$, strikes an identical stationary ball such that after the collision the direction of each ball makes an angle of $30°$ with the original line of motion. The speeds of the two balls after the collision are

(A) $3\sqrt{3}\ \text{ms}^{-1}$ and $3\ \text{ms}^{-1}$

(B) $3\ \text{ms}^{-1}$ and $3\sqrt{3}\ \text{ms}^{-1}$

(C) $3\ \text{ms}^{-1}$ and $3\ \text{ms}^{-1}$

(D) $3\sqrt{3}\ \text{ms}^{-1}$ and $3\sqrt{3}\ \text{ms}^{-1}$

121. A ball of mass m_1 collides head-on and elastically with an identical ball of mass m_2, initially at rest. The transfer of energy will be maximum when

(A) $m_1 = m_2$ (B) $m_1 = \frac{m_2}{2}$

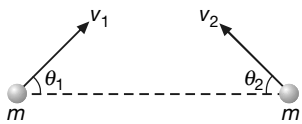
(C) $m_1 = 2m_2$ (D) None of these

122. A meter rule weighing $100\ \text{g}$ rests on a table with a part projecting over the edge. The length of the part to be projected out if a $5\ \text{g}$ body hung at the end just tilts the rule is

(A) 26.8 cm (B) 37.5 cm

(C) 40.2 cm (D) 47.6 cm

123. Two particles of equal mass m are projected form the ground with speeds v_1 and v_2 at angles θ_1 and θ_2 as shown in figure. The centre of mass of the two particles



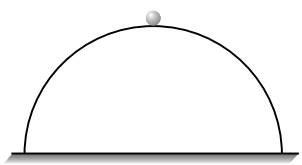
(A) will move in a parabolic path for any values of v_1, v_2, θ_1 and θ_2

(B) can move in a vertical line

(C) can move in a horizontal line

(D) will move in a straight line for any values of v_1, v_2, θ_1 and θ_2

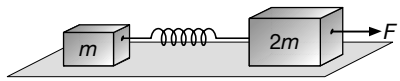
124. A hemisphere of radius R and of mass $4m$ is free to slide with its base on a smooth horizontal table. A particle of mass m is placed on the top of the hemisphere. The angular velocity of the particle relative to hemisphere at an angular displacement θ, when velocity of hemisphere has become v is



(A) $\frac{2v}{R\cos\theta}$ (B) $\frac{3v}{R\sin\theta}$

(C) $\frac{5v}{R\cos\theta}$ (D) $\frac{5v}{R\sin\theta}$

125. Two blocks of masses m and $2m$, kept on a smooth horizontal surface, are connected by an ideal spring of force constant k. Initially the spring is unstretched. A constant force is applied to the heavier block in the direction shown in figure. If the displacement of the smaller block at time t is x, then displacement of the heavier block at this moment is



(A) $\frac{x}{2}$ (B) $\frac{Ft^2}{4m} - \frac{x}{2}$

(C) $\frac{x}{3}$ (D) $\frac{Ft^2}{6m} + \frac{x}{3}$

126. A space-craft of mass M is moving in free space with velocity V. It explodes and breaks into two. One part, which has a mass m, is left stationary. The velocity of the other part is

(A) $\frac{MV}{m+m}$ (B) $\frac{mV}{M+m}$

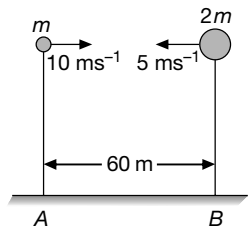
(C) $\frac{MV}{M-m}$ (D) $\frac{mV}{M-m}$

127. A boy of mass $60\ \text{kg}$ is standing over a platform of mass $40\ \text{kg}$ placed over a smooth horizontal surface. He throws a stone of mass $1\ \text{kg}$ with velocity $u = 10\ \text{ms}^{-1}$ at an angle of $45°$ with respect to ground. The displacement of platform (with boy) on the horizontal surface, when the stone lands on the ground is $\left(\text{Take } g = 10\ \text{ms}^{-2}\right)$

(A) 5 cm (B) 10 cm

(C) 25 cm (D) 50 cm

128. Two particles one of mass m and the other of mass $2m$ are projected horizontally towards each other from the same level above the ground with velocities $10\ \text{ms}^{-1}$ and $5\ \text{ms}^{-1}$ respectively. They collide in air and stick to each other. The distance from A, where the combined mass finally land is



(A) 40 m (B) 20 m

(C) 30 m (D) 45 m

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. A ball B of mass m is lying at rest on the top surface of a smooth horizontal table $5\ \text{m}$ high. Another moving ball A of same mass makes an elastic collision with B and B slides off the table and strikes the floor at a horizontal distance of $10\ \text{m}$ from the table. Then select the correct alternative(s) $\left(g = 10\ \text{ms}^{-2}\right)$

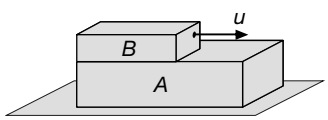
(A) The velocity of the ball A before collision is $5\ \text{ms}^{-1}$

(B) The kinetic energy of the ball B at the time of striking the ground is $(m \times 100)$ J

(C) The velocity of the ball A before collision is $10\ \text{ms}^{-1}$

(D) The kinetic energy of the ball B at the time when it strikes the ground is $(50 \times m)$ J

2. A horizontal block A is at rest on a smooth horizontal surface. A small block B, whose mass is half of A, is placed on A at one end and projected along other end with some velocity u. The coefficient of friction between blocks is μ. Then



(A) the blocks will reach a final common velocity $\frac{u}{3}$

(B) the work done against friction is two third of the initial kinetic energy of B

(C) before the blocks reach a common velocity, the acceleration of A relative to B is $\frac{2}{3}\mu g$

(D) before the blocks reach a common velocity, the acceleration of A relative to B is $\frac{3}{2}\mu g$

3. Two identical particles A and B each of mass m are connected together by a light and inextensible string of length l. The particles are held at rest in air in same horizontal level at a separation l. Both particles are released simultaneously, however particle A is given speed v_0 vertically upwards. Select the correct statement(s). Ignore air resistance.

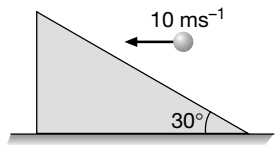
(A) The maximum height attained by the centre of mass of the system of A and B is $\frac{v_0^2}{8g}$

(B) The kinetic energy of the system of A and B when the centre of mass is at its highest point, is $\frac{mv_0^2}{2}$.

(C) The maximum height attained by the centre of mass of the system of A and B is $\frac{v_0^2}{4g}$

(D) The kinetic energy of the system of A and B when the centre of mass is its highest point, is $\frac{mv_0^2}{4}$

4. A ball of mass $1\ \text{kg}$ strikes a wedge of mass $4\ \text{kg}$, horizontally with a velocity of $10\ \text{ms}^{-1}$. Just after collision, the velocity of wedge becomes $4\ \text{ms}^{-1}$. Assuming friction to be absent everywhere and collision to be elastic, select the correct statement(s)



(A) Speed of ball after collision is $6\ \text{ms}^{-1}$.

(B) Speed of ball after collision is $8\ \text{ms}^{-1}$.

(C) Impulse between ball and wedge during collision is 16 Ns.

(D) Impulse between ball and wedge during collision is 32 Ns.

5. A cannon and a supply of cannon balls are inside a sealed rail road car at rest. The cannon fires to the right, the car recoils to the left. The balls remain in the car after hitting the farther wall. Assume friction to be absent. If L is the length of the rail road car, D is the distance traversed by the rail road car relative to the ground, m is the total mass of cannon balls fired and M is the total mass of remaining system (cannon + car) then

(A) $D > L$ (B) $D < L$

(C) $D = \frac{ML}{M+m}$ (D) $D = \frac{mL}{M+m}$

6. A particle of mass m moving with a velocity $\left(3\hat{i} + 2\hat{j}\right)\ \text{ms}^{-1}$ collides with another body of mass M and finally moves with velocity $\left(-2\hat{i} + \hat{j}\right)\ \text{ms}^{-1}$. Then during the collision

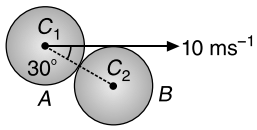
(A) impulse imparted to m is $m\left(5\hat{i} + \hat{j}\right)$

(B) impulse imparted to m is $m\left(-5\hat{i} - \hat{j}\right)$

(C) impulse imparted to M is $m\left(-5\hat{i} - \hat{j}\right)$

(D) impulse imparted to M is $m\left(5\hat{i} + \hat{j}\right)$

7. A ball A collides elastically with another identical ball B with a velocity of $10\ \text{ms}^{-1}$ making an angle of $30°$ with the line joining their centres C_1 and C_2. Select the correct statement(s).



(A) Velocity of ball A after collision is $5\ \text{ms}^{-1}$.

(B) Velocity of ball B after collision is $5\sqrt{3}\ \text{ms}^{-1}$.

(C) Kinetic energy will not be conserved here, because collision is not head on.

(D) Both the balls move at right angles after collision.

8. Ball A of mass m strikes a stationary ball B of mass M and undergoes an elastic collision. After collision ball A has a speed one third of its initial speed. The ratio of $\frac{M}{m}$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) 2 (D) 4

9. Velocity of a particle of mass $2\ \text{kg}$ changes from $\vec{v}_1 = 2\hat{i} - 2\hat{j}\ \text{ms}^{-1}$ to $\vec{v}_2 = (\hat{i} - \hat{j})\ \text{ms}^{-1}$ after colliding with a plane surface. Select the correct statement(s).

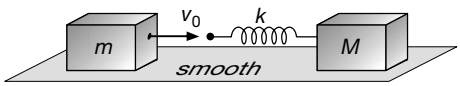
(A) The angle made by the plane surface with the positive x-axis is $\tan^{-1}\left(\frac{1}{3}\right)$

(B) The angle made by the plane surface with the positive x-axis is $90° + \tan^{-1}\left(\frac{1}{3}\right)$

(C) The direction of change in momentum makes an angle $90° + \tan^{-1}\left(\frac{1}{3}\right)$ with the plane surface

(D) The direction of change in momentum makes an angle $\tan^{-1}\left(\frac{1}{3}\right)$ with the $+$ve x-axis

10. A block of mass m moving with a velocity v_0 collides with a stationary block of mass M at the back of which a spring of spring constant k is attached, as shown in the figure. Select the correct alternative(s)



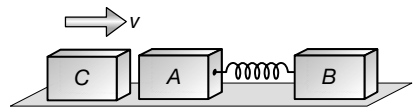
(A) The velocity of centre of mass is $\frac{m}{m+M}v_0$

(B) The initial kinetic energy of the system in the centre of mass frame is $\frac{1}{4}\left(\frac{mM}{M+m}\right)v_0^2$

(C) The maximum compression in the spring is $v_0\sqrt{\frac{mM}{m+M}\frac{1}{k}}$

(D) When the spring is in the state of maximum compression the kinetic energy in the centre of mass frame is zero

11. Two blocks A and B each of mass m are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown in figure. A third identical block C, also of mass m, moves on the floor with a speed v along the line joining A to B and collides with A. Then



(A) the kinetic energy of the $A-B$ system, at maximum compression of the spring is zero

(B) the kinetic energy of the $A-B$ system, at maximum compression of the spring is $\frac{mv^2}{4}$

(C) the maximum compression of the spring is $v\sqrt{\frac{m}{k}}$

(D) the maximum compression of the spring is $v\sqrt{\frac{m}{2k}}$

12. For the case of rocket propulsion, select the correct statements.

(A) Momentum of system always remains constant.

(B) Newton's Third Law is applicable.

(C) If exhaust velocity and rate of burning of mass is kept constant, then acceleration of rocket keeps on increasing.

(D) Newton's Second Law is applicable.

13. A bag of mass M hangs by a long thread. A bullet of mass m comes horizontally with a velocity v and gets embedded in the bag. Then for the combined system, immediately after the collision, the

(A) momentum is $\frac{mvM}{M+m}$

(B) momentum is mv

(C) kinetic energy is $\frac{m^2v^2}{2(M+m)}$

(D) kinetic energy is $\frac{mv^2}{2}$

14. In a one-dimensional collision between two particles A and B, the particle B is stationary and A has momentum p before impact. During impact, A gives an impulse J to B. Then select the incorrect statement(s)

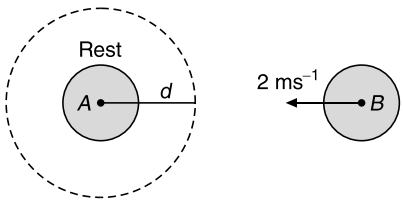
(A) The total momentum of A plus B system before and after the impact is p and during the impact is $(p-J)$

(B) During the impact B gives an impulse J to A

(C) The coefficient of restitution is $\frac{2J}{p}-1$

(D) The coefficient of restitution is $\frac{2J}{p}+1$

15. The elastic collision between two bodies, A and B can be considered using the model in which A and B are free to move along a common line without friction. When separation between the surfaces is greater than $d=1$ m, the interacting force is zero. However, when their distance less than d, a constant repulsive force $F=6$ N is present. The mass of body A is $m_A=1$ kg and it is initially at rest. The mass of body B is $m_B=3$ kg and it is approaching towards A with a speed $v_0=2$ ms^{-1}. Then select the correct statement(s).



(A) The minimum separation between the bodies is 0.25 m

(B) The common velocity attained by the bodies are 1.5 ms^{-1}

(C) The common velocity attained by the bodies are 2.0 ms^{-1}

(D) The minimum separation between the bodies is 0.75 m

16. Two identical balls are thrown obliquely from the ground simultaneously. The acceleration of centre of mass

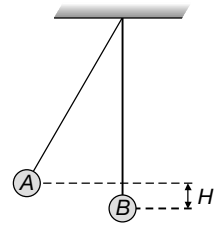
(A) does not depend on the mass of the two balls

(B) depends on the direction in which two balls are projected

(C) is equal to g

(D) is zero

17. Two small balls A and B of mass M and $3M$ hang from the ceiling by strings of equal length. The ball is drawn aside so that it is raised to a height H. Now the ball A is released such that it collides with ball B. Select the correct statement(s).



(A) If collision is perfectly elastic, ball B will rise to a height $\frac{H}{4}$

(B) If the collision is perfectly elastic, ball A will rise upto a height $\frac{H}{4}$

(C) If the collision is perfectly inelastic, the combined mass will rise to a height $\frac{H}{4}$

(D) If the collision is perfectly inelastic, the combined mass will rise to a height $\frac{H}{16}$

18. A particle of mass m strikes a horizontal smooth floor with a velocity u making an angle θ with the floor and rebound with velocity v making an angle ϕ with the floor. The coefficient of restitution between the particle and the floor is e. Then

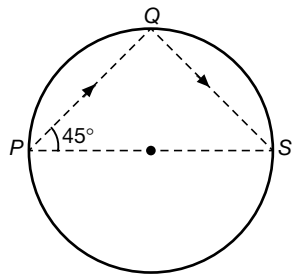
(A) the impulse delivered by the floor to the body is $mu(1+e)\sin\theta$

(B) $\tan\phi=e\tan\theta$

(C) $v=u\sqrt{1-(1-e^2)\sin^2\theta}$

(D) the ratio of the final kinetic energy to the initial kinetic energy is $\cos^2\theta+e^2\sin^2\theta$

19. A body is fired from point P and strikes at Q inside a smooth circular wall as shown in the figure. It rebounds to point S (diametrically opposite to P), then



(A) the coefficient of restitution is zero

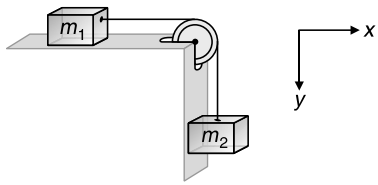
(B) the coefficient of restitution is 1

(C) kinetic energy is conserved in this collision

(D) the coefficient of restitution is $\frac{1}{\sqrt{3}}$

20. A particle of mass $0.1\ \text{kg}$ moving with an initial speed v collides with another particle of same mass kept at rest. If after collision total energy becomes $0.2\ \text{J}$. Then
(A) minimum value of v is $3\ \text{ms}^{-1}$
(B) minimum value of v is $2\ \text{ms}^{-1}$
(C) maximum value of v is $2\sqrt{2}\ \text{ms}^{-1}$
(D) maximum value of v is $6\ \text{ms}^{-1}$

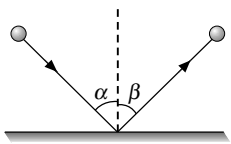
21. In the arrangement shown, all surfaces are smooth and system is released from rest. The x and y components of acceleration of centre of mass are



(A) $(a_{cm})_x = \dfrac{m_1 m_2 g}{m_1 + m_2}$ (B) $(a_{cm})_x = \dfrac{m_1 m_2 g}{(m_1 + m_2)^2}$
(C) $(a_{cm})_y = \left(\dfrac{m_2}{m_1 + m_2}\right)^2 g$ (D) $(a_{cm})_y = \left(\dfrac{m_2}{m_1 + m_2}\right) g$

22. A point mass of $1\ \text{kg}$ collides elastically with a stationary point mass of $5\ \text{kg}$. After their collision, the $1\ \text{kg}$ mass reverses its direction and moves with a speed of $2\ \text{ms}^{-1}$. Which of the following statement(s) is (are) correct for the system of these two masses?
(A) Total momentum of the system is $3\ \text{kgms}^{-1}$
(B) Momentum of $5\ \text{kg}$ mass after collision is $4\ \text{kgms}^{-1}$
(C) Kinetic energy of the centre of mass is $0.75\ \text{J}$
(D) Total kinetic energy of the system is $4\ \text{J}$

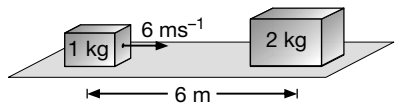
23. A ball strikes the ground at an angle α and rebound at an angle β with the vertical as shown in the figure. Then



(A) coefficient of restitution is $\dfrac{\tan\alpha}{\tan\beta}$
(B) if $\alpha < \beta$ the collision is inelastic
(C) if $\alpha = \beta$ the collision is elastic
(D) if $\alpha > \beta$ the collision is inelastic

24. A particle of mass m, moving with velocity v collides with a stationary particle of mass $2m$. As a result of collision, the lighter particle deviates by $45°$ and has final speed of $\dfrac{v}{2}$. For this situation, select the correct statement(s).
(A) The angle of divergence between particles after collision is $90°$
(B) The angle of divergence between particles after collision is less than $90°$
(C) Collision is elastic
(D) Collision is inelastic

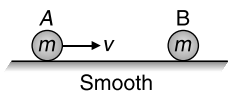
25. On a frictionless surface a block of mass $1\ \text{kg}$ is pushed with a velocity of $6\ \text{ms}^{-1}$ towards another block of mass $2\ \text{kg}$ placed at a distance of $6\ \text{m}$ from it as shown in Figure. Just after collision, the velocity of $2\ \text{kg}$ block becomes $4\ \text{ms}^{-1}$. Select the correct statement(s).



(A) Coefficient of restitution between two blocks is $\dfrac{1}{2}$
(B) Coefficient of restitution between two blocks is 1
(C) Velocity of centre of mass at $t = 2\ \text{s}$ is $2\ \text{ms}^{-1}$
(D) Velocity of centre of mass at $t = 0.5\ \text{s}$ is $2\ \text{ms}^{-1}$

26. A ball of mass $10\ \text{g}$ is dropped on the ground from a height of $10\ \text{m}$. It rebounds to a height of $2.5\ \text{m}$. If the ball is in contact with the ground for $0.01\ \text{s}$, then which statement(s) is incorrect?
(A) Impulse force between the ground and ball is $15\ \text{N}$
(B) Impulse force between the ground and ball is $5\sqrt{2}\ \text{N}$
(C) Coefficient of restitution between the ground and ball is 0.25
(D) Coefficient of restitution between the ground and ball is 0.5

27. If the coefficient of restitution between A and B (shown in Figure) is $e = \dfrac{1}{2}$, then select the correct statement(s).



(A) Velocity of B after collision is $\dfrac{v}{2}$
(B) Impulse between the bodies during collision is $\dfrac{3}{4}mv$
(C) Loss in kinetic energy during the collision is $\dfrac{3}{16}mv^2$
(D) Loss in kinetic energy during the collision is $\dfrac{1}{4}mv^2$

28. A charged particle A moves directly towards another charged particle B. For the $(A+B)$ system, the total momentum is p and the total energy is E.

(A) p and E are conserved if both A and B are free to move

(B) Option (A) is true only if particles A and B have similar charges

(C) If B is fixed, E is conserved but not p.

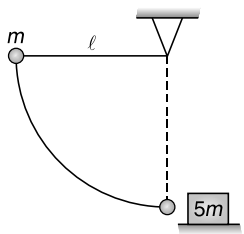
(D) If B is fixed, neither E nor p is conserved

29. A projectile is fired from a point on the horizontal ground. The coefficient of restitution between the projectile and the ground is e. Let a, b and c be the ratio of the time of flights, the maximum heights and the horizontal ranges for the first two collisions with the horizontal ground, then

(A) $a=\frac{1}{e}$ (B) $b=\frac{1}{e^2}$

(C) $c=\frac{1}{e^2}$ (D) $a=be=c$

30. A pendulum bob of mass m connected to the end of an ideal string of length l is released from rest from horizontal position as shown in Figure. At the lowest point, the bob makes an elastic collision with a stationary block of mass $5m$, which is kept on a frictionless surface. Select the correct statement(s) for the instant just after the impact



(A) Tension in the string is $3mg$

(B) Tension in the string is $\frac{17mg}{9}$

(C) The velocity of the block is $\frac{\sqrt{2gl}}{3}$

(D) The maximum height attained by the pendulum bob after impact is (measured from the lowest position) $\frac{4l}{9}$

31. A particle moving with kinetic energy K makes a head on elastic collision with an identical particle at rest. During collision, the

(A) elastic potential energy of the system is always zero.

(B) maximum elastic potential energy of the system is $\frac{K}{2}$.

(C) kinetic energy of the system is constant.

(D) minimum kinetic energy of the system is $\frac{K}{2}$.

32. Two particles having equal masses are projected simultaneously from the roof of a tower of height $20\ \text{m}$ with same speed $20\ \text{ms}^{-1}$, one horizontally and the other vertically upwards. Select the correct alternative(s).

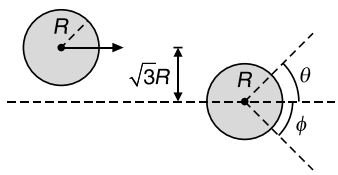
(A) The acceleration of centre of mass is $\frac{g}{2}$ downwards.

(B) The acceleration of centre of mass is g downwards.

(C) Maximum height of centre of mass from the ground is 25 m.

(D) Maximum height of centre of mass from the ground is 40 m.

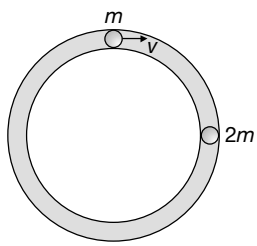
33. A disk moving on a frictionless horizontal table collides elastically with another identical disk as shown in Figure. The direction of motion of the two disks makes angle θ and ϕ with the initial line of motion. Select the correct values of θ and ϕ.



(A) $\theta=60°$ (B) $\theta=30°$

(C) $\phi=60°$ (D) $\phi=30°$

34. A particle of mass m moving with a speed v collides elastically with another particle of mass $2m$ on a horizontal circular tube of radius R, then select the correct alternative(s)



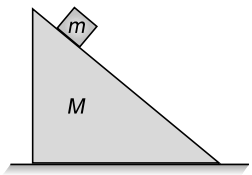
(A) The time after which the next collision will take place is $\frac{2\pi R}{v}$

(B) The time after which the next collision will take place is proportional to m

(C) The time after which the next collision will take place is inversely proportional to m

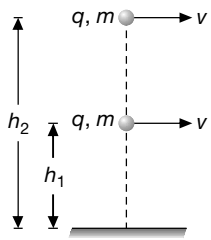
(D) The time after which the next collision will take place is independent of the mass of the balls

35. A block of mass m is lying at rest on a smooth wedge of mass M also lying at rest on a smooth horizontal surface. When the system is released select the correct statements.



(A) The CM of the system remains stationary.
(B) The CM of the system has an acceleration g vertically downward.
(C) Momentum of the system is conserved along the horizontal direction.
(D) Acceleration of CM is vertically downwards and $a < g$.

36. Two particles having same mass and charge are both thrown along the horizontal with same velocity v from two different height h_1 and h_2 $(h_1 < h_2)$. Initially they were located in the same vertical line. Select the correct alternative(s).



(A) Both the particles always lie on a vertical line.
(B) Acceleration of the centre of mass of the two particles is g downwards.
(C) Horizontal displacement of particle lying at h_1 is less than the value, which would had been in the absence of charges on them.
(D) Horizontal displacement of particle lying at h_2 is more than the value, which would had been in the absence of charges on them.

37. A man of mass m is standing on a stationary flat car. The car can move without friction along horizontal rails. The man starts walking with velocity v relative to the car. The work done by him

(A) is greater than $\frac{1}{2}mv^2$, if he walks along the rails.

(B) is less than $\frac{1}{2}mv^2$, if he walks along the rails.

(C) is equal to $\frac{1}{2}mv^2$, if he walks normal to rails.

(D) can never be less than $\frac{1}{2}mv^2$.

REASONING BASED QUESTIONS

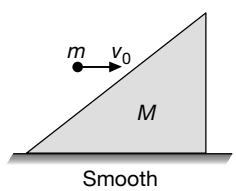
This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** In case of bullet fired from a gun, the ratio of kinetic energy of gun and bullet is equal to ratio of masses of bullet and gun.
Statement-2: In firing, momentum is conserved.

2. **Statement-1:** The position of centre of mass of a body does not depend upon shape and size of the body.
Statement-2: The centre of mass of a body may lie where there is no mass.

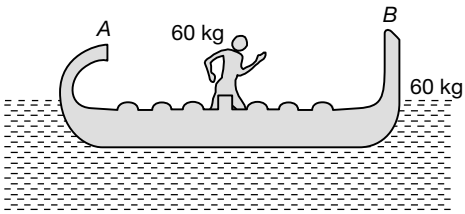
3. **Statement-1:** A particle of mass m strikes a smooth wedge of mass M as shown in the Figure.



Linear momentum of particle along the surface of wedge is conserved during collision.
Statement-2: Wedge exerts a force perpendicular to inclined face of wedge on particle during collision.

4. **Statement-1:** A body of mass m_1 collides head on elastically with another stationary body of mass m_2. After the collision, velocity of mass m_2 is maximum, when $m_1 \gg m_2$.
Statement-2: Velocity of second body is always maximum, when its mass m_2 is greater than mass of the hitting body.

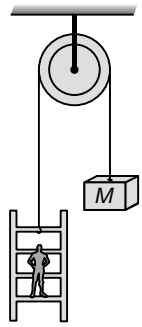
5. **Statement-1:** On a $8\ \text{m}$ boat as shown in figure, when man moves from centre to end B, the boat moves backward on water by $4\ \text{m}$.
Statement-2: Conservation of momentum principle is being followed.



6. **Statement-1:** If no external force acts on a system of particles, then the centre of mass will not move in any direction.
Statement-2: If net external force is zero, then the linear momentum of the system remains constant.

7. **Statement-1:** A block is kept at the top of a smooth wedge which is kept on a smooth horizontal surface. As the block slides down the wedge, centre of the mass of system will be accelerated.
Statement-2: When external force acting on the system is zero, centre of mass is in rest.

8. **Statement-1:** A block of mass M, balances a man at rest on a stationary ladder. If man moves upwards with respect to the ladder, the centre of mass will not move. Assume string to be massless and pulley to be smooth.
Statement-2: For a system on which no net external force acts and if $u_{cm} = 0$ then position of centre of mass will remain fixed.



9. **Statement-1:** Change in momentum of a particle in small time interval depends upon net impulse acting on it.
Statement-2: Internal force on the particle must be zero.

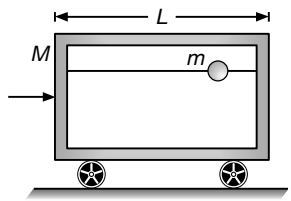
10. **Statement-1:** In a two body collision, the momenta of the particles are equal and opposite to one another, before as well as after the collision when measured in the center of mass frame.
Statement-2: The momentum of the system is zero when measured from the centre of mass frame.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

A horizontal frictionless string is threaded through a bead of mass m. The string is pulled between two vertical opposite sides of a cart of mass M, as shown in figure. The length of the cart is L and the radius of the bead, r, is very small in comparison with $L(r \ll L)$. Initially, the bead is at the right edge of the cart. The cart is struck and as a result, it moves with velocity v_0. When the bead collides with the cart's walls, assume the collisions to be completely elastic always.



Based on above information, answer the following questions.

1. The velocity of the centre of mass of the cart and the bead is

(A) $\frac{mv_0}{m+M}$ (B) $\frac{Mv_0}{m+M}$

(C) $\frac{Mv_0}{M-m}$ (D) $\frac{mv_0}{M-m}$

2. The first collision of the bead with the cart's wall takes place at the time t_1 given by

(A) $\frac{L}{v_0}$ (B) $\frac{L}{v_{cm}}$

(C) $\frac{2L}{v_{cm}}$ (D) $\frac{2L}{v_0}$

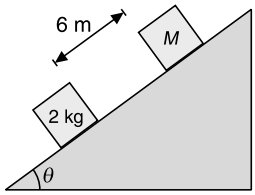
3. The velocity of the bead after the first collision, in the centre of mass frame is

(A) $\frac{2Mv_0}{M+m}$ (B) $\frac{Mv_0}{M+m}$

(C) $\frac{2mv_0}{m+M}$ (D) $\frac{2mMv_0}{m^2+M^2}$

Comprehension 2

Two blocks of mass 2 kg and M are at rest on an inclined plane and are separated by a distance of 6.0 m as shown in Figure.



The coefficient of friction between each block and the inclined plane is 0.25. The 2 kg block is given a velocity of 10.0 ms^{-1} up the inclined plane. It collides with M, comes back and has a velocity of 1.0 ms^{-1} when it reaches its initial position. The other block M after the collision moves 0.5 m up and comes to rest. Take $\sin\theta \approx \tan\theta = 0.05$ and $g = 10\ ms^{-2}$. Based on the above facts, answer the following questions.

4. The approximate velocity of the 2 kg block just before collision is

(A) 4 ms^{-1} (B) 8 ms^{-1}

(C) 16 ms^{-1} (D) 32 ms^{-1}

5. The approximate velocity of 2 kg block just after collision is

(A) 1 ms^{-1} (B) 3 ms^{-1}

(C) 5 ms^{-1} (D) 7 ms^{-1}

6. The approximate velocity of block M just after collision is

(A) 1 ms^{-1} (B) 1.25 ms^{-1}

(C) 1.75 ms^{-1} (D) 2 ms^{-1}

7. If e be the coefficient of restitution between the blocks, then e equals

(A) 0.24 (B) 0.44

(C) 0.64 (D) 0.84

8. The approximate value of the mass M is

(A) 2.5 kg (B) 5 kg

(C) 10 kg (D) 15 kg

Comprehension 3

Two pendulum bobs of mass m and $2m$ are attached to threads of length L suspended from a rigid support. Both the bobs are taken to a height H from the lowest point and then released so they collide elastically at the lowest point in their motion. Based on the above facts, answer the following questions.

9. Velocity of bob of mass m just after impact is

(A) $\frac{4}{3}\sqrt{2gH}$ (B) $\frac{5}{3}\sqrt{2gH}$

(C) $\frac{7}{3}\sqrt{2gH}$ (D) $\sqrt{2gH}$

10. The height to which $2m$ will rise after impact is

(A) $\frac{H}{9}$ (B) $\frac{4H}{9}$

(C) H (D) $\frac{16H}{9}$

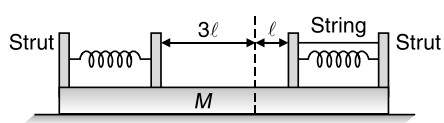
11. The height to which m will rise after impact is

(A) $\frac{H}{9}$ (B) H

(C) $\frac{25H}{9}$ (D) $\frac{16H}{9}$

Comprehension 4

A plank of mass M is placed on a smooth horizontal surface. Two light identical springs, each of stiffness K, are rigidly connected to struts at the end of the plank as shown in Figure.



When the springs are in their natural position, the distance between their free ends is $3l$. A block of mass m is placed on the plank and pressed against one of the springs so that it is compressed to l. To keep the block at rest it is connected to the strut by means of a light string. Initially, the system is at rest. Now if the string is burnt, then answer the following questions.

12. The maximum displacement of the plank is

(A) $\frac{5ml}{M}$ (B) $\frac{3ml}{M+m}$

(C) $\frac{4ml}{M+m}$ (D) $\frac{5ml}{M+m}$

13. The maximum velocity of the plank is

(A) $l\sqrt{\frac{km}{(M+M)}}$ (B) $l\sqrt{\frac{k}{(M+m)}}$

(C) $l\sqrt{\frac{km}{M(M+m)}}$ (D) $l\sqrt{\frac{kM}{m(M+m)}}$

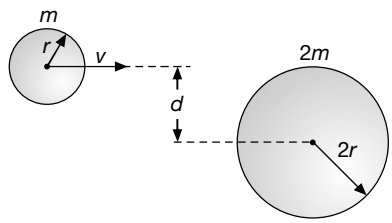
14. The maximum kinetic energy of the block m is

(A) $\frac{kml^2}{2M(M+m)}$ (B) $\frac{kMl^2}{2(M+m)}$

(C) $\frac{kml^2}{M(M+m)}$ (D) $\frac{2kMl^2}{m(M+m)}$

Comprehension 5

A uniform sphere with mass $2m$ and radius $2r$ is at rest in a place where no external forces are applied. A second sphere of mass m and radius r moves towards it with a velocity v (without rotating). The two spheres stick together and then continue their motion as one body. Given that $d = 2r$.



Based on above information, answer the following questions.

15. The distance of the centre of mass of system from the centre of bigger sphere at the moment of collision is

(A) r (B) $2r$

(C) $3r$ (D) $4r$

16. The velocity of centre of mass immediately after the collision is

(A) $\frac{2v}{3}$ (B) $\frac{2v}{5}$

(C) $\frac{3v}{2}$ (D) $\frac{v}{3}$

17. The moment of inertia about the center of mass of the unified body after the collision is

(A) $\frac{24}{5}mr^2$ (B) $\frac{24}{3}mr^2$

(C) $\frac{48}{5}mr^2$ (D) $\frac{48}{3}mr^2$

Comprehension 6

To describe the momentum and kinetic energy of a particle, we need a reference frame. A very popular reference frame is the one which is attached with the centre of mass of a system. To describe the motion of the constituent particles of a system we have to find the velocity of centre of mass and then the velocities of the constituent particles with respect to centre of mass frame called C-frame. For simplicity consider a two particle system having masses m_1, m_2 and their velocities in a stationary frame of reference to be $\vec{v}_1$, $\vec{v}_2$ respectively. Based on information given, answer the following questions.

18. The total momentum of the system in the C-frame is

(A) $m_1\vec{v}_1 + m_2\vec{v}_2$ (B) $m_1\vec{v}_2 + m_2\vec{v}_1$

(C) $\left(\frac{m_1m_2}{m_1+m_2}\right)(\vec{v}_2 - \vec{v}_1)$ (D) ZERO

19. The total kinetic energy of the system in the C-frame is

(A) $\frac{1}{2}\left(m_1v_1^2 + m_2v_2^2\right)$

(B) $\frac{1}{2}(m_1+m_2)(\vec{v}_1 - \vec{v}_2)^2$

(C) $\frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(\vec{v}_1 - \vec{v}_2)^2$

(D) ZERO

20. Let the velocity of centre of mass of two particles is v and the sum of the masses of two particles is m. Kinetic energy of the system in stationary frame

(A) Will be equal to $\frac{1}{2}mv^2$

(B) Will always be less than $\frac{1}{2}mv^2$

(C) Will be greater than or equal to $\frac{1}{2}mv^2$

(D) Will always be greater than $\frac{1}{2}mv^2$

Comprehension 7

Two friends A and B (each weighing 40 kg) are sitting on a frictionless platform some distance d apart. A rolls a ball of mass 4 kg on the platform towards B, which B catches. Then B rolls the ball towards A and A catches it. The ball keeps on moving back and forth between A and B.

The ball has a fixed speed of $5\ \text{ms}^{-1}$ on the platform. Based on above information, answer the following questions.

21. The speed of A after he rolls the ball for the first time is

(A) $0.25\ \text{ms}^{-1}$ (B) $0.5\ \text{ms}^{-1}$

(C) $1\ \text{ms}^{-1}$ (D) $1.5\ \text{ms}^{-1}$

22. The speed of A after he catches the ball for the first time is

(A) $\frac{10}{11}\ \text{ms}^{-1}$ (B) $\frac{25}{11}\ \text{ms}^{-1}$

(C) $\frac{5}{11}\ \text{ms}^{-1}$ (D) $\frac{50}{11}\ \text{ms}^{-1}$

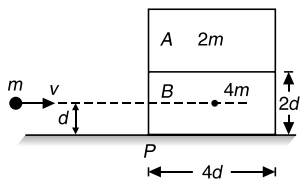
23. The speed of A and B after the ball has made four round trips and is held by A are

(A) $\frac{50}{11}\ \text{ms}^{-1}, \frac{25}{11}\ \text{ms}^{-1}$ (B) $\frac{40}{11}\ \text{ms}^{-1}, \frac{20}{11}\ \text{ms}^{-1}$

(C) $\frac{30}{11}\ \text{ms}^{-1}, \frac{25}{11}\ \text{ms}^{-1}$ (D) $\frac{50}{11}\ \text{ms}^{-1}, \frac{30}{11}\ \text{ms}^{-1}$

Comprehension 8

A block A of mass $2m$ is placed on another block B of mass $4m$ which in turn is placed of a fixed table. The two blocks have a same length $4d$ and they are placed as shown in figure. The coefficient of friction (both static and kinetic) between the block B and table is μ. There is no friction between the two blocks. A small object of mass m moving horizontally along a line passing through the centre of mass (CM) of the block B and perpendicular to its face with a speed v collides elastically with the block B at a height d above the table.



Based on the above facts, answer the following questions.

24. The minimum value of v (say v_0) so as to make the block A topple is

(A) $v_0 = \frac{2}{5}\sqrt{6\mu g d}$ (B) $v_0 = \frac{5}{2}\sqrt{3\mu g d}$

(C) $v_0 = \frac{5}{2}\sqrt{6\mu g d}$ (D) $v_0 = \frac{2}{5}\sqrt{3\mu g d}$

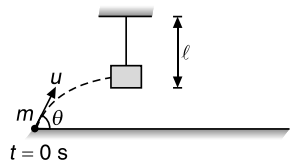
25. Assume $v = 2v_0$, the distance measured from point P at which the mass m falls on the table after collision (ignoring the role of friction during the collision) is

(A) $3d\sqrt{6\mu}$ (B) $6d\sqrt{3\mu}$

(C) $2d\sqrt{3\mu}$ (D) $3d\sqrt{2\mu}$

Comprehension 9

A small bullet of mass m is fired with velocity u at an angle θ with the horizontal, freely under gravity. At the highest point of its path, it strikes with a wooden block (mass M) and emerges out with half of its velocity (just before collision). During the impact (collision), the string (supporting the wooden block) remains almost vertical. (Given $(M > m)$)



26. Velocity of block as the bullet emerges out of it

(A) $\frac{Mu\cos\theta}{2m}$ (B) $\frac{mu}{2M}$

(C) $\frac{mu\cos\theta}{2M}$ (D) $\frac{mu\cos\theta}{M}$

27. Loss in K.E. of (bullet + block) system as the bullet emerges out of block

(A) $\frac{m(3M-2m)}{8M}u^2\cos^2\theta$

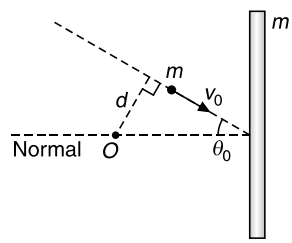
(B) $\frac{m(3M-m)}{8M}u^2\cos^2\theta$

(C) $\frac{m}{8M}(3M+m)u^2\cos^2\theta$

(D) $\frac{3m}{8M}(M-m)u^2\cos^2\theta$

Comprehension 10

A uniform rod of mass m is lying on a frictionless horizontal surface. A particle of same mass as that of rod collides with rod at its centre with an angle θ_0 from the normal at mid-point of the rod as shown in the figure. Assume that there is no friction between the rod and particle and that the collision is elastic.



Based on above information, answer the following questions.

28. The magnitude of change in velocity of rod after collision is

(A) v_0 (B) $v_0 \sin\theta_0$

(C) $v_0 \cos\theta_0$ (D) None of these

29. The magnitude of change in velocity of particle after collision is

(A) v_0 (B) $v_0 \sin\theta_0$

(C) $v_0 \cos\theta_0$ (D) None of these

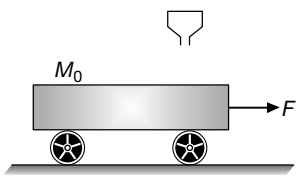
30. The magnitude of change in angular momentum of the particle about the point O after collision is

(A) $mv_0 d$ (B) ZERO

(C) $2mv_0 d$ (D) None of these

Comprehension 11

The figure shows a long cart moving on a smooth horizontal surface due to an external constant force of magnitude F. Initial mass of the cart is M_0 and its initial velocity is zero. At $t = 0$, sand starts falling from a stationary hopper onto the cart at constant rate μ kgs^{-1} and sticks to the cart. After time t_0 the sand starts leaking from the bottom at the same constant rate μ kgs^{-1}. Eventually at time $t = 2t_0$, the sand stops falling from the hopper on to the cart and force F also stops acting.



Based on above information, answer the following questions.

31. The velocity of the cart at time $t(< t_0)$

(A) $\dfrac{Ft}{M_0}$ (B) $\dfrac{Ft}{M_0}e^{\mu t}$

(C) $\dfrac{Ft}{M_0+\mu t}e^{\mu t}$ (D) $\dfrac{Ft}{M_0+\mu t}$

32. In the same model of the above question if the cart is to be moved with constant velocity v, then the power supplied by external agent applying that force is

(A) $2\mu v$ (B) μv

(C) μv^2 (D) $\dfrac{1}{2}\mu v^2$

33. In the above question the rate of increase of the kinetic energy of the cart (with sand) is

(A) $2\mu v^2$ (B) μv

(C) μv^2 (D) $\dfrac{1}{2}\mu v^2$

34. In the above model (for $t_0 < t < 2t_0$) what additional force over and above the force μv already applied to the right is required to move the cart with the speed acquired at t_0,

(A) μv towards right (B) $2\mu v$ towards right

(C) ZERO (D) μv towards left

Comprehension 12

A body of mass m_1 moving with velocity u_1, collides with a body of mass m_2 having velocity u_2, elastically. The collision is observed to be head on. Based on the above facts, answer the following questions.

35. Assume a massive body to collide with a light body at rest, then immediately after impact

(A) massive body comes to rest and the light body has velocity u_1

(B) massive body continues to move with the same velocity but light body attains velocity of massive body

(C) massive body continues to move with the same velocity but light body attains twice the velocity of massive body

(D) None of above seems to be correct

36. It is observed that for a ratio $\dfrac{m_1}{m_2} = (3 - x^2 + x)$, maximum transfer of momentum takes place from body 1 to body 2. Then

(A) $x = 1$ (B) $x = 2$

(C) $x = 3$ (D) $x = -1$

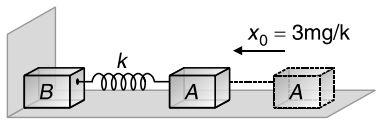
37. The fraction of total energy lost by m_1 is

(A) $\dfrac{4m_1m_2}{(m_1+m_2)^2}$ (B) $\dfrac{m_1}{m_1+m_2}$

(C) $\left(\dfrac{m_1-m_2}{m_1+m_2}\right)^2$ (D) $\dfrac{m_1-m_2}{m_1+m_2}$

Comprehension 13

A system consists of block A and B each of mass m connected by a light spring with block B in contact with a wall as shown in Figure.



The block A compresses the spring by $3mg/k$ from natural length of spring and then released from rest. Neglect friction everywhere, answer the following questions.

38. Acceleration of centre of mass of system comprising A and B just after A is released is

(A) g (B) $\frac{g}{2}$

(C) $\frac{3g}{2}$ (D) $\frac{g}{4}$

39. Velocity of centre of mass of system comprising A and B when block B just loose contact with the wall is

(A) $\frac{g}{2}\sqrt{\frac{m}{k}}$ (B) $\frac{g}{4}\sqrt{\frac{m}{k}}$

(C) $\frac{3g}{2}\sqrt{\frac{m}{k}}$ (D) $\frac{3g}{4}\sqrt{\frac{m}{k}}$

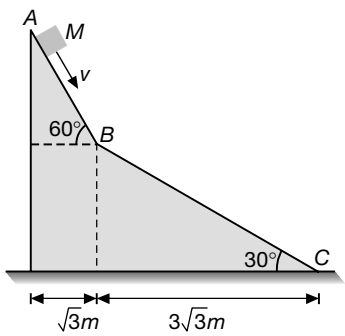
40. Maximum extension in the spring after system loses contact with wall is

(A) $\frac{mg}{k\sqrt{2}}$ (B) $\frac{mg}{3k\sqrt{2}}$

(C) $\frac{3mg}{k\sqrt{2}}$ (D) $\frac{mg}{k}$

Comprehension 14

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from $60°$ to $30°$ at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic $\left(g = 10\ \text{ms}^{-2}\right)$.



41. The speed of the block at point B immediately after it strikes the second incline is

(A) $\sqrt{60}\ \text{ms}^{-1}$ (B) $\sqrt{45}\ \text{ms}^{-1}$

(C) $\sqrt{30}\ \text{ms}^{-1}$ (D) $\sqrt{15}\ \text{ms}^{-1}$

42. The speed of the block at point C, immediately before it leaves the second incline is

(A) $\sqrt{120}\ \text{ms}^{-1}$ (B) $\sqrt{105}\ \text{ms}^{-1}$

(C) $\sqrt{90}\ \text{ms}^{-1}$ (D) $\sqrt{75}\ \text{ms}^{-1}$

43. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is

(A) $\sqrt{30}\ \text{ms}^{-1}$ (B) $\sqrt{15}\ \text{ms}^{-1}$

(C) 0 (D) $-\sqrt{15}\ \text{ms}^{-1}$

Comprehension 15

An experiment is done on two metal balls of equal masses moving with unequal speeds u_1 and u_2 collide inelastically head on. The experiment is repeated with initial same speeds of the two balls u_1 and u_2 but balls are charged with unequal charges of same polarity moving in the same line from a large separation. Assume no radiation loss during collision when balls are charged. Assume in second case balls do not touch each other.

Based on above information, answer the following questions.

44. Which of the following statements is correct for potential energy between balls?

(A) When balls are charged the maximum potential energy is more than the case when balls are uncharged during collision

(B) When balls are charged the maximum potential energy is less than the case when balls are uncharged during collision

(C) When balls are charged the maximum potential energy is equal to the case when balls are uncharged during collision

(D) Nothing can be said about the maximum potential energy as it depends on the magnitude of charges of the ball

45. Which of the following statement is correct about center of mass of the two balls during collision

1. In first case when balls are uncharged, the centre of mass of balls moves at uniform speed during collision
2. In second case when balls are charged, the centre of mass of balls moves at uniform speed during collision
3. In second case, centre of mass of balls will move at uniform speed only when the balls are equally charged

(A) only 1 is correct (B) 1 and 2 are correct

(C) 1 and 3 are correct (D) None of these

46. If an observer moving with constant velocity parallel to the line of motion of balls observes the collision when balls are uncharged

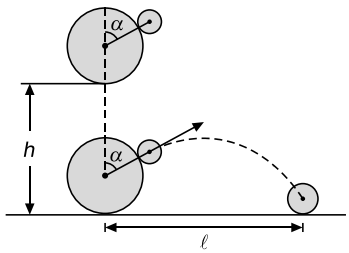
1. The kinetic energy of the system in the frame of observer remains conserved

2. The velocity of center of mass in the frame of observer remains constant

(A) only (1) is true
(B) only (2) is true
(C) both (1) and (2) are true
(D) both (1) and (2) are wrong

Comprehension 16

A large heavy sphere and a small light sphere are dropped onto a flat surface from a height h. The radii of both spheres is much smaller than height h. The large sphere collides with the surface with velocity v_0 and immediately thereafter with the small sphere. The spheres are dropped so that all motion is vertical before the second collision and the small sphere hits the larger sphere at an angle α from its uppermost point, as shown in Figure. If all collisions are perfectly elastic and there is no surface friction between the spheres.



47. The angle made by velocity vector of small sphere with the vertical just after the second collision in the frame of large sphere is

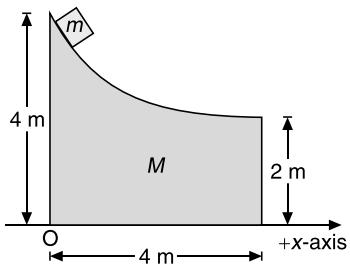
(A) α (B) 2α
(C) $\frac{3\alpha}{2}$ (D) ZERO

48. The vertical velocity of smaller sphere just after the collision with respect to ground is

(A) $2v_0\cos\alpha$ (B) $v_0\cos\alpha + v_0$
(C) $2v_0\cos 2\alpha + v_0$ (D) $v_0\cos 2\alpha + v_0$

Comprehension 17

A small block of mass $m = 1$ kg is placed over a wedge of mass $M = 4$ kg as shown in figure. Mass m is released from rest. All surfaces are smooth and the origin O is fixed on the ground shown. Based on above information, answer the following questions.



49. Final velocity of the wedge is

(A) $\frac{1}{\sqrt{3}}$ ms^{-1} (B) $\sqrt{3}$ ms^{-1}
(C) $\frac{1}{\sqrt{2}}$ ms^{-1} (D) $\sqrt{2}$ ms^{-1}

50. The block strikes the x-axis at

(A) 4.2 (B) 5.6
(C) 6.8 (D) 7.6

51. At an instant when absolute acceleration of m is $5\sqrt{3}$ ms^{-2} at $60°$ with horizontal, the normal reaction (N) between the two blocks make an angle of $30°$ with horizontal. Then N (in newton) equals

(A) 4 (B) 5
(C) 6 (D) 10

52. At the same instant reaction on the wedge from the ground (in newton) is

(A) 38.5 (B) 40
(C) 42.5 (D) 45

Comprehension 18

A piece of wood of mass 0.03 kg is dropped from the top of a building 100 m high. At the same time a bullet of mass 0.02 kg is fired vertically upwards with a velocity of 100 ms^{-1} from the ground. The bullet gets embedded in the wooden piece after striking it. Taking $g = 10$ ms^{-2}.

Based on above information, answer the following questions.

53. The time after which bullet strikes the wooden block is

(A) 1 sec (B) 0.5 sec
(C) 1.5 sec (D) 2 sec

54. If the time of collision is 100 μs and $\vec{u}_1$, $\vec{u}_2$ and $\vec{v}_1$, $\vec{v}_2$ are velocities of block, bullet before and after the collision respectively, then the value of $\left|(m_1\vec{u}_1 + m_2\vec{u}_2) - (m_1\vec{v}_1 + m_2\vec{v}_2)\right|$ is

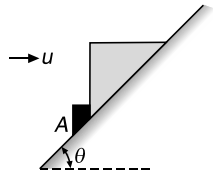
(A) 5×10^{-4} Ns (B) 5×10^{-5} Ns
(C) 5×10^{-3} Ns (D) 5×10^{-6} Ns

55. The velocity of the (block + bullet) system after the collision is

(A) 14 ms^{-1} (B) 21 ms^{-1}
(C) 42 ms^{-1} (D) 30 ms^{-1}

Comprehension 19

A block of mass M is held at rest on the smooth inclined plane by the stop block at A. The bullet of mass m is travelling at speed u, when it becomes embedded in the block. Based on this information, answer the following questions.



56. For the collision between block and bullet, which of the following conservative law can be applied?
(A) Conservation of mechanical energy
(B) Conservation of linear momentum along horizontal
(C) Conservation of linear momentum along vertical
(D) Conservation of linear momentum in a direction parallel to inclined surface

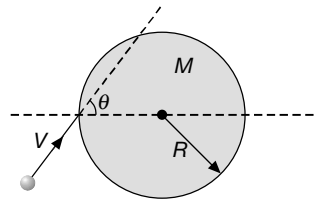
57. In which of the following directions, net impulsive force on the system is zero?
(A) Along horizontal
(B) Along vertical
(C) Along the inclined surface
(D) Along a normal to the inclined surface

58. What is the impulse generated between the block and the inclined plane?
(A) $mu\sin\theta$
(B) $mu\cos\theta$
(C) $(M+m)u\cos\theta$
(D) $(M+m)u\sin\theta$

59. The distance up to which the block slide before momentarily stopping is
(A) $\dfrac{u^2}{2g\sin\theta}$ (B) $\dfrac{m^2u^2}{2g\sin\theta(m+M)^2}$
(C) $\dfrac{m^2u^2\cos^2\theta}{2g\sin\theta(m+M)^2}$ (D) $\dfrac{m^2u^2\sin^2\theta}{2g\sin\theta(m+M)^2}$

Comprehension 20

A disc of mass M and radius of curvature R is lying on a frictionless ground. A small ball of mass M moving on the ground with a velocity V strikes the disc and collides with it. The coefficient of restitution being e.



Based on above information, answer the following questions.

60. Velocity of approach for the system (ball + disc) along line of impact is
(A) V (B) $V\sin\theta$
(C) $V\cos\theta$ (D) ZERO

61. Velocity of the disc after collision is
(A) $\dfrac{V\cos\theta}{2}$ (B) $\dfrac{eV\cos\theta}{2}$
(C) $\dfrac{V\cos\theta(1-e)}{2}$ (D) $\dfrac{V\cos\theta(1+e)}{2}$

62. The speed of the ball after the collision is
(A) $eV\cos\theta$
(B) $V\sqrt{\dfrac{\cos^2\theta(1-e)^2}{4}+\sin^2\theta}$
(C) $V\sqrt{\cos^2\theta(1+e)^2+\dfrac{\sin^2\theta}{4}}$
(D) $V\sqrt{\dfrac{\cos^2\theta(1+e)^2}{4}+\sin^2\theta}$

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

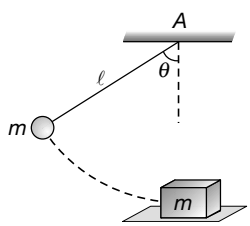
If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

	p	q	r	s	t
A	●	○	○	●	●
B	○	●	●	○	○
C	●	●	○	○	○
D	○	○	○	●	●

1. Match the quantities in **COLUMN-I** to their respective values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Momentum is increased by 200%, corresponding change in kinetic energy as multiple of 100%	(p) 8
(B) Kinetic energy is increased by 300%, corresponding change in momentum as multiple of 100%	(q) 2
(C) Momentum is increased by 1%, corresponding change in kinetic energy as multiple of $\frac{1}{2}\%$	(r) 1
(D) Kinetic energy is increased by 1%, corresponding change in momentum as multiple of $\frac{1}{2}\%$	(s) 4
	(t) None of these

2. In the arrangement shown in figure, ball and block have the same mass $m = 1$ kg each, $\theta = 60°$ and length $l = 2.5$ m. The coefficient of friction between block and floor is 0.5. When the ball is released from the position shown in the figure, it collide with the block and the block stops after moving a distance 2.5 m. Match the quantities in **COLUMN-I** with their respective values in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) velocity of ball just before collision (ms^{-1})	(p) 5
(B) velocity of block just after collision (ms^{-1})	(q) 3
(C) velocity of block after travelling 1.6 m (ms^{-1})	(r) 1
(D) coefficient of restitution (dimensionless)	(s) 0

(*Continued*)

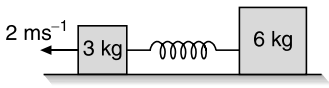
3. A particle is projected from the ground upwards with a velocity of $20\ \text{ms}^{-1}$. At the same instant, another identical particle is dropped from a height of 180 m but not along the same vertical line. Assume that collision of first particle with ground is perfectly inelastic, match the quantities given in **COLUMN-I** for the centre of mass of the two-particle system with their respective magnitudes in SI system given in **COLUMN-II**. Take $g = 10\ \text{ms}^{-2}$.

COLUMN-I	COLUMN-II
(A) Initial acceleration	(p) 5
(B) Initial velocity	(q) 10
(C) Acceleration at $t = 5$ s	(r) 20
(D) Velocity at $t = 5$ s	(s) 25

4. A particle of mass 1 kg has velocity $\vec{v}_1 = (2t)\hat{i}$ and another particle of mass 2 kg has velocity $\vec{v}_2 = (t^2)\hat{j}$. Match the quantities in **COLUMN-I** at $t = 2$ s to their respective values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Magnitude of net force on centre of mass	(p) $\frac{20}{9}$ unit
(B) Magnitude of momentum of centre of mass	(q) $\sqrt{68}$ unit
(C) Magnitude of velocity of centre of mass	(r) $\frac{\sqrt{80}}{3}$ unit
(D) Distance of centre of mass from origin	(s) $\sqrt{80}$ unit

5. Two blocks having masses 3 kg and 6 kg placed on a horizontal frictionless surface are connected by an ideal spring as shown in Figure. The 3 kg block is imparted a speed of $2\ \text{ms}^{-1}$ towards left. Considering leftward to be the positive direction, match the quantities in **COLUMN-I** with their respective answers in **COLUMN-II**.

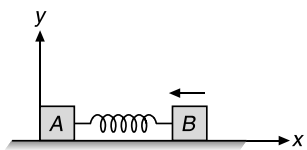


COLUMN-I	COLUMN-II
(A) When the velocity of 3 kg block is $\frac{2}{3}$ ms^{-1}	(p) velocity of centre of mass is $\frac{2}{3}$ ms^{-1}
(B) When the velocity of 6 kg block is $\frac{2}{3}$ ms^{-1}	(q) deformation of the spring is zero
(C) When the speed of 3 kg block is minimum	(r) deformation of the spring is maximum
(D) When the speed of 6 kg block is maximum	(s) both the blocks are at rest with respect to each other

6. A particle of mass m, kinetic energy K and momentum p collides head on elastically with another particle of mass $2m$ at rest. Match the quantities in **COLUMN-I** (quantities after collision) to their respective values in **COLUMN-II**.

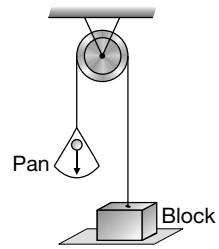
COLUMN-I	COLUMN-II
(A) Momentum of first particle (multiple of p)	(p) $\frac{4}{3}$
(B) Momentum of second particle (multiple of p)	(q) $\frac{1}{9}$
(C) Kinetic energy of first particle (multiple of K)	(r) $-\frac{1}{3}$
(D) Kinetic energy of second particle (multiple of K)	(s) $\frac{8}{9}$

7. Two identical blocks A and B lying on a smooth horizontal surface (along the x-axis) are connected by a spring as shown in Figure. The block A is not connected to the wall (along the y-axis). When the block B is moved to the left and released, then match arguments in **COLUMN-I** to the results in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) Acceleration of centre of mass of two blocks	(p) remains constant
(B) Velocity of centre of mass of two blocks	(q) first increases then becomes constant
(C) x-coordinate of centre of mass of two blocks	(r) first decreases then become zero
(D) y-coordinate of centre of mass of two blocks	(s) continuously increases

8. A pan of mass $m = 1.5$ kg and a block of mass $M = 3$ kg are connected with each other by a flexible, light and inextensible string, passing over a small, light and frictionless pulley. Initially the block is resting over a horizontal floor as shown in Figure.

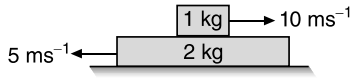


At $t = 0$, an inelastic ball of mass $m_0 = 0.5$ kg collides with the pan with velocity $v_0 = 16$ ms^{-1} (vertically downward). Take $g = 10$ ms^{-2}, match the quantities in **COLUMN-I** with the respective values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) velocity of pan in ms^{-1} just after collision	(p) 0.64
(B) acceleration in ms^{-2} of pan just after collision	(q) 1.6
(C) maximum height rise in m of block	(r) 2
(D) time in s at which block strikes the floor	(s) 9.8

9. A 1 kg block is lying on a 2 kg platform which lies on a smooth horizontal surface. Friction is present between the block and the platform. At some instant the block has a horizontal velocity of 10 ms^{-1} towards right

and the platform has a horizontal velocity of $5\ ms^{-1}$ towards left. Match the contents of **COLUMN-I** with their respective answers in **COLUMN-II**.

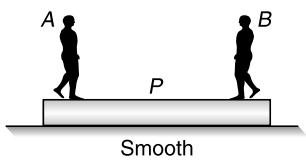


COLUMN-I	COLUMN-II
(A) Velocity of centre of mass	(p) constant
(B) Momentum of centre of mass	(q) first decreases then become zero
(C) Momentum of block	(r) is ZERO
(D) Kinetic energy platform	(s) keeps changing all the time

10. For a system of particles, if the net force is zero, then match the quantities of **COLUMN-I** with their best matches in **COLUMN-II**.

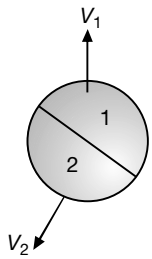
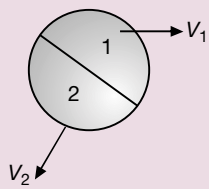
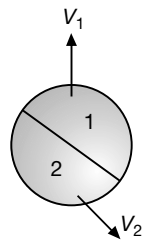
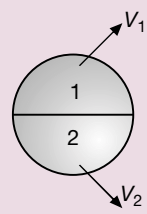
COLUMN-I	COLUMN-II
(A) Acceleration of centre of mass	(p) Constant
(B) Velocity of centre of mass	(q) Zero
(C) Momentum of centre of mass	(r) May be zero
(D) Velocity of an individual particle of the system	(s) May be constant
	(t) None of these

11. Two boys A and B having masses $30\ kg$ and $60\ kg$ are standing on a plank P of mass $30\ kg$ lying on the smooth horizontal surface as shown in Figure. Match the displacement of plank of **COLUMN II** with the conditions given in **COLUMN I**. Note that the displacements mentioned in two columns are with respect to ground.

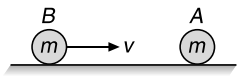


COLUMN-I	COLUMN-II
(A) A moves x towards right	(p) x, towards right
(B) B moves x towards left	(q) $2x$, towards left
(C) A moves x towards right and B moves x towards left	(r) $3x$, towards left
(D) A and B both move x towards right	(s) $\frac{x}{3}$, towards left

12. A body is initially moving towards right explodes into two pieces 1 and 2. Match the direction of motion of the pieces shown in **COLUMN-I** to the possible mass relationships in **COLUMN-II**.

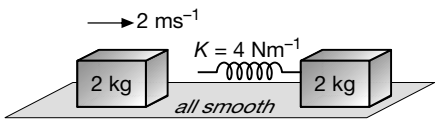
COLUMN-I	COLUMN-II
(A) 	(p) $m_1 > m_2$
(B) 	(q) $m_1 = m_2$
(C) 	(r) $m_1 < m_2$
(D) 	(s) Impossible for any masses

13. Two identical balls A and B are kept on a smooth table as shown. The ball B moving with a speed v collides with the ball A at rest. For different conditions mentioned in **COLUMN-I**, give the corresponding match for the speed of ball A after collision given in **COLUMN II**.



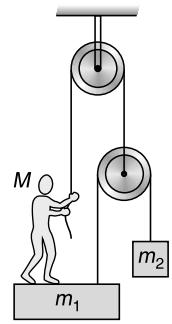
COLUMN-I	COLUMN-II
(A) Elastic collision	(p) $\frac{3v}{4}$
(B) Perfectly inelastic collision	(q) $\frac{5v}{8}$
(C) Inelastic collision with $e=\frac{1}{2}$	(r) v
(D) Inelastic collision with $e=\frac{1}{4}$	(s) $\frac{v}{2}$

14. In the arrangement shown in figure, match the quantities in **COLUMN-I** with the respective values in **COLUMN-II**.



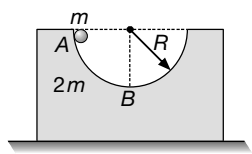
COLUMN-I	COLUMN-II
(A) Velocity of centre of mass (in ms^{-1})	(p) 2
(B) Velocity of combined mass, (in ms^{-1}) when compression in the spring is maximum	(q) 1
(C) Maximum compression (in m) in the spring	(r) 4
(D) Maximum potential energy, in joule stored in the spring	(s) 0.5
	(t) None of these

15. A man of mass M is standing on a platform of mass $m_1 = 10$ kg connected to a mass $m_2 = 20$ kg. The man is holding a string passing over a system of ideal pulleys as shown in Figure. Taking $g = 10\ ms^{-2}$, match the quantities given in **COLUMN-I** with their respective values in SI system in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) Weight of man for equilibrium	(p) 100 N
(B) Force exerted by man on string so that the centre of mass of system accelerates upwards	(q) 150 N
(C) Force exerted by man on string so that the centre of mass of system accelerates downwards	(r) 500 N
(D) Force exerted by platform on man in equilibrium is	(s) 600 N

16. In the system shown in figure, mass m is released from rest from position A. Suppose potential energy of m at point A with respect to point B is E. Size of m is negligible and all surfaces are smooth. When mass m reaches at point B.

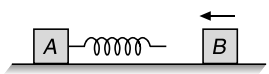


COLUMN-I	COLUMN-II
(A) Kinetic energy of m	(p) $\frac{E}{3}$
(B) Kinetic energy of $2m$	(q) $\frac{2E}{3}$
(C) Momentum of m	(r) $\sqrt{\frac{4mE}{3}}$
(D) Momentum of $2m$	(s) $\sqrt{\frac{2mE}{3}}$

17. Match the quantities in **COLUMN-I** with the respective quantities in **COLUMN-II**.

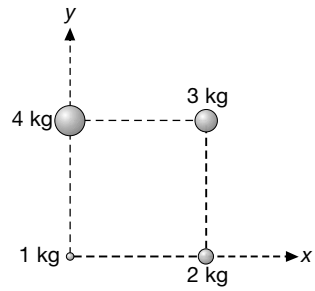
COLUMN-I	COLUMN-II
(A) In the absence of external force	(p) Inside the body
(B) If a man walks on a platform, which is placed on frictionless horizontal surface then	(q) The state of centre of mass will remain unchanged
(C) Centre of mass of a body can be	(r) The linear momentum of system remains conserved
(D) Centre of mass of solid cube is	(s) Outside a body

18. Consider two identical blocks A and B each of mass 0.5 kg. A light elastic spring is connected to block A as shown in Figure. The block B is moving towards A with a kinetic energy of 4 J. Match the momenta in **COLUMN-I** to their respective values in SI system in **COLUMN-II**. Assume friction to be absent everywhere.



COLUMN-I	COLUMN-II
(A) Initial momentum of B	(p) 0
(B) Momentum of centre of mass of two blocks	(q) 1
(C) Momentum of A at maximum compression	(r) 2
(D) Momentum of B when spring is relaxed after compression	(s) 4

19. Four point masses are placed at four corners of a square of side 4 m as shown. Match the quantities in **COLUMN-I** to their respective values in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) x co-ordinate of centre of mass of 4 kg and 2 kg	(p) $\frac{7}{2}$ m
(B) x co-ordinate of centre of mass of 4 kg , 2 kg and 3 kg	(q) $\frac{4}{3}$ m
(C) y co-ordinate of centre of mass of 1 kg , 4 kg and 3 kg	(r) 3 m
(D) y co-ordinate of centre of mass of 1 kg and 3 kg	(s) $\frac{20}{9}$ m

(*Continued*)

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

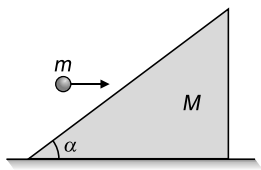
1. After perfectly inelastic collision between two identical particles moving with same speed in different directions, the speed of the particles becomes half the initial speed. The angle between the two colliding bodies just before impact is ϕ. Find $\frac{\phi}{20}$, in degree.

2. Block A has a mass of 3 kg and is sliding on a rough horizontal surface with a velocity $v_A = 2\ \text{ms}^{-1}$ when it makes a direct collision with block B, which has a mass of 2 kg and is originally at rest. If the collision is perfectly elastic, calculate the distance, in cm, between the blocks when they stop sliding. The coefficient of kinetic friction between the blocks and the plane is $\mu_k = 0.4$ and given $g = 10\ \text{ms}^{-2}$.

3. The maximum tension that an inextensible rope of mass density $0.2\ \text{kgm}^{-1}$ can bear is 40 N. Length of rope is 2 m. Calculate the maximum angular velocity,

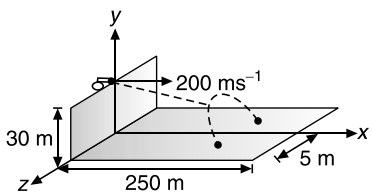
in rads^{-1}, with which it can be rotated horizontally in a circular path on a frictionless table.

4. A wedge (free to move) of mass M has one face making an angle α with horizontal and is resting on a smooth rigid floor. A particle of mass m hits the inclined face of the wedge with a horizontal velocity v_0. It is observed that the particle rebounds in vertical direction after impact. Neglect friction between particle and the wedge and take $M = 2m$, $v_0 = 10\ \text{ms}^{-1}$, $\tan\alpha = 2$, $g = 10\ \text{ms}^{-2}$. Calculate the coefficient of restitution for the impact.

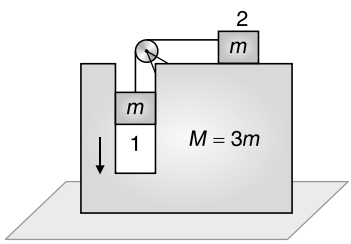


5. Two particles A and B of equal masses lie close together on a horizontal table and are connected by a light inextensible string of length l. A is projected vertically upwards with a velocity $\sqrt{10gl}$. The velocity with which it reaches the table again is $\sqrt{xgl}$. Find x.

6. Two $20\ \text{kg}$ cannon balls are chained together and fired horizontally with a velocity of $200\ \text{ms}^{-1}$ from the top of a $30\ \text{m}$ wall. The chain breaks during the flight of the cannon balls and one of them strikes the ground at $t = 2\ \text{s}$, at a distance of $250\ \text{m}$ from the foot of the wall and $5\ \text{m}$ to the right of the line of fire. The position of the other cannon ball at that instant is $(x_2, y_2, z_2)\ \text{m}$. Calculate $x_2 + y_2 + z_2$. Neglect the resistance of air.



7. A wedge having a vertical slot in it is placed on smooth horizontal surface. Two blocks are arranged as shown in Figure.

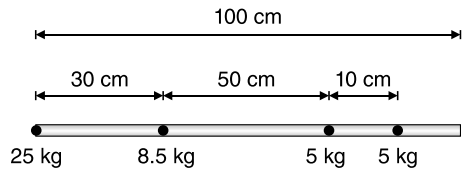


The system is released from rest. The speed of the wedge when block 1 comes down a distance h is $\sqrt{\frac{2gh}{\alpha}}$. Calculate α.

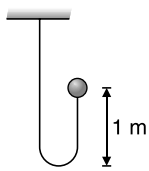
8. A system consists of two particles. At $t = 0$, one particle is at the origin and the other particle having a mass of $600\ \text{kg}$, is on the y-axis at $y = 80\ \text{m}$. At $t = 0$ the centre of mass of the system is on the y-axis at $y = 24\ \text{m}$ and has a velocity given by $(6\ \text{ms}^{-3})t^2\hat{j}$.

(a) Find the total mass of the system in kg.
(b) Find the acceleration of the centre of mass, in ms^{-2}, at time $t = 2\ \text{s}$.
(c) Find the net external force, in N, acting on the system at $t = 3\ \text{s}$.

9. A rod of mass $10\ \text{kg}$ of length $100\ \text{cm}$ with point masses tied to it at different positions as shown in Figure. Find the location of a point on the rod from its left end at which if the rod is placed over a knife edge, it will be in equilibrium about that knife edge.



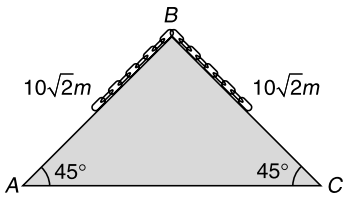
10. A ball of mass $1\ \text{kg}$ is attached to an inextensible string. The ball is released from the position shown in Figure. The impulse imparted by the string to the ball immediately after the string becomes taut is $\sqrt{\alpha}\ \text{kgms}^{-1}$. Calculate α.



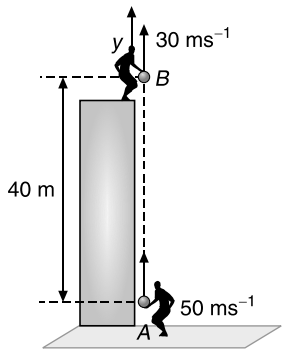
11. A ball of mass m is just disturbed from the top of a fixed smooth circular tube of radius a in a vertical plane and falls impinging on a ball of mass $2m$ at the bottom. If the coefficient of restitution between the balls is $\frac{1}{2}$ then the heights to which the balls m and $2m$ rise after the second impact are $\frac{a}{x}$ and $\frac{a}{y}$. Then find x and y.

12. A flexible chain of length $L = 20\sqrt{2}\ \text{m}$ and weight $W = 10\ \text{kg}$ is initially placed at rest on a smooth frictionless wedge surface ABC. It is given a slight

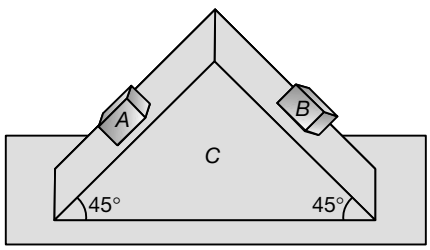
push on one-side so that it will start sliding on that side. Calculate the speed of the chain, in ms^{-1}, when its one end will leave the vertex of the wedge. Take $g = 10\ \text{ms}^{-2}$



13. Two balls A and B of equal masses are projected upward simultaneously, one from the ground with speed $50\ \text{ms}^{-1}$ and other from a lower of height $40\ \text{m}$ above the first ball with initial speed $30\ \text{ms}^{-1}$. Calculate the maximum height attained by their centre of mass in metre.

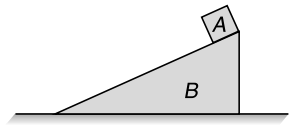


14. Two blocks A and B of equal mass are released on two sides of a fixed wedge C as shown in Figure.

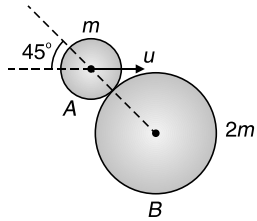


Calculate the acceleration of centre of mass of blocks A and B, in ms^{-2}. Neglect friction.

15. A block A (mass $= 4M$) is placed on the top of a wedge block B of base length $12\ \text{m}$ (mass $= 20M$) as shown in Figure. When the system is released from rest. Find the distance moved by the block B (in metre) till the block A reaches ground. Assume all surfaces are frictionless.



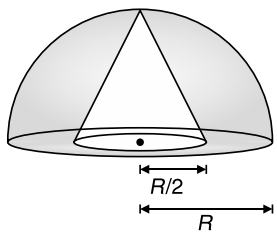
16. Disc A of mass m collides with stationary disc B of mass $2m$ as shown in Figure.



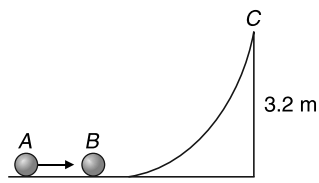
The value of coefficient of restitution for which the two discs move in perpendicular direction after collision is e. Calculate $\dfrac{1}{e}$.

17. In an elastic head-on collision of bodies of mass m and $(x^2 - 9x + 21)m$, the second body is at rest and the maximum transfer of momentum takes place. Find the value(s) of x.

18. From a hemisphere of radius R, a cone of base radius $\dfrac{R}{2}$ and height R is removed as shown in Figure. Calculate the height of centre of mass of the remaining object in cm if $R = 28\ \text{cm}$.

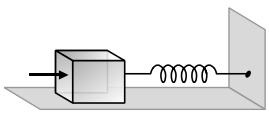


19. Two identical balls A and B lie on a smooth horizontal surface, which gradually merges into a curve to a height $3.2\ \text{m}$. Ball A is given a velocity of $10\ \text{ms}^{-1}$, to collide head-on with ball B, which then takes up the curved path. Calculate the minimum coefficient of restitution, e, for the collision between A and B so that ball B reaches the highest point C of the curve.

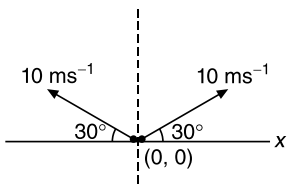


20. A balloon having mass m is filled with gas and is held in hands of a boy. Then suddenly it get released and gas starts coming out of it with a constant rate. The velocities of the ejected gases is also constant $2\ \text{ms}^{-1}$ with respect to the balloon. Find out the velocity of the balloon, in ms^{-1}, when the mass of gas is reduced to half. Take $\ln(2) \approx 0.7$

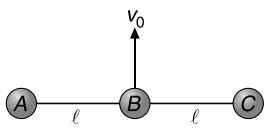
21. A block of mass $0.18\ \text{kg}$ is attached to a spring of force constant $2\ \text{Nm}^{-1}$. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of $0.06\ \text{m}$ and comes to rest for the first time. The initial velocity of the block in ms^{-1} is $V = \frac{N}{10}$. Find the value of N.



22. Two particles A and B of equal mass are projected simultaneously with speed of $10\ \text{ms}^{-1}$ each as shown in Figure. Find the co-ordinate of centre of mass at $t = 10\ \text{s}$ is (x, y) m. Calculate $x - y$. Take $g = 10\ \text{ms}^{-2}$.

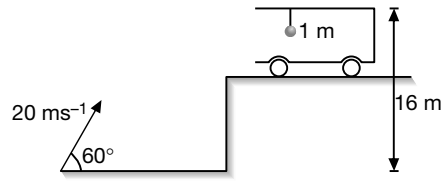


23. Three identical balls each of mass $m = 0.5\ \text{kg}$ are connected with each other as shown in Figure and rest over a smooth horizontal table. At moment $t = 0$, ball B is imparted a horizontal velocity $v_0 = 9\ \text{ms}^{-1}$. Calculate velocity of A just before it collides with ball C.

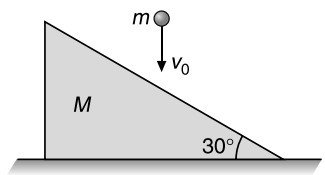


24. Rahul having mass $50\ \text{kg}$ and Mohit having mass $60\ \text{kg}$ are sitting at the two extremes of a $4\ \text{m}$ long floating platform having mass $40\ \text{kg}$ standing still in water. To discuss a problem, they come to the middle of the platform. Neglecting friction with water, how far does the platform move in the water during the process (in centimetre).

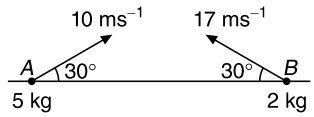
25. A shell of mass 1 kg is projected with velocity $20\ \text{ms}^{-1}$ at an angle $60°$ with horizontal. It collides inelastically with a ball of mass 1 kg which is suspended through a thread of length 1 m. The other end of the thread is attached to the ceiling of a trolley of mass 18 kg as shown in figure. Initially the trolley is stationary and it is free to move along horizontal rails without any friction. The maximum deflection of the thread with vertical is θ, such that $\cos\theta = \frac{1}{*}$, where $*$ is not readable. Taking $g = 10\ \text{ms}^{-2}$, find $*$.



26. A ball of mass $m = 1\ \text{kg}$ falling vertically with a velocity $v_0 = 2\ \text{ms}^{-1}$ strikes a wedge of mass $M = 2\ \text{kg}$ kept on a smooth, horizontal surface as shown in figure. The coefficient of restitution between the ball and the wedge is $e = \frac{1}{2}$. The velocity of the wedge and the ball immediately after collision is $\frac{\alpha}{\sqrt{3}}\ \text{ms}^{-1}$. Calculate α.

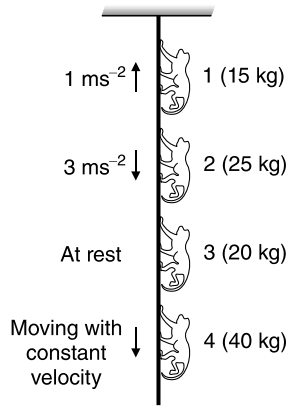


27. Two particles A and B of mass $5\ \text{kg}$ and $2\ \text{kg}$ are projected simultaneously from same ground level as shown in Figure. Find the maximum height (in cm) reached by the centre of mass. Assume that only gravitational force of attraction by the earth is present and take $g = 10\ \text{ms}^{-2}$.



28. A sphere A is of mass m and another sphere B of identical size but of mass $2m$, move towards each other with velocity $\hat{i} + 2\hat{j}$ and $-\hat{i} + 3\hat{j}$ respectively. They collide when their line of centres is parallel to $\hat{i} - \hat{j}$. Taking $e = \frac{1}{2}$ the velocities of A and B after impact are $\frac{1}{\alpha}\left(-x_1\hat{i} + y_1\hat{j}\right)$ and $\frac{1}{\beta}\left(-x_2\hat{i} + y_2\hat{j}\right)$. Find α, β, x_1, x_2, y_1 and y_2.

29. A light rope supporting four monkeys, is hanging from roof as shown in Figure.



Monkey 1 of mass $15\ \text{kg}$ is going upwards with an acceleration $1\ \text{ms}^{-2}$, monkey 2 of mass $25\ \text{kg}$ is going downwards with acceleration $3\ \text{ms}^{-2}$, monkey 3 of mass $20\ \text{kg}$ is at rest and monkey 4 of mass $40\ \text{kg}$ is moving downward with constant speed. Calculate the force exerted by ceiling on the rope in newton. Take $g = 9.8\ \text{ms}^{-2}$

30. A gun is mounted on a railroad car. The mass of the car with all of its components is $80m$ and mass of each shell to be fired is $5m$. If the muzzle velocity of the shells is $100\ \text{ms}^{-1}$ horizontally, then calculate the recoil speed of the car after second shot (in ms^{-1}). Consider car to be at rest initially.

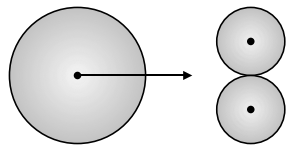
31. A 4 g bullet travelling horizontally with a velocity of magnitude $600\ \text{ms}^{-1}$ is fired into a wooden block with a mass of 1 kg, initially at rest on a level surface. The bullet passes through the block and emerges with speed $100\ \text{ms}^{-1}$. The block slides a distance of 0.4 m along the surface from its initial position.

(a) The coefficient of kinetic friction between the block and surface is $\mu_k = \frac{1}{*}$, where $*$ is not readable. Find $*$.

(b) What is the decrease in kinetic energy of the bullet, in joule?

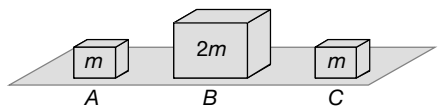
(c) What is the kinetic energy of the block, in joule, at the instant after the bullet passed through it? Neglect friction during collision of bullet with the block. Take $g = 10\ \text{ms}^{-2}$.

32. Two equal discs are in contact on a table. A third disc of same mass but of double radius strikes them symmetrically and remains at rest after impact. Find the coefficient of restitution.



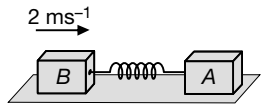
33. A space shuttle of mass M, moving at $4000\ \text{kph}$ relative to earth ejects a capsule backward of mass $\frac{M}{5}$. If speed of ejection of capsule is $120\ \text{kmph}$ relative to state of shuttle before ejection, find the final velocity of the shuttle in kmph.

34. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed $9\ \text{ms}^{-1}$ and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in ms^{-1}) of the object C.

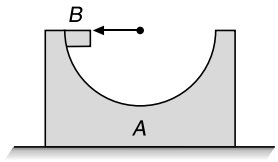


35. Two balls A and B having different but unknown masses, collide elastically. A is initially at rest when B has a speed v. After collision B as a speed $\frac{v}{2}$ and moves at right angles to its original motion. The direction in which ball A moves after collision is $\tan^{-1}\left(\frac{1}{\alpha}\right)$. Calculate α. If the speed of A is $\sqrt{\beta}v$. Calculate β.

36. Two blocks A and B each of mass $m = 1\ \text{kg}$ are lying on a smooth horizontal surface as shown in figure. A spring of force constant $k = 200\ \text{Nm}^{-1}$ is fixed at one end of block A. Block B collides with block A with velocity $v_0 = 2\ \text{ms}^{-1}$. Find the maximum compression, in cm, of the spring.



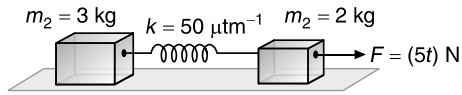
37. A block A of mass $6m$ having a smooth semi-circular groove of radius a placed on a smooth horizontal surface as shown in Figure.



A block B of mass m is released from a position in grove where its radius is horizontal. If the speed of bigger block when smaller block reaches its bottommost position is $\sqrt{\frac{ga}{*}}$ where $*$ is not readable. Calculate $*$.

38. A particle of mass m having collided a stationary particle of mass $M = 5\text{ m}$ deviated by an angle $\frac{\pi}{2}$ whereas the particle M recoiled at an angle $\theta = 30°$ to the direction of the initial motion of the particle m. Find the percentage change in kinetic energy of this system after collision.

39. On a spring block system, a time varying force $F = (5t)\text{ N}$ is applied on 2 kg mass as shown in Figure.



After 10 s, the velocity of 3 kg mass is 30 ms^{-1}. Calculate velocity of 2 kg mass at this instant in ms^{-1}.

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1. **[Online September 2020]**

A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass $3m$ at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by

(A) $v = \sqrt{\frac{2}{3}}u$

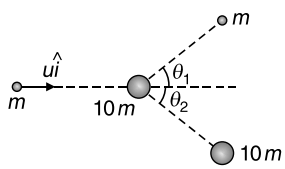
(B) $v = \frac{1}{\sqrt{6}}u$

(C) $v = \frac{u}{\sqrt{3}}$

(D) $v = \frac{u}{\sqrt{2}}$

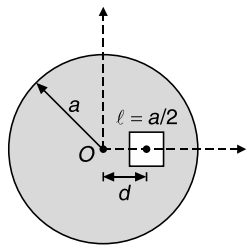
2. **[Online September 2020]**

A particle of mass m is moving long the x-axis with initial velocity $u\hat{i}$. It collides elastically with a particle of mass $10m$ at rest and then moves with half its initial kinetic energy (see figure). If $\sin\theta_1 = \sqrt{n}\sin\theta_2$ then value of n is ……



3. **[Online September 2020]**

A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from the center O of a uniform circular disk of radius a. If the distance of the center of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is ……



4. **[Online September 2020]**

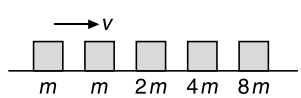
A block of mass 1.9 kg is at rest at the edge of a table, of height 1 m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20 ms^{-1} in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take $g = 10\text{ ms}^{-2}$. Assume there is no rotational motion and loss of energy after the collision is negligible]

(A) 21 J (B) 23 J

(C) 19 J (D) 20 J

5. **[Online September 2020]**

Blocks of masses m, $2m$, $4m$ and $8m$ are arranged in a line on a frictionless floor. Another block of mass m, moving with speed v along the same line (see figure) collides with mass m in perfectly inelastic manner. All the subsequent collisions are also perfectly inelastic. By the time the last block of mass $8m$ starts moving the total energy loss is $p\%$ of the original energy. Value of p is close to



(A) 77 (B) 37

(C) 87 (D) 94

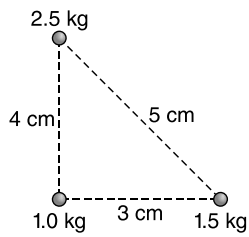
6. **[Online September 2020]**
Two bodies of the same mass are moving with the same speed, but in different directions in a plane. They have a completely inelastic collision and move together thereafter with a final speed which is half of their initial speed. The angle between the initial velocities of the two bodies (in degree) is

7. **[Online September 2020]**
Particle A of mass m_1 moving with velocity $\left(\sqrt{3}\hat{i}+\hat{j}\right)$ ms^{-1} collides with another particle B of mass m_2 which is at rest initially. Let $\vec{V}_1$ and $\vec{V}_2$ be the velocities of particle A and B after collision respectively. If $m_1 = 2m_2$ and after collision $\vec{V}_1 = \left(\hat{i}+\sqrt{3}\hat{j}\right)$ ms^{-1}, the angle between $\vec{V}_1$ and $\vec{V}_2$ is

(A) 60° (B) 15°
(C) –45° (D) 105°

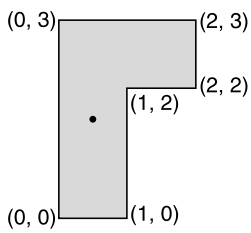
8. **[Online September 2020]**
The center of mass of a solid hemisphere of radius 8 cm is x cm from the center of the flat surface. Then value of x is ……

9. **[Online January 2020]**
Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in Figure. The center of mass of the system is at a point



(A) 1.5 cm right and 1.2 cm above 1 kg mass
(B) 0.9 cm right and 2.0 cm above 1 kg mass
(C) 0.6 cm right and 2.0 cm above 1 kg mass
(D) 2.0 cm right and 0.9 cm above 1 kg mass

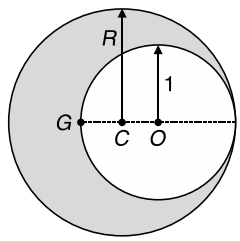
10. **[Online January 2020]**
The coordinates of center of mass of a uniform flag shaped lamina (thin flat plate) of mass 4 kg. (The coordinates of the same are shown in Figure) are



(A) (1.25 m, 1.50 m) (B) (1 m, 1.75 m)
(C) (0.75 m, 0.75 m) (D) (0.75 m, 1.75 m)

11. **[Online January 2020]**
A body A, of mass $m = 0.1$ kg has an initial velocity of $3\hat{i}$ ms^{-1}. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j}$ ms^{-1}. After collision, A moves with a velocity $\vec{v} = 4\left(\hat{i}+\hat{j}\right)$. The energy of B after collision is written as $\frac{x}{10}$ J. The value of x is ……

12. **[Online January 2020]**
A spherical cavity (centered at O) of radius 1 is cut out of a uniform sphere of radius R (centered at C) as shown in Figure. The center of mass of remaining (shaded) part of sphere is at G, i.e., on the surface of the cavity. R can be determined by the equation



(A) $(R^2 - R + 1)(2 - R) = 1$
(B) $(R^2 + R - 1)(2 - R) = 1$
(C) $(R^2 + R + 1)(2 - R) = 1$
(D) $(R^2 - R - 1)(2 - R) = 1$

13. **[Online January 2020]**
Two particles of equal mass m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is

(A) $\frac{3}{4}mu^2$ (B) $\frac{1}{8}mu^2$
(C) $\sqrt{\frac{2}{3}}mu^2$ (D) $\frac{1}{3}mu^2$

14. **[Online January 2020]**
A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the center of mass of the rod is at

(A) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$ (B) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$
(C) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$ (D) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

15. [Online January 2020]

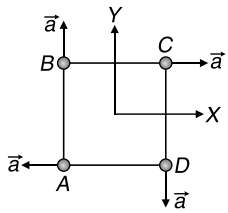
A particle of mass m is projected with a speed u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is

(A) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$ (B) $2\sqrt{2}\frac{u^2}{g}$

(C) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (D) $\frac{5}{8}\frac{u^2}{g}$

16. [Online April 2019]

Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is



(A) ZERO (B) $a\left(\hat{i}+\hat{j}\right)$

(C) $\frac{a}{5}\left(\hat{i}+\hat{j}\right)$ (D) $\frac{a}{5}\left(\hat{i}-\hat{j}\right)$

17. [Online April 2019]

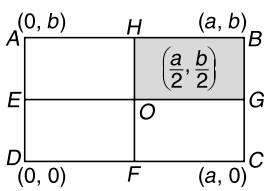
A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively. If $m_2 = 0.5\ m_1$ and $v_3 = 0.5\ v_1$, then v_1 is

(A) $v_4 - \frac{v_2}{4}$ (B) $v_4 - v_2$

(C) $v_4 + v_2$ (D) $v_4 - \frac{v_2}{2}$

18. [Online April 2019]

A uniform rectangular thin sheet $ABCD$ of mass M has length a and breadth b, as shown in the figure. If the shaded portion $HBGO$ is cut-off, the coordinates of the centre of mass of the remaining portion will be



(A) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (B) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

(C) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (D) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

19. [Online April 2019]

A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body?

(A) 1.5 kg (B) 1.8 kg

(C) 1.0 kg (D) 1.2 kg

20. [Online April 2019]

A wedge of mass $M = 4m$ lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by

(A) $\frac{v^2}{g}$ (B) $\frac{2v^2}{5g}$

(C) $\frac{2v^2}{7g}$ (D) $\frac{v^2}{2g}$

21. [Online April 2019]

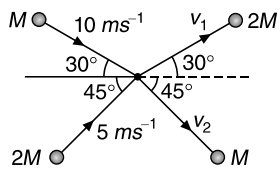
A particle of mass m is moving with speed $2v$ and collides with a mass $2m$ moving with speed v in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass m, which move at angle 45° with respect to the original direction. The speed of each of the moving particle will be

(A) $\frac{v}{2\sqrt{2}}$ (B) $\frac{v}{\sqrt{2}}$

(C) $\sqrt{2}v$ (D) $2\sqrt{2}v$

22. [Online April 2019]

Two particles, of masses M and $2M$, moving, as shown, with speeds of $10\ \text{ms}^{-1}$ and $5\ \text{ms}^{-1}$, collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2, respectively. The values of v_1 and v_2 are nearly



(A) $3.2\ \text{ms}^{-1}$ and $12.6\ \text{ms}^{-1}$

(B) $3.2\ \text{ms}^{-1}$ and $6.3\ \text{ms}^{-1}$

(C) $6.5\ \text{ms}^{-1}$ and $3.2\ \text{ms}^{-1}$

(D) $6.5\ \text{ms}^{-1}$ and $6.3\ \text{ms}^{-1}$

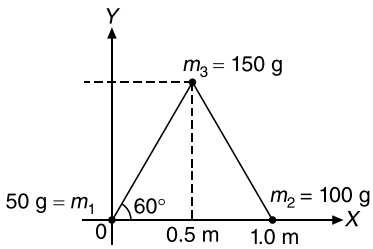
23. [Online April 2019]

A man $(\text{mass} = 50\ \text{kg})$ and his son $(\text{mass} = 20\ \text{kg})$ are standing on a frictionless surface facing each other. The man pushed his son so that he starts moving at a speed of $0.70\ \text{ms}^{-1}$ with respect to the man. The speed of the man with respect to the surface is

(A) $0.20\ \text{ms}^{-1}$ (B) $0.14\ \text{ms}^{-1}$
(C) $0.47\ \text{ms}^{-1}$ (D) $0.28\ \text{ms}^{-1}$

24. [Online April 2019]

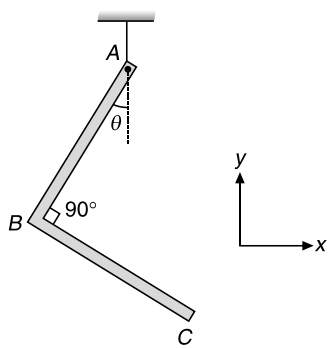
Three particles of masses, $50\ \text{g}$, $100\ \text{g}$ and $150\ \text{g}$ are placed at the vertices of an equilateral triangle of side $1\ \text{m}$ (as shown in the figure). The (x, y) coordinates of the centre of mass will be



(A) $\left(\frac{\sqrt{3}}{8}\ \text{m}, \frac{7}{12}\ \text{m}\right)$ (B) $\left(\frac{\sqrt{3}}{4}\ \text{m}, \frac{5}{12}\ \text{m}\right)$
(C) $\left(\frac{7}{12}\ \text{m}, \frac{\sqrt{3}}{8}\ \text{m}\right)$ (D) $\left(\frac{7}{12}\ \text{m}, \frac{\sqrt{3}}{4}\ \text{m}\right)$

25. [Online January 2019]

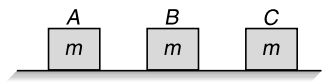
An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If $AB = BC$, and the angle made by AB with downward vertical is θ, then



(A) $\tan\theta = \frac{1}{2}$ (B) $\tan\theta = \frac{2}{\sqrt{3}}$
(C) $\tan\theta = \frac{1}{3}$ (D) $\tan\theta = \frac{1}{2\sqrt{3}}$

26. [Online January 2019]

Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of $\frac{M}{m}$?



(A) 3 (B) 4
(C) 2 (D) 5

27. [Online January 2019]

A piece of wood of mass $0.03\ \text{kg}$ is dropped from the top of a $100\ \text{m}$ height building. At the same time, a bullet of mass $0.02\ \text{kg}$ is fired vertically upwards, with a velocity $100\ \text{ms}^{-1}$, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) 30 m (B) 40 m
(C) 20 m (D) 10 m

28. [Online January 2019]

A body of mass $1\ \text{kg}$ falls freely from a height of $100\ \text{m}$, on a platform of mass $3\ \text{kg}$ which is mounted on a spring having spring constant $k = 1.25 \times 10^6\ \text{Nm}^{-1}$. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10\ \text{ms}^{-2}$, the value of x will be close to

(A) 80 cm (B) 8 cm
(C) 4 cm (D) 40 cm

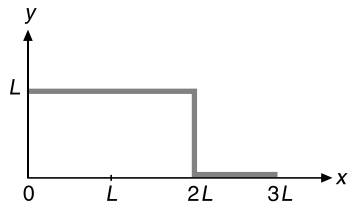
29. [Online January 2019]

A simple pendulum, made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then M is given by

(A) $m\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$ (B) $\frac{m}{2}\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$
(C) $\frac{m}{2}\left(\frac{\theta_0 + \theta_1}{\theta_0 - \theta_1}\right)$ (D) $m\left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)$

30. [Online January 2019]

The position vector of the center of mass $\vec{r}_{cm}$ of an asymmetric uniform bar of negligible area of cross-section as shown in Figure is



(A) $\vec{r}_{cm} = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$ (B) $\vec{r}_{cm} = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$

(C) $\vec{r}_{cm} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$ (D) $\vec{r}_{cm} = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$

31. [2018]

In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

(A) $\frac{v_0}{4}$ (B) $\sqrt{2}v_0$

(C) $\frac{v_0}{2}$ (D) $\frac{v_0}{\sqrt{2}}$

32. [Online 2018]

A proton of mass m collides elastically with a particle of unknown mass at rest. After the collision, the proton and the unknown particle are seen moving at an angle of $90°$ with respect to each other. The mass of unknown particle is

(A) m (B) $2m$

(C) $\frac{m}{\sqrt{3}}$ (D) $\frac{m}{2}$

33. [Online 2017]

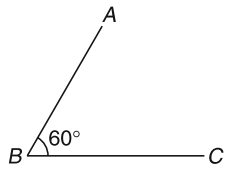
An object is dropped from a height h from the ground. Every time it hits the ground it looses 50% of its kinetic energy. The total distance covered at $t \to \infty$ is

(A) $2h$ (B) $\frac{8}{3}h$

(C) $\frac{5}{3}h$ (D) ∞

34. [Online 2016]

In the figure shown ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\frac{BC}{AB}$ is close to



(A) 1.85 (B) 1.5

(C) 1.37 (D) 3

35. [2015]

Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to

(A) h (B) $\frac{3h}{4}$

(C) $\frac{5h}{8}$ (D) $\frac{3h^2}{8R}$

36. [Online 2015]

A uniform thin rod AB of length L has linear mass density $\mu(x) = a + \frac{bx}{L}$, where x is measured from A. If the CM of the rod lies at a distance of $\frac{7L}{12}$ from A, then a and b are related as

(A) $a = b$ (B) $a = 2b$

(C) $2a = b$ (D) $3a = 2b$

37. [2015]

A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to

(A) 44% (B) 50%

(C) 56% (D) 62%

38. [2010]

Statement-1: Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2: Principle of conservation of momentum holds true for all kinds of collisions.

(A) Statement-1 is true, Statement-2 is false.

(B) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.

(C) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1.

(D) Statement-1 is false, Statement-2 is true.

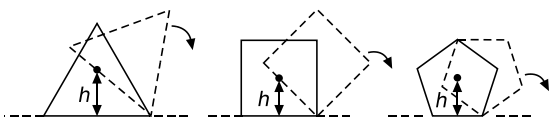
ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct)

1. [JEE (Advanced) 2017]

Consider regular polygons with number of sides $n = 3$, 4, 5…….as shown in the figure. The centre of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the centre of mass for each polygon is Δ. Then, Δ depends on n and h as

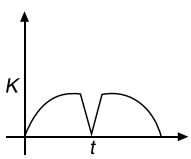


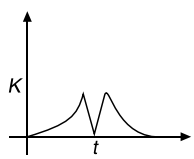
(A) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$ (B) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$

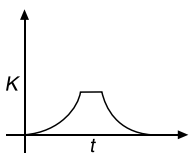
(C) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$ (D) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$

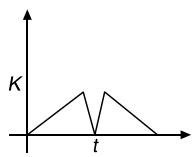
2. [JEE (Advanced) 2014]

A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale.

(A)


(B)


(C)


(D)


3. [JEE (Advanced) 2013]

A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mV and the speed of light is 3×10^8 ms^{-1}. The final momentum of the object is

(A) 0.3×10^{-17} kgms^{-1} (B) 1.0×10^{-17} kgms^{-1}

(C) 3.0×10^{-17} kgms^{-1} (D) 9.0×10^{-17} kgms^{-1}

4. [JEE (Advanced) 2013]

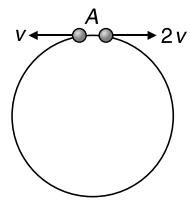
A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{4}+\alpha$

(C) $\frac{\pi}{4}-\alpha$ (D) $\frac{\pi}{2}$

5. [JEE (Advanced) 2009]

Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$ respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A?

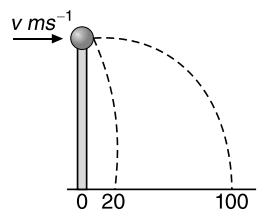


(A) 4 (B) 3

(C) 2 (D) 1

6. [IIT-JEE 2011]

A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity v ms^{-1} in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity v of the bullet is

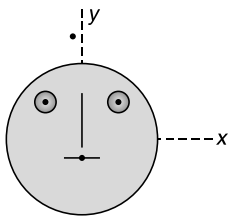


(A) 250 ms^{-1} (B) $250\sqrt{2}$ ms^{-1}

(C) 400 ms^{-1} (D) 500 ms^{-1}

7. [IIT-JEE 2009]

Look at the drawing given in the figure, which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles and each of the two line segments is m. The mass of the ink used to draw the outer circle is $6\ m$. The coordinates of the centres of the different parts are: outer circle (0, 0), left inner circle $(-a, a)$, right inner circle (a, a), vertical line (0, 0), and horizontal line $(0, -a)$. The y coordinate of the centre of mass of the ink in this drawing is



(A) $\frac{a}{10}$ (B) $\frac{a}{8}$

(C) $\frac{a}{12}$ (D) $\frac{a}{3}$

8. [IIT-JEE 2009]

If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

(A) linear momentum of the system does not change in time

(B) kinetic energy of the system does not change in time

(C) angular momentum of the system does not change in time

(D) potential energy of the system does not change in time

9. [IIT-JEE 2007]

A particle moves in the x-y plane under the influence of a force such that its linear momentum is $\vec{p}(t) = A\left[\hat{i}\cos(kt) - \hat{j}\sin(kt)\right]$, where, A and k are constants. The angle between the force the momentum is

(A) $0°$ (B) $30°$

(C) $45°$ (D) $90°$

10. [IIT-JEE 2002]

Two blocks of masses $10\ \text{kg}$ and $4\ \text{kg}$ are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of $14\ \text{ms}^{-1}$ to the heavier block in the direction of the lighter block. The velocity of the centre of mass is

(A) $30\ \text{ms}^{-1}$ (B) $20\ \text{ms}^{-1}$

(C) $10\ \text{ms}^{-1}$ (D) $5\ \text{ms}^{-1}$

11. [IIT-JEE 2001]

Two particles of masses m_1 and m_2 in projectile motion have velocities v_1 and v_2 respectively at time $t = 0$. They collide at time t_0. Their velocities become $\vec{v}_1'$ and $\vec{v}_2'$ at time $2t_0$, while still moving in air. The value of $\left|\left(m_1\vec{v}_1' + m_2\vec{v}_2'\right) - \left(m_1\vec{v}_1 + m_2\vec{v}_2\right)\right|$ is

(A) ZERO (B) $(m_1 + m_2)gt_0$

(C) $2(m_1 + m_2)gt_0$ (D) $\frac{1}{2}(m_1 + m_2)gt_0$

12. [IIT-JEE 1997]

An isolated particle of mass m is moving in horizontal plane $(x\text{-}y)$, along the x-axis, at a certain height above the ground. It suddenly explodes into two fragment of masses $\frac{m}{4}$ and $\frac{3m}{4}$. An instant later, the smaller fragment is at $y = +15\ \text{cm}$. The larger fragment at this instant is at

(A) $y = -5\ \text{cm}$ (B) $y = +20\ \text{cm}$

(C) $y = +5\ \text{cm}$ (D) $y = -20\ \text{cm}$

13. [IIT-JEE 1986]

A shell is fired from a canon with a velocity v at an angle θ with the horizontal. At the highest point it explodes into two pieces of equal masses. One of the pieces retraces its path to the canon. The speed of the other piece immediately after the explosion is

(A) $3v\cos\theta$ (B) $2v\cos\theta$

(C) $\frac{3}{2}v\cos\theta$ (D) $v\cos\theta$

14. [IIT-JEE 1986]

A ball hits the floor and rebounds after an inelastic collision. In this case

(A) the momentum of the ball just after the collision is the same as that just before the collision

(B) the mechanical energy of the ball remains the same in the collision

(C) the total momentum of the ball and the earth is conserved

(D) the total energy of the ball and the earth is conserved

15. [IIT-JEE 1982]

Two particles A and B, initially at rest, move towards each other under a mutual force of attraction. At the instant when the speed of A is V and the speed of B is $2V$, the speed of the centre of mass of the system is

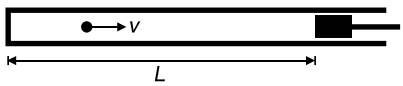
(A) zero (B) V
(C) $1.5V$ (D) $3V$

Multiple Correct Choice Type Problems

(In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct)

1. [JEE (Advanced) 2019]

A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V \ll \frac{dL}{L} v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct?



(A) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

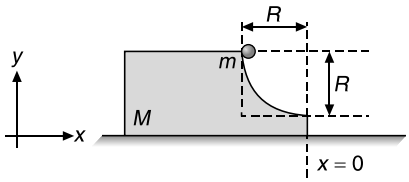
(B) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

(C) After each collision with the piston, the particle speed increases by $2V$

(D) The rate at which the particle strikes the piston $\frac{v}{L}$

2. [JEE (Advanced) 2017]

A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surfaced of a fixed table. Initially the right edge of the block is at $x = 0$, in a coordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following option is/are correct?



(A) The velocity of the point mass m is $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$

(B) The x component of displacement of the centre of mass of the block M is $-\frac{mR}{M+m}$

(C) The position of the point mass is $x = -\sqrt{2}\frac{mR}{M+m}$

(D) The velocity of the block M is $V = -\frac{m}{M}\sqrt{2gR}$

3. [IIT-JEE 2010]

A point mass of $1\,\text{kg}$ collides elastically with a stationary point mass of 5 kg. After their collision, the $1\,\text{kg}$ mass reverses its direction and moves with a speed of $2\ \text{ms}^{-1}$. Which of the following statement(s) is (are) correct for the system of these two masses?

(A) Total momentum of the system is $3\ \text{kgms}^{-1}$

(B) Momentum of 5 kg mass after collision is $4\ \text{kgms}^{-1}$

(C) Kinetic energy of the centre of mass is 0.75 J

(D) Total kinetic energy of the system is 4 J

4. [IIT-JEE 2008]

Two balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}_1'$ and $\vec{p}_2'$ be their final momenta. The following option figure(s) is (are) not allowed for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2.

(A) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j}$

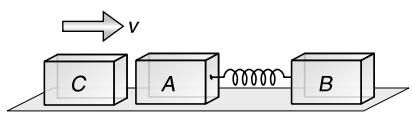
(B) $\vec{p}_1' = c_1\hat{k}$, $\vec{p}_2' = c_2\hat{k}$

(C) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

(D) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j}$, $\vec{p}_2' = a_2\hat{i} + b_1\hat{j}$

5. [IIT-JEE 1993]

Two blocks A and B each of mass m are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown in figure. A third identical block C, also of mass m, moves on the floor with a speed v along the line joining A to B and collides with A. Then



(A) the kinetic energy of the A-B system, at maximum compression of the spring is zero

(B) the kinetic energy of the A-B system, at maximum compression of the spring is $\frac{mv^2}{4}$

(C) the maximum compression of the spring is $v\sqrt{\frac{m}{k}}$

(D) the maximum compression of the spring is $v\sqrt{\frac{m}{2k}}$

Reasoning Based Questions

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. [IIT-JEE 2007]
Statement-1: If there is no external torque on a body about its centre of mass, then the velocity of the centre of mass remains constant.
Statement-2: The linear momentum of an isolated system remains constant.

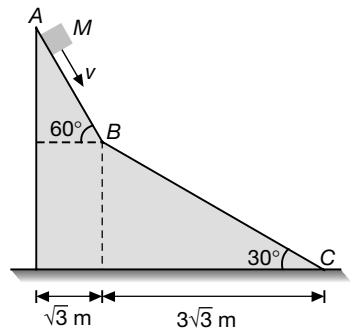
2. [IIT-JEE 2007]
Statement-1: In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision.
Statement-2: In an elastic collision, the linear momentum of the system is conserved.

Linked Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from $60°$ to $30°$ at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic $\left(g = 10\ \text{ms}^{-2}\right)$.



1. [IIT-JEE 2008]
The speed of the block at point B immediately after it strikes the second incline is

(A) $\sqrt{60}\ \text{ms}^{-1}$ (B) $\sqrt{45}\ \text{ms}^{-1}$
(C) $\sqrt{30}\ \text{ms}^{-1}$ (D) $\sqrt{15}\ \text{ms}^{-1}$

2. [IIT-JEE 2008]
The speed of the block at point C, immediately before it leaves the second incline is

(A) $\sqrt{120}\ \text{ms}^{-1}$ (B) $\sqrt{105}\ \text{ms}^{-1}$
(C) $\sqrt{90}\ \text{ms}^{-1}$ (D) $\sqrt{75}\ \text{ms}^{-1}$

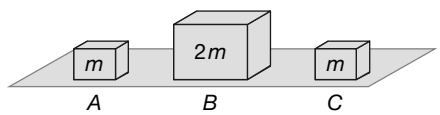
3. [IIT-JEE 2008]
If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is

(A) $\sqrt{30}\ \text{ms}^{-1}$ (B) $\sqrt{15}\ \text{ms}^{-1}$
(C) 0 (D) $-\sqrt{15}\ \text{ms}^{-1}$

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. [IIT-JEE 2009]
Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed $9\ \text{ms}^{-1}$ and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed $\left(\text{in ms}^{-1}\right)$ of the object C.



ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Centre of Mass)

1. $\frac{m_1\ell_1 + m_2\ell_2}{m_1 + m_2}$
2. $\frac{m_2 x}{m_1}$
3. $\frac{1}{2}\left(3\hat{i} + \hat{j} - \hat{k}\right)$ m
4. $\frac{6R}{\pi}$
5. $\left(-\frac{a}{12}, -\frac{b}{12}\right)$
6. $\left(\frac{2r}{3(4-\pi)}, 0\right)$
7. 2 kg
8. $\frac{(\rho_1 + 3\rho_2)L}{4(\rho_1 + \rho_2)}$
9. $\left[\frac{l}{2}\left(\frac{4+\sqrt{3}+3\pi}{4+\sqrt{3}+\pi}\right), \frac{l}{2}\left(\frac{5+2\sqrt{3}+\pi}{4+\sqrt{3}+\pi}\right)\right]$
10. $\frac{3\ell}{4}$
11. $\left(\frac{L}{8}, \frac{5L}{8}\right)$
12. $\frac{\pi a}{\pi + 4}$
13. $\frac{R}{2(2\pi - 1)}$
14. $\left(\frac{\lambda_1 + 2\lambda_2}{\lambda_1 + \lambda_2}\right)\frac{L}{3}$

Test Your Concepts-II (Based on Motion of Centre of Mass)

1. $\frac{m_1 L}{2(m_1 + m_2)}$
2. $\sqrt{31}$ ms^{-2}
3. 5 m
4. 45 m, 10 ms^{-1}
5. 13.6 m
6. $l/12$
7. $5l/28$
8. (a) $\left(1.5\hat{i} + 2\hat{j}\right)$ ms^{-1}, (b) $\left(-3\hat{i} + 4\hat{j}\right)$ m
9. $\frac{mR\sin\theta}{m+M}$
10. 1.25 ms^{-1}
11. (a) 0, (b) not accelerate, (c) yes, upwards (d) $\frac{ml}{2M}$, upwards
12. (a) 0.3 kg , (b) $2.4\hat{j}$ kgms^{-1}, (c) $8\hat{j}$ ms^{-1}
13. $\frac{ML}{m+M}$
14. (a) 40 cm , (b) 2.67 ms^{-1}
15. $\frac{ml}{M+m}$
16. $\frac{(m_1 + m_2)x_1 - m_2 l}{m_1 + m_2 + m_0}$
17. $-1.5\hat{i} + 2\hat{j}$ ms^{-1}, $\left(-3\hat{i} + 4\hat{j}\right)$ m
18. $\frac{2nv\cos\left(\frac{\theta}{2}\right)}{1+n}$

Test Your Concepts-III (Based on Work Energy Theorem for System of Particles and its Applications from Centre of Mass Reference Frame)

1. 10 J
2. $\sqrt{2gR\left(\frac{2}{\pi} + \frac{\pi}{2}\right)}$
3. $3v_0^2/g$
4. $2\sqrt{\frac{3m}{k}}v_0$
5. $x\sqrt{\frac{k}{2m}}$
6. $\frac{2(m_1F_2 + m_2F_1)}{k(m_1 + m_2)}$
7. $-\frac{3}{8}mv_0^2$
8. $mgl/18$
9. $2v_0\sqrt{\frac{m}{k}}$

Test Your Concepts-IV (Based on Conservation of Linear Momentum)

1. $\frac{m_1 v_r}{m_1 + m_2}$
2. $\frac{mu}{m+M}$
3. $R\sqrt{\frac{2(M+m)}{M}}$
4. $\sqrt{2gh}\tan\theta, m\sqrt{2gh}\sec\theta, mgh(1-\tan^2\theta)$
5. 2.55 ms^{-1}, 24.1 ms^{-1}
6. $\frac{5}{3}v_0\cos\theta_0$, along x direction
7. $2m\sin\left(\frac{\alpha}{2}\right)\sqrt{\frac{g\ell}{(M+m)M}}$
8. (a) $v' = \frac{u}{2}$ and $\int T dt = \frac{mu}{2}$ (b) $v' = \frac{u\sqrt{13}}{8}$ and $\int T dt = \frac{mu\sqrt{13}}{8}$ (c) $v' = \frac{u\sqrt{3}}{4}$ and $\int T dt = \frac{mu\sqrt{3}}{4}$
9. $\frac{mv}{m+M}$
10. $\frac{v}{2}$
11. (a) $\frac{2mu}{M+2m}$, (b) $\frac{mu}{M+2m} + \frac{mu}{M+m}$
12. $\frac{2k^2+2k+1}{k}$
13. $\sqrt{\frac{2MgR}{M+m}}$
14. $mv_0\cos\theta, \frac{mv_0\sin\theta}{M}$
15. $\frac{2m}{M-m}\sqrt{\frac{2h}{g}}, \frac{Mmgh}{M+m}$
16. $mv_0\,\text{cosec}\,\theta, mv_0\cot\theta, v_0\cot\theta, \frac{mv_0}{M}, \mu mv_0\cot\theta$
17. 1.5 ms^{-1}, 0.1 m
18. $\frac{Mu^2}{2g(M+m)}$

Test Your Concepts-V (Based on Variable Mass Systems)

1. $\frac{F}{m_0 - \mu t}, \frac{F}{\mu}\log_e\left(\frac{m_0}{m_0 - \mu t}\right)$
2. $\rho A(v+u)^2$
3. (a) (i) 2.5 kgs^{-1}, (ii) 7.5 kgs^{-1} (b) 4.156 kms^{-1}
4. $\frac{(m_1 - m_0 + bt)g + bv_0}{m_0 + m_1 - bt}$
5. $\lambda v^2 + \lambda g(h-y)$
6. $\frac{M}{k} - \frac{v_0}{g}$
7. $m_0 e^{-\left(\frac{a}{u}\right)t}$
8. $\frac{F}{m_0 - \mu t}, \frac{F}{\mu}\ln\left(\frac{m_0}{m_0 - \mu t}\right)$
9. $\left(\frac{2M_0 g + \mu v_0}{\mu}\right)\log_e\left(\frac{2M_0}{2M_0 - \mu t}\right) + gt$

Test Your Concepts-VI (Based on Head-On Collisions)

1. $\frac{1}{\sqrt{2}}$
2. **64 J**
3. 4 ms^{-1}
4. $0.23d$
5. $v_1 = 0$ and $v_2 = 6$ ms^{-1}
6. $\frac{\sqrt{2\mu g d} - \sqrt{v_0^2 - 2\mu g d}}{2}$
7. $\left(\frac{1+e}{2}\right)^{n-1}u$
8. $9h$
9. $\sqrt{m_1 m_3}$
10. $\frac{m_2 g}{k}\left(\frac{m_2 + 2m_1}{2m_1}\right)$
11. $-2v$, $-v$
12. $x\sqrt{\frac{kM}{m(M+m)}}, x\sqrt{\frac{km}{M(M+m)}}$

Test Your Concepts-VII (Based on Oblique Collisions)

1. $\frac{H^5}{L^4}$
2. $-\frac{3}{2}\hat{i} + \hat{j}, \frac{27m}{8}, -\left(\frac{9m}{2}\right)\hat{i}$
3. $\frac{mv_0\cos\theta}{M+m}, \frac{mv_0^2}{2}\left(\frac{M + m\sin^2\theta}{M+m}\right)$

4. $m\left(-3\hat{i}+4\hat{j}\right)$, $\frac{9}{16}$
5. $\sqrt{u^2+v^2}$
6. $\left[\frac{(1+e)m\sin\theta}{M+m\sin^2\theta}\right]v_0$, $\left(\frac{m\sin^2\theta-Me}{M+m\sin^2\theta}\right)v_0$
7. eL_1
8. $v \cong 8\ \text{ms}^{-1}$
9. $e=0$, $e=\tan^2\theta=\tan^2\frac{\pi}{6}=\frac{1}{3}$
10. (a) e^2h_1, (b) e^2h_1
11. $\frac{1}{\left(\frac{u^2\sin(2\alpha)}{ag}-1\right)}$

Single Correct Choice Type Questions

1. B	2. A	3. C	4. B	5. B	6. b	7. D	8. A	9. B	10. B
11. C	12. D	13. A	14. A	15. C	16. B	17. B	18. A	19. C	20. B
21. C	22. C	23. B	24. B	25. D	26. B	27. D	28. A	29. B	30. A
31. C	32. B	33. B	34. C	35. D	36. D	37. B	38. C	39. D	40. A
41. C	42. C	43. C	44. A	45. D	46. A	47. D	48. A	49. C	50. C
51. B	52. C	53. A	54. D	55. D	56. A	57. C	58. C	59. D	60. D
61. C	62. D	63. B	64. C	65. C	66. D	67. A	68. B	69. D	70. C
71. B	72. B	73. A	74. D	75. B	76. D	77. A	78. B	79. C	80. B
81. C	82. B	83. B	84. D	85. D	86. B	87. C	88. C	89. B	90. C
91. A	92. A	93. D	94. B	95. A	96. A	97. B	98. C	99. D	100. C
101. C	102. C	103. C	104. C	105. A	106. B	107. D	108. C	109. C	110. D
111. C	112. C	113. D	114. A	115. D	116. B	117. C	118. D	119. C	120. D
121. A	122. D	123. B	124. C	125. B	126. C	127. B	128. A		

Multiple Correct Choice Type Questions

1. B, C	2. A, B, D	3. A, D	4. A, D	5. B, D
6. B, D	7. A, B, D	8. B, C	9. B, D	10. A, C, D
11. B, D	12. B, C, D	13. B, C	14. A, B, D	15. B, D
16. A, C	17. A, B, D	18. A, B, C, D	19. B, C	20. B, C
21. B, C	22. A, C	23. A, B, C	24. B, D	25. B, C, D
26. A, B, C	27. B, C	28. A, C	29. A, B, D	30. B, C, D
31. B, D	32. B, C	33. B, C	34. A, D	35. C, D
36. A, B, C, D	37. B, C			

Reasoning Based Questions

1. A	2. D	3. A	4. C	5. D	6. D	7. C	8. D	9. C	10. A

Linked Comprehension Type Questions

1. B	2. A	3. B	4. B	5. C	6. C	7. D	8. D	9. B	10. A
11. C	12. D	13. C	14. B	15. A	16. D	17. C	18. D	19. C	20. C

21. B	**22.** C	**23.** D	**24.** C	**25.** B	**26.** C	**27.** D	**28.** C	**29.** C	**30.** B
31. D	**32.** C	**33.** D	**34.** C	**35.** C	**36.** B, D	**37.** A	**38.** C	**39.** C	**40.** C
41. B	**42.** B	**43.** C	**44.** C	**45.** B	**46.** B	**47.** B	**48.** C	**49.** D	**50.** C
51. B	**52.** C	**53.** A	**54.** B	**55.** D	**56.** D	**57.** C	**58.** A	**59.** C	**60.** C
61. D	**62.** B								

Matrix Match/Column Match Type Questions

1.	A → (p)	B → (r)	C → (s)	D → (r)
2.	A → (p)	B → (p,	C → (q)	D → (r)
3.	A → (q)	B → (q)	C → (p)	D → (s)
4.	A → (q)	B → (s)	C → (r)	D → (p)
5.	A → (p, r, s)	B → (p, r, s)	C → (p)	D → (p, q)
6.	A → (r)	B → (p)	C → (q)	D → (s)
7.	A → (r)	B → (q)	C → (s)	D → (p)
8.	A → (q)	B → (r)	C → (p)	D → (p)
9.	A → (p, r)	B → (p, r)	C → (q)	D → (q)
10.	A → (q)	B → (p, r)	C → (p, r)	D → (r, s)
11.	A → (s)	B → (p)	C → (p)	D → (r)
12.	A → (s)	B → (s)	C → (p, q, r)	D → (p, q, r)
13.	A → (r)	B → (s)	C → (p)	D → (q)
14.	A → (q)	B → (q)	C → (q)	D → (p)
15.	A → (r)	B → (r, s)	C → (p, q)	D → (p)
16.	A → (q)	B → (p)	C → (r)	D → (r)
17.	A → (q, r)	B → (q, r)	C → (p, s)	D → (p)
18.	A → (r)	B → (r)	C → (q)	D → (p)
19.	A → (q)	B → (s)	C → (p)	D → (r)

Integer/Numerical Answer Type Questions

1. 6	**2.** 70	**3.** 10	**4.** 0.75	**5.** 4
6. 35	**7.** 45	**8.** (a) 2, (b) 24, (c) 72	**9.** 30	**10.** 40
11. 2, 8	**12.** 10	**13.** 100	**14.** 5	**15.** 2
16. 2	**17.** 4, 5	**18.** 11	**19.** 16	**20.** 1, 4
21. 4	**22.** 450	**23.** 6	**24.** 13	**25.** 8
26. 2	**27.** 180	**28.** 2, 4, 1, 1, 7, 9	**29.** 920	**30.** 13
31. (a) 2, (b) 700, (c) 2	**32.** 0.56	**33.** 4030	**34.** 4	**35.** 2, 0.45
36. 10	**37.** 21	**38.** 40	**39.** 80	

ARCHIVE: JEE MAIN

1. D	**2.** 10.00	**3.** 23.00	**4.** A	**5.** D	**6.** 120.00	**7.** D	**8.** 3	**9.** B	**10.** D
11. 1.00	**12.** C	**13.** B	**14.** D	**15.** C	**16.** D	**17.** B	**18.** B	**19.** D	**20.** B
21. D	**22.** D	**23.** A	**24.** D	**25.** C	**26.** B	**27.** B	**28.** *	**29.** A	**30.** C
31. B	**32.** A	**33.** *	**34.** C	**35.** B	**36.** C	**37.** C	**38.** B		

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Single Correct Choice Type Problems

1. D	**2.** B	**3.** B	**4.** A	**5.** C	**6.** D	**7.** A	**8.** A	**9.** D	**10.** C
11. C	**12.** A	**13.** A	**14.** C	**15.** A					

Multiple Correct Choice Type Problems

1. B, C	**2.** A, B	**3.** A, C	**4.** A, D	**5.** B, D

Reasoning Based Questions

1. D	**2.** D

Linked Comprehension Type Questions

1. B	**2.** B	**3.** C

* No given option is correct.

CHAPTER

3 Rotational Dynamics

Learning Objectives

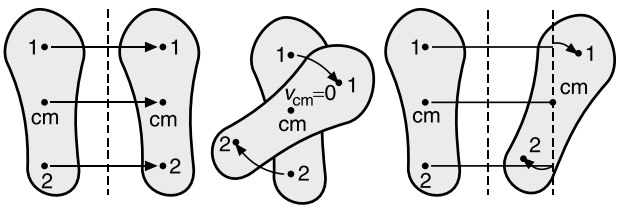
After reading this chapter, you will be able to understand concepts and problems based on:

(a) Moment of Inertia
(b) Parallel and Perpendicular Axis Theorem
(c) Rotational Kinematics
(d) Combined Effect of Rotation and Translation Motion
(e) Instantaneous Axis of Rotation
(f) Pure Rolling
(g) Conservation of Energy
(h) Concept of Torque
(i) General Motion of a Rigid Body
(j) Concept of Toppling
(k) Shifting of Normal Reaction
(l) Work-Energy Principle
(m) Modified Newton's Second Law for Fixed Axis Rotation
(n) Uniform and Accelerated Pure Rolling
(o) Angular Momentum and its Conservation
(p) Rolling with Slipping.

All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

INTRODUCTION

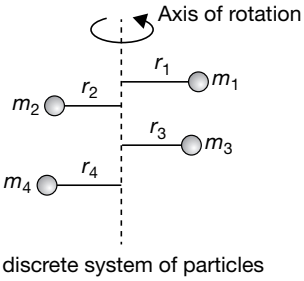
In the previous chapters we have studied the translator motion. In this chapter we will study the rotational motion of a rigid body about a fixed axis.



A rigid body is defined as an object that has fixed size and shape. In other words, the relative positions of its constituent particles remain constant. In actual, a rigid body does not exist – it is a useful idealization. By the term **fixed axis**, we mean that the axis must be fixed relative to the body and fixed in direction relative to an inertial position.

MOMENT OF INERTIA (I)

Moment of Inertia (MI) of a rigid n-particle system about an axis shown in Figure is the sum of the products of the masses of the particles with the square of their respective distances from the axis of rotation.



A discrete system of particles

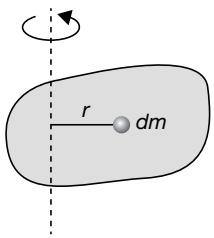
$$\Rightarrow \quad I = m_1 r_1^2 + m_2 r_2^2 + ... + m_n r_n^2 = \Sigma m_i r_{i\perp}^2$$

where m_i is the mass of the i^{th} particle and $r_{i\perp}$ is the perpendicular distance of the i^{th} particle from the Axis of Rotation (AOR). It is a tensor quantity with SI unit kgm^2.

Moment of Inertia depends upon the mass of the body and the manner the mass is distributed in the system with respect to the AOR. So, the moment of inertia depends on the location of the axis, that is, on how the mass of the body is distributed relative to the axis. Thus, a body does not possess a unique moment of inertia and different axes through the body are associated with different moments of inertia.

The role played by MI in rotational dynamics is actually the same as that played by inertia in linear dynamics. Moment of Inertia opposes any change in rotational motion and hence is also called **rotational inertia**.

To find the Moment of Inertia of a continuous mass distribution, we consider an element of mass dm at a perpendicular distance r from the AOR as shown in Figure.



The moment of inertia of this infinitesimal element is dI, given by the moment of inertia of the whole body is given by

$$I = \int r_\perp^2 dm = \int r^2 dm$$

where $r_\perp$ is the perpendicular distance of the element dm from the axis of rotation.

The integration has then to be carried within specified limits.

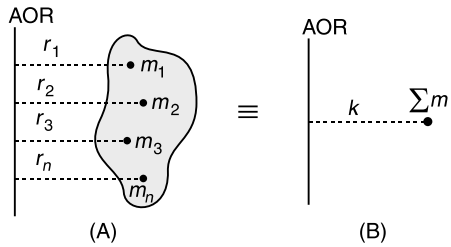
RADIUS OF GYRATION (k)

Consider the Figures shown. Figure (A) has the system with masses m_1, m_2, …, m_n at distances r_1, r_2,…, r_n from AOR. If we take a point mass $M = \Sigma m = m_1 + m_2 + ... + m_n$ and place it at a distance k from AOR, then we must get same value of Moment of Inertia from both the cases. So,

$$\Sigma m_i r_i^2 = Mk^2$$

$$\Rightarrow \quad k = \sqrt{\frac{m_1 r_1^2 + m_2 r_2^2 + ... + m_n r_n^2}{M}}$$

So, radius of gyration for a system is that distance from the AOR, where if the entire mass of the system is placed, then we get same value of MI as obtained from the original mass distribution.



SI unit of k is metre.

Further, for a symmetrical mass distribution we have

$$m_1 = m_2 = ... = m_n = m \text{ (say)}$$

$$M = \Sigma m = nm$$

$$\Rightarrow \quad k = \sqrt{\frac{r_1^2 + r_2^2 + ... + r_n^2}{n}}$$

So, for a symmetrical mass distribution, the radius of gyration equals the Root Mean Squares (RMS) of the distances of the particles from the AOR.

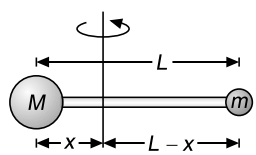
(The operation Root Mean Square is always to be read as Root of Mean of Squares).

Finally, in general for a system of total mass M rotating about an axis, we have

$$k = \sqrt{\frac{I}{M}}$$

ILLUSTRATION 1

Two balls with masses M and m are connected by a rigid rod of length L and negligible mass as in Figure.



For an axis perpendicular to the rod, show that the system has the minimum moment of inertia when the

axis passes through the centre of mass. Show that this moment of inertia is $I = \mu L^2$, where $\mu = \dfrac{mM}{m+M}$ is the reduced mass of system.

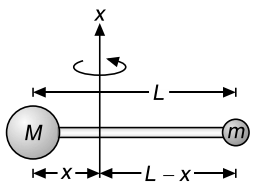
SOLUTION

$$I = Mx^2 + m(L-x)^2$$

For I to be minimum, $\dfrac{dI}{dx} = 0$

$$\Rightarrow \quad \frac{dI}{dx} = 2Mx - 2m(L-x) = 0$$

$$\Rightarrow \quad x = \frac{mL}{M+m} = (\text{Centre of Mass of System})$$



MI about an axis passing through x is

$$I_{CM} = M\left(\frac{mL}{M+m}\right)^2 + m\left(1-\frac{m}{M+m}\right)^2 L^2$$

$$\Rightarrow \quad I_{CM} = \left(\frac{Mm}{M+m}\right)L^2 = \mu L^2, \text{ where } \mu = \frac{Mm}{M+m}$$

So, we see that the moment of inertia of the system is minimum about the axis passing through the axis of rotation.

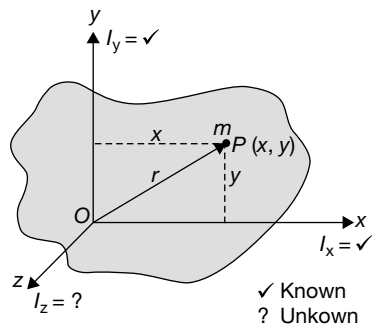
PERPENDICULAR AXIS THEOREM

This theorem is distinct for a 2-D body (such as a lamina or a planar body) and a 3-D body (such as cube, prism etc).

For a Lamina: The theorem states that the moment of inertia of a lamina about an axis perpendicular to the plane of lamina (say z-axis) is equal to the sum of the moments of inertia of lamina about two mutually perpendicular axis lying in its own plane (say x and y-axis) intersecting each other at the point where the two perpendicular axis meet.

If I_x and I_y be the moment of inertia of lamina about two mutually perpendicular axis x and y lying in its plane and I_z be the moment of inertia of lamina about third axis perpendicular to lamina and passing through intersection of x and y, then

$$I_z = I_x + I_y$$



Proof of Perpendicular Axis Theorem

Consider a lamina that lies in x-y plane. So, z axis will be perpendicular to x, y and hence normal to lamina. Now consider a particle of mass m lying at the point $P(x, y)$

Now, MI of particle about x axis is my^2

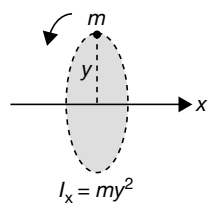
MI of body/lamina about x axis is

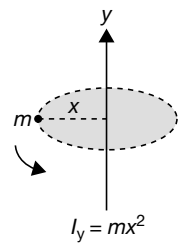
$$I_x = \Sigma my^2 \qquad \ldots(1)$$

Similarly, MI of particle about y axis is mx^2

MI of body/lamina about y axis is

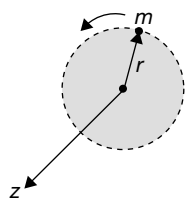
$$I_y = \Sigma mx^2 \qquad \ldots(2)$$





Finally, MI of particle about z axis is mr^2

MI of body/lamina about z axis is



$$I_z = \Sigma mr^2 \qquad \ldots(3)$$

Since, $r^2 = x^2 + y^2$

$$\Rightarrow \quad I_z = \Sigma mx^2 + \Sigma my^2$$

$$\Rightarrow \quad I_z = I_y + I_x$$

Conceptual Note(s)

For a 3-D body: The theorem, here states that the sum of the moments of inertia of a three-dimensional body about three mutually perpendicular axes is equal to twice the summation Σmr^2 about the origin, where $r^2 = x^2 + y^2 + z^2$

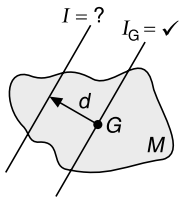
Mathematically,

$$I_x + I_y + I_z = 2\Sigma mr^2$$

$$\Rightarrow \quad I_x + I_y + I_z = 2\Sigma m\left(x^2 + y^2 + z^2\right)$$

PARALLEL AXIS THEOREM

This theorem has the same statement for a planar (laminar) body as well as a three-dimensional body.



It states that the moment of inertia of a body about any axis is equal to its moment of inertia about a parallel axis through its centre of gravity (CG) (or centre of mass (CM)) plus the product of the mass of the body and the square of the perpendicular distance between the two parallel axes.

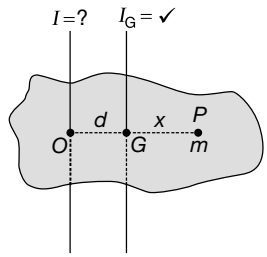
Mathematically, $I = I_G + (\Sigma m)d^2$

$$\Rightarrow \quad I = I_G + Md^2$$

where, M is the total mass of the body, I_G is the MI of body about CG and d is the perpendicular distance between two parallel axes.

Proof of Parallel Axis Theorem

Consider a body of mass M having centre of gravity G. Let I_G be the MI of body about an axis passing through the centre of gravity (G). Let us find the MI of the body about another axis parallel to axis passing through centre of gravity (G) at perpendicular distance d from it as shown in Figure.



For this let us consider a particle of mass m lying at a point P at perpendicular distance x from G. Then

$$I = I_G + Md^2$$

MI of particle about G is mx^2

MI of body about G is $I_G = \Sigma mx^2$ …(1)

MI of particle about O is $m(x+d)^2$

MI of body about O is $I = \Sigma m(x+d)^2$ …(2)

$$\Rightarrow \quad I = \Sigma m\left(x^2 + d^2 + 2xd\right)$$

$$\Rightarrow \quad I = \Sigma mx^2 + \Sigma md^2 + \Sigma m(2xd)$$

$$\Rightarrow \quad I = I_G + d^2\Sigma m + 2d\Sigma mx \qquad \ldots(3)$$

Now, since a body always balances itself when held from centre of gravity (CG), i.e., net torque due to weights of all the particles about CG is zero. Hence

$$\Sigma\left(mg\right)x = g\Sigma mx = 0$$

$$\Rightarrow \quad \Sigma mx = 0$$

$$\Rightarrow \quad I = I_G + Md^2 \qquad \{\text{from equation (3)}\}$$

Conceptual Note(s)

The moment of inertia of a body is minimum about an axis that passes through the centre of mass. So $\frac{dI}{dx} = 0$ will give the location of the centre of mass of the system.

For example, if the moment of inertia I of a system varies with x as $I = \left(8x^2 - 4x + 5\right)$ kgm^2, then to find the location of the centre of mass of the system, we have

$$\frac{dI}{dx} = 0$$

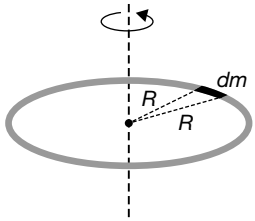
$$\Rightarrow \quad \frac{d}{dx}\left(8x^2 - 4x + 5\right) = 0$$

$$\Rightarrow \quad 16x - 4 = 0$$

$$\Rightarrow \quad x = 4 \text{ m}$$

MOMENT OF INERTIA OF A THIN RING

CASE-1: About an axis passing through centre perpendicular to plane of ring

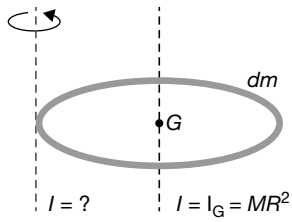


Consider an element of mass dm on perimeter of ring. Then MI of element about said AOR is

$$dI = R^2 dm$$

$$\Rightarrow \quad I = R^2 \int dm = MR^2$$

CASE-2: About an axis tangential to ring and perpendicular to the plane of ring



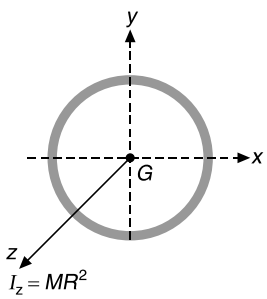
Using Parallel Axis Theorem, we get

$$I = I_G + Md^2$$

$$\Rightarrow \quad I = MR^2 + MR^2 = 2MR^2$$

CASE-3: About the diameter

Consider two diameters mutually perpendicular to each other i.e., along x and y axis (say) as shown in Figure.



Then by Perpendicular Axis Theorem, we have

$$I_z = I_x + I_y \text{ (by perpendicular axis theorem)}$$

Since both diameters are identical, so

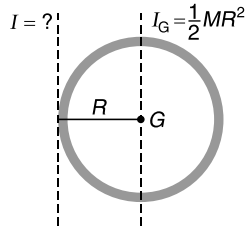
$$I_x = I_y = I_d$$

Also, $I_z = MR^2$

$$\Rightarrow \quad MR^2 = I_d + I_d$$

$$\Rightarrow \quad I_d = \frac{1}{2}MR^2$$

CASE-4: About an axis tangential to ring lying in its plane



Using Parallel Axis Theorem, we get

$$I = I_G + Md^2$$

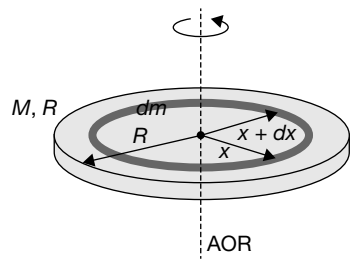
Since $I_G = \frac{1}{2}MR^2$

$$\Rightarrow \quad I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

MOMENT OF INERTIA OF A THIN DISC

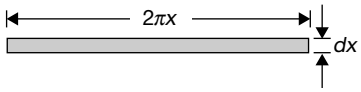
CASE-1: About axis passing through centre perpendicular to its plane

Consider a disc of mass M, radius R having surface mass density $\sigma\left(=\frac{M}{\pi R^2}\right)$. Consider a circular concentric element of inner radius x and outer radius $x+dx$ (i.e., thickness dx) as shown in Figure.



If dm be the mass of element of area dA, then

$$dA = (2\pi x)dx$$



Mass of element is

$$dm = \sigma A = \frac{M}{\pi R^2} dA = \frac{M}{\pi R^2}(2\pi x dx)$$

$$\Rightarrow \quad dm = \frac{2M}{R^2} x dx$$

So, MI of infinitesimal ring element about the specified AOR is

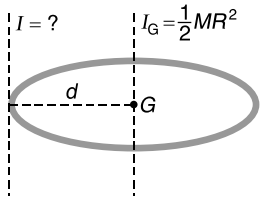
$$dI = (dm)x^2$$

$$\Rightarrow \quad dI = \frac{2M}{R^2} x^3 dx$$

$$\Rightarrow \quad I = \frac{2M}{R^2}\int_0^R x^3 dx = \frac{2M}{R^2}\left(\frac{x^4}{4}\Big|_0^R\right) = \frac{2M}{R^2}\left(\frac{R^4}{4} - 0\right)$$

$$\Rightarrow \quad I = \frac{1}{2}MR^2$$

CASE-2: About an axis tangential to disc perpendicular to disc

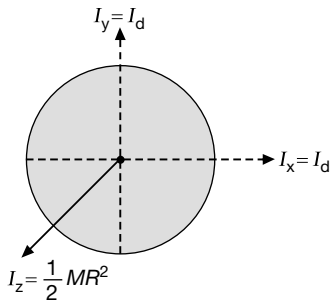


Using Parallel Axis Theorem, we get

$$I = I_G + Md^2$$

$$\Rightarrow \quad I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

CASE-3: About the diameter



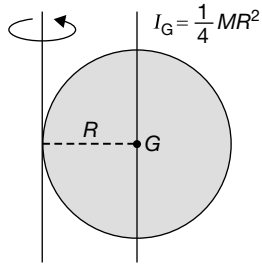
Using Perpendicular Axis Theorem, we get

$$I_x + I_y = I_z$$

$$\Rightarrow \quad I_d + I_d = \frac{1}{2}MR^2$$

$$\Rightarrow \quad I_d = \frac{1}{4}MR^2$$

CASE-4: About an axis tangential to disc lying in its plane



Using Parallel Axis Theorem, we get

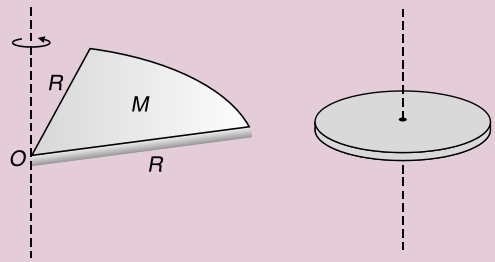
$$I = I_G + Md^2$$

$$\Rightarrow \quad I = \frac{1}{4}MR^2 + MR^2$$

$$\Rightarrow \quad I = \frac{5}{4}MR^2$$

Conceptual Note(s)

Moment of inertia of a part of a rigid body (symmetrically cut from the whole mass) is the same as that of the whole body. e.g., in Figure (a) moment of inertia of the section shown (a part of a circular disc) about an axis perpendicular to its plane and passing through point O is $\frac{1}{2}MR^2$ as the moment of inertia of the complete disc is also $\frac{1}{2}MR^2$.



This can be shown as in Figure. Suppose the given section is $\frac{1}{n}$th part of the disc, then mass of the disc will be nM.

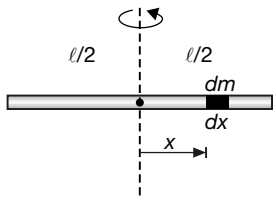
$$I_{disc} = \frac{1}{2}(nM)R^2$$

$$\Rightarrow \quad I_{section} = \frac{1}{n}I_{disc} = \frac{1}{2}MR^2$$

MOMENT OF INERTIA OF A THIN ROD

CASE-1: About axis passing through centre perpendicular to its length

Consider a thin uniform rod of mass M, length l having mass per unit length λ.



Consider an element of length dx, mass dm at perpendicular distance x from AOR. If dm be the mass of this element, then

$$dm = \lambda dx = \frac{M}{l}dx$$

Moment of inertia due to this element about said axis is

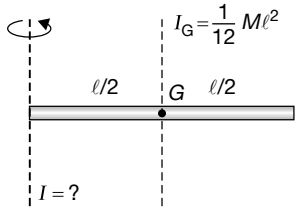
$$dI = x^2 dm = x^2\left(\frac{M}{l}dx\right)$$

$$\Rightarrow \quad I = \frac{M}{l}\int_{-l/2}^{l/2} x^2 dx = \frac{M}{l}\left(\frac{x^3}{3}\Big|_{-l/2}^{l/2}\right)$$

$$\Rightarrow \quad I = \frac{M}{3l}\left[\frac{l^3}{8} - \left(-\frac{l^3}{8}\right)\right] = \frac{M}{3l}\left(\frac{l^3}{4}\right)$$

$$\Rightarrow \quad I = \frac{Ml^2}{12}$$

CASE-2: About an axis passing through end and perpendicular to its length



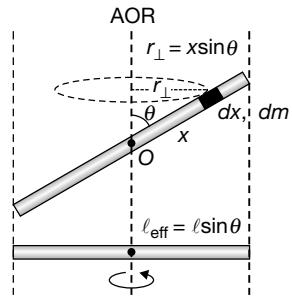
Using Parallel Axis Theorem, we get

$$I = I_G + Md^2$$

$$\Rightarrow \quad I = \frac{Ml^2}{12} + M\left(\frac{l}{2}\right)^2 = \frac{Ml^2}{12} + \frac{Ml^2}{4}$$

$$\Rightarrow \quad I = \frac{Ml^2}{3}$$

CASE-3: About an axis passing through centre, inclined to the rod at an angle θ.



$$I = \frac{1}{12}M\ell_{eff}^2 = \frac{1}{12}M\ell^2\sin^2\theta$$

Consider an element of length dx at a distance x from centre of rod. If dm be mass of element then

$$dm = \frac{M}{l}dx$$

Now, when the rod rotates about the said AOR, then the element rotates about AOR in a circle of radius $r_\perp = x\sin\theta$. Since

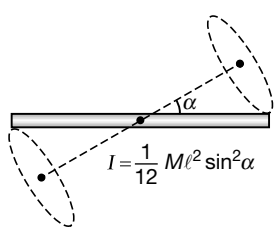
$$dI = (dm)r_\perp^2$$

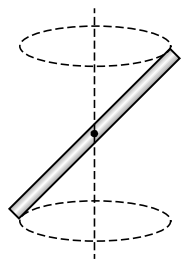
where, $dm = \frac{M}{l}dx$

$$\Rightarrow \quad dI = \frac{M}{l}x^2\sin^2\theta dx$$

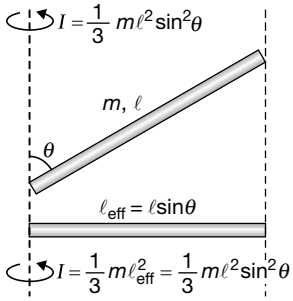
$$\Rightarrow \quad I = \frac{M}{l}\sin^2\theta\int_{-l/2}^{l/2} x^2 dx = \frac{M}{12}l^2\sin^2\theta$$

$$\Rightarrow \quad I = \frac{1}{12}M(l\sin\theta)^2 = \frac{1}{12}Ml_{eff}^2$$





Similarly, if the axis would have passed through one end of rod as shown, then $I = \frac{1}{3}Ml^2\sin^2\theta$



MOMENT OF INERTIA OF A SHELL ABOUT DIAMETER AND TANGENT

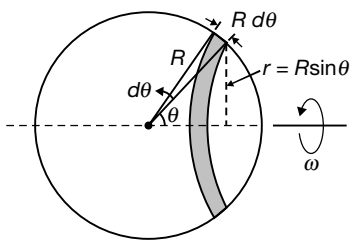
Consider a uniform shell of mass M, radius R. Let us find the moment of inertia of shell about diameter shown. For that, let us consider an infinitesimal ring element of radius $r(=R\sin\theta)$ thickness $Rd\theta$ having mass dm. Then

$$dm = \left(\frac{M}{4\pi R^2}\right)(\text{Area of Element})$$

$$\Rightarrow \quad dm = \left(\frac{M}{4\pi R^2}\right)(2\pi r)(Rd\theta)$$

$$\Rightarrow \quad dm = \left(\frac{M}{4\pi R^2}\right)(2\pi R\sin\theta)(Rd\theta)$$

$$\Rightarrow \quad dm = \frac{M}{2}\sin\theta d\theta$$



The moment of inertia of this ring element about diameter is

$$dI = r^2dm = (R^2\sin^2\theta)\left(\frac{M}{2}\sin\theta d\theta\right)$$

$$\Rightarrow \quad I = \int dI = \frac{MR^2}{2}\int_0^{\pi}\sin^3\theta d\theta$$

$$\Rightarrow \quad I = \frac{MR^2}{2}\int_0^{\pi}(1-\cos^2\theta)\sin\theta d\theta$$

Since, $d(\cos\theta) = -\sin\theta d\theta$

$$\Rightarrow \quad I = -\frac{MR^2}{2}\int_0^{\pi}(1-\cos^2\theta)d(\cos\theta)$$

$$\Rightarrow \quad I = -\frac{MR^2}{2}\left[\int_0^{\pi}d(\cos\theta) - \int_0^{\pi}d\left(\frac{\cos^3\theta}{3}\right)\right]$$

$$\Rightarrow \quad I = -\frac{MR^2}{2}\left[\cos\theta\Big|_0^{\pi} - \frac{1}{3}\cos^3\theta\Big|_0^{\pi}\right]$$

$$\Rightarrow \quad I = -\frac{MR^2}{2}\left[-2-\frac{1}{3}(-2)\right] = MR^2\left(1-\frac{1}{3}\right)$$

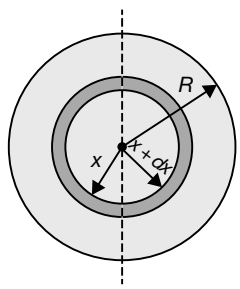
$$\Rightarrow \quad I = \frac{2}{3}MR^2$$

Applying Parallel Axis Theorem, the MI of the shell about the diameter is

$$I = I_{cm} + Md^2 = \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$$

MOMENT OF INERTIA OF A SPHERE ABOUT DIAMETER AND TANGENT

Consider a uniform sphere of mass M and radius R. Let us find the moment of inertia of the sphere about diameter as shown in Figure.



For that, let us consider an infinitesimal shell element of inner radius x, outer radius $x+dx$ and mass dm. If ρ be the mass per unit volume of sphere, then

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3}$$

If dV be the volume of infinitesimal shell element, then

$$dm = \rho dV = \rho(4\pi x^2 dx)$$

$$\Rightarrow \quad dm = \left(\frac{3M}{4\pi R^3}\right)(4\pi x^2 dx)\rho = \left(\frac{3M}{R^3}\right)(x^2 dx)$$

If dI be the moment of inertia of element about said axis, then

$$dI = \frac{2}{3}(dm)x^2 = \frac{2}{3}\left(\frac{3M}{R^3}x^2 dx\right)x^2 = \frac{2M}{R^3}x^4 dx$$

$$\Rightarrow \quad I = \frac{2M}{R^3}\int_0^R x^4 dx I = \frac{2M}{R^3}\left(\frac{R^5}{5}\right)$$

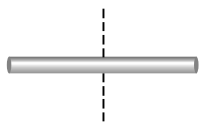
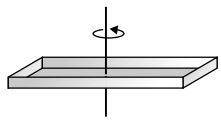
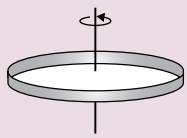
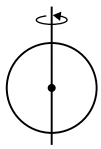
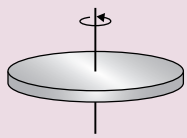
$$\Rightarrow \quad I = \frac{2}{5}MR^2$$

Applying Parallel Axis Theorem, the MI of the sphere about the diameter is

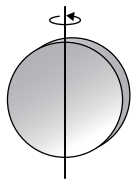
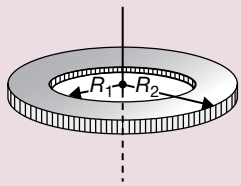
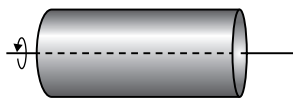
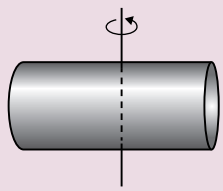
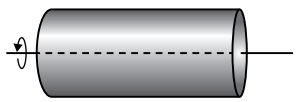
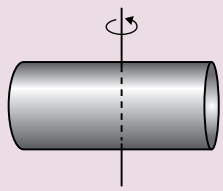
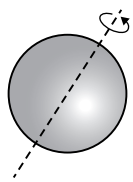
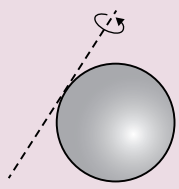
$$I = I_{cm} + Md^2 = \frac{2}{5}MR^2 + MR^2$$

$$\Rightarrow \quad I = \frac{7}{5}MR^2$$

TABLE : MOMENT OF INERTIA OF DIFFERENT BODIES

Body	Axis of Rotation	Diagram Showing Axis of Rotation	Moment of Inertia
1. Uniform thin bar	a) Through centre of gravity and perpendicular to length.		$\frac{Ml^2}{12}$
	b) Through one end and perpendicular to length.		$\frac{Ml^2}{3}$
2. Rectangular lamina	Passing through its C.G. and perpendicular to its plane of length and breadth.		$\frac{M(l^2+b^2)}{12}$
3. Ring or Hoop	a) Passing through its centre and perpendicular to its plane.		MR^2
	b) About diameter.		$\frac{MR^2}{2}$
4. Disc	a) Passing through its centre and perpendicular to its plane.		$\frac{MR^2}{2}$

(*Continued*)

Body	Axis of Rotation	Diagram Showing Axis of Rotation	Moment of Inertia
	b) About diameter.		$\frac{MR^2}{4}$
5. Hollow disc/ Annular disc of radii R_1 and R_2	Passing through its centre and perpendicular to its plane.		$I = \frac{M\left(R_1^2 + R_2^2\right)}{2}$
6. Solid cylinder	a) About its own geometric axis.		$\frac{MR^2}{2}$
	b) Passing through C.G. and perpendicular to its geometric axis.		$M\left(\frac{l^2}{12} + \frac{R^2}{4}\right)$
7. Hollow cylinder	a) About its own geometrical axis.		MR^2
	b) Passing through C.G. and perpendicular to length.		$M\left(\frac{l^2}{12} + \frac{R^2}{2}\right)$
8. Thin spherical shell	a) About diameter.		$\frac{2}{3}MR^2$
	b) About tangent.		$\frac{5}{3}MR^2$

(*Continued*)

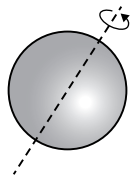
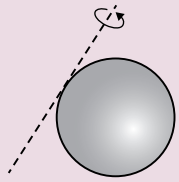
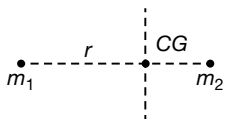
Body	Axis of Rotation	Diagram Showing Axis of Rotation	Moment of Inertia
9. Solid sphere	a) About diameter.		$\frac{2}{5}MR^2$
	b) About tangent.		$\frac{7}{5}MR^2$
10. Diatomic molecule	Passing through centre of gravity and perpendicular to bond length.		$\left(\frac{m_1m_2}{m_1+m_2}\right)r^2$

ILLUSTRATION 2

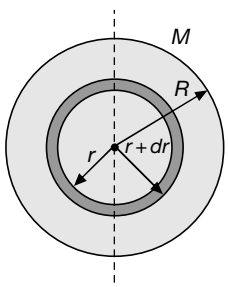
The density of a sphere of mass M, radius R, varies with distance r from its centre as $\rho=\rho_0\left(1+\frac{r}{R}\right)$, where ρ_0 is a positive constant. Calculate the moment of inertia of this sphere about its diameter.

SOLUTION

Consider an infinite shell element of mass dm having inner radius r and outer radius $r+dr$ i.e., thickness dr. If dV be the volume of the element, then $dV=4\pi r^2dr$

$$\Rightarrow \quad dm=\rho dV$$

$$\Rightarrow \quad dm=\rho_0\left(1+\frac{r}{R}\right)4\pi r^2dr \qquad \ldots(1)$$



If dI be the moment of inertia of this element about diameter, then

$$dI=\frac{2}{3}(dm)r^2$$

$$\Rightarrow \quad dI=\frac{2}{3}\rho_0\left(1+\frac{r}{R}\right)4\pi r^4dr$$

$$\Rightarrow \quad dI=\frac{8\pi\rho_0}{3}r^4dr+\frac{8\pi\rho_0}{3R}r^5dr$$

$$\Rightarrow \quad I=\frac{8\pi\rho_0}{3}\left(\frac{R^5}{5}\right)+\frac{8\pi\rho_0}{3R}\left(\frac{R^6}{6}\right)$$

$$\Rightarrow \quad I=\frac{8\pi\rho_0R^5}{3}\left(\frac{1}{5}+\frac{1}{6}\right)$$

$$\Rightarrow \quad I=\frac{88\pi\rho_0R^5}{90} \qquad \ldots(2)$$

However, from (1), we get, on integrating

$$M=\int dm=4\pi\rho_0\left(\int_0^R r^2dr+\int_0^R\frac{r^3}{R}dr\right)$$

$$\Rightarrow \quad M=4\pi\rho_0\left(\frac{R^3}{3}+\frac{R^3}{4}\right)$$

$$\Rightarrow \quad M=4\pi\rho_0R^3\left(\frac{7}{12}\right)=\frac{7}{3}\pi\rho_0R^3$$

$$\Rightarrow \quad \pi\rho_0R^3=\frac{3M}{7} \qquad \ldots(3)$$

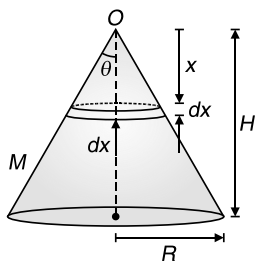
Substituting (3) in (2), we get

$$I = \frac{88}{90}\left(\frac{3M}{7}\right)R^2$$

$$\Rightarrow \quad I = \frac{44}{105}MR^2$$

MOMENT OF INERTIA OF A HOLLOW CONE

To calculate the MI of a hollow cone of mass M, radius R and height H, we consider elemental strips of vertical width dx at a distance x from the vertex of the cone as shown in Figure.



The actual width of this strip is $dx\sec\theta$, where θ is the half angle of the cone. So, area dA of the strip is

$$dA = (2\pi r)(dx\sec\theta)$$

where, r is the radius of the ring. Since,

$$\tan\theta = \frac{r}{x} = \frac{R}{H}$$

$$\Rightarrow \quad r = \left(\frac{R}{H}\right)x$$

If the cone mass has a mass M, then mass dm of the elemental strip ring is given by

$$dm = \sigma dA = \left(\frac{M}{\pi R\sqrt{R^2+H^2}}\right)(2\pi r dx\sec\theta)$$

The moment of inertia of this ring is

$$dI = r^2 dm$$

$$\Rightarrow \quad I = \int_0^H r^2 dm = \sigma\int_0^H\left(\frac{R^2x^2}{H^2}\right)dA$$

$$\Rightarrow \quad I = \sigma\left(\frac{R^2}{H^2}\right)\int_0^H x^2(2\pi r dx\sec\theta)$$

$$\Rightarrow \quad I = \left(\frac{2\pi\sigma R^3}{H^3}\right)\left(\frac{\sqrt{R^2+H^2}}{H}\right)\int_0^H x^3 dx$$

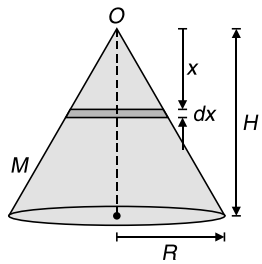
$$\Rightarrow \quad I = \left(\frac{2\pi MR^3}{\pi RH^3\sqrt{R^2+H^2}}\right)\left(\frac{\sqrt{R^2+H^2}}{H}\right)\left(\frac{H^4}{4}\right)$$

$$\Rightarrow \quad I = \frac{2MR^2}{H^4}\int_0^H x^3 dx = \frac{2MR^2}{H^4}\left(\frac{x^4}{4}\right)\Big|_0^H$$

$$\Rightarrow \quad I = \frac{1}{2}MR^2$$

MOMENT OF INERTIA OF A SOLID CONE

To calculate the moment of inertia of a solid cone of mass M, radius R and height H, we consider an infinitesimal disc at a distance x from the vertex of the cone as shown in Figure.



The mass dm of this disc is

$$dm = \left(\frac{M}{\pi R^2H/3}\right)(\pi r^2 dx) = \left(\frac{3Mr^2}{R^2H}\right)dx$$

where, r is the radius of the infinitesimal disc. Since,

$$\tan\theta = \frac{r}{x} = \frac{R}{H}$$

$$\Rightarrow \quad r = \left(\frac{R}{H}\right)x$$

The moment of inertia of this infinitesimal disc about the specified axis of rotation (AOR) is

$$dI = \frac{1}{2}(dm)r^2$$

Moment of inertia of complete cone is evaluated by integrating the above expression for the total height of the cone. So, we get

$$I = \int dI = \frac{1}{2}\int(dm)r^2 = \frac{1}{2}\int\left(\frac{3M}{R^2H}\right)r^4 dx$$

$$\Rightarrow \quad I=\int_0^H \frac{1}{2}\left(\frac{3M}{R^2H}\right)\left(\frac{Rx}{H}\right)^4 dx=\frac{3MR^2}{2H^4}\int_0^H x^4dx$$

$$\Rightarrow \quad I=\frac{3MR^2}{2H^4}\left(\frac{x^5}{5}\right)\Big|_0^H=\frac{3MR^2}{2H^4}\left(\frac{H^5}{5}\right)$$

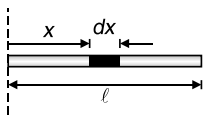
$$\Rightarrow \quad I=\frac{3}{10}MR^2$$

ILLUSTRATION 3

A rod of length l is pivoted about an end. The linear mass density of rod varies with distance x from its one end as $\lambda=(ax^2+b)$ kgm^{-1}. Calculate the moment of inertia of the rod about axis passing through this end and perpendicular to the rod.

SOLUTION

Since linear mass density of the rod varies with distance, so here we cannot use the expression $\frac{Ml^2}{3}$ because it is only applicable for uniformly distributed mass along the length of rod pivoted at an end.
So, let us consider an infinitesimal element of mass dm, length dx from the AOR at a distance x as shown in Figure.



Since, $dm=\lambda dx$

$$\Rightarrow \quad dm=(ax^2+b)dx$$

During rotation of rod about the specified AOR, this dm revolves in a circle of radius x, hence its moment of inertia dI is given by

$$dI=x^2dm$$

$$\Rightarrow \quad dI=(ax^2+b)x^2dx$$

The moment of inertia of the whole rod is obtained by integrating the above expression within limits from zero to l. So, we get

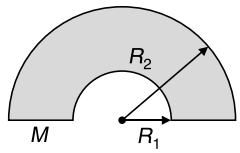
$$I=\int_0^l (ax^2+b)x^2dx$$

$$\Rightarrow \quad I=\left(\frac{ax^5}{5}+\frac{bx^3}{3}\right)\Big|_0^l$$

$$\Rightarrow \quad I=\frac{al^5}{5}+\frac{bl^3}{3}$$

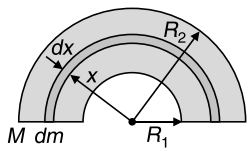
ILLUSTRATION 4

Find the moment of inertia of a semi-circular annular half disc (of mass M, outer radius R_2, inner radius R_1), about an axis passing through its centre and perpendicular to its plane as shown in Figure.



SOLUTION

Consider an infinitesimal half ring of radius x, width dx and mass dm as shown in Figure.



So, $dm=\sigma dA=\left(\frac{M}{\pi R_2^2-\pi R_1^2}\right)(\pi x dx)$

Moment of inertia of this elemental half ring about the specified is given by

$$dI=x^2dm$$

$$\Rightarrow \quad I=\int dI=\left(\frac{M\pi}{\pi R_2^2-\pi R_1^2}\right)\int_{R_1}^{R_2} x^3dx$$

$$\Rightarrow \quad I=\left(\frac{M}{R_2^2-R_1^2}\right)\left(\frac{x^4}{4}\right)\Big|_{R_1}^{R_2}=\frac{M}{4\left(R_2^2-R_1^2\right)}\left(R_2^4-R_1^4\right)$$

$$\Rightarrow \quad I=\frac{1}{4}M\left(R_1^2+R_2^2\right)$$

ILLUSTRATION 5

Calculate the moment of inertia of a rod whose linear density changes from λ to $\eta\lambda$ from the thinner end to the thicker end. The mass of the rod is equal to M and length L. Consider the axis of rotation perpendicular to the rod and passing through the thinner end and express your answer in terms of M, L and η.

SOLUTION

Consider an element at a distance x from thinner end of width dx, we use its mass dm as

$$dm = \left[\frac{\lambda(\eta-1)}{L}x + \lambda\right]dx$$

Total mass of rod is

$$M = \int dm\left(\frac{\lambda(\eta-1)}{L}x + \lambda\right)dx$$

$$\Rightarrow \quad M = \frac{\lambda(\eta-1)}{L}\left(\frac{L^2}{2}\right) + \lambda L$$

$$\Rightarrow \quad M = \frac{\lambda(\eta-1)L}{2} + \lambda L$$

$$\Rightarrow \quad M = \frac{1}{2}\lambda(\eta+1)L \qquad \ldots(1)$$

Moment of inertia of element is

$$dI = x^2 dm$$

Moment of inertia of rod is

$$I = \int dI = \int_0^L \left[\lambda + \frac{\lambda(\eta-1)}{L}x\right]x^2 dx$$

$$\Rightarrow \quad I = \frac{\lambda L^3}{3} + \frac{\lambda(\eta-1)}{L}\frac{L^4}{4}$$

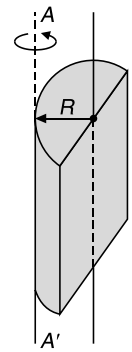
$$\Rightarrow \quad I = \frac{1}{12}\lambda L^3(3\eta+1) \qquad \ldots(2)$$

Substituting equation (1) in (2), we get

$$I = \frac{1}{6}ML^2\left(\frac{3\eta+1}{\eta+1}\right)$$

ILLUSTRATION 6

Find moment of inertia of the half cylinder of mass M shown in Figure, about the axis AA'.



SOLUTION

From Parallel Axis Theorem, we have

$$I = I_{cm} + Md^2$$

The moment of inertia of the half cylinder about the axis through its centre of mass at distance $4R/3\pi$ from the axis passing through the centre of mass is

$$I_{cm} = I - M\left(\frac{4R}{3\pi}\right)^2, \text{ where } I = \frac{1}{2}MR^2$$

$$\Rightarrow \quad I_{cm} = \frac{1}{2}MR^2 - \frac{16}{9\pi^2}MR^2 = MR^2\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)$$

The moment of inertia about axis AA' is again calculated by using the parallel axis theorem.

$$\Rightarrow \quad I_{AA'} = I_{cm} + M\left(R - \frac{4R}{3\pi}\right)^2 = I_{cm} + MR^2\left(1 - \frac{4}{3\pi}\right)^2$$

$$\Rightarrow \quad I_{AA'} = MR^2\left[\left(\frac{1}{2} - \frac{16}{9\pi^2}\right) + \left(1 + \frac{16}{9\pi^2} - \frac{8}{3\pi}\right)\right]$$

$$\Rightarrow \quad I_{AA'} = \left(\frac{3}{2} - \frac{8}{3\pi}\right)MR^2$$

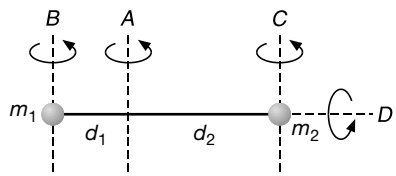
$$\Rightarrow \quad I_{AA'} = \left(\frac{9\pi-16}{6\pi}\right)MR^2$$

Test Your Concepts-I

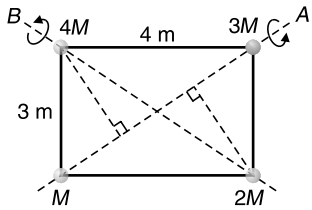
Based on Moment of Inertia and Applications

(Solutions on page H.147)

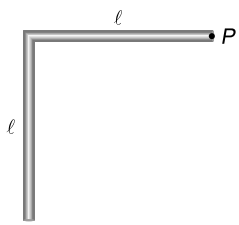
1. Many molecules have a simple diatomic, dumb-bell structure. Let us find the moments of inertia about four axes. We treat the bodies as point particles with mass $m_1 = 3$ kg and $m_2 = 5$ kg. Take $d_1 = 1$ m and $d_2 = 2$ m in Figure.



2. Four-point masses lie at the corners of a rectangle with sides of length 3 m and 4 m, as shown in Figure. Find the moment of inertia about each of the diagonals. Take $M = 1\,\text{kg}$.



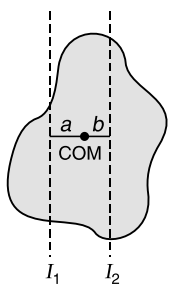
3. Calculate the moment of inertia of the two uniform joint rods having mass m each about point P as shown in Figure using parallel axis theorem.



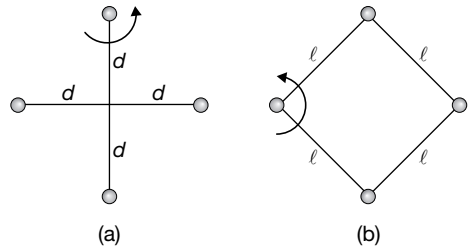
4. A non-uniform rod AB has a mass M and length $2L$. The mass per unit length of the rod varies with the distance x from the end A as $\lambda = \lambda_0 x$ where λ_0 is a positive constant. Find the moment of inertia of this rod about an axis perpendicular to the rod
(a) through A
(b) through the mid-point of AB.

5. If I_1 is the moment of inertia of a thin rod about an axis perpendicular to its length and passing through its centre of mass and I_2 the moment of inertia of the ring formed by the same rod about an axis tangent to the ring and perpendicular to the plane of the ring. Then find the ratio $\frac{I_1}{I_2}$.

6. Find the difference between I_1 and I_2, where I_1 and I_2 moment of inertia of a rigid body mass m about an axis as shown in Figure.

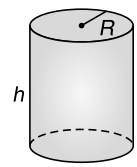


7. Four particles with equal masses m are connected by rods of negligible mass. Find the moment of inertia, about the indicated axis, of
(a) the cross in Figure (a)
(b) the square in Figure (b)

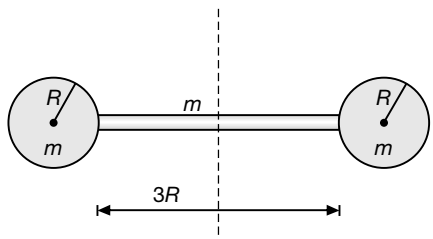


(a) (b)

8. A can is a hollow cylinder of radius R and height h. Its ends are sealed and it has no seams. The can is made from sheet of surface mass density σ. Calculate the moment of inertia about the central axis of symmetry?

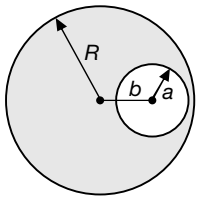


9. Two solid spheres of mass m and radius R are stuck to the ends of a thin rod of mass m and length $3R$. Find the moment of inertia of the system about the axis at the mid-point of the rod and perpendicular to it, as shown in Figure.



10. A uniform sphere of radius a has a concentric spherical cavity of radius b. Find the moment of inertia about a diameter. The mass of the object is M.

11. A uniform disk of mass M and radius R has a hole of radius a drilled through it, as in Figure. The center of the hole is at a distance b from the centre of the original disk. What is the moment of inertia of the disk about an axis through the center perpendicular to its plane?



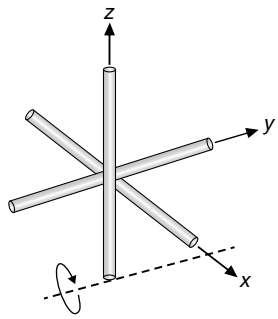
12. Calculate the moment of inertia of a semi-circular disc about an axis passing through its centre of mass and perpendicular to the plane.

13. Three masses m_1, m_2 and m_3 are located at the vertices of an equilateral triangle of side length a. Calculate the moment of inertia of the system about an axis along the altitude of the triangle passing through m_1?

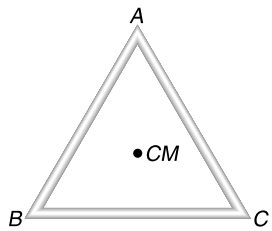
14. The radius of gyration of a uniform disc about a line perpendicular to the disc equals to its radius R. Find the distance of the line from the centre.

15. A circular lamina of radius R and centre O has a mass per unit area of kx^2, where x is the distance from O and k is a constant. If the mass of the lamina is M, find in terms of M and R, the moment of inertia of the lamina about an axis through O and perpendicular to the lamina.

16. Three identical thin rods, each of length L and mass m, are welded perpendicular to one another as shown in Figure. The assembly is rotated about an axis that passes through the end of one rod and is parallel to another. Find the moment of inertia of this structure.



17. Three rods each of mass m and length ℓ are joined together to form an equilateral triangle as shown in Figure. Calculate the moment of inertia of the system about an axis passing through its centre of mass and perpendicular to the plane of the triangle. Also, find the radius of gyration of system about said axis.



COMBINED TRANSLATIONAL AND ROTATIONAL MOTION OF A RIGID BODY

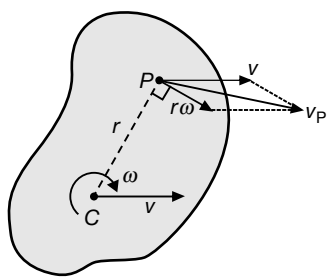
It has been observed that frequent attempts have been made by JEE Advanced paper setters to ask questions related to combined translational and rotational motion of a rigid body. The combined translational and rotational motion of a rigid body can be thought to be made up of two parts.

1. Pure translational motion in which centre of mass has velocity $v_{cm} = v$ and acceleration $a_{cm} = a$.
2. Pure rotational motion about an axis passing through the centre of mass with angular velocity ω and angular acceleration α.

In this type of motion, different particles of the rigid body will possess different linear velocities and linear accelerations. The following discussion is about finding the linear velocity and linear acceleration of any general particle P on the rigid body in such a motion.

LINEAR VELOCITY OF A GENERAL POINT *P* UNDER COMBINED TRANSLATIONAL AND ROTATIONAL MOTION OF A RIGID BODY

Now consider a rigid body having centre of mass at C having velocity of centre of mass v and angular velocity ω as shown in Figure.



Velocity of point P w.r.t. centre of mass C is $\vec{v}_{PC}$. By definition

$$\vec{v}_{PC} = \vec{v}_P - \vec{v}_C$$

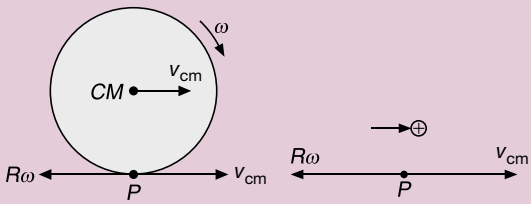
$\Rightarrow \quad \vec{v}_P = \vec{v}_{PC} + \vec{v}_C$

Now $|\vec{v}_C| = |\vec{v}_{CM}| = v$ and $|\vec{v}_{PC}| = r\omega$, perpendicular to the line CP. So, $|\vec{v}_P|$ is actually the magnitude of the resultant of $\vec{v}_{PC}$ and $\vec{v}_C$ as shown in Figure. If θ is the angle between $\vec{v}_{PC}$ and $\vec{v}_C$, then

$$|\vec{v}_P| = \sqrt{v^2 + r^2\omega^2 + 2v(r\omega)\cos\theta}$$

Conceptual Note(s)

For uniform pure rolling i.e. rolling without slipping, we must have the velocity of point of contact to be zero.



$\vec{v}_P = \vec{v}_{CM} + \vec{v}_{tangential}$

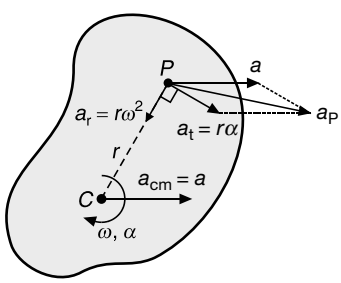
$\Rightarrow \quad \vec{v}_P = \vec{v}_{CM} + (-R\omega) = 0$

$\Rightarrow \quad v_{CM} = R\omega$

Hence, $(v_P)_{net} = v_{CM} - R\omega = 0$

LINEAR ACCELERATION OF A GENERAL POINT *P* UNDER COMBINED TRANSLATIONAL AND ROTATIONAL MOTION OF A RIGID BODY

Now consider a rigid body having centre of mass at C having velocity of centre of mass v and angular velocity ω and angular acceleration α as shown in Figure.



Acceleration of point P w.r.t. the centre of mass C is $\vec{a}_{PC}$, given by

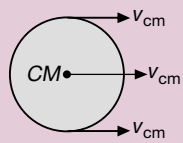
$$\vec{a}_{PC} = \vec{a}_P - \vec{a}_C$$

$\Rightarrow \quad \vec{a}_P = \vec{a}_{PC} + \vec{a}_C$

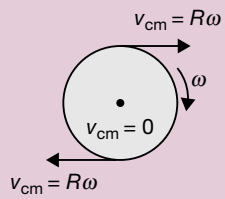
Now $|\vec{a}_C| = |\vec{a}_{CM}| = a$ and in the frame of the centre of mass, the point P has tangential acceleration $a_t = R\alpha$ perpendicular to the radius and the radial acceleration $a_r = r\omega^2$ along the radius towards the centre. *So, net acceleration of the point P is the vector resultant of a, a_t and a_r.* For different particles of the body, we observe that a, ω and α are the same, whereas r may be different, so different particles of the body possess different linear accelerations.

Conceptual Note(s)

(a) Linear Motion or Rectilinear Motion in which all particles move with the same translational velocity that equals the velocity of centre of mass. Also, angular velocity $\omega = 0$ for the case of pure translational motion as shown in Figure.

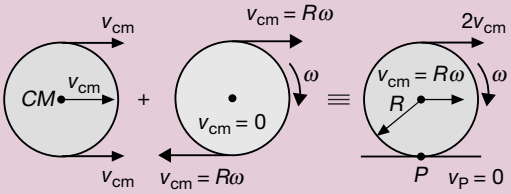


(b) Rotational Motion in which the velocity of centre of mass of the particles is zero because the diametrically opposite points have velocities equal in magnitude but opposite in direction as shown in Figure.

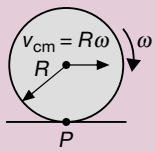


(c) Pure Rolling Motion in which the centre of mass has a velocity v_{cm} and the body has an angular velocity ω *such that velocity of the point of contact equals the velocity of the surface on which the body is rolling.*

Pure rolling can be regarded as the combined effect of translational motion and the rotational motion as shown in Figure.



If the surface is at rest on which the body is in pure rolling motion, then



$$v_{\text{point of contact}} = v_{\text{surface}} = 0$$

$$\Rightarrow \quad v_{cm} - R\omega = 0$$

$$\Rightarrow \quad v_{cm} = R\omega$$

Uniform pure rolling can also be taken as pure rotation of body about the point of contact, so

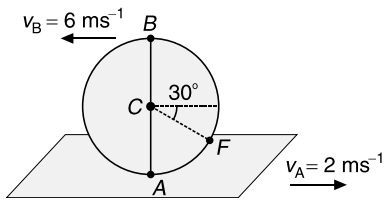
velocity of point of contact is $v_P = 0$

velocity of CM is $v_{cm} = R\omega$

velocity of topmost point is $v_Q = (2R)\omega$

ILLUSTRATION 7

Due to slipping, points A and B on the rim of the disk have the velocities shown. Determine the velocities of the centre point C and point F at this instant.

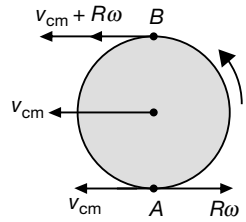


SOLUTION

Since $v_B > v_A$, so ω should be counterclockwise and v_{cm} should be leftwards. Hence, we have

$$v_B = v_{cm} + R\omega = 6 \qquad \ldots(1)$$

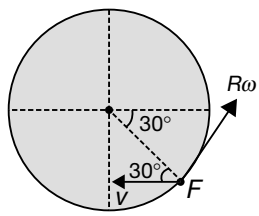
and $\quad v_A = R\omega - v_{cm} = 2 \qquad \ldots(2)$



Solving (1) and (2), we get

$$R\omega = 4 \text{ ms}^{-1} \text{ and } v_{cm} = 2 \text{ ms}^{-1}$$

So, velocity of point C, i.e., centre of mass is 2 ms^{-1}, leftwards



Now, for the point F, we have

$$v_F^2 = v_{cm}^2 + R^2\omega^2 + 2(v_{cm})(R\omega)\cos(90° + 30°)$$

$$\Rightarrow \quad v_F^2 = 4 + 16 + 2(2)(4)\left(-\frac{1}{2}\right)$$

$$\Rightarrow \quad v_F^2 = 4 + 16 - 8$$

$$\Rightarrow \quad v_F = \sqrt{12} = 2\sqrt{3} \text{ ms}^{-1}$$

Conceptual Note(s)

This problem will give the same result even when we would have taken v_{cm} rightwards. In that case equations would be

$$v_{cm} + R\omega = 2 \text{ and}$$

$$R\omega - v_{cm} = 6$$

Solving, we get

$$R\omega = 4 \text{ ms}^{-1} \text{ and } v_{cm} = -2 \text{ ms}^{-1}$$

The negative sign with v_{cm} indicates that it is directed leftwards.

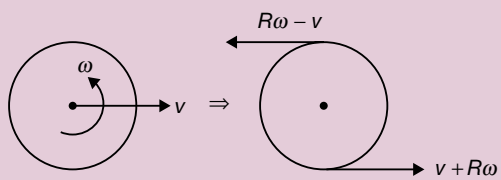
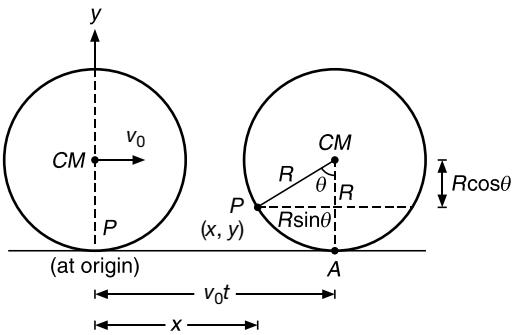


ILLUSTRATION 8

Consider a hoop of radius R having centre of mass velocity v_0. Let the hoop roll on a surface without sliding. Assuming a point P on the hoop to be at the origin initially. Find the velocity of this point as a function of time. Also find the acceleration of this point P. Prove that the trajectory followed by this point is a cycloid and the total distance travelled by this point is $8R$ (in one complete cycle).

SOLUTION

Since the hoop is rolling without slipping, therefore the arc length AP is equal to the displacement PA from the origin to point A i.e., $v_0 t$



$$\widehat{AP} = v_0 t$$

$$\Rightarrow \quad R\theta = v_0 t$$

$$\Rightarrow \quad \theta = \frac{v_0 t}{R} \qquad \ldots(1)$$

Co-ordinates of P at t are (x, y) where

$$x = v_0 t - R\sin\theta$$

$$\Rightarrow \quad x = v_0 t - R\sin\left(\frac{v_0 t}{R}\right) \qquad \ldots(2)$$

$$y = R - R\cos\theta$$

$$\Rightarrow \quad y = R - R\cos\left(\frac{v_0 t}{R}\right) \qquad \ldots(3)$$

The (2) and (3) combination of x and y will give rise to the trajectory called **Cycloid**.

$$\text{Since, } v_x = \frac{dx}{dt} = v_0 - \frac{v_0}{R}\left[R\cos\left(\frac{v_0 t}{R}\right)\right]$$

$$\Rightarrow \quad v_x = v_0 - v_0\cos\left(\frac{v_0 t}{R}\right) \qquad \ldots(4)$$

$$\text{and} \quad v_y = \frac{dy}{dt} = v_0\sin\left(\frac{v_0 t}{R}\right) \qquad \ldots(5)$$

Since $\vec{v} = \vec{v}_p = v_x\hat{i} + v_y\hat{j}$

and $|\vec{v}| = v_P = \sqrt{v_x^2 + v_y^2}$

$$\Rightarrow \quad v_P = v_0\sqrt{1+\cos^2\left(\frac{v_0 t}{R}\right) - 2\cos\left(\frac{v_0 t}{R}\right) + \sin^2\left(\frac{v_0 t}{R}\right)}$$

$$\Rightarrow \quad v_P = v_0\sqrt{2 - 2\cos\left(\frac{v_0 t}{R}\right)}$$

$$\Rightarrow \quad v_P = \sqrt{2}v_0\sqrt{1-\cos\left(\frac{v_0 t}{R}\right)}$$

$$\Rightarrow \quad v_P = \sqrt{2}v_0\sqrt{2\sin^2\left(\frac{v_0 t}{2R}\right)}$$

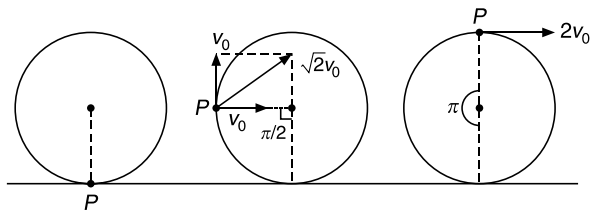
$$\Rightarrow \quad v_P = 2v_0\sin\left(\frac{v_0 t}{2R}\right)$$

Since $\theta = \dfrac{v_0 t}{R}$

$$v_P = 2v_0\sin\left(\frac{v_0 t}{2R}\right) = 2v_0\sin\left(\frac{\theta}{2}\right) \qquad \ldots(6)$$

Now let us calculate the acceleration of the point P. Since

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(v_0 - v_0\cos\left(\frac{v_0 t}{R}\right)\right)$$



$\theta = 0°$, $v_P = 0$

$\theta = \dfrac{\pi}{2}$, $v_P = 2v_0\sin\left(\dfrac{\pi}{4}\right)$, $v_P = \sqrt{2}v_0$

$\theta = \pi$, $v_P = 2v_0\sin\left(\dfrac{\pi}{2}\right)$, $v_P = 2v_0$

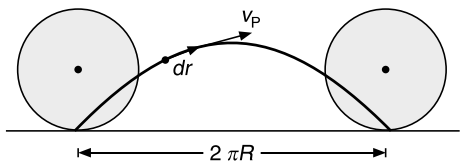
$$\Rightarrow \quad a_x = \frac{dv_x}{dt} = \frac{v_0^2}{R}\sin\left(\frac{v_0 t}{R}\right) \qquad \ldots(7)$$

Similarly

$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}\left(v_0\sin\left(\frac{v_0 t}{R}\right)\right) = \frac{v_0^2}{R}\cos\left(\frac{v_0 t}{R}\right) \qquad \ldots(8)$$

Since, $a = \sqrt{a_x^2 + a_y^2}$

$$\Rightarrow \quad a = \frac{v_0^2}{R} \qquad \ldots(9)$$



In time dt,

$$dr = v_P dt$$

$$\Rightarrow \quad dr = 2v_0 \sin\left(\frac{v_0 t}{2R}\right) dt$$

$$\Rightarrow \quad r = 2v_0 \int_0^{\frac{2\pi R}{v_0}} \sin\left(\frac{v_0 t}{2R}\right) dt = 2v_0 \left[\frac{-\cos\left(\frac{v_0 t}{2R}\right)}{\left(\frac{v_0}{2R}\right)}\right]_0^{\frac{2\pi R}{v_0}}$$

$$\Rightarrow \quad r = -4R\left[\cos\left(\frac{v_0}{2R} \times \frac{2\pi R}{v_0}\right) - \cos 0\right]$$

$$\Rightarrow \quad r = -4R(-2) = 8R$$

$$\Rightarrow \quad r = 8R$$

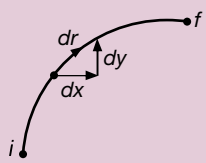
(a) Curve Rectification

$$\Rightarrow \quad (dr)^2 = (dx)^2 + (dy)^2$$

$$\Rightarrow \quad dr = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\Rightarrow \quad dr = \sqrt{1 + m^2}\, dx$$

$$\Rightarrow \quad r = \int dr = \int \sqrt{1 + m^2}\, dx = \int_i^f \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$



(b) If at $t = 0$, the point P lies at the bottom of the hoop, then

$$v_P = 2v_0 \sin\left(\frac{v_0 t}{2R}\right) = 2v_0 \sin\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad v_P = 2v_0 \sin\left(\frac{\omega_0 t}{2}\right)$$

where, θ is the angle between the point P at time t and the vertical line passing through the centre of the hoop.

(c) If at $t = 0$, the point P lies at the top of the hoop, then

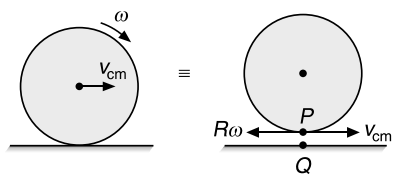
$$v_P = 2v_0 \cos\left(\frac{v_0 t}{2R}\right) = 2v_0 \cos\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad v_P = 2v_0 \cos\left(\frac{\omega_0 t}{2}\right)$$

where θ is the angle between the point P at time t and the vertical line passing through the centre of the hoop.

UNIFORM PURE ROLLING

Uniform Pure Rolling or simply "pure rolling" means that no relative motion exists at the point of contact between the body and the surface. Let a disc of radius R rolls without slipping on a horizontal stationary surface/ground. For the disc to roll without slipping, we must have



$$(v_{net})_P = v_{surface}$$

$$\Rightarrow \quad v_{cm} - R\omega = 0$$

$$\Rightarrow \quad v_{cm} = R\omega$$

So, $v_{cm} = R\omega$ is the condition for a body to be in pure rolling on a stationary horizontal surface/ground. It is sometimes simply called as ROLLING.

CASE OF FORWARD SLIPPING

If $v_P > v_Q$

$$\Rightarrow \quad v_{cm} - R\omega > 0$$

$$\Rightarrow \quad v_{cm} > R\omega$$

CASE OF BACKWARD SLIPPING (also called FORWARD ENGLISH)

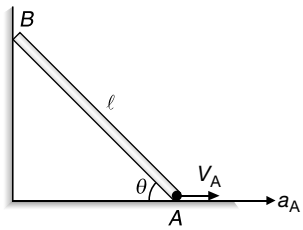
If $v_P < v_Q$

$\Rightarrow \quad v_{cm} - R\omega < 0$

$\Rightarrow \quad v_{cm} < R\omega$

ILLUSTRATION 9

A rod AB of length l is lying as shown in the Figure. At the instant shown, the point A has a speed V_A and acceleration a_A. Calculate the acceleration of point B and angular acceleration of the rod.



SOLUTION

By definition of relative velocity, we have

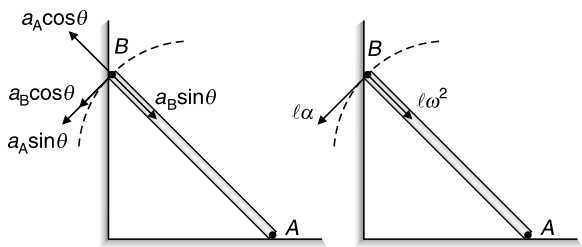
$$\vec{a}_{BA} = \vec{a}_{BG} - \vec{a}_{AG}$$

$$\Rightarrow \quad \vec{a}_{BG} = \vec{a}_{BA} + \vec{a}_{AG}$$

The angular velocity ω of the rod is

$$\omega = \frac{V_A}{l\sin\theta}$$

The components of the accelerations along the rod and perpendicular to the rod are shown in Figure.



So, we have

$$a_B \sin\theta - a_A \cos\theta = l\omega^2 \qquad \ldots(1)$$

and $a_A \sin\theta + a_B \cos\theta = l\alpha \qquad \ldots(2)$

$$\Rightarrow \quad a_B = \frac{a_A \cos\theta + l\omega^2}{\sin\theta} = \left(\frac{V_A^2}{l\sin^2\theta} + a_A \cos\theta\right)\frac{1}{\sin\theta} \ldots(3)$$

Substituting the value of a_B from equation (3) in equation (2), we get

$$\alpha = \frac{1}{l}\left[a_A \sin\theta + \left(\frac{V_A^2}{l\sin^2\theta} + a_A \cos\theta\right)\cot\theta\right]$$

$$l\omega^2 = \frac{V_A^2}{l\sin^2\theta}$$

In ground frame, net acceleration of point B will be the vector sum of all the three accelerations shown in Figure.

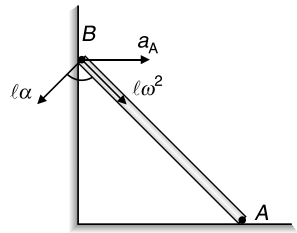
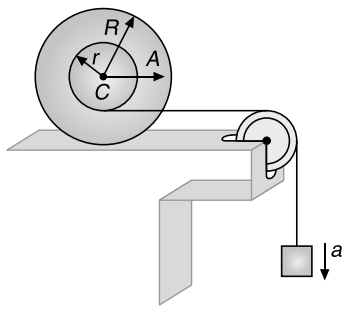


ILLUSTRATION 10

A bobbin having inner radius r and outer radius R is placed on a rough horizontal surface. A light string wrapped on its inner core connects a block with the bobbin as shown in Figure.



Now system is released from rest and bobbin moves on the horizontal surface without sliding. If the string does not slide from bobbin, calculate a/A.

SOLUTION

Since the bobbin is in pure rolling, point P should be instantaneous centre (IC) of zero velocity. So, for pure rolling acceleration of the point of contact is zero, hence we have

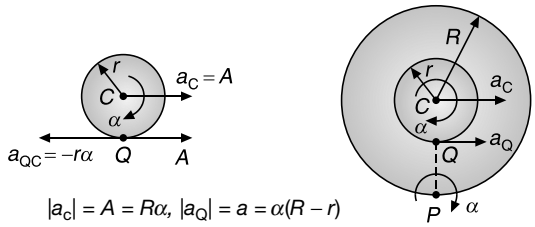
$$\vec{a}_P = \vec{0}$$

$$\Rightarrow \quad \vec{a}_P = \vec{a}_{PC} + \vec{a}_C = \vec{0}$$

$\Rightarrow \quad R\alpha - A = 0$

$\Rightarrow \quad A = R\alpha \qquad \ldots(1)$

Acceleration of the point Q with respect to the centre of mass C is shown in Figure.



$|a_c| = A = R\alpha,\ |a_Q| = a = \alpha(R - r)$

$\vec{a}_{QC} = \vec{a}_Q - \vec{a}_C$

$\Rightarrow \quad \vec{a}_Q = \vec{a}_{QC} + \vec{a}_C$

where, $a_Q = a$, $a_{QC} = -r\alpha$ and $a_C = A = R\alpha$

$a_Q = a = A - r = (R - r)\alpha$

Since point Q is directly connected with block, it means the magnitude of acceleration of the point Q should be equal to the acceleration of the block

$a_Q = a = \alpha(R - r) \qquad \ldots(2)$

From Equation (1) and (2), we get

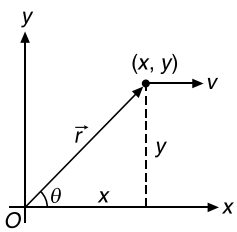
$$\frac{a}{A} = \frac{R - r}{R} = 1 - \frac{r}{R}$$

ILLUSTRATION 11

A particle of mass m is moving in x-y plane with a speed v, along x-axis. At a given instant of time, its position co-ordinates are (x, y). Calculate the angular velocity of the position vector of the particle and the angular acceleration of the particle with respect to origin.

SOLUTION

The position of x and velocity of the particle are shown in Figure.



As the particle moves along x-axis, its position vector turns clockwise, thus decreasing the angle θ. The time rate of change of this angle, i.e. $\frac{d\theta}{dt}$ equals the angular velocity. So,

$$\vec{\omega} = \left(\frac{d\theta}{dt}\right)\hat{k}$$

Since $y = x\tan\theta$

$\Rightarrow \quad x = y\cot\theta$

$\Rightarrow \quad \frac{dx}{dt} = v = \left(-y\,\text{cosec}^2\theta\right)\frac{d\theta}{dt}$

$$\left\{\because \ \frac{d}{d\theta}(\cot\theta) = -\text{cosec}^2\theta\right\}$$

$\Rightarrow \quad \frac{d\theta}{dt} = -\frac{v\sin^2\theta}{y} = -\frac{vy}{x^2 + y^2} \quad \left\{\because \ \sin\theta = \frac{y}{\sqrt{x^2 + y^2}}\right\}$

$\Rightarrow \quad \vec{\omega} = -\left(\frac{vy}{x^2 + y^2}\right)\hat{k}$

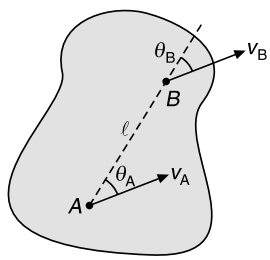
Also, angular acceleration $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$

$\Rightarrow \quad \vec{\alpha} = -vy\hat{k}\left[(-1)\left(x^2 + y^2\right)^{-2}\left(2x\frac{dx}{dt}\right)\right]$

$\Rightarrow \quad \vec{\alpha} = \frac{2xv^2y}{\left(x^2 + y^2\right)^2}\hat{k}$

RIGID BODY CONSTRAINT

Consider a rigid body on which two points A and B having velocities $\vec{v}_A$ and $\vec{v}_B$ are taken as shown in Figure.

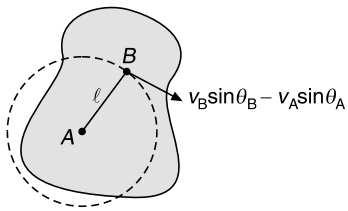


Since the body is rigid, so the distance between points A and B is constant and hence the component of velocity of A and B along the line joining A to B must be equal, i.e.

$$v_A\cos\theta_A = v_B\cos\theta_B$$

However, the perpendicular components may or not be equal.

If $v_A \sin\theta_A \neq v_B \sin\theta_B$ and if we observe the motion of the rigid body from the frame of point A, then the only possible way in which point B and all other points on the body can move is in a circle with A as the centre.



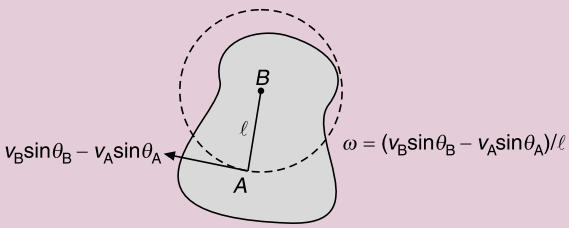
In the reference frame of point A

So, angular velocity of the line AB (or angular) velocity of the body is given by

$$\omega = \frac{v_B \sin\theta_B - v_A \sin\theta_A}{\ell}$$

Conceptual Note(s)

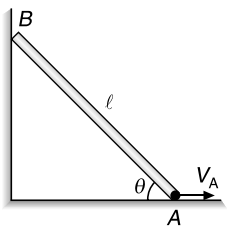
Angular velocity of the body means the rate of change of orientation of any line drawn on the body. Now let us try to find the angular velocity of the body from the frame of point B.



Thus, it can be concluded that angular velocity of the body remains same for all the points on the body.

ILLUSTRATION 12

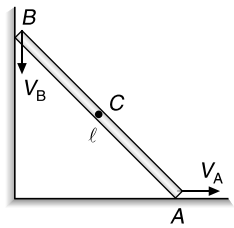
A rod AB of length l has its one end lying on the horizontal floor and the other end leaning against the vertical wall as shown in Figure.



If the point A is moving with a speed V_A, calculate the speed of point B and angular velocity of the rod at this instant.

SOLUTION

Let us consider the point B to move downwards with speed V_B. According to the rigid body constraint, the component of velocity of A and B along the AB must be equal. So, we have

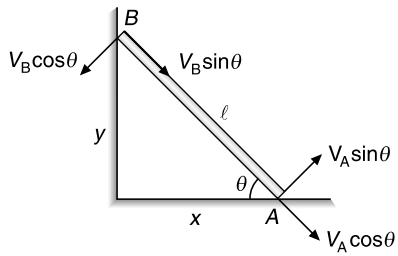


$$V_B \sin\theta = V_A \cos\theta$$

$$\Rightarrow \quad V_B = V_A \frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \quad V_B = V_A \cot\theta \qquad \ldots(1)$$

The components of velocity of A and B perpendicular to the rod will contribute to the rotation of the body, so we have



$$\omega = \frac{(v_{rel})_\perp}{l} = \frac{V_B \cos\theta + V_A \sin\theta}{l}$$

Using equation (1), we get

$$\omega = \frac{\left(V_A \cos^2\theta + V_A \sin^2\theta\right)/l}{\sin\theta}$$

$$\Rightarrow \quad \omega = \frac{V_A}{l \sin\theta}$$

Also, we see that, $\tan\theta = \frac{y}{x}$

$$\Rightarrow \quad x = l\cos\theta$$

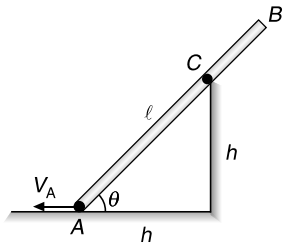
$$\Rightarrow \quad \frac{dx}{dt} = -l\sin\theta\left(\frac{d\theta}{dt}\right)$$

$$\Rightarrow \quad |\omega| = \frac{v_A}{l\sin\theta}$$

Both the methods fetch us same results.

ILLUSTRATION 13

A rod AB of length l is lying as shown in the Figure. The end A of the rod has speed V_A as shown in Figure.



Calculate the speed of point B and the angular velocity of the rod

SOLUTION

Since we observe that the velocity of the point C is along the rod, so by using the rigid body constraint, we get

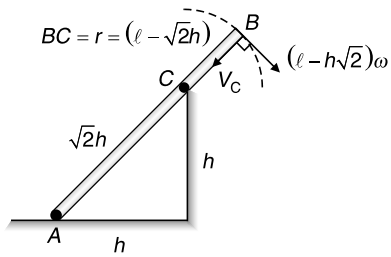
$$V_A \cos 45° = V_C$$

$$\Rightarrow \quad V_C = \frac{V_A}{\sqrt{2}}$$

Since $AC = h\sqrt{2}$, so the angular velocity ω is

$$\omega = \frac{V_A \sin 45°}{h\sqrt{2}} = \frac{V_A}{2h}$$

Once we know the angular velocity, then we can calculate the speed of point B by using the concept of relative motion and the rigid body constraint.



So, the linear tangential velocity of the point B is $r\omega = (l - h\sqrt{2})\omega$ and hence speed of point B is

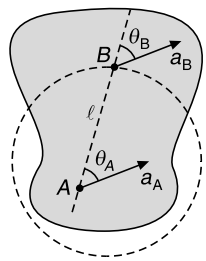
$$V_B = \sqrt{V_C^2 + (l - h\sqrt{2})^2\,\omega^2}$$

$$\Rightarrow \quad V_B = V_A\sqrt{\frac{1}{2} + \left(\frac{l - h\sqrt{2}}{2h}\right)^2}$$

RIGID BODY CONSTRAINT FOR ACCELERATION

Just like we saw the constraint relation between the velocities of any two points on a rigid body, we can also find some relations between the acceleration, angular acceleration, velocity and angular velocity.

Consider two points A and B, separated by a distance l, having accelerations a_A and a_B as shown in Figure.



Since, B is moving in a circle with angular velocity ω when observed from reference frame attached to A. So, we have

$$a_A \cos\theta_A - a_B \cos\theta_B \neq 0$$

As a matter of fact, $a_A \cos\theta_A - a_B \cos\theta_B$ directed radially inverse provides centripetal acceleration or normal acceleration and hence

$$a_A \cos\theta_A - a_B \cos\theta_B = l\omega^2$$

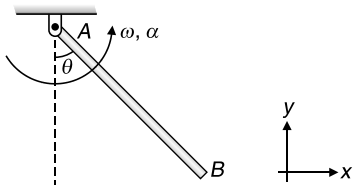
The components of accelerations perpendicular to the line joining A to B will provide the tangential acceleration α to the body, so tangential acceleration of B in frame of A is

$$|a_A \sin\theta_A - a_B \sin\theta_B| = l\alpha$$

Just like angular velocity, it can also be proved that angular acceleration of the body remains same in the frame of all the points on the rigid body.

ILLUSTRATION 14

The angular velocity and angular acceleration of the pivoted rod are ω and α respectively. Calculate the x and y components of acceleration of the end B of the rod.



SOLUTION

The acceleration of the point A is zero. The tangential and radial accelerations of the point B w.r.t. the point A are

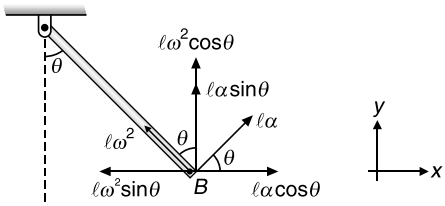
$$(a_{BA})_T = l\alpha \text{ and } (a_{BA})_R = l\omega^2$$

Since, by definition, we have

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$

$$\Rightarrow \quad \vec{a}_B = \vec{a}_{BA} + \vec{a}_A$$

Then resolving the vectors in x and y directions, we get

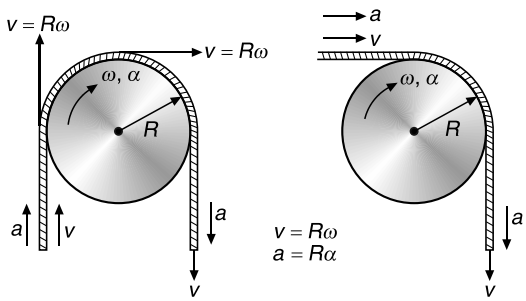


$$(a_B)_x = l\alpha\cos\theta - l\omega^2\sin\theta$$

and $$(a_B)_y = l\alpha\sin\theta + l\omega^2\cos\theta$$

ATWOOD'S MACHINE (SIMPLY PULLEY)

It is a simple pulley, hinged at the centre so as to turn freely without friction. The pulley has a mass M, radius R and it can be in the shape of a disc or a ring or a cylinder. Sufficient friction is present between the rope and the pulley surface so that the rope passing over its surface does not slip on the pulley surface. Due to this, the speed of the rope will be same as the speed of a point on the surface of the pulley as shown in Figure.



Since, $v = R\omega$

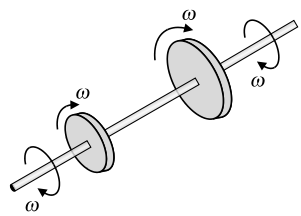
$$\Rightarrow \quad \frac{dv}{dt} = R\frac{d\omega}{dt}$$

$$\Rightarrow \quad a = R\alpha$$

So, acceleration of the ends of rope is related to angular acceleration of the pulley as $a = R\alpha$.

COAXIAL PULLEYS

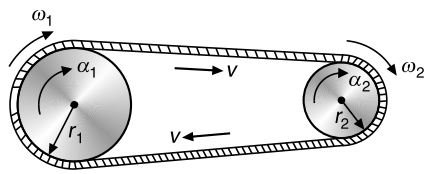
Consider an arrangement of two pulleys fixed on the same shaft (co-axial). When the shaft turns with angular speed ω, the two pulleys will also turn with angular speed ω. Consider the following Figure.



As the shaft turns, the two pulleys also turn with same angular speed ω.

DISCS CONNECTED BY A ROPE OR CHAIN

Consider two discs/wheels of radii r_1 and r_2 mounted on two separate fixed shafts turning with angular speeds ω_1 and ω_2. Let the two discs be connected to each other through a rope or a belt or a chain as shown in Figure.

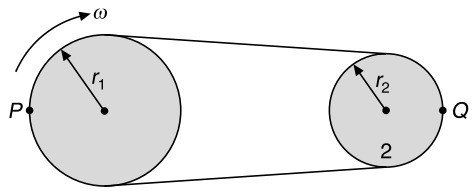


If the rope or belt or the chain does not slip over the two discs, the linear speed of any section of the rope is same as that of a point on the periphery of the two discs. So, we have

$$v = \omega_1 r_1 = \omega_2 r_2 \text{ and } a = \alpha_1 r_1 = \alpha_2 r_2$$

ILLUSTRATION 15

Two wheels 1 and 2 of radii r_1 and r_2 are connected by a belt as shown in Figure.

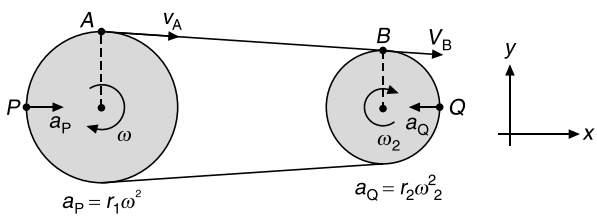


If the wheel 1 is rotating with uniform angular velocity ω and the belt does not slide over the wheels, calculate the speed of a point on the belt, the angular velocity of the wheel 2 and acceleration of the point P with respect to the point Q.

SOLUTION

Since the belt is not sliding, so we have

$$v_A = v_B$$



$$\Rightarrow \quad r_1\omega_1 = r_2\omega_2$$

$$\Rightarrow \quad \omega_2 = \frac{r_1\omega}{r_2}$$

Since ω is constant, so tangential acceleration of any point on the wheels is zero and hence we have

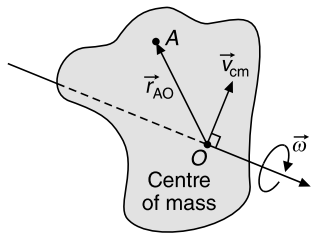
$$\vec{a}_P = r_1\omega^2(\hat{i}) \text{ and } \vec{a}_Q = r_2\omega_2^2(-\hat{i})$$

So, acceleration of point P w.r.t. point Q is

$$\vec{a}_{PQ} = \vec{a}_P - \vec{a}_Q = r_1\omega^2\left(1+\frac{r_1}{r_2}\right)\hat{i}$$

ACCELERATION AND VELOCITY OF ANY POINT ON A RIGID BODY

Consider a rigid body moving in space such that at any instant, the velocity of its centre of mass is $\vec{v}_{cm}$ and angular velocity is $\vec{\omega}$. To describe the motion of any point A with respect to a stationary point (or ground) we must understand that in a rigid body, the relative position of the points cannot change. This simply implies that, the relative velocity of point A (or any other point that lies on the body) w.r.t. centre of mass is tangential and is given by



$$\vec{v}_{A/cm} = \vec{\omega}\times\vec{r}_{AO}$$

$$\Rightarrow \quad \vec{v}_A - \vec{v}_{cm} = \vec{\omega}\times\vec{r}_{AO}$$

$$\Rightarrow \quad \vec{v}_A = \vec{v}_{cm} + \vec{\omega}\times\vec{r}_{AO}$$

Differentiating, we get

$$\vec{a}_A = \frac{d}{dt}\left(\vec{v}_{cm} + \vec{\omega}\times\vec{r}_{AO}\right) = \frac{d\vec{v}_{cm}}{dt} + \frac{d}{dt}\left(\vec{\omega}\times\vec{r}_{AO}\right)$$

$$\Rightarrow \quad \vec{a}_A = \vec{a}_{cm} + \frac{d\vec{\omega}}{dt}\times\vec{r}_{AO} + \vec{\omega}\times\frac{d\vec{r}_{AO}}{dt}$$

$$\Rightarrow \quad \vec{a}_A = \vec{a}_{cm} + \vec{\alpha}\times\vec{r}_{AO} + \vec{\omega}\times\vec{v}_{A/cm}$$

$$\Rightarrow \quad \vec{a}_A = \vec{a}_{cm} + \vec{\alpha}\times\vec{r}_{AO} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{AO}\right)$$

Also, we note that the velocity of any point A w.r.t. B is given by

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = \vec{\omega}\times\vec{r}_{AB}$$

where, $\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$ is the position vector of A w.r.t. B.

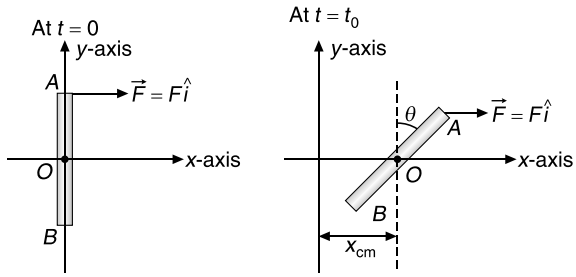
In general, we can say that the general motion of a body can be called as a combination of rotational motion and translational motion.

ILLUSTRATION 16

A thin uniform rod AB of mass m and length L is placed on a smooth horizontal table. A constant horizontal force of magnitude F starts acting on the rod at one of the ends AB. Initially, the force is perpendicular to the length of the rod. Taking the moment at which force starts acting as $t = 0$, calculate the distance moved by centre of mass of the rod in time t_0, magnitude of initial acceleration of the end A of the rod.

SOLUTION

Consider the rod to be placed in x-y plane (horizontal) with initial orientation along y-axis. Further, assuming that force acting on the rod is along the x-axis as shown in Figure.



The acceleration of centre of mass is given by

$$\vec{a}_{cm} = \frac{\vec{F}}{m} = \frac{F}{m}\hat{i} = \text{constant}$$

The displacement is given by

$$\Rightarrow \quad x_{cm} = \frac{1}{2}a_{cm}t_0^2 = \frac{1}{2}\frac{F}{m}t_0^2 \qquad \{\because a_{cm} = \text{constant}\}$$

The initial acceleration of end A of the rod is

$$\vec{a}_A = \vec{a}_{cm} + \vec{\alpha}\times\vec{r}_{AO} \qquad \{\because \omega = 0\}$$

$$\Rightarrow \quad \vec{a}_A = \frac{F}{m}\hat{i} + \left(-\frac{6F}{mL}\hat{k}\right)\times\left(\frac{L}{2}\hat{j}\right)$$

$$\Rightarrow \quad \vec{a}_A = \frac{4F}{m}\hat{i}$$

Test Your Concepts-II

Based on Rotational Kinematics, Combined Effect of Rotation and Translation Motion

(Solutions on page H.150)

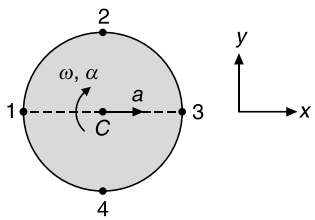
1. The angular position of a point on the rim of a rotating wheel is given by $\theta = 4t - 3t^2 + t^3$, where θ is in radian and t is in second

(a) Calculate the angular velocity at $t = 2$ s and $t = 4$ s.

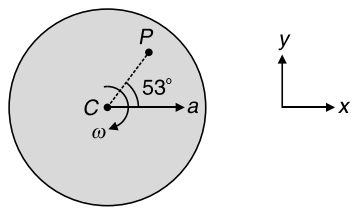
(b) Calculate the average angular acceleration for the time interval that begins at $t = 2$ s and ends at $t = 4$ s

(c) Calculate the instantaneous angular acceleration at the beginning and the end of this time interval.

2. A uniform disc of radius r spins with angular velocity ω and angular acceleration α. If the centre of mass of the disc has linear acceleration a, calculate the magnitude and direction of acceleration of the points 1, 2, 3 and 4.



3. Calculate the linear acceleration of the particle P at $t = 1$ s if $a = 2$ ms^{-2}, $\omega = (2t)$ rads^{-1} and $CP = 1$ m as shown in Figure. Express your result in terms of $\hat{i}$ and $\hat{j}$.



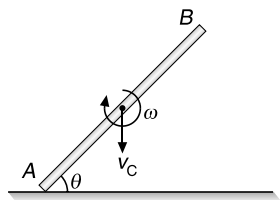
4. A disc starts rotating with constant angular acceleration of π rads^{-2} about a fixed axis perpendicular to its plane and through its centre.

(a) Find the angular velocity of the disc after 4 s.

(b) Find the angular displacement of the disc after 4 s and

(c) Find the number of turns accomplished by the disc in 4 s.

5. A rod AB of length l is supported against smooth horizontal surface. The rod is inclined at angle θ with horizontal. Now it released so that end A starts sliding on horizontal surface. Find the velocity of centre of mass of the rod in terms of its angular velocity ω.

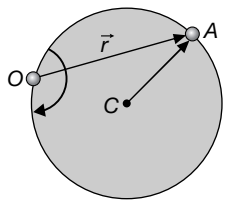


6. A flywheel of radius 20 cm starts from rest, and has a constant angular acceleration of 60 rads^{-2}. Find

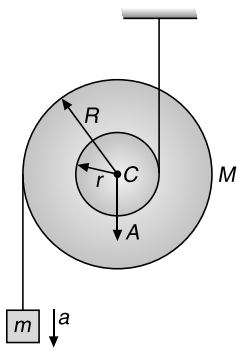
(a) the magnitude of the net linear acceleration of a point on the rim after 0.15 s

(b) the number of revolutions completed in 0.25 s

7. A particle A moves along a circle of radius $R = 10$ cm so that its radius vector $\vec{r}$ relative to O rotates with constant angular velocity $\omega_0 = 0.2$ rads^{-1}. Find the modulus of the velocity of the particle and modulus and direction of its total acceleration.

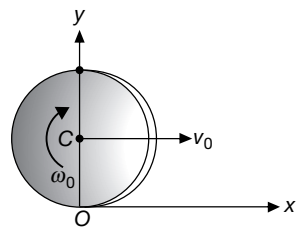


8. A bobbin with inner radius r and outer radius R is arranged with light strings and a block as shown in Figure.

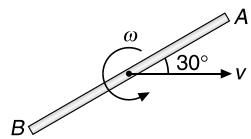


The string does not slide over the cylinder and when the system is released from rest, then the block and bobbin move down with accelerations a and A respectively. Calculate the ratio of the acceleration of the bobbin to the acceleration of the block.

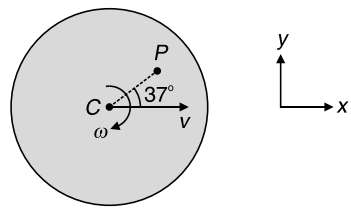
9. A disc of radius R start at time $t = 0$ moving along the positive x-axis with linear speed v_0 and angular speed ω_0. Find the x and y co-ordinates of the bottommost point at any time t.



10. A uniform rod of length l is spinning with an angular velocity $\omega = \frac{2v}{l}$ while its centre of mass moves with a velocity v. Calculate the velocity of the end A of the rod.

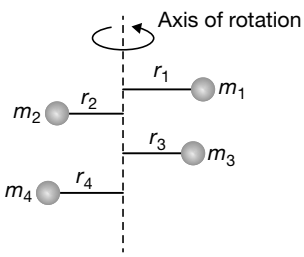


11. Calculate the linear velocity of particle P shown in Figure, if $v = 2$ ms^{-1}, $\omega = 5$ rads^{-1} and $CP = 1$ m. Express your result in terms of $\hat{i}$ and $\hat{j}$.



ROTATIONAL KINETIC ENERGY (R.K.E.)

Consider a system made of particles of masses m_1, m_2, m_3, m_4, ..., m_n placed at distances r_1, r_2, …, r_n from the axis of rotation. Let the particles rotate about the axis of rotation with uniform angular velocity ω such that the tangential velocities of the particles are v_1, v_2, …, v_n respectively. From above consideration we conclude that the system possesses kinetic energy due to rotational motion of the particles and hence this kinetic energy is called **rotational kinetic energy** denoted by K_R.



A discrete system of particles

$$\Rightarrow \quad K_{\text{rotational}} = K_R = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + ... + \frac{1}{2}m_nv_n^2$$

But $v_1 = r_1\omega$, $v_2 = r_2\omega$, ..., $v_n = r_n\omega$

$$\Rightarrow \quad K_R = \frac{1}{2}\left(m_1r_1^2 + m_2r_2^2 + ... + m_nr_n^2\right)\omega^2$$

$$\Rightarrow \quad K_R = \frac{1}{2}I\omega^2$$

For a system rotating about a point, we must keep in mind that the translational kinetic energy of a rigid body having mass M and centre of mass velocity v_{cm} is given by

$$K_{translational} = K_T = \frac{1}{2}Mv_{cm}^2$$

and the rotational kinetic energy of the body about its centre of mass is

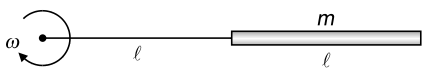
$$K_R = \frac{1}{2}I_{cm}\omega^2$$

So, the total energy of a body in rotation about an axis can be written as

$$K = K_R + K_T$$

ILLUSTRATION 17

A rod of mass m and length l is connected with a light rod of length l. The composite rod is made to rotate with angular velocity ω as shown in Figure.



Calculate the translational kinetic energy, rotational kinetic energy and total kinetic energy of rod.

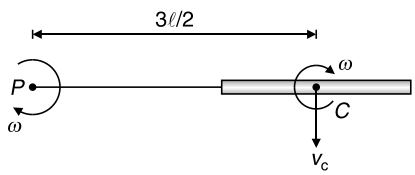
SOLUTION

Translational kinetic energy of rod,

$$K_{translational} = \frac{1}{2}mv_{cm}^2$$

Velocity of centre of mass is

$$v_{cm} = \omega\left(\frac{3}{2}l\right)$$



The translational kinetic energy of rod is

$$K_{translational} = K_T = \frac{1}{2}m\left(\frac{3\omega l}{2}\right)^2 = \frac{9}{8}m\omega^2l^2$$

The rotational kinetic energy of rod is

$$K_{rotational} = \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}\left(\frac{ml^2}{12}\right)\omega^2 = \frac{1}{24}m\omega^2l^2$$

Total kinetic energy of the rod is

$$K_{total} = K_{translational} + K_{rotational}$$

$$\Rightarrow \quad K_{total} = \frac{9}{8}m\omega^2l^2 + \frac{1}{24}m\omega^2l^2 = \frac{7}{6}m\omega^2l^2$$

The total kinetic energy of the rod can also be thought to be the rotational kinetic energy of the rod about the fixed axis. Hence, we have

$$K_{total} = \frac{1}{2}I_P\omega^2$$

where I_P is the moment of inertial of the rod about the axis passing through the point P. According to parallel axis theorem, we have

$$I_P = I_{cm} + md^2 = I_{cm} + m\left(\frac{3}{2}l\right)^2$$

$$\Rightarrow \quad I_P = \frac{ml^2}{12} + \frac{9}{4}ml^2 = \frac{7}{3}ml^2$$

Hence total kinetic energy of the rod is

$$K_{total} = \frac{1}{2}\left(\frac{7}{3}ml^2\right)\omega^2 = \frac{7}{6}m\omega^2l^2$$

MODIFIED WORK ENERGY THEOREM (MWET) AND CONSERVATION OF MECHANICAL ENERGY

If W_{ext} is the work done by external forces, W_{nc} is the work done by non-conservative forces, W_{ps} is the work done by pseudo forces and W_{int} is the work done by internal forces, then according to Modified Work Energy Theorem (MWET) studied earlier, we have

$$W_{ext} + W_{nc} + W_{ps} + W_{int} = \Delta U + \Delta K$$

However, in the absence of dissipative (non-conservative), external, pseudo forces, work done by thems is zero and

if work done by internal forces is zero, then the total mechanical energy of a system is conserved, i.e.

$$\Delta K + \Delta U = 0$$

$$\Rightarrow \quad K_f + U_f = K_i + U_i$$

$$\Rightarrow \quad (U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

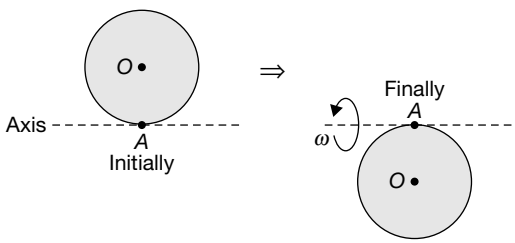
ILLUSTRATION 18

A uniform circular disc of mass m, radius R and centre O is free to rotate about a smooth, horizontal axis which is tangential to the disc at a point A. The disc is held in a vertical plane with A below O and is then slightly displaced from this position. Find the angular velocity of the disc when its plane is next vertical.

SOLUTION

By Law of Conservation of Mechanical Energy, we have

$$\begin{pmatrix}\text{Loss in Gravitational}\\ \text{Potential Energy of}\\ \text{Centre of Mass}\\ \text{of the Disc}\end{pmatrix} = \begin{pmatrix}\text{Gain in Rotational}\\ \text{Kinetic Energy of}\\ \text{Disc about the said}\\ \text{Axis of Rotation}\end{pmatrix}$$



$$\Rightarrow \quad mgh = \frac{1}{2} I\omega^2$$

$$\Rightarrow \quad mg(2R) = \frac{1}{2}\left(\frac{1}{4}mR^2 + mR^2\right)\omega^2$$

$$\Rightarrow \quad \omega = 4\sqrt{\frac{g}{5R}}$$

ILLUSTRATION 19

A uniform disk of mass M and radius R is pivoted so that it can rotate freely about a horizontal axis through its centre and normal to the plane of the disk. A small particle of mass m is attached to the rim of the disk at the top directly above the pivot. The system is given a gentle start and the disk begins to rotate.

(a) What is the angular velocity of the disk when the particle is at its lowest point?

(b) At this point, what force must be exerted on the particle by the disk to keep it on the disk?

SOLUTION

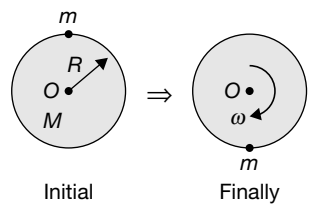
(a) By Law of Conservation of Mechanical Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{Gravitational}\\ \text{Potential Energy}\\ \text{of Particle}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{Rotational}\\ \text{Kinetic Energy}\\ \text{of disc + Particle}\end{pmatrix}$$

$$\Rightarrow \quad mg(2R) = \frac{1}{2}\left(\frac{1}{2}MR^2 + mR^2\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{8mg}{(2m+M)R}}$$

(b) If F be the force exerted by disk on the particle (upwards), then



$$F - mg = mR\omega^2$$

$$\Rightarrow \quad F = mg + \frac{8m^2 g}{(2m+M)}$$

$$\Rightarrow \quad F = \frac{mg(10m+M)}{(2m+M)}$$

ILLUSTRATION 20

ABC is a triangular framework of three uniform rods each of mass m and length $2l$. It is free to rotate in its own plane about a smooth horizontal axis through A normal to the plane ABC. If it is released from rest when AB is horizontal and C is above AB, find the maximum velocity of C in the subsequent motion.

SOLUTION

$$I_A = I_{AB} + I_{AC} + I_{BC}$$

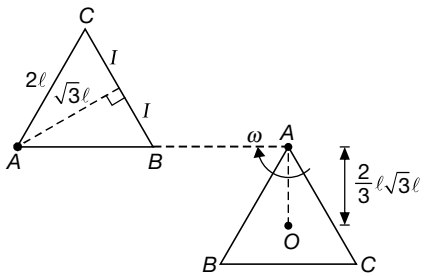
$$\Rightarrow \quad I_A = \frac{1}{3}m(2\ell)^2 + \frac{1}{3}m(2\ell)^2 + \left(\frac{1}{12}m(2\ell)^2 + md^2\right)$$

where $d = \sqrt{3}\ell$

$$\Rightarrow \quad I_A = \frac{4}{3}m\ell^2 + \frac{4}{3}m\ell^2 + \left(\frac{1}{3}m\ell^2 + m\left(\ell\sqrt{3}\right)^2\right)$$

$$\Rightarrow \quad I_A = 6m\ell^2$$

Assume the horizontal line passing through A to be the zero Potential Energy Level (ZPEL) as shown in Figure.



Then initially CG is $\frac{1}{3}\left(\ell\sqrt{3}\right)$ above ZPEL and finally i.e., when C attains maximum velocity, the CG is $\frac{2}{3}\left(\ell\sqrt{3}\right)$ below ZPEL. So, in the process the CG has fallen through

$$h = \frac{2}{3}\left(\ell\sqrt{3}\right) + \frac{1}{3}\left(\ell\sqrt{3}\right) = \ell\sqrt{3}$$

Applying Law of Conservation of Mechanical Energy, we get

$$\begin{pmatrix}\text{Loss in}\\ \text{Gravitational}\\ \text{Potential Energy}\\ \text{of CG}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{Rotational}\\ \text{Kinetic Energy}\\ \text{of system}\end{pmatrix}$$

$$\Rightarrow \quad (3m)g\left(\ell\sqrt{3}\right) = \frac{1}{2}\left(6m\ell^2\right)\omega^2$$

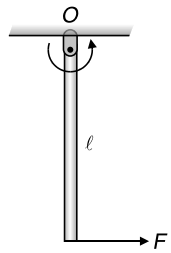
$$\Rightarrow \quad \omega = \sqrt{\frac{g\sqrt{3}}{\ell}}$$

So, velocity of C at this instant is

$$v_C = (2\ell)\omega = 2\sqrt{g\ell\sqrt{3}}$$

ILLUSTRATION 21

A uniform rod of mass m and length l is pivoted smoothly at O. A horizontal force acts at the bottom of the rod.

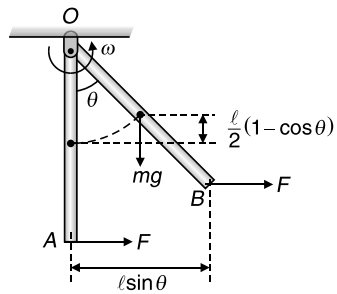


Calculate the angular velocity of the rod as the function of angle of rotation θ and the maximum angular displacement of the rod.

SOLUTION

According to modified work energy theorem, we have $W_{ext} = \Delta U + \Delta K$, where

$$F_{ext} = F\Delta r = Fl\sin\theta$$



ΔU is the rise in potential energy of the centre of mass of the rod, so

$$\Delta U = mg\left(\frac{l}{2}\right)(1-\cos\theta)$$

ΔK is the rotational kinetic energy of the rod about rotational axes passing through hinge and can also be called as the total kinetic energy of the rod, so

$$\Delta K = \frac{1}{2}I_O\omega^2$$

$$\Rightarrow \quad Fl\sin\theta = \frac{mgl}{2}(1-\cos\theta) + \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6F\sin\theta}{ml} - \frac{3g}{l}(1-\cos\theta)} \qquad \ldots(1)$$

At maximum angular displacement, the angular velocity of the rod is zero. Substituting $\omega = 0$ in equation (1), we get

$$0 = \sqrt{\frac{6F}{ml}\sin\theta - \frac{3g}{l}(1-\cos\theta)}$$

$$\Rightarrow \quad \frac{6F}{ml}\sin\theta = \frac{3g}{l}(1-\cos\theta)$$

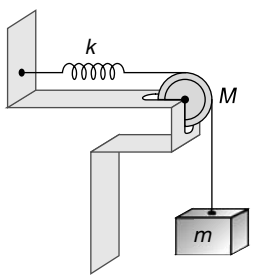
$$\Rightarrow \quad \frac{2F}{m}\left[2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right] = g\left(2\sin^2\left(\frac{\theta}{2}\right)\right)$$

$$\Rightarrow \quad \tan\left(\frac{\theta}{2}\right) = \frac{2F}{mg}$$

$$\Rightarrow \quad \theta = 2\tan^{-1}\left(\frac{2F}{mg}\right)$$

ILLUSTRATION 22

A block of mass $m = 4$ kg is attached to a spring of spring constant $(k = 32\ \text{Nm}^{-1})$ by a rope that hangs over a pulley of mass $M = 8$ kg. If the system starts from rest with the spring unstretched, find the speed of the block after it falls 1 m. Treat the pulley as a disc, so $I = \frac{1}{2}MR^2$.



SOLUTION

Since the rim of the pulley moves at the same speed as the block, the speed of the block and the angular velocity of the pulley related by $v = \omega R$.

When the block falls by a distance x, its potential energy decreases, so $\Delta U_g = -mgx$, the potential energy of the spring increases, so $\Delta U_s = +\frac{1}{2}kx^2$, both the block and the pulley gain kinetic energy, so $\Delta K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. Applying Law of Conservation of Mechanical Energy, we get

$$\Delta K + \Delta U = 0,$$

$$\Rightarrow \quad \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 + \frac{1}{2}kx^2 - mgx = 0$$

$$\Rightarrow \quad \frac{1}{2}\left(m + \frac{M}{2}\right)v^2 + \frac{1}{2}kx^2 - mgx = 0$$

Please note that in this problem, the radius R was not required. Substituting $m = 4$ kg, $M = 8$ kg, $k = 32\ \text{Nm}^{-1}$ and $x = 1$ m, we get

$$\frac{1}{2}\left(4 + \frac{8}{2}\right)v^2 + \frac{1}{2}(32)(1)^2 - (4)(10)(1) = 0$$

$$\Rightarrow \quad 4v^2 + 16 - 40 = 0$$

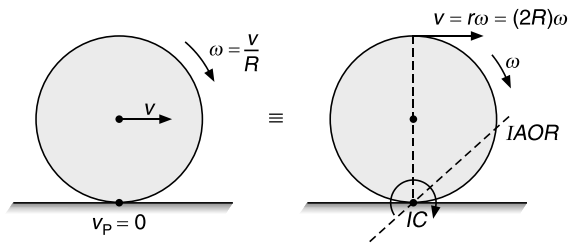
$$\Rightarrow \quad v = 2.4\ \text{ms}^{-1}$$

TOTAL ENERGY OF A BODY IN PURE ROLLING

When a body rolls without slipping, then it possesses simultaneous translational motion (of the CM) and rotational motion of the entire body. So,

Total K.E. = Rotational K.E. + Translational K.E.

$$\Rightarrow \quad \text{Total K.E. } = E = \frac{1}{2}I_{CM}\omega^2 + \frac{1}{2}mv_{CM}^2$$



Axis normal to the plane of rolling body and passing through *IC*

$$\Rightarrow \quad E = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad E = \frac{1}{2}(mk^2)\omega^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad E = \frac{1}{2}(mk^2)\omega^2 + \frac{1}{2}m(R\omega)^2$$

$$\Rightarrow \quad E = \frac{1}{2}(mk^2 + mR^2)\omega^2 \qquad \ldots(1)$$

Version I

$$E = \frac{1}{2}(mk^2)\left(1 + \frac{R^2}{k^2}\right)\omega^2$$

$$\Rightarrow \quad E = \frac{1}{2}I\omega^2\left(1 + \frac{R^2}{k^2}\right)$$

Version II

$$E=\frac{1}{2}mk^2\left(1+\frac{R^2}{k^2}\right)\left(\frac{v^2}{R^2}\right)$$

$$\Rightarrow \quad E=\frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$$

Conceptual Note(s)

If K_R stands for rotational kinetic energy and K_T for translational kinetic energy, then we have

$$K_R=\frac{1}{2}I_{cm}\omega^2 \text{ and } K_T=\frac{1}{2}mv_{cm}^2, \text{ then}$$

(i) for a ring, $\frac{K_R}{K_T}=1$

(ii) for a disc, $\frac{K_R}{K_T}=\frac{1}{2}$

(iii) for a cylinder, $\frac{K_R}{K_T}=\frac{1}{2}$

(iv) for a shell, $\frac{K_R}{K_T}=\frac{2}{3}$

(v) for a solid sphere, $\frac{K_R}{K_T}=\frac{2}{5}$

ILLUSTRATION 23

A hoop of mass m and radius R rolls without slipping with velocity v_0. Find its kinetic energy.

SOLUTION

METHOD I:

$$K=\frac{1}{2}mv_c^2+\frac{1}{2}I_c\omega^2$$

$$K=\frac{1}{2}mv_o^2+\frac{1}{2}(mR^2)\omega^2$$

Since $v_0=\omega R$

$$\Rightarrow \quad K=\frac{1}{2}mv_o^2+\frac{1}{2}mv_o^2$$

$$\Rightarrow \quad K=mv_o^2$$

METHOD II:

$$K=\frac{1}{2}I_0\omega^2 \text{ where } I_0=I_c+mR^2$$

$$\Rightarrow \quad I_0=mR^2+mR^2=2mR^2$$

$$\Rightarrow \quad K=\frac{1}{2}(2mR^2)\omega^2$$

$$\Rightarrow \quad K=mR^2\omega^2=mv_0^2$$

ILLUSTRATION 24

A carpet of mass M of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to $\frac{R}{2}$.

SOLUTION

Let M be the initial mass of the carpet of radius R and length l (say), then

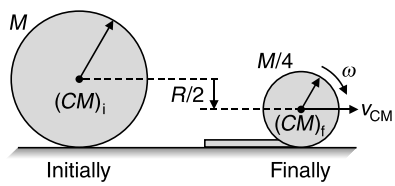
$$M=(\pi R^2 l)\rho$$

When the gentle push is given to the carpet and it unrolls such that its radius reduces to $R/2$, then the new mass of the rolling portion is

$$M'=\pi\left(\frac{R}{2}\right)^2 l\rho=\frac{M}{4}$$

By law of conservation of energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{GPE of CM}\end{pmatrix}=\begin{pmatrix}\text{Gain in}\\ \text{RKE of CM}\end{pmatrix}+\begin{pmatrix}\text{Gain in}\\ \text{TKE of CM}\end{pmatrix}$$



$$\Rightarrow \quad MgR-M'gR'=\left(\frac{1}{2}I'\omega^2\right)+\frac{1}{2}M'v_{cm}^2$$

$$\Rightarrow \quad \frac{7}{8}MgR=\frac{1}{2}\left(\frac{1}{2}M'R'^2\right)\left(\frac{v_{cm}}{R'}\right)^2+\left(\frac{1}{2}\left(\frac{M}{4}\right)v_{cm}^2\right)$$

$$\Rightarrow \quad \frac{7}{8}MgR=\frac{1}{2}\left(\frac{M}{8}\right)v_{cm}^2+\left(\frac{M}{8}\right)v_{cm}^2$$

$$\Rightarrow \quad \frac{7}{8}MgR=\left(\frac{M}{8}\right)\left(\frac{3}{2}\right)v_{cm}^2$$

$$\Rightarrow \quad v_{cm}=\sqrt{\frac{14}{3}gR}$$

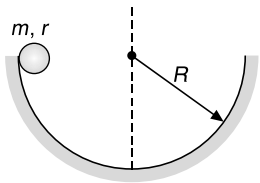
ILLUSTRATION 25

A small steel sphere of mass m and radius r rolls without slipping on the surface of a large hemisphere of radius $R(\gg r)$ whose axis of symmetry is vertical. It starts at the top from the rest.

(a) What is the kinetic energy at the bottom?
(b) What fraction is the rotational kinetic energy of the total kinetic energy?
(c) What fraction is the translational kinetic energy of the total kinetic energy?
(d) Calculate the normal force that the small sphere will exert on the hemisphere at its bottom. How the results will be affected if r is not very small as compared to R.

SOLUTION

Dotted line is the axis of symmetry, which is vertical.



(a) In pure rolling (on stationary ground) work done by friction is zero. Hence, mechanical energy remains conserved. So

K.E. at bottom = decrease in P.E. $= mg(R-r)$

(b) In pure rolling $\frac{K_R}{K_T}=\frac{2}{5}$ (for a sphere)

$$\Rightarrow \frac{K_R}{K_R+K_T}=\frac{2}{7}$$

(c) $\frac{K_T}{K_R+K_T}=\frac{5}{7}$

(d) $K_T=\frac{5}{7}$ (Total energy)

$$\Rightarrow \frac{1}{2}mv^2=\frac{5}{7}mg(R-r)$$

$$\Rightarrow \frac{mv^2}{R-r}=\frac{10}{7}mg$$

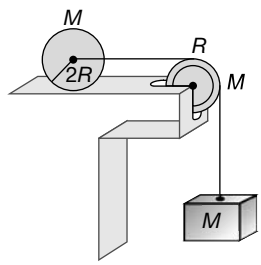
$$\Rightarrow N-mg=\frac{mv^2}{R-r}=\frac{10}{7}mg$$

$$\Rightarrow N=\frac{17}{7}mg$$

The normal reaction will still be $\frac{17}{7}mg$ even if r is not very small as compared to R.

ILLUSTRATION 26

A uniform solid cylinder of mass M and radius $2R$ rests on a horizontal table top. A string is attached by a yoke to a frictionless axle through the centre of the cylinder so that the cylinder can rotate about the axle. The string runs over a pulley in the shape of a disk of mass M and radius R that is mounted on a frictionless axle through its centre. A block of mass M is suspended from the free end of the string. The string does not slip over the pulley surface and the cylinder rolls without slipping on the table top. After the system is released from rest, what is the magnitude of the downward acceleration of the block?



SOLUTION

When the block moves down by h (say), in time t, then it loses potential energy. This potential energy lost by block is gained as

(a) translational kinetic energy, $\frac{1}{2}Mv^2$ by the block

(b) rotational kinetic energy, $\frac{1}{2}I_1\omega_1^2$ by the pulley disk

(c) rolling kinetic energy, $\frac{1}{2}Mv^2+\frac{1}{2}I_2\omega_2^2$ by the cylinder

So, by Law of Conservation of Energy, we get

$$Mgh=\frac{1}{2}Mv^2+\frac{1}{2}I_1\omega_1^2+\left(\frac{1}{2}Mv^2+\frac{1}{2}I_2\omega_2^2\right)$$

where $h=\frac{1}{2}at^2$, $v=at$, $\omega_1=\frac{v}{R}=\frac{at}{R}$, $\omega_2=\frac{v}{2R}=\frac{at}{2R}$

$$I_1=\frac{1}{2}MR^2 \text{ and } I_2=\frac{1}{2}M(2R)^2$$

So, we get

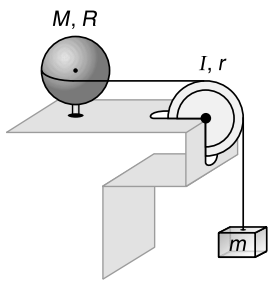
$$g\left(\frac{1}{2}at^2\right)=(at)^2+\frac{R^2}{4}\left(\frac{at}{R}\right)^2+\frac{(2R)^2}{4}\left(\frac{at}{2R}\right)^2$$

$$\Rightarrow \quad g=2a+\frac{a}{2}+\frac{a}{2}$$

$$\Rightarrow \quad a=\frac{g}{3}$$

ILLUSTRATION 27

A uniform spherical shell of mass M and radius R rotates about a vertical axis on frictionless bearing. A light cord passes around the equator of the shell, over a pulley of rotational inertia I and radius r and is attached to a small object of mass m that is otherwise free to fall under the influence of gravity. There is no friction of pulley's axle and the cord does not slip on the pulley. Calculate speed of the object after it has fallen a distance h from rest.



SOLUTION

By law of conservation of energy, we observe that the loss in gravitational potential energy of the block equals the sum of gain in kinetic energy of the block, gain in rotational kinetic energy of the pulley and the gain in rotational kinetic energy of the shell.

$$\Rightarrow \quad mgh=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2+\frac{1}{2}\left(\frac{2MR^2}{3}\right)(\omega')^2$$

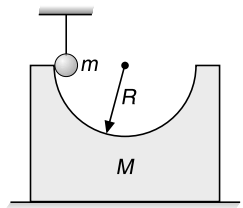
where, $\omega=\frac{v}{r}$ and $\omega'=\frac{v}{R}$

After substituting these values in the above equation and solving, we get

$$v=\sqrt{\frac{mgh}{\frac{m}{2}+\frac{I}{2r^2}+\frac{M}{3}}}$$

ILLUSTRATION 28

A semi-circular track of radius $R=62.5$ cm is cut in a block. Mass of block, having track, is $M=1$ kg and rests over a smooth horizontal floor. A cylinder of radius $r=10$ cm and mass $m=0.5$ kg is hanging by thread such that axes of cylinder and track are in same level and surface of cylinder is in contact with the track as shown in Figure.



When the thread is burnt, cylinder starts to move down the track. Sufficient friction exists between surface of cylinder and track, so that cylinder does not slip.
Calculate velocity of axis of cylinder and velocity of the block when it reaches bottom of the track. Also find force applied by block on the floor at that moment $\left(g=10\ \text{ms}^{-2}\right)$.

SOLUTION

Using Law of Conservation of Linear Momentum, we get

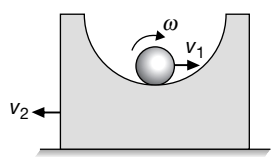
$$mv_1=Mv_2 \qquad \ldots(1)$$

Applying Law of Conservation of Mechanical Energy, we get

$$mg(R-r)=\frac{1}{2}mv_1^2+\frac{1}{2}I\omega^2+\frac{1}{2}Mv_2^2 \qquad \ldots(2)$$

where, $I=\frac{1}{2}mr^2$ and $\omega=\frac{v_r}{r}$, where v_r is the velocity of cylinder axis relative to block

$$v_r=v_1+v_2 \qquad \ldots(3)$$



Solving equations (1), (2) and (3) with given data, we get

$$v_1=2\ \text{ms}^{-1}$$

and $v_2 = 1.5\ \text{ms}^{-1}$

Further, we have

$$N - mg = \frac{mv_r^2}{R-r}$$

$$\Rightarrow \quad N = mg + \frac{mv_r^2}{R-r} = (0.5)(10) + \frac{(0.5)(3.5)^2}{0.525}$$

$$\Rightarrow N = 16.67\ \text{N}$$

Conceptual Note(s)

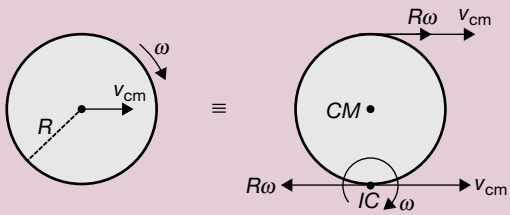
AN OBSERVATION

For a body in pure rolling motion, we have

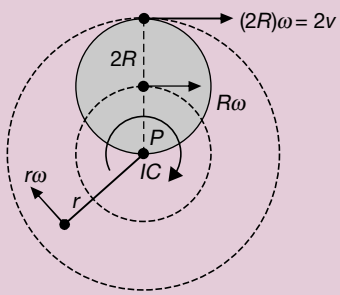
$$E = \frac{1}{2}(mk^2 + mR^2)\omega^2$$

Now $(mk^2 + mR^2)$ is actually moment of inertia of body (in combined effect of rotation and translation) about axis that passes through the point of contact P, which happens to be at rest (in case of pure rolling), so

$$E = \frac{1}{2}(I_{\text{about } P})\omega^2 = \frac{1}{2}(mk^2 + mR^2)\omega^2$$



This axis passing through P is always normal to the plane used to represent motion and the intersection of this axis with the plane is the location of instantaneous centre of zero velocity (IC).



If I_{IAOR} is the Moment of Inertia of body about **Instantaneous Axis of Rotation (IAOR)**, then

$$I_{IAOR} = I_{CG} + md^2 = mk^2 + mR^2$$

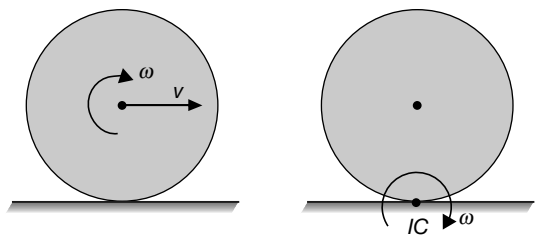
$$KE = \frac{1}{2}(I_{IAOR})\omega^2$$

The **Instantaneous Axis of Rotation (IAOR) passes through a point called the Instantaneous Centre of Zero Velocity (IC).**

So, from above we conclude that pure rolling is equivalent to the case of pure rotation about a new axis (called Instantaneous Axis of Rotation, IAOR) that passes through the point of contact (because it was the only point which is at rest in pure rolling).

INSTANTANEOUS AXIS OF ROTATION (IAOR)

The combined effect of translation of the centre of mass and rotation of a rigid body about an axis through the centre of mass is equivalent to a pure rotation with the same angular speed about an axis passing through a point of zero velocity. Such an axis is called the Instantaneous Axis of Rotation (IAOR).



$$\text{So, } (\text{Rotation}) + (\text{Translation}) \equiv \begin{pmatrix} \text{Pure rotation} \\ \text{about } IAOR \\ \text{passing} \\ \text{through } IC \end{pmatrix}$$

$$\Rightarrow \quad E = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

$$\Rightarrow \quad E = \frac{1}{2}(mk^2 + mR^2)\omega^2 = \frac{1}{2}I_{IAOR}\omega^2$$

Problem Solving Technique(s)

WORD OF ADVICE

(a) Although the concept of IC is conveniently used to determine the velocity of any point in a body, **however generally, IC does not have zero acceleration and therefore, it should not be used for finding the acceleration of any point in the body.**

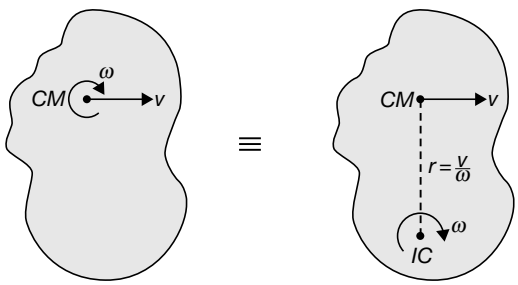
(b) When a body is subjected to general plane motion, the point determined as the instantaneous centre of zero velocity for the body can only be used for an instant of time. Since the body changes its position from one instant to the next, then for each position of the body a unique instantaneous centre must be determined. The locus of points which defines the IC during the body's motion is called a **centrode**. Thus, each point on the centrode acts as the IC for the body only for an instant of time.

LOCATION OF THE IC

The location of the IC is determined by using the fact that the relative position vector extending from the IC to a point is always perpendicular to the velocity of the point. Then the following three possibilities exist.

CASE-1

When the velocity of a point (generally the centre of mass) on the body and the angular velocity of the body are known.



Combined Effect of Rotation & Tranlation

Single Effect of Rotation about *IC*

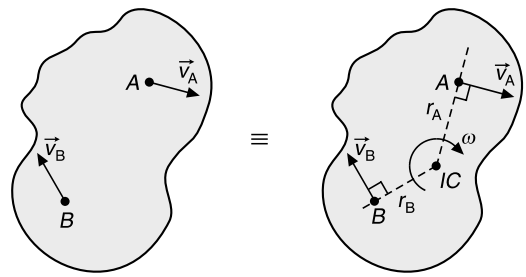
When v and ω are known, then IC is located along the line drawn perpendicular to $\vec{v}$ at CM, such that the distance from CM to IC is, $r = \frac{v}{\omega}$.

Note that IC lie on that side of CM which causes rotation about the IC, which is consistent with the direction of motion caused by $\vec{\omega}$ and $\vec{v}$.

CASE-2

When the lines of action of two non-parallel velocities are known.

Consider the body shown in Figure where the line of action of the velocities $\vec{v}_A$ and $\vec{v}_B$ are known.



STEP-1: Draw perpendiculars at A and B to these lines of action.

STEP-2: The point of intersection of these perpendiculars, locates the IC at the instant considered. If v_A and v_B originate at perpendicular distances r_A and r_B from IC, then

$$v_A = r_A\omega$$

and $v_B = r_B\omega$

CASE-3

When the magnitude and direction of two parallel velocities are known.

When the velocities of points A and B are parallel separated by perpendicular distance d and have known magnitudes v_A and v_B, then the location of the IC is determined by the following steps.

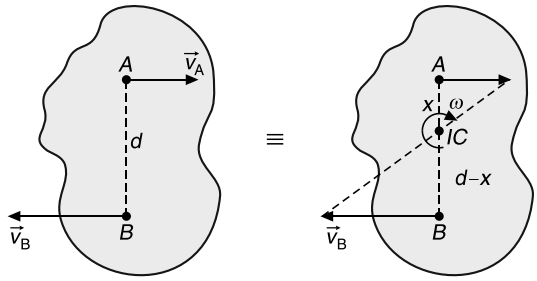
STEP-1: Always draw the velocities to the scale.

STEP-2: Join the points of origin of the two velocities i.e., tails of velocity vectors by a straight line.

STEP-3: Now, join the heads of the two velocity vectors by straight line.

STEP-4: IC is located at the intersection of the lines in STEP-2 and STEP-3.

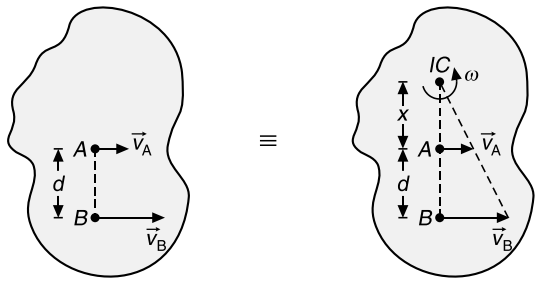
To locate position of IC in (a), we have



(a) When velocities are anti-parallel

$v_A = x\omega$ and $v_B = (d-x)\omega$

To locate position of IC in (b), we have



(b) When velocities are parallel

$v_A = x\omega$ and $v_B = (d+x)\omega$

As a special case, if the body is translating, $v_A = v_B$ and the IC would be located at infinity, in which case $\omega = 0$.

Conceptual Note(s)

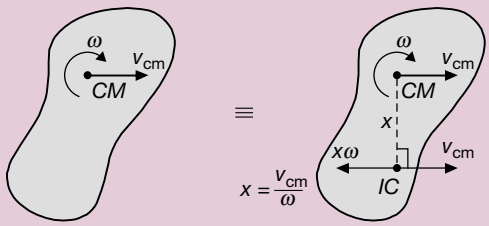
Q. Why do we use the concept of IAOR for bodies in combined rotation and translation?

A. When a body is under the combined influence of rotational and translational effect, then there exists a point (on the body or off the body) which happens to have zero net velocity, see diagram. **This makes us think about a point called Instantaneous Centre (IC) of zero velocity and the axis passing through IC, normal to plane of motion, is called Instantaneous Axis of Rotation (IAOR).** So, the combined effect of rotation and translation is just equivalent to a single effect of rotation about IAOR (as if the body is pinned to IC and is just rotating about IC.

So, $\begin{pmatrix}\text{Combined Effect}\\ \text{of Rotation}\\ \text{and Translation}\end{pmatrix} \equiv \begin{pmatrix}\text{Single Effect of}\\ \text{Rotation about}\\ \text{IC or IAOR}\end{pmatrix}$

Hence, if IC is at a distance x from centre of mass, then $(v_{IC})_{net} = 0$ i.e., $v_{cm} - x\omega = 0$

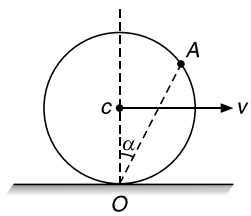
$\Rightarrow \quad x = \dfrac{v_{cm}}{\omega}$



IC lies in the portion or IC lies towards the portion that rotates opposite (i.e., has tangential velocity opposite) to v_{cm}.

ILLUSTRATION 29

A hoop of radius R rolls over a horizontal plane with a constant velocity v without slipping. Find the velocity of point A of the loop as shown in Figure.



SOLUTION

The velocity of the point A can be obtained by two methods.

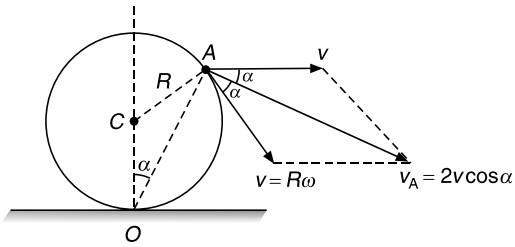
METHOD I:

Velocity of point A w.r.t. centre of mass C is $\vec{v}_{AC}$ given by

$$\vec{v}_{AC} = \vec{v}_A - \vec{v}_C$$

$$\Rightarrow \quad \vec{v}_A = \vec{v}_{AC} + \vec{v}_C$$

Now $|\vec{v}_C| = |\vec{v}_{CM}| = v$ and $|\vec{v}_{AC}| = R\omega$, perpendicular to the line CA. So, $|\vec{v}_A|$ is actually the magnitude of the resultant of $\vec{v}_{AC}$ and $\vec{v}_C$ as shown in Figure.



If θ is the angle between $\vec{v}_{PC}$ and $\vec{v}_C$, then $\theta = 2\alpha$.

$$\Rightarrow \quad |\vec{v}_A| = \sqrt{v^2 + R^2\omega^2 + 2v(R\omega)\cos(2\alpha)}$$

For pure rolling condition, $v = R\omega$

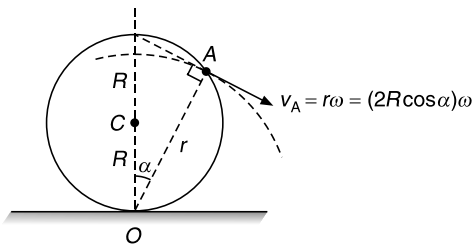
$$\Rightarrow \quad |\vec{v}_A| = \sqrt{v^2 + v^2 + 2v^2\cos(2\alpha)}$$

$$\Rightarrow \quad |\vec{v}_A| = v\sqrt{2(1+\cos 2\alpha)} = v\sqrt{2\times 2\cos^2\left(\frac{2\alpha}{2}\right)}$$

$$\Rightarrow \quad |\vec{v}_A| = 2v\cos\alpha$$

METHOD II:

A body in pure rolling can be considered to be in pure rotation about a point having zero velocity (in this case the point of contact), i.e. instantaneous centre (IC) of zero velocity. So, we have



$$v_A = r\omega = (2R\cos\alpha)\omega = 2(R\omega)\cos\alpha$$

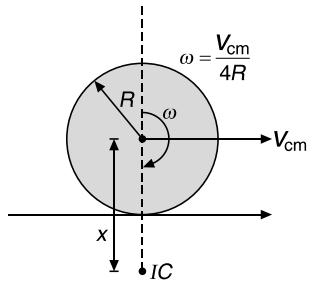
$$\Rightarrow \quad v_A = 2v\cos\alpha \qquad \{\because\ v_{cm} = v = R\omega\}$$

ILLUSTRATION 30

A wheel of radius R is moving on the ground. If v_{cm} be the velocity of centre of mass of the wheel and $\omega = \frac{v_{cm}}{4R}$ be the angular velocity of the wheel, then locate its instantaneous centre of zero velocity.

SOLUTION

Let the instantaneous centre of zero velocity be located at a distance x from the centre of the wheel as shown in Figure.



Since velocity of IC is zero, so we have $v_{IC} = 0$

$$\Rightarrow \quad v_{cm} - x\omega = 0$$

$$\Rightarrow \quad v_{cm} = x\left(\frac{v_{cm}}{4R}\right)$$

$$\Rightarrow \quad x = 4R$$

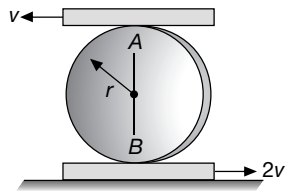
Problem Solving Technique(s)

If the body is placed on a surface which is in motion, then **for no slipping, the point of contact must move with the velocity of the surface.**

$(v_P)_{net}$ = velocity of surface

ILLUSTRATION 31

The disc of mass m, radius r is confined to roll without slipping at A and B. If the plates have the velocities shown, determine the angular velocity of the disc. Also find the velocity of centre of mass of disc, the location of instantaneous centre of zero velocity and total energy of the disc.



SOLUTION

Let v_0 be the velocity of centre of mass and ω_0 the angular velocity of the disc as shown in Figure then, for no slipping at A and B, we have

$$(v_B)_{net} = 2v$$

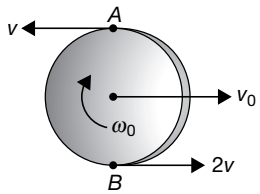
$$\Rightarrow \quad v_0 - r\omega_0 = 2v \qquad \ldots(1)$$

$$(v_A)_{net} = -v$$

$$\Rightarrow \quad v_0 + r\omega_0 = -v \qquad \ldots(2)$$

Solving (1) and (2), we get

$$\omega_0 = -\frac{3v}{2r} \text{ and } v_0 = \frac{v}{2}$$



Also, we ca directly find the value of ω by using

$$\omega = \frac{(v_r)_{\perp}}{\perp \text{ separation between the velocities}}$$

$$\Rightarrow \quad \omega = \frac{3v}{2r}$$

Now let IC be located at a distance x from A or $(2r - x)$ from B, then

$$v = x\omega$$

$$\Rightarrow \quad x = \frac{v}{3v/2r} = \frac{2r}{3}, \text{ from } A$$

OR $\quad (2r - x) = \frac{4r}{3}$, from B

The total energy of the disc is

$$E = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2$$

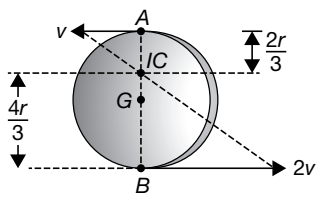
$$\Rightarrow \quad E = \frac{1}{2}m\left(\frac{v^2}{4}\right) + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{9v^2}{4r^2}\right)$$

$$\Rightarrow \quad E = \frac{mv^2}{8} + \frac{9mv^2}{16} = \frac{11}{16}mv^2$$

Alternatively, we have

$$E = \frac{1}{2}(I_{\text{IAOR}})\omega^2$$

where $I_{\text{IAOR}} = I_G + md^2$



where $I_G = \frac{1}{2}mr^2$ and $d = r - \frac{2r}{3} = \frac{r}{3}$

$$\Rightarrow \quad E = \frac{1}{2}\left(\frac{1}{2}mr^2 + \frac{1}{9}mr^2\right)\omega^2$$

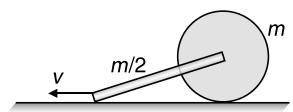
$$\Rightarrow \quad E = \frac{1}{2}\left(\frac{11}{18}mr^2\right)\left(\frac{3v}{2r}\right)^2$$

$$\Rightarrow \quad E = \frac{1}{2}\left(\frac{11}{18}mr^2\right)\left(\frac{9v^2}{4r^2}\right)$$

$$\Rightarrow \quad E = \frac{11}{16}mv^2$$

ILLUSTRATION 32

A uniform disc of mass m is fitted (pivoted smoothly) with a rod of mass $\frac{m}{2}$. If the bottom of the rod is pulled with a velocity v, it moves without changing its angle of orientation and the disc rolls without sliding. Calculate the kinetic energy of the rod disc system.



SOLUTION

Rolling can be considered as pure rotation about point of contact, so

$$K_{\text{rolling}} = \frac{1}{2}I_P\omega^2$$

$$\Rightarrow \quad K_{\text{rolling}} = \frac{1}{2}\left(\frac{mR^2}{2} + mR^2\right)\omega^2 = \frac{3}{4}mR^2\omega^2 \qquad \ldots(1)$$

The rod translates with the velocity v, hence velocity of centre of disc will also be v.

$$\Rightarrow \quad v = \omega R \qquad \ldots(2)$$

From (1) and (2), we get $K_{\text{rolling}} = \frac{3}{4}mv^2$

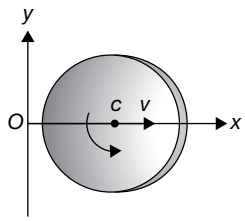
Kinetic energy of the rod is

$$K_{\text{rod}} = \frac{1}{2}\left(\frac{m}{2}\right)v^2 = \frac{mv^2}{4} \qquad \ldots(3)$$

$$\Rightarrow \quad K_{\text{total}} = K_{\text{rolling}} + K_{\text{rod}} = mv^2$$

ILLUSTRATION 33

A rotating disc moves in the positive direction of the x-axis. Find the equation $y(x)$ describing the position of the instantaneous axis of rotation, if at the initial moment, the centre c of the disc was located at the point O after which it moved with constant velocity v while the disc started rotating counterclockwise with a constant angular acceleration α. Assume the initial angular velocity to be zero.



SOLUTION

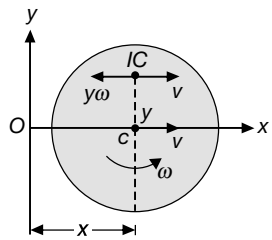
The position of disc at time t is $x = vt$

$$\Rightarrow \quad t = \frac{x}{v}$$

Since the disc is rotation with constant angular acceleration α, so

$$\omega = \omega_0 + \alpha t$$

$$\Rightarrow \quad \omega = \alpha t = \frac{\alpha x}{v} \qquad \{\because\ \omega_0 = 0\}$$



Now, if IC is located at a distance y from centre of disc, then we have

$$v_{IC} = 0$$

$$\Rightarrow \quad y\omega - v = 0$$

$$\Rightarrow \quad y = \frac{v}{\omega}$$

$$\Rightarrow \quad y = \frac{v}{\alpha x/v} = \frac{v^2}{\alpha x}$$

$$\Rightarrow \quad xy = \frac{v^2}{\alpha} = \text{constant}$$

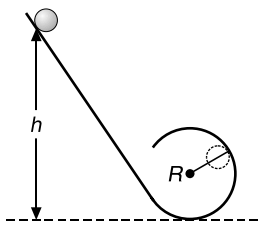
This is the desired x-y equation. This equation represents a rectangular hyperbola.

Test Your Concepts-III

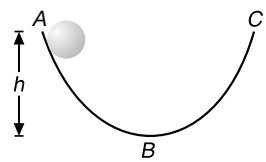
Based on Instantaneous Axis of Rotation, Pure Rolling and Conservation of Energy

(Solutions on page H.152)

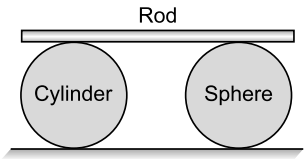
1. A uniform ball of radius r rolls without slipping along the loop-the-loop track in Figure. It starts from rest at height h above the bottom of the loop. If the ball is not to leave the track at the top of the loop, what is the least value h can have (in terms of the radius $R \gg r$ of the loop)? What would h have been if the ball were to slide along a frictionless track instead of rolling?



2. A solid ball rolls down a parabolic path ABC from a height h as shown in Figure. The portion AB of the path is rough while BC is smooth. How high will the ball climb in BC?

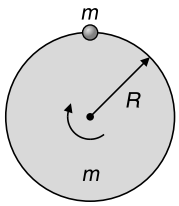


3. A rod of mass m is kept on a cylinder and sphere each of radius R. The masses of the sphere and cylinder are $m_1 = 4m$ and $m_2 = 5m$ respectively. If the speed of the rod is v, find the KE of the system. Assume that the surfaces do not slide relative to each other.



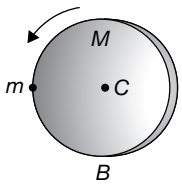
4. A uniform sphere of radius r starts rolling down without slipping from the top of another sphere of radius R. Find the angular velocity of the sphere after it leaves the surface of the larger sphere.

5. A bead of mass m is welded at the periphery of the smoothly pivoted disc mass m and radius R. Find the speed of the bead at its lowest position.

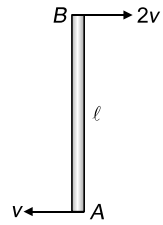


6. A uniform rod of length $2l$ and mass m is free to rotate in a vertical plane about a smooth fixed horizontal axis perpendicular to the rod through one end of the rod. Initially, the rod is held in horizontal position and is then released. Find the maximum angular velocity of the rod in the subsequent motion.

7. A uniform disc of mass M and radius R is pivoted about the horizontal axis normal to its plane and passing through its centre C. A point of mass m glued to the disc at its rim, as shown in Figure, is released from rest. Find the angular velocity of the disc when m reaches the point B.



8. A uniform rod of mass m, length l is kept on a smooth horizontal surface. The ends A and B of the rod move with speeds v and $2v$ respectively perpendicular to the rod as shown in Figure.

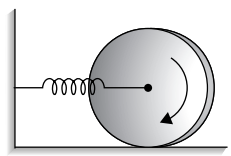


Calculate the angular velocity of the rod, linear velocity of centre of mass of the rod and the kinetic energy of the rod.

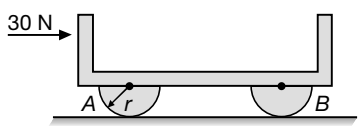
9. A uniform ring of radius a and mass m is free to rotate in a vertical plane about a fixed smooth axis which is perpendicular to the plane of the ring and passes through a point A on the ring. A particle of mass m is attached to the ring at B, where AB is a diameter. When the ring is hanging in a position of stable equilibrium the particle is struck a blow which gives it a velocity $3\sqrt{ga}$. Find the vertical height above A to which the particle rises.

10. A solid disk is rolling without slipping on a level surface at a constant speed of $2\ \text{ms}^{-1}$. How far can it roll up a $30°$ ramp before it stops? Take $g = 10\ \text{ms}^{-2}$.

11. A disc of mass m and radius R has a spring of constant k attached to its centre, the other end of the spring being fixed to a vertical wall (shown in Figure). If the disc rolls without slipping on a level floor, how far to the right does the centre of mass move, if initially the spring was unstretched and the angular speed of the disc was ω_0?



12. The 9 kg cradle is supported as shown by two uniform disks that roll without sliding at all surfaces of contact. The mass of each disk is $m = 6$ kg and the radius of each disk is $r = 80$ mm. Assuming that the system is initially at rest, find the velocity of the cradle after it has moved 250 mm.



TORQUE

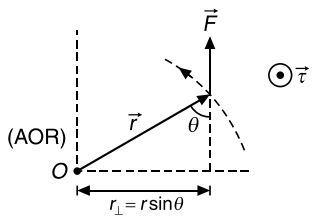
The rotational effect of force is called Torque. It is also called as the rotational analogue of force. Force acting on a body provides linear acceleration to it and correspondingly torque acting on a body provides angular acceleration to it.

It is **an axial vector** whose **direction** can be **found by** using the **Right-Hand Thumb Rule**. To find the direction of torque simply curl the fingers of the right hand in the sense of rotation in which the force tends to rotate the system about a given axis, then the direction of the thumb gives the direction of the torque.

Mathematically, the moment of force is called **torque i.e. torque is equal to the product of force and the perpendicular distance of the force from the axis of rotation (AOR).** Mathematically, if θ is angle between $\vec{r}$ and $\vec{F}$, then

$$\tau = Fr_{\perp} = F(r\sin\theta)$$

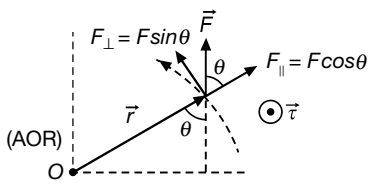
where $r_{\perp}$ is the perpendicular distance from the origin to the line of action of the force as shown in Figure. It is also called the lever arm.



The above definition of torque i.e. $\tau = rF\sin\theta$, may be also be interpreted as

$$\tau = r(F\sin\theta) = rF_{\perp}$$

The turning effect of a force about the origin is produced only by the perpendicular component $(F_{\perp} = F\sin\theta)$ as shown in Figure.



Vectorially,

$$\vec{\tau} = \vec{r}\times\vec{F}$$

So, if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$, then

$$\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \overset{\oplus}{\hat{i}} & \overset{\ominus}{\hat{j}} & \overset{\oplus}{\hat{k}} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

In the most simplest manner, if we know the sense of rotation (Clockwise or Anticlockwise) then, to find the direction of $\vec{\tau}$ we just curl the fingers of Right Hand along the sense of rotation and then the thumb gives direction of $\vec{\tau}$.

For a system with moment of inertia I having an angular acceleration $\vec{\alpha}$, we have

$$\vec{\tau} = I\vec{\alpha}$$

In Magnitude $\tau = I\alpha$ (Direction can be found by using Right Hand Thumb Rule). **This is also called Newton's Second Law of Rotational Motion.**

ILLUSTRATION 34

The position vector of point of application of force $\vec{F} = (\hat{i} + 2\hat{j} - 3\hat{k})$ N about O is $\vec{r} = (2\hat{i} + 3\hat{j} - \hat{k})$ m. Calculate the torque of a force about a point O.

SOLUTION

Torque $\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & 2 & -3 \end{vmatrix}$

$$\Rightarrow \quad \vec{\tau} = \hat{i}(-9+2) + \hat{j}(-1+6) + \hat{k}(4-3)$$

$$\Rightarrow \quad \vec{\tau} = (-7\hat{i} + 5\hat{j} + \hat{k})\ \text{Nm}$$

Conceptual Note(s)

(a) Cases for zero torque

Since, $|\vec{\tau}| = rF\sin\theta$

So, for $|\vec{\tau}| = 0$, we have the following three cases:

CASE-1:

$F = 0$ i.e., no force acts on system

CASE-2:

$r = 0$ i.e., force acts at AOR

CASE-3:

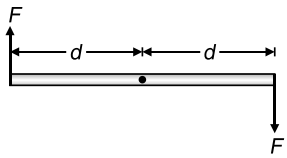
$F \neq 0$, $r \neq 0$ but angle between $\vec{F}$ and $\vec{r}$ is either $0°$ or $180°$ i.e., $\vec{F}$ acts along $\vec{r}$ or opposite to $\vec{r}$. **Such forces are called Radial Forces. So, torque due to Radial Forces is always zero.**

(b) SI unit of $|\vec{\tau}|$ is Nm (not joule)

(c) $\vec{\tau}=\vec{r}\times\vec{F}$, **so** $\vec{\tau}$ **is** $\perp$ **to** $\vec{r}$ **as well as** $\vec{F}$ i.e., $\vec{\tau}\cdot\vec{r}=0$ and $\vec{\tau}\cdot\vec{F}=0$

FORCE COUPLE

A pair of forces each of same magnitude and acting in opposite direction is called a force couple. The torque due to couple is the product of magnitude of either force and the perpendicular separation between the forces.



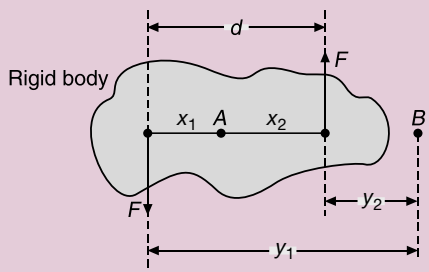
$$\begin{pmatrix}\text{Torque}\\ \text{due to}\\ \text{couple}\end{pmatrix}=\begin{pmatrix}\text{Magnitude}\\ \text{of one}\\ \text{force}\end{pmatrix}\times\begin{pmatrix}\perp\text{ Distance}\\ \text{between their}\\ \text{lines of action}\end{pmatrix}$$

$\Rightarrow \quad \tau=F(2d)$

Conceptual Note(s)

(a) A couple does not exert a net force on an object even though it exerts a torque.

(b) Net torque due to a force couple is same about any point.



Torque about A is $\tau_A=x_1F+x_2F$

$\Rightarrow \quad \tau_A=F(x_1+x_2)=Fd$

Torque about B is $\tau_B=y_1F-y_2F$

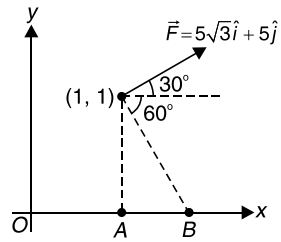
$\Rightarrow \quad \tau_B=F(y_1-y_2)=Fd$

(c) If net force acting on a system is zero, torque is same about any point.

(d) A consequence is that, if $F_{net}=0$ and $\tau_{net}=0$ about one point, then $\tau_{net}=0$ about any point.

ILLUSTRATION 35

Calculate the torque about point O and A due to the force as shown in Figure.



SOLUTION

Torque about point O

$$\vec{\tau}=\vec{r}_0\times\vec{F},\ \vec{r}_0=\hat{i}+\hat{j},\ \vec{F}=5\sqrt{3}\hat{i}+5\hat{j}$$

$$\Rightarrow \quad \vec{\tau}=\left(\hat{i}+\hat{j}\right)\times\left(5\sqrt{3}\hat{i}+5\hat{j}\right)=5\left(1-\sqrt{3}\right)\hat{k}$$

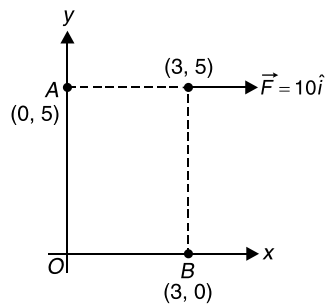
Torque about point A

$$\vec{\tau}=\vec{r}_a\times\vec{F},\ \vec{r}_a=\hat{j},\ \vec{F}=5\sqrt{3}\hat{i}+5\hat{j}$$

$$\Rightarrow \quad \vec{\tau}=\hat{j}\times\left(5\sqrt{3}\hat{i}+5\hat{j}\right)=5\left(-\sqrt{3}\right)\hat{k}$$

ILLUSTRATION 36

Calculate the torque about point A, O and B due to the force as shown in Figure.



SOLUTION

Torque about point A

$$\vec{\tau}_A=\vec{r}_A\times\vec{F},\ \vec{r}_A=3\hat{i},\ \vec{F}=10\hat{i}$$

$$\Rightarrow \quad \vec{\tau}_A=3\hat{i}\times10\hat{i}=0$$

Torque about point B

$$\vec{\tau}_B=\vec{r}_B\times\vec{F},\ \vec{r}_B=5\hat{j},\ \vec{F}=10\hat{i}$$

$$\Rightarrow \quad \vec{\tau}_B=5\hat{j}\times10\hat{i}=-50\hat{k}$$

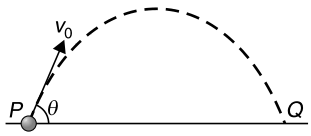
Torque about point O

$$\vec{\tau}_O = \vec{r}_O \times \vec{F},\ \vec{r}_O = 3\hat{i} + 5\hat{j},\ \vec{F} = 10\hat{i}$$

$$\Rightarrow \quad \vec{\tau}_O = \left(3\hat{i} + 5\hat{j}\right) \times 10\hat{i} = -50\hat{k}$$

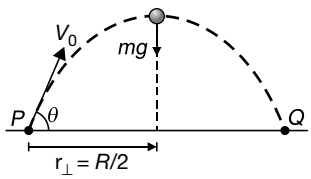
ILLUSTRATION 37

A particle having mass m is projected with a velocity v_0 from a point P on a horizontal ground making an angle θ with horizontal. Calculate the torque about the point of projection acting on the particle when it is at its maximum height.



SOLUTION

When the particle reaches the maximum height as shown in Figure, then

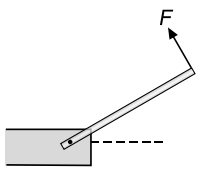


$$\tau = Fr_\perp = mg\left(\frac{R}{2}\right) = mg\left(\frac{v_0^2 \sin 2\theta}{2g}\right)$$

$$\Rightarrow \quad \tau = \frac{mv_0^2 \sin 2\theta}{2}$$

ILLUSTRATION 38

A uniform bar of mass m and length l is hinged at one end and held in the position shown by applying a constant force F at the other end, perpendicular to the rod.



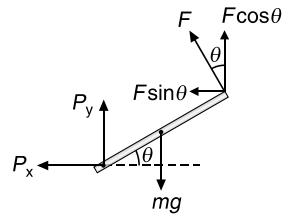
Calculate the magnitude and direction of the force exerted by the hinge on the rod.

SOLUTION

The two known forces that act on the rod are its weight mg acting vertically downward and the applied force F. Let the force applied by the hinge be having two components, the horizontal component P_x and the vertical component P_y. Then

$$P = \sqrt{P_x^2 + P_y^2}$$

Please note that the force due to the hinge can be assumed to be in any direction. The forces acting on the rod are drawn in the FBD shown in Figure.



Please note that, P_x and P_y have been assigned arbitrary directions. If the answers for P_x, P_y or both come out to be negative, then the direction will be opposite to the direction assumed.

For translatory equilibrium, we have

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

When $\Sigma F_x = 0$, we get

$$P_x + F\sin\theta = 0$$

$$\Rightarrow \quad P_x = -F\sin\theta \qquad \ldots(1)$$

Negative sign simply shows that P_x is directed to the right.

When $\Sigma F_y = 0$, we get

$$P_y + F\cos\theta - mg = 0$$

$$\Rightarrow \quad P_y = mg - F\cos\theta \qquad \ldots(2)$$

To calculate P_x, we must find the angle θ. For that, let us apply the condition for rotational equilibrium. For rotational equilibrium, we have

$$\Sigma\tau = 0 \qquad \text{(about any point)}$$

So, lets calculate the net torque acting on the rod about the hinge, then we get

$$-(mg)\left(\frac{l}{2}\cos\theta\right) + Fl = 0$$

$$\Rightarrow \quad F = \frac{mg}{2}\cos\theta \qquad \ldots(3)$$

From equations (1) and (3), we get

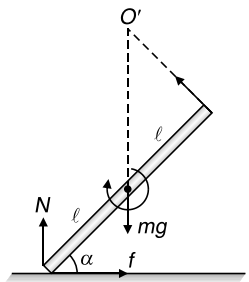
$$P_x = -F\sqrt{1 - \frac{4F^2}{m^2g^2}} = -\frac{F}{mg}\sqrt{m^2g^2 - 4F^2}$$

ILLUSTRATION 39

Calculate minimum coefficient of friction between a thin homogeneous rod and the floor for which a person can slowly lift the rod upwards without slipping on the floor by applying a force perpendicular to the rod.

SOLUTION

Consider the equilibrium of the rod when it is inclined at an angle α with the horizontal. The forces acting on the rod are shown in Figure.



Let us consider torque about the point of intersection O' of the force of gravity mg and the force F applied by the person perpendicular to the rod. Since the torque due to these forces about this point O' is zero, so doing this eliminates the unknown force F. At equilibrium, we have

$$(\Sigma\tau)_{\text{about } O'} = 0$$

Since, moment arm for N about O' is $l\cos\alpha$, while that for friction about O' is $\left(\frac{l}{\sin\alpha}+l\sin\alpha\right)$

$$\Rightarrow \quad Nl\cos\alpha - fl\left(\frac{1}{\sin\alpha}+\sin\alpha\right)=0$$

$$\Rightarrow \quad f = N\left(\frac{\cos\alpha\sin\alpha}{1+\sin^2\alpha}\right) = N\left(\frac{\cos\alpha\sin\alpha}{2\sin^2\alpha+\cos^2\alpha}\right)$$

$$\Rightarrow \quad f = N\left(\frac{1}{2\tan\alpha\cot\alpha}\right)$$

Since, $f \le \mu N$

$$\Rightarrow \quad N\left(\frac{1}{2\tan\alpha\cot\alpha}\right) \le \mu N$$

$$\Rightarrow \quad \mu \ge \frac{1}{2\tan\alpha+\cot\alpha} \qquad \ldots(1)$$

For μ to be minimum, we must have

$$2\tan\alpha+\cot\alpha = \text{MAXIMUM}$$

$$\Rightarrow \quad \frac{d}{d\alpha}(2\tan\alpha+\cot\alpha)=0$$

$$\Rightarrow \quad 2\sec^2\alpha-\text{cosec}^2\alpha=0$$

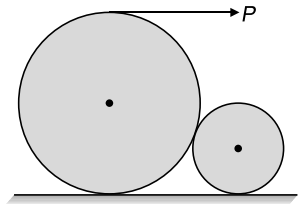
$$\Rightarrow \quad \tan\alpha=\frac{1}{\sqrt{2}}$$

Substituting this value in equation (1), we get

$$\mu_{\min}=\frac{1}{2\sqrt{2}}$$

ILLUSTRATION 40

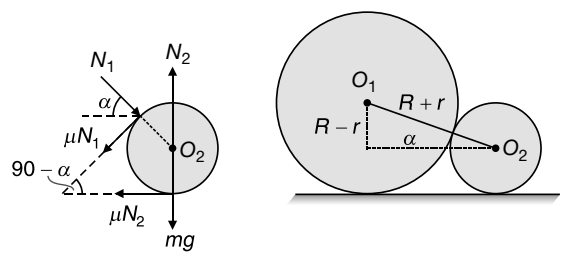
Consider two heavy right circular rollers of radii R and r respectively at rest on a rough horizontal plane as shown in Figure.



The larger roller has a string wound around it to which a horizontal force P can be applied. Assume the coefficient of friction μ has the same value for all surfaces in contact, calculate the necessary condition under which the larger roller can be pulled over the smaller one. Assume the smaller cylinder should neither roll nor slide.

SOLUTION

Let N_1 be the reaction between the two cylinders and N_2 be the normal reaction between the smaller cylinder and the horizontal surface. The forces acting on the smaller cylinder have been shown in Figure.



At equilibrium, we have

$$\Sigma F_x = N_1\cos\alpha-\mu N_2-\mu N_1\cos(90-\alpha)=0 \quad \ldots(1)$$

Also, torque about O_2 (centre of smaller sphere) is zero, so we get

$$\Sigma\tau = \mu N_1 r - \mu N_2 r = 0$$

$$\Rightarrow \quad N_1 = N_2 \qquad \ldots(2)$$

Substituting equation (2) in equation (1), we get

$$\mu N_1 + \mu N_1 \sin\alpha = N_1 \cos\alpha$$

$$\Rightarrow \quad \mu(1+\sin\alpha) = \cos\alpha \qquad \ldots(3)$$

Also, we see that

$$\sin\alpha = \sqrt{\frac{R-r}{R+r}}$$

$$\Rightarrow \quad \cos\alpha = \sqrt{1-\sin^2\alpha} = \frac{2\sqrt{Rr}}{R+r}$$

Substituting these values in equation (3), we get

$$\mu\left(1+\frac{R-r}{R+r}\right) = \frac{2\sqrt{Rr}}{R+r}$$

$$\Rightarrow \quad \mu = \sqrt{\frac{r}{R}}$$

Hence the required condition is $\mu \geq \sqrt{\frac{r}{R}}$

ILLUSTRATION 41

A uniform rod of length L and mass M is pivoted freely at one end as shown in Figure.

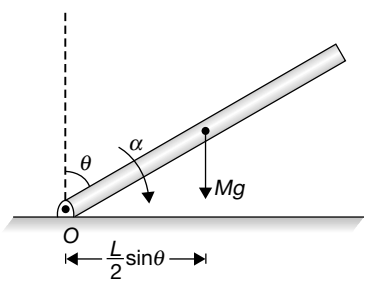
(a) Find the angular acceleration of the rod when it is at angle θ to the vertical.

(b) Assuming the rod to start from the vertical position, find the angular velocity as the function of θ.

(c) Find the tangential linear acceleration of the free end when the rod is horizontal.

SOLUTION

(a) Figure shows the rod at an angle θ to the vertical.



Net torque about the point O is

$$\tau_O = Mg\left(\frac{L}{2}\sin\theta\right)$$

Using the Second Law of Motion $\tau_O = I_O\alpha$

$$\frac{MgL}{2}\sin\theta = \frac{ML^2}{3}\alpha$$

$$\Rightarrow \quad \alpha = \frac{3g\sin\theta}{2L}$$

(b) From above, we have

$$\alpha = \frac{3g\sin\theta}{2L}$$

$$\Rightarrow \quad \omega\frac{d\omega}{d\theta} = \frac{3g\sin\theta}{2L}$$

$$\Rightarrow \quad \omega d\omega = \frac{3g}{2L}\sin\theta d\theta$$

Integrating within appropriate limits, we get

$$\int_0^\omega \omega d\omega = \frac{3g}{2L}\int_0^\theta \sin\theta d\theta$$

$$\Rightarrow \quad \frac{\omega^2}{2} = -\frac{3g}{2L}\cos\theta\Big|_0^\theta = \frac{3g}{2L}(1-\cos\theta)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3g}{L}(1-\cos\theta)}$$

The above result can also be obtained by using the Law of Conservation of Mechanical Energy, where we use,

$$\begin{pmatrix}\text{Loss in GPE}\\ \text{of CM of Rod}\end{pmatrix} = \begin{pmatrix}\text{Gain in RKE}\\ \text{of Rod}\end{pmatrix}$$

$$\Rightarrow \quad Mg\frac{L}{2}(1-\cos\theta) = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3g}{L}(1-\cos\theta)}$$

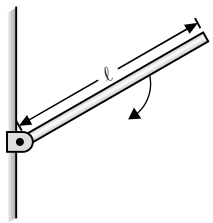
(c) When the rod is horizontal $\theta = \frac{\pi}{2}$, so $\alpha = \frac{3g}{2L}$. So, the tangential linear acceleration is

$$a_t = \alpha L = \frac{3g}{2}$$

This is greater than the acceleration of an object falling freely.

ILLUSTRATION 42

A thin rod of mass M and length l is pivoted at one end. Initially the rod makes an angle of $60°$ with the vertical and is then released. Calculate the magnitude and direction of the force on the pivot when the rod is horizontal.



SOLUTION

Let ω be the angular velocity of rod in horizontal position, then

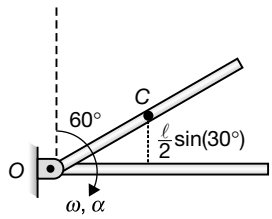
$$\alpha = \frac{\tau}{I} = \frac{(Mg)\frac{l}{2}}{\frac{Ml^2}{3}} = \frac{3}{2}\frac{g}{l} \qquad \ldots(1)$$

By Law of Conservation of Energy

$$\begin{pmatrix}\text{Loss in GPE of}\\ \text{CM of Rod}\end{pmatrix} = \begin{pmatrix}\text{Gain in RKE}\\ \text{of the Rod}\end{pmatrix}$$

$$\Rightarrow \quad Mg\left(\frac{l}{2}\sin(30°)\right) = \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad Mg\left(\frac{l}{4}\right) = \frac{1}{2}\left(\frac{1}{3}Ml^2\right)\omega^2$$

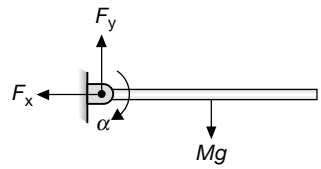


$$\Rightarrow \quad \omega = \sqrt{\frac{3g}{2l}} \qquad \ldots(2)$$

$$F_x = M\left(\frac{l}{2}\right)\omega^2 = M\left(\frac{l}{2}\right)\left(\frac{3g}{2l}\right) = \frac{3}{4}Mg$$

$$Mg - F_y = Ma_t = M(\alpha)\left(\frac{l}{2}\right)$$

$$\Rightarrow \quad F_y = Mg - \frac{3}{4}Mg = \frac{Mg}{4}$$



So, $\quad F = \sqrt{F_x^2 + F_y^2} = \frac{\sqrt{10}}{4}Mg$

$$\tan\alpha = \frac{F_y}{F_x} = \frac{\left(\frac{Mg}{4}\right)}{\left(\frac{3Mg}{4}\right)} = \frac{1}{3}$$

$$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{1}{3}\right)$$

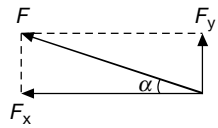
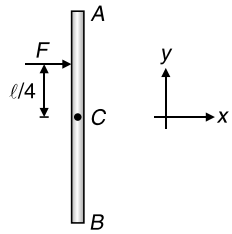


ILLUSTRATION 43

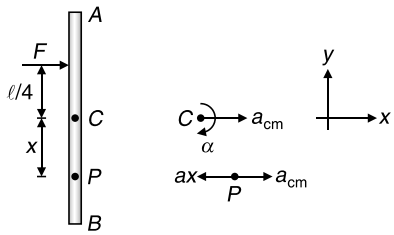
A thin uniform rod of mass m and length l is placed on smooth horizontal surface. It is acted upon by a force F as shown in Figure.



Calculate linear and angular acceleration of the rod. Find the location of a point on the rod which has zero acceleration.

SOLUTION

According to the problem, the external force is acting on rod along the x-direction, hence the centre of mass of the rod will accelerate along the x-direction.
Also, we observe that due to this force, a net torque will act on the rod in clockwise sense. Hence the motion of the rod will be combined rotational and translational.



According to Newton's Second Law for translational motion, we have

$$F = ma$$

$$\Rightarrow \quad a = \frac{F}{m} \qquad \ldots(1)$$

According to Newton's Second Law for rotational motion, we have

$$\tau = I_{cm}\alpha$$

$$\Rightarrow \quad F\left(\frac{l}{4}\right) = \frac{ml^2}{12}\alpha$$

$$\Rightarrow \quad \alpha = \frac{3F}{ml} \qquad \ldots(2)$$

Point of zero acceleration on the rod should be between C and B, as the vectors due to acceleration due to translation and acceleration due to rotation act in opposite directions as shown in Figure.

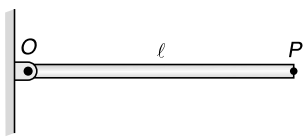
At point P, $a_P = 0$

$$\Rightarrow \quad a_{cm} - \alpha x = \frac{F}{m} - \left(\frac{3F}{ml}\right)x = 0$$

$$\Rightarrow \quad x = \frac{l}{3}$$

ILLUSTRATION 44

A rod of mass m and length l (hinged at O) is released from the rest from horizontal position as shown in Figure.

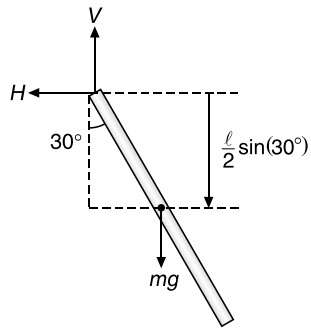


When the rod makes an angle of $30°$ with the vertical, calculate the angular acceleration, velocity of point P, acceleration of centre of mass and the horizontal component of force applied by hinge.

SOLUTION

Torque about hinged point is

$$\tau_{net} = \tau_{mg} = mg\left(\frac{l}{2}\sin 30°\right) = \frac{mgl}{4}$$



Since $\tau_{net} = I_O\alpha = \left(\frac{ml^2}{3}\right)\alpha$

$$\Rightarrow \quad \alpha = \frac{\tau}{I_O} = \frac{mgl/4}{ml^2/3} = \frac{3g}{4l} \qquad \ldots(1)$$

By conservation of mechanical energy, the loss in gravitational potential energy of the centre of mass of the rod equals the gain in rotational kinetic energy of the rod.

Loss of potential energy of the centre of mass of the rod is

$$-\Delta U = mg\left(\frac{l}{2}\right)\cos 30° = mgl\frac{\sqrt{3}}{4}$$

Gain in rotational kinetic energy of the rod is

$$K_R = \frac{1}{2}I_O\omega^2 = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$$\Rightarrow \quad \frac{mgl\sqrt{3}}{4} = \left(\frac{ml^2}{6}\right)\omega^2$$

$$\Rightarrow \quad \omega^2 = \frac{6g\sqrt{3}}{4l}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6\sqrt{3}g}{4l}}$$

$$\Rightarrow \quad v_p = l\omega = \sqrt{\frac{6\sqrt{3}gl}{4}}$$

The tangential acceleration of the centre of mass of the rod is

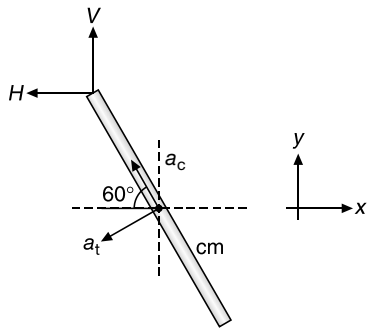
$$a_t = \frac{l}{2}\alpha = \left(\frac{l}{2}\right)\left(\frac{3g}{4l}\right) = \frac{3g}{8}$$

The centripetal acceleration of the centre of mass of the rod is

$$a_c = r\omega^2 = \left(\frac{l}{2}\right)\left(\frac{6\sqrt{3}g}{4l}\right) = \frac{3\sqrt{3}g}{4}$$

So, total acceleration of centre of mass of rod is

$$\Rightarrow \quad a_{cm} = \sqrt{a_c^2 + a_t^2} = \frac{g}{8}\sqrt{9+108} = \frac{g\sqrt{117}}{8}$$



To find the x and y components of $\vec{a}_{cm}$, we know that $\vec{a}_{cm} = \vec{a}_c + \vec{a}_t$, where

$$\vec{a}_c = a_c\left(-\cos 60°\hat{i} + \sin 60°\hat{j}\right)$$

$$\Rightarrow \quad \vec{a}_c = \frac{3\sqrt{3}g}{8}\left(-\hat{i} + \sqrt{3}\hat{j}\right)$$

and $\vec{a}_t = -a_t\left(\cos 30°\hat{i} + \sin 30°\hat{j}\right)$

$$\Rightarrow \quad \vec{a}_t = -\frac{3g}{16}\left(\sqrt{3}\hat{i} + \hat{j}\right)$$

$$\vec{a}_{cm} = \frac{3\sqrt{3}g}{8}\left(-\hat{i} + \sqrt{3}\hat{j}\right) - \frac{3g}{16}\left(\sqrt{3}\hat{i} + \hat{j}\right)$$

$$\Rightarrow \quad \vec{a}_{cm} = -\frac{1}{2}\left(a_c + a_t\sqrt{3}\right)\hat{i} + \frac{1}{2}\left(a_c\sqrt{3} - a_t\right)\hat{j}$$

$$\Rightarrow \quad \vec{a}_{cm} = -\frac{9\sqrt{3}g}{16}\hat{i} + \frac{15g}{16}\hat{j}$$

$$\Rightarrow \quad |\vec{a}_{cm}| = g\sqrt{\frac{243+225}{256}} = \frac{g\sqrt{117}}{8}$$

If horizontal component of force applied by the hinge is H and the vertical component force applied by the hinge is V, then from Newton's Second Law, we have

$$\vec{F}_{net} = \vec{m}a_{cm}$$

$$\Rightarrow \quad -H\hat{i} + V\hat{j} - mgj = m\left(\frac{-9\sqrt{3}}{16}\hat{i} + \frac{15g}{8}\hat{j}\right)$$

$$\Rightarrow \quad -H\hat{i} + (V - mg)\hat{j} = \frac{-mg\sqrt{3}}{16}\hat{i} + \frac{15mg}{8}\hat{j}$$

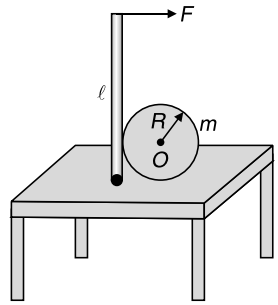
Comparing, we get

$$V - mg = \frac{15mg}{8}, \; H = \frac{mg\sqrt{3}}{16}$$

$$\Rightarrow \quad V = \frac{23mg}{8}, \; H = \frac{mg\sqrt{3}}{16}$$

ILLUSTRATION 45

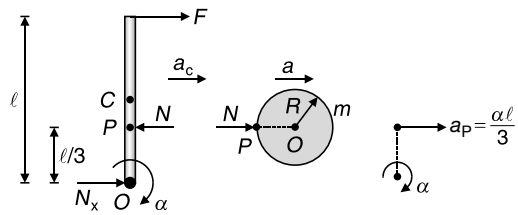
A uniform rod of mass m and length l which can rotate freely in vertical plane without friction, is hinged at its lower end on a table. A sphere of mass m and radius one third the length of the rod is placed in contact with the vertical rod and a horizontal force F is applied at the upper end of the rod as shown in Figure.



Calculate the acceleration of the sphere just after the force starts acting. Also, calculate the horizontal component of hinge reaction acting on the rod just after force F starts acting.

SOLUTION

The forces acting on the rod and the sphere are shown in Figure.



The torque equation for rod about O is

$$\tau = Fl - N\left(\frac{l}{3}\right) = I_O\alpha$$

$$\Rightarrow \quad Fl - \frac{Nl}{3} = \frac{ml^2}{3}\alpha$$

$$\Rightarrow \quad \frac{ml\alpha}{3} = F - \frac{N}{3} \qquad \ldots(1)$$

The motion of the sphere is translational only, so the acceleration (a) of the sphere equals the acceleration of point P, i.e.

$$a = \frac{l\alpha}{3} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$ma = F - \frac{N}{3} \qquad \ldots(3)$$

For sphere, $N = ma$ $\ldots(4)$

Substituting equation (3) in (4), we get

$$a = \frac{3F}{4m} \text{ and } N = \frac{3F}{4}$$

Acceleration of centre of mass of rod is

$$a_{cm} = \left(\frac{l}{2}\right)\alpha = \frac{3a}{2} = \frac{9F}{8m}$$

If the horizontal component of hinge reaction is N_x, then for the rod we have

$$F - \frac{3F}{4} + N_x = ma_{cm}$$

$$\Rightarrow \quad F - \frac{3F}{4} + N_x = m\left(\frac{9F}{8m}\right)$$

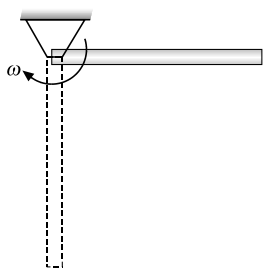
$$\Rightarrow \quad N_x = \frac{9F}{8} - \frac{F}{4} = \frac{7F}{8}$$

ILLUSTRATION 46

A rod of mass m and length l is hinged about its one of the ends. The rod is released from horizontal position. When the rod becomes vertical, find the angular speed of the rod and the hinge reaction

SOLUTION

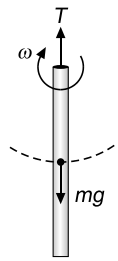
Applying conservation of mechanical energy, we see that the loss in gravitational potential energy equals gain in rotational kinetic energy.



$$\Rightarrow \quad mg\left(\frac{L}{2}\right) = \frac{1}{2}\left(\frac{mL^2}{3}\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3g}{L}}$$

Since the centre of mass of rod is moving in a circular path of radius $\frac{L}{2}$, so we have



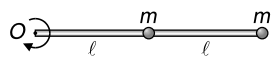
$$T - mg = m\omega^2\frac{L}{2}$$

$$\Rightarrow \quad T = mg + m\left(\frac{L}{2}\right)\left(\frac{3g}{L}\right)$$

$$\Rightarrow \quad T = 4mg$$

ILLUSTRATION 47

A light rod is connected rigidly with two identical particles each of mass m. The free end of the rod is smoothly pivoted at O. The rod is released from rest from its horizontal position at $t = 0$ as shown in Figure.

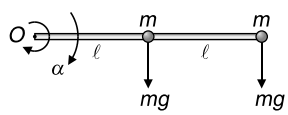


Calculate the initial angular acceleration of the rod and the initial reaction offered by the pivot.

SOLUTION

The net torque about O is

$$\tau_O = mgl + mg(2l) = 3mgl$$



This torque produces an angular acceleration α given by the relation

$$\alpha = \frac{\tau}{I_O}$$

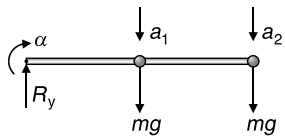
where, $I_O = ml^2 + m(2l)^2 = 5ml^2$

$$\Rightarrow \quad \alpha = \frac{3mgl}{5ml^2} = \frac{3g}{5l}$$

Initially just when the rod is released, then $\omega = 0$ but $\alpha = \frac{3g}{5l}$, so

$$R_x = ml\omega^2 + m(2l)\omega^2 = 0$$

The various forces acting on the rod are shown in Figure.



$$mg + mg - R_y = ma_1 + ma_2$$

where $a_1 = l\alpha$ and $a_2 = 2l\alpha$

$$\Rightarrow \quad R_y = m(2g - 3l\alpha)$$

Substituting $\alpha = \frac{3g}{5l}$, we get

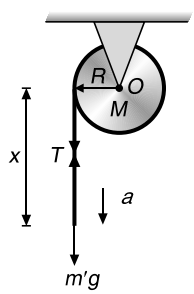
$$R_y = \frac{mg}{5}$$

ILLUSTRATION 48

A uniform cylinder of radius R and mass M can rotate freely about a stationary horizontal axis O. A thin cord of length l and mass m is wound on the cylinder in a single layer. Calculate the angular acceleration of the cylinder as a function of the length x of the hanging part of the cord. Assume that the wound part of cord has its centre of mass at the axis of the cylinder.

SOLUTION

At any instant, let m' be the mass of cord hanging from the cylinder as shown in Figure.



$$\Rightarrow \quad m' = \frac{m}{l}x$$

The mass of the cord wound on the perimeter of the cylinder will be

$$m'' = \frac{m}{l}(l - x)R^2$$

Let tension at the upper end of the cord in contact with the cylinder be T and its downward acceleration be a, then according to Newton's Second Law, we have

$$\left(\frac{m}{l}x\right)g - T = \left(\frac{m}{l}x\right)a \qquad \ldots(1)$$

When the cord accelerates downwards, the cylinder will be rotating with an angular acceleration

$$\alpha = \frac{a}{R}$$

The tension in the string provides necessary torque to the cylinder to rotate, so we have

$$\tau = TR = I\alpha \qquad \ldots(2)$$

where, I is the moment of inertia of the cylinder plus the part of cord wound on it, so

$$I = \frac{1}{2}MR^2 + m''R^2 = \frac{1}{2}MR^2 + \frac{m}{l}(l - x)R^2 \qquad \ldots(3)$$

Substituting equation (3) in (2), we get

$$TR = \left(\frac{1}{2}MR^2 + \frac{m}{l}(l - x)R^2\right)\alpha$$

$$\Rightarrow \quad T = \left(\frac{MR}{2} + \frac{m}{l}(l - x)R\right)\alpha \qquad \ldots(4)$$

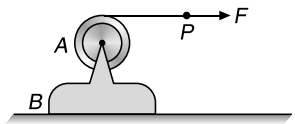
Substituting equation (4) in (1), we get

$$\left(\frac{m}{l}xg\right) - \left(\frac{MR}{2} + \frac{m}{l}(l - x)R\right)\alpha = \left(\frac{m}{l}x\right)R\alpha$$

$$\Rightarrow \quad \alpha = \frac{2mgx}{(2m + M)Rl}$$

ILLUSTRATION 49

A uniform solid cylinder A of mass m_1 can freely rotate about a horizontal axis fixed to a mount B of mass m_2. A constant horizontal force F is applied to end P of a light thread tightly wound on the cylinder as shown in Figure.



The friction between the mount and the supporting horizontal surface is supposed to be absent. Calculate the acceleration of this point P and the kinetic energy of this system t seconds after beginning of motion.

SOLUTION

Applying Newton's Second Law to the cylinder mount system, we get

$$F = (m_1 + m_2)a$$

$$\Rightarrow \quad a = \frac{F}{m_1 + m_2}$$

For cylinder, we have

$$\tau = FR = I\alpha = \left(\frac{1}{2}m_1R^2\right)\alpha$$

$$\Rightarrow \quad \alpha = \frac{2F}{m_1R}$$

Acceleration of point P is

$$a_P = a + R\alpha$$

$$\Rightarrow \quad a_P = \frac{F}{m_1 + m_2} + \frac{2F}{m_1}$$

$$\Rightarrow \quad a_P = \frac{F(3m_1 + 2m_2)}{m_1(m_1 + m_2)}$$

In t seconds, displacement of point P is

$$s = \frac{1}{2}a_Pt^2$$

The kinetic energy of system equals the work done by external force, so we get

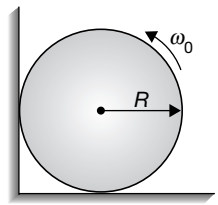
$$\Delta K = K - 0 = Fs$$

$$\Rightarrow \quad K = F\left[\frac{1}{2}\left(\frac{F(3m_1 + 2m_2)}{m_1(m_1 + m_2)}\right)t^2\right]$$

$$\Rightarrow \quad K = \frac{F^2t^2(3m_1 + 2m_2)}{2m_1(m_1 + m_2)}$$

ILLUSTRATION 50

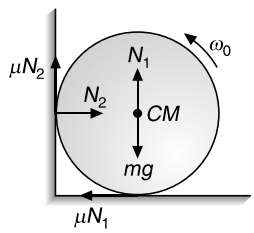
A uniform cylinder of mass m, radius R is spun about its axis to the angular velocity ω_0 and then placed into a corner, as shown in Figure.



The coefficient of kinetic friction between the corner walls and the cylinder is μ. Calculate the normal reaction imparted by the wall to the cylinder, by the ground to the cylinder. Also calculate the angular acceleration of the cylinder and the number of turns accomplish by the cylinder before stopping.

SOLUTION

Let us draw the free body diagram for the situation shown.



Since the centre of mass of cylinder does not accelerate, so

$$N_2 - \mu N_1 = 0$$

Also, $N_1 + \mu N_2 - mg = 0$

$$\Rightarrow \quad N_2 = \mu N_1 \qquad \ldots(1)$$

$$N_1 + \mu N_2 = mg \qquad \ldots(2)$$

From (1) and (2), we get

$$N_1 = \frac{mg}{1+\mu^2} \text{ and } N_2 = \frac{\mu mg}{1+\mu^2}$$

So, $\tau_{cm} = (\mu N_1)R + (\mu N_2)R$

$$\Rightarrow \quad \tau_{cm} = \mu mgR\left(\frac{1+\mu}{1+\mu^2}\right)$$

$$\Rightarrow \quad \left(\frac{1}{2}mR^2\right)\alpha = \mu mgR\left(\frac{1+\mu}{1+\mu^2}\right)$$

$$\Rightarrow \quad \alpha = \left(\frac{2\mu g}{R}\right)\left(\frac{1+\mu}{1+\mu^2}\right)$$

$$\Rightarrow \quad \alpha = \frac{2\mu(1+\mu)g}{(1+\mu^2)R}, \text{ clockwise sense}$$

Now $n_0 = \dfrac{\theta}{2\pi}$

Since, $\omega^2 - \omega_0^2 = 2\alpha\theta$

$$\Rightarrow \quad 0^2 - \omega_0^2 = 2\left(\frac{-2\mu(1+\mu)g}{(1+\mu^2)R}\right)\theta$$

$$\Rightarrow \quad \theta = \frac{(1+\mu^2)R\omega_0^2}{4\mu(1+\mu)g}$$

$$\Rightarrow \quad n_0 = \frac{(1+\mu^2)R\omega_0^2}{8\pi\mu(1+\mu)g}$$

GENERAL MOTION OF A RIGID BODY

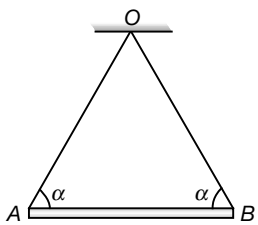
Till now, we have been learning ways to analyse the rotatory motion of a rigid body about a fixed axis of rotation. Now, let us describe the motion of a rigid body free to move in a plane. The basic laws of dynamics are applicable in this case also. You can use the following two results.

1. $\sum \vec{F}_{\text{net}} = m\vec{a}_{\text{cm}}$
2. $\sum \vec{\tau}_{\text{net}} = I\vec{\alpha}$

Please note that the second result must be applied about an axis taken through the centre of mass. This is so as the points on the body are accelerating and thus non-inertial. Although centre of mass also accelerates but the torque due to pseudo force about the centre of mass will be zero

ILLUSTRATION 51

A uniform rod AB of mass m and length $2l$ is suspended by two strings OA and OB of equal length attached to a fixed point O. The rod is at rest in a horizontal position and each string makes an angle α with the horizontal. If the string OB is cut, in what ratio the tension in OA will be instantaneously reduced?



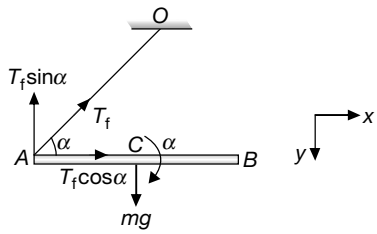
SOLUTION

Initially $2T_i \sin\alpha = mg$

$$\Rightarrow \quad T_i = \frac{mg}{2\sin\alpha} \qquad \ldots(1)$$

When the string OB is cut, let the tension in string OA be T_f, then equations of motion for centre of mass of rod are

$$a_x = \frac{T_f \cos\alpha}{m} \qquad \ldots(2)$$



$$a_y = \frac{mg - T_f \sin\alpha}{m} \qquad \ldots(3)$$

If α be the angular acceleration of the rod about its centre of mass C, then

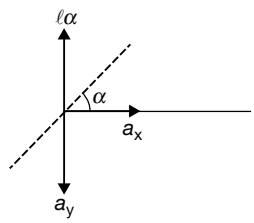
$$\alpha = \frac{(T_f \sin\alpha)l}{\frac{m(2l)^2}{12}} = \frac{3T_f \sin\alpha}{ml} \qquad \ldots(4)$$

At the instant OB is cut, point A moves perpendicular to OA, so acceleration of point A along AO should be zero, so

$$a_x \cos\alpha + l\alpha \sin\alpha = a_y \sin\alpha$$

Substituting the values, we get

$$\frac{T_f \cos^2\alpha}{m} + \frac{3T_f \sin^2\alpha}{m} = \frac{(mg - T_f \sin\alpha)\sin\alpha}{m}$$



Solving this equation, we get

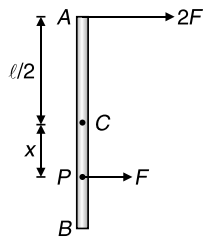
$$T_f = \frac{mg\sin\alpha}{\cos^2\alpha + 4\sin^2\alpha} \qquad \ldots(5)$$

From equations (1) and (5), we get

$$\frac{T_f}{T_i}=\frac{2\sin^2\alpha}{\cos^2\alpha+4\sin^2\alpha}=\frac{2\sin^2\alpha}{1+3\sin^2\alpha}$$

ILLUSTRATION 52

A uniform rod of mass m and length l is acted upon by the forces F and $2F$ as shown in Figure.



Calculate the linear and angular acceleration of the rod. Also find the value of x for which the point P does not accelerate.

SOLUTION

Applying Newton's Second Law for translational motion, we get

$$2F+F=ma$$

$$\Rightarrow \quad a=\frac{3F}{m} \qquad \ldots(1)$$

Applying Newton's Second Law for rotational motion to get the torque equation about centre of mass, we have

$$2F\left(\frac{l}{2}\right)-Fx=\left(\frac{ml^2}{12}\right)\alpha$$

$$\Rightarrow \quad \alpha=\frac{12F(l-x)}{ml^2} \qquad \ldots(2)$$

If point P does not accelerate, then we have

$$\vec{a}_P=\vec{a}_{P,C}+\vec{a}_C=\vec{0}$$

$$\Rightarrow \quad a_P=0=-\alpha x+a$$

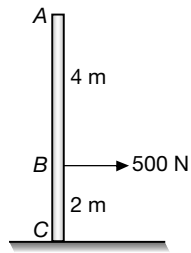
$$\Rightarrow \quad \alpha x=a$$

$$\Rightarrow \quad \frac{12F(l-x)x}{ml^2}=\frac{3F}{m}$$

$$\Rightarrow \quad x=\frac{l}{2}$$

ILLUSTRATION 53

The uniform 50 kg pole ABC is balanced in the vertical position. A 500 N horizontal force is suddenly applied at B. If the coefficient of kinetic friction between the pole and the ground is 0.3, determine the initial acceleration of point A. Take $g=10\ \text{ms}^{-2}$.



SOLUTION

$$N=mg=500$$

$$500-\mu N=ma_x$$

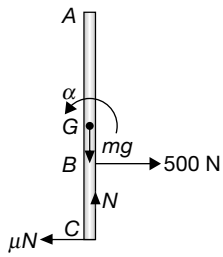
$$\Rightarrow \quad a_x=\frac{500-\mu N}{m}$$

$$\Rightarrow \quad a_x=\frac{(500)-(0.3)(500)}{50}=7\ \text{ms}^{-2}$$

Now, let us calculate torque due to forces about the centre of mass or centre of gravity G of the rod, then

$$\tau=(500)(1)-(0.3)(500)(3)$$

$$\Rightarrow \quad \tau=500-450=50\ \text{Nm}$$



Since $\alpha=\frac{\tau}{I}$

$$\Rightarrow \quad \alpha=\frac{50}{\frac{1}{12}(50)(6)^2}$$

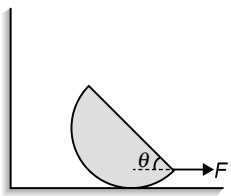
$$\Rightarrow \quad \alpha=\frac{1}{3}\ \text{rads}^{-2}$$

So, acceleration of point A is

$$a_A=a_x-3\alpha=7-(3)\left(\frac{1}{3}\right)=6\ \text{ms}^{-2}\ \text{(to the right)}$$

ILLUSTRATION 54

A hemisphere of radius r and mass M is pulled by means of a string (Figure) so that it moves with a uniform velocity. If μ is the coefficient of friction with the surface, find the angle of inclination of the hemisphere. Given that the centre of gravity of the hemisphere lies at a distance $\frac{3r}{8}$ from centre.

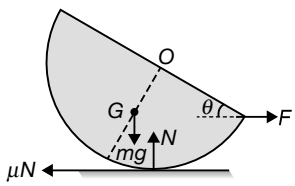


SOLUTION

The hemisphere is moving with uniform velocity. Hence, it is in equilibrium, both rotational as well as translational. So,

$$N = mg \qquad \ldots(1)$$

$$F = \mu N \qquad \ldots(2)$$



For rotational equilibrium about the bottom-most point,

$$\sum \tau_{\substack{\text{bottom-most}\\ \text{point}}} = 0$$

$$\Rightarrow \quad F(r - r\sin\theta) = mg\left(\frac{3r}{8}\sin\theta\right) \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$\sin\theta = \frac{8\mu}{3+8\mu}$$

ILLUSTRATION 55

A rough uniform rod, of mass m and length $4a$, is held on a horizontal table perpendicularly to an edge of the table, with a length $3a$ projecting horizontally over the edge. If the rod is released from rest and allowed to turn about the edge without slipping then find its angular speed after turning through an angle θ. Assuming that the rod has not started to slip, deduce an expression, in terms of θ, for the angular acceleration, and hence determine the reaction normal to the rod. Show that the rod begins to slip when $\tan\theta = \frac{4\mu}{13}$, where μ is the coefficient of friction.

SOLUTION

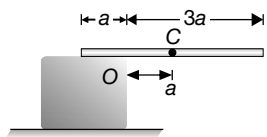
Let C be the centre of mass of rod then by Law of Conservation of Mechanical Energy, we have

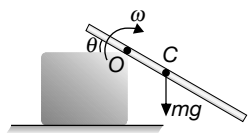
$$\frac{1}{2}I_0\omega^2 = mg(a\sin\theta)$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{m(4a)^2}{12} + ma^2\right)\omega^2 = mga\sin\theta$$

$$\Rightarrow \quad \frac{7}{6}a\omega^2 = g\sin\theta$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6g\sin\theta}{7a}}$$



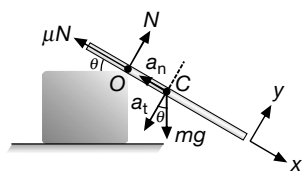


Since, $\alpha = \frac{\tau}{I} = \frac{mga\cos\theta}{\frac{7}{3}ma^2} = \frac{3g\cos\theta}{7a}$

Applying Newton's Second Law for translation motion along y-axis, we get

$$\Sigma F_y = ma_y$$

$$\Rightarrow \quad mg\cos\theta - N = ma_t$$



Since $a_t = r\alpha = a\alpha$

$$\Rightarrow \quad N = mg\cos\theta - ma_t = mg\cos\theta - m(a\alpha)$$

$$\Rightarrow \quad N = mg\cos\theta - \frac{3mg\cos\theta}{7} = \frac{4}{7}mg\cos\theta$$

Rod begins to slip when

$$\mu N - mg\sin\theta = ma_n$$

Since $a_n = r\omega^2 = a\omega^2$

$$\Rightarrow \quad \frac{4}{7}\mu mg\cos\theta - mg\sin\theta = m(a\omega^2)$$

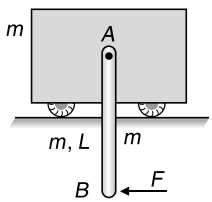
$$\Rightarrow \quad \frac{4}{7}\mu mg\cos\theta - mg\sin\theta = \frac{6mg\sin\theta}{7}$$

$$\Rightarrow \quad \frac{4}{7}\mu mg\cos\theta = \frac{13}{7}mg\sin\theta$$

$$\Rightarrow \quad \tan\theta = \frac{4\mu}{13}$$

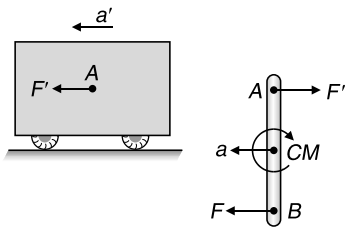
ILLUSTRATION 56

A uniform slender bar AB of mass m is suspended as shown from a small cart of the small cart of the same mass m. Neglecting the effect of friction, determine the accelerations of points A and B immediately after a horizontal force F has been applied at B.



SOLUTION

Let the acceleration of bar be a, cart be a' and α be the angular acceleration of the bar, then



for the bar, we have

$$F - F' = ma \qquad \ldots(1)$$

for the cart, we have

$$F' = ma' \qquad \ldots(2)$$

Applying the torque equation for rod about the centre of mass, we get

$$(F + F')\frac{L}{2} = \frac{ML^2}{12}\alpha$$

$$\Rightarrow \quad F + F' = \frac{ML\alpha}{6} \qquad \ldots(3)$$

The acceleration of point A on the rod will be same as the acceleration of the cart, so we have

$$\vec{a}_A = \vec{a}_{AC} + \vec{a}_C$$

$$\Rightarrow \quad -a' = \frac{\alpha L}{2} - a$$

$$\Rightarrow \quad a' = a - \frac{L\alpha}{2} \qquad \ldots(4)$$

Solving equations (1), (2), (3), and (4), we get

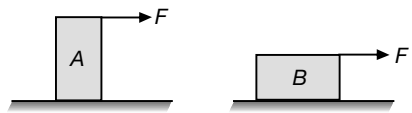
$$a = \frac{7F}{5m}, \ \alpha = \frac{18F}{5mL}$$

$$\Rightarrow \quad |a'| = |a_A| = a - \frac{L\alpha}{2} = -\frac{2F}{5m}$$

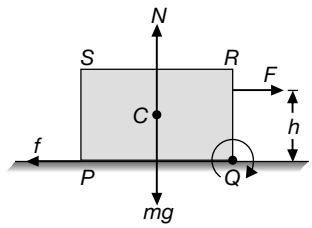
$$\Rightarrow \quad |a_B| = a + \frac{L\alpha}{2} = \frac{16F}{5m}$$

CONCEPT OF TOPPLING

In our practical life, we have observed that when a force F is applied on a block A of smaller width and greater height, it is more likely to topple down before sliding in comparison to another block B of broader base/bigger width and lower height where the chances of sliding are more compared to toppling.



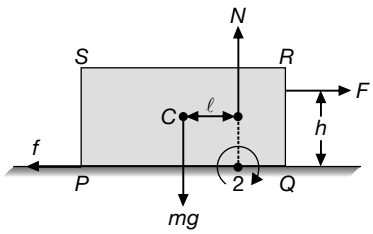
So, let us analyse these situations with the help of examples using basic concepts of torque. For this let us consider a block $PQRS$ having centre of mass at C.



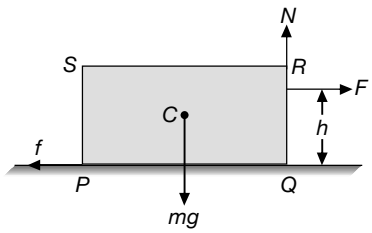
Let a force F be applied normally to face QR at height h above ground. Let the frictional force f be sufficient enough to prevent sliding. Now, if we say that the normal reaction also passes through C, then we have to face a problem that in spite of the fact that translational equilibrium exists i.e., $F = f$ and $N = mg$, we have an unbalanced CW torque due to

F about the point Q. This unbalanced CW torque has a tendency to topple the block about point Q. So, to cancel the effect of unbalanced torque due to F about the point Q, the normal reaction N shifts towards the right, say by l, such that

$$(mg)l = Fh$$



Now, if F or h (or both) are increased then correspondingly l also increases, however cannot go beyond the right side of the block (QR). So, in the extreme case the normal reaction passes through Q and beyond this the block will topple down.



Now, if F or h (or both) are further increased, then the block will topple down. It is due to this reason, that the block with broader base has less chances of toppling in comparison to a block of smaller base because the larger base has more margin to accommodate the shifting of the normal reaction.

Conceptual Note(s)

Why rolling is easy in comparison to sliding?

From arguments and reasons discussed, we observe that in case of a rolling body the normal reaction has a zero margin to shift. So, even if the body is in translational equilibrium $(F = f \text{ and } N = mg)$, still an unbalanced torque (due to F and f) is left behind that rotates the body in clockwise sense. The instant the body starts rolling, the force of friction gets adjusted in magnitude and direction such that pure rolling starts (if it is sufficient enough) or the body starts sliding if it is not sufficient enough).

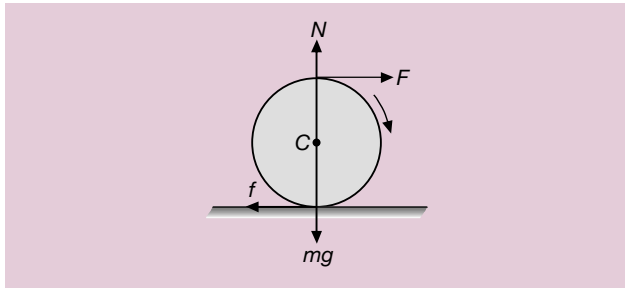


ILLUSTRATION 57

A uniform cylinder of height h and radius r is placed with its circular face on a rough inclined plane and the inclination of the plane to the horizontal is gradually increased. If μ is the coefficient of friction, then under what conditions the cylinder will

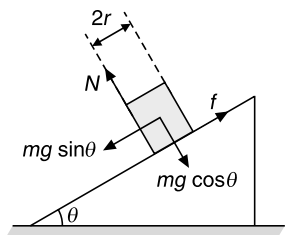
(a) slide before toppling
(b) topple before sliding.

SOLUTION

(a) The cylinder will slide if

$$mg\sin\theta > \mu mg\cos\theta$$

$$\Rightarrow \tan\theta > \mu \qquad \ldots(1)$$



The cylinder will topple if

$$(mg\sin\theta)\frac{h}{2} > (mg\cos\theta)r$$

$$\Rightarrow \tan\theta > \frac{2r}{h} \qquad \ldots(2)$$

Thus, the condition of sliding is $\tan\theta > \mu$ and condition of toppling is $\tan\theta > \frac{2r}{h}$. Hence, the cylinder will slide before toppling if

$$\mu < \frac{2r}{h}$$

(b) The cylinder will topple before sliding if

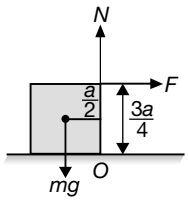
$$\mu > \frac{2r}{h}$$

ILLUSTRATION 58

A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point directly above the centre of the face, at a height $\frac{3a}{4}$ above the base. What is the minimum value of F for which the cube begins to topple about an edge?

SOLUTION

In the limiting case normal reaction will pass through O. The cube will topple about O if torque of F exceeds the torque of mg.



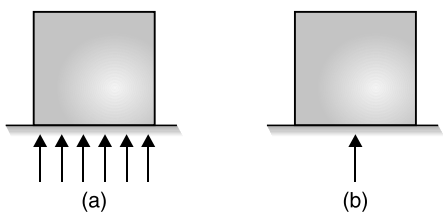
$$\Rightarrow \quad F\left(\frac{3a}{4}\right) > mg\left(\frac{a}{2}\right)$$

$$\Rightarrow \quad F > \frac{2}{3}mg$$

So, the minimum value of F is $\frac{2}{3}mg$

SHIFTING OF NORMAL REACTION AND TOPPLING

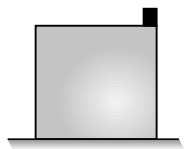
When a surface comes into contact with other surface then one surface applies normal reaction on other surface. The normal reaction does not act only at one point but it is distributed over the whole surface in contact. Consider a uniform block lying on a horizontal surface. Then the normal reaction is distributed uniformly over the surface as shown in figure (a). Now as the body is in translatory and rotatory equilibrium thus the net force and torque on the body should be zero.



To balance the torque and force of gravitational pull, normal reaction should have magnitude equal to the weight of the block and effectively normal reaction should pass through the centre of the block as shown in figure (b).

ILLUSTRATION 59

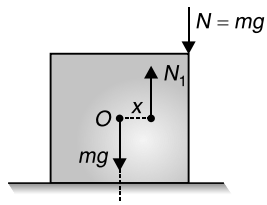
A uniform square block of side a and mass m is lying on a horizontal surface. A small block of mass m is placed on the top corner of the block. Find the distance by which normal reaction is shifted.



SOLUTION

For translational equilibrium of the system, we have

$$N_1 = 2mg$$



For rotational equilibrium of the system, we have

$$\tau_0 = \left(N_1 x - mg\frac{a}{2}\right) = 0$$

$$\Rightarrow \quad x = \frac{\left(mg\frac{a}{2}\right)}{2mg} = \frac{a}{4}$$

Here the normal reaction is still distributed but now its distribution is non uniform as shown in Figure.

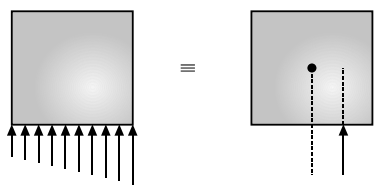


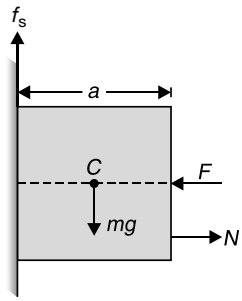
ILLUSTRATION 60

A cubical block of side a is held at rest, against a rough vertical wall by applying a force F acting along the centre. The mass of the block is m. Taking acceleration due to gravity as g, calculate the

(a) minimum coefficient of friction between block and the wall.
(b) torque by normal reaction about centre of mass.

SOLUTION

The situation is shown in the Figure.



Let C be the centre of mass of the block. For equilibrium of the block, we have

$$\Sigma F_x = 0 \quad \Rightarrow \quad F - N = 0 \qquad \ldots(1)$$

$$\Sigma F_y = 0 \quad \Rightarrow \quad f_s = mg \qquad \ldots(2)$$

$$\Sigma \tau = 0 \quad \Rightarrow \quad -f_s\left(\frac{a}{2}\right) + \tau_N = 0 \qquad \ldots(3)$$

While writing equation (3), we have taken clockwise torque to be negative.
Please note that the normal reaction N due to the wall on the block does not pass through the centre C.

$\Rightarrow \quad N = F$ and $f_s = mg$

(a) Since $f_s \le \mu N$

$\Rightarrow \quad mg \le \mu F$

$\Rightarrow \quad \mu \ge \dfrac{mg}{F}$

(b) $-f_s\left(\dfrac{a}{2}\right) + \tau_N = 0$

$\tau_N = f_s \times \dfrac{a}{2} = mg\dfrac{a}{2}$ (in counter clockwise sense)

TOPPLING

Consider a square plate shown in figure. A force is applied at the top as shown in figure. Now normal reaction will shift to balance the torque produced by F and friction about centre of mass. Now if force F increases then normal will shift further but shifting of normal has a limit. It cannot shift beyond point A. Now if F is further increased then normal cannot shift further to balance the torque. Hence now the block will start rotating about point A. This is called toppling.

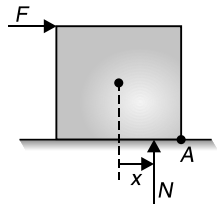


ILLUSTRATION 61

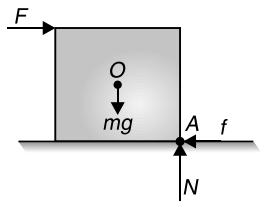
A square plate of side a and mass m is lying on a horizontal floor. A force F is applied at the top. Find the maximum force that can be applied on the square plate so that the plate does not topple about A.

SOLUTION

For translational equilibrium of the system, we have

$N = mg$

$f = F$



For rotational equilibrium of the system, we have

$$\tau_0 = 0 = F\frac{a}{2} + f\frac{a}{2} - N\frac{a}{2}$$

$\Rightarrow \quad Fa = mg\dfrac{a}{2}$

$\Rightarrow \quad F = \dfrac{mg}{2}$

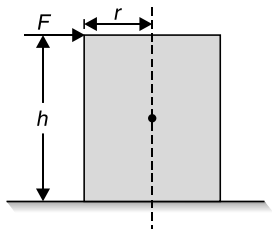
We can also think that when the body is in complete equilibrium, then net torque will be zero about any point taken. So net torque about the point A is zero.

$\Rightarrow \quad Fa = mg\dfrac{a}{2}$

$\Rightarrow \quad F = \dfrac{mg}{2}$

ILLUSTRATION 62

A force F is applied at the top most point of a cylinder of radius r and height h as shown in Figure.



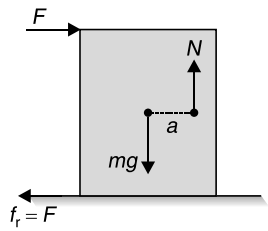
(a) If the cylinder remains at rest, then find distance of line of action of normal reaction from centre of mass.

(b) Find coefficient of friction required so that the cylinder topples before sliding.

SOLUTION

(a) Since body is at rest, it is in translational equilibrium

$$f_r = F$$



For rotational equilibrium,

$$\tau_{net} = 0 \qquad \text{(about centre)}$$

$$\Rightarrow \frac{Fh}{2} + \frac{Fh}{2} - Na = 0 \qquad \{\because f_r = F\}$$

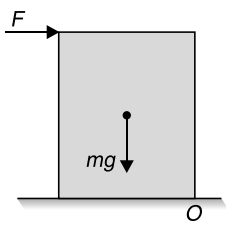
$$\Rightarrow a = \frac{Fh}{N} = \frac{Fh}{mg}$$

(b) For toppling about point O

$$\tau_F > \tau_{mg}$$

$$\Rightarrow Fh > mgr$$

$$\Rightarrow F > \frac{mgr}{h}$$



To avoid sliding, $F < \mu mg$

$$\Rightarrow \frac{mgr}{h} < F < \mu mg$$

$$\Rightarrow \frac{r}{h} < \mu$$

WORK DONE BY A TORQUE

If a torque $\vec{\tau}$ rotates a body through infinitesimal displacement $d\vec{\theta}$ then the infinitesimal work done is

$$dW = \vec{\tau} \cdot d\vec{\theta}$$

If $\vec{\tau}$ and $d\vec{\theta}$ are in the same direction, then

$$dW = \tau\, d\theta$$

$$\Rightarrow W = \int dW = \int_{\theta_1}^{\theta_2} \tau\, d\theta$$

If a constant torque τ acts on the body, then

$$W = \tau(\theta_2 - \theta_1)$$

$$\Rightarrow W = \tau \Delta\theta$$

STRING CONSTANT (TORSIONAL CONSTANT) AND ENERGY STORED IN A STRING

If a string is given an angular displacement θ (measured in radian) then the torsion produced tries to restore it to its original configuration and this torque (torsion produced) is directly proportional to θ. Mathematically,

$$\tau \propto \theta$$

$$\Rightarrow \tau = -C\theta$$

where, C is called the Torsional Constant or the String Constant of the string. *The negative sign shows that the torque is restoring in nature.*

If dW is the infinitesimal work done to give the string an angular displacement θ, then

$$dW = \tau\, d\theta \cos(180°) = -\tau\, d\theta$$

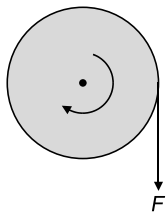
$$\Rightarrow W = \int dW = \frac{1}{2} C(\theta_2^2 - \theta_1^2)$$

If $\theta_1 = 0$ and $\theta_2 = \theta$, then

$$W = \frac{1}{2} C\theta^2$$

ILLUSTRATION 63

A uniform cylinder of mass M and radius R, initially at rest is mounted so as to rotate freely about a horizontal axis that passes through the central axis of the cylinder. A constant force F acts on the cylinder as shown in Figure.



Calculate the angular velocity of the cylinder as it rotates by an angle of π.

SOLUTION

Work done by a constant force F is $W = \int \tau d\theta$, where

$$\tau = Fr_{\perp} = FR$$

$$\Rightarrow \quad W = \int FRd\theta = FR\int_0^{\pi} d\theta = FR\pi$$

According to Work-Energy theorem, we have

$$W = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

$$\Rightarrow \quad \frac{1}{2}I\omega_f^2 = FR\pi \qquad \{\text{cylinder was initially at rest}\}$$

$$\Rightarrow \quad \omega_f = \sqrt{\frac{2FR\pi}{I}} = \sqrt{\frac{2FR\pi}{MR^2/2}} = \sqrt{\frac{4FR\pi}{MR^2}}$$

$$\Rightarrow \quad \omega_f = \sqrt{\frac{4F\pi}{MR}}$$

POWER

The rotational power or simply the **Power** is defined as the rate at which work is done by a torque. The SI unit of power is watt and here too $1\ \text{W} = 1\ \text{Js}^{-1}$

Average Power $P_{\text{av}} = \dfrac{W}{t} = \tau\left(\dfrac{\theta}{t}\right) = \tau\omega_{\text{av}}$

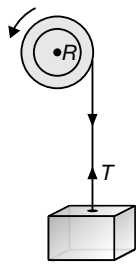
Instantaneous Power $P_{\text{ins}} = \dfrac{dW}{dt} = \vec{\tau}\cdot\dfrac{d\vec{\theta}}{dt} = \vec{\tau}\cdot\vec{\omega}$

ILLUSTRATION 64

A motor rotates a pulley of radius $25\ \text{cm}$ at $20\ \text{rpm}$. A rope around the pulley lifts a $50\ \text{kg}$ block, as shown in Figure. What is the power output of the motor?

SOLUTION

The tension in the rope is equal to the weight since there is no acceleration. Thus, $T = 500\ \text{N}$.



Therefore, $\tau = TR = (500)(0.25) = 125\ \text{Nm}$.

Angular velocity, $\omega = \dfrac{2\pi N}{60}$

$$\omega = \frac{2\pi(20)}{60} = \frac{2\pi}{3}\ \text{rads}^{-1}$$

The power required is

$$P = \tau\omega = \left(125\ \text{Nm}^{-1}\right)\left(\frac{2\pi}{3}\ \text{rads}^{-1}\right) = 260\ \text{W}$$

WORK-ENERGY PRINCIPLE

According to this theorem, work done by a torque equals the change in rotational kinetic energy.

i.e. $\quad W = \tau\theta = \dfrac{1}{2}I\omega^2 - \dfrac{1}{2}I\omega_0^2$

So, in complete analogue to the Work Energy Theorem studied in Translational Motion, we can say that the net rotational work done by the forces is equal to the change in rotational kinetic energy of the body.

$$W_{\text{rot}} = \Delta K_{\text{rot}}$$

For a rolling body, we have, work done equals the change in total energy possessed by the rolling body. So,

$$W_{\text{total}} = \Delta K = \left(\frac{1}{2}mv^2 - \frac{1}{2}mu^2\right)\left(1 + \frac{k^2}{R^2}\right)$$

MODIFIED NEWTON'S SECOND LAW FOR FIXED AXIS ROTATION

Since torque is a rotational analogue of force, therefore, Newton's Second Law for rotational motion is given by

$$\tau_{net} = I_{cm}\alpha = I_{cm}\left(\frac{a_{cm}}{R}\right)$$

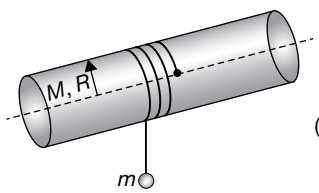
It is valid in two situations.

(a) The axis is fixed in position and direction.
(b) The axis passes through the centre of mass and is fixed in direction only.

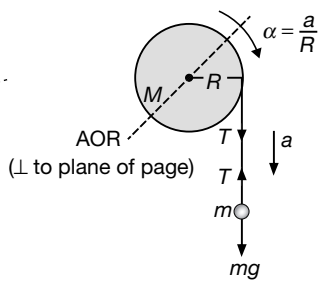
The equation $\tau_{cm} = I_{cm}\alpha_{cm}$ is valid even if the centre of mass is accelerating.

MOTION OF A POINT MASS ATTACHED TO A CYLINDER WITH A THREAD

Consider a point mass m attached to a thread wound over a cylinder of radius R, mass M, moment of inertia $I\left(=\frac{1}{2}MR^2\right)$ and radius of gyration k. The point mass ascends down with acceleration a and the cylinder rotates while the thread unwinds. The tension in the thread provides necessary torque to the cylinder to rotate with angular acceleration α. So,



Front view



Cross Sectional View

$$\tau = TR\sin 90 = TR$$

Further $\tau = I\alpha$

$$\Rightarrow \quad TR = I\alpha$$

$$\Rightarrow \quad TR = I\left(\frac{a}{R}\right) \qquad \left\{\because \alpha = \frac{a}{R}\right\}$$

$$\Rightarrow \quad T = \frac{Ia}{R^2} \qquad \ldots(1)$$

Further,

$$mg - T = ma$$

$$\Rightarrow \quad mg - \frac{Ia}{R^2} = ma$$

$$\Rightarrow \quad mg = \left(\frac{I}{R^2} + m\right)a$$

$$\Rightarrow \quad a = \frac{g}{\left(1 + \frac{I}{mR^2}\right)} = \frac{g}{\left(1 + \frac{M}{2m}\right)}$$

Put this value of a in (1), we get

$$T = \frac{mg}{\left(1 + \frac{mR^2}{I}\right)} = \frac{mg}{\left(1 + \frac{2m}{M}\right)}$$

Special Case

If both the cylinder and the point mass possess equal mass, then $m = M$ and hence,

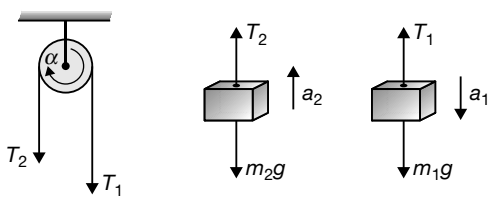
$$T = \frac{mg}{3} \text{ and } a = \frac{2g}{3}$$

ATWOOD'S MACHINE

The free body diagrams of the pulley and the blocks are shown in Figure. Note that tensions on two sides of the pulley are different. Why? Applying Newton's Second Law on the pulley, we get

$$\tau = T_1R - T_2R = (T_1 - T_2)R$$

Since, $\tau = I\alpha$



Therefore, $(T_1 - T_2)R = I\alpha$

$$\Rightarrow \quad T_1 - T_2 = \frac{Ia}{R^2} \qquad \ldots(1)$$

Applying Newton's Law on the blocks, we get

$$T_2 - m_2g = m_2a \qquad \ldots(2)$$

$$m_1g - T_1 = m_1a \qquad \ldots(3)$$

Solving equations (1), (2), (3), we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2 + \frac{I}{R^2}} \right) g$$

If the pulley had no mass, then

$$I = \frac{1}{2} MR^2 \longrightarrow 0$$

$$a \cong \left(\frac{m_1 - m_2}{m_1 + m_2} \right) g$$

For pulley (generally assumed to be a disc or a cylinder)

$$I = \frac{1}{2} MR^2$$

$$\Rightarrow \quad a = \left(\frac{m_1 - m_2}{m_1 + m_2 + \frac{M}{2}} \right) g$$

APPLICATION OF NEWTON'S SECOND LAW IN ROLLING MOTION

(a) **Ignoring Rotation,** write $F_{net} = Ma_{cm}$ for the object as if it were a point–mass.

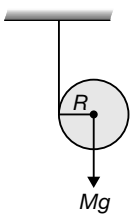
(b) **Ignoring Translation,** write $\tau = I_{cm}\alpha$ as if the object were only rotating about the centre of mass.

(c) Use of no slip condition i.e. $v_{cm} = R\omega$ or $a_{cm} = R\alpha$

(d) Solve the resulting equations simultaneously for any unknown.

ILLUSTRATION 65

A solid cylinder of mass M has a string wrapped several times around its circumference. The free end of string is attached to the ceiling and the cylinder is released from rest. Find the acceleration of the cylinder and the tension in the string.



SOLUTION

$$Mg - T = Ma \qquad \ldots(1)$$

Also, $\tau = TR = I\alpha$

$$\Rightarrow \quad T = \frac{I\alpha}{R} \text{ where } I = \frac{1}{2} MR^2$$

$$\Rightarrow \quad T = \frac{MR\alpha}{2} \qquad \ldots(2)$$

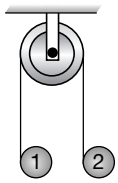
Condition of no slipping, $a = \alpha r$ $\ldots(3)$

Solving equation (1), (2) and (3) we obtain

$$a = \frac{2g}{3} \text{ and } T = \frac{Mg}{3}$$

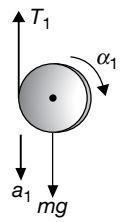
ILLUSTRATION 66

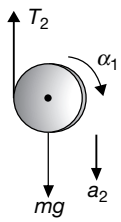
A thread is wound around two discs on either side. The pulley and the two discs have the same mass and radius. There is no slipping at the pulley and no friction at the hinge. Find out the accelerations of the two discs and the angular acceleration of the pulley.



SOLUTION

Let R be the radius of the discs and T_1 and T_2 be the tensions in the left and right segments of the rope.





Acceleration of disc 1,

$$a_1 = \frac{mg - T_1}{m} \qquad \ldots(1)$$

Acceleration of disc 2,

$$a_2 = \frac{mg - T_2}{m} \qquad \ldots(2)$$

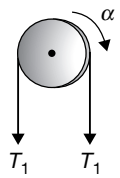
Angular acceleration of disc 1,

$$\alpha_1 = \frac{\tau}{I} = \frac{T_1 R}{\frac{1}{2}mR^2} = \frac{2T_1}{mR} \qquad \ldots(3)$$

Similarly, angular acceleration of disc 2,

$$\alpha_2 = \frac{2T_2}{mR} \qquad \ldots(4)$$

where, both α_1 and α_2 are clockwise.



Angular acceleration of pulley,

$$\alpha = \frac{(T_2 - T_1)R}{\frac{1}{2}mR^2} = \frac{2(T_2 - T_1)}{mR} \qquad \ldots(5)$$

For no slipping, $R\alpha_1 - a_1 = a_2 - R\alpha_2 = R\alpha$ $\ldots(6)$

Solving these equations, we get

$$\alpha = 0 \text{ and } a_1 = a_2 = \frac{2g}{3}$$

OBSERVATION

Since, both the discs are in identical situation, $T_1 = T_2$ and $\alpha = 0$, i.e., each of the discs falls independently and identically.

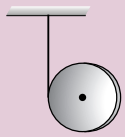
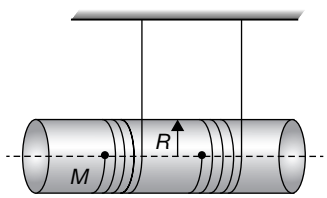


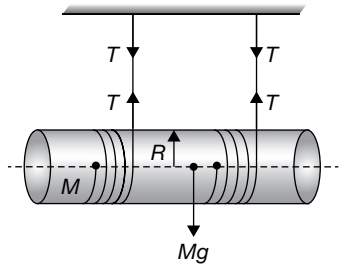
ILLUSTRATION 67

Find the acceleration and tension in the system shown. Assume that the cylinder remains straight and the strings are same.



SOLUTION

The forces acting on the system are shown in Figure.



Applying Newton's Second Law for rotational motion, we get

$$2(TR) = I\alpha$$

$$\Rightarrow \quad 2T = \frac{Ia}{R^2} \qquad \ldots(1)$$

Applying Newton's Second Law for translational motion, we get

$$Mg - 2T = Ma$$

$$\Rightarrow \quad Mg - \frac{Ia}{R^2} = Ma$$

$$\Rightarrow \quad a = \frac{Mg}{\left(M + \frac{I}{R^2}\right)}$$

Since, $I = \frac{1}{2}MR^2$

$$\Rightarrow \quad a = \frac{Mg}{M + \frac{M}{2}} = \frac{2g}{3}$$

Put this value of a in (1), we get

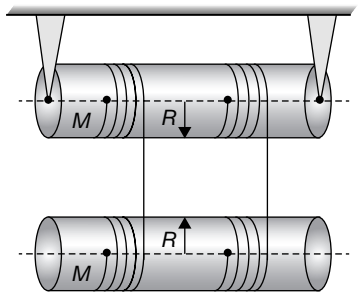
$$2T = \frac{I}{R^2}\left(\frac{2g}{3}\right)$$

$$\Rightarrow \quad 2T = \frac{M}{2}\left(\frac{2g}{3}\right) \qquad \left\{\because I = \frac{1}{2}MR^2\right\}$$

$$\Rightarrow \quad T = \frac{Mg}{6}$$

ILLUSTRATION 68

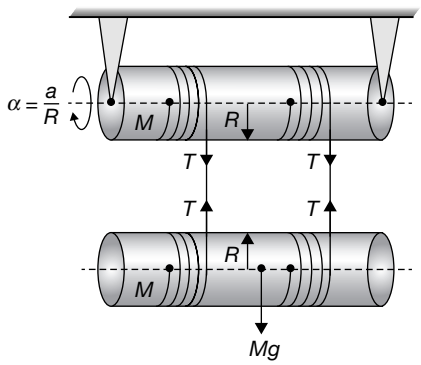
The arrangement shown in Figure consists of two identical uniform solid cylinders, each of mass M, radius R on which two light threads are wound symmetrically. Find the tension of each thread in the process of motion. The friction in the axle of the upper cylinder is assumed to be absent.



SOLUTION

If a be the acceleration at which the thread from the upper cylinder is released and A be the total acceleration of the lower cylinder, then

$$A = \begin{pmatrix}\text{Acceleration}\\ \text{of } CM\\ \text{of lower}\\ \text{cylinder}\end{pmatrix} + \begin{pmatrix}\text{Acceleration at}\\ \text{which thread}\\ \text{is being released}\\ \text{by upper cylinder}\end{pmatrix}$$



$$\Rightarrow \quad A = R\alpha + R\alpha = 2a = 2R\alpha$$

Applying Newton's Second Law for rotational motion, we get

$$\tau = (2T)R = I\alpha$$

$$\Rightarrow \quad 2T = \frac{Ia}{R^2} \qquad \ldots(1)$$

Applying Newton's Second Law for translational motion, we get

$$Mg - 2T = M(A) = M(2a)$$

$$\Rightarrow \quad Mg - \frac{Ia}{R^2} = 2Ma$$

$$\Rightarrow \quad Mg - \frac{M}{2}a = 2Ma$$

$$\Rightarrow \quad Mg = \left(2M + \frac{M}{2}\right)a$$

$$\Rightarrow \quad a = \frac{2g}{5}$$

$$\Rightarrow \quad A = 2a = \frac{4g}{5} \text{ and } 2T = \frac{M}{2}\left(\frac{2g}{5}\right)$$

$$\Rightarrow \quad T = \frac{Mg}{10}$$

Test Your Concepts-IV

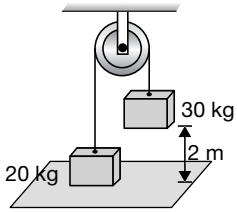
Based on Torque and Applications

(Solutions on page H.155)

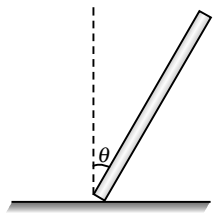
1. A uniform rod pivoted at its upper end hangs vertically. It is displaced through an angle of $60°$ and then released. Find the magnitude of the force acting on a particle of mass dm at the tip of the rod when the rod makes an angle of $37°$ with the vertical. Given, $\cos(37°) = \frac{4}{5}$ and $g = 10\ \text{ms}^{-2}$.

2. A thin horizontal uniform rod AB of mass m and length ℓ can rotate freely about a vertical fixed axis passing through its end A. At a certain moment the end B starts experiencing a constant force F which is always perpendicular to the original position of the stationary rod and directed in the horizontal plane. Find the angular velocity of the rod as a function of its rotation angle ϕ counted relative to the initial position.

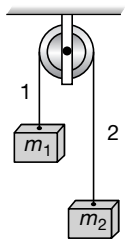
3. The system in Figure is released from rest. The $30\ \text{kg}$ body is $2\ \text{m}$ above the floor and is connected through an ideal string passing over the pulley (a uniform disk with a radius of $10\ \text{cm}$ and mass $5\ \text{kg}$) to another body of mass $20\ \text{kg}$. Find the speed of the $30\ \text{kg}$ body just before it hits the floor and the angular speed of the pulley at that time, the tensions in the strings and the time it takes for the $30\ \text{kg}$ body to reach the floor.



4. A slender rod of mass m and length ℓ is released from rest at $\theta = 0°$. Calculate the normal and frictional forces, which are exerted on the rod by the ground as it falls downward as a function of θ. Is it possible for the rod to slip as it falls? Give reason in support of your answer.



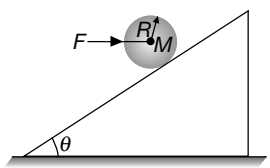
5. In the arrangement shown in Figure the mass of the uniform solid cylinder of radius R is equal to m and the masses of two bodies are equal to m_1 and m_2. The thread slipping and the friction in the axle of the cylinder are supposed to be absent. Find the angular acceleration of the cylinder and the ratio of tensions $\frac{T_1}{T_2}$ of the vertical sections of the thread in the process of motion.



6. A horizontal force F is applied to the sphere shown in Figure. Calculate the

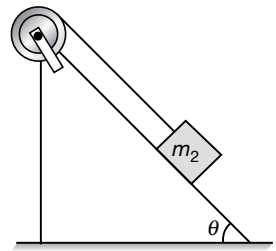
(a) value of F required to hold the sphere in equilibrium.

(b) frictional force of the incline on the sphere.



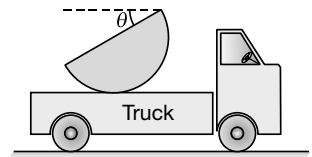
7. A uniform solid cylinder of mass m and radius a is free to rotate about its axis which is smooth and vertical. A light inextensible string is wrapped around the cylinder and its free end is pulled horizontally with a constant force $2mg$. Calculate the angular velocity of the cylinder when the free end of the string has moved through a distance $4a$.

8. A uniform cylinder of mass m_1 and radius R is pivoted on frictionless bearings. A string wrapped around the cylinder connects to a mass m_2, which is on a frictionless incline of angle θ, as shown in Figure.

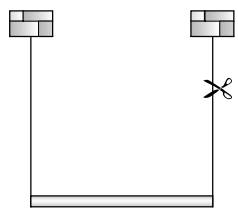


This system is released from rest with m_2 at a height h above the bottom of the incline. Find the acceleration of m_2, tension in the string, total energy of the system when m_2 is at height h, total energy when m_2 reaches bottom of the incline with speed v and the speed v.

9. A solid hemisphere rests on a truck as shown in Figure. If the coefficient of friction μ is just sufficient to prevent slipping, calculate the acceleration of the truck.

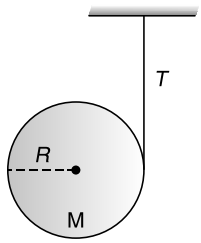


10. A uniform rod of mass m and length ℓ is held horizontally by two vertical strings of negligible mass, as shown in Figure. Immediately after the right string is cut, find the



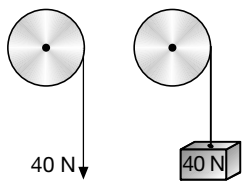
(a) linear acceleration of the free end of the rod.
(b) linear acceleration of the centre of mass of rod.
(c) tension in the left string.

11. A uniform cylinder of mass M and radius R has a string wrapped around it. The string is held fixed and the cylinder falls vertically, as shown in Figure. Calculate the acceleration of the cylinder and tension in the string.



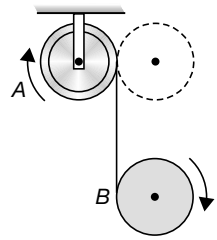
12. A uniform solid cylinder of mass m and radius a starts rotating from rest freely about its axis of symmetry under the action of a constant torque $4mga$. Find the angular velocity of the cylinder at $t = 4s$ from the start.

13. A cord is wrapped around the rim of a solid cylinder of radius 0.25 m and a constant force of 40 N is exerted on the cord as shown in Figure.

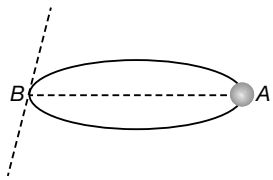


If the cylinder is mounted on frictionless bearings and moment of inertia of cylinder is 4 kgm^2, then
(a) use the work-energy principle to calculate the angular velocity of the cylinder after 5 m of cord have been unwound.
(b) if 40 N force is replaced by a 40 N weight, calculate the angular velocity of cylinder after 5 m of cord has unwound.
Take $g = 10$ ms^{-2}.

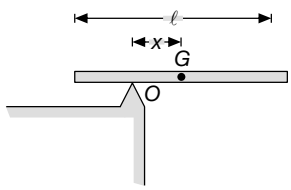
14. Two identical uniform discs A and B each of mass m and radius R are held, as shown in Figure with the help of a long massless string which is wrapped around the discs in opposite directions. Disc A is attached to the ceiling in such a way that it can rotate freely about its axis. The disc, B, initially held at same height as A, is then released to fall so that string unwinds from both the discs. Find the angular and linear accelerations of falling disc and tension in the string. Assume that string does not slip and motion is confined in the same vertical plane.



15. A ring of mass m and radius r has a particle of mass m attached to it at a point A. The ring can rotate about a smooth horizontal axis which is tangential to the ring at a point B diametrically opposite to A. The ring is released from rest when AB is horizontal. Find the angular velocity and the angular acceleration of the body when AB has turned through an angle $\frac{\pi}{3}$.



16. A uniform slender bar is released from rest in the horizontal position as shown in Figure. For what value of x the angular acceleration is maximum? Determine the corresponding angular acceleration α.



ACCELERATED PURE ROLLING

Till now we had been discussing uniform pure rolling where v and ω are constants. However, if an external force is applied to a rigid body, then the motion no longer is uniform pure rolling. Now for pure rolling on a stationary rough ground we must have

$$v = R\omega \qquad \ldots(1)$$

Taking derivative on both sides of (1) w.r.t. time t, we get

$$\frac{dv}{dt} = R\left(\frac{d\omega}{dt}\right)$$

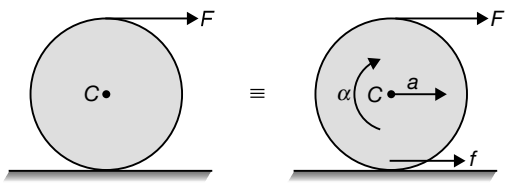
$$\Rightarrow \quad a = R\alpha \qquad \ldots(2)$$

So, in addition to $v = R\omega$ to be obeyed at every instant of time, the relation $a = R\alpha$ is also obeyed for pure rolling to take place and this is called Accelerated Pure Rolling.

Problem Solving Technique(s)

Here friction plays an important role in maintaining the pure rolling. The friction sometimes may act backward and sometimes may act forward or under certain conditions may also be zero, because the basic nature of friction is to self-adjust (up to a certain maximum limit) and it has a tendency to stop the relative motion between the two surfaces / bodies in contact.

Let us now have a more clear understanding of this concept by applying a force F to the top most point of a rigid body of mass M, radius R and moment of inertia I about an axis passing through its centre of mass, as shown in Figure.



Now, this applied force F produces in the body a/an

(a) linear acceleration (a) and

(b) angular acceleration (α)

Now three cases arise

(i) If $a = R\alpha$, then no need of friction and $f = 0$.

(ii) If $a < R\alpha$, then to support the linear motion and to oppose the angular motion, friction f acts forwards (along a).

(iii) If $a > R\alpha$, then to support the angular motion and to oppose the linear motion, friction f acts backwards (opposite to a).

So, we observe that the friction f can be zero, forward or backward depending upon value of I, M and R.
Applying Newton's Second Law for linear motion, we get

$$F + f = Ma \qquad \ldots(3)$$

Applying Newton's Second Law for rotational motion, we get

$$\tau = I\alpha$$

$$\Rightarrow \quad FR(CW) + fR(CCW) = I\alpha$$

$$\Rightarrow \quad FR - fR = I\alpha \qquad \ldots(4)$$

For pure rolling to take place, we have

$$a = R\alpha$$

$$\Rightarrow \quad FR - fR = I\left(\frac{a}{R}\right)$$

$$\Rightarrow \quad F - f = \left(\frac{I}{R^2}\right)a \qquad \ldots(5)$$

From (3) and (5), we get

$$f = \left(\frac{MR^2 - I}{MR^2 + I}\right)F \qquad \ldots(6)$$

From (6), we observe that if we have

(i) $I = MR^2$, i.e., a force F is applied at top of a ring, then $f = 0$ and the ring will roll without slipping.

(ii) $I < MR^2$ i.e., a force F is applied at top of a solid sphere, a shell, a cylinder or a disc, then f is positive i.e., f acts forwards.

(iii) If $I > MR^2$ i.e., f acts backwards. Although for the axis passing through CM, **we cannot have** $I > MR^2$.

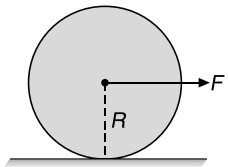
So, we conclude that f is either zero or forwards i.e., along F and so it supports linear motion.

ILLUSTRATION 69

A force F is applied at centre of a uniform round object of mass m radius R and moment of inertia about its centre of mass I_{cm}. Find (if $\frac{I_{cm}}{mR^2} = k$).

(a) Acceleration of centre of the round object if it rolls without slipping.

(b) Minimum coefficient of friction required so that the round object rolls without slipping.



SOLUTION

Equation for translatory motion

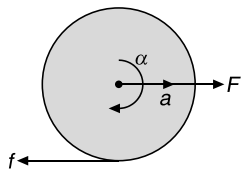
$$F - f = ma \quad \ldots(1)$$

Equation for rotatory motion

$$fR = I_{cm}\alpha \quad \ldots(2)$$

Condition for no slipping

$$a = R\alpha \quad \ldots(3)$$



By solving the equations

$$F - \frac{I_{cm}a}{R^2} = ma$$

$$a = \frac{F}{m + \frac{I_{cm}}{R^2}} = \frac{F}{m\left[1 + \frac{I_{cm}}{mR^2}\right]} = \frac{F}{m(1+k)}$$

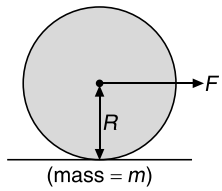
$$f = \frac{I_{cm}F}{mR^2(1+k)} = \frac{kF}{(1+k)}$$

Thus $\mu_{min}mg = \frac{kF}{1+k}$

$$\Rightarrow \quad \mu_{min} = \left(\frac{k}{1+k}\right)\frac{F}{mg}$$

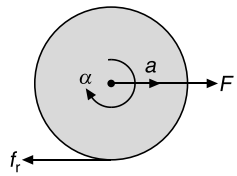
ILLUSTRATION 70

A force F is applied on a disc at its centre. Find acceleration of centre of mass in the case of pure rolling and also find minimum coefficient of friction required for pure rolling.



SOLUTION

Let the friction force acting on disc be f_r, acceleration of centre of mass is a and angular acceleration is α. We have three unknowns



(1) a **(2)** α **(3)** f_r

We require 3 equations to solve them

Translatory Motion

$$F - f_r = ma \quad \ldots(1)$$

Rotatory Motion

$$rf_r = \frac{mr^2}{2}\alpha \quad \ldots(2)$$

Condition of no slipping

$$a = r\alpha \quad \ldots(3)$$

By solving (2) and (3)

$$f_r = \frac{ma}{2}, \; F = \frac{3ma}{2}$$

$$\Rightarrow \quad a = \frac{2F}{3m}, \; f_r = \frac{m}{2} \times \frac{2F}{3m}, \; f_r = \frac{F}{3}$$

Now, $f_r \le \mu N$

$$\Rightarrow \quad \frac{F}{3} \le \mu mg$$

$$\Rightarrow \quad \mu \ge \frac{F}{3mg}$$

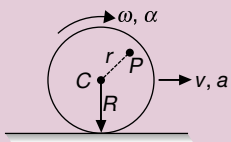
Problem Solving Technique(s)

(a) In accelerated pure rolling the velocity of the bottommost point is zero but despite the relation $a = R\alpha$, the acceleration of the bottommost point is not zero. Because acceleration of any point P can be given as

$$\vec{a}_P = \vec{a}_C + \vec{a}_{PC}$$

Here, $\vec{a}_{PC}$ has two components:

Tangential acceleration $a_t = r\alpha$ (which is perpendicular to CP) and radial or normal acceleration $a_n = r\omega^2$ (which is along PC)



Since, $\vec{a}_{PC} = \vec{a}_P - \vec{a}_C$

$\Rightarrow \vec{a}_P = \vec{a}_{PC} + \vec{a}_C$

$\Rightarrow \vec{a}_P = \vec{a}_C + (\vec{a}_{PC})_t + (\vec{a}_{PC})_n$

For the bottommost point, $\vec{a}_C + (\vec{a}_{PC})_t = 0$

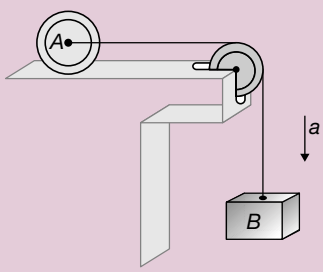
Because $\vec{a}_C = a$ {in forward direction}

and $(\vec{a}_{PC})_t = R\alpha$ {in backward direction}

and since, $a = R\alpha$ therefore, $\vec{a}_C + (\vec{a}_{PC})_t = 0$

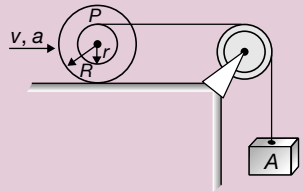
But $(\vec{a}_{PC})_n \neq 0$, because it is $R\omega^2$ towards centre. Thus, acceleration of bottommost point is $R\omega^2$ towards centre.

(b) If a_{cm} is acceleration of CM of A then acceleration of B is also $a_{CM} = a$.



(c) Similarly, in the problems like shown in Figure, it is wrong to say that acceleration of point P is equal to acceleration of block A. Although we can write,

$$a_A = a + r\alpha$$



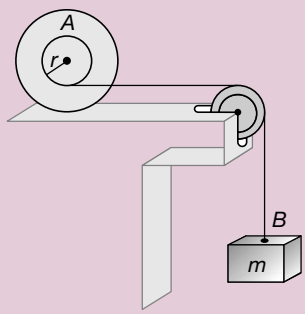
In case of pure rolling, problems can also be solved by using the energy conservation principle (provided no other dissipative forces are present). So, in this case we can also use the Law of Conservation of Energy to get

$$\begin{pmatrix}\text{Decrease}\\ \text{in GPE}\\ \text{of A}\end{pmatrix} = \begin{pmatrix}\text{Increase}\\ \text{in KE}\\ \text{of A}\end{pmatrix} + \begin{pmatrix}\text{Increase in}\\ \text{KE of Rolling}\\ \text{of Spool}\end{pmatrix}$$

(d) If a_{CM} is acceleration of CM of spool, then acceleration of B is

$$a_B = \begin{pmatrix}\text{Acceleration}\\ \text{of CM}\\ \text{of spool}\end{pmatrix} + \begin{pmatrix}\text{Acceleration at}\\ \text{which thread is}\\ \text{being released}\\ \text{from spool}\end{pmatrix}$$

$$a_B = a_{CM} + r\alpha$$



If $a_{CM} = a$

$a_B = a + r\alpha$

In this particular case,

$a_B = -a + r\alpha$

{CM of spool moves backwards}

(e) In cases where pulley is having some mass and friction is sufficient enough to prevent slipping, the tension on two sides of the pulley will be different and rotational motion of the pulley is also to be considered.

(f) At a given instant the value of ω for a rigid body will be same for every point.

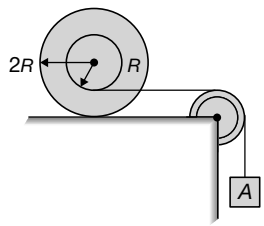
(g) The torque equation $(\tau = I\alpha)$ can be applied only about two points. These are,

(i) centre of mass

(ii) point about which body is in pure rotation.

ILLUSTRATION 71

A thin light thread is wound on a reel of mass 3 kg and moment of inertia 0.6 kgm^2. The inner radius is $R = 10$ cm and peripheral radius is $2R = 20$ cm. The reel is placed on a rough table and the friction is enough to prevent slipping. Find the acceleration of the centre of reel and of hanging mass of 1 kg.



SOLUTION

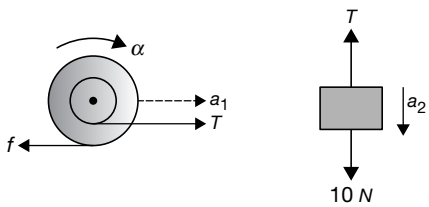
Let, a_1 = acceleration of centre of mass of reel
a_2 = acceleration of 1 kg block
α = angular acceleration of reel (clockwise)
T = tension in the string
and f = force of friction

Free body diagram of reel is as shown where, only horizontal forces are taken.
Equations of motion for reel are

$$T - f = 3a_1 \quad \ldots(1)$$

$$\alpha = \frac{\tau}{I} = \frac{f(2R) - TR}{I} = \frac{0.2f - 0.1T}{0.6} = \frac{f}{3} - \frac{T}{6} \quad \ldots(2)$$

Free body diagram of mass is also shown here.



Equation of motion for mass is

$$10 - T = a_2 \quad \ldots(3)$$

For no slipping condition, we have

$$a_1 = 2R\alpha$$

Since $2R = 0.2$ m

$$\Rightarrow \quad a_1 = 0.2\alpha \quad \ldots(4)$$

and $a_2 = a_1 - R\alpha$

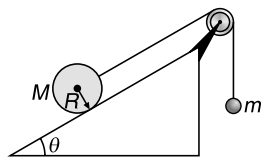
$$\Rightarrow \quad a_2 = a_1 - 0.1\alpha \quad \ldots(5)$$

Solving equations (1), (2), (3), (4) and (5), we get

$$a_1 = 0.27 \text{ ms}^{-2} \text{ and } a_2 = 0.135 \text{ ms}^{-2}$$

ILLUSTRATION 72

A uniform solid cylinder of mass M and radius R rolls a rough inclined plane with its axis perpendicular to the line of greatest slope as shown in Figure.



As the cylinder rolls it winds up a light string which passes over a light and smooth pulley and attached to a mass m, the part of the string between pulley and cylinder being parallel to the line of greatest slope. Prove that the tension in the string is

$$T = \left(\frac{(3 + 4\sin\theta)Mmg}{3M + 8m}\right)$$

SOLUTION

If cylinder rolls down at acceleration a, mass m goes up at acceleration $2a$ so we use

$$T - mg = m(2a) \quad \ldots(1)$$

$$Mg\sin\theta - T - f = Ma \quad \ldots(2)$$

$$fR - TR = \frac{1}{2}MR^2\left(\frac{a}{R}\right) \quad \ldots(3)$$

Adding equations (2) and (3), we get

$$Mg\sin\theta - 2T = \frac{3}{2}Ma$$

$$\Rightarrow \quad a = \frac{2}{3}g\sin\theta - \frac{4}{3}\frac{T}{M}$$

From equation (1), we use

$$T - mg = 2m\left(\frac{2}{3}g\sin\theta - \frac{4}{3}\frac{T}{M}\right)$$

$$\Rightarrow \quad T - mg = \frac{4}{3}mg\sin\theta - \frac{8m}{3M}T$$

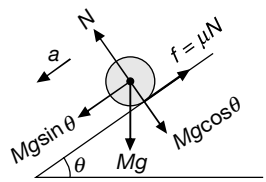
$$\Rightarrow \quad T\left(1+\frac{8m}{3M}\right) = mg\left(\frac{4}{3}\sin\theta + 1\right)$$

$$\Rightarrow \quad T = \frac{Mmg(4\sin\theta + 3)}{3M + 8m}$$

BODY ROLLING WITHOUT SLIPPING ON AN INCLINED PLANE

METHOD I: Using the concept of Newton's Second Law (for translation and rotation motion)

Consider a body of M radius R and moment of inertia I rolling without slipping on an inclined plane (making an angle θ with the horizontal). For the body to roll without slipping necessary friction (f) must be present. From the Figure we observe that all forces (other than friction) are acting along the radius and hence will not produce torque in the body.



However, friction being a tangential force will produce a torque τ given by

$$\tau = fR\sin 90$$

$$\Rightarrow \quad \tau = fR$$

Further

$$\tau = I\alpha$$

$$\Rightarrow \quad I\alpha = fR$$

$$\Rightarrow \quad I\frac{a}{R} = fR$$

$$\Rightarrow \quad f = \frac{Ia}{R^2} \qquad \ldots(1)$$

Further

$$Mg\sin\theta - f = Ma$$

$$\Rightarrow \quad Mg\sin\theta - \frac{Ia}{R^2} = Ma$$

$$\Rightarrow \quad Mg\sin\theta = \left(\frac{I}{R^2} + M\right)a$$

$$\Rightarrow \quad a = \frac{Mg\sin\theta}{\left(M + \frac{I}{R^2}\right)}$$

If k is the radius of gyration of the rolling body, then

$$I = Mk^2$$

$$\Rightarrow \quad a = \frac{g\sin\theta}{\left(1+\frac{k^2}{R^2}\right)} \qquad \ldots(2)$$

Put (2) in (1), we get

$$f = \frac{Mg\sin\theta}{\left(1+\frac{R^2}{k^2}\right)} \qquad \ldots(3)$$

METHOD II: Using the concept of IAOR

At the point of contact velocity is zero, so the net torque due to the various forces about the IAOR is

$$\tau = (Mg\sin\theta)R = I_{IAOR}\alpha$$

$$\Rightarrow \quad (\cancel{M}g\sin\theta)R = (\cancel{M}R^2 + \cancel{M}k^2)\frac{a}{R}$$

$$\Rightarrow \quad a = \frac{g\sin\theta R^2}{R^2 + k^2}$$

$$\Rightarrow \quad a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$$

$$\Rightarrow \quad f = \frac{Ia}{R^2} = \frac{(MR^2)}{R^2}\frac{g\sin\theta}{\left(1+\frac{k^2}{R^2}\right)}$$

$$\Rightarrow \quad f = \frac{Mg\sin\theta}{\left(1+\frac{R^2}{k^2}\right)}$$

If μ be the coefficient of friction, then

$$\mu = \frac{f}{N}, \text{ where } N = Mg\cos\theta$$

Using (3), we get

$$\mu = \frac{\tan\theta}{\left(1+\frac{R^2}{k^2}\right)} \qquad \ldots(4)$$

CONDITION FOR A BODY TO ROLL WITHOUT SLIPPING

For a body to roll without slipping, the force of friction f calculated above must be less than or equal to the maximum value of friction i.e. $\mu Mg\cos\theta$.

$$\Rightarrow \quad \frac{Mg\sin\theta}{\left(1+\frac{R^2}{k^2}\right)} \le \mu Mg\cos\theta$$

$$\Rightarrow \quad \mu \ge \frac{\tan\theta}{\left(1+\frac{R^2}{k^2}\right)}$$

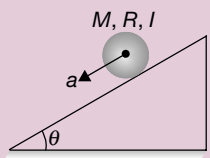
For various rolling bodies a, f and the condition to roll without slipping are shown in the table.
From table, we conclude

$$a_{SPHERE} > a_{DISC} = a_{CYLINDER} > a_{SHELL} > a_{RING}$$

Hence $t_{SPHERE} < t_{DISC} = t_{CYLINDER} < t_{SHELL} < t_{RING}$

i.e. the sphere reaches the first and the ring reaches the last down an incline (while rolling).

OBSERVATION



For a body rolling on a surface,

(a) if the surface is smooth, then, $a = g\sin\theta$ if surface is smooth

(b) if pure rolling takes place, then friction must be sufficient enough to prevent slipping, i.e. $\mu > \mu_{min}$ then

$$a = \frac{g\sin\theta}{\left(1+\frac{I}{MR^2}\right)} = \frac{g\sin\theta}{\left(1+\frac{k^2}{R^2}\right)}$$

and $f = \dfrac{mg\sin\theta}{\left(1+\frac{R^2}{k^2}\right)}$

(c) if the surface is rough, but friction is insufficient to prevent slipping i.e. $\mu < \mu_{min}$ then forward slipping will take place and maximum friction will act in this case and then the acceleration of the body is $a = g\sin\theta - \mu g\cos\theta$

The various parameters for bodies of different shapes are given in the Table.

Condition to Roll Without Slipping	Ring $\left(\frac{k^2}{R^2}=1\right)$	Disc $\left(\frac{k^2}{R^2}=\frac{1}{2}\right)$	Cylinder $\left(\frac{k^2}{R^2}=\frac{1}{2}\right)$	Shell $\left(\frac{k^2}{R^2}=\frac{2}{3}\right)$	Sphere $\left(\frac{k^2}{R^2}=\frac{2}{5}\right)$
$a=\frac{g\sin\theta}{\left(1+\frac{k^2}{R^2}\right)}$	$a=\frac{1}{2}g\sin\theta$	$a=\frac{2}{3}g\sin\theta$	$a=\frac{2}{3}g\sin\theta$	$a=\frac{3}{5}g\sin\theta$	$a=\frac{5}{7}g\sin\theta$
$f=\frac{Mg\sin\theta}{\left(1+\frac{R^2}{k^2}\right)}$	$f=\frac{1}{2}Mg\sin\theta$	$f=\frac{1}{3}Mg\sin\theta$	$f=\frac{1}{3}Mg\sin\theta$	$f=\frac{2}{5}Mg\sin\theta$	$f=\frac{2}{7}Mg\sin\theta$
Condition to Roll without Slipping is $\theta=\tan^{-1}\left[\mu\left(1+\frac{R^2}{k^2}\right)\right]$	$\theta=\tan^{-1}(2\mu)$	$\theta=\tan^{-1}(3\mu)$	$\theta=\tan^{-1}(3\mu)$	$\theta=\tan^{-1}\left(\frac{5\mu}{2}\right)$	$\theta=\tan^{-1}\left(\frac{7\mu}{2}\right)$

ILLUSTRATION 73

A shell is released from the top of an inclined plane of inclination θ. Calculate the minimum coefficient of friction between the shell and the plane to prevent sliding. Assuming the friction coefficient between the shell to be half the calculated minimum value, then calculate kinetic energy of the shell as it moves a length l down the incline.

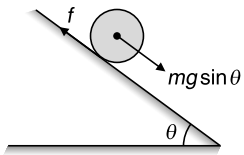
SOLUTION

For pure rolling of shell, we have

$$mg\sin\theta - f = ma_{cm}$$

where, $a_{cm} = \dfrac{g\sin\theta}{1+\left(k^2/R^2\right)}$

For hollow sphere $\dfrac{k^2}{R^2} = \dfrac{2}{3}$, so we get



$$mg\sin\theta - f = m\left(\frac{g\sin\theta}{1+(2/3)}\right)$$

$$\Rightarrow \quad f = \frac{2}{5}mg\sin\theta$$

For no sliding to take place, we must have

$$f < f_l\left(=\mu mg\cos\theta\right)$$

$$\Rightarrow \quad \frac{2}{5}mg\sin\theta < \mu mg\cos\theta$$

$$\Rightarrow \quad \mu > \frac{2}{5}\tan\theta$$

$$\Rightarrow \quad \mu_{min} = \frac{2}{5}\tan\theta$$

If $\mu = \dfrac{\mu_{min}}{2} = \dfrac{1}{5}\tan\theta$, then friction on shell is

$$f = \mu mg\cos\theta = \frac{1}{5}mg\sin\theta$$

Acceleration of shell is

$$a = g\sin\theta - \mu g\cos\theta = \frac{4}{5}g\sin\theta$$

Angular acceleration of shell is

$$\alpha = \frac{fR}{I} = \frac{\frac{1}{5}mgR\sin\theta}{\frac{2}{3}mR^2} = \frac{3}{10}\frac{g\sin\theta}{R}$$

Time of sliding is

$$t = \sqrt{\frac{2l}{a}} = \sqrt{\frac{5l}{2g\sin\theta}}$$

Speed attained by shell as it travels a distance l of inclined is

$$v = at$$

$$\Rightarrow \quad v = \sqrt{\frac{8}{5}gl\sin\theta}$$

Angular speed attained by shall as it travels a distance l

$$\omega = \alpha t$$

$$\Rightarrow \quad \omega = \frac{3g\sin\theta}{10R}\sqrt{\frac{5l}{2g\sin\theta}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6gl\sin\theta}{40R^2}}$$

Final kinetic energy of balls is

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

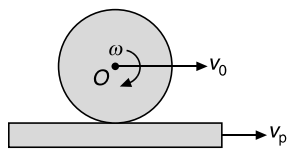
$$\Rightarrow \quad K = \frac{1}{2}m\left(\frac{8gl\sin\theta}{5}\right) + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{9gl\sin\theta}{40R^2}\right)$$

$$\Rightarrow \quad K = \left(\frac{4}{5}+\frac{3}{40}\right)gl\sin\theta$$

$$\Rightarrow \quad K = \frac{7}{8}mgl\sin\theta$$

ROLLING WHEEL ON A MOVABLE PLANK

Till now we have seen round objects rolling on a fixed surface. Surfaces being either horizontal or inclined but fixed. But what if the surfaces themself move.



Consider a round body of radius R moving on a plank. Let the centre of the round object has velocity v_0 and the round object has angular velocity ω. The plank is moving with a speed v_p as shown. Now for pure rolling, the two points in contact should have same velocity. Thus

$$v_0 - R\omega = v_p$$

and $$a_0 - R\alpha = a_p$$

where a_0 is acceleration of centre, α is angular acceleration and a_p is acceleration of plank.

For forward slipping, we have

$$v_0 - R\omega > v_p$$

$$a_0 - R\alpha > a_p$$

if v_0, ω and v_p satisfy the relation $v_0 - R\omega = v_p$.

For backward slipping, we have

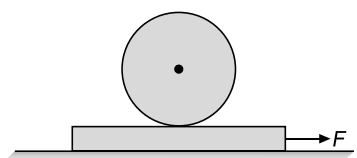
$$v_0 - R\omega < v_p$$

$$a_0 - R\alpha < a_p$$

if v_0, ω and v_p satisfy the relation $v_0 - R\omega = v_p$.

ILLUSTRATION 74

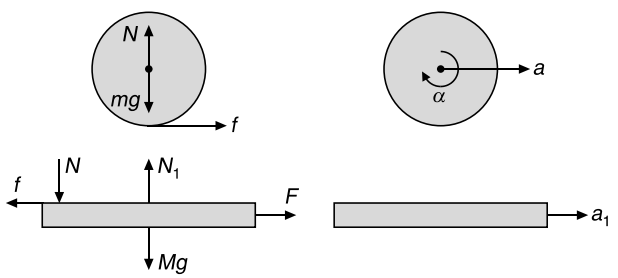
Consider a disc of mass m and radius R placed on a rough plank of mass M which in turn is placed on a smooth horizontal surface.



Now plank is pulled by a force F and disc starts to roll on the plank. If there is no friction anywhere then calculate the force of friction acting on the disc. Also calculate the angular acceleration of the disc.

SOLUTION

Let us apply Newton's laws on disc in horizontal direction



$$f = ma \qquad \ldots(1)$$

For plank, we have

$$F - f = Ma_1 \qquad \ldots(2)$$

Applying torque equation on disc about its centre of mass, we get

$$\tau = I\alpha$$

$$\Rightarrow \quad fR = \frac{mR^2}{2}\alpha \qquad \ldots(3)$$

For pure rolling, we have

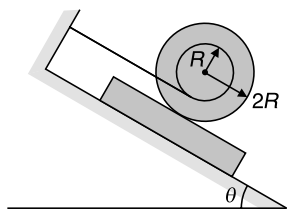
$$a_1 = a + R\alpha \qquad \ldots(4)$$

Solving the four equations, we get

$$f = \frac{Fm}{m+3M} \text{ and } \alpha = \frac{2F}{(m+3M)R}$$

ILLUSTRATION 75

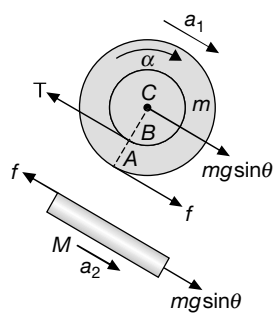
In the arrangement shown a spool of mass m, moment of inertia I is placed on a plank of mass M. There is no friction between the plank and the surface inclined at an angle θ. If friction between spool and plank is sufficient enough to prevent slipping, find the angular acceleration of the spool and the value of ratio $\left(\frac{m}{M}\right)$ for which plank will



(i) ascend
(ii) descend
(iii) will remain stationary

SOLUTION

The free body diagrams for spool and plank are shown here. According to Newton's Second Law, we have



for m, $mg\sin\theta + f - T = ma_1$ …(1)

for M, $Mg\sin\theta - f = Ma_2$ …(2)

Taking moment of forces about centre of spool, we get

$$T(R) - f(2R) = I\alpha \quad \ldots(3)$$

For no slipping at point B, we have

$$a_1 - R\alpha = 0 \quad \ldots(4)$$

For no slipping at point A, we have

$$a_1 - 2R\alpha = a_2 \quad \ldots(5)$$

From equations (4) and (5), we get

$$a_2 = -R\alpha \quad \ldots(6)$$

Solving above equations, we get

$$\alpha = \left(\frac{(m-M)g\sin\theta}{mR^2 + MR^2 + 1}\right)R$$

CASE-1:

For $m < M$ i.e., $\left(\frac{m}{M}\right) < 1$

$\alpha \rightarrow$ negative, $a_2 \rightarrow$ positive

So, the plank will descend.

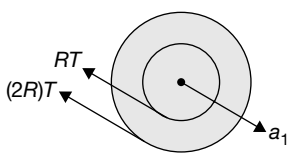
CASE-2:

For $m > M$ i.e., $\left(\frac{m}{M}\right) > 1$

α is positive, a_2 is negative

So, the plank will ascend.

CASE-3:



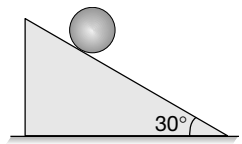
For $m = M$ i.e., $\frac{m}{M} = 1$,

$\alpha = 0$, $a_2 = 0$

So, the plank will remain stationary.

ILLUSTRATION 76

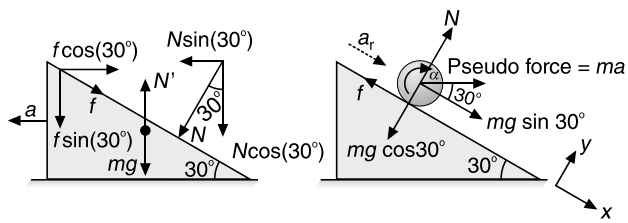
A sphere of mass 10 kg is placed on the inclined surface of a rough wedge of inclination $30°$. If the mass of the wedge is also 10 kg, calculate the acceleration of the wedge when sphere is allowed to roll down without slipping. Neglect friction between ground and wedge. Take $g = 10\ \text{ms}^{-2}$.



SOLUTION

When the sphere rolls down without slipping on the wedge, the wedge exerts a normal reaction N on sphere. Consequently, the sphere exerts the same force N on wedge as shown. Furthermore, if friction f on the sphere due to the wedge is tangentially upwards, then friction on wedge due to sphere acts tangentially downwards as shown. Also N' be the normal exerted on wedge by horizontal frictionless ground and mg be the weight of the wedge. Due to these forces, if a be the acceleration of wedge towards left a_r be the acceleration of centre of mass of sphere down the plane relative to wedge. α be the angular acceleration of the sphere.

Free body diagrams for the wedge with respect to ground and the sphere with respect to wedge are shown in Figure.



For wedge, the equations of motion are

$$N' = N\cos(30°) + mg + f\sin(30°)$$

$$N\sin 30° - f\cos 30° = ma$$

$$\Rightarrow \quad \frac{N}{2} - \frac{\sqrt{3}f}{2} = 10a$$

$$\Rightarrow \quad N - \sqrt{3}f = 20a \quad \ldots(1)$$

For sphere, the equations of motion are, $(m = 10\ \text{kg})$

$$\Sigma F_x = ma_r$$

$$\Rightarrow \quad mg\sin 30° + ma\cos 30° - f = ma_r$$

$$\Rightarrow \quad 50 + 5\sqrt{3}a - f = 10a_r \quad \ldots(2)$$

Also, $\Sigma F_y = 0$

$\Rightarrow \quad N + ma\sin 30° = mg\cos 30°$

$\Rightarrow \quad N + 5a = 50\sqrt{3}$ …(3)

Further, $\alpha = \dfrac{\tau}{I} = \dfrac{fR}{\frac{2}{5}mR^2} = \dfrac{f}{4R}$ …(4)

For no slipping between the wedge and the sphere, we have

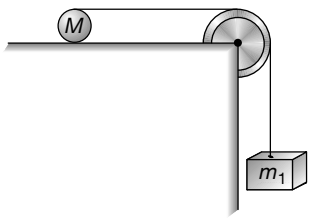
$a_r = R\alpha$ …(5)

Solving equations (1), (2), (3), (4) and (5), we get

$a = 2.11 \text{ ms}^{-2}$

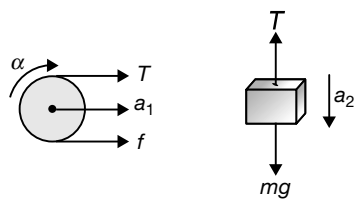
ILLUSTRATION 77

In the arrangement shown in Figure, the string is wrapped around a uniform cylinder which rolls without slipping. The other end of the string is passed over a light, frictionless pulley and is connected to a falling weight. Find the acceleration of the falling mass m in terms of only the mass of the cylinder M, the mass m and g.



SOLUTION

Let T be the tension in the string and f the force of (static) friction, between the cylinder and the surface. If a_1 be the acceleration of centre of mass of cylinder towards right a_2 be the downward acceleration of block m and α be the angular acceleration of cylinder (clockwise)



For block, $mg - T = ma_2$ …(1)

For cylinder, $T + f = Ma_1$ …(2)

$\alpha = \dfrac{(T - f)R}{\frac{1}{2}MR^2}$ …(3)

Since the string attached the mass m to the highest point of the cylinder, so

$v_m = v_{CM} + R\omega$

Differentiating we get

$a_2 = a_1 + R\alpha$ …(4)

For M to roll without slipping, we have

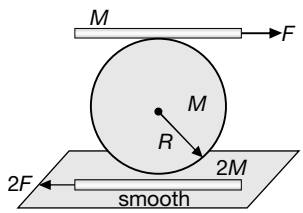
$a_1 = R\alpha$ …(5)

Solving equations (1), (2), (3), (4) and (5), we get

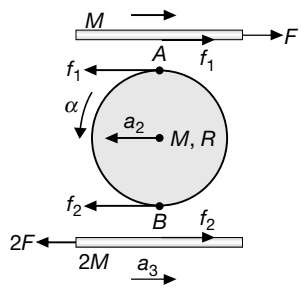
$a_2 = \dfrac{8mg}{3M + 8m}$

ILLUSTRATION 78

A cylinder of mass M, radius R is sandwiched between two planks having masses M and $2M$. Two constant horizontal forces F and $2F$ are applied on the planks as shown. Determine the acceleration of the centre of mass of cylinder, top plank and bottom plank if there is no slipping at the top and bottom of cylinder. Also find the friction between the planks and cylinder and the angular acceleration of the cylinder.



SOLUTION



Equations of motion

for plank M is, $F + f_1 = Ma_1$ …(1)

for cylinder M is, $f_1 + f_2 = Ma_2$ …(2)

for plank $2M$ is, $2F - f_2 = 2Ma_3$ …(3)

Further for the cylinder, we have

$$\alpha = \frac{(f_1 - f_2)R}{\frac{1}{2}MR^2}$$

$$\Rightarrow \quad \alpha = \frac{2(f_1 - f_2)}{MR} \qquad \ldots(4)$$

For no slipping condition, at the points of contact A and B, we have

$$a_2 + R\alpha = -a_1 \qquad \ldots(5)$$

and $$a_2 - R\alpha = a_3 \qquad \ldots(6)$$

We have six unknowns, f_1, f_2, a_1, a_2, a_3 and α. Solving the above six equations, we get

$$a_1 = \frac{21F}{26M},\ a_2 = \frac{F}{26M},\ a_3 = \frac{23F}{26M}$$

$$f_1 = -\frac{5F}{26},\ f_2 = \frac{3F}{13} \text{ and } \alpha = -\frac{11F}{13MR}$$

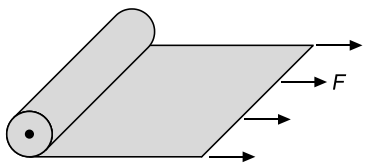
Negative sign with f_1 indicates its direction is opposite to the direction shown in Figure and negative sign with α shows it is in CW sense.

Test Your Concepts-V

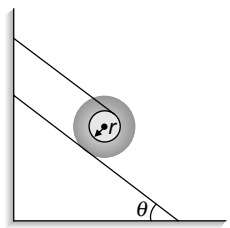
Based on Uniform and Accelerated Pure Rolling

(Solutions on page H.159)

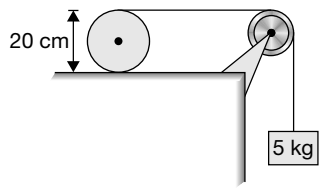
1. A heavy roll of wrapping paper of mass M, radius R, in the form of a solid cylinder is resting on a horizontal tabletop. If a horizontal force F is applied uniformly to the paper, as shown in Figure, find the linear acceleration of the centre of the roll and the angular acceleration around the centre of the roll. The coefficient of kinetic friction between the paper and the table is μ_K. Assume the roll of paper does slip on the surface.



2. A spool of thread of mass m is placed on an inclined smooth plane set at an angle θ to the horizontal. The free end of the thread is attached to the wall as shown in Figure. Find the acceleration of the centre of mass of the spool, if its moment of inertia about its axis is I and the radius of the wound thread layer is r.



3. In Figure the cylinder of mass 10 kg and radius 10 cm has a tape wrapped round it. The pulley weighs 100 N and has a radius 5 cm. When the system is released, the 5 kg mass comes down and the cylinder rolls without slipping. Find the acceleration and velocity of the mass as a function of time.



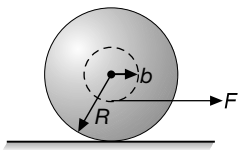
4. A lawn roller in the form of a thin-walled hollow cylinder of mass M is pulled horizontally with a constant horizontal force F applied by a handle attached to the axle. If it rolls without slipping, find the acceleration and the friction force.

5. A solid cylindrical wheel of mass M and radius R is pulled by a force F applied to the centre of the wheel at 37° to the horizontal. If the wheel is to roll without slipping, what is the maximum value of $|F|$? The coefficients of static and kinetic friction are $\mu_S = 0.40$ and $\mu_K = 0.30$. $\sin(37°) = \frac{3}{5}$.

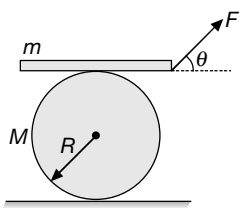
6. A sphere, a disc, and a hoop made of homogeneous materials have the same radius (10 cm) and mass (3 kg). They are released from rest at the top of a 30° incline and roll down without slipping through a vertical distance of 2 m.

(a) What are their speeds at the bottom?
(b) Find the frictional force f in each case.
(c) If they start together at $t = 0$, at what time does each reach the bottom?

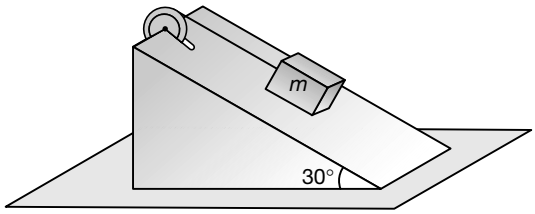
7. A Yo-Yo of mass M has an axle of radius b and a spool of radius R. Its moment of inertia can be taken to be $\frac{MR^2}{2}$. The Yo-Yo is placed upright on a table and the string is pulled with a horizontal force F as shown. The coefficient of friction between the Yo-Yo and the table is μ. What is the maximum value of F for which the Yo-Yo will roll without slipping?



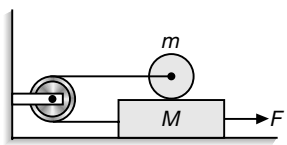
8. A cylinder of mass M and radius R is lying on a rough horizontal plane. It has a plank lying at its top as shown in Figure. A force F is applied on the plank such that the plank moves and causes the cylinder to roll. The plank always remains horizontal. There is no slipping at any point of contact. Find the acceleration of the cylinder and the frictional forces at the two contacts.



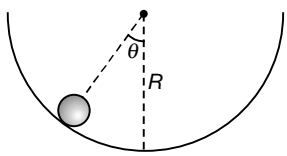
9. A block of mass $m = 1\,\text{kg}$ slides down the surface of a smooth incline as shown in Figure. The block is tied to a string which is wrapped around a disk capable of rotating about a horizontal axis. The disk has a mass $M = 5\,\text{kg}$ and a radius $R = 0.2\,\text{m}$. Initially the string is taut. If the mass is released, calculate its acceleration. Take $g = 9.8\,\text{ms}^{-2}$.



10. Find the acceleration of the cylinder of mass m and radius R and that of plank of mass M placed on smooth surface if pulled with a force F shown in Figure. Given that sufficient friction is present between cylinder and the plank surface to prevent sliding of cylinder.



11. A ball of mass m and radius r rolls along a circular path of radius R. Its speed at the bottom $(\theta = 0)$ of the path is v_0. Find the tangential and the normal force exerted by the path on the ball as a function of θ.



ANGULAR MOMENTUM ($\vec{L}$)

The rotational effect of linear momentum is called Angular Momentum. When a body rotates about some point/axis, then the momentum associated with the body due to its rotation is called Angular Momentum. An external torque is required to change angular momentum just like an external force is required to change the linear momentum.

CASE-1: Angular Momentum of a Particle About Some Point

Angular Momentum, $\vec{L}$ of a particle about an arbitrary point O is the moment of linear momentum taken about that point.

$$L = \begin{pmatrix}\text{Linear}\\ \text{Momentum}\end{pmatrix} \perp \begin{pmatrix}\text{Distance of}\\ \vec{p} \text{ from } O\end{pmatrix}$$

$$\Rightarrow \quad L = pr_{\perp}$$

$\Rightarrow \quad L = pr\sin\theta$ {in magnitude}

where $r_\perp = r\sin\theta$ is called the moment arm.

Vectorially

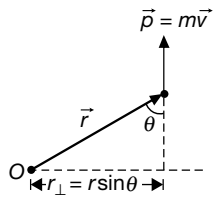
$$\vec{L} = \vec{r} \times \vec{p}$$

$\Rightarrow \quad \vec{L} = m(\vec{r} \times \vec{v})$

If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, then

$$\vec{L} = m\begin{vmatrix} \overset{\oplus}{\hat{i}} & \overset{\ominus}{\hat{j}} & \overset{\oplus}{\hat{k}} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix}$$

The direction of $\vec{L}$ is also found by Right Hand Thumb Rule.

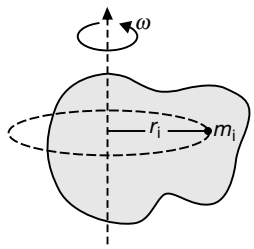


The SI unit of angular momentum is kgms^{-2}.

Please note that angular momentum is defined always with respect to a point.

CASE-2: Angular Momentum of a Rigid Body Rotating About a Fixed Axis

The total angular momentum $\vec{L}$ of a system of particles relative to a given origin is the sum of the angular momentum of the particles.



A rigid body rotating about a fixed axis

$$\vec{L} = \Sigma(\vec{r}_i \times \vec{p}_i)$$

Since $\vec{r}_i$ and $\vec{p}_i$ are perpendicular, so

$L = \Sigma r_i p_i$, where $p_i = m_i v_i = m_i r_i \omega$

$\Rightarrow \quad L = (\Sigma m_i r_i^2)\omega$

$\Rightarrow \quad L = I\omega$, where $I = \Sigma m_i r_i^2$

CASE-3: Angular Momentum of System in Combined Rotation and Translation

If a body is in combined rotation and translation like rolling and we are asked to find the angular momentum of the body about any fixed point or a reference point, then

$$\vec{L}_{\text{total}} = \vec{L}_{\text{CM}} + M(\vec{r}_{\text{CM}} \times \vec{v}_{\text{CM}})$$

i.e., total angular momentum of system/body is equal to the sum of the angular momentum of the CM about that point and the angular momentum of the system about the CM.

So, for a rigid body undergoing linear and rotational motion, the total angular momentum may be split into two parts

(a) the orbital angular momentum, $\vec{L}_O$ and

(b) the spin angular momentum $\vec{L}_{\text{cm}}$.

The orbital angular momentum is the angular momentum of the centre of mass motion about an origin O in an inertial frame.

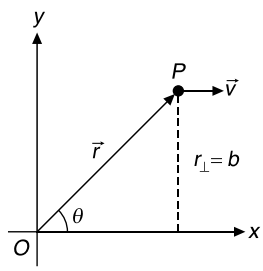
The spin angular momentum is the angular momentum relative to the centre of mass.

The orbital term treats the system as a point particle at the centre of mass, whereas the spin term is the sum of the angular momenta of the particles relative to the centre of mass. The total angular momentum relative to the origin O in an inertial frame is the sum of both the angular momenta i.e.,

$$\vec{L} = \vec{L}_{\text{Orbital}} + \vec{L}_{\text{Spin}} = \vec{L}_O + \vec{L}_{\text{cm}}$$

ILLUSTRATION 79

A particle of mass m is projected from origin O with speed u at an angle θ with positive x-axis. Positive y-axis is in vertically upward direction. Find the angular momentum of particle at any time t about O before the particle strikes the ground again.



SOLUTION

$$\vec{L} = m(\vec{r} \times \vec{v})$$

Here, $\vec{r}(t) = x\hat{i} + y\hat{j} = (u\cos\theta)t\hat{i} + \left(ut\sin\theta - \frac{1}{2}gt^2\right)\hat{j}$

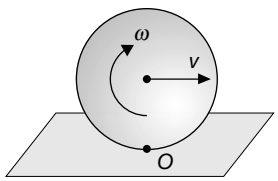
and $\vec{v}(t) = v_x\hat{i} + v_y\hat{j} = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$

$$\Rightarrow \quad \vec{L} = m(\vec{r} \times \vec{v}) = m\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (u\cos\theta)t & (u\sin\theta)t - \frac{1}{2}gt^2 & 0 \\ u\cos\theta & u\sin\theta - gt & 0 \end{vmatrix}$$

$$\Rightarrow \quad \vec{L} = -\frac{1}{2}m(u\cos\theta)gt^2\hat{k}$$

ILLUSTRATION 80

A solid sphere of mass M and radius R rolls without slipping on a horizontal surface as shown in Figure. Find the total angular momentum of the sphere with respect to the origin O fixed on the ground.



SOLUTION

Let us assume the clockwise sense of rotation positive.

Orbital angular momentum about O is $L_O = MvR$

Spin angular momentum about centre of mass is

$$L_{cm} = I\omega = \frac{2}{5}MR^2\omega$$

The total angular momentum is

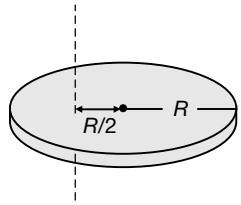
$$L = L_O + L_{cm} = MvR + \frac{2}{5}MvR$$

For pure rolling, we have $v = \omega R$

$$\Rightarrow \quad L = \frac{2}{5}MvR$$

ILLUSTRATION 81

A disc of mass M and radius R rotating at an angular velocity ω about an axis perpendicular to its plane at a distance $\frac{R}{2}$ from the centre, as shown in Figure. Calculate the angular momentum of the disc about the axis shown.



SOLUTION

The moment of inertia of the disc about the given axis may be found from the parallel axes theorem, equation $I = I_{cm} + Md^2$, where h is the distance between the given axis and a parallel axis through the centre of mass.

Here $h = \frac{R}{2}$, therefore, $I = \frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2 = \frac{3}{4}MR^2$

The angular momentum is $L = I\omega = \frac{3}{4}MR^2\omega$

RELATION BETWEEN $\vec{L}$ AND $\vec{\tau}$

As force changes the linear momentum of a particle, torque changes the angular momentum of a particle.

Since, $\vec{L} = \vec{r} \times \vec{p}$ …(1)

The rate of change of angular with time is found by taking the derivative on both sides of (1) with respect to time. So

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p})$$

Using, $\frac{d}{dt}(\vec{A} \times \vec{B}) = \vec{A} \times \frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt} \times \vec{B}$, we get

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

$$\Rightarrow \quad \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} + \vec{v} \times \vec{p} \qquad \left\{\because \frac{d\vec{p}}{dt} = \vec{F} \text{ and } \frac{d\vec{r}}{dt} = \vec{v}\right\}$$

$$\Rightarrow \quad \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} + m(\vec{v} \times \vec{v}) \qquad \{\because \ \vec{p} = m\vec{v}\}$$

Since, $\vec{v} \times \vec{v} = \vec{0}$

$$\Rightarrow \quad \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

$$\Rightarrow \quad \vec{\tau} = \frac{d\vec{L}}{dt}$$

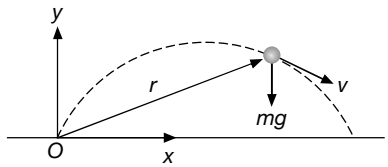
ILLUSTRATION 82

Show that the equation $\vec{\tau} = \frac{d\vec{L}}{dt}$, can be applied to the motion of a projectile.

SOLUTION

The change in angular momentum of the projectile is produced by the torque exerted by the force of gravity.

In Figure, we take the initial point as the origin. At a later time, $\vec{r} = x\hat{i} + y\hat{j}$.



Since the force on the particle is $\vec{F} = -mg\hat{j}$, the gravitational torque on it is

$$\vec{\tau} = \left(x\hat{i} + y\hat{j}\right) \times (-mg\hat{j}) = -mgx\hat{k}$$

The rate of change of the angular momentum $\vec{L} = \vec{r} \times \vec{p}$ is

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} = m\vec{r} \times \frac{d\vec{v}}{dt}$$

But the acceleration is $\frac{d\vec{v}}{dt} = -g\hat{j}$. So,

$$\frac{d\vec{L}}{dt} = m\vec{r} \times \frac{d\vec{v}}{dt} = m\left(x\hat{i} + y\hat{j}\right) \times \left(-g\hat{j}\right) = -\left(mgx\right)\hat{k}$$

Hence the equation $\vec{\tau} = \frac{d\vec{L}}{dt}$ is applicable here.

ANGULAR IMPULSE

In complete analogy with the linear momentum, angular impulse is defined as

$$\Delta\vec{L} = \int \vec{\tau}_{\text{ext}} dt$$

Since we know that linear impulse $\vec{J}$ equals the change in linear momentum, so we have

$$\vec{J} = \vec{F}\Delta t = \Delta\vec{p} = m\left(\vec{v}_f - \vec{v}_i\right)$$

In one dimension, we can simply write this as

$$J = \Delta p = p_f - p_i = m\left(v_f - v_i\right)$$

If $v_i = 0$ and $v_f = v$, then we have $J = mv$

$$\Rightarrow \quad v = \frac{J}{m}$$

Just like translational the impulse momentum theorem in translational motion, we have angular impulse angular momentum theorem in rotation according to which angular impulse equals the change in angular momentum of the body and hence

$$\text{Angular Impulse (AI)} = \tau\Delta t = \Delta L$$

$$\Rightarrow \quad \text{AI} = \tau\Delta t = \Delta L = L_f - L_i = I\left(\omega_f - \omega_i\right)$$

Since, $\tau = Fr_\perp$

$$\Rightarrow \quad \text{AI} = \left(Fr_\perp\right)\Delta t = Jr_\perp \qquad \{\because F\Delta t = J\}$$

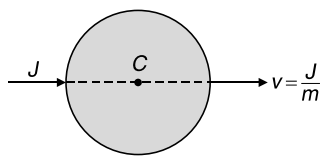
$$\Rightarrow \quad \text{AI} = Jr_\perp = L_f - L_i = I\left(\omega_f - \omega_i\right)$$

If $L_i = 0$, then $L_f = L = I\omega$

$$\Rightarrow \quad \text{AI} = Jr_\perp = I\omega$$

$$\Rightarrow \quad \omega = \frac{Jr_\perp}{I}$$

In Figure (a): A linear impulse J is applied at centre of mass C of the rigid body.

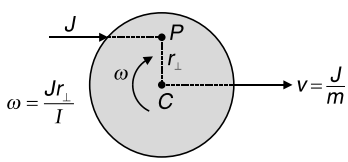


(a)

Just after hitting, it will have only translational motion and its linear velocity will be given by

$$v = \frac{J}{m}$$

In Figure (b): A linear impulse J is applied at point P, at a perpendicular distance $r_\perp = CP$.



(b)

Just after hitting it will have both translational and rotational motion. Its linear velocity v and angular velocity ω will be given by

$$v = \frac{J}{m} \text{ and } \omega = \frac{Jr_\perp}{I}$$

If $r_\perp$ is increased (keeping J to be constant) then v will remain same but ω will increase. So, the translational kinetic energy will have the same value but rotational kinetic energy will be more.

Problem Solving Technique(s)

Since, Angular Impulse (AI) $= \tau\Delta t = \Delta L$

Now, following three cases can be considered.

(a) If torque is constant, then angular impulse (AI) is obtained by directly multiplying this constant torque with the given time interval.

(b) If torque is a function of time, then angular impulse (AI) is given by

$$\text{Angular Impulse (AI)} = \int_{t_i}^{t_f} \tau dt$$

(c) If torque versus time graph is given, then angular impulse (AI) can be obtained by the area under that graph.

In all three cases, angular impulse is equal to the change in angular momentum.

ANGULAR IMPULSE-ANGULAR MOMENTUM THEOREM

This theorem is the rotational analogue of the Impulse Momentum theorem studied already in Newton's Laws of Motion. Using Newton's Second Law for Rotation Motion, we have

$$\vec{\tau}_{\text{ext}} = \frac{d\vec{L}}{dt}$$

$$d\vec{L} = \vec{\tau}dt$$

$$\Rightarrow \quad \Delta\vec{L} = \vec{L}_f - \vec{L}_i = I\left(\vec{\omega} - \vec{\omega}_0\right) = \int \vec{\tau}dt$$

The net angular impulse acting on a rigid body is equal to the change in angular momentum of the body. This is called the **Angular Impulse Angular Momentum** theorem for rotational dynamics.

Please be careful to understand and see the hidden fundamental of the axis of rotation i.e. all $\vec{L}_f$, $\vec{L}_i$ and $\vec{\tau}$ must be about identical AOR to use the results as they are. Else suitable modifications have to be made in the results.

LAW OF CONSERVATION OF ANGULAR MOMENTUM

Since, by definition we know that

$$\vec{\tau} = \frac{d\vec{L}}{dt} \qquad \ldots(1)$$

For no external torque acting on system, we have

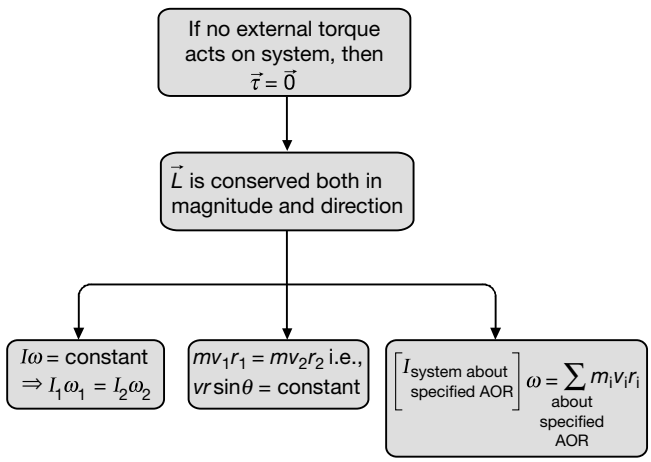
$$\vec{\tau} = \vec{0}$$

So, (1) gives

$$\vec{0} = \frac{d\vec{L}}{dt}$$

$\Rightarrow \quad \vec{L}$ = constant (both in magnitude and direction)

So, for no external torque acting on the system $\vec{L}$ is conserved both in magnitude and direction.
This law is the rotational analogue of the Law of Conservation of Linear Momentum.


If no external torque acts on system, then $\vec{\tau} = \vec{0}$
$\vec{L}$ is conserved both in magnitude and direction
$I\omega$ = constant $\Rightarrow I_1\omega_1 = I_2\omega_2$
$mv_1r_1 = mv_2r_2$ i.e., $vr\sin\theta$ = constant
$I_{\text{system about specified AOR}}\ \omega = \sum m_i v_i r_i$ about specified AOR

Analogy between Rotational and Linear Dynamics.

Quantity	Linear	Rotational
1. Inertia	m	$\sum m_i r_i^2$ or $\int r^2 dm$
2. Newton's Second Law	$F_{\text{ext}} = ma$ $\vec{F}_{\text{ext}} = \frac{d\vec{p}}{dt}$	$\tau_{\text{ext}} = I\alpha$ $\vec{\tau}_{\text{ext}} = \frac{d\vec{L}}{dt}$

(*Continued*)

Quantity	Linear	Rotational
3. Work	$W_{\text{lin}} = \int \vec{F} \cdot d\vec{l}$	$W_{\text{rot}} = \int \tau d\theta$
4. Kinetic Energy	$K_{\text{lin}} = \frac{1}{2} mv^2$	$K_{\text{rot}} = \frac{1}{2} I\omega^2$
5. Work Energy Theorem	$W_{\text{lin}} = \Delta K_{\text{lin}}$	$W_{\text{rot}} = \Delta K_{\text{rot}}$
6. Impulse	$I = \int F_{\text{ext}} dt = \Delta p$	$J = \int \tau_{\text{ext}} dt = \Delta L$
7. Momentum	$p = mv$	$L = I\omega$
8. Impulse Momentum Theorem	$\vec{I} = \Delta\vec{p}$	$\vec{J} = \Delta\vec{L}$
9. Power	$P = \vec{F} \cdot \vec{v}$	$P = \vec{\tau} \cdot \vec{\omega}$

ILLUSTRATION 83

A disc of moment of inertia $4\ \text{kgm}^2$ is spinning freely at $3\ \text{rads}^{-1}$. A second disc of moment of inertia $2\ \text{kgm}^2$ slides down the spindle and they rotate together.

(a) What is the angular velocity of the combination?
(b) What is the change in kinetic energy of the system?

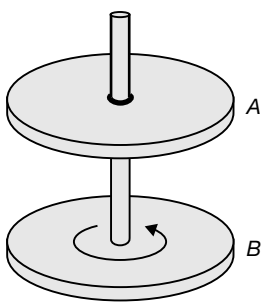
SOLUTION

(a) Since there are no external torques acting, we may apply the conservation of angular momentum.

$$I_f\omega_f = I_i\omega_i$$

$$(6\ \text{kgm}^2)\omega_f = (4\ \text{kgm}^2)(3\ \text{rads}^{-1})$$

Thus, $\omega_f = 2\ \text{rads}^{-2}$



(b) The kinetic energies before and after the collision are

$$K_i = \frac{1}{2} I_i\omega_i^2 = 18\ \text{J}$$

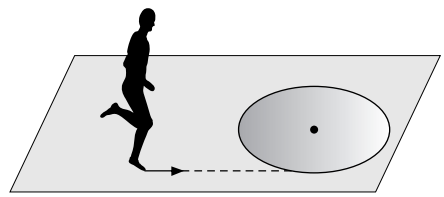
$$K_f = \frac{1}{2} I_f\omega_f^2 = 12\ \text{J}$$

The change is $\Delta K = K_f - K_i = -6\ \text{J}$.

In order for the two discs to spin together at the same rate, there had to be friction between them. The lost kinetic energy is converted with thermal energy.

ILLUSTRATION 84

A man of mass $m = 80\ \text{kg}$ runs at a speed $u = 4\ \text{ms}^{-1}$ along the tangent to a disc-shaped platform of mass $M = 160\ \text{kg}$ and radius $R = 2\ \text{m}$. The platform is initially at rest but can rotate freely about an axis through its center. Take $I = \frac{1}{2} MR^2$.



Calculate the angular velocity of the platform after the man jumps on. If the man now walks to the centre, then calculate the new angular velocity. Treat the man as a point particle.

SOLUTION

Before trying to attempt this problem let us have a self-analysis done and answer the following questions.

Q. Can we apply the conservation of linear momentum?
A. No, it cannot be applied because the axle exerts an external force on the system i.e. man + platform.
Q. Can we apply the conservation of angular momentum?
A. Yes, since the axle does not exert any torque, we may use the conservation of angular momentum.
Q. Can we apply conservation of kinetic energy for the collision between the man and the platform?
A. No.

Let us consider the origin to be at the centre of platform. When the man runs in a straight line along

the tangent, then his initial angular momentum about this origin is $L = r_\perp p$, where $r_\perp = R$ so

$$L_i = muR$$

After he jumps on, one must take into account his contribution mR^2 to the moment of inertia. The final angular momentum, $L = I\omega$, is,

$$L_f = \left(\frac{1}{2}MR^2 + mR^2\right)\omega$$

By Law of Conservation of Angular Momentum, we have $L_f = L_i$, wo

$$\omega = \frac{mu}{\left(\frac{M}{2} + m\right)R}$$

Substituting, $m = 80$ kg, $M = 160$ kg, $u = 4$ ms^{-1}, $R = 2$ m, we get

$$\omega = \frac{(80)(4)}{\left(\frac{160}{2} + 80\right)2} = 1 \text{ rads}^{-1}$$

When the man reaches the center, his contribution to the moment of inertia is zero. The final angular momentum and the initial momentum in this case are given by

$$L_i = \left(\frac{1}{2}MR^2 + mR^2\right)\omega_1 = 640 \text{ kgm}^2s^{-1}$$

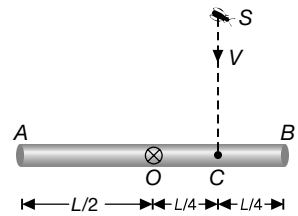
$$L_f = \left(\frac{MR^2}{2}\right)\omega_2 = 320\omega_2$$

Applying angular momentum conservation $L_f = L_i$ we get

$$\omega_2 = 2 \text{ rads}^{-2}$$

ILLUSTRATION 85

A homogeneous rod AB of length $L = 1.8$ m and mass M is pivoted at the centre O in such a way that it can rotate freely in the vertical plane (shown in Figure). The rod is initially in the horizontal position. An insect S of the same mass M falls vertically with speed V on the point C, midway between the points O and B. Immediately after falling, the insect moves towards the end B such that the rod rotates with a constant angular velocity ω.

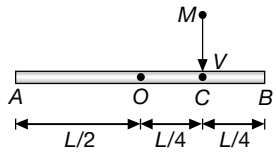


(a) Determine the angular velocity ω in terms of V and L.

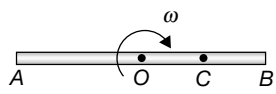
(b) If the insect reaches the end B when the rod has turned through an angle of $90°$, determine V.

SOLUTION

In this problem we will denote angular momentum by its standard symbol H because L has been used for length of the rod.



Just before collision



Just after collision

Angular momentum of the system (rod + insect) about the centre of the rod O will remain conserved just before collision and after collision i.e., $H_i = H_f$.

$$\Rightarrow \quad MV\frac{L}{4} = \text{I}\omega = \left[\frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2\right]\omega$$

$$\Rightarrow \quad MV\frac{L}{4} = \frac{7}{48}ML^2\omega$$

$$\Rightarrow \quad \omega = \frac{12}{7}\frac{V}{L} \qquad \ldots(1)$$

Due to the torque of weight of insect about O, angular momentum of the system will not remain conserved (although angular velocity ω is constant). As the insect moves towards B, moment of inertia of the system increases, hence, the angular momentum of the system will increase.

Let at time t the insect be at a distance x from O and by then the rod has rotated through an angle θ. Then, angular momentum at that moment,

$$H = \left[\frac{ML^2}{12} + Mx^2\right]\omega$$

$$\Rightarrow \quad \frac{dH}{dt} = 2M\omega x\frac{dx}{dt} \qquad \{\omega = \text{constant}\}$$

$$\Rightarrow \quad \tau = 2M\omega x\frac{dx}{dt} \qquad \left\{\because \frac{dH}{dt} = \tau\right\}$$

$$\Rightarrow \quad Mgx\cos\theta = 2M\omega x\frac{dx}{dt}$$

$$\Rightarrow \quad dx = \left(\frac{g}{2\omega}\right)\cos\theta\, dt \qquad \{\because \theta = \omega t\}$$

$$\Rightarrow \quad \frac{dx}{d\theta}\frac{d\theta}{dt} = \frac{g}{2\omega}\cos\theta$$

$$\Rightarrow \quad \omega\frac{dx}{d\theta} = \frac{g}{2\omega}\cos\theta$$

$$\Rightarrow \quad \int_{\frac{L}{4}}^{\frac{L}{2}} dx = \frac{g}{2\omega^2}\int_0^{\frac{\pi}{2}}\cos\theta\, d\theta$$

$$\Rightarrow \quad x\Big|_{\frac{L}{4}}^{\frac{L}{2}} = \frac{g}{2\omega^2}\sin\theta\Big|_0^{\frac{\pi}{2}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{2g}{L}}$$

Substituting in equation (1), we get

$$\sqrt{\frac{2g}{L}} = \frac{12}{7}\frac{V}{L}$$

$$\Rightarrow \quad V = \frac{7}{12}\sqrt{2gL} = \frac{7}{12}\sqrt{2\times10\times1.8} = 3.5\ \text{ms}^{-1}$$

$$\Rightarrow \quad V = 3.5\ \text{ms}^{-1}$$

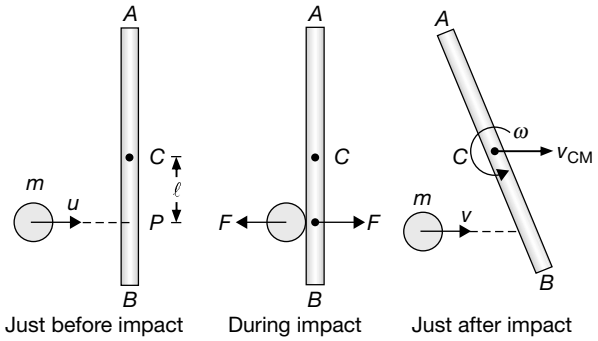
BALL ROD COLLISIONS

Consider a uniform rod AB of mass M, length L. Let a ball of mass m moving with initial velocity v, hit the rod at a point P (other than centre of mass of the rod). Now two cases arise.

CASE-1: When the Rod is not Hinged

For Ball

$$mv - mu = -Ft \qquad \ldots(1)$$



For Rod

$$mv_{cm} - 0 = Ft \qquad \ldots(2)$$

From (1) and (2), we get

$$mv - mu = -mv_{cm}$$

$$\Rightarrow \quad mu = mv_{cm} + mv \qquad \ldots(3)$$

$$\begin{pmatrix}\text{Total Initial}\\ \text{Momentum of}\\ \text{Ball + Rod}\end{pmatrix} = \begin{pmatrix}\text{Total Final}\\ \text{Momentum of}\\ \text{Ball + Rod}\end{pmatrix}$$

The total linear momentum of Ball + Rod system is conserved.

Using Angular Impulse – Angular Momentum Theorem, we get

For Ball

$$(mv)l - (mu)l = -\tau t$$

$$\Rightarrow \quad (mv)l - (mu)l = -(Fl)t \qquad \ldots(4)$$

For Rod

$$I\omega - 0 = (Fl)t \qquad \ldots(5)$$

From (4) and (5), we get

$$(mv)l - (mu)l = -I\omega$$

$$\Rightarrow \quad (mu)l = I\omega + (mv)l \qquad \ldots(6)$$

$$\begin{pmatrix}\text{Total Initial}\\ \text{Angular}\\ \text{Momentum}\\ \text{of Ball + Rod}\end{pmatrix} = \begin{pmatrix}\text{Total Final}\\ \text{Angular}\\ \text{Momentum}\\ \text{of Ball + Rod}\end{pmatrix}$$

So, the total Angular Momentum of Ball + Rod is conserved.

So, if the rod is not hinged, then we observe both linear and angular momentum of Rod + Ball to be conserved.

Subcase-1(a)

If the collision is elastic, then we have

$$\begin{pmatrix}\text{Total Initial Energy}\\ \text{of Ball + Rod}\end{pmatrix}=\begin{pmatrix}\text{Total Final Energy}\\ \text{of Ball + Rod}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}mu^2+0=\frac{1}{2}mv^2+\frac{1}{2}mv_{cm}^2+\frac{1}{2}I\omega^2$$

Subcase-1(b)

If collision is inelastic

$$e=-\left[\frac{(v_2)_n-(v_1)_n}{(u_2)_n-(u_1)_n}\right]_{\text{at the point of impact}}$$

$$\Rightarrow \quad e=-\left[\frac{(v_{cm}+l\omega)-v}{0-u}\right]$$

(Net velocity of rod at P will be $v_{cm}+l\omega$, as it is under combined influence of translation and rotation.

Subcase-1(c)

For perfectly inelastic collision, we have

$$e=0$$

$$\Rightarrow \quad v=v_{cm}+l\omega$$

CASE-2: When the Rod is Hinged

For Ball

Using Impulse – Momentum Theorem, we get

$$mv-mu=-Ft \qquad \ldots(1)$$

Using Angular Impulse – Angular Momentum Theorem, about hinge, we get

$$(mv)l-(mu)l=-\tau t=-(Fl)t \qquad \ldots(2)$$

For Rod

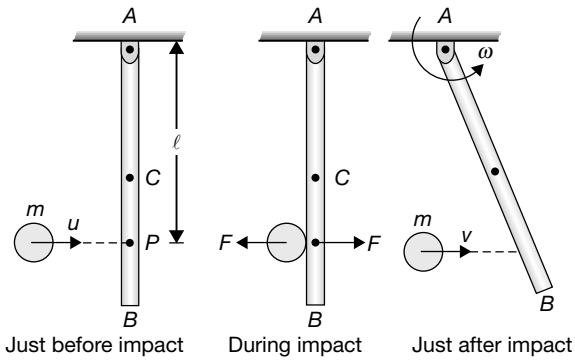
Using Angular Impulse – Angular Momentum Theorem, about hinge, we get

$$I\omega-0=(Fl)t \qquad \ldots(3)$$

From (2) and (3), we get

$$(mu)l=(mv)l+I\omega \qquad \{\text{about Hinge}\}$$

So, if the rod is hinged, then only angular momentum is conserved only about the hinge because τ due to the forces acting on the hinge about the hinge is zero.



Subcase-2(a)

If the collision is elastic, then we have

$$\frac{1}{2}mv^2=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$$

Subcase-2(b)

If the collision is inelastic, then we have

$$e=-\left[\frac{(v_2)_n-(v_1)_n}{(u_2)_n-(u_1)_n}\right]_{\text{at the point of impact}}$$

$$\Rightarrow \quad e=-\left(\frac{l\omega-v}{0-u}\right)$$

Subcase-2(c)

If the collision is perfectly inelastic, then

$$e=0$$

$$\Rightarrow \quad v=l\omega$$

ILLUSTRATION 86

A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end 'A' of the rod with a velocity v_0 in a direction perpendicular to AB. The collision is elastic. After the collision the particle comes to rest.

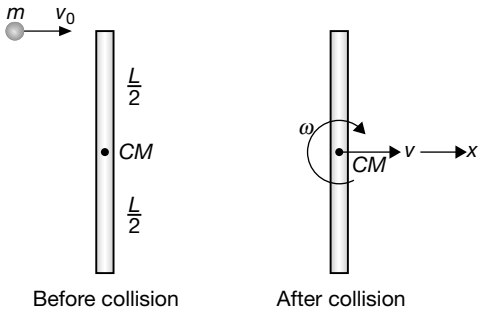
(a) Find the ratio $\frac{m}{M}$

(b) A point P on the rod is at rest immediately after collision. Find the distance AP.

(c) Find the linear speed of the point P after a time $\frac{\pi L}{3V_0}$ after the collision.

SOLUTION

(a) Let just after collision, velocity of centre of mass of rod is v and angular velocity about centre of mass is ω. Applying following three laws:



(i) External force on the system (rod + mass) in horizontal plane along x-axis is zero.

$\therefore$ Applying Conservation of Linear Momentum in x-direction.

$$mv_o = mv \qquad \ldots(1)$$

(ii) Net torque on the system about CM of rod is zero

$\therefore$ Applying Conservation of Angular Momentum about CM of rod, we get

$$mv_o\left(\frac{L}{2}\right) = I\omega$$

$$\Rightarrow \quad mv_o\frac{L}{2} = \frac{ML^2}{12}\omega$$

$$\Rightarrow \quad mv_o = \frac{ML\omega}{6} \qquad \ldots(2)$$

(iii) Since, the collision is elastic, kinetic energy is also conserved. So,

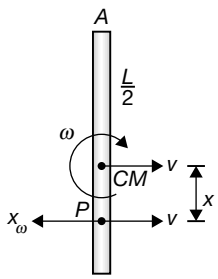
$$\frac{1}{2}mv_o^2 = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad mv_o = Mv^2 + \frac{ML^2}{12}\omega^2 \qquad \ldots(3)$$

From equations (1), (2) and (3), we get the following results

$$\frac{m}{M} = \frac{1}{4},\ v = \frac{mv_o}{M} \text{ and } \omega = \frac{6mv_o}{ML}$$

(b) Point P will be at rest if $x\omega = v$



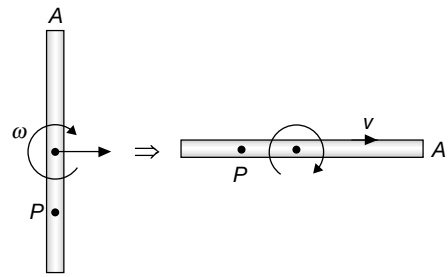
$$\Rightarrow \quad x = \frac{v}{\omega} = \frac{\frac{mv_o}{M}}{\frac{6mv_o}{ML}}$$

$$\Rightarrow \quad x = \frac{L}{6}$$

$$\Rightarrow \quad AP = \frac{L}{2} + \frac{L}{6}$$

$$\Rightarrow \quad AP = \frac{2}{3}L$$

(c) After time $t = \frac{\pi L}{3v_o}$



angle rotated by rod, $\theta = \omega t = \frac{6mv_o}{ML}\cdot\frac{\pi L}{3v_o}$

$$\Rightarrow \quad \theta = 2\pi\left(\frac{m}{M}\right) = 2\pi\left(\frac{1}{4}\right)\frac{m}{M} = \frac{1}{4}$$

$$\Rightarrow \quad \theta = \frac{\pi}{2}$$

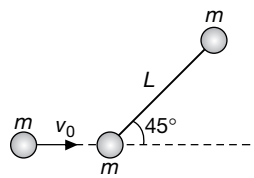
Therefore, situation will be as shown below:
So, resultant velocity of point P will be

$$\left|\vec{v}_p\right| = \sqrt{2}v = \sqrt{2}\left(\frac{m}{M}\right)v_o = \frac{\sqrt{2}}{4}v_o = \frac{v_o}{2\sqrt{2}}$$

$$\Rightarrow \quad \left|\vec{v}_p\right| = \frac{v_o}{2\sqrt{2}}$$

ILLUSTRATION 87

A rigid massless rod of length L joins two particles each of mass m. The rod lies on a frictionless table, and is struck by a particle of mass m and velocity v_0, moving as shown. After the collision, the projectile moves straight back.



Find the angular velocity of the rod about its centre of mass after the collision, assuming that mechanical energy is conserved.

SOLUTION

Applying Law of Conservation of Linear Momentum, we get

$$mv_0 = 2mv_1 - mv_2$$

$$\Rightarrow \quad 2v_1 - v_2 = v_0 \qquad \ldots(1)$$

Applying Law of Conservation of Angular Momentum about centre of mass C of light rod and the two identical particles, we get

$$mv_0\left(\frac{L}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = 2m\left(\frac{L}{2}\right)^2\omega - mv_2\left(\frac{L}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad v_0 = \sqrt{2}L\omega - v_2 \qquad \ldots(2)$$

Since the mechanical energy is conserved so, the collision is elastic, hence $e = 1$ at point of impact along common normal direction

$$\Rightarrow \quad \begin{pmatrix}\text{Relative Speed}\\ \text{of Approach}\end{pmatrix} = \begin{pmatrix}\text{Relative Speed}\\ \text{of separation}\end{pmatrix}$$

$$\Rightarrow \quad v_0 = v_2 + v_1 + \left(\frac{L}{2}\omega\right)\left(\frac{1}{\sqrt{2}}\right)$$

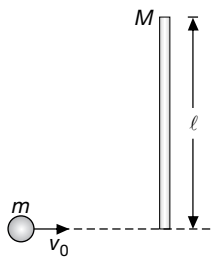
$$\Rightarrow \quad v_0 = v_1 + v_2 + \frac{\omega L}{2\sqrt{2}} \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$\omega = \frac{4\sqrt{2}}{7}\frac{v_0}{L}$$

ILLUSTRATION 88

A boy of mass m runs on ice with velocity v_0 and steps on the end of a plank of length l and mass M which is perpendicular to his path.



(a) Describe quantitatively the motion of the system after the boy is on the plank. Neglect friction with the ice.

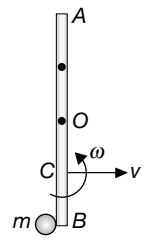
(b) One point on the plank is at rest immediately after the collision. Where is it?

SOLUTION

Let C be the centre of mass of boy plus plank. Let C be at a distance x from the end where the boy steps on the plank, then

$$x = \frac{m(0) + M\left(\frac{l}{2}\right)}{m + M}$$

$$\Rightarrow \quad BC = x = \left(\frac{M}{M+m}\right)\frac{l}{2}$$



So, distance of centre of mass C from the middle of the rod (O) is

$$OC = \left(\frac{l}{2} - x\right) = \left(\frac{m}{M+m}\right)\frac{l}{2}$$

Applying the Law of Conservation of Linear Momentum, we get

$$(M+m)v = mv_0$$

$$\Rightarrow \quad v = \left(\frac{m}{M+m}\right)v_0 \qquad \ldots(1)$$

Applying the Law of Conservation of Angular Momentum about point C, we get

$$mv_0(BC) = I_{\text{system}}\omega$$

$$\Rightarrow \quad \frac{mMv_0 l}{2(M+m)} = \left[m\left(\frac{M}{M+m}\right)^2\left(\frac{l}{4}\right)^2 + \frac{Ml^2}{12} + M\left(\frac{m}{M+m}\right)^2\left(\frac{l^2}{4}\right)\right]\omega$$

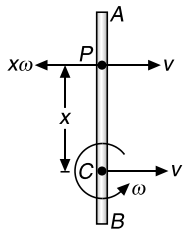
Substituting, $\frac{mv_0}{M+m} = v$ from equation (1), we get

$$\frac{v}{\omega}=\frac{l}{6}\left(\frac{4m+M}{M+m}\right)$$

Now, we have the plank divided in two portions

(a) White portion, below C till the end B.
(b) Grey shaded portion, above C till the end A.

Since $v\left(=\frac{mv_0}{M+m}\right)$ is actually the velocity of centre of mass of boy plus plank and so every point of the boy + plank system has a forward velocity v.
However, the **lower white portion** has a tangential velocity $r\omega$ where r is measured from C to B. This $r\omega$ is forward and so we cannot expect the resultant of v and $r\omega$ both forwards to be zero.



However, in the **upper grey portion**, all points move forward with velocity v but simultaneously the upper grey portion has a tangential velocity backwards. Now wherever (say the point P) the forward velocity v equals the backwards tangential velocity $x\omega$ (where x is the distance of point P from C), then at that point net velocity is zero, so

$$v=x\omega$$

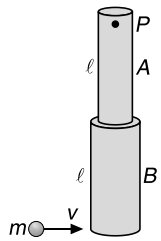
$$\Rightarrow \quad x=\frac{v}{\omega}$$

$$\Rightarrow \quad BP=BC+x$$

$$\Rightarrow \quad BP=\left(\frac{M}{M+m}\right)\left(\frac{l}{2}\right)+\frac{l}{6}\left(\frac{4m+M}{M+m}\right)=\frac{2l}{3}$$

ILLUSTRATION 89

Two uniform rods A and B of length 0.6 m each and of masses 0.01 kg and 0.02 kg respectively are rigidly joined end to end. The combination is pivoted at the lighter end, P as shown in Figure, such that it can freely rotate about point P in a vertical plane. A small object of mass 0.05 kg, moving horizontally, hits the lower end of the combination and sticks to it. What should be the velocity of the object so that the system could just be raised to the horizontal position?

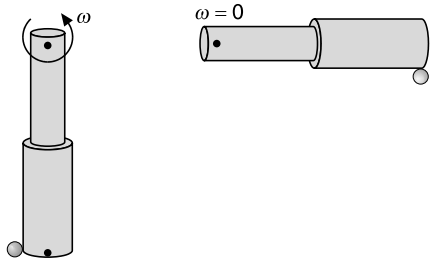


SOLUTION

System is free to rotate but not free to translate. During collision, net torque on the system $(\text{rod } A+\text{rod } B+\text{mass } m)$ about point P i.e., hinge is zero, so angular momentum is conserved about P. If ω be the angular velocity of system just after collision, then

$$L_i=L_f$$

$$\Rightarrow \quad mv(2l)=I\omega$$



where, I is the moment of inertia of system about P, so

$$I=m(2l)^2+m_A\left(\frac{l^2}{3}\right)+m_B\left(\frac{l^2}{12}+\left(\frac{l}{2}+l\right)^2\right)$$

Given $l=0.6$ m, $m=0.05$ kg, $m_A=0.01$ kg and $m_B=0.02$ kg.

Substituting the values, we get

$$I=0.09 \text{ kgm}^2$$

Therefore, from equation (1), we get

$$\omega=\frac{2mvl}{I}=\frac{(2)(0.05)(v)(0.6)}{0.09}$$

$$\Rightarrow \quad \omega=0.67v \qquad \ldots(1)$$

Now, after collision, mechanical energy will be conserved.

$$\text{Therefore, } \begin{pmatrix}\text{Decrease in}\\ \text{RKE}\end{pmatrix}=\begin{pmatrix}\text{Increase in}\\ \text{GPE of CM}\\ \text{of both Rods}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}I\omega^2 = mg(2l)+m_Ag\left(\frac{l}{2}\right)+m_Bg\left(l+\frac{l}{2}\right)$$

$$\Rightarrow \quad \omega^2 = \frac{gl(4m+m_A+3m_B)}{I}$$

$$\Rightarrow \quad \omega^2 = \frac{(9.8)(0.6)(4\times0.05+0.01+3\times0.02)}{0.09} = 17.64$$

$$\Rightarrow \quad \omega = 4.2 \text{ rads}^{-1} \qquad \ldots(2)$$

Equating equations (1) and (2), we get

$$v = \frac{4.2}{0.67} \text{ ms}^{-1}$$

$$\Rightarrow \quad v = 6.3 \text{ ms}^{-1}$$

ILLUSTRATION 90

An ice cube of mass M and with sides of length a is sliding without friction across a countertop with a speed v_0 when it hits a ridge E at the edge of the counter (see the Figure). This collision causes the cube to tilt as shown. Show that the minimum value of v_0 needed for the cube to fall off the table is given by

$$v_0 = \sqrt{1.1ag}$$

Given that the moment of inertia of the cube about an axis passing through its centre of gravity and parallel to horizontal surface is $\frac{1}{6}Ma^2$

SOLUTION

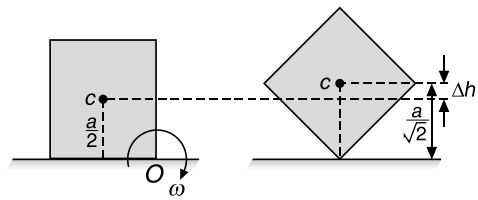
By Law of Conservation of Angular Momentum around the axis through E, we get

$$L_i = L_f$$

$$\Rightarrow \quad Mv_0\left(\frac{a}{2}\right) = I_0\omega = \left(\frac{Ma^2}{6}+M\left(\frac{a}{\sqrt{2}}\right)^2\right)\omega$$

$$\Rightarrow \quad Mv_0\left(\frac{a}{2}\right) = \frac{2}{3}Ma^2\omega$$

$$\Rightarrow \quad \omega = \frac{3}{4}\frac{v_0}{a}$$



Further by Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in RKE}\\ \text{of Block about O}\end{pmatrix} = \begin{pmatrix}\text{Gain in GPE}\\ \text{of CM of Block}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}I\omega^2 = Mg(\Delta h)$$

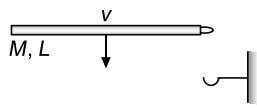
where $\Delta h = \frac{a}{\sqrt{2}}-\frac{a}{2}$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{2}{3}Ma^2\right)\left(\frac{3}{4}\frac{v_0}{a}\right)^2 = Mg\left(\frac{a}{\sqrt{2}}-\frac{a}{2}\right)$$

$$\Rightarrow \quad v_0 = \sqrt{1.1ag}$$

ILLUSTRATION 91

A rod of mass M and length L is falling vertically with speed v. Suddenly its one end gets stuck in a frictionless hook. Find the angular velocity of the rod just after its end gets stuck.



SOLUTION

Since all the impulse forces during the impact are applied by the hook so they don't produce any torque about the hook and hence angular momentum of the rod about the hook is conserved.

Initial angular momentum about hook is

$$H_i = mvr_\perp = Mv\left(\frac{L}{2}\right)$$

Final angular momentum about the hook is

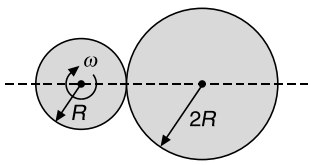
$$H_f = I\omega = \left(\frac{ML^2}{3}\right)\omega$$

$$\Rightarrow \quad \frac{MvL}{2} = \frac{ML^2}{3}\omega$$

$$\Rightarrow \quad \omega = \frac{3v}{2L}$$

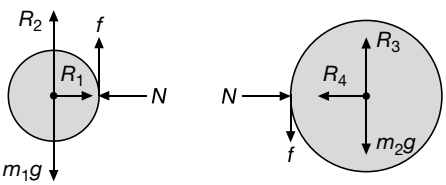
ILLUSTRATION 92

Two discs of radii R and $2R$ are pressed against each other. Initially disc with radius R is rotating with angular velocity ω and another disc was stationary. Both discs are hinged at their respective centres and free to rotate about them. Moment of inertia of smaller is I and bigger disc is $2I$ about their respective axis of rotation. Find the angular velocity of the bigger disc after long time.

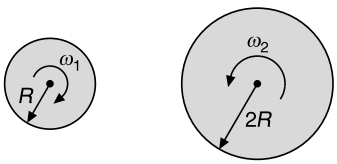


SOLUTION

Let R_1, R_2 be the horizontal and vertical components of the reaction offered by the first hinge to the pulley and R_3, R_4 be the horizontal and vertical components of the reaction offered by the second hinge to the pulley. The only force which is producing any torque about the centre of first disk is friction as shown in Figure.



Let the disc rotate with angular velocity ω_1 and ω_2 as shown in Figure.



Since v is same at the point of contact, so we have

$$\omega_1 R = \omega_2 (2R)$$

$$\Rightarrow \quad \omega_2 = \frac{\omega_1}{2}$$

Now, total angular impulse provided by the friction is equal to change in angular momentum of the disc. So, for disc 1, we get

$$\int fR\,dt = I(\omega - \omega_1)$$

For disc 2, we get

$$\int f(2R)\,dt = 2I\omega_2$$

$$\Rightarrow \quad 2I(\omega - \omega_1) = 2I\omega_2$$

$$\Rightarrow \quad \omega_1 + \omega_2 = \omega$$

$$\Rightarrow \quad 2\omega_2 + \omega_2 = \omega$$

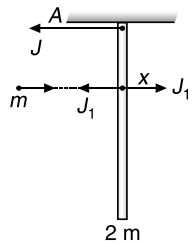
$$\Rightarrow \quad \omega_2 = \frac{\omega}{3}$$

ILLUSTRATION 93

A uniform rod AB of length $2l$ and mass $2m$ is suspended freely at A and hangs vertically at rest when a particle of mass m is fired horizontally with speed v to strike the rod at its mid-point. If the particle is brought to rest by the impact, calculate the impulsive reaction at A, the initial angular speed of the rod and the maximum angle the rod makes with the vertical in the subsequent motion.

SOLUTION

At the instant of collision, if impulsive reaction at A is J and impulse between particle and rod be J_1, then we have



by impulse-momentum theorem

$$J_1 = mv$$

and by angular impulse-angular momentum theorem

$$J_1 l = \frac{2m(2l)^2}{3}\omega$$

$$\Rightarrow \quad \omega = \frac{3J_1}{8ml} = \frac{3v}{8l}$$

For rod, we have

$$J_1 - J = 2m(l\omega)$$

$$\Rightarrow \quad J = J_1 - 2ml\left(\frac{3v}{8l}\right) = \frac{mv}{4}$$

If rod gets displaced by an angle before coming to rest, then by law of conservation of energy, we have

$$\frac{1}{2}I\omega^2 = mgh$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{(2m)(2l)}{3}\right)^2\left(\frac{3v}{8l}\right)^2 = 2mgl(1-\cos\phi)$$

$$\Rightarrow \quad \cos\phi = 1-\frac{3v^2}{32gl}$$

$$\Rightarrow \quad \phi = \cos^{-1}\left(1-\frac{3v^2}{32gl}\right)$$

ILLUSTRATION 94

A thin uniform square plate with side l and mass M can rotate freely about a stationary vertical axis coinciding with one of its sides. A small ball of mass m flying with velocity v at right angles to the plate strikes elastically at centre of the square plate. Calculate the velocity of the ball v' after the impact and the horizontal component of the force which the axis will exert on the plate after the impact.

SOLUTION

Applying conservation of angular momentum, we get

$$mv\left(\frac{l}{2}\right) = mv'\left(\frac{l}{2}\right)+\frac{Ml^2}{3}\omega$$

$$\Rightarrow \quad mv = mv'+\frac{2}{3}Ml\omega \qquad \ldots(1)$$

Since collision is elastic, so we have

$$v = \frac{\omega l}{2}-v' \qquad \ldots(2)$$

From equations (1) and (2), we get

$$mv = mv'+\frac{2}{3}M(v+v')(2)$$

$$\Rightarrow \quad mv-\frac{4}{3}Mv = \left(m+\frac{4}{3}M\right)v'$$

$$\Rightarrow \quad v' = \left(\frac{3m-4M}{3m+4M}\right)v$$

From, equation (2), we get

$$\omega = \frac{2}{l}(v+v') = \frac{2v}{l}\left(1+\frac{3m-4M}{3m+4M}\right)$$

$$\Rightarrow \quad \omega = \frac{12mv}{l(3m+4M)}$$

Force due to axis on the plate is

$$F = M\left(\frac{l}{2}\right)\omega^2 = \frac{72Mm^2v^2}{l(3m+4M)^2}$$

Test Your Concepts-VI

Based on Angular Momentum and Its Conservation

(Solutions on page H.162)

1. A small ball is suspended from a point O by a light thread of length ℓ. Then the ball is drawn aside so that the thread deviates through an angle θ from the vertical and set in motion in a horizontal direction at right angles to the vertical plane in which the thread is located. What is the initial velocity that has to be imparted to the ball so that it could deviate through the maximum angle of $\frac{\pi}{2}$ with the vertical in the process of motion?

2. A uniform rod of length L rests on a frictionless horizontal surface. The rod is pivoted about a fixed frictionless axis at one end. The rod is initially at rest. A bullet travelling parallel to the horizontal surface and perpendicular to the rod with speed v strikes the rod at its centre and becomes embedded in it. The mass of the bullet is one-sixth the mass of the rod.

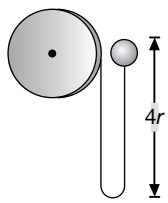
(a) What is the final angular velocity of the rod?

(b) What is the ratio of the kinetic energy of the system after the collision to the kinetic energy of the bullet before the collision?

3. A wheel of moment of inertia I and radius R is rotating about its axis at an angular speed ω_0. It picks up a stationary particle of mass m at its edge. Find the new angular speed of the wheel.

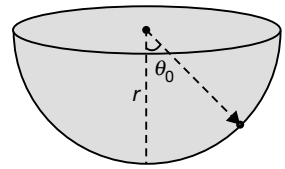
4. A uniform circular disc of mass m and radius a is rotating with constant angular velocity ω in a horizontal plane about a vertical axis through its centre A. A particle P of mass $2m$ is placed gently on the disc at a point distant $\frac{a}{2}$ from A. If the particle does not slip on the disc, find the new angular velocity of the rotating system.

5. A pulley in the form of a uniform disc of mass $2m$ and radius r, is free to rotate in a vertical plane about a fixed horizontal axis through its centre. A light inextensible string has one end fastened to a point on the rim of the pulley and is wrapped several times round the rim. The portion of string not wrapped round the pulley length $8r$ and carries a particle of mass m at its free end. The particle is held close to the rim of the pulley and level with its centre. If the particle is released from this position find the initial angular velocity of the pulley and the impulse of the sudden tension in the string when it becomes taut.



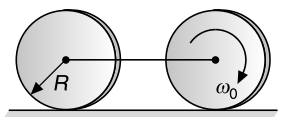
6. A horizontally oriented uniform disc of mass M and radius R rotates freely about a stationary vertical axis passing through its centre. The disc has a radial guide along which a small body of mass m can slide without friction. A light thread running down through the hollow axle of the disc is tied to the body. Initially the body is located at the edge of the disc and the whole system is rotated with an angular velocity ω_0. Then, by means of a force F applied to the lower end of the thread the body is slowly pulled towards the axis of rotation. Find the
(a) angular velocity of the system in its final state.
(b) work performed by the force F.

7. A particle is projected horizontally along the interior of a smooth hemispherical bowl of radius r which is kept at rest. Find the minimum initial speed v_0 required for the particle to just reach the top of the bowl. The initial angular position of the particle is θ_0.



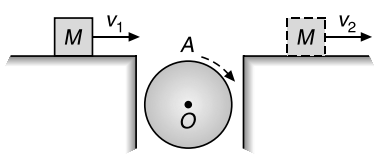
8. A uniform rod AB of length 2ℓ and mass m is rotating in a horizontal plane about a vertical axis through A, with angular velocity ω, when the midpoint of the rod strikes a fixed nail and is brought immediately to rest. Find the angular impulse exerted by the nail.

9. The assembly of two discs as shown in Figure is placed on a rough horizontal surface and the front disc is given an initial angular velocity ω_0. Determine the final linear and angular velocity when both the discs start rolling. It is given that friction is sufficient to sustain rolling in the rear wheel from the beginning of motion.



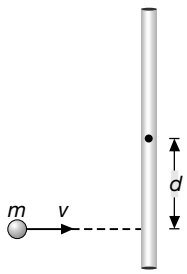
10. A man of mass m_1 stands on the edge of a horizontal uniform disc of mass m_2 and radius R which is capable of rotating freely about a stationary vertical axis passing through its centre. The man walks along the edge of the disc through an angle θ relative to the disc and then stops. Find the angle ϕ through which the disc turned by the time the man stopped.

11. The axis of a cylinder of radius R and moment of inertia about its axis I is fixed at centre O as shown in Figure. Its highest point A is in level with two plane horizontal surfaces. A block of mass M is initially moving to the right without friction with speed v_1. It passes over the cylinder to the dotted position. Calculate the speed v_2 in the dotted position and the angular velocity acquired by the cylinder if at the time of detaching from cylinder block stops slipping on it.



12. A uniform rod of mass m and length ℓ rests on a smooth horizontal surface. One of the ends of the rod is struck in a horizontal direction at right angles to the rod. As a result, the rod obtains velocity v_0. Find the force with which one half of the rod will act on the other in the process of motion.

13. A rod of mass M, length ℓ lies on horizontal table and is free to move on the table. A ball of mass m, moving perpendicularly to the rod at a distance d from its centre with speed v collides elastically with it as shown in Figure. What quantities are conserved in the collision? What must be the mass of the ball so that it remains at rest immediately after collision?



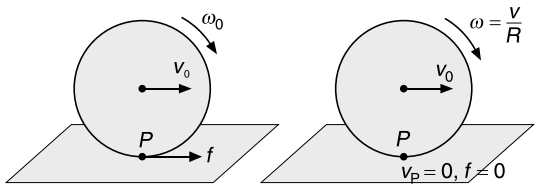
14. A uniform rod AB, of mass m and length $4a$, is smoothly pivoted at a point O of its length, where $AO = a$ and hangs at rest in equilibrium position with A above pivot O. A horizontal impulse of magnitude J is imparted to the rod at its centre of mass. Find the initial angular velocity of the rod. If the rod describes complete revolutions in the subsequent motion, find an inequality for J in terms of a, m and g.

15. A smooth rod rotates freely in a horizontal plane with the angular velocity ω_0 about a stationary vertical axis O, relative to which the rod's moment of inertia is I. A small ring the mass m is located on the rod close to the rotation axis and is tied to it by a thread. When the thread is burned, the ring starts sliding along the rod. Find the velocity v_r of the ring relative to the rod as a function of its distance r from the rotation axis.

ROLLING WITH SLIPPING ($v_0 > R\omega_0$)

CASE-1: When, $v_0 > R\omega_0$

Since $v_0 > R\omega_0$, so the point P has a tendency to slip forward and hence sliding friction (later on called as friction) acts backwards as shown.



ROLE OF FRICTION

Since f is acting opposite to v_0, therefore it will have a tendency to decrease v_0. Also, when f acts backwards then it will provide a torque that will be acting in the clockwise sense and hence will have a tendency to increase ω_0, i.e., at some later time, t (say), we can have $v = r\omega$

Since, $v_0 > R\omega_0$ {at $t = 0$}

So, to attain $v = R\omega$ at some later time t, f must be directed such that the role of f is

(a) to decrease v_0 to v (say)
(b) to increase ω_0 to ω (say)

such that at t, we get $v = R\omega$ (condition for pure rolling)

From impulse momentum theorem and angular impulse angular momentum theorem, we get

$$mv - mv_0 = -ft \quad \ldots(1)$$

$$I\omega - I\omega_0 = +\tau t = (fR)t \quad \ldots(2)$$

But at t, we have

$$v = R\omega \quad \ldots(3)$$

So, we use (1), (2), (3) to get the desired results. Also, work done by friction equals change in KE, so

$$W_f = \underbrace{\left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)}_{\omega = v/R} - \underbrace{\left(\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2\right)}_{\omega_0 \neq v/R} \quad \ldots(4)$$

Since, $f = \mu N = \mu mg$, retardation $a = \dfrac{\mu N}{m} = \mu g$, so distance travelled in time t is

$$s = v_0 t + \frac{1}{2}(-a)t^2 = v_0 t + \frac{1}{2}\left(-\frac{\mu mg}{m}\right)t^2 \qquad \ldots(5)$$

If θ is the total angle traversed, then

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta = \omega_0 t + \frac{1}{2}\left(\frac{\tau}{I}\right)t^2 \qquad \{\because\ \tau = I\alpha\}$$

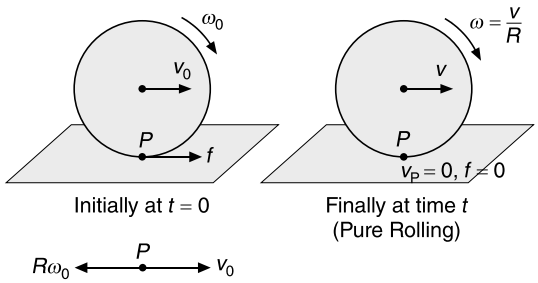
where $\tau = fR = I\alpha$

$$\Rightarrow \quad \alpha = \frac{fR}{I} = \frac{(\mu mg)}{I}R$$

So, $\theta = \omega_0 t + \frac{1}{2}\left(\dfrac{\mu mgR}{I}\right)t^2 \qquad \ldots(6)$

CASE-2: When, $v_0 < R\omega_0$

Since $v_0 < R\omega_0$, so the point P has a tendency to slip backward and hence sliding friction acts forward, as shown in Figure.



ROLE OF FRICTION

Since f is acting along v_0, so it will have a tendency to increase v_0. Also, when f acts forwards, along v_0, then it will provide a torque that will be acting in the counter-clockwise sense and hence will have a tendency to decrease ω_0, i.e., at some later time, t (say), we can have $v = r\omega$

Since, $v_0 < R\omega_0$ {at $t = 0$}

So, to attain $v = R\omega$ at some later time t, f must be directed such that the role of f is

(a) to increase v_0 to v (say)
(b) to decrease ω_0 to ω (say)

Such that at t, we get $v = R\omega$ (condition for pure rolling)

From impulse momentum theorem and angular impulse angular momentum theorem, we get

$$mv - mv_0 = ft \qquad \ldots(1)$$

$$I\omega - I\omega_0 = -\tau t = -(fR)t \qquad \ldots(2)$$

But at t, we have

$$v = R\omega \qquad \ldots(3)$$

So, we use (1), (2), (3) to get the desired results. Also work done by friction equals change in KE, so

$$\Rightarrow \quad W_f = \underbrace{\left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)}_{\omega = v/R} - \underbrace{\left(\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2\right)}_{\omega_0 \neq v/R} \qquad \ldots(4)$$

Since, $f = \mu N = \mu mg$ and $a = \dfrac{\mu N}{m} = \mu g$, so distance travelled in time t is

$$s = v_0 t + \frac{1}{2}(a)t^2 \qquad \{\because\ f \text{ accelerates } v_0\}$$

$$\Rightarrow \quad s = v_0 t + \frac{1}{2}\left(\frac{\mu mg}{m}\right)t^2 \qquad \ldots(5)$$

If θ is the angle traversed, then

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Rightarrow \quad \theta = \omega_0 t + \frac{1}{2}\left(-\frac{\tau}{I}\right)t^2 \qquad \{\because\ \tau = I\alpha\}$$

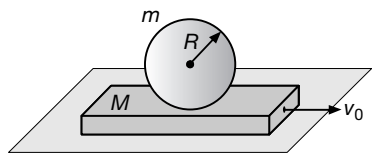
where, $\tau = fR = I\alpha$

$$\Rightarrow \quad \alpha = \frac{fR}{I} = \frac{(\mu mg)R}{I}$$

$$\Rightarrow \quad \theta = \omega_0 t - \frac{1}{2}\left(\frac{\mu mgR}{I}\right)t^2 \qquad \ldots(6)$$

ILLUSTRATION 95

A sphere of mass m and radius R is placed at rest on a plank of mass M which is placed on a smooth horizontal surface as shown in Figure. The coefficient of friction between the sphere and the plank is μ. At $t = 0$, a horizontal velocity v_0 is given to the plank. Find the time after which the sphere starts rolling.

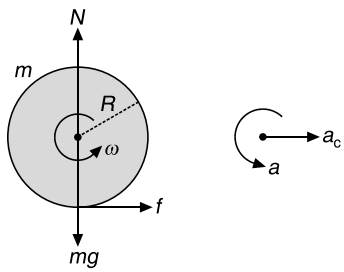


SOLUTION

For the Sphere:

$$a_c = \frac{f}{m} = \mu g$$

$$\alpha = \frac{\tau_c}{I_c} = \frac{f R}{\frac{2}{5} mR^2} = \frac{5}{2}\frac{\mu g}{R}$$



After time t

$$v_c = a_c t = \mu g t$$

$$\omega = \alpha t = \frac{5}{2}\frac{\mu g}{R} t$$

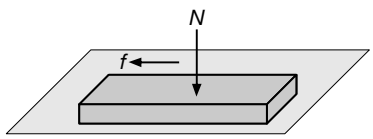
The velocity of the point of contact is

$$v = v_c + \omega R = \mu g t + \frac{5}{2}\mu g t = \frac{7}{2}\mu g t$$

For the Plank:

Retardation $a = \frac{f}{M} = \frac{\mu mg}{M}$

Instantaneous velocity $v = v_0 - \frac{\mu mg}{M} t$



Condition of pure rolling

$$v = \frac{7}{2}\mu g t = v_0 - \frac{\mu mg}{M} t$$

$$\Rightarrow \quad t = \frac{v_0}{\left(\frac{7}{2} + \frac{m}{M}\right)\mu g}$$

ILLUSTRATION 96

A wheel is held by a handle on its axle and given an initial angular velocity ω_0. The wheel is then placed in contact with the ground. At first the wheel remains stationary, spinning in place. After a short time it begins to move forward and eventually reaches the point where it rolls without slipping. Find the final velocity of the wheel in terms of the initial angular velocity ω_0. Take $I = \frac{MR^2}{2}$.

SOLUTION

We assume that wheel is initially rotating clockwise. Let the wheel starts rolling after a time t.
Then, Using Impulse Momentum Theorem

For translation:

$$\text{Impulse} = \Delta p = p_f - p_i$$

$$\Rightarrow \quad ft = Mv - 0 \qquad \ldots(1)$$

For rotation:

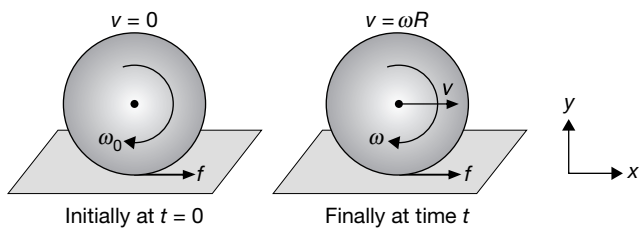
$$\text{Angular Impulse} = \Delta L = L_f - L_i$$

$$\Rightarrow \quad -fRt = I\omega - I\omega_0$$

Note that clockwise angular momentum is considered as positive.

Since $I = \frac{MR^2}{2}$

$$-(fR)t = \frac{MR^2}{2}(\omega - \omega_0) \qquad \ldots(2)$$



For the condition of pure rolling, we have

$$v = \omega R \qquad \ldots(3)$$

Using equations (1), (2) and (3), we get

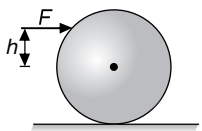
$$-M\omega R^2 = \frac{MR^2}{2}(\omega - \omega_0)$$

$$\Rightarrow \quad \omega = \frac{\omega_0}{3}$$

The linear velocity of the wheel is $v = R\omega = \frac{R\omega_0}{3}$

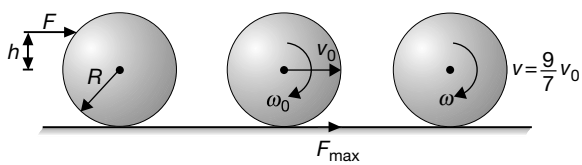
ILLUSTRATION 97

A billiard ball of radius R, initially at rest, is given a sharp impulse by a cue. The cue is held horizontally a distance h above the centre line as shown in Figure. The ball leaves the cue with a speed v_0 and because of its forward English (backward slipping) eventually acquires a final speed $\frac{9}{7}v_0$. Find h.



SOLUTION

Let ω_0 be the angular speed of the ball just after it leaves the cue. The maximum friction acts in forward direction till the slipping continues. Let v be the linear speed and ω the angular speed when slipping ceases, so



$$v = R\omega$$

$$\Rightarrow \quad \omega = \frac{v}{R}$$

Given, $v = \frac{9}{7}v_0$ …(1)

$$\Rightarrow \quad \omega = \frac{9}{7}\frac{v_0}{R} \qquad \ldots(2)$$

Applying impulse momentum theorem, we get

$$Fdt = mv_0 \qquad \ldots(3)$$

Applying angular impulse angular momentum theorem, we get

$$\tau dt = I\omega_0$$

$$\Rightarrow \quad Fhdt = \frac{2}{5}mR^2\omega_0 \qquad \ldots(4)$$

Angular momentum about bottommost point will remain conserved, so

$$L_i = L_f$$

$$\Rightarrow \quad I\omega_0 + mRv_0 = I\omega + mRv$$

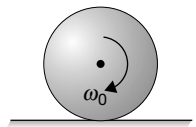
$$\Rightarrow \quad \frac{2}{5}mR^2\omega_0 + mRv_0 = \frac{2}{5}mR^2\left(\frac{9}{7}\frac{v_0}{R}\right) + \frac{9}{7}mRv_0 \qquad \ldots(5)$$

Solving equations (3), (4) and (5) we get,

$$h = \frac{4}{5}R$$

ILLUSTRATION 98

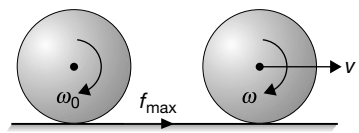
A solid sphere of radius r is gently placed on a rough horizontal ground with an initial angular speed ω_0 and no linear velocity. If the coefficient of friction is μ, find the time t when the slipping stops. In addition, state the linear velocity v and angular velocity ω at the end of slipping.



SOLUTION

METHOD I:

Let m be the mass of the sphere. According to the problem $v_0 < r\omega_0$, so it is a case of backward slipping and hence force of friction is in forward direction. Limiting friction will act in this case.



Linear acceleration $a = \frac{f}{m} = \frac{\mu mg}{m} = \mu g$

Angular retardation $\alpha = \frac{\tau}{I} = \frac{fr}{\frac{2}{5}mr^2} = \frac{5}{2}\frac{\mu g}{r}$

Slipping ceases when $v = r\omega$

$$\Rightarrow \quad (at) = r(\omega_0 - \alpha t)$$

$$\Rightarrow \quad \mu gt = r\left(\omega_0 - \frac{5}{2}\frac{\mu gt}{r}\right)$$

$$\Rightarrow \quad \frac{7}{2}\mu gt = r\omega_0$$

$$\Rightarrow \quad t = \frac{2}{7}\frac{r\omega_0}{\mu g}$$

$$\Rightarrow \quad v = at = \mu gt = \frac{2}{7}r\omega_0$$

and $\omega = \frac{v}{r} = \frac{2}{7}\omega_0$

METHOD II:

Net torque on the sphere about the bottommost point i.e., the point of contact is zero. Therefore, angular momentum of the sphere will remain conserved about the bottommost point.

$$L_i = L_f$$

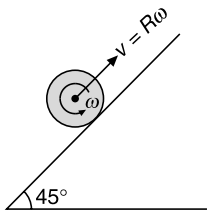
$$\Rightarrow \quad I\omega_0 = I\omega + mrv$$

$$\Rightarrow \quad \frac{2}{5}mr^2\omega_0 = \frac{2}{5}mr^2\omega + mr(\omega r)$$

$$\Rightarrow \quad \omega = \frac{2}{7}\omega_0 \text{ and } v = r\omega = \frac{2}{7}r\omega_0$$

ILLUSTRATION 99

A solid sphere of radius R is projected along an inclined plane as shown in Figure.

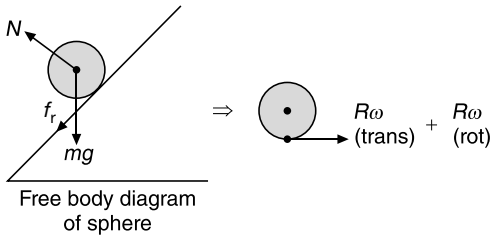


If coefficient of friction is $\mu = 0.5$ then calculate the time when linear velocity of the sphere becomes zero. Also calculate the angular velocity of the sphere at that instant.

SOLUTION

Velocity of lower most point is not zero, hence it is not the case of pure rolling and friction between the sphere and the incline is kinetic in nature. Since velocity of lower most point in forward direction therefore friction will act in backward direction.

Let retardation is a and angular retardation is α



$$\Rightarrow \quad mg\sin\theta + \mu mg\cos\theta = ma$$

$$\Rightarrow \quad a = g(\sin\theta + \mu\cos\theta) = \frac{g}{\sqrt{2}}(1+0.5) = \frac{1.5g}{\sqrt{2}} \quad \ldots(1)$$

Now, τ due to friction is

$$\tau = R(\mu mg\cos\theta)$$

$$\Rightarrow \quad \alpha = \frac{(r\mu mg\cos\theta)}{\frac{2}{5}mR^2} = \frac{5g\mu\cos\theta}{2R}$$

$$\Rightarrow \quad \alpha = \frac{5g(0.5)(1/\sqrt{2})}{2R}$$

$$\Rightarrow \quad \alpha = \frac{5g}{4R\sqrt{2}} \quad \ldots(2)$$

For translatory motion

$$v' = v - at = R\omega - at$$

When $v' = 0$, then

$$t = \frac{R\omega}{a} = \frac{R\omega\sqrt{2}}{1.5g} = \frac{\sqrt{2}}{1.5}\left(\frac{R\omega}{g}\right)$$

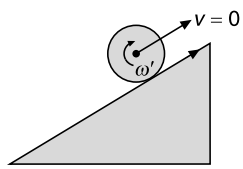
For rotatory motion, we have

$$\omega' = \omega - \alpha t$$

$$\Rightarrow \quad t = \frac{\omega}{\alpha} = \frac{\omega}{\frac{5g}{4R\sqrt{2}}} = \frac{4\sqrt{2}R\omega}{5g}$$

Linear velocity of sphere becomes zero before angular velocity become zero

Angular velocity at time t when linear velocity is zero.



$$\omega' = \omega - \left(\frac{5g}{4R\sqrt{2}}\right)\left(\frac{\sqrt{2}R\omega}{1.5g}\right)$$

$$\Rightarrow \quad \omega' = \omega - \left(\frac{5}{4}\right)\left(\frac{\omega}{1.5}\right) = \frac{\omega}{6}$$

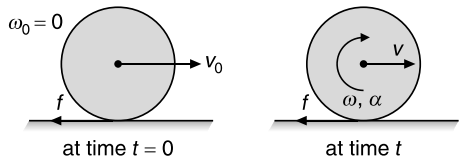
ILLUSTRATION 100

A uniform round object of mass m, radius R and moment of inertia about its centre of mass I_{cm} is thrown with speed v_0 without any rotation on a rough horizontal surface of coefficient of friction μ. Assuming $\frac{I_{cm}}{mR^2} = k$, calculate the time after which

slipping stops. Also calculate the speed and angular speed of the object when the slipping stops.

SOLUTION

The direction of friction is such that it opposes the translational motion of object and provides torque that supports the rotational motion of the object as shown in Figure.



Applying Newton's Second Law for translational motion, we get

$$f = \mu N = \mu mg$$

$$\Rightarrow \quad a = \frac{f}{m} = \mu g$$

$$\Rightarrow \quad v = v_0 - at = v_0 - \mu gt \qquad \ldots(1)$$

Applying Newton's Second Law for rotational motion, we get

$$fR = I_{cm}\alpha$$

$$\Rightarrow \quad \alpha = \frac{\mu mgR}{I_{cm}}$$

$$\Rightarrow \quad \omega = \alpha t = \left(\frac{\mu mgR}{I_{cm}}\right)t$$

For pure rolling, we have

$$v = R\omega$$

$$\Rightarrow \quad v_0 - \mu gt = \frac{R\mu mgRt}{I_{cm}}$$

$$\Rightarrow \quad v_0 = \mu gt\left(1 + \frac{1}{k}\right) \qquad \left\{\because \frac{I_{cm}}{mR^2} = k\right\}$$

$$\Rightarrow \quad t = \frac{v_0 k}{\mu g(1+k)}$$

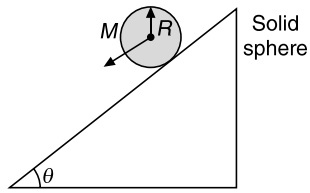
Substituting this value in equation (1), we get

$$v = v_0 - \mu g\frac{v_0 k}{(1+k)\mu g} = v_0\left(\frac{1+k-k}{1+k}\right)$$

$$\Rightarrow \quad v = \frac{v_0}{1+k} \text{ and } \omega = \frac{v}{R} = \frac{v_0}{R(1+k)}$$

ILLUSTRATION 101

A solid sphere starts to roll without slipping on a rough inclined plane as shown in Figure.

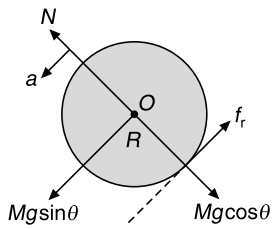


Calculate the friction force acting on the sphere, minimum coefficient of friction μ_0 for pure rolling of the sphere and the work done by friction if $\mu = \frac{\mu_0}{4}$ and the displacement of centre of mass is x.

SOLUTION

Acceleration of centre of mass of the sphere is

$$a = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5g\sin\theta}{7}$$



Applying Newton's Second Law for translational motion, we get

$$Mg\sin\theta - f_r = Ma$$

$$\Rightarrow \quad f_r = Mg\sin\theta - \frac{5Mg\sin\theta}{7} = \frac{2Mg\sin\theta}{7}$$

Since, $f_r = \frac{2Mg\sin\theta}{7} \le \mu N$

$$\Rightarrow \quad \frac{2Mg\sin\theta}{7} \le \mu Mg\cos\theta$$

$$\Rightarrow \quad \mu \ge \frac{2}{7}\tan\theta$$

$$\Rightarrow \quad \mu_0 = \frac{2}{7}\tan\theta$$

If $\mu < \mu_{min}$, then there will be slipping and hence friction will be kinetic in nature, so we have

$$f_k = \mu Mg\cos\theta = \left(\frac{2}{7}\frac{\tan\theta}{4}\right)Mg\cos\theta$$

$$\Rightarrow \quad f_k = \frac{Mg\sin\theta}{14}$$

$$\Rightarrow \quad a = \frac{Mg\sin\theta - f_k}{M} = \frac{Mg\sin\theta - \dfrac{Mg\sin\theta}{14}}{M}$$

$$\Rightarrow \quad a = \frac{13g\sin\theta}{14}$$

$$\Rightarrow \quad \alpha = \frac{\mu MgR\cos\theta}{\frac{2}{5}MR^2} = \frac{5\mu g\cos\theta}{2R}$$

$$\Rightarrow \quad \alpha = \left(\frac{5g\cos\theta}{2R}\right)\left(\frac{2}{7}\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{4}\right) = \frac{5g\sin\theta}{28R}$$

$$\Rightarrow \quad \frac{a}{\alpha} = \frac{\dfrac{13g\sin\theta}{14}}{\dfrac{5g\sin\theta}{28R}} = \left(\frac{13}{14}\right)\left(\frac{28R}{5}\right)R = \frac{26R}{5}$$

Since, $\dfrac{x}{\theta} = \dfrac{a}{\alpha} = 5.2R$

$$\Rightarrow \quad \frac{x}{\theta} = 5.2R$$

Work done by friction is

$$W_{fr} = \left(W_{fr}\right)_{\text{Rotation}} + \left(W_{fr}\right)_{\text{Trans}}$$

$$\Rightarrow \quad W_{fr} = \left(\mu mg\cos\theta\right)R\theta + \left(\mu mg\cos\theta\right)x\cos(180^\circ)$$

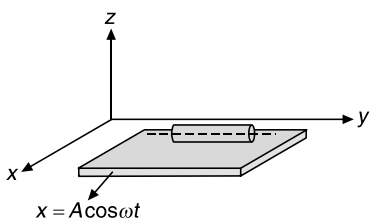
$$\Rightarrow \quad W_{fr} = \mu mg\left(R\theta - x\right)\cos\theta$$

$$\Rightarrow \quad W_{fr} = \mu mg\cos\theta\left(\frac{x}{5.2} - x\right) = -\frac{4.2}{5.2}\left(\mu mg\cos\theta\right)x$$

$$\Rightarrow \quad W_{fr} = -\frac{21}{26}\mu mgx\cos\theta$$

ILLUSTRATION 102

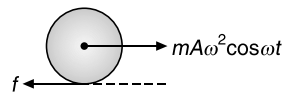
A cylinder of mass m and radius R is resting on a horizontal platform (which is parallel to xy plane) with its axis fixed along the y-axis and free to rotate about its axis as shown in Figure.



The platform is given a motion in the x-direction given by $x = A\cos(\omega t)$. There is no slipping between the cylinder and platform. Calculate the maximum torque acting on the cylinder during its motion.

SOLUTION

We must know that maximum torque will act on cylinder when it is ta the extreme positions of oscillatory motion. The cylinder moves under the influence of pseudo force with respect to platform as shown in Figure.



Thus, equation of motion of cylinder is

$$mA\omega^2 - f = ma_{\max} \qquad \ldots(1)$$

and $$fR = \frac{1}{2}mR^2\left(\frac{a_{\max}}{R}\right) \qquad \ldots(2)$$

Adding (1) and (2), we get

$$A\omega^2 = \frac{3}{2}a_{\max}$$

$$\Rightarrow \quad a_{\max} = \frac{2A\omega^2}{3}$$

Maximum angular acceleration is

$$\alpha_{\max} = \frac{a_{\max}}{R} = \frac{2A\omega^2}{3R}$$

Maximum torque on cylinder is

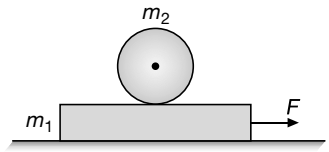
$$\tau_{\max} = I\alpha_{\max}$$

$$\Rightarrow \quad \tau_{\max} = \frac{1}{2}MR^2\left(\frac{2A\omega^2}{3R}\right)$$

$$\Rightarrow \quad \tau_{\max} = \frac{1}{3}mAR\omega^2$$

ILLUSTRATION 103

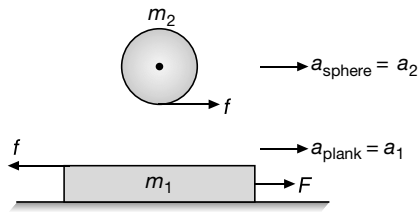
A plank of mass m_1 with a uniform sphere of mass m_2 placed on it rests on a smooth horizontal plane as shown in Figure. A constant horizontal force F is applied to the plank. Calculate the acceleration of the plank and the centre of the sphere if there is no sliding between the plank and the sphere.



SOLUTION

FROM FRAME ATTACHED TO GROUND

For the two bodies, equations of motion *from the ground reference frame* for accelerations a_1 and a_2 shown in Figure are



$$F - f = m_1 a_1 \quad \ldots(1)$$

$$f = m_2 a_2 \quad \ldots(2)$$

$$fR = -\frac{2}{5} m_2 R^2 \left(\frac{a_1 - a_2}{R} \right) \quad \ldots(3)$$

From (1) and (2), we get

$$F = m_1 a_1 + m_2 a_2 \quad \ldots(4)$$

From (2) and (3), we get

$$5 m_2 a_2 = 2 m_2 a_1 - 2 m_2 a_2$$

$$\Rightarrow \quad 7 a_2 = 2 a_1 \quad \ldots(5)$$

Using equation (4), we get

$$F = m_1 a_1 + m_2 \left(\frac{2 a_1}{7} \right)$$

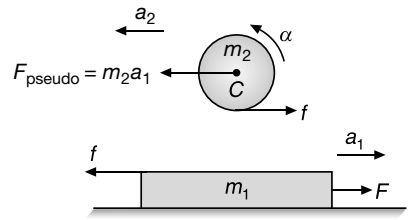
$$\Rightarrow \quad a_1 = \frac{7F}{7m_1 + 2m_2}$$

From equation (5), we get

$$a_2 = \frac{2F}{7m_1 + 2m_2}$$

FROM FRAME ATTACHED TO PLANK

Since the sphere does not slip over the plank surface, so it is the case of pure rolling and hence we can take friction on sphere in any direction, say towards right as shown in Figure.



Also note that since friction on sphere is acting towards right, so it must act on the plank towards left. Let plank moves toward right with an acceleration a_1, due to which the sphere will experience a pseudo force $m_2 a_1$ towards left, and hence it will roll towards left with respect to plank with an acceleration a_2.

Since we have used the concept of pseudo force, so we must say that the acceleration a_2 must be with respect to the plank. Let the angular acceleration of the sphere during rolling be α, so we have

$$a_2 = R\alpha$$

For translational motion of plank, we have

$$F - f = m_1 a_1 \quad \ldots(1)$$

For translational motion of sphere with respect to plank we have

$$m_2 a_1 - f = m_2 a_2 \quad \ldots(2)$$

For rotational motion of sphere with respect to plank, we have

$$fR = I\alpha$$

$$\Rightarrow \quad fR = \left(\frac{2}{5} m_2 R^2 \right) \left(\frac{a_2}{R} \right)$$

$$\Rightarrow \quad f = \frac{2}{5} m_2 a_2 \quad \ldots(3)$$

From equation (2) and (3), we get

$$m_2 a_1 - \frac{2}{5} m_2 a_2 = m_2 a_2$$

$$\Rightarrow \quad a_1 = \frac{7}{5} a_2 \quad \ldots(4)$$

Using equations (1), (2) and (3), we get

$$F - \frac{2}{5} m_2 a_2 = m_1 a_1$$

$$\Rightarrow \quad F = \frac{2}{5} m_2 a_2 + m_1 a_1 = \frac{2}{5} m_2 \left(\frac{5}{7} a_1 \right) + m_1 a_1$$

$$\Rightarrow \quad a_1 = \frac{7F}{7m_1 + 2m_2}$$

From equation (4), we get

$$a_2 = \frac{5}{7}a_1 = \frac{5}{7}\left(\frac{7F}{7m_1 + 2m_2}\right)$$

$$\Rightarrow \quad a_2 = \frac{5F}{7m_1 + 2m_2}$$

Since, a_2 is acceleration of sphere relative to the plank, so net acceleration of the sphere is

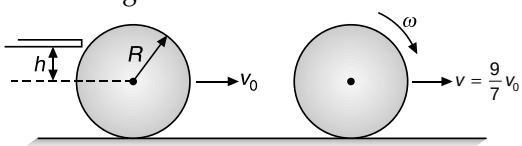
$$(a_{net})_{sphere} = a_1 - a_2 = \frac{2F}{7m_1 + 2m_2}$$

Please note that in the equation $a_2 = R\alpha$, a_2 is the acceleration of cylinder with respect to plank.

In problems where rolling of a body takes place in an inertial (ground) frame or a non-inertial (accelerated) frame, then in the equation $a = R\alpha$, the acceleration a must be relative, i.e. with respect to inertial (ground) frame or with respect to the non-inertial (accelerated) frame.

ILLUSTRATION 104

A billiard ball of mass m, radius R initially at rest is given a sharp horizontal impulse by a cue. The cue is held horizontally a distance h above the centre line as shown in Figure.



The ball leaves the cue with a speed v_0 and because of its forward English eventually acquires a final speed of $\frac{9}{7}v_0$, calculate the value of h.

SOLUTION

Let the impulse imparted by the cue to the ball be J and initial angular speed of ball be ω_0, then applying the impulse momentum theorem, we get

$$J = mv_0 \qquad \ldots(1)$$

Applying the angular impulse angular momentum theorem, we get

$$Jh = \frac{2}{5}mR^2\omega_0 \qquad \ldots(2)$$

From equations (1) and (2), we get

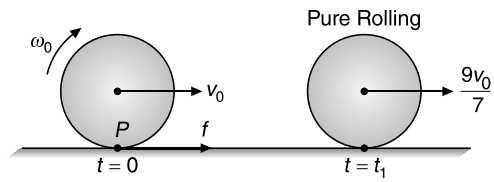
$$\omega_0 = \frac{5v_0h}{2R^2}$$

Applying conservation of angular momentum about point of contact P on the ground, we get

$$L_{initial} = L_{final}$$

where, $L_{initial} = mv_0R + \left(\frac{2mR^2}{5}\right)\left(\frac{5v_0h}{2R^2}\right)$ and

$$L_{final} = m\left(\frac{9v_0}{7}\right)R + \frac{2}{5}mR^2\left(\frac{9v_0}{7R}\right)$$



$$\Rightarrow \quad mv_0R + \frac{2}{5}mR^2\left(\frac{5v_0h}{2R^2}\right) = m\left(\frac{9v_0}{7}\right)R + \frac{2}{5}mR^2\left(\frac{9v_0}{7R}\right)$$

$$\Rightarrow \quad \left(1 + \frac{h}{R}\right) = \left(\frac{9}{7} + \frac{18}{35}\right)$$

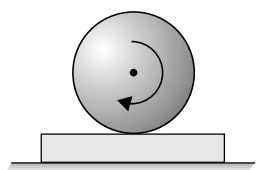
$$\Rightarrow \quad h = \left(\frac{63}{35} - 1\right)R = \frac{4}{5}R$$

Test Your Concepts-VII

Based on Rolling with Slipping

(Solutions on page H.165)

1. A horizontal plank having mass m lies on a smooth horizontal surface. A sphere of same mass and radius r is spun to an angular frequency ω_0 and gently placed on the plank as shown in Figure. If coefficient of friction between the plank and the sphere is μ, find the distance moved by the plank till the sphere starts pure rolling on the plank. The plank is long enough.

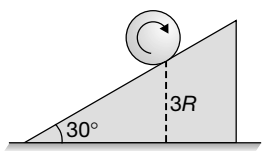


2. A disc of radius R and mass m is projected on to a horizontal floor with a backward spin such that its centre of mass speed is v_0 and angular velocity is ω_0. What must be the minimum value of ω_0 so that the disc eventually returns?

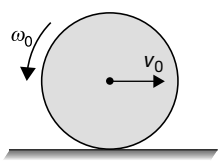
3. A solid cylinder of mass m and radius R is set in rotation about its axis with an angular velocity ω_0, then lowered with its lateral surface onto a horizontal plane and released. The coefficient of friction between the cylinder and the plane is μ. Calculate the time for which the cylinder will move with sliding and the total work done by friction.

4. A bowling ball is thrown straight down the alley. When it starts, its centre of mass has a speed v_0 and it is sliding without rotating. Determine how far the ball moves down the alley before it starts rolling without slipping. Express your answer in terms of v_0, g and μ_K.

5. A cylinder of mass m and radius R is rotated about its axis with angular velocity ω_0 (as shown in Figure). It is lowered on a rough inclined plane at an angle $30°$ with horizontal and having the coefficient of friction $\mu = \frac{1}{\sqrt{3}}$. The point of initial contact of cylinder and incline is at a height of $3R$ from horizontal. Calculate the total time taken by the cylinder to reach the bottom of incline.



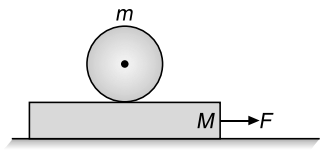
6. A sphere of radius R, mass M is projected along a rough horizontal surface (having coefficient of friction μ), with initial velocities shown in Figure.



If the final velocity of the sphere is zero, then calculate (in terms of v_0, R, ω_0, μ and M) the required magnitude of ω_0, the time t required by sphere to come to rest and the distance travelled by sphere before it stops.

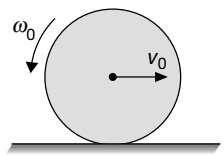
7. A bowling ball is thrown straight down the alley. When it starts, its centre of mass has a speed v_0 and it is sliding without rotating. Determine how far the ball moves down the alley before it starts rolling without slipping. Express your answer in terms of v_0, g and μ_K.

8. A plank of mass M is placed on a smooth surface over which a cylinder of mass m and radius R is placed as shown in Figure.



The plank is now pulled towards right with an external force F. If cylinder does not slip over the surface of plank, calculate the linear acceleration of plank and cylinder. Also calculate the angular acceleration of the cylinder.

9. A bowler projects a ball of mass M, radius R along an alley with forward velocity v_0 and a backspin ω_0. The coefficient of friction between the alley and the ball is μ. Calculate the time t at which the ball will start rolling with slipping and the speed of the ball at time t.



10. A uniform circular cylinder of mass m and radius r is given an initial angular velocity ω_0 and no initial translational velocity. It is placed in contact with a plane inclined at an angle α to the horizontal. If there is a coefficient of friction μ for sliding between the cylinder and plane, find the distance the cylinder moves up before sliding stops. Also calculate the maximum distance it travels up the plane. Assume $\mu > \tan\alpha$.

SOLVED PROBLEMS

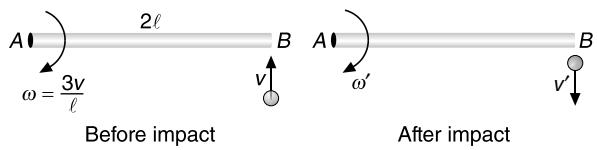
PROBLEM 1

A uniform rod AB of length $2l$ and mass m is turning freely in a horizontal plane about a vertical axis through A, with angular velocity $\frac{3v}{l}$. A particle P, also having mass m, is moving with constant velocity v in the same horizontal plane. The particle and the rod are moving towards each other and when AB is perpendicular to the path of the particle, P collides with point B of the rod. If the coefficient of restitution between the rod and the particle is $\frac{1}{2}$. Find the speed of the particle after impact.

SOLUTION

Angular momentum of the system about A is conserved, so

$$I\omega - mv(2l) = I\omega' + mv'(2l)$$



$$\Rightarrow \quad \frac{m(2l)^2}{3}\left(\frac{3v}{l}\right) - 2mvl = \frac{m(2l)^2}{3}\omega' + 2mv'l$$

$$\Rightarrow \quad 2mvl = \frac{4}{3}ml^2\omega' + 2mv'l$$

$$\Rightarrow \quad 2v = \frac{4}{3}l\omega' + 2v' \qquad \ldots(1)$$

$$\Rightarrow \quad 2v = \frac{4}{3}l\omega' + 2v'$$

At point B, we have

$$e = \frac{\text{Relative Velocity of Separation}}{\text{Relative Velocity of Approach}}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{v' - 2\omega' l}{v + \left(\frac{3v}{l}\right)(2l)} = \frac{v' - 2\omega' l}{7v}$$

$$\Rightarrow \quad 7v = 2v' - 4\omega' l$$

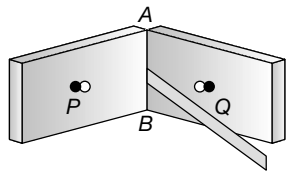
$$\Rightarrow \quad 4\omega' l = 2v' - 7v \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v' = \frac{13}{8}v$$

PROBLEM 2

Two heavy metallic plates are joined together at $90°$ to each other. A laminar sheet of mass 30 kg is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its centre of mass is 1.2 kgm^2. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This distance is chosen so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacles with an angular velocity 1 rads^{-1} and turns back. If the impulse on the sheet due to each obstacle is 6 Ns.



(a) find the location of the centre of mass of the laminar sheet from AB.

(b) at what angular velocity does the laminar sheet come back after the first impact?

(c) after how many impacts, does the laminar sheet come to rest?

SOLUTION

Let r be the perpendicular distance of centre of mass from the line AB and ω be the angular velocity of the sheet just after colliding with rubber obstacle for the first time.

The linear velocity of centre of mass before and after collision are

$$v_i = (r)(1 \text{ rads}^{-1}) = r \text{ and } v_f = r\omega$$

where, $\vec{v}_i$ and $\vec{v}_f$ will be in opposite directions.

Now, by Impulse – Momentum Theorem, we have

$$\begin{pmatrix}\text{Linear Impulse}\\ \text{on CM}\end{pmatrix} = \begin{pmatrix}\text{Change in Linear}\\ \text{Momentum of CM}\end{pmatrix}$$

$$\Rightarrow \quad 6 = m\left(v_f + v_i\right) = 30\left(r + r\omega\right)$$

$$\Rightarrow \quad r\left(1+\omega\right) = \frac{1}{5} \qquad \ldots(1)$$

Similarly, by Angular Impulse – Angular Momentum Theorem, we have

$$\begin{pmatrix}\text{Angular Impulse}\\ \text{about } AB\end{pmatrix} = \begin{pmatrix}\text{Change in Angular}\\ \text{Momentum about } AB\end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix}\text{Angular}\\ \text{impulse}\end{pmatrix} = \begin{pmatrix}\text{Linear}\\ \text{impulse}\end{pmatrix} \times \begin{pmatrix}\text{Perpendicular}\\ \text{distance of}\\ \text{impulse from } AB\end{pmatrix}$$

$$\Rightarrow \quad 6\left(0.5\,m\right) = I_{AB}\left(\omega+1\right)$$

(Initial angular velocity $= 1\ \text{rads}^{-1}$)

$$\Rightarrow \quad 3 = \left(I_{CM} + Mr^2\right)\left(1+\omega\right)$$

$$\Rightarrow \quad 3 = \left(1.2 + 30r^2\right)\left(1+\omega\right) \qquad \ldots(2)$$

Solving equations (1) and (2) for r, we get

$$r = 0.4\ \text{m} \ \text{OR} \ r = 0.1\ \text{m}$$

But at $r = 0.4$ m, ω comes out to be negative $\left(-0.5\ \text{rads}^{-1}\right)$ which is not acceptable. Therefore,

(a) $r =$ distance of CM from $AB = 0.1\ m$

(b) Substituting $r = 0.1m$ in equation (1), we get $\omega = 1\ \text{rads}^{-1}$ i.e., the angular velocity with which sheet comes back after the first impact is $1\ \text{rads}^{-1}$.

(c) Since, the sheet returns with same angular velocity of $1\ \text{rads}^{-1}$, the sheet will never come to rest.

PROBLEM 3

A uniform circular disc of mass M and radius a is pivoted at a point O on its circumference so that it can rotate about the tangent at O, which is horizontal, the centre of the disc describing a vertical circle of centre O in a plane perpendicular to the tangent. The point diametrically opposite to O is A and the disc is just displaced from rest when A is vertically above O. Find the angular velocity of the disc when A is vertically below O. At this instant a particle of mass M travelling with velocity u in the opposite direction of motion of the centre of the disc hits the disc at its centre and adheres to it. Find the angular velocity of the system immediately before and after the impact. If the disc just reaches its initial position show that

$$u = \left(3\sqrt{2} + \sqrt{5}\right)\sqrt{ag}$$

SOLUTION

(i) By Law of Conservation of Mechanical Energy, we get

$$Mg\left(2a\right) = \frac{1}{2}I\omega^2 \qquad \ldots(1)$$

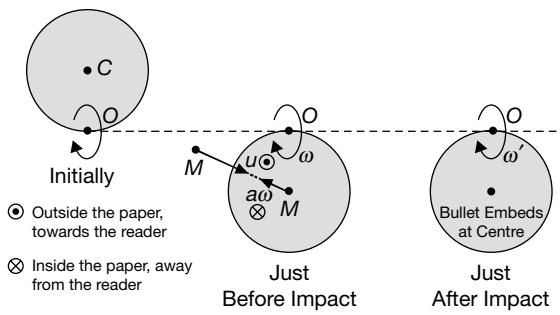
where, $I = \frac{1}{4}Ma^2 + Ma^2 = \frac{5}{4}Ma^2$

Substituting in equation (1), we get

$$2Mga = \frac{1}{2}\left(\frac{5}{4}Ma^2\right)\omega^2$$

$$\Rightarrow \quad \omega = 4\sqrt{\frac{g}{5a}}$$

(ii) By Law of Conservation of Angular Momentum about an axis through O, we get



$$Mu\left(a\right) - \left(\frac{5}{4}Ma^2\right)\omega = \left(\frac{5}{4}Ma^2 + Ma^2\right)\omega'$$

$$\Rightarrow \quad u - \frac{5}{4}\omega a = \frac{9}{4}\omega' a$$

$$\Rightarrow \quad \omega' = \frac{4u - 5\omega a}{9a} = \frac{4}{9a}\left(u - \frac{5}{4}a\omega\right)$$

Substituting the value of ω we get

$$\omega' = \frac{4}{9a}\left(u - 5a\sqrt{\frac{g}{5a}}\right) = \frac{4}{9a}\left(u - \sqrt{5ag}\right)$$

(iii) Again, applying Law of Conservation of Mechanical Energy, we get

$$2Mg\left(2a\right) = \frac{1}{2}\left(\frac{5}{4}Ma^2 + Ma^2\right)\left(\omega'\right)^2$$

$$\Rightarrow \quad 4Mga = \frac{9}{8}Ma^2\left(\omega'\right)^2$$

$$\Rightarrow \quad 32g = 9a\left(\omega'\right)^2$$

$$\Rightarrow \omega' = \sqrt{\frac{32}{9}\frac{g}{a}} = 4\sqrt{\frac{2}{9}\frac{g}{a}}$$

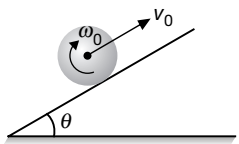
$$\Rightarrow 4\sqrt{\frac{2}{9}\frac{g}{a}} = \frac{4}{9a}\left(u - \sqrt{5ag}\right)$$

$$\Rightarrow u - \sqrt{5ag} = 9\sqrt{\frac{2}{9}ag}$$

$$\Rightarrow u = \left(3\sqrt{2} + \sqrt{5}\right)\sqrt{ag}$$

PROBLEM 4

A sphere of radius r is projected up an inclined plane for which $\mu = \left(\frac{1}{7}\right)\tan\theta$ with a velocity v_0 and initial angular velocity ω_0 such that, $v_0 > r\omega_0$. Prove that friction firstly acts down the incline and afterwards acts down the incline. Further prove that the total time of rise is $T = \frac{17v_0 + 4r\omega_0}{18g\sin\theta}$.



SOLUTION

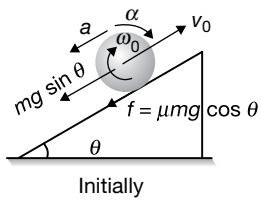
Initially i.e., at $t = 0$, we have $v_0 > r\omega_0$, therefore, there is forward slipping. Hence, friction will be down the incline.

Once, $v = r\omega$, force of friction becomes up the incline, so

$$a = \frac{\mu mg\cos\theta + mg\sin\theta}{m} = \frac{8}{7}g\sin\theta \qquad \left\{\because \mu = \frac{1}{7}\tan\theta\right\}$$

Also, $\alpha = \frac{\tau}{I} = \frac{\mu rmg\cos\theta}{\frac{2}{5}mr^2}$

$$\Rightarrow \alpha = \frac{5}{14}\frac{g\sin\theta}{r} \qquad \left\{\because \mu = \frac{1}{7}\tan\theta\right\}$$



Initially

This will last till pure rolling begins, so

$$v = r\omega$$

$$\Rightarrow \left(v_0 - at_1\right) = r\left(\omega_0 + \alpha t_1\right)$$

Substituting the values, we get

$$t_1 = \frac{2}{3}\frac{\left(v_0 - r\omega_0\right)}{g\sin\theta}$$

Since $v = v_0 - at_1$

$$\Rightarrow v = v_0 - \left(\frac{8}{7}g\sin\theta\right)\left(\frac{2}{3}\right)\left(\frac{v_0 - r\omega_0}{g\sin\theta}\right)$$

$$\Rightarrow v = \frac{5}{21}v_0 + \frac{16}{21}r\omega_0$$

Once $v = r\omega$ i.e., the condition of pure rolling is obtained, then minimum value of μ required to maintain pure rolling is

$$\mu_{\min} = \frac{\tan\theta}{1 + \frac{mr^2}{I}} = \frac{\tan\theta}{1 + \frac{5}{2}} = \frac{\tan\theta}{3.5}$$

However, $\mu_{\text{given}} = \mu = \frac{1}{7}\tan\theta$ and since $\mu_{\text{given}} < \mu_{\min}$ so slipping will occur even after that and so the force of friction is upwards and maximum.

So, linear retardation is

$$a' = g\sin\theta - \mu g\cos\theta = \frac{6}{7}g\sin\theta$$

If t_2 be the further time taken by sphere to rise, then

$$0 = v - a't_2$$

$$\Rightarrow t_2 = \frac{v}{a'} = \frac{\frac{5}{21}v_0 + \frac{16}{21}r\omega_0}{\frac{6}{7}g\sin\theta}$$

$$\Rightarrow t_2 = \frac{5v_0 + 16r\omega_0}{18g\sin\theta}$$

So, we get, $T = t_1 + t_2 = \frac{17v_0 + 4r\omega_0}{18g\sin\theta}$

PROBLEM 5

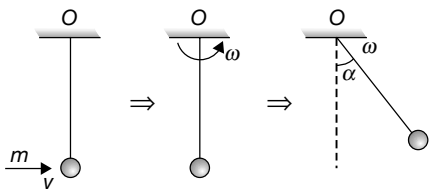
A vertically oriented uniform rod of mass M and length l can rotate about its upper end. A horizontally flying bullet of mass m strikes the lower end of the rod and gets embedded in it as a result of which the rod swings through an angle α. Assuming that $m \ll M$, find the

(a) velocity of the flying bullet.
(b) momentum increment in the system "bullet-rod" during the impact and the cause that changes the momentum.
(c) distance x from the upper end of the rod the bullet must strike for the momentum of the system "bullet-rod" to remain constant during the impact.

SOLUTION

(a) Applying Law of Conservation of Angular Momentum about its upper end, we get

$$\sum mvr_{\perp} = \left(I_{\text{system}}\right)\omega$$



$$\Rightarrow \quad (mv)l = \left(\frac{Ml^2}{3}\right)\omega \qquad \ldots(1)$$

$$\Rightarrow \quad \omega = \frac{3m}{M}\frac{v}{l}$$

After the bullet gets embedded in the rod, the rod swings through an angle α, so by Law of Conservation of Energy we have

$$\begin{pmatrix}\text{Loss in}\\ \text{RKE of}\\ \text{Rod + Bullet}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{GPE of}\\ \text{CM of Rod}\end{pmatrix} + \begin{pmatrix}\text{Gain in}\\ \text{GPE of}\\ \text{Bullet}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{Ml^2}{3}\right)\omega^2 + ml^2\omega^2 = Mg\frac{l}{2}(1-\cos\alpha) + mgl(1-\cos\alpha)$$

Since $m \ll M$, so the underlined terms are neglected

$$\Rightarrow \quad \frac{1}{2}\left(\frac{Ml^2}{3}\right)\omega^2 = Mg\frac{l}{2}(1-\cos\alpha) \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v = \frac{M}{m}\sqrt{\frac{2}{3}gl}\sin\left(\frac{\alpha}{2}\right)$$

(b) $\Delta P = P_f - P_i = M\left(\frac{l}{2}\omega\right) - mv$

$$\Rightarrow \quad \Delta P = \frac{Ml}{2}\left(\frac{3m}{M}\right)\left(\frac{v}{l}\right) - mv = \frac{1}{2}mv$$

$$\Rightarrow \quad \Delta P = \frac{1}{2}m\left(\frac{M}{m}\right)\sqrt{\frac{2}{3}gl}\sin\left(\frac{\alpha}{2}\right)$$

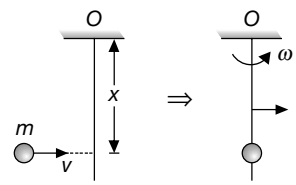
$$\Rightarrow \quad \Delta P = M\sqrt{\frac{1}{6}gl}\sin\left(\frac{\alpha}{2}\right)$$

The force exerted by the support is responsible for the change in momentum.

(c) For Angular Momentum to be conserved, we have

$$L_i = L_f$$

$$\Rightarrow \quad mvx = I\omega = \left(\frac{Ml^2}{3}\right)\omega \qquad \ldots(3)$$



For Linear Momentum to be conserved, we have

$$P_i = P_f$$

$$\Rightarrow \quad mv = Mv_c = M\left(\frac{l}{2}\omega\right) \qquad \ldots(4)$$

From equations (3) and (4), we have

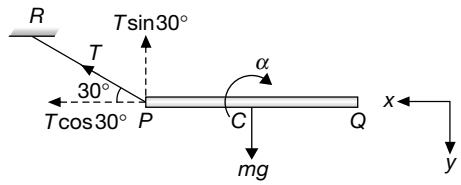
$$x = \frac{2}{3}l$$

PROBLEM 6

A uniform rod of mass m and length l is suspended by the two wires at P and Q. The wire at Q suddenly breaks. Find the tension T in the wire at P immediately after the wire at Q is broken.

SOLUTION

Equation of motion for the centre of mass of rod are



$$a_x = \frac{T\cos 30°}{m} = \frac{\sqrt{3}T}{2m} \quad \ldots(1)$$

$$a_y = \frac{mg - T\sin 30°}{m} = g - \frac{T}{2m} \quad \ldots(2)$$

Angular acceleration of rod about its centre of mass C is

$$\alpha = \frac{\tau}{I} = \frac{(T\sin 30°)\frac{l}{2}}{\frac{ml^2}{12}} = \frac{3T}{ml} \quad \ldots(3)$$

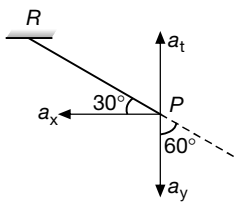
Acceleration of point P immediately after the wire Q breaks will be perpendicular to RP. Thus, along RP its component should be zero. Further

$$\vec{a}_{PC} = \vec{a}_P - \vec{a}_C$$

$$\Rightarrow \quad \vec{a}_P = \vec{a}_C + \vec{a}_{PC}$$

where, $\vec{a}_C$ has two components a_x and a_y.

Similarly $\vec{a}_{PC}$ has also two components a_t and a_n, where $a_n = 0$, because $a_n = \frac{l}{2}\omega^2$ and ω is zero at the instant wire Q breaks. Now, $a_t = \frac{l}{2}\alpha$ in the direction shown in Figure.



Since, acceleration of P along RP is zero so

$$a_t \sin(30°) + a_x \cos(30°) = a_y \cos(60°)$$

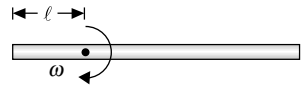
$$\Rightarrow \quad \frac{l}{4}\alpha + \frac{\sqrt{3}}{2}a_x = \frac{a_y}{2} \quad \ldots(4)$$

Solving above equations we get,

$$T = \frac{2}{7}mg$$

PROBLEM 7

A uniform rod of length $4l$ and mass m is free to rotate about a horizontal axis passing through a point distant l from its one end. When the rod is horizontal, its angular velocity is ω as shown in Figure. Calculate



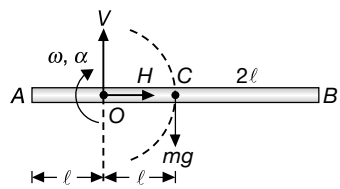
(a) reaction of axis at this instant,
(b) acceleration of centre of mass of the rod at this instant,
(c) reaction of axis and acceleration of centre mass of the rod when rod becomes vertical for the first time.
(d) minimum value of ω so that centre of rod can complete circular motion.

SOLUTION

(a) $\alpha = \frac{mgl}{\frac{m(4l)^2}{12} + ml^2} = \frac{3}{7}\frac{g}{l}$

If a_t be the tangential acceleration of the centre of mass of rod, then

$$a_t = l\alpha = \frac{3}{7}g \quad \{\text{downwards}\}$$



Let V be the vertical reaction (upwards) at axis O, then

$$mg - V = ma_t = \frac{3mg}{7}$$

$$\Rightarrow \quad V = \frac{4}{7}mg \quad \ldots(1)$$

If H be the horizontal reaction (towards C) at axis, then

$$H = ma_n = ml\omega^2 \quad \ldots(2)$$

So, total reaction at axis is

$$N = \sqrt{H^2 + V^2} = \frac{4}{7}mg\sqrt{1 + \left(\frac{7l\omega^2}{4g}\right)^2}$$

(b) The acceleration of centre of mass of rod is

$$a_C = \sqrt{a_t^2 + a_n^2}$$

$$\Rightarrow \quad a_C = \sqrt{\left(\frac{3g}{7}\right)^2 + (l\omega^2)^2}$$

(c) Let ω' be the angular speed of the rod when it becomes vertical for the first time. Then by Law of Conservation of Mechanical Energy, we have

$$\frac{1}{2} I(\omega'^2 - \omega^2) = mgl$$

$$\Rightarrow \quad \omega'^2 = \omega^2 + \frac{2mgl}{I}$$

$$\Rightarrow \quad \omega'^2 = \omega^2 + \frac{2mgl}{\frac{7}{3}ml^2} = \omega^2 + \frac{6g}{7l}$$

Acceleration of centre of mass at this instance is

$$a_C = l\omega'^2 = l\omega^2 + \frac{6g}{7}$$

Let V be the reaction (upwards) at axis at this instant, then,

$$V - mg = ma_C = ml\omega^2 + \frac{6mg}{7}$$

$$\Rightarrow \quad V = \frac{13}{7}mg + ml\omega^2$$

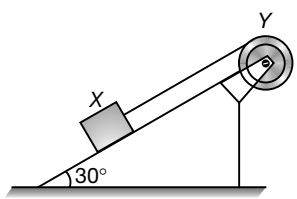
(d) By Law of Conservation of Mechanical Energy, we have

$$mgl = \frac{1}{2} I\omega_{\min}^2$$

$$\Rightarrow \quad \omega_{\min} = \sqrt{\frac{2mgl}{I}} = \sqrt{\frac{2mgl}{\frac{7}{3}ml^2}} = \sqrt{\frac{6g}{7l}}$$

PROBLEM 8

A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination $30°$ to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and of radius 0.2 m as shown in Figure.



The drum is given an initial angular velocity such that the block X starts moving up the plane.

(i) Find the tension in the string during the motion. Take $g = 9.8$ ms^{-2}.

(ii) At a certain instant of time the magnitude of the angular velocity of Y is 10 rads^{-1}. Calculate the distance travelled by X from the instant of time until it comes to rest.

SOLUTION

Mass of block X is $m = 0.5$ kg

Mass of drum Y, $M = 2$ kg

Radius of drum, $R = 0.2$ m

Angle of inclined plane, $\theta = 30^o$

Let 'a' be the linear retardation of block X and α the angular retardation of drum Y. Then,

$$a = R\alpha$$

Since, $mg\sin 30^o - T = ma$ …(1)

$$\Rightarrow \quad \frac{mg}{2} - T = ma \qquad \ldots(2)$$

Also $\alpha = \frac{\tau}{I} = \frac{TR}{\frac{1}{2}MR^2}$

$$\Rightarrow \quad \alpha = \frac{2T}{MR} \qquad \ldots(3)$$

Solving equations (1), (2) and (3) for T, we get

$$T = \frac{1}{2}\left[\frac{Mmg}{M+2m}\right]$$

Substituting the value, we get

$$T = \left(\frac{1}{2}\right)\left\{\frac{(2)(0.5)(9.8)}{2+(0.5)(2)}\right\} = 1.63 \text{ N}$$

$$\Rightarrow \quad T = 1.63 \text{ N}$$

From equation (3), angular retardation of drum

$$\alpha = \frac{2T}{MR} = \frac{(2)(1.63)}{(2)(0.2)} = 8.15 \text{ rads}^{-2}$$

Linear retardation of block is

$$a = R\alpha = (0.2)(8.15) = 1.63 \text{ ms}^{-2}$$

At the moment when angular velocity of drum is

$$\omega_o = 10 \text{ rads}^{-1}$$

The linear velocity of block will be

$$v_o = \omega_o R = (10)(0.2) = 2 \text{ ms}^{-1}$$

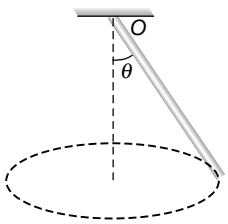
Now, the distance(s) travelled by the block until it comes to rest will be given by

$$s = \frac{v_o^2}{2a} \qquad \{\text{using } v^2 = v_o^2 - 2as \text{ with } v = 0\}$$

$$\Rightarrow \quad s = \frac{(2)^2}{2(1.63)} m = 1.22 \text{ m}$$

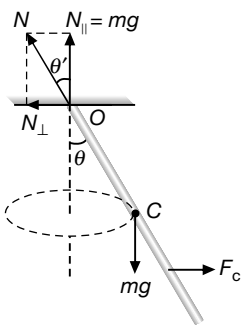
PROBLEM 9

A thin uniform rod of mass m and length l rotates with the constant angular velocity ω about the vertical axis passing through the rod's suspension point O. In doing so, the rod describes a conical surface with a half aperture angle θ. Find the angle θ as well as the magnitude and direction of the reaction force N at the point O.



SOLUTION

Let us consider the frame rotating about the vertical axis along with the rod. In this reference frame, the rod experiences not only the gravity mg and the reaction force N but also the centrifugal force F_c, directed radially outwards as shown in Figure.



Since the rod is observed to be in equilibrium in the given reference frame, the resultant torques of all forces relative to any point and the resultant of all the forces are equal to zero.

The gravity and the centrifugal forces are the only torque producing forces relative to the point O. So, if τ_C is the total torque produced due to centrifugal forces acting on the rod, then

$$\tau_{mg} = \tau_C$$

$$\Rightarrow \quad \frac{mgl}{2}\sin\theta = \tau_c \qquad \ldots(1)$$

Let us now calculate τ_c by integration

The elementary torque due to the centrifugal force that acts on the infinitesimal rod element of length dx located at the distance x from the point O and having mass $dm\left(=\frac{m}{l}dx\right)$ is equal to,

$$d\tau_C = (dF_c)\, x\cos\theta$$

where $dF_C = (dm)(x\sin\theta)\omega^2 = \left(\frac{m}{l}dx\right)(x\sin\theta)\omega^2$

$$\Rightarrow \quad d\tau_c = \frac{m\omega^2}{l}\sin\theta\cos\theta x^2 dx$$

Integrating this expression over the whole length of the rod, we get

$$\tau_c = \frac{m\omega^2 l^2}{3}\sin\theta\cos\theta \qquad \ldots(2)$$

From equation (1) and (2), we get

$$\cos\theta = \frac{3g}{2\omega^2 l} \qquad \ldots(3)$$

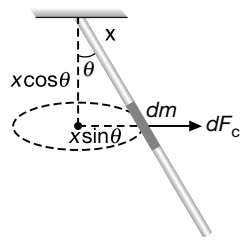
$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{3g}{2\omega^2 l}\right)$$

Further, writing equations for centre of mass of the rod, we get

$$N_{\parallel} = mg$$

and $\quad N_{\perp} = ma_n = \frac{m\omega^2 l}{2}\sin\theta \qquad \ldots(4)$

$$\Rightarrow \quad N = \sqrt{N_{\parallel}^2 + N_{\perp}^2} = \sqrt{(mg)^2 + \left(\frac{m\omega^2 l}{2}\sin\theta\right)^2}$$



Substituting the value of θ from equation (3), we get

$$N = \frac{m\omega^2 l}{2}\sqrt{1+\frac{7g^2}{4\omega^4 l^2}}$$

For its direction, we observe that

$$\cos\theta' = \frac{mg}{N}$$

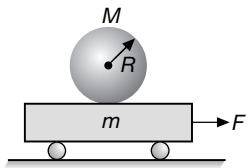
Substituting the values, we get

$$\cos\theta' = 4\cos\theta\sqrt{9+7\cos^2\theta}$$

The centrifugal force F_c does not pass through point C but below it. (Think, why?)

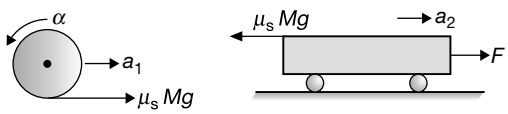
PROBLEM 10

Determine the maximum horizontal force F that may be applied to the plank of mass m for which the solid sphere does not slip as it begins to roll on the plank. The sphere has a mass M and radius R. The coefficient of static and kinetic friction between the sphere and the plank are μ_s and μ_k respectively.



SOLUTION

The free body diagrams for the sphere and the plank are shown in Figure.



For sphere: Linear acceleration

$$a_1 = \frac{\mu_s Mg}{M} = \mu_s g \qquad \ldots(1)$$

Angular acceleration

$$\alpha = \frac{(\mu_s Mg)R}{\frac{2}{5}MR^2} = \frac{5}{2}\frac{\mu_s g}{R} \qquad \ldots(2)$$

For plank: Linear acceleration

$$a_2 = \frac{F-\mu_s Mg}{m} \qquad \ldots(3)$$

For no slipping, we have

$$a_2 = a_1 + R\alpha \qquad \ldots(4)$$

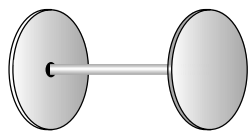
Solving the above four equations, we get

$$F = \mu_s g\left(M+\frac{7}{2}m\right)$$

Thus, maximum value of F can be $\mu_s g\left(M+\frac{7}{2}m\right)$

PROBLEM 11

Two thin circular discs of mass $2\ \text{kg}$ and radius $10\ \text{cm}$ each are joined by a rigid massless rod of length $20\ \text{cm}$. The axis of the rod is along the perpendicular to the planes of the disc through their centres. This object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of motion of the truck. Its friction with the floor of the truck is large enough so that the object can roll on the truck without slipping. Take X-axis as the direction of motion of the truck and Z-axis as the vertically upwards direction. If the truck has an acceleration $9\ \text{ms}^{-2}$, calculate



(i) the force of friction on each disc.

(ii) the magnitude and direction of the friction torque acting on each disc about the centre of mass O of the object. Express the torque in the vector form in terms of unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ in X, Y and Z directions.

SOLUTION

Given mass of disc $m = 2\ \text{kg}$ and radius $R = 0.1\ \text{m}$

(i) FBD of any one disc is

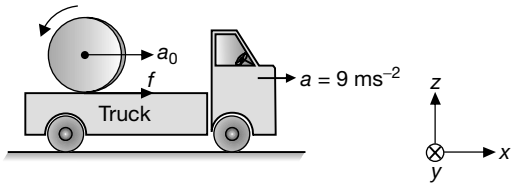
Frictional force on the disc should be in forward direction.

Let a_0 be the linear acceleration of centre of mass of disc and α the angular acceleration about its centre of mass. Then,

$$a_0 = \frac{f}{m} = \frac{f}{2} \qquad \ldots(1)$$

$$\alpha = \frac{\tau}{I} = \frac{fR}{mR^2/2} = \frac{2f}{mR} = \frac{2f}{2(0.1)} = 10f \qquad \ldots(2)$$

Since, there is no slipping between disc and truck.



Therefore,

$$\left(\begin{array}{c}\text{Acceleration of the}\\ \text{contact point}\end{array}\right)=\left(\begin{array}{c}\text{Acceleration of}\\ \text{the truck}\end{array}\right)$$

$$\Rightarrow\ a_0 + R\alpha = a$$

$$\Rightarrow\ \left(\frac{f}{2}\right)+(0.1)(10f)=a$$

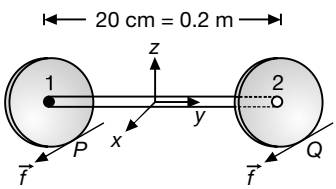
$$\Rightarrow\ \frac{3}{2}f = a$$

$$\Rightarrow\ f=\frac{2a}{3}=\frac{2\times 9}{3}\ N$$

$$\Rightarrow\ f = 6\ N$$

Since, this force is acting in positive x-direction, so in vector form,

$$\vec{f}=\left(6\hat{i}\right)N$$



(ii) $\vec{\tau}=\vec{r}\times\vec{f}$

where, $\vec{f}=\left(6\hat{i}\right)N$ (for both the discs)

$$\vec{r}_P=\vec{r}_1=-0.1\hat{j}-0.1\hat{k}$$

and $\vec{r}_Q=\vec{r}_2=0.1\hat{j}-0.1\hat{k}$

Therefore, frictional torque on disk 1 about point O (centre of mass).

$$\vec{\tau}_1=\vec{r}_1\times\vec{f}=\left(-0.1\hat{j}-0.1\hat{k}\right)\times\left(6\hat{i}\right)\text{Nm}$$

$$\vec{\tau}_1=\left(0.6\hat{k}-0.6\hat{j}\right)$$

$$\Rightarrow\ \vec{\tau}_1=0.6\left(\hat{k}-\hat{j}\right)\ \text{Nm}$$

and $|\vec{\tau}_1|=\sqrt{(0.6)^2+(0.6)^2}=0.85\ \text{Nm}$

Similarly, $\vec{\tau}_2=\vec{r}_2\times\vec{f}=0.6\left(-\hat{j}-\hat{k}\right)$

and $|\vec{\tau}_1|=|\vec{\tau}_2|=0.85\ \text{Nm}$

PROBLEM 12

Show that if a rod held at angle θ to the horizontal and released, its lower end will not slip if the friction coefficient between rod and ground is greater than $\frac{3}{2}\left(\frac{\sin 2\theta}{1+3\sin^2\theta}\right)$.

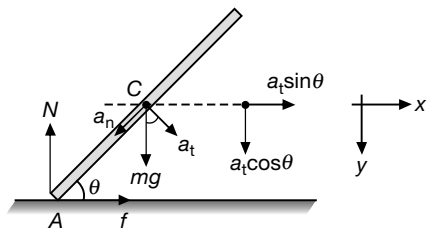
SOLUTION

Point A is momentarily at rest, so

$$\alpha=\frac{\tau}{I}=\frac{mg(l/2)\cos\theta}{ml^2/3}=\frac{3g\cos\theta}{2l}$$

If a_t be the tangential acceleration of the centre of mass of rod C, then

$$a_t=\frac{l}{2}\alpha=\frac{3}{4}g\cos\theta$$



Now, $\mu N = ma_x$ or $\mu N = ma_C\sin\theta$

$$\Rightarrow\quad \mu N=\frac{3}{4}mg\sin\theta\cos\theta \qquad \ldots(1)$$

Further, $mg-N=ma_y$

$$\Rightarrow\quad N=mg-ma_t\cos\theta$$

$$\Rightarrow\quad N=mg-\frac{3}{4}mg\cos^2\theta \qquad \ldots(2)$$

Dividing equation (1) by (2), we get

$$\mu=\frac{\frac{3}{4}\sin\theta\cos\theta}{1-\frac{3}{4}\cos^2\theta}=\frac{3\sin\theta\cos\theta}{4-3\cos^2\theta}$$

$$\Rightarrow \quad \mu = \frac{3\sin\theta\cos\theta}{1+3\sin^2\theta} = \frac{3}{2}\left(\frac{2\sin\theta\cos\theta}{1+3\sin^2\theta}\right)$$

$$\Rightarrow \quad \mu = \frac{3}{2}\left(\frac{\sin(2\theta)}{1+3\sin^2\theta}\right)$$

PROBLEM 13

A uniform stick of length L, mass M hinged at one end is released from rest at an angle θ_0 with the vertical. When the angle with the vertical is θ, the hinge exerts a force F_r along the stick and F_t perpendicular to the stick. Calculate F_r and F_t.

SOLUTION

Let C be the centre of mass of the rod and ω be the angular speed of rod about point O at angle θ. By Law of Conservation of Mechanical Energy, we have

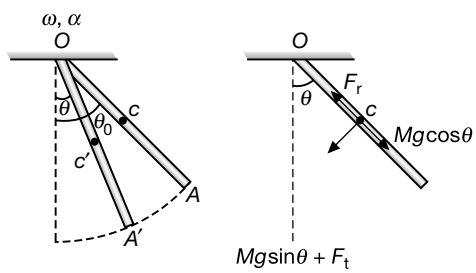
$$Mg\frac{L}{2}(\cos\theta - \cos\theta_0) = \frac{1}{2}\left(\frac{ML^2}{3}\right)\omega^2$$

$$\Rightarrow \quad \omega^2 = \frac{3g}{L}(\cos\theta - \cos\theta_0) \qquad \ldots(1)$$

Since, $F_r - Mg\cos\theta = M\left(\frac{L}{2}\right)\omega^2$ $\qquad \ldots(2)$

From equations (1) and (2), we get

$$F_r = \frac{1}{2}Mg(5\cos\theta - 3\cos\theta_0)$$



Angular acceleration of rod at this instant,

$$\alpha = \frac{\tau}{I} = \frac{Mg\left(\frac{L}{2}\sin\theta\right)}{\frac{ML^2}{3}} = \frac{3}{2}\frac{g\sin\theta}{L}$$

Tangential acceleration of centre of mass C is

$$a_t = r\alpha = (\alpha)\left(\frac{L}{2}\right) = \frac{3}{4}g\sin\theta \qquad \ldots(3)$$

Now, $F_t + Mg\sin\theta = Ma_t$ $\qquad \ldots(4)$

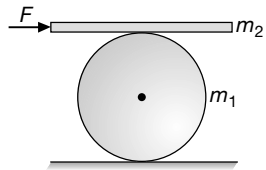
From equation (3) and (4), we get

$$F_t = -\frac{1}{4}Mg\sin\theta$$

Here negative sign implies that direction of F_t is opposite to the component $Mg\sin\theta$.

PROBLEM 14

A man pushes a cylinder of mass m_1 with the help of a plank of mass m_2 as shown. There is no slipping at any contact. The horizontal component of the force applied by the man is F, find



(a) the acceleration of the plank and the centre of mass of the cylinder, and

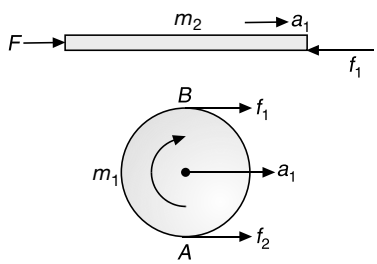
(b) the magnitudes and directions of frictional forces at contact points.

SOLUTION

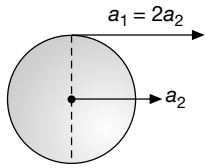
We can choose my arbitrary directions of frictional forces at different contacts. In the final answer the negative values will show the opposite directions.

Let the friction between plank and cylinder, i.e. at contact point B be f_1, the friction between cylinder and ground, i.e. at contact point A be f_2, acceleration of the plank be $a_p = a_1$, acceleration of centre of mass of cylinder be $a_c = a_2$ and angular acceleration of cylinder about its CM be α.

Directions of f_1 and f_2 are as shown in Figure.



Since, there is no slipping anywhere, so acceleration of plank equals the acceleration of top point of cylinder, so we have



$$a_1 = 2a_2 \quad \ldots(1)$$

Also, $a_1 = \dfrac{F - f_1}{m_2}$ $\ldots(2)$

$$a_2 = \frac{f_1 + f_2}{m_1} \quad \ldots(3)$$

If I be moment of inertia of cylinder about CM, then

$$\alpha = \frac{(f_1 - f_2)R}{I} = \frac{(f_1 - f_2)R}{m_1R^2/2}$$

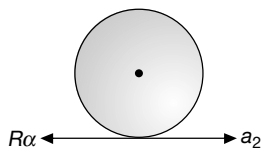
$$\Rightarrow \quad \alpha = \frac{2(f_1 - f_2)}{m_1R} \quad \ldots(4)$$

$$\Rightarrow \quad a_2 = R\alpha = \frac{2(f_1 - f_2)}{m_1} \quad \ldots(5)$$

Acceleration of bottom most point of cylinder is zero.

(a) Solving equations (1), (2), (3) and (5), we get

$$a_1 = \frac{8F}{3m_1 + 8m_2} \text{ and } a_2 = \frac{4F}{3m_1 + 8m_2}$$



(b) $f_B = f_1 = \dfrac{3m_1F}{3m_1 + 8m_2}$ and $f_A = f_2 = \dfrac{m_1F}{3m_1 + 8m_2}$

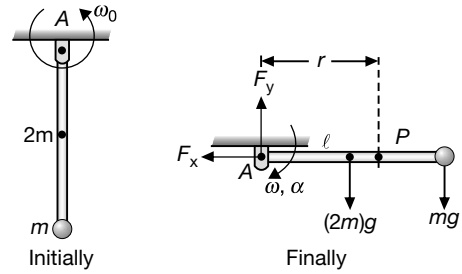
Since, all quantities are positive, they are correctly shown in the above Figures.

PROBLEM 15

A uniform rod AB of length $2l$ and mass $2m$ has a particle of mass m attached at B. The rod is free to rotate in a vertical plane about a horizontal axis through A. When the rod is hanging at rest, with B below A, it is given an angular velocity $\dfrac{7}{2}\sqrt{\dfrac{g}{l}}$. Find the reaction at the axis when the rod first becomes horizontal. Take $g = 10\ \text{ms}^{-2}$.

SOLUTION

When the rod first becomes horizontal, this situation is shown in Figure.



By Law of Conservation of Mechanical Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{RKE of}\\ \text{Particle + Rod}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{GPE of}\\ \text{CM of Rod}\end{pmatrix} + \begin{pmatrix}\text{Gain in}\\ \text{GPE of}\\ \text{Particle}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}I_A(\omega_0^2 - \omega^2) = (2m)gl + mg(2l)$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{2m(2l)^2}{3} + m(2l)^2\right)\left(\frac{49g}{4l} - \omega^2\right) = 4mgl$$

$$\Rightarrow \quad \omega^2 = \frac{221g}{20l}$$

In the horizontal position of the rod, we have

$$\tau = (2m)gl + mg(2l) = 4mgl$$

Since $\tau = I\alpha$

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{2mgl + mg(2l)}{\left(\dfrac{2m(2l)^2}{3} + m(2l)^2\right)} = \frac{3}{5}\frac{g}{l}$$

Let the CM of Rod plus Particle be at point P, a distance r from A. Then

$$r = \frac{(2m)l + m(2l)}{3m} = \frac{4l}{3}$$

$$\Rightarrow \quad a_y = a_t = r\alpha = \left(\frac{4l}{3}\right)\left(\frac{3g}{5l}\right) = 0.8g$$

Since $3mg - F_y = (3m)a_t$

$$\Rightarrow \quad F_y = 3mg - 3m(0.8g)$$

$$\Rightarrow \quad F_y = 0.6mg \text{, upwards}$$

Also, $a_x = a_c = r\omega^2 = \left(\dfrac{4l}{3}\right)\left(\dfrac{221g}{20l}\right) = 14.73g$

Force exerted by hinge in x-direction is

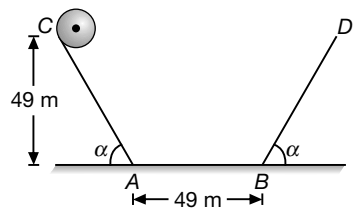
$$F_x = F_c = (3m)a_c = 44.2mg$$

So, total hinge force is given by

$$F = \sqrt{F_x^2 + F_y^2} = 442 \text{ mN}$$

PROBLEM 16

A solid sphere is released from a height of 49 m from horizontal on an inclined plane as shown in Figure. All the surfaces are smooth except AB which has $\mu = \frac{2}{3}$ and length of AB is 49 m. After passing over the rough surface, the sphere climbs to the right portion BD of incline.



(a) Find the height from horizontal to which it climbs on BD.

(b) Now the sphere returns and goes up the incline AC. Find the linear velocity of sphere at A during the left-out motion.

SOLUTION

From C to A we have, velocity at A is

$$v_A = \sqrt{2 \times g \times h} = \sqrt{2 \times 10 \times 49} = 31.3 \text{ ms}^{-1}$$

From A to B, let pure rolling starts after time t.

$$v = R\omega$$

$$\Rightarrow \quad v_A - at = R(\alpha t)$$

$$\Rightarrow \quad v_A - \mu g t = R\left(\frac{\mu mgR}{2mr^2/5}\right)t = 2.5\mu g t$$

$$\Rightarrow \quad t = \frac{v_A}{3.5\mu g} = \frac{31.3}{3.5 \times (2/3) \times 10} = 1.34 \text{ s}$$

Now, $s = v_A t - \frac{1}{2}(\mu g)t^2$

$$\Rightarrow \quad s = (31.3)(1.34) - \frac{1}{2}\left(\frac{2}{3}\right)(10) \times (1.34)^2$$

$$\Rightarrow \quad s = 36 \text{ m}$$

Since velocity $v = v_A - at$, so we have

$$v = v_A - \mu g t = 31.3 - \frac{2}{3} \times 10 \times 1.34 = 22.36 \text{ ms}^{-1}$$

The remaining distance of $49 - 36 = 13$ m the velocity is maintained at 22.36 ms^{-1}, because of pure rolling the friction is zero. So, the height raised above the horizontal is

$$H = \frac{v^2}{2g} = \frac{(22.36)^2}{2 \times 10} = 25 \text{ m}$$

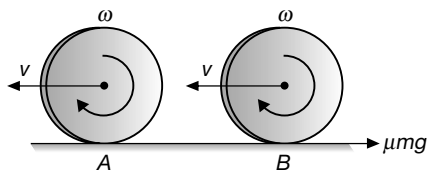
From B to A, let pure rolling start at time t', then

$$v' = R\omega'$$

where $\omega' = -\omega + \alpha t'$ and $v' = v - \mu g t'$

$$\Rightarrow \quad v - \mu g t' = -\omega R + \alpha R t'$$

$$\Rightarrow \quad t' = \frac{v + R\omega}{3.5\mu g} = \frac{2v}{3.5\mu g}$$



$$\Rightarrow \quad t' = \frac{2 \times 22.36}{3.5 \times (2/3) \times 10} = 1.91 \text{ s}$$

So, $v' = v - \mu g t$

$$\Rightarrow \quad v' = 22.36 - \frac{2}{3} \times 10 \times 1.91 = 9.62 \text{ ms}^{-1}$$

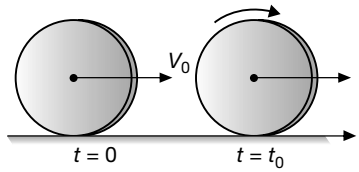
$$\Rightarrow \quad s' = 22.36 \times 1.91 - \frac{1}{2} \times \frac{2}{3} \times 10 \times (1.91)^2$$

$$\Rightarrow \quad s' = 30.53 \text{ m}$$

Since s' is less than 49 m, velocity of sphere at A during the remaining journey is 9.62 ms^{-1}.

PROBLEM 17

A uniform disc of mass m and radius R is projected horizontally with velocity V_0 on a rough horizontal floor, so that it starts off with a purely sliding motion at $t = 0$. After t_0 second, it acquires a purely rolling motion as shown in Figure.



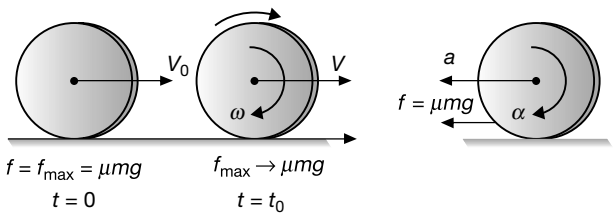
(a) Calculate the velocity of the centre of mass of the disc at t_0.

(b) Assuming the coefficient of friction to be μ, calculate t_0. Also calculate the work done by the frictional force as a function of time and the total work done by it over a time t much longer than t_0.

SOLUTION

Between the time $t = 0$ to $t = t_0$, there is forward sliding, so friction f is leftwards and maximum, i.e. μmg.

For time $t > t_0$, friction f will become zero, because now pure rolling has started i.e., there is no sliding (no relative motion) between the points of contact.



So, for time $t < t_0$, the linear retardation and the angular acceleration respectively are

$$a = \frac{f}{m} = \mu g \qquad \{\because f = \mu mg\}$$

$$\alpha = \frac{\tau}{I} = \frac{fR}{mR^2/2} = \frac{2\mu g}{R}$$

Now let V be the linear velocity and ω, the angular velocity of the disc at time $t = t_0$ then

$$V = V_0 - at_0 = V_0 - \mu g t_0 \qquad \ldots(1)$$

and $$\omega = \alpha t_0 = \frac{2\mu g t_0}{R} \qquad \ldots(2)$$

For pure rolling to take place

$$V = R\omega$$

i.e., $$V_0 - \mu g t_0 = 2\mu t_0$$

$$\Rightarrow \quad t_0 = \frac{V_0}{3\mu g}$$

Substituting in equation (1), we get

$$V = V_0 - \mu g\left(\frac{V_0}{3\mu g}\right)$$

$$\Rightarrow \quad V = \frac{2}{3}V_0$$

For $t \le t_0$, the linear velocity of disc at any time t is $V = V_0 - \mu g t$ and angular velocity is $\omega = \alpha t = \frac{2\mu g t}{R}$.

From work-energy theorem, work done by friction up to time t equals the change in kinetic energy, so we have

$$\Rightarrow \quad W = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 - \frac{1}{2}mV_0^2$$

$$\Rightarrow \quad W = \frac{1}{2}m\left(V_0 - \mu g t\right)^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{2\mu g t}{2}\right)^2 - \frac{1}{2}mV_0^2$$

$$\Rightarrow \quad W = \frac{m}{2}\left(V_0^2 + \mu^2 g^2 t^2 - 2V_0\mu g t + 2\mu^2 g^2 t^2 - V_0^2\right)$$

$$\Rightarrow \quad W = \frac{\mu mgt}{2}\left(3\mu g t - 2V_0\right)$$

For $t > t_0$, friction force is zero i.e., work done in friction is zero. Hence the energy will be conserved. Therefore, total work done by friction over a time t much longer then t_0 is total work done up to time t_0 (because beyond the work done by friction is zero) which is equal to

$$W = \frac{m\mu g t_0}{2}\left(3\mu g t_0 - 2V_0\right)$$

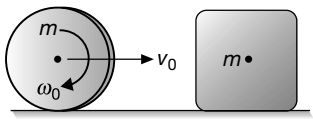
Substituting $t_0 = \frac{V_0}{3\mu g}$, we get

$$W = \frac{mV_0}{6}\left(V_0 - 2V_0\right)$$

$$\Rightarrow \quad W = -\frac{mV_o^2}{6}$$

PROBLEM 18

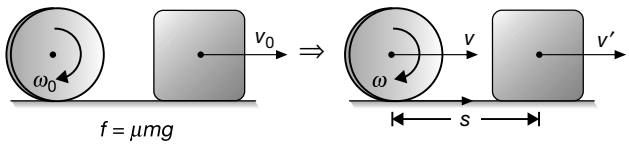
A solid cylinder of mass m is rolling on a rough horizontal surface with velocity v_0. It collides elastically with a cubical block of same mass at rest. The height of centre of mass of both the bodies is same. Assume that there is no friction between the cylinder and the block and coefficient of friction between all other surfaces is μ, find the



(a) velocity of block when cylinder starts pure rolling again.
(b) time after first collision at which the second collision will take place.

SOLUTION

In head on elastic collision between two equal masses velocities are exchanged.



Let the cylinder have radius R and let it start rolling after time t_1, then

$$v = R\omega \qquad \ldots(1)$$

where $v = at_1 = (\mu g)t_1$ and

$$\omega = \omega_0 - \alpha t = \omega_0 - \left(\frac{fR}{I}\right)t_1$$

$$\Rightarrow \quad \omega = \omega_0 - \left(\frac{2\mu g}{R}\right)t_1$$

So, from (1) i.e., $v = R\omega$, we get

$$\mu g t_1 = R\left(\omega_0 - \frac{2\mu g t_1}{R}\right)$$

$$\Rightarrow \quad \mu g t_1 = R\left(\frac{v_0}{R} - \frac{2\mu g t_1}{R}\right)$$

$$\Rightarrow \quad t_1 = \frac{v_0}{3\mu g}$$

Since, $v = \mu g t_1$

$$\Rightarrow \quad v = (\mu g)\left(\frac{v_0}{3\mu g}\right) = \frac{v_0}{3}$$

During this time t_1, the block moves forward under a retardation μg, so if v' be the velocity of block when cylinder starts pure rolling at t_1, then

$$v' = v_0 - \mu g t_1 = v_0 - \frac{v_0}{3} = \frac{2v_0}{3}$$

After the first collision, we have

$$v_r = v_0, \; a_r = -2\,\mu g$$

Since, $s = v_r t_1 + \frac{1}{2}a_r t_1^2$

$$\Rightarrow \quad s = \frac{v_0^2}{3\mu g} - \frac{v_0^2}{9\mu g} = \frac{2}{9}\frac{v_0^2}{\mu g}$$

After the cylinder starts pure rolling friction on cylinder becomes zero and $v_r = v' - v = \frac{v_0}{3}$, $a_r = -\mu g$

Let the second collision takes place after time t_2, then

$$-s = v_r t + \frac{1}{2}a_r t^2$$

Substituting the values, we get

$$-\frac{2}{9}\frac{v_0^2}{\mu g} = \frac{v_0}{3}t_2 - \frac{1}{2}\mu g t_2^2$$

$$\Rightarrow \quad t_2 = \left(\frac{\sqrt{5}+1}{3\mu g}\right)v_0$$

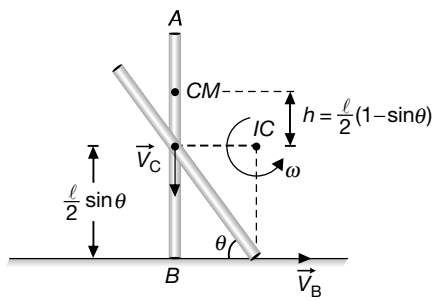
$$\Rightarrow \quad t = t_1 + t_2 = \frac{(\sqrt{5}+2)v_0}{3\mu g}$$

PROBLEM 19

A uniform thin rod of mass m and length l is standing on a smooth horizontal surface. A slight disturbance causes the lower end to slip on the smooth surface and the rod starts falling. Describe the trajectory of the centre of mass. Find the velocity of centre of mass of the rod at the instant when it makes an angle θ with horizontal. Also find the velocity of centre of mass when it touches the surface.

SOLUTION

Since the floor is smooth so the mechanical energy of the rod will remain conserved. Further, no horizontal force acts on the rod, hence the centre of mass moves vertically downwards in a straight line. Thus, velocities of centre of mass and the lower end B are in the directions shown in Figure.



The location of IC at this instant is found by drawing perpendiculars to $\vec{v}_C$ and $\vec{v}_B$ at respective points and IC is located at the point of intersection of these two perpendiculars. So, the rod may now be assumed to be in pure rotational motion about $IAOR$ passing through IC with angular speed ω.

Applying Law of Conservation of Mechanical Energy. we get

$$\begin{pmatrix}\text{Decrease in GPE}\\ \text{of CM of Rod}\end{pmatrix}=\begin{pmatrix}\text{Increase in RKE}\\ \text{of Rod about } IAOR\end{pmatrix}$$

$$\Rightarrow \quad mgh=\frac{1}{2}I_{IAOR}\omega^2$$

$$\Rightarrow \quad mg\frac{l}{2}(1-\sin\theta)=\frac{1}{2}\left(\frac{ml^2}{12}+\frac{ml^2}{4}\cos^2\theta\right)\omega^2$$

Solving this equation, we get

$$\omega=\sqrt{\frac{12g(1-\sin\theta)}{l(1+3\cos^2\theta)}}$$

Now, $|\vec{v}_c|=\left(\frac{l}{2}\cos\theta\right)\omega$

$$\Rightarrow \quad |\vec{v}_c|=\sqrt{\frac{3gl(1-\sin\theta)\cos^2\theta}{(1+3\cos^2\theta)}}$$

When the rod touches the surface, then $\theta=0°$, so

$$|\vec{v}_c|=\sqrt{\frac{3gl}{4}}$$

PROBLEM 20

A uniform rod AB of mass m and length $2a$ has a particle of mass m attached to the end B. The rod can rotate in a vertical plane about a smooth axis through A. If the body is slightly displaced from the position in which B is vertically above A, find the magnitude of the reaction at the axis, which is horizontal, when the rod has rotated through $\frac{\pi}{3}$ radians.

SOLUTION

By Law of Conservation of Energy, we get

$$\begin{pmatrix}\text{Gain in}\\ \text{RKE of}\\ \text{Rod}\end{pmatrix}=\begin{pmatrix}\text{Loss in}\\ \text{GPE of}\\ \text{CM of Rod}\end{pmatrix}+\begin{pmatrix}\text{Loss in}\\ \text{GPE of}\\ \text{Particle}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}\left[\frac{m(2a)^2}{3}+m(2a)^2\right]\omega^2=$$
$$mg(2a)(1-\cos 60°)+mg(a)(1-\cos 60°)$$

$$\Rightarrow \quad \frac{8}{3}ma^2\omega^2=\frac{3mga}{2}$$

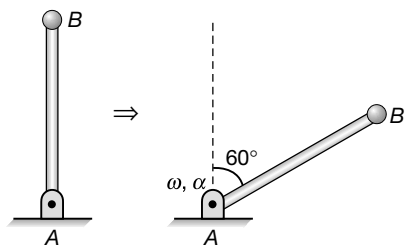
$$\Rightarrow \quad \omega^2=\frac{9}{16}\frac{g}{a} \qquad \ldots(1)$$

Now, $\alpha=\frac{\tau}{I}=\frac{mg(a\sin 60°)+mg(2a\sin 60°)}{\frac{16}{3}ma^2}$

$$\Rightarrow \quad \alpha=\frac{9\sqrt{3}}{32}\frac{g}{a} \qquad \ldots(2)$$

Now centre of mass of the system (point C) is at a distance of $\frac{3a}{2}$ from point A. Acceleration of C has two components a_t and a_n, where

$$a_t=\left(\frac{3a}{2}\right)\alpha=\frac{27\sqrt{3}}{64}g \text{ and } a_n=\left(\frac{3a}{2}\right)\omega^2=\frac{27}{32}g$$

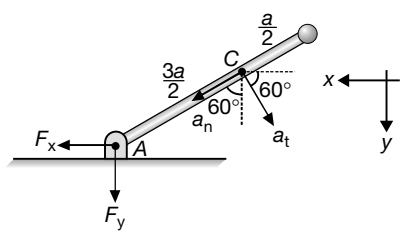


Force applied by hinge in x-direction

$$F_x=2ma_x=2m(a_n\sin 60°-a_t\cos 60°)$$

$$\Rightarrow \quad F_x=2m\left(\frac{27\sqrt{3}}{64}-\frac{27\sqrt{3}}{128}\right)g$$

$$\Rightarrow \quad F_x=\frac{54\sqrt{3}}{128}mg$$



Similarly, $F_y+(2m)g=(2m)a_y$

$$\Rightarrow \quad F_y=2m(a_t\sin 60°+a_n\cos 60°)-2mg$$

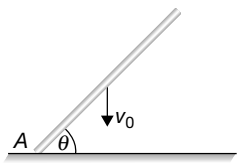
$$\Rightarrow \quad F_y=2m\left(\frac{81}{128}+\frac{27}{64}\right)g-2mg=\frac{270}{128}mg-2mg$$

$$\Rightarrow \quad F_y = \frac{14}{128}mg$$

So, $$F = \sqrt{F_x^2 + F_y^2} = \frac{mg}{32}\sqrt{559}$$

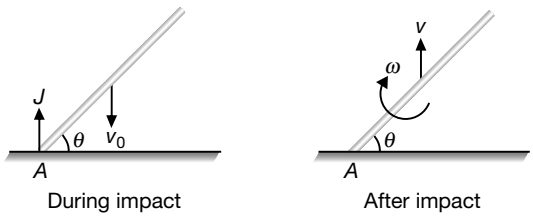
PROBLEM 21

A rod of length l forming an angle θ with the horizontal strikes a frictionless floor at A with its centre of mass velocity v_0 and no angular velocity. Assuming that the impact at A is perfectly elastic, find the angular velocity of the rod immediately after the impact.



SOLUTION

If v be the linear velocity of rod after impact (upwards), ω be the angular velocity of rod and J be the linear impulse at A during impact, then by Impulse – Momentum Theorem, we have



$$J = \Delta p = p_f - p_i$$

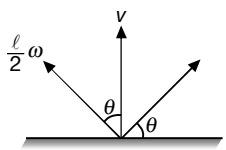
$$\Rightarrow \quad J = mv - (-mv_0)$$

$$\Rightarrow \quad J = m(v + v_0) \qquad \ldots(1)$$

Further by angular impulse angular momentum theorem, we have

$$J\left(\frac{l}{2}\cos\theta\right) = I\omega = \frac{ml^2}{12}\omega \qquad \ldots(2)$$

Since the collision is elastic, so at the point of impact, $e = 1$.



$$\Rightarrow \quad \begin{pmatrix}\text{Relative speed}\\ \text{of approach}\end{pmatrix} = \begin{pmatrix}\text{Relative speed}\\ \text{of separation}\end{pmatrix}$$

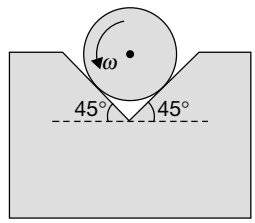
$$\Rightarrow \quad v_0 = v + \frac{l}{2}\omega\cos\theta \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$\omega = \frac{6v_0\cos\theta}{l(1+3\cos^2\theta)}$$

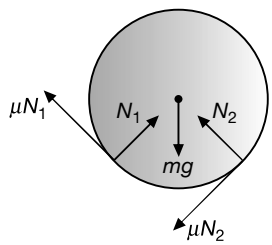
PROBLEM 22

A cylinder of mass M and radius R is rotated in a uniform V shaped groove with constant angular velocity ω. The coefficient of friction between the cylinder and each surface is μ. What torque must be applied to the cylinder to keep it rotating?



SOLUTION

The forces acting on the cylinder are shown in Figure.



For vertical equilibrium of cylinder, we have

$$N_1\cos 45° + N_2\cos 45° + \mu N_1\cos 45° = \mu N_2\cos 45° + Mg$$

$$\Rightarrow \quad N_1(1+\mu) + N_2(1-\mu) = \sqrt{2}Mg \qquad \ldots(1)$$

For horizontal equilibrium of cylinder,

$$N_1\sin 45° = N_2\sin 45° + \mu N_1\sin 45° + \mu N_2\sin 45°$$

$$\Rightarrow \quad N_1(1-\mu) - N_2(1+\mu) = 0 \qquad \ldots(2)$$

Solving these two equations, we get

$$2N_1(1+\mu^2) = \sqrt{2}Mg(1+\mu)$$

$$\Rightarrow \quad N_1 = \frac{\sqrt{2}Mg(1+\mu)}{2(1+\mu^2)}$$

From (2), we get

$$N_2 = \frac{\sqrt{2}Mg(1-\mu)}{2(1+\mu^2)}$$

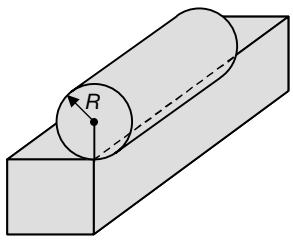
The required torque is,

$$\tau = \mu N_1 R + \mu N_2 R = \mu R(N_1 + N_2)$$

$$\Rightarrow \quad \tau = \frac{\sqrt{2}\mu MgR}{(1+\mu^2)}$$

PROBLEM 23

A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane as shown in Figure.

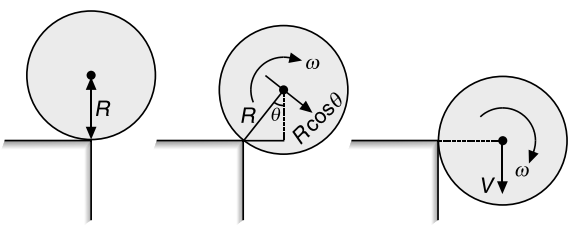


There is sufficient friction present at the edge so that a very small displacement causes the cylinder to roll off the edge without slipping. Determine:

(a) the angle θ_c through which the cylinder rotates before it leaves contact with the edge.
(b) the speed of the centre of mass of the cylinder before leaving contact with the edge, and
(c) the ratio of the translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.

SOLUTION

The cylinder rotates about the point of contact as shown in Figure.



Hence, the mechanical energy of the cylinder will be conserved i.e.,

$$(PE + KE) = (PE + KE)_2$$

$$\Rightarrow \quad mgR + 0 = mgR\cos\theta + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

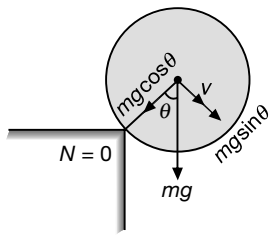
where, $I = \frac{1}{2}mR^2$ and for no slipping at point of contact, we have $\omega = \frac{v}{R}$

$$\Rightarrow \quad mgR = mgR\cos\theta + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right) + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{3}{4}v^2 = gR(1-\cos\theta)$$

$$\Rightarrow \quad v^2 = \frac{4}{3}gR(1-\cos\theta)$$

$$\Rightarrow \quad \frac{v^2}{R} = \frac{4}{3}g(1-\cos\theta) \qquad \ldots(1)$$



At the time of leaving contact, normal reaction $N = 0$ and $\theta = \theta_c$ hence,

$$mg\cos\theta = \frac{mv^2}{R}$$

$$\Rightarrow \quad \frac{v^2}{R} = g\cos\theta \qquad \ldots(2)$$

From equations (1) and (2),

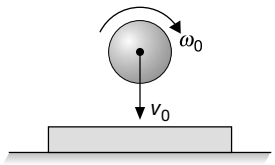
$$\frac{4}{3}g(1-\cos\theta_c) = g\cos\theta_c$$

$$\Rightarrow \quad \frac{7}{4}\cos\theta_c = 1$$

$$\Rightarrow \quad \cos\theta_c = \frac{4}{7}$$

PROBLEM 24

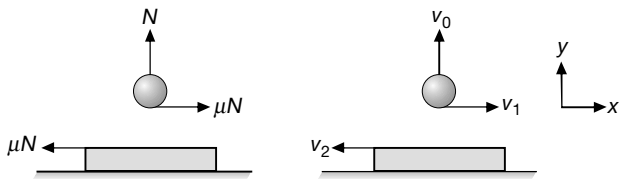
A spherical ball of radius r and mass m collides with a plank of mass M kept on a smooth horizontal surface. Before impact, the centre of the ball has a velocity v_0 and angular velocity ω_0 as shown in Figure.



The normal velocity is reversed with same magnitude and the ball stops rotating after the impact. Find the distance on the plank between first two impacts of the ball. The coefficient of friction between the ball and the plank is μ. Assume that plank is large enough.

SOLUTION

The forces during impact are shown in Figure.



Let the horizontal velocities of the ball and the plank be v_1 and v_2 in opposite directions as shown in Figure, then by Law of Conservation of Linear Momentum in horizontal direction, we get

$$mv_1 = Mv_2 \qquad \ldots(1)$$

According to Impulse Momentum Theorem,

$$\begin{pmatrix}\text{Linear Impulse}\\ \text{of the ball in}\\ \text{vertical direction}\end{pmatrix} = \begin{pmatrix}\text{Change in}\\ \text{Linear Momentum}\\ \text{in vertical direction}\end{pmatrix}$$

$$\Rightarrow \quad J = Ndt$$

$$\Rightarrow \quad J = 2mv_0 \qquad \ldots(2)$$

$$\begin{pmatrix}\text{Linear Impulse}\\ \text{on the ball in}\\ \text{horizontal}\\ \text{direction}\end{pmatrix} = \begin{pmatrix}\text{Change in}\\ \text{Linear Momentum}\\ \text{in horizontal}\\ \text{direction}\end{pmatrix}$$

$$\Rightarrow \quad \mu J = mv_1 \qquad \ldots(3)$$

According to Angular Impulse Angular Momentum Theorem, we have

$$\begin{pmatrix}\text{Angular Impulse}\\ \text{on the ball about}\\ \text{Centre of Mass}\end{pmatrix} = \begin{pmatrix}\text{Change in}\\ \text{Angular Momentum}\\ \text{about Centre of Mass}\end{pmatrix}$$

$$\Rightarrow \quad (\mu J)r = I\omega_0 = \left(\frac{2}{5}mr^2\right)\omega_0 \qquad \ldots(4)$$

Solving equations (1), (2), (3) and (4), we get

$$v_1 = \frac{2}{5}r\omega_0 \text{ and } v_2 = \frac{m}{M}\left(\frac{2}{5}r\omega_0\right)$$

Now, actual path of the ball is a projectile whose time of flight will be

$$T = \frac{2v_y}{g} = \frac{2v_0}{g}$$

Relative velocity of ball with respect to plank in horizontal direction is

$$v_r = v_1 + v_2 = \left(\frac{M+m}{M}\right)v_1 = \frac{2}{5}\left(\frac{M+m}{M}\right)r\omega_0$$

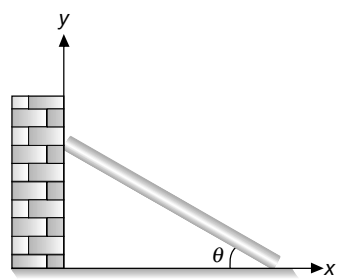
Therefore, the required distance is

$$s = v_r T$$

$$\Rightarrow \quad s = \frac{4}{5}\left(\frac{M+m}{M}\right)\frac{v_0 r\omega_0}{g}$$

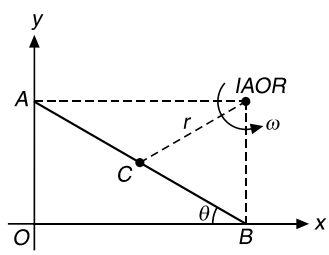
PROBLEM 25

A uniform rod of length L and weight W stands vertically touching a vertical wall (y-axis). When slightly displaced, its lower end begins to slide along the floor (x-axis). Obtain an expression for the angular velocity (ω) of the rod as a function of θ. Determine the distance moved by the lower end at which the rod no longer touches the vertical wall. Neglect friction everywhere.



SOLUTION

The position of instantaneous axis of rotation (IAOR) is shown in Figure.



$$C \equiv \left(\frac{l}{2}\cos\theta, \frac{l}{2}\sin\theta\right)$$

$$r = \frac{l}{2} = \text{half of the diagonal}$$

All surfaces are smooth. Therefore, mechanical energy will remain conserved. So,

$$\begin{pmatrix}\text{Decrease in GPE}\\ \text{of CM of Rod}\end{pmatrix} = \begin{pmatrix}\text{Increase in RKE}\\ \text{of Rod about } IAOR\end{pmatrix}.$$

$$\Rightarrow \quad mg\frac{l}{2}(1-\sin\theta) = \frac{1}{2}I\omega^2 \qquad \ldots(1)$$

where, $I = \frac{ml^2}{12} + mr^2$ {about IAOR}

$$\Rightarrow \quad I = \frac{ml^2}{12} + \frac{ml^2}{4} = \frac{ml^2}{3}$$

Substituting in equation (1), we get

$$mg\frac{l}{2}(1-\sin\theta) = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3g(1-\sin\theta)}{l}}$$

$$\Rightarrow \quad \omega^2 = \frac{3g(1-\sin\theta)}{l}$$

Differentiating w.r.t. time, we get

$$2\omega\left(\frac{d\omega}{dt}\right) = -\frac{3g}{l}\cos\theta\left(\frac{d\theta}{dt}\right) \qquad \ldots(2)$$

Since, $\frac{d\omega}{dt} = \alpha$ {the angular acceleration of rod}

and $\frac{d\theta}{dt} = \omega$ {the angular velocity of rod}

Substituting in equation (2), we have

$$\alpha = -\frac{3g}{2l}\cos\theta \qquad \ldots(3)$$

The x-coordinate of centre of mass at angle θ is

$$x = \frac{l}{2}\cos\theta$$

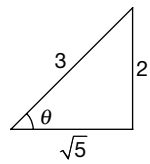
Differentiating twice w.r.t. time, we get

$$a_x = -\frac{\omega^2 l}{2}\cos\theta - \frac{\alpha l}{2}\sin\theta \qquad \ldots(4)$$

The rod will lose contact with the vertical wall at the instant when normal reaction by the wall (along x-axis) becomes zero. So,

$$F_x = 0$$

$$\Rightarrow \quad a_x = 0$$



Substitute, $a_x = 0$ in equation (4), we get

$$-\omega^2\cos\theta - \alpha\sin\theta = 0$$

$$\Rightarrow \quad -\frac{3g}{l}\cos\theta(1-\sin\theta) + \frac{3g}{2l}\sin\theta\cos\theta = 0$$

Solving this, we get

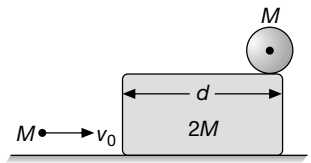
$$\sin\theta = \frac{2}{3}$$

At this moment the lower end has moved a distance d given by,

$$d = l\cos\theta = \frac{\sqrt{5}}{3}l$$

PROBLEM 26

A cylinder of mass M is placed on top of a block of mass $2M$. The friction coefficient between the block and the horizontal plane is zero and the friction coefficient between the cylinder and the top surface of block is μ. A small body of mass M travelling with a velocity v_0 hits the block horizontally such that the collision is completely elastic. Find the maximum value of v_0 for which the cylinder does start pure rolling before it topples from the top of the block.

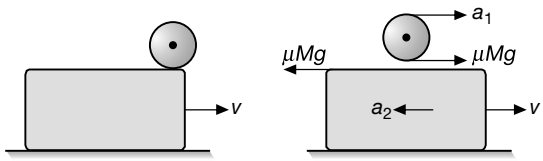


SOLUTION

Let v be the velocity of block after collision. Applying the equation of head on elastic collision between two particles, we get

$$v = \frac{2M}{M+2M}v_0 = \frac{2}{3}v_0$$

The forces acting on the cylinder and the block till pure rolling starts are shown in Figure.



$$a_1 = \frac{\mu Mg}{M} = \mu g$$

$$a_2 = \frac{\mu Mg}{2M} = \frac{\mu g}{2}$$

$$\alpha = \frac{\mu MgR}{MR^2/2} = \frac{2\mu g}{R}$$

Let the pure rolling starts after time t, then

$$v - a_2 t = a_1 t + (\alpha t) R$$

$$\Rightarrow \quad t = \frac{v}{a_1 + a_2 + \alpha R} = \frac{v}{\mu g + \frac{\mu g}{2} + 2\mu g}$$

$$\Rightarrow \quad t = \frac{2v}{7\mu g} = \frac{4v_0}{21\mu g}$$

Now, $d = \left(vt - \frac{1}{2} a_2 t^2 \right) - \frac{1}{2} a_1 t^2$

$$\Rightarrow \quad d = vt - \frac{1}{2}(a_1 + a_2) t^2$$

$$\Rightarrow \quad d = \frac{8v_0^2}{63\mu g} - \frac{12v_0^2}{441\mu g}$$

$$\Rightarrow \quad 441\mu g d = 56v_0^2 - 12v_0^2$$

$$\Rightarrow \quad 44v_0^2 = 441\mu g d$$

$$\Rightarrow \quad v_0 = \frac{21}{2}\sqrt{\frac{\mu g d}{11}}$$

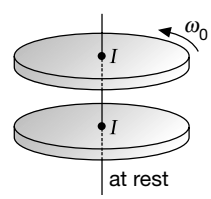
$$\Rightarrow \quad (v_0)_{\min} = \frac{21}{2}\sqrt{\frac{\mu g d}{11}}$$

PROBLEM 27

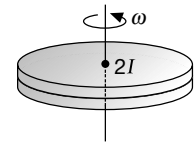
A uniform disc of radius r_0 lies on a smooth horizontal plane. A similar disc spinning with the angular velocity ω_0 is carefully lowered onto the first disc. How soon do both discs spin with the same angular-velocity if the friction coefficient between them is equal to μ.

SOLUTION

If I be the moment of inertia of each disc relative to common rotation axis as shown in Figure.



Initially (at $t = 0$)



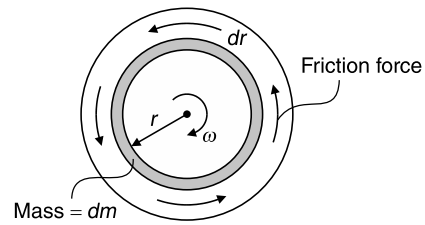
Finally (at t_0) when the discs attain common angular velocity

Then by law of conservation of angular momentum applied to the initial and the final situations, we get

$$I\omega_0 = 2I\omega$$

$$\Rightarrow \quad \omega = \frac{\omega_0}{2}$$

The angular velocity of each disc varies due to the torque τ of the friction forces. To calculate τ, let us take an elementary ring with radii r and $r + dr$ as shown in Figure.



The torque of the friction forces acting on the given ring is equal to

$$d\tau = fr_{\perp} = \mu(dN) r = \left[\mu(dm) g \right] r$$

$$\Rightarrow \quad d\tau = \mu \left(\frac{mg}{\pi r_0^2} \right) (2\pi r dr) r = \left(\frac{2\mu mg}{r_0^2} \right) r^2 dr$$

where, m is the mass of each disc. Integrating this with respect to r between 0 and r_0, we get

$$\tau = \frac{2}{3}\mu mg r_0 = \text{constant}$$

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{(2\mu mg r_0/3)}{(mr_0^2/3)} = \frac{4\mu g}{3r_0} = \text{constant}$$

The angular speed of lower disc increases from zero to $\frac{\omega_0}{2}$ with constant angular acceleration α, so

$$\frac{\omega_0}{2} = 0 + \alpha t$$

$$\Rightarrow \quad t = \frac{\omega_0}{2\alpha} = \frac{3r_0\omega_0}{8\mu g}$$

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

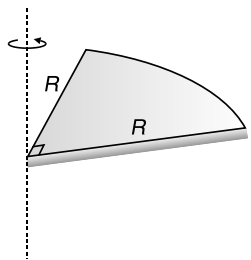
This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. Three identical rods, each of length l, are joined to form a rigid equilateral triangle. Its radius of gyration about an axis passing through a corner and perpendicular to the plane of the triangle is

(A) $\frac{l}{2}$ (B) $\sqrt{\frac{3}{2}}l$

(C) $\frac{l}{\sqrt{2}}$ (D) $\frac{l}{\sqrt{3}}$

2. One quarter sector is cut from a uniform disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the center of the original disc. Its moment of inertia about the axis of rotation is



(A) $\frac{MR^2}{2}$ (B) $\frac{MR^2}{4}$

(C) $\frac{MR^2}{8}$ (D) $\sqrt{2}MR^2$

3. Two particles of equal mass m at A and B are connected by a rigid light rod AB, lying on a smooth horizontal table. An impulse J is applied at A in the plane of the table and perpendicular at AB. Then the velocity of particle at A is

(A) ZERO (B) $\frac{J}{2m}$

(C) $\frac{J}{m}$ (D) $\frac{2J}{m}$

4. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of

(A) 0.42 m from mass of 0.3 kg
(B) 0.70 m from mass of 0.7 kg
(C) 0.98 m from mass of 0.3 kg
(D) 0.98 m from mass of 0.7 kg

5. A spherical body of radius R is allowed to roll without slipping down an incline to reach the bottom with a speed v_0. The incline is then made smooth by waxing and the body is allowed to slide without rolling now to reach the bottom with a speed $\frac{5}{4}v_0$. The radius of gyration of the body about an axis passing through its centre is

(A) $\frac{2}{5}R$ (B) $\frac{3}{4}R$

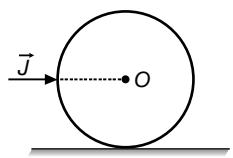
(C) $\frac{4}{3}R$ (D) $\frac{5}{2}R$

6. A uniform disc of radius R lies in x-y plane with its centre at origin. Its moment of inertia about the axis $x = 2R$ and $y = 0$ is equal to the moment of inertia about the axis $y = d$ and $z = 0$, then d equals

(A) $\frac{4R}{3}$ (B) $\frac{R\sqrt{17}}{2}$

(C) $R\sqrt{13}$ (D) $\frac{R\sqrt{15}}{2}$

7. An impulse J is applied on a ring of mass m along a line passing through its centre O. The ring is placed on a rough horizontal surface. The linear velocity of centre of ring once it starts rolling without slipping is



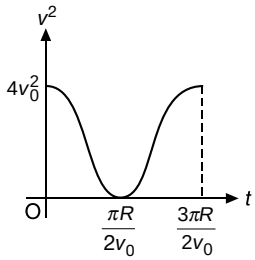
(A) $\frac{J}{m}$ (B) $\frac{J}{2m}$

(C) $\frac{J}{4m}$ (D) $\frac{J}{3m}$

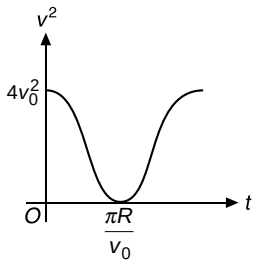
8. A wheel is rolling without sliding on a horizontal surface. The centre of the wheel moves with a constant speed v_0. Consider a point P on the rim which is at

the top at time $t = 0$. The square of speed of point P is plotted against time t. The correct plot is (R is radius of the wheel)

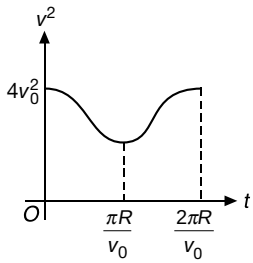
(A)



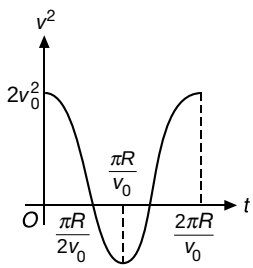
(B)



(C)



(D)



9. A thin uniform rod of mass m moves translationally with acceleration a due to two antiparallel forces of lever arm l. One force is of magnitude F and acts at one extreme end. The length of the rod is

(A) $l\left(1+\frac{F}{ma}\right)$ (B) $2l\left(1+\frac{F}{ma}\right)$

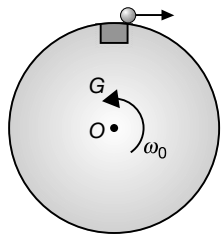
(C) $\frac{l}{2}\left(1+\frac{F}{ma}\right)$ (D) $\frac{mal}{ma+F}$

10. A ladder of length l and mass m is placed against a smooth vertical wall, but the ground is not smooth. Coefficient of friction between the ground and the ladder is μ. The angle θ at which the ladder will stay in equilibrium is

(A) $\theta = \tan^{-1}(\mu)$ (B) $\theta = \tan^{-1}(2\mu)$

(C) $\theta = \tan^{-1}\left(\frac{\mu}{2}\right)$ (D) $\theta = \tan^{-1}\left(\frac{1}{2\mu}\right)$

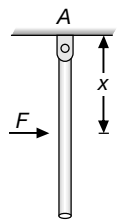
11. A horizontal turn table in the form of a disc of radius r carries a gun at G and rotates with angular velocity ω_0 about a vertical axis passing through the centre O. The increase in angular velocity of the system if the gun fires a bullet of mass m with a tangential velocity v with respect to the gun is (moment of inertia of gun + table about O is I_0)



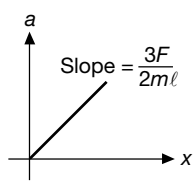
(A) $\frac{v}{2r}$ (B) $\frac{mvr}{2I_0}$

(C) $\frac{2mvr}{I_0}$ (D) $\frac{mvr}{I_0+mr^2}$

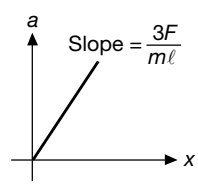
12. A rod of mass m and length l is hinged at one of its end A as shown in figure. A force F is applied at a distance x from A. The acceleration of centre of mass (a) varies with x as



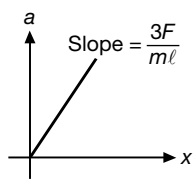
(A)



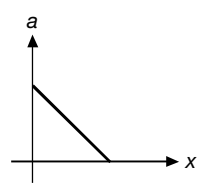
(B)



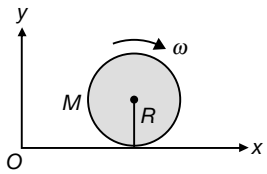
(C)



(D)



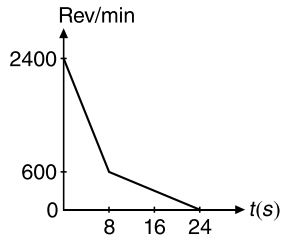
13. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in figure. The magnitude of angular momentum of the disc about the origin O is



(A) $\left(\frac{1}{2}\right)MR^2\omega$ (B) $MR^2\omega$

(C) $\left(\frac{3}{2}\right)MR^2\omega$ (D) $2MR^2\omega$

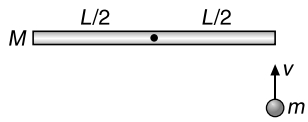
14. A table fan, rotating at a speed of 2400 rpm is switched off and the resulting variation of the rpm with time is shown in the figure. The total number of revolutions of the fan before it comes to rest is



(A) 420 (B) 280

(C) 190 (D) 380

15. A stick of length L and mass M lies on a frictionless horizontal surface on which it is free to move in any way. A ball of mass m moving with speed v collides elastically with the stick as shown in Figure. If after the collision ball comes to rest, then what should be the mass of the ball?



(A) $m = 2M$ (B) $m = M$

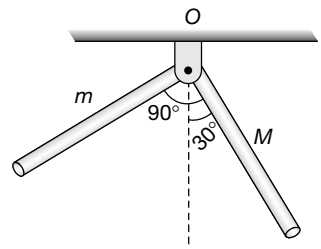
(C) $m = \frac{M}{2}$ (D) $m = \frac{M}{4}$

16. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

(A) I (B) $I\sin^2\theta$

(C) $I\cos^2\theta$ (D) $I\cos^2\left(\frac{\theta}{2}\right)$

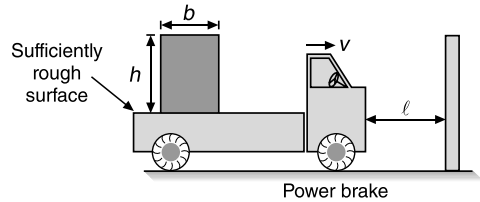
17. Two uniform rods of equal length but different masses are rigidly joined to form an L-shaped body, which is then pivoted as shown. If in equilibrium the body is in the shown configuration, ratio $\frac{M}{m}$ will be



(A) $\sqrt{2}$ (B) 2

(C) $\sqrt{3}$ (D) 3

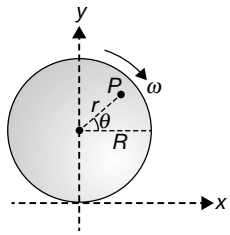
18. The minimum value of l so that truck can avoid hitting the dead end, without toppling the block kept on it is



(A) $\frac{2hv^2}{bg}$ (B) $\frac{bv^2}{2hg}$

(C) $\frac{2bv^2}{hg}$ (D) $\frac{hv^2}{2bg}$

19. A disc of radius R, having angular velocity ω rolls without slipping along positive x-axis. The velocity of point P at the instant shown in figure is



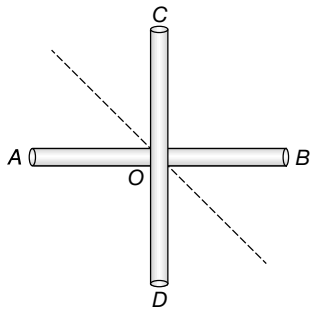
(A) $\vec{v}_P = \omega\left[(r\sin\theta)\hat{i} + (r\cos\theta)\hat{j}\right]$

(B) $\vec{v}_P = \omega\left[(r\sin\theta)\hat{i} - (r\cos\theta)\hat{j}\right]$

(C) $\vec{v}_P = \omega\left[(R + r\sin\theta)\hat{i} - (r\cos\theta)\hat{j}\right]$

(D) $\vec{v}_P = \omega\left[(R + r\sin\theta)\hat{i} + (r\cos\theta)\hat{j}\right]$

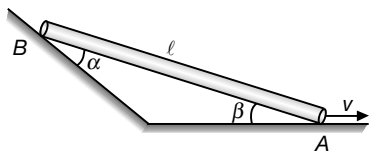
20. AB and CD are two identical rods each of length l and mass m joined to form a cross. The moment of inertia of these two rods about a bisector of the angle between the rods is



(A) $\frac{ml^2}{6}$ (B) $\frac{ml^2}{3}$

(C) $\frac{ml^2}{12}$ (D) $\frac{2ml^2}{3}$

21. A rod of length l slides down along the inclined wall as shown in figure. At the instant shown in figure the speed of end A is v, then the speed of B will be



(A) $\frac{v\sin\alpha}{\sin\beta}$ (B) $\frac{v\sin\beta}{\sin\alpha}$

(C) $\frac{v\cos\alpha}{\cos\beta}$ (D) $\frac{v\cos\beta}{\cos\alpha}$

22. A solid sphere and a hollow sphere of equal mass and radius are placed over a rough horizontal surface after rotating them about their respective centre of mass with same angular velocity ω_0. Once the pure rolling starts let v_1 and v_2 be the linear speeds of their centres of mass. Then

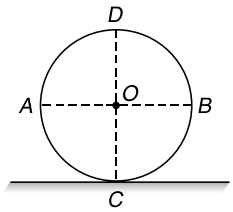
(A) $v_1 = v_2$

(B) $v_1 > v_2$

(C) $v_1 < v_2$

(D) data is insufficient

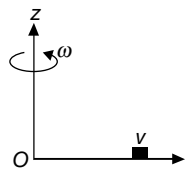
23. A uniform ring of mass M and radius R is in uniform pure rolling motion on a horizontal surface. The velocity of the centre of ring is v_0. The kinetic energy of the segment ACB is



(A) $\frac{Mv_0^2}{2} - \frac{Mv_0^2}{\pi}$ (B) $\frac{Mv_0^2}{2} + \frac{Mv_0^2}{\pi}$

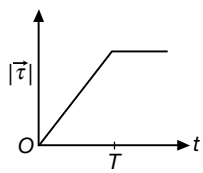
(C) $\frac{Mv_0^2}{2}$ (D) Mv_0^2

24. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure.

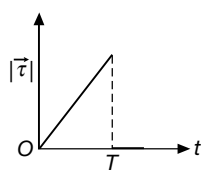


At time $t = 0$, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\vec{\tau}|)$ on the system about O, as a function of time is best represented by which plot?

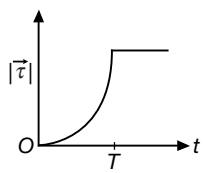
(A)



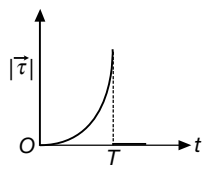
(B)



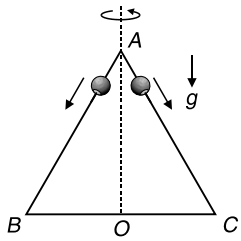
(C)



(D)



25. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as beads slide down are



(A) angular velocity and total energy (kinetic and potential).
(B) total angular momentum and total energy.
(C) angular velocity and moment of inertia about the axis of rotation.
(D) total angular momentum and moment of inertia about the axis of rotation.

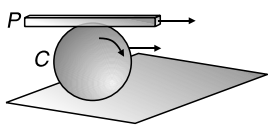
26. A uniform rod of length $2a$ and mass m lies at rest on a smooth horizontal table. A perfectly elastic particle having same mass as the rod moving with speed u on the table in a direction perpendicular to the rod, strikes one end of the rod. The final kinetic energy of the rod is

(A) $\frac{4}{13}mu^2$ (B) $\frac{1}{4}mu^2$
(C) $\frac{4}{25}mu^2$ (D) $\frac{8}{25}mu^2$

27. A uniform horizontal circular platform of mass 200 kg is rotating at 10 rpm about a vertical axis passing through its centre. A boy of mass 50 kg is standing on its edge. If the boy moves to the centre of the platform, the frequency of rotation would be

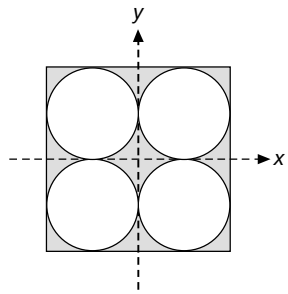
(A) 7.5 rpm (B) 12.5 rpm
(C) 15 rpm (D) 20 rpm

28. A plank P is placed on a hollow cylinder C, which rolls on a horizontal surface as shown. No slippage is there at any of the surfaces in contact. Both have equal mass say M (each) and if v is the velocity of centre of mass of the cylinder C, then the ratio of the kinetic energy of plank P to the cylinder C is



(A) 1 : 1 (B) 2 : 1
(C) 3 : 8 (D) 8 : 11

29. Four holes of radius R are cut from a thin square plate of side $4R$ and mass M. The moment of inertia of the remaining portion about z-axis is



(A) $\frac{\pi}{12}MR^2$ (B) $\left(\frac{4}{3}-\frac{\pi}{4}\right)MR^2$
(C) $\left(\frac{8}{3}-\frac{10\pi}{16}\right)MR^2$ (D) $\left(\frac{4}{3}-\frac{\pi}{6}\right)MR^2$

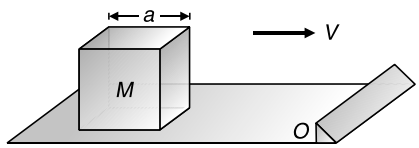
30. A flat rail road car is accelerating along the positive x-axis with an acceleration a_p. A sphere is placed over the car. The friction between car and sphere is not sufficient to support pure rolling of sphere. The correct statement is

(A) The sphere will slip and force of friction on sphere is along $-x$ direction
(B) The sphere will slip and force of friction on sphere is along $+x$ direction
(C) Acceleration of sphere is along $-x$ direction
(D) None of the above

31. A hollow straight tube of length $2l$ and mass m can turn freely about its centre on a smooth horizontal table. Another smooth uniform rod of same length and mass is fitted into the tube so that their centres coincide. The system is set in motion with an initial angular velocity ω_0. The angular velocity of the tube at the instant when the rod slips out of the tube

(A) $\frac{\omega_0}{4}$ (B) $\frac{\omega_0}{5}$
(C) $\frac{\omega_0}{7}$ (D) $\frac{\omega_0}{2}$

32. A cubical block of side a is moving with velocity v on a horizontal smooth plane as shown in figure. It hits a ridge at point O. If the moment of inertia of the block about an axis passing through centre of gravity is $\frac{1}{6}ma^2$, then the angular speed of the block after it hits O is



(A) $\frac{3v}{4a}$ (B) $\frac{3v}{2a}$

(C) $\sqrt{\frac{3}{2}}\frac{v}{a}$ (D) ZERO

33. In a simple pendulum, the breaking strength of the string is double the weight of the bob. The bob is released from rest when the string is horizontal. The string breaks when it makes an angle θ with the vertical

(A) $\theta = \cos^{-1}\left(\frac{1}{3}\right)$

(B) $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

(C) $\theta = 60°$

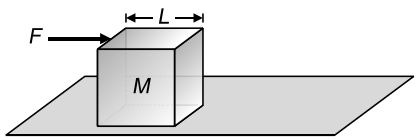
(D) $\theta = 0$

34. Two men each of mass m stand on the rim of a horizontal circular disc, diametrically opposite to each other. The disc has a mass M and is free to rotate about a vertical axis passing through its centre of mass. Each man starts simultaneously along the rim clockwise and reaches its original starting position on the disc. The angle turned through by the disc with respect to the ground (in radian) is

(A) $\frac{\pi m}{M+m}$ (B) $\frac{2\pi m}{4m+M}$

(C) $\frac{4\pi m}{2M+m}$ (D) $\frac{8\pi m}{4m+M}$

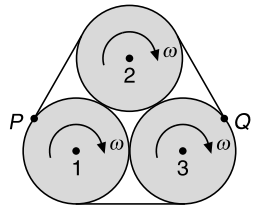
35. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is



(A) infinitesimal (B) $\frac{mg}{4}$

(C) $\frac{mg}{2}$ (D) $mg(1-\mu)$

36. Three identical cylinders of radius R are in contact. Each cylinder is rotating with angular velocity ω. A thin belt is moving without sliding on the cylinders. If P and Q are two points of belt which are in contact with the cylinder, then the magnitude of velocity of point P with respect to Q.



(A) $2R\omega$ (B) $R\omega$

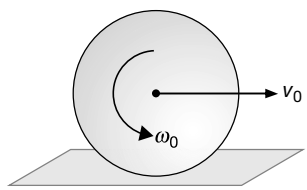
(C) $\frac{R\omega}{2}$ (D) $\sqrt{3}R\omega$

37. A balance is made of a rigid rod free to rotate about a point not at the centre of the rod. When an unknown mass m is placed in the left hand pan, it is balanced by a mass m_1 placed in the right hand pan, and similarly when the mass m is placed in the right pan, it is balanced by a mass m_2 in the left hand pan. Neglecting the masses of the pans, m is

(A) $\frac{1}{2}(m_1+m_2)$ (B) $\sqrt{m_1 m_2}$

(C) $\frac{1}{2}\sqrt{m_1^2+m_2^2}$ (D) $\sqrt{\frac{m_1^2+m_2^2}{2}}$

38. A uniform sphere of radius R is placed on a rough horizontal surface and given a linear velocity v_0 and angular velocity ω_0 as shown. For the sphere to come to rest after moving some distance to the right, we have



(A) $v_0 = R\omega_0$ (B) $5v_0 = 2R\omega_0$

(C) $2v_0 = 5R\omega_0$ (D) $2v_0 = R\omega_0$

39. A stone of mass 16 kg is attached to a string 144 m long and is whirled in a horizontal circle. The maximum tension the string can stand is 16 N. The maximum velocity of revolution that can be given to the stone without breaking the string is

(A) 20 ms^{-1} (B) 16 ms^{-1}

(C) 14 ms^{-1} (D) 12 ms^{-1}

40. At some instant, a particle is moving along a straight line $2x-3y=2$ and its co-ordinates on that line are $(4, 2)$. Now at another instant the same particle is moving along a straight line $3x+4y=7$ and its co-ordinate are $(1, 1)$. The co-ordinates of the axis about which it is in pure rotation are

(A) $\left(\frac{8}{9}, \frac{16}{9}\right)$ (B) $\left(\frac{5}{17}, \frac{6}{17}\right)$

(C) $\left(\frac{30}{17}, \frac{35}{17}\right)$ (D) $\left(\frac{50}{17}, \frac{61}{17}\right)$

41. A disc is rotating with an angular velocity ω_0. A constant retarding torque is applied on it to stop the disc. The angular velocity becomes $\frac{\omega_0}{2}$ after n rotations. How many more rotations will it make before coming to rest?

(A) $\frac{n}{2}$ (B) $\frac{n}{3}$

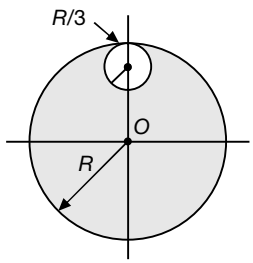
(C) n (D) $2n$

42. A wire of mass m and length l is bent in the form of a quarter circle. The moment of inertia of this wire about an axis passing through the centre of the quarter circle and perpendicular to the plane of the quarter circle is $\left(\text{Take } \pi^2 = 10\right)$

(A) $\frac{3ml^2}{5}$ (B) ml^2

(C) $\frac{ml^2}{5}$ (D) $\frac{2ml^2}{5}$

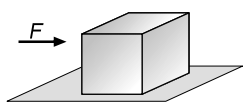
43. A small disc of radius $\frac{R}{3}$ is removed from a circular disc of radius R and mass $9M$. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is



(A) $4MR^2$ (B) $\frac{40}{9}MR^2$

(C) $10MR^2$ (D) $\frac{37}{9}MR^2$

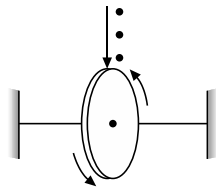
44. A force F is applied on the top of a cube as shown in figure. The coefficient of friction between the cube and the ground is μ. If F is gradually increased, the cube will topple before sliding if



(A) $\mu > 1$ (B) $\mu < \frac{1}{2}$

(C) $\mu < 1$ (D) $\mu > \frac{1}{2}$

45. A disc of mass m_0 rotates freely about a fixed horizontal axis through its centre. A thin cotton pad is fixed to its rim, which can absorb water. The mass of water dripping onto the pad is μ per second. The time after which the angular velocity of the disc gets reduced to half of its initial value is



(A) $\frac{2m_0}{\mu}$ (B) $\frac{m_0}{2\mu}$

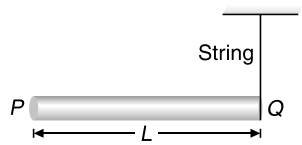
(C) $\frac{m_0}{\mu}$ (D) $\frac{3m_0}{\mu}$

46. Three point masses each of mass m, are placed at the corners of an equilateral triangle of side l. The moment of inertia of this system about an axis along one side of the triangle is

(A) ml^2 (B) $\frac{3}{4}ml^2$

(C) $3ml^2$ (D) $\frac{3}{2}ml^2$

47. A uniform rod PQ of length L is hinged at one end P. The rod is kept in the horizontal position by a massless string tied to point Q as shown in the figure. If the string is cut, the initial angular acceleration of the rod will be

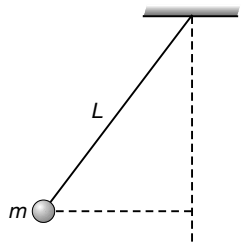


(A) $\frac{g}{L}$ (B) $\frac{2g}{L}$

(C) $\frac{2g}{3L}$ (D) $\frac{3g}{2L}$

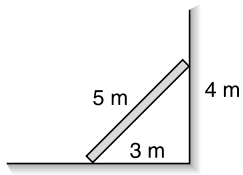
48. A ball of mass $m = 0.5$ kg is attached to the end of a string having length $L = 0.5$ m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N.

The maximum possible value of angular velocity of ball, in rads^{-1} is



(A) 9 (B) 18
(C) 27 (D) 36

49. A uniform ladder of length 5 m is placed against the wall as shown in the Figure.



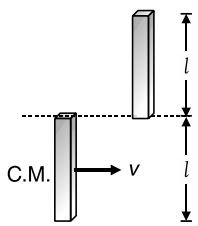
If coefficient of friction μ is the same for both the walls, the minimum value of μ for the ladder not to slip is

(A) $\mu = \frac{1}{2}$ (B) $\mu = \frac{1}{3}$
(C) $\mu = \frac{1}{4}$ (D) $\mu = \frac{1}{5}$

50. A thin circular ring of mass M is rotating about its axis with a constant angular velocity ω. The two objects, each of mass m, are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity

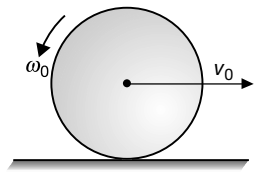
(A) $\frac{\omega M}{M+m}$ (B) $\frac{\omega(M-2m)}{M+2m}$
(C) $\frac{\omega M}{M+2m}$ (D) $\frac{\omega(M+2m)}{M}$

51. A bar of mass m, length l is in pure translatory motion with its centre of mass velocity v. It collides with and sticks to another identical bar at rest as shown in figure. Assuming that after collision it becomes one composite bar of length $2l$, the angular velocity of the composite bar will be



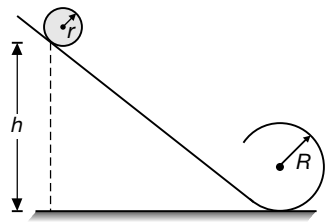
(A) $\frac{3}{4}\frac{v}{l}$, anticlockwise
(B) $\frac{4}{3}\frac{v}{l}$, anticlockwise
(C) $\frac{3}{4}\frac{v}{l}$, clockwise
(D) $\frac{4}{3}\frac{v}{l}$, clockwise

52. A uniform circular disc of radius r placed on a rough horizontal plane has initial velocity v_0 and an angular velocity ω_0 as shown. The disc comes to rest after moving some distance in the direction of motion. Then



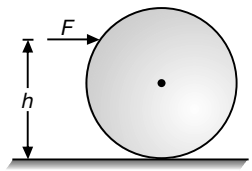
(A) the friction force acts in the forward direction
(B) the point of contact of disc with ground has zero velocity
(C) v_0 must be equal to $\frac{r\omega_0}{2}$ in magnitude
(D) v_0 must be equal to $2r\omega_0$ in magnitude

53. A solid sphere of radius r rolls without slipping from rest from a height h of an inclined track at the bottom of which there is a loop of radius R much larger than the radius of sphere, as shown in figure. The minimum value of h for the sphere to complete the loop is



(A) $2.1R$ (B) $2.3R$
(C) $2.5R$ (D) $2.7R$

54. A solid sphere of radius R is resting on a smooth horizontal surface. A constant force F is applied at a height h from the bottom. Choose the correct alternative

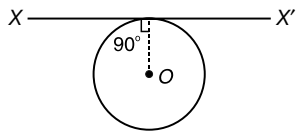


(A) Sphere will always slide whatever be the value of h
(B) Sphere will roll without slipping when $h \geq 1.4R$
(C) Sphere will roll without slipping if $h = 1.4R$
(D) None of the above

55. A wire of length l and mass m is first bent in a circle, then in a square and then in an equilateral triangle. The moment of inertia in these three cases about an axis perpendicular to their planes and passing through their centres of mass are I_1, I_2 and I_3 respectively. Then maximum of them is

(A) I_1 (B) I_2
(C) I_3 (D) data insufficient

56. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is



(A) $\frac{\rho L^3}{8\pi^2}$ (B) $\frac{\rho L^3}{16\pi^2}$
(C) $\frac{5\rho L^3}{16\pi^2}$ (D) $\frac{3\rho L^3}{8\pi^2}$

57. A playground merry-go-round is at rest, pivoted about a frictionless axis. A child of mass m runs along the path tangential to the rim with speed v and jumps on to merry-go-round. If R be the radius of merry-go-round and I is the moment of inertia, then the angular velocity of the merry-go-round and the child is

(A) $\frac{mvR}{mR^2 + I}$ (B) $\frac{mvR}{I}$
(C) $\frac{mR^2 + I}{mvR}$ (D) $\frac{I}{mvR}$

58. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(A) $\frac{M\omega^2 L}{2}$ (B) $M\omega^2 L$
(C) $\frac{M\omega^2 L}{4}$ (D) $\frac{M\omega^2 L^2}{2}$

59. A solid sphere, a ring and a disc all having same mass and radius are placed at the top of an incline and released. The coefficient of friction between the objects and the incline is same but not sufficient to allow pure rolling. The least time will be taken in reaching the bottom by

(A) the solid sphere
(B) the ring
(C) the disc
(D) All will take the same time

60. In PROBLEM 59, the least value of kinetic energy at the bottom of the incline will be achieved by

(A) the ring
(B) the disc
(C) the solid sphere
(D) All will achieve the same kinetic energy

61. The string of a pendulum, having bob of mass m, is displaced through 90° from the vertical and then released. The minimum strength of the string in order to withstand the tension as the pendulum passes through the mean position is

(A) mg (B) $3mg$
(C) $5mg$ (D) $6mg$

62. Two men A and B are carrying a uniform bar of length L on their shoulders. The bar is held horizontally such that A gets $\left(\frac{1}{4}\right)$th load. If A is at one end of the bar, the distance of B from that end is

(A) $\frac{L}{3}$ (B) $\frac{L}{2}$
(C) $\frac{2L}{3}$ (D) $\frac{3L}{4}$

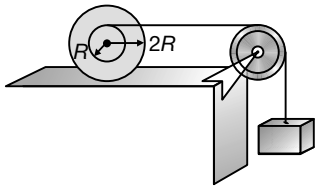
63. A body having moment of inertia about its axis of rotation equal to $3\ \text{kgm}^2$ is rotating with angular velocity equal to $3\ \text{rads}^{-1}$. The kinetic energy of this body is the same as that of a body of mass 27 kg moving with a speed of

(A) $1\ \text{ms}^{-1}$ (B) $0.5\ \text{ms}^{-1}$
(C) $1.5\ \text{ms}^{-1}$ (D) $2.0\ \text{ms}^{-1}$

64. A particle is moved in a circle with a constant angular velocity. Its angular momentum is L. If the radius of the circle is halved keeping the angular velocity same, the angular momentum of the particle will become

(A) $\frac{L}{4}$ (B) $\frac{L}{2}$
(C) L (D) $2L$

65. In the figure shown, the spherical body and block each have a mass m. The moment of inertia of the spherical body about centre of mass is $2mR^2$. The spherical body rolls without slipping on the horizontal surface. The ratio of kinetic energy of the spherical body to that of block is



(A) $\frac{1}{3}$ (B) $\frac{1}{2}$

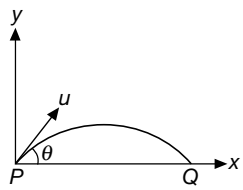
(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

66. A stone of mass m, tied to the end of a string, is whirled around in a horizontal circle (Neglect the force due to gravity). The length of the string is reduced gradually, keeping the angular momentum of the stone about the centre of the circle constant. Then, the tension in the string is given by $T = Ar^n$, where A is a constant, r is the instantaneous radius of the circle, then n is

(A) 1 (B) –1

(C) –2 (D) –3

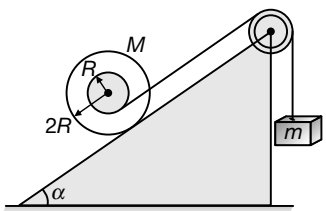
67. Average torque on a projectile of mass m, initial speed u and angle of projection θ is launched between initial and final positions P and Q as shown in figure. The average torque on the projectile about the point of projection is



(A) $mu^2\cos\theta$ (B) $mu^2\sin\theta$

(C) $\frac{mu^2\sin(2\theta)}{2}$ (D) $\frac{mu^2\cos\theta}{2}$

68. A spool of mass M and radius $2R$ lies on an inclined plane as shown in figure. A light thread is wound around the connecting tube of the spool and its free end carries a weight of mass m. The value of m so that system is in equilibrium is



(A) $2M\sin\alpha$ (B) $M\sin\alpha$

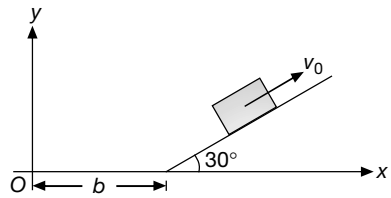
(C) $2M\tan\alpha$ (D) $M\cos\alpha$

69. Two discs have same mass and thickness. Their materials are of densities d_1 and d_2. The ratio of their moments of inertia about and axis passing through the centre and perpendicular to the plane

(A) $d_1 : d_2$ (B) $d_2 : d_1$

(C) $1 : d_1 d_2$ (D) $d_1 d_2 : 1$

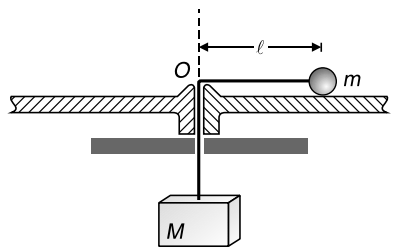
70. A cube of mass m and side a is moving along a plane with constant speed v_0 as shown in figure. The magnitude of angular momentum of the cube about z-axis would be



(A) $\frac{mv_0 b}{2}$ (B) $\frac{\sqrt{3}mv_0 b}{2}$

(C) $mv_0\left(b-\frac{a}{2}\right)$ (D) None of these

71. Two masses m and M are connected by a light string that passes through a smooth hole O at the centre of a table. Mass m lies on the table and M hangs vertically. m is moved round in a horizontal circle with O as the centre. If l is the length of the string from O to m then the frequency with which m should revolve so that M remains stationary is



(A) $\frac{1}{2\pi}\sqrt{\frac{Mg}{ml}}$ (B) $\frac{1}{\pi}\sqrt{\frac{Mg}{ml}}$

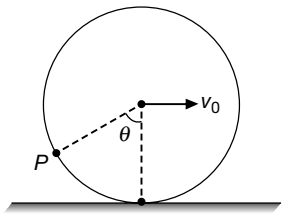
(C) $\frac{1}{2\pi}\sqrt{\frac{Ml}{mg}}$ (D) $\frac{1}{\pi}\sqrt{\frac{Ml}{mg}}$

72. A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and moment of inertia about its axis is I. A weight mg is attached to the end of the cord and is allowed to fall from rest. The angular velocity of the wheel, when the weight has fallen through a distance h, is

(A) $\sqrt{\frac{2gh}{I+mr}}$ (B) $\sqrt{\frac{2mgh}{I+mr^2}}$

(C) $\sqrt{\frac{2mgh}{I+2mr^2}}$ (D) $\sqrt{2gh}$

73. A hoop rolls without slipping on a horizontal ground having centre of mass speed v_0. The speed of a particle P on the circumference of the hoop at angle θ is



(A) $v_0 \sin\theta$ (B) $v_0 \cos\theta$

(C) $2v_0 \sin\left(\frac{\theta}{2}\right)$ (D) $2v_0 \cos\left(\frac{\theta}{2}\right)$

74. Two rings each of mass m and radius r are placed such that their centres are at a common point and their planes are normal to each other. The moment of inertia of the system about an axis passing through the centre and perpendicular to plane of one of the ring is

(A) $2mr^2$ (B) mr^2

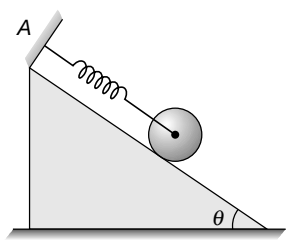
(C) $\frac{1}{2}mr^2$ (D) $\frac{3}{2}mr^2$

75. A particle of mass 2 kg located at the position $\left(\hat{i}+\hat{j}\right)$ m has a velocity $2\left(\hat{i}-\hat{j}+\hat{k}\right)$ ms^{-1}. Its angular momentum about z-axis in kgm^2s^{-1} is

(A) ZERO (B) +8

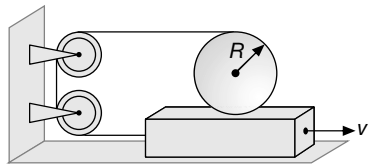
(C) –4 (D) –8

76. A uniform cylinder of mass M and radius R, rolls without slipping down an incline making an angle θ with horizontal. The cylinder is connected to a spring of force constant k at the centre, the other side of which is connected to a fixed support at A. The cylinder is released when the spring is unstretched. The force of friction (f) is



(A) always down the incline.
(B) always up the incline.
(C) initially up the incline and then becomes zero.
(D) initially up the incline and then becomes down the incline.

77. In the figure shown, the plank is being pulled to the right with a constant speed v. If the cylinder does not slip then



(A) the speed of the centre of mass of the cylinder is $2v$
(B) the speed of the centre of mass of the cylinder is v
(C) the angular velocity of the cylinder is $\frac{v}{R}$
(D) the angular velocity of the cylinder is zero

78. A cylinder is released from rest from the top of an incline plane of inclination 60° where friction coefficient varies with distance x as $\mu = \frac{2-3x}{\sqrt{3}}$. Find the distance travelled by the cylinder on incline before it starts slipping

(A) $\frac{1}{3}$ m (B) $\frac{1}{\sqrt{3}}$ m

(C) 3 m (D) $\sqrt{3}$ m

79. A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B. Then

(A) $\omega_A < \omega_B$ (B) $\omega_A = \omega_B$

(C) $\omega_A = \omega$ (D) $\omega = \omega_B$

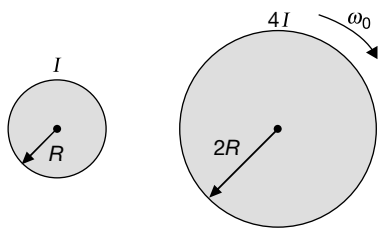
80. Masses of 1 g, 2 g, 3 g, ... 100 g are suspended from the 1 cm, 2 cm, 3 cm, ... 100 cm marks of a light metre scale. The system will be supported in equilibrium at

(A) 60 cm (B) 67 cm

(C) 55 cm (D) 72 cm

81. Two cylinders having radii $2R$ and R and moment of inertias $4I$ and I about their central axes are supported by axles perpendicular to their planes. The large cylinder is initially rotating clockwise with angular velocity ω_0. The small cylinder is moved to the right until it touches the large cylinder and is made to rotate

by the frictional force between the two. Eventually slipping ceases and the two cylinders rotate at constant rates in opposite directions. During this process



(A) angular momentum of system is conserved
(B) kinetic energy is conserved
(C) neither the angular momentum nor the kinetic energy is conserved
(D) both the angular momentum and kinetic energy are conserved

82. A uniform rod AB of mass m and length l is at rest on a smooth horizontal surface. An impulse J is applied to the end B perpendicular to the rod in horizontal direction. The speed of particle P at a distance $\frac{l}{6}$ from the centre towards A of the rod after time $t = \frac{\pi ml}{12J}$ is

(A) $\frac{J}{m}$ (B) $\frac{2J}{m}$
(C) $\frac{J\sqrt{2}}{m}$ (D) $\frac{J}{\sqrt{2}m}$

83. A circular platform of radius 2 m and moment of inertia 200 kgm^2 is mounted on a vertical frictionless axle. It is initially at rest. A 70 kg man stands on the edge of the platform and begins to walk along the edge at speed O relative to the ground. The angular velocity of the platform is

(A) 0.4 rads^{-1} (B) CD
(C) $2ma^2$ (D) O

84. In PROBLEM 83, when the man has walked once around the platform, so that he is at his original position on it, then his angular displacement relative to ground is

(A) $\frac{6\pi}{5}$ (B) $\frac{5\pi}{6}$
(C) $\frac{4\pi}{5}$ (D) $\frac{5\pi}{4}$

85. A ring of radius R is first rotated with an angular velocity ω_0 and then carefully placed on a rough horizontal surface. The coefficient of friction between the surface and the ring is μ. The time after which its angular speed is reduced to half is

(A) $\frac{\omega_0 \mu R}{2g}$ (B) $\frac{\omega_0 g}{2\mu R}$
(C) $\frac{2\omega_0 R}{\mu g}$ (D) $\frac{\omega_0 R}{2\mu g}$

86. A uniform thin bar of mass $\frac{\omega_0}{8}$ and length $\frac{3v}{4a}$ is bent to make a regular hexagon. Its moment of inertia about an axis passing through the centre of mass and normal to the plane of hexagon is

(A) $\frac{12}{5}mL^2$ (B) $6mL^2$
(C) $20mL^2$ (D) $30mL^2$

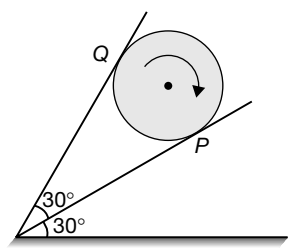
87. A uniform disc of radius R lies in x-y plane with its centre at the origin. Its moment of inertia about z-axis is equal to its moment of inertia about line $y = x + c$. Then c equals

(A) $\frac{R}{\sqrt{2}}$ (B) $-\frac{R}{2}$
(C) $+\frac{R}{4}$ (D) $-R$

88. Four spheres each of mass M and diameter $2r$, are placed with their centres on four corners of a square of side $a(> 2r)$. The moment of inertia of the system about one side of square

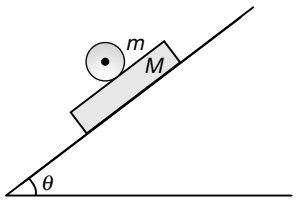
(A) $\frac{2}{5}M(5r^2 + 4a^2)$ (B) $\frac{2}{5}M(5r^2 + 2a^2)$
(C) $\frac{2}{5}M(2r^2 + 5a^2)$ (D) $\frac{2}{5}M(4r^2 + 5a^2)$

89. A sphere is rotating between two rough inclined plane as shown in figure. The coefficient of friction between each plane and the sphere is $\frac{1}{3}$. If f_1 and f_2 be the friction forces at P and Q. Then $\frac{f_1}{f_2}$ is



(A) $\frac{1}{2} + \sqrt{3}$ (B) $2 + \frac{1}{\sqrt{3}}$
(C) $1 + \frac{4}{\sqrt{3}}$ (D) $1 + 2\sqrt{3}$

90. A plank of mass M is placed on a smooth inclined, plane and a sphere of mass m is then placed on the plank. Assuming sufficient friction to be present between the sphere and plank the frictional force on sphere when the plank and sphere are released from rest is

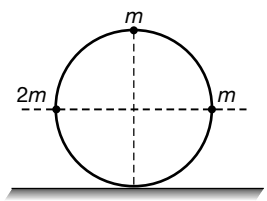


(A) zero
(B) horizontal
(C) up the incline
(D) down the incline

91. A thin rod of mass m and length $2l$ is made to rotate about an axis passing through its centre and perpendicular to it. If its angular velocity changes from 0 to ω in time t, the torque acting on it is

(A) $\frac{ml^2\omega}{12t}$
(B) $\frac{ml^2\omega}{3t}$
(C) $\frac{ml^2\omega}{t}$
(D) $\frac{4ml^2\omega}{3t}$

92. A ring of mass m and radius R has three particles attached to the ring as shown in the figure. The centre of the ring has a speed v_0. Assume slipping to be absent, the kinetic energy of the system is



(A) $4mv_0^2$
(B) $6mv_0^2$
(C) $8mv_0^2$
(D) $12mv_0^2$

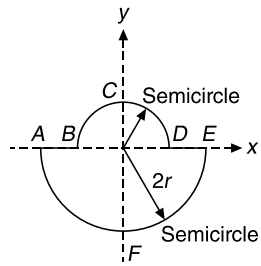
93. Two cones A and B are made of two different materials, the density of A being greater than that of B. The height of B is greater than that of A but their base areas and masses are the same. The correct statement about the moment of inertia of the two cones about their axis is

(A) A will have larger moment of inertia than B
(B) B will have larger moment of inertia than A
(C) in such a situation, it is dependent up on the height of the cone, the mass of the cone and radius of the base
(D) the moment of inertia of the two will be the same as it is not dependent on height of the cone but depends only upon the mass and the base area

94. A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is

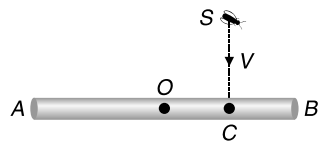
(A) $2K$
(B) $\frac{K}{2}$
(C) $\frac{K}{4}$
(D) $4K$

95. A uniform thin rod is bent in the form of closed loop $ABCDEFA$ as shown in Figure. The ratio of moment of inertia of the loop about x-axis to that about y-axis is



(A) >1
(B) <1
(C) $=1$
(D) $=\frac{1}{2}$

96. A homogeneous rod AB of length L and mass M is pivoted at the centre O in such a way that it can rotate freely in a vertical plane. The rod AB is initially in horizontal position. An insect S of same mass falls vertically with a speed V at the point C, midway between points O and B. Immediately after the insect falls, the angular velocity of the system is

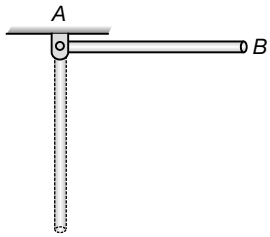


(A) $\frac{7}{12}\frac{V}{L}$
(B) $\frac{12}{7}\frac{V}{L}$
(C) $\frac{3}{2}\frac{V}{L}$
(D) $\frac{2}{3}\frac{V}{L}$

97. A circular loop of wire of mass m and radius r is making n revolutions per second about a point on its rim. Its rotational kinetic energy is

(A) $\pi^2mr^2n^2$
(B) $2\pi^2mr^2n^2$
(C) $4\pi^2mr^2n^2$
(D) $8\pi^2mr^2n^2$

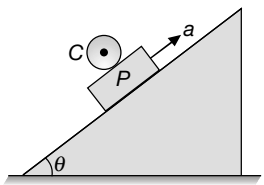
98. One end of a uniform rod of length l and mass m is hinged at A. It is released from rest from horizontal position AB as shown in figure. The force exerted by the rod on the hinge when it becomes vertical is



(A) $\frac{3}{2}mg$ (B) $\frac{5}{2}mg$

(C) $3\,mg$ (D) $5\,mg$

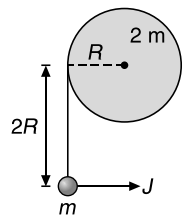
99. The acceleration a of the plank P required to keep the centre C of a cylinder which rolls without slipping on the plank in a fixed position during the motion is



(A) $\frac{g}{2}\sin\theta$ (B) $g\sin\theta$

(C) $2g\sin\theta$ (D) $\sqrt{2g}\sin\theta$

100. A uniform circular disc of mass $2m$ and radius R is placed freely on a horizontal smooth surface. A particle of mass m is connected to the circumference of the disc with a massless string. Now an impulse J is applied on the particle in the direction as shown in Figure.



If $J = 10$ Ns, $m = \sqrt{10}$ kg and $R = 25$ cm, then acceleration of centre of mass of the disc just after application of impulse is

(A) $1\ \text{ms}^{-2}$ (B) $2\ \text{ms}^{-2}$

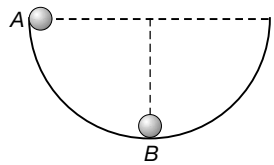
(C) $3\ \text{ms}^{-2}$ (D) $4\ \text{ms}^{-2}$

101. A solid sphere of mass 2 kg rolls up a 30° incline with an initial speed of $10\ \text{ms}^{-1}$. The maximum height reached by the sphere is ($g = 10\ \text{ms}^{-2}$)

(A) 3.5 m (B) 7.0 m

(C) 10.5 m (D) 14.0 m

102. A small ball of mass m, radius r is released from rest from a point A to roll inside a hemispherical shell of radius R as shown in figure. The angular velocity of centre of the ball in position B about the centre of the shell is



(A) $2\sqrt{\frac{g}{5(R-r)}}$ (B) $\sqrt{\frac{10g}{7(R-r)}}$

(C) $\sqrt{\frac{2g}{5(R-r)}}$ (D) $\sqrt{\frac{5g}{2(R-r)}}$

103. In PROBLEM 102, the normal force between the ball and the shell in position $\frac{\vec{dL}}{dt}$ is

(A) mg (B) $\frac{10}{7}mg$

(C) $\frac{12}{7}mg$ (D) $\frac{17}{7}mg$

104. An inclined plane makes an angle of 60^o with horizontal. A disc rolling down this inclined plane without slipping has a linear acceleration equal to

(A) $\frac{g}{3}$ (B) $\frac{3g}{4}$

(C) $\frac{g}{\sqrt{3}}$ (D) $\frac{g}{2}$

105. Locus of all the points in a plane on which the moment of inertia of a rigid body is same throughout is

(A) a straight line (B) a circle

(C) a parabola (D) an ellipse

106. A rigid spherical body is spinning around an axis without any external torque. Due to temperature its volume increases by 3%. Then percentage change in its angular speed is

(A) –2% (B) –1%

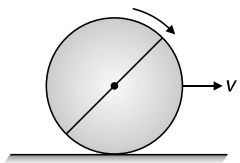
(C) –3% (D) 1%

107. A uniform rod AB of mass m and length $2a$ is falling with velocity v without rotation in gravity free space with AB horizontal. Suddenly the end A gets hit by an obstacle when the speed of the rod is v. Assume the end A to hinge about the obstacle, the angular speed with which the rod begins to rotate is

(A) $\frac{v}{2a}$ (B) $\frac{4v}{3a}$

(C) $\frac{v}{3a}$ (D) $\frac{3v}{4a}$

108. A ring of mass m is rolling without slipping with linear velocity v as shown in figure. A rod of identical mass is fixed along one of its diameter. The total kinetic energy of the system is



(A) $\frac{7}{5}mv^2$ (B) $\frac{2}{3}mv^2$

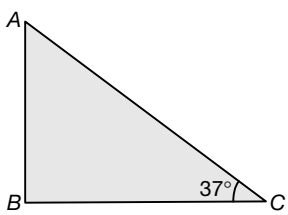
(C) $\frac{4}{3}mv^2$ (D) $\frac{5}{3}mv^2$

109. A uniform rod of mass m and length l is suspended by means of two light inextensible strings as shown in figure. Tension in one string immediately after the other string is cut is

(A) $\frac{mg}{4}$ (B) $\frac{mg}{2}$

(C) mg (D) $2mg$

110. ABC is a right angled triangular plate of uniform thickness. I_1, I_2 and I_3 are moments of inertia about AB, BC and AC respectively. Given that $\sin(37°) = \frac{3}{5}$, then which of the following relation is correct?



(A) $I_2 > I_1 > I_3$

(B) $I_3 > I_1 > I_2$

(C) $I_3 < I_2 < I_1$

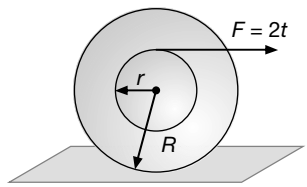
(D) $I_1 = I_2 = I_3$

111. A solid sphere and a solid cylinder of same mass are rolled down on two inclined planes of heights h_1 and h_2 respectively. At the bottom of the plane the two objects have same linear velocities, then $h_1 : h_2$ is

(A) 2 : 3 (B) 7 : 5

(C) 14 : 15 (D) 15 : 14

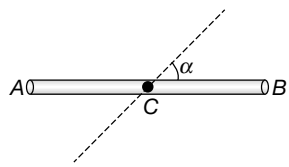
112. A time varying force $F = 2t$ is applied on a spool as shown in figure. The angular momentum of the spool at time t about bottommost point is



(A) $\frac{r^2t^2}{R}$ (B) $\frac{(R+r)^2}{r}t^2$

(C) $(R+r)t^2$ (D) $\frac{R^2t^2}{r}$

113. The moment of inertia of a uniform rod of length $2l$ and mass m about an axis passing through its centre and inclined at an angle α is



(A) $\frac{ml^2}{3}\sin^2\alpha$ (B) $\frac{ml^2}{12}\sin^2\alpha$

(C) $\frac{K}{4}$ (D) v_c

114. A rod of length l is given two velocities v_1 and v_2 in opposite directions at its two ends at right angles to the length. The distance of the instantaneous axis of rotation from v_1 is

(A) ZERO (B) $\frac{l}{2}$

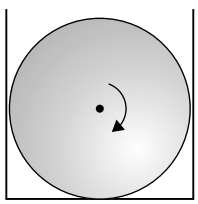
(C) $\left(\frac{v_1}{v_1+v_2}\right)l$ (D) $\left(\frac{v_2}{v_1+v_2}\right)l$

115. If the radius of the earth suddenly contracts to $\frac{1}{n^{\text{th}}}$ of its present radius without any change in its mass, the duration of the day will approximately become

(A) $\frac{24}{n}$ hour (B) $\frac{24}{n^2}$ hour

(C) $24n$ hour (D) $24n^2$ hour

116. A solid sphere of mass 5 kg and radius 1 m after rotating with angular speed $\omega_0 = 40$ rads^{-1} is placed between two smooth walls on a rough ground. Distance between the walls is slightly greater than the diameter of the sphere. If the coefficient of friction between the sphere and the ground is $\mu = 0.1$, then the sphere will stop rotating after time



(A) 8 s (B) 12 s

(C) 16 s (D) 20 s

117. A wheel rotates with constant acceleration of 2.0 rads^{-2}. If the wheel starts from rest, the number of revolutions it makes in the first ten second will be approximately

(A) 8 (B) 16

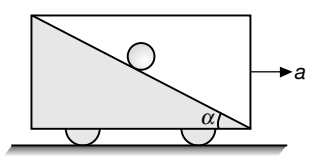
(C) 24 (D) 32

118. Two loops P and Q are made from a uniform wire. The radii of P and Q are R_1 and R_2 respectively, and their moments of inertia are I_1 and I_2 respectively. If $\frac{I_2}{I_1} = 4$, then $\frac{R_2}{R_1}$ is

(A) $4^{2/3}$ (B) $4^{1/3}$

(C) $4^{-2/3}$ (D) $4^{-1/3}$

119. In the arrangement shown, a smooth inclined plane of inclination θ is fixed in a car and a sphere is set in pure rolling on the incline. The value of a (the acceleration of car in horizontal direction) for which the sphere will continue pure rolling is



(A) $g\cos\theta$ (B) $g\sin\theta$

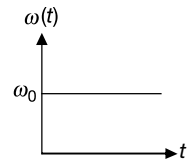
(C) $g\cot\theta$ (D) $g\tan\theta$

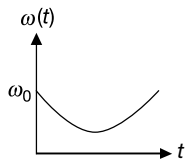
120. A homogeneous cylinder of mass M and radius R is pulled on a horizontal plane by a horizontal force F acting through its centre of mass. Assuming the cylinder to roll without slipping, the angular acceleration of the cylinder is

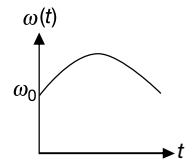
(A) $\frac{3F}{2MR}$ (B) $\frac{2F}{3MR}$

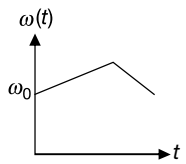
(C) $\frac{F}{2MR}$ (D) $\frac{3F}{4MR}$

121. Moment of inertia I of a solid sphere about an axis parallel to a diameter and at a distance x from it varies as

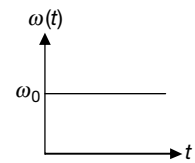
(A) 

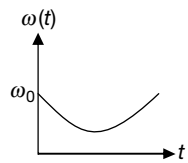
(B) 

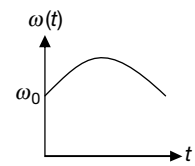
(C) 

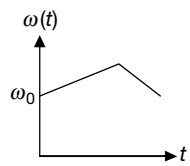
(D) 

122. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now the platform is given an angular velocity ω_0. When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform). The angular velocity of the platform $\omega(t)$ will vary with time t as

(A) 

(B) 

(C) 

(D) 

123. A particle of mass 5 g is moving with a uniform speed of $3\sqrt{2}$ cms^{-1} in the xy-plane along the line $y = x + 4$. Magnitude of its angular momentum about origin in gcms^{-2} is

(A) ZERO (B) 30

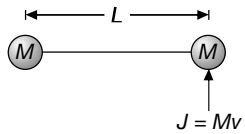
(C) $30\sqrt{2}$ (D) 60

124. When a mass is rotated in a plane about a fixed point, its angular momentum is directed along the
(A) radius
(B) tangent to orbit
(C) axis of rotation
(D) line at an angle of 45° to plane of rotation

125. A uniform thin wire of length $\omega = \frac{v}{5a}$ and mass m is bent to form of a rectangular frame $ABCD$ with $\frac{AB}{BC} = 2$. The moment of inertia of this wire frame about the smaller side is

(A) $\frac{6}{81}ml^2$ (B) $\frac{6}{217}ml^2$
(C) $\frac{7}{81}ml^2$ (D) $\frac{7}{162}ml^2$

126. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = Mv$ is imparted to the body at one of its end, the its angular velocity is



(A) $\frac{v}{L}$ (B) $\frac{2v}{L}$
(C) $\frac{v}{3L}$ (D) $\frac{v}{4L}$

127. A cylinder rolls up an inclined plane, reaches some height and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are
(A) up the incline while ascending and down the incline while descending
(B) up the incline while ascending as well as descending
(C) down the incline while ascending and up the incline while descending
(D) down the incline while ascending as well as descending

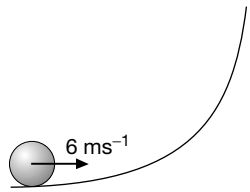
128. A thin uniform circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω. Another disc of same dimensions but of mass $\frac{1}{4}M$ is placed gently on the first disc co-axially. The angular velocity of the system is

(A) $\sqrt{2}\omega$ (B) $\frac{4}{5}\omega$
(C) $\frac{3}{4}\omega$ (D) $\frac{1}{3}\omega$

129. A uniform ring of mass m and radius R is released from top of an inclined plane. The plane makes an angle θ with horizontal. The coefficient of friction between the ring and the plane is μ. Initially, the point of contact of ring and plane is P. Angular momentum of the ring about point P as a function of time t is

(A) $mgRt\sin\theta - \mu mgRt\cos\theta$
(B) $mgRt\sin\theta$
(C) $mgRt\sin\theta + \mu mgRt\cos\theta$
(D) $mgRt(1-\mu^2)\sin\theta$

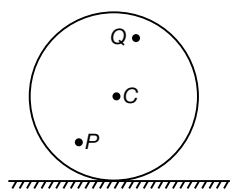
130. A disc of radius 0.1 m rolls without sliding on a horizontal surface with a velocity of 6 ms^{-1}. It then ascends a smooth continuous track as shown in figure. Taking $g = 10\ ms^{-2}$, the height upto which it will ascend is



(A) 0.9 m (B) 1.8 m
(C) 2.4 m (D) 2.7 m

131. Two equal and opposite forces act on a rigid body at a certain distance. Then
(A) the body is in equilibrium
(B) the body will rotate about its centre of mass
(C) the body may rotate about any point other than its centre of mass
(D) the body cannot rotate about its centre of mass

132. A disc is rolling (without slipping) on a horizontal surface. C is its centre and Q and P are two points equidistant from C. Let v_P, v_Q and v_C be the magnitude of velocities of points P, Q and C respectively, then



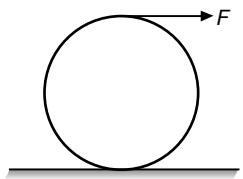
(A) $v_Q > v_C > v_P$
(B) $v_Q < v_C < v_P$
(C) $v_Q = v_P, v_C = \frac{1}{2}v_P$
(D) $v_Q < v_C > v_P$

133. Two particles A and B are moving with constant velocities $\vec{v}_1 = 1\ \text{ms}^{-1}\,\hat{j}$ and $\vec{v}_2 = 2\ \text{ms}^{-1}\,\hat{i}$ respectively in XY plane. At time $t = 0$, the particle A is at co-ordinates $(0,\ 0)$ m and B is at $(0,\ -4)$ m. The angular velocity of B with respect to A at $t = 2$ s is

(A) $\frac{1}{2}$ rads^{-1} (B) 1 rads^{-1}

(C) 2 rads^{-1} (D) 4 rads^{-1}

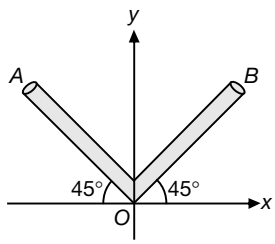
134. A force F is applied at the top of a ring of mass M and radius R placed on a rough horizontal surface as shown in figure. Friction is sufficient to prevent slipping. The friction force acting on the ring is



(A) $\frac{F}{2}$ towards right (B) $\frac{F}{3}$ towards left

(C) $\frac{2F}{3}$ towards right (D) ZERO

135. Two rods OA and OB of equal length and mass are lying in xy plane as shown in figure. Let I_x, I_y and I_z be the moment of inertias of both the rods about x, y and z-axis respectively. Then



(A) $I_x = I_y > I_z$ (B) $I_x = I_y < I_z$

(C) $I_x > I_y > I_z$ (D) $I_z > I_y > I_x$

136. A carpet of mass M made of inextensible material is rolled along its length in the form of a cylinder of radius R and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. The horizontal velocity of the axis of cylindrical part of the carpet when its radius reduces to $\frac{R}{2}$ is

(A) $\sqrt{gR}$ (B) $\sqrt{\frac{7}{3}gR}$

(C) $\sqrt{\frac{8}{3}gR}$ (D) $\sqrt{\frac{14}{3}gR}$

137. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is

(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$

(C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) infinitesimal

138. A solid sphere rolls down two different inclined planes of same height but of different inclinations. In both cases

(A) the speed and time of descend will be same

(B) the speed will be same but time of descend will be different

(C) the speed will be different but time of descend will be same

(D) speed and time of descend both are different

139. The linear velocity of a particle moving with angular velocity $\vec{\omega} = 2\hat{k}$ at position vector $\vec{r} = 2\hat{i} + 2\hat{j}$ is

(A) $4\left(\hat{i} - \hat{j}\right)$ (B) $4\left(\hat{j} - \hat{i}\right)$

(C) $4\hat{i}$ (D) $-4\hat{i}$

140. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved?

(A) Centre of circle

(B) On the circumference of the circle

(C) Inside the circle

(D) Outside the circle

141. A force $\vec{F} = a\hat{i} + 3\hat{j} + 6\hat{k}$ is acting at a point $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$. The value of a for which angular momentum is conserved is

(A) 0 (B) 1

(C) –1 (D) 2

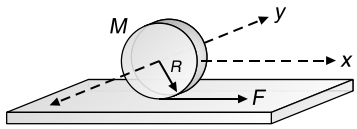
142. A solid cylinder of mass 50 kg and radius 0.5 mis free to rotate about its axis which is horizontal. A string is wound around the cylinder with one end attached to it and other hanging freely. The tension in the string required to produce an angular acceleration of 2 revs^{-2} in the cylinder is

(A) 78.5 N (B) 157 N

(C) 314 N (D) 628 N

143. A uniform cylindrical disc of radius R and mass M is pulled over a horizontal frictionless surface by a constant force. The force is applied by means of a string wound around the disc as shown in the

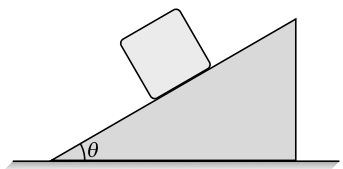
figure. If it starts from rest at $t = 0$, the linear and angular displacements respectively at time t are



(A) $\left(\frac{F}{M}\right)t^2, \left(\frac{F}{M}\right)t^2$ (B) $\left(\frac{F}{2M}\right)t, \left(\frac{F}{2MR}\right)t^2$

(C) $\left(\frac{F}{M}\right)t^2, \left(\frac{F}{MR}\right)t^2$ (D) $\left(\frac{2F}{M}\right)t^2, \left(\frac{2F}{MR}\right)t^2$

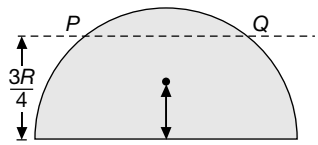
144. A cube is placed on a rough inclined plane of inclination θ as shown in figure. The coefficient of friction between the cube and the plane is μ. If the angle θ is gradually increased, the cube slides before toppling when



(A) $\mu < 1$ (B) $\mu < \frac{1}{2}$

(C) $\mu > 1$ (D) $\mu > \frac{1}{2}$

145. Assuming the centre of mass of a hemisphere to lie at a height $\frac{3R}{8}$ from the base, the radius of gyration of a solid hemisphere of mass M and radius R about an axis parallel to the diameter at a distance $\frac{3}{4}R$ from this plane is given by



(A) $\frac{3R}{\sqrt{10}}$ (B) $\frac{5R}{4}$

(C) $\frac{5R}{8}$ (D) $\sqrt{\frac{2}{5}}R$

146. A particle moves in a circular path with decreasing speed. Choose the correct statement

(A) Angular momentum remains constant

(B) Acceleration $(\vec{a})$ is towards the centre

(C) Particle moves in a spiral path with decreasing radius

(D) The direction of angular momentum remains constant

147. A sphere of mass M and radius R moves on a horizontal surface with a velocity V and then climbs up an inclined plane up to a height h where it stops. The height upto which it rises will be

(A) directly proportional to the square of the velocity and inversely proportional to the angle of the inclination

(B) directly proportional to the velocity and inversely proportional to its mass

(C) directly proportional to the square of and the angle of the velocity and independent of mass the inclination

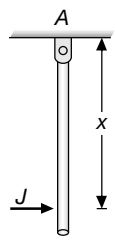
(D) directly proportional to its velocity and inversely proportional to the angle of the inclination

148. The moment of inertia of the body about an axis is $1.2\ \text{kgm}^2$. Initially the body is at rest. In order to produce a rotational kinetic energy of $1500\ \text{J}$, an angular acceleration of $25\ \text{rads}^{-2}$must be applied about the axis for the duration of

(A) 2 s (B) 4 s

(C) 8 s (D) 10 s

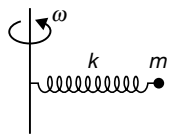
149. A uniform rod of length l is pivoted at point A. It is struck by a horizontal force which delivers an impulse J at a distance x from point A as shown in figure. The impulse delivered by pivot is zero when x equals



(A) $\frac{l}{2}$ (B) $\frac{l}{3}$

(C) $\frac{2l}{3}$ (D) $\frac{3l}{4}$

150. A particle of mass m is fixed to one end of a light spring of force constant k and unstretched length l. The system is rotated about the other end of the spring with an angular velocity ω, in gravity free space. The increase in length of the spring will be



(A) $\frac{ml\omega^2}{k}$ (B) $\frac{ml\omega^2}{k-m\omega^2}$

(C) $\frac{ml\omega^2}{k+m\omega^2}$ (D) None of these

151. A uniform cube of side a and mass m rests on a rough horizontal surface. A horizontal force F is applied normal to one face at a point that lies directly above the centre of the face at a height $\frac{a}{4}$ above the centre. The minimum value of F for which the cube begins to topple above an edge without sliding is

(A) $\frac{1}{4}mg$ (B) $\frac{1}{2}mg$

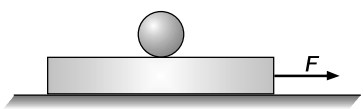
(C) $\frac{2}{3}mg$ (D) $2mg$

152. The ratio of moment of inertia of the ring about an axis passing through its rim and perpendicular to its plane and that about its diameter is

(A) 1 : 4 (B) 4 : 1

(C) 1 : 2 (D) 2 : 1

153. A plank with a uniform sphere placed on it resting on a smooth horizontal plane. Plank is pulled to right by a constant force F. If sphere does not slip over the plank. Which of the following is incorrect?



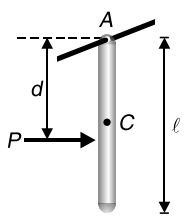
(A) Acceleration of the centre of sphere is less than that of the plank

(B) Work done by friction acting on the sphere is equal to its total kinetic energy

(C) Total kinetic energy of the system is equal to work done by the force F

(D) None of the above

154. The uniform bar of length l and mass m is suspended from a very thin axle that passes through a hole near the top end A of the bar as shown. An impulsive blow having magnitude P is applied at right angles to the bar in order to start the bar rotating about A without breaking the axle. For this



(A) $d=\frac{l}{2}$ (B) $d=\frac{l}{3}$

(C) $d=l$ (D) $d=\frac{2l}{3}$

155. A railway track is banked for a speed v, by making the height of the outer rail h higher than that of the inner rail. The distance between the rails is d. The radius of curvature of the track is r.

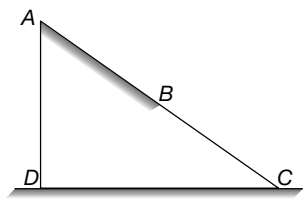
(A) $\frac{h}{d}=\frac{v^2}{rg}$

(B) $\tan\left\{\sin^{-1}\left(\frac{h}{d}\right)\right\}=\frac{v^2}{rg}$

(C) $\tan^{-1}\left(\frac{h}{d}\right)=\frac{v^2}{rg}$

(D) $\frac{h}{r}=\frac{v^2}{dg}$

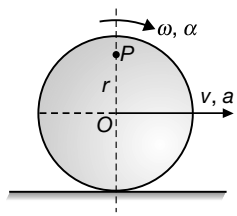
156. Of a wedge shown in figure the portion AB is rough and BC is smooth. A solid cylinder rolls without slipping from A to B. If $AB=BC$, then ratio of translational kinetic energy to rotational kinetic energy, when the cylinder reaches the point C is



(A) $\frac{3}{5}$ (B) $\frac{7}{5}$

(C) $\frac{8}{3}$ (D) 5

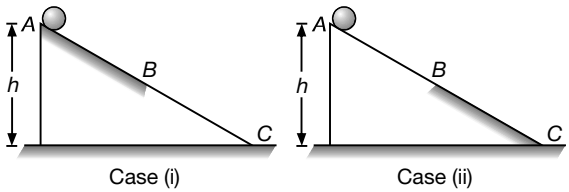
157. A disc of radius R rolls on a horizontal ground with linear acceleration a and angular acceleration α as shown in figure. The magnitude of acceleration of point P shown in figure at an instant when its linear velocity is v and angular velocity is ω will be



(A) $\sqrt{(a+r\alpha)^2+(r\omega^2)^2}$ (B) $\frac{ar}{R}$

(C) $\sqrt{r^2\alpha^2+r^2\omega^4}$ (D) $r\alpha$

158. In both the cases, B is mid-point of A and C, while all other factors are same, except that in Case (i) AB is rough and BC is smooth while in Case (ii) AB is smooth and BC is rough. The kinetic energy of the ball on reaching the bottom

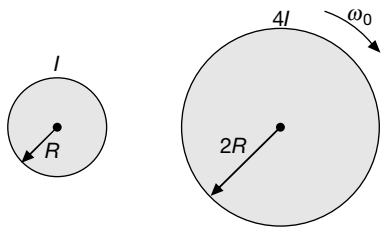


(A) is same in both the cases
(B) is greater in Case (i)
(C) is greater in Case (ii)
(D) None of these

159. A wheel of radius R rolls on the ground with a uniform velocity v. The relative acceleration of topmost point of the wheel with respect to the bottommost point is

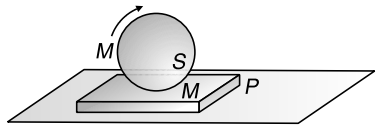
(A) $\frac{v^2}{R}$ (B) $\frac{2v^2}{R}$
(C) $\frac{v^2}{2R}$ (D) $\frac{4v^2}{R}$

160. Two cylinders having radii $2R$ and R and moment of inertias $4I$ and I about their central axes are supported by axles perpendicular to their planes. The large cylinder is initially rotating clockwise with angular velocity ω_0. The small cylinder is moved to the right until it touches the large cylinder and is made to rotate by the frictional force between the two. Eventually slipping ceases and the two cylinders rotate at constant rates in opposite directions. The final angular velocity of the small cylinder is



(A) $\frac{\omega_0}{4}$ (B) ω_0
(C) $\frac{\omega_0}{2}$ (D) $\frac{\omega_0}{8}$

161. A sphere S of mass M is given a finite angular velocity about a horizontal axis through its centre. Now it is gently placed on a plank P of same mass. The coefficient of friction between the two is μ and the plank rests on a smooth horizontal surface as shown. The initial acceleration of the sphere relative to the plank will be



(A) μg (B) $2\mu g$
(C) $\frac{2}{5}\mu g$ (D) ZERO

162. A solid cylinder R is free to rotate about its axis which is horizontal. A string is wound around it and a mass m is attached to its free end. When m falls through a distance h, its speed at that instant is proportional to

(A) R (B) $\frac{1}{R}$
(C) $\frac{1}{R^2}$ (D) None of these

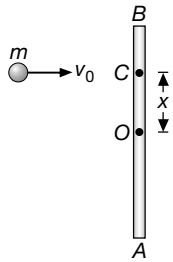
163. A wheel of mass 40 kg and radius of gyration 0.5 m comes to rest from a speed of 1800 rpm in 30 s. Assuming that the retardation is uniform, the value of the retarding torque, in Nm, is

(A) 10π (B) 20π
(C) 30π (D) 40π

164. A solid cylinder of mass M and radius R rolls down the inclined plane from height h without slipping. The speed of its centre of mass when it reaches the bottom is

(A) $\sqrt{2gh}$ (B) $\sqrt{\frac{4}{3}gh}$
(C) $\sqrt{\frac{3}{4}gh}$ (D) $\sqrt{\frac{4g}{h}}$

165. A uniform rod AB of length L having mass m are lying on a smooth table. A small particle of mass m strikes the rod with a velocity v_0 at point C, a distance x from the centre O. The particle comes to rest after collision. The value of x, so that point A of the rod remains stationary just after collision is



(A) $\frac{L}{3}$ (B) $\frac{L}{6}$

(C) $\frac{L}{4}$ (D) $\frac{L}{12}$

166. A wheel of radius r rolls without slipping with a speed v on a horizontal road. When it is at a point A on the road, a small blob of mud separates from the wheel at its highest point and lands at point B on the road

(A) $AB = v\sqrt{\frac{r}{g}}$ (B) $AB = 2v\sqrt{\frac{r}{g}}$

(C) $AB = 4v\sqrt{\frac{r}{g}}$ (D) $AB = 8v\sqrt{\frac{r}{g}}$

167. A mass M is moving with a constant velocity parallel to the x-axis. Its angular momentum with respect to the origin

(A) is zero (B) remains constant

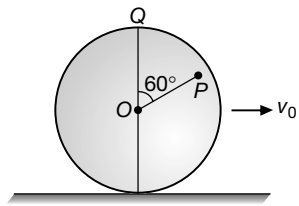
(C) increases (D) decreases

168. A billiard ball of mass m and radius r, hit by a cue at a height h above the centre, acquires a linear velocity v_0. The angular velocity ω_0 acquired by the ball is

(A) $\frac{2v_0h}{5r^2}$ (B) $\frac{5v_0h}{2r^2}$

(C) $\frac{2v_0r^2}{5h}$ (D) $\frac{5v_0r^2}{2h}$

169. A disc of radius r rolls without slipping on a rough horizontal floor. If velocity of its centre of mass is v_0, then velocity of point P at distance $\frac{r}{2}$ such that $\angle POQ = 60°$ as shown in the figure is



(A) v_0 (B) $\frac{v_0}{2}$

(C) $\frac{v_0\sqrt{7}}{2}$ (D) $\frac{v_0\sqrt{3}}{2}$

170. A particle is projected with velocity v at an angle of ϕ with horizontal. The average angular velocity of the particle from the point of projection to impact equals

(A) $\frac{g\cos\phi}{\phi v}$ (B) $\frac{g}{v\sin\phi}$

(C) $\frac{g}{v\phi}$ (D) $\frac{g\phi}{v\sin\phi}$

171. A uniform stick of length F_3 and mass m lies on a smooth table. It rotates with angular velocity ω about an axis perpendicular to the table and passes through one end of the stick. The angular momentum of the stick about the end is

(A) $ml^2\omega$ (B) F_2

(C) $I_1 + I_2$ (D) $\frac{ml^2\omega}{6}$

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. The moment of inertia of a solid cylinder of mass M, length $2R$ and radius R about an axis passing through the centre of mass and perpendicular to the axis of the cylinder is I_1 and about an axis passing through one end of the cylinder and perpendicular to the axis of cylinder is I_2

(A) $I_2 - I_1 = MR^2$ (B) $I_2 > I_1$

(C) $\frac{I_2}{I_1} = \frac{19}{12}$ (D) $I_1 - I_2 = MR^2$

2. The torque $\vec{\tau}$ on a body about a given point is found to be $\vec{A} \times \vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is angular momentum of the body about that point. From this it follows that

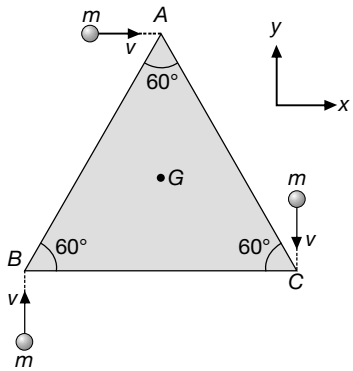
(A) $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instants of time.

(B) the component of $\vec{L}$ in the direction of $\vec{A}$ does not change with time.

(C) the magnitude of $\vec{L}$ does not change with time.

(D) $\vec{L}$ does not change with time.

3. A triangular wedge ABC of mass m and sides $2a$ lies on a smooth horizontal plane as shown. Three point masses of mass m each strike the wedge at A, B and C with speeds v as shown. After the collision, the particles come to rest. Select the correct alternative(s).



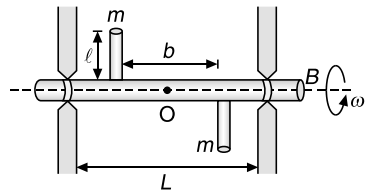
(A) The centre of mass of ABC remains stationary after collision

(B) The centre of mass of ABC moves with a velocity v along x-axis after collision

(C) The triangular wedge rotates with an angular velocity $\omega = \frac{2\sqrt{3}mva}{I}$ about its centre of mass (here, I is the moment of inertia of triangular wedge about its centroid axis perpendicular to its plane)

(D) A point lying at a distance of $\left(\frac{I}{2\sqrt{3}ma}\right)$ from G on perpendicular bisector of BC (below G) is at rest just after collision

4. Figure shows a horizontal rod AB which is free to rotate about two smooth bearing system. Two identical uniform rods each of mass m are attached to rod AB. All the dimensions are given in the Figure. The system is rotating with constant angular velocity ω in such a way that the upper rod is coming outward from the plane of the paper in the position shown. Gravity can be assumed to be absent in the experiment, then choose the correct option(s).



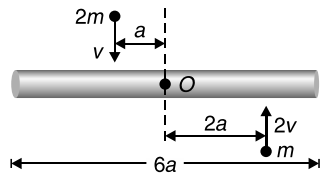
(A) The hinge reaction at A on the rod AB is downward

(B) The hinge reaction at B on the rod AB is upward

(C) The hinge reaction at B on the rod AB is downward

(D) The angular momentum of the system about point O is not along the rod AB.

5. A uniform bar of length $6a$ and mass $8m$ lies on a smooth horizontal table. Two point masses m and $2m$ moving in the same horizontal plane with speeds $2v$ and v, respectively, strike the bar as shown in the figure and stick to it after collision. Denoting angular velocity (about the centre of mass), total energy and the centre of mass velocity by ω, E and V_c respectively, we have after collision



(A) $V_c = 0$ (B) $\omega = \frac{3v}{5a}$

(C) $\omega = \frac{v}{5a}$ (D) $E = \frac{3mv^2}{5}$

6. A particle of mass m is travelling with a constant velocity $\vec{v} = v_0\hat{i}$ along the line $y = b$, $z = 0$. If dA be the area swept out by the position vector from origin to the particle in time dt and L be the magnitude of angular momentum of particle about origin at any time t. Then

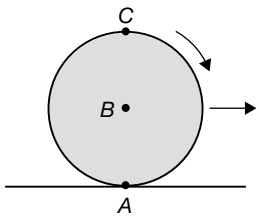
(A) $L \neq$ constant

(B) $L =$ constant

(C) $\frac{dA}{dt} = \frac{2L}{m}$

(D) $\frac{dA}{dt} = \frac{L}{2m}$

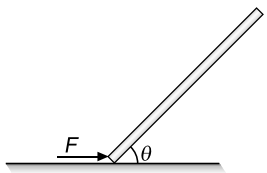
7. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,



(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$ (B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(C) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$ (D) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

8. A rod of length l is balanced in a vertical plane by applying a horizontal force F to its end lying on the surface as shown in Figure.



The linear mass density of the rod varies with distance r from the end lying on the surface as $\lambda(r) = \left(\frac{\lambda_0}{l^2}\right)r$, where λ_0 is a positive constant. If the applied force equals 4/3 times the weight of the rod and the rod only translates, then select the correct option(s).

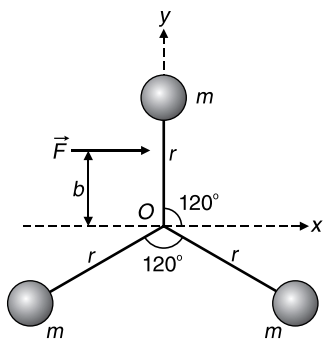
(A) $\theta = 37°$

(B) Acceleration of rod is $\frac{4g}{3}$

(C) normal force is $\frac{\lambda_0 g}{2}$

(D) normal force is $\lambda_0 g$

9. Each of the three balls has a mass m and is welded to the rigid equiangular from of negligible mass. The assembly rests on a smooth horizontal surface. A force F is suddenly applied to one bar as shown.



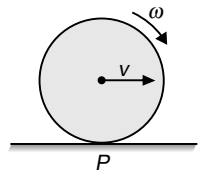
(A) Acceleration of point O is $\vec{a} = \frac{1}{3}\frac{\vec{F}}{m}\hat{i}$.

(B) Acceleration of point O is $\vec{a} = \frac{\vec{F}}{m}\hat{i}$.

(C) The magnitude of angular acceleration of the frame is $\alpha = \frac{1}{3}\frac{Fb}{mr^2}$.

(D) The magnitude of angular acceleration of the frame is $\alpha = \frac{Fb}{mr^2}$.

10. A body of mass m, radius R rolls on a horizontal surface with linear velocity v as shown in Figure.



If L_1 and L_2 be the magnitudes of the respective angular momenta of the body about centre of mass and the point of contact P. Then

(A) $L_2 = 2L_1$, for all cases

(B) $L_2 = 2L_1$, if radius of gyration $K = R$

(C) $L_2 > 2L_1$, if radius of gyration $K < R$

(D) $L_2 > 2L_1$, if radius of gyration $K > R$

11. A and B are two solid spheres of equal masses. A rolls down an incline plane without slipping from a height of 3 m. B falls vertically from the same height. Then on reaching the ground

(A) A can do more work than B

(B) B can do more work than A

(C) both perform equal work

(D) both will have different linear speeds

12. A solid body rotates about a stationary axis so that its angular velocity depends upon the rotation angle ϕ as $\omega = \omega_0 - a\phi$, where a and ω_0 are positive constants. Initially that is at $t = 0$, $\phi = 0$.

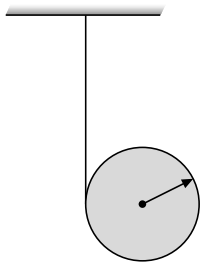
(A) $\phi = \frac{\omega_0}{a}(1 - e^{-at})$

(B) $\phi = \frac{\omega_0}{a}(1 + e^{at})$

(C) $\omega = \omega_0 e^{-at}$

(D) $\omega = \omega_0 e^{at}$

13. A string is wrapped over a uniform cylinder, as shown in Figure (side view). When cylinder is released, string unwraps without any slipping and cylinder comes down. Select the correct statements(s).



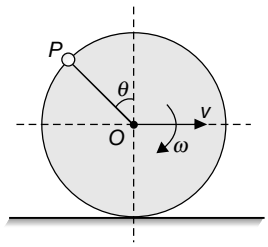
(A) Work done by tension force on the cylinder is zero.
(B) Work done by tension is negative
(C) Ratio of rotational kinetic energy and translational kinetic energy is $\frac{1}{2}$
(D) Ratio of rotational kinetic energy to translational kinetic energy is 2

14. The density of a rod AB increases linearly from A to B. Its midpoint is O and its centre of mass is at C. Four axes pass through A, B, O and C, all perpendicular to the length of the rod. The moments of inertia of the rod about these axes are I_A, I_B, I_O and I_C respectively

(A) $I_A > I_B$ (B) $I_A < I_B$
(C) $I_O > I_C$ (D) $I_O < I_C$

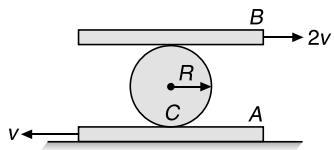
15. A thin uniform rod of mass m and length l is free to rotate about its upper end. When it is at rest, it receives an impulse J at its lowest point, normal to its length. Immediately after impact
(A) the angular momentum of the rod is Jl
(B) the angular velocity of the rod is $\frac{3J}{ml}$
(C) the kinetic energy of the rod is $\frac{3J^2}{2m}$
(D) the linear velocity of the midpoint of the rod is $\frac{3J}{2m}$

16. A disc of radius R rolls on a horizontal surface with linear velocity v and angular velocity ω. A point P on the circumference of the disc at angle θ shown has a vertical velocity. Then,



(A) $\theta = \pi + \sin^{-1}\left(\frac{v}{R\omega}\right)$ (B) $\theta = \frac{\pi}{2} - \sin^{-1}\left(\frac{v}{R\omega}\right)$
(C) $\theta = \pi - \cos^{-1}\left(\frac{v}{R\omega}\right)$ (D) $\theta = \pi + \cos^{-1}\left(\frac{v}{R\omega}\right)$

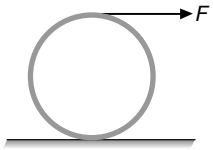
17. A cylinder C of mass $8m$ is rolling without sliding over two horizontal planks A and B having masses $2m$ and m, moving with uniform velocities $-v\hat{i}$ and $2v\hat{i}$ respectively. Select the correct option(s).



(A) The instantaneous axis of rotation is located at a distance of $\frac{4R}{3}$ below the topmost contact point.
(B) Angular velocity of the cylinder is $\frac{3v}{2R}$
(C) Translational kinetic energy of the system is $4mv^2$
(D) Kinetic energy of the system is $\frac{9}{2}mv^2$

18. A uniform disc is rotating at a constant speed in a vertical plane about a fixed horizontal axis passing through the centre of the disc. A piece of the disc from its rim detaches itself from the disc at the instant when it is at horizontal level with the centre of the disc and moving upward. Then about the fixed axis, the angular speed of the
(A) remaining disc decreases
(B) remaining disc increases
(C) remaining disc stays constant
(D) broken away piece decreases initially and later increases

19. A constant force F is applied at the top of a ring of mass M and radius is R as shown. The angular momentum of particle about point of contact at time t



(A) is constant
(B) increases linearly with time
(C) is $2FRt$
(D) decreases linearly with time

20. A particle moves in a circle of radius r with angular velocity $\vec{\omega}$. At some instant its velocity is $\vec{v}$ and radius vector with respect to centre of the circle is $\vec{r}$. At this particular instant centripetal acceleration $\vec{a}_c$ of the particle is

(A) $\vec{v}\times\vec{\omega}$ (B) $\vec{\omega}\times\vec{v}$
(C) $\vec{v}\times(\vec{r}\times\vec{\omega})$ (D) $\vec{\omega}\times(\vec{\omega}\times\vec{r})$

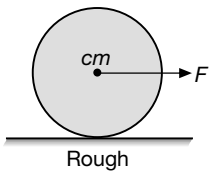
21. A disc rolls without slipping in the absence any external force on a

(A) rough inclined plane
(B) smooth inclined plane
(C) rough horizontal surface
(D) smooth horizontal surface

22. A pan containing a layer of uniform thickness of ice is placed on a circular turntable with its centre coinciding with the centre of the turntable. The turntable is now rotated at a constant angular velocity about a vertical axis passing through its centre and the driving torque is withdrawn. There is no friction between the table and the pivot. The pan rotates with the table. As the ice melts

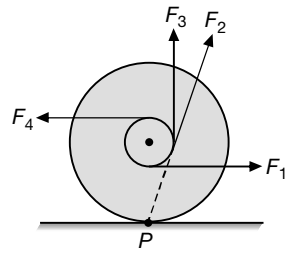
(A) the angular velocity of system increases
(B) the angular velocity of system decreases
(C) the angular velocity remains unchanged
(D) the moment of inertia of system increases

23. A solid cylinder of mass M and radius R is rolled horizontally on a rough surface as shown in the figure. Choose the correct alternative(s)



(A) The acceleration of the centre of mass is $\frac{F}{M}$
(B) The acceleration of the centre of mass is $\frac{2}{3}\frac{F}{M}$
(C) The friction force on the cylinder acts backward
(D) The magnitude of the friction force is $\frac{F}{3}$

24. A spool of wire rests on a horizontal surface as shown in figure. As the wire is pulled, the spool does not slip at contact point P. On separate trials, each one of the forces F_1, F_2, F_3 and F_4 is applied to the spool. For each one of these forces the spool

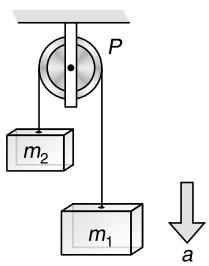


(A) will rotate anticlockwise if F_1 is applied
(B) will rotate anticlockwise if F_3 is applied
(C) will rotate clockwise if F_4 is applied
(D) will not rotate if F_2 is applied

25. A rod of length l is standing vertically on a frictionless surface. It is disturbed slightly from this position. Let ω and α be the angular speed and angular acceleration of the rod when the rod turns through an angle θ with the vertical. If the speed of the centre of mass is v and the value of acceleration of centre of mass of the rod is a, then

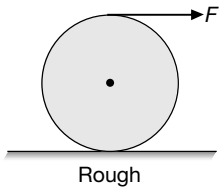
(A) $a=\frac{l\alpha}{2}\sin\theta+\frac{l\omega^2}{2}\cos\theta$
(B) $a=\frac{l\omega^2}{2}\sin\theta+\frac{l\alpha}{2}\cos\theta$
(C) $v=\frac{l\omega}{2}\cos\theta$
(D) $v=\frac{l\omega}{2}\sin\theta$

26. In the figure the blocks have unequal masses m_1 and $m_2(m_1>m_2)$. m_1 has a downward acceleration a. The pulley P has a radius R, and some mass. The string does not slip on the pulley



(A) the two sections of the string have unequal tensions
(B) the two blocks have accelerations of equal magnitude
(C) the angular acceleration of P is $\frac{a}{R}$
(D) $a < \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$

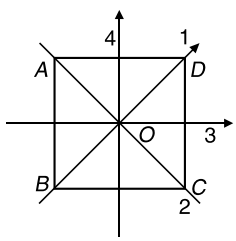
27. A solid sphere of radius R is rolled by a force F acting at the top of the sphere as shown in the figure. There is no slipping and initially sphere is in the rest position then



(A) work done by force F when the centre of mass moves a distance S is $2FS$
(B) speed of centre of mass (CM) when CM moves a distance S is $\sqrt{\frac{20FS}{7M}}$
(C) work done by the force F when CM moves a distance S is FS
(D) speed of the CM when CM moves a distance S is $\sqrt{\frac{4FS}{M}}$

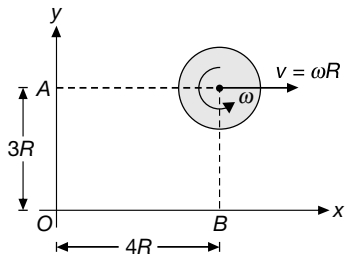
28. If a circular concentric hole is made in a disc, then about an axis passing through the centre of the disc and perpendicular to its plane the
(A) radius of gyration increases
(B) radius of gyration decreases
(C) moment of inertia increases
(D) moment of inertia decreases

29. The moment of inertia of a thin square plate $ABCD$ of uniform thickness about an axis passing through the centre O and perpendicular to the plate is



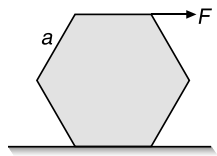
(A) $I_1 + I_2$ (B) $I_3 + I_4$
(C) $I_1 + I_3$ (D) $I_1 + I_2 + I_3 + I_4$

30. A disc of mass M and radius R moves in the x-y plane as shown in the figure. The angular momentum of the disc at the instant shown is



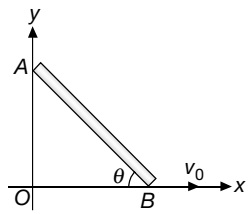
(A) $\frac{5MR^2\omega}{2}$ about O (B) $\frac{7MR^2\omega}{2}$ about O
(C) $\frac{MR^2\omega}{2}$ about A (D) $4MR^2\omega$ about A

31. A horizontal force F acts on one side of a hexagonal body that lies on a horizontal rough surface as shown in Figure. Select the correct statement(s).



(A) For toppling, the minimum value of coefficient of friction is $\frac{1}{2\sqrt{3}}$.
(B) For toppling, the minimum value of coefficient of friction is $\frac{2}{\sqrt{3}}$.
(C) If $\mu = 2\mu_{\min}$ and applied force $F = \frac{mg}{\sqrt{3}}$, then angular acceleration of the body is $\frac{6g}{17a}$.
(D) If $\mu = 2\mu_{\min}$ and applied force $F = \frac{mg}{\sqrt{3}}$, then angular acceleration of the body is $\frac{6g}{5a}$.

32. The end B of the rod AB which makes angle θ with the floor is being pulled with a constant velocity v_0 as shown. The length of the rod is l. At the instant when $\theta = 37°$, the



(A) velocity of end A is constant

(B) velocity of end A is $\frac{4}{3}v_0$ downwards

(C) angular velocity of rod is constant

(D) angular velocity of rod is $\frac{5}{3}\frac{v_0}{l}$

33. Four particles each of mass m are placed at four corners of a square $ABCD$ of side a. If the point O happens to be the centre of the square, then the moment of inertia of all four particles about an axis passing through

(A) A and B is $2ma^2$

(B) A and C is ma^2

(C) O and perpendicular to plane of square is $2ma^2$

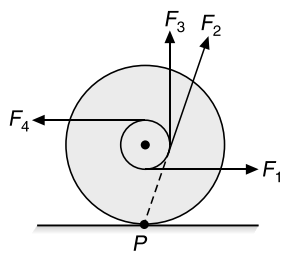
(D) O and parallel to CD is ma^2

34. In pure rolling, the fraction of total energy associated with rotation is α for a ring and β for a solid sphere. Then

(A) $\alpha = \frac{1}{2}$ (B) $\alpha = \frac{1}{4}$

(C) $\beta = \frac{2}{5}$ (D) $\beta = \frac{2}{7}$

35. A spool of wire rests on a horizontal surface as shown in Figure.



As the wire is pulled, the spool does not slip at contact point P. On separate trials, each one of the forces F_1, F_2, F_3 and F_4 is applied to the spool. The direction of friction force is

(A) towards left if F_1 is applied

(B) towards left if F_2 is applied

(C) towards right if F_3 is applied

(D) may be right or left or friction may be zero if F_4 is applied

36. A student holds the axle of a spinning bicycle wheel while seated on a pivoted stool. The student and the stool are initially at rest while the wheel is spinning in a horizontal plane with an angular momentum $\vec{L}_0$ pointing upward. The wheel is inverted about its centre by $180°$.

(A) The angular momentum of the system is conserved.

(B) The angular momentum of the system is not conserved.

(C) The final angular momentum of student plus stool will be $2\vec{L}_0$.

(D) The final angular momentum of student plus stool will be zero.

37. Which of the following statement(s) is/are correct for a spherical body rolling without slipping on a rough horizontal surface at rest?

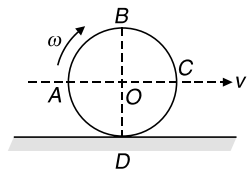
(A) The speed of some of the point(s) is (are) zero

(B) The acceleration of a point in contact with ground is zero

(C) Work done by friction may or may not be zero

(D) Friction force may or may not be zero

38. A ring rolls without slipping on a horizontal surface. At any instant, its position is as shown in the figure



(A) section ABC has greater kinetic energy than section ADC

(B) section BC has greater kinetic energy than section CD

(C) section BC has the same kinetic energy as section DA

(D) the section AB, BC, CD and DA have the same kinetic energy

39. A particle of mass m is projected with velocity v making an angle of $45°$ with the horizontal. The magnitude of angular momentum of the projectile about point of projection when the particle is at maximum height h is

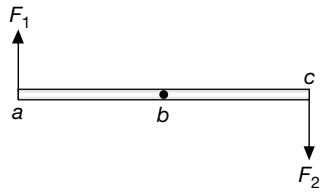
(A) ZERO

(B) $\frac{mv^3}{4\sqrt{2}g}$

(C) $\frac{mv^3}{\sqrt{2}g}$

(D) $m\sqrt{2gh^3}$

40. Two forces F_1 and F_2 are acting on a rod abc as shown in figure. Select the correct statement(s).



(A) If $F_1 \neq F_2$, then $\tau_a \neq \tau_b \neq \tau_c$

(B) If $F_1 \neq F_2$, then $\tau_a = \tau_c \neq \tau_b$

(C) If $F_1 = F_2$, then $\tau_a = \tau_b = \tau_c$ (for both the forces)

(D) If $F_1 = F_2$, then $\tau_a = \tau_c \neq \tau_b$

41. A horizontal disc rotates freely about a vertical axis through its centre. A ring, having the same mass and radius as the disc, is now gently placed on the disc. After some time, the two rotate with a common angular velocity

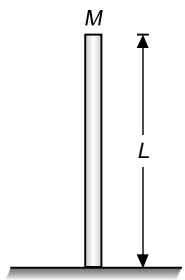
(A) some friction exists between the disc and the ring.

(B) the angular momentum of the 'disc plus ring' is conserved.

(C) the final common angular velocity is $\frac{2}{3}$ times initial angular velocity of the disc.

(D) $\frac{2}{3}$ times the initial kinetic energy changes to heat.

42. A uniform rod of mass M and length L is held vertically on a smooth horizontal surface. When the rod is released, choose the correct alternative(s).



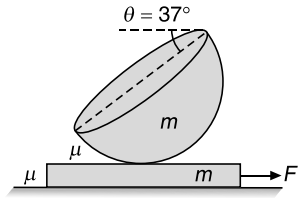
(A) The centre of mass of the rod accelerates in the vertical direction

(B) Initially, the magnitude of the normal reaction is Mg

(C) When the rod becomes just horizontal, the magnitude of the normal reaction becomes $\frac{Mg}{2}$

(D) When the rod becomes just horizontal, the magnitude of the normal reaction becomes $\frac{Mg}{4}$

43. A force F is applied on the plank such that the hollow hemispherical shell of mass $m = 5$ kg is in equilibrium as shown in Figure.



The coefficient of friction μ between hemispherical shell and plank is same as between plank and ground. Friction is just sufficient to prevent the slipping. Taking $g = 10\ \text{ms}^{-2}$, select the correct statement(s).

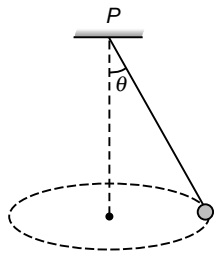
(A) Minimum coefficient of friction to prevent slipping is $\mu_{\min} = \frac{1}{4}$

(B) Minimum coefficient of friction to prevent slipping is $\mu_{\min} = \frac{1}{2}$

(C) Acceleration of plank is $10\ \text{ms}^{-2}$

(D) Magnitude of applied force is 100 N

44. A particle of mass 2 kg is attached to a string of length 1 m, moves in a horizontal circle just like a conical pendulum. The string makes an angle $\theta = 30^o$ with the vertical. Select the correct alternative(s) $\left(g = 10\ \text{ms}^{-2}\right)$.



(A) The vertical component of angular momentum of mass about the point of support P is approximately $1.7\ \text{kgm}^2\text{s}^{-1}$.

(B) The horizontal component of angular momentum of mass about the point of support P is approximately $2.9\ \text{kgm}^2\text{s}^{-1}$.

(C) Magnitude of $\frac{d\vec{L}}{dt}$, where $\vec{L}$ is the angular momentum of mass about point of support P, is approximately $10\ \text{kgm}^2\text{s}^{-2}$.

(D) $\vec{\tau} = \frac{d\vec{L}}{dt}$ will not hold good in this case.

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** If earth shrink (without change in mass) to half it's present size. Length of the day would become 6 hours.
Statement-2: As size of earth changes its moment of inertia changes.

2. **Statement-1:** A ring moving down on a smooth inclined plane will be in slipping motion.
Statement-2: Work done by friction in pure rolling motion is zero.

3. **Statement-1:** A non-uniform sphere is placed such that its centre is at the origin of coordinate system. If I_x and I_y be moment of inertia about x-axis and y-axis respectively then moment of inertia about z-axis is $I_x + I_y$.
Statement-2: According to perpendicular axis theory $I_z = I_x + I_y$ when object is lying in x-y plane.

4. **Statement-1:** A sphere is performing pure rolling on a rough horizontal surface with constant angular velocity. Frictional force acting on the sphere is zero.
Statement-2: Velocity of contact point is zero.

5. **Statement-1:** If momentum of system is zero than kinetic energy must be zero.
Statement-2: If kinetic energy of system is zero than momentum must be zero.

6. **Statement-1:** A ladders is more likely to slip when a person is near the top than when he is near the bottom.
Statement-2: The friction between the ladder and floor decreases as he climbs up.

7. **Statement-1:** If rod is thrown upward with initial angular velocity and velocity of centre of mass then its momentum changes but angular velocity remains same.
Statement-2: Torque on rod about centre of mass due to gravitational force is zero.

8. **Statement-1:** The mass of a body cannot be considered to be concentrated at the centre of mass of the body for the purpose of computing its moment of inertia.
Statement-2: For then the moment of inertia of every-body about an axis passing through its centre of mass would be zero.

9. **Statement-1:** A disc is rolling on a rough horizontal surface. The instantaneous speed of the point of contact during perfect rolling is zero with respect to ground.
Statement-2: The force of friction can help in achieving pure rolling condition.

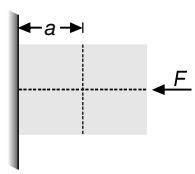
10. **Statement-1:** A solid sphere rolling on a rough horizontal surface. Acceleration of contact point is zero.
Statement-2: A solid sphere can roll on the smooth surface.

11. **Statement-1:** A disc is rolling on an inclined plane without slipping. The velocity of centre of mass is V. These other points on the disc lie on a circular arc having same speed as centre of mass.
Statement-2: When a disc is rolling on an inclined plane. The magnitude of velocities of all the point from the contact point is same, having distance equal to radius r.

12. **Statement-1:** The velocity of a body at the bottom of an inclined place of given height, is more when it slides down the plane, compared to, when it rolling down the same plane.
Statement-2: In rolling down, a body acquires both, kinetic energy of translation and rotation.

13. **Statement-1:** The force of friction in the case of a disc rolling without slipping down on inclined plane is $\frac{1}{3} g \sin\alpha$.
Statement-2: When the disc rolls without slipping, friction is required because for rolling condition velocity of point of contact is zero.

14. **Statement-1:** A horizontal force F is applied such that the block remains stationary because N will produce torque.
Statement-2: The torque produced by friction force is equal and opposite the torque produce due to normal reaction (N).



15. **Statement-1:** When a diver dives, the rotational kinetic energy of diver increases, during several somersaults.
Statement-2: When diver pulls his limbs, the moment of inertia decreases and on account of conservation of angular momentum his angular speed increases.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

A uniform rod AB of mass $3m$ and length $4l$, which is free to turn in a vertical plane about a smooth horizontal axis through A, is released from rest from the horizontal position. When the rod first becomes vertical, a point C of the rod, where $AC = 3l$, strikes a fixed peg. In Situation-I, for an impulse J_1 exerted by the peg on the rod, the rod is brought to rest by the peg. However, in Situation- II, for an impulse J_2 exerted by the peg on the rod, the rod rebounds and next comes to instantaneous rest when inclined to the downward vertical at an angle of $\frac{\pi}{3}$ radian. Based on above information, answer the following questions.

1. The value of J_1 in Situation-I is

(A) $m\sqrt{3gl}$ (B) $\frac{m}{8}\sqrt{3gl}$

(C) $8m\sqrt{\frac{gl}{3}}$ (D) $3m\sqrt{\frac{gl}{8}}$

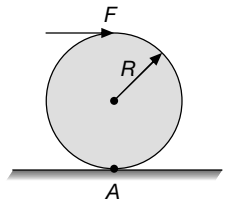
2. The value of J_2 in Situation-II is

(A) $m\sqrt{\frac{gl}{3}}$ (B) $4m\left(\sqrt{2}+1\right)\sqrt{\frac{2gl}{3}}$

(C) $2m\left(\sqrt{2}+1\right)\sqrt{\frac{gl}{3}}$ (D) $4m\sqrt{\frac{gl}{3}}$

Comprehension 2

When a body is hinged at a point and a force is acting on the body, in such a way that the line of action of force is at some distance from the hinged point, the body will start rotating about the hinged point. The angular acceleration of the body can be calculated by finding the torque of that force about the hinged point. A disc of mass m and radius R is hinged at point A at its bottom and is free to rotate in the vertical plane. A force of magnitude F is acting on the ring at the top most point. Based on above information, answer the following questions.



3. Tangential acceleration of centre of mass is

(A) $\frac{3F}{4m}$ (B) $\frac{F}{m}$

(C) $\frac{2F}{3m}$ (D) $\frac{4F}{3m}$

4. The component of reaction at hinge in vertical direction is

(A) $\frac{4}{3}mg$ (B) mg

(C) $\frac{mg}{2}$ (D) $\frac{2}{3}mg$

5. Component of reaction at hinge in horizontal direction is

(A) $\frac{F}{4}$ (B) F

(C) $\frac{F}{3}$ (D) $\frac{F}{2}$

Comprehension 3

A uniform rod AB of mass $3m$ and length $2l$ is lying at rest on a smooth horizontal table with a smooth vertical axis through the end A. A particle of mass $2m$ moves with speed $2u$ across the table and strikes the rod at its midpoint C. In Situation-I, the particle strikes the rod normally, the impact is perfectly elastic such that the speed of the particle after impact is v_1. In Situation-II, the particle strikes the rod such that its path before impact is inclined at $60°$ to AC, the impact is still perfectly elastic such that the speed of the particle after impact is v_2. Based on above information, answer the following questions.

6. The value of v_1 in Situation-I is

(A) u (B) $\frac{u}{2}$

(C) $\frac{2u}{3}$ (D) $\frac{u}{4}$

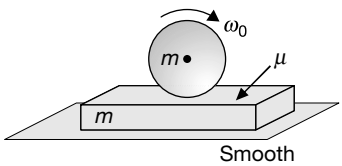
7. The value of v_2 in Situation-II is

(A) $\frac{u}{\sqrt{3}}$ (B) $\frac{2u}{\sqrt{3}}$

(C) $\frac{u}{\sqrt{2}}$ (D) $\frac{3u}{\sqrt{2}}$

Comprehension 4

A long horizontal plank of mass m is lying on a smooth horizontal surface. A sphere of same mass m and radius r is spinned about its own axis with angular velocity ω_0 and gently placed on the plank. The coefficient of friction between the plank and the sphere is μ. After some time, the pure rolling of the sphere on the plank starts. Based on above information, answer the following questions.



8. The time t at which the pure rolling begins is

(A) $\frac{r\omega_0}{9\mu g}$ (B) $\frac{2r\omega_0}{9\mu g}$

(C) $\frac{r\omega_0}{3\mu g}$ (D) $\frac{2r\omega_0}{\mu g}$

9. The velocity of the sphere, after the pure rolling begins is

(A) $\frac{2}{9}r\omega_0$ (B) $\frac{r\omega_0}{9}$

(C) $\frac{r\omega_0}{3}$ (D) $\frac{2}{3}r\omega_0$

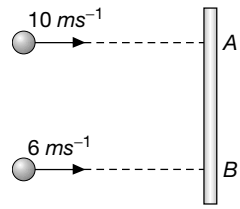
10. The displacement of the plank, till the sphere starts pure rolling is

(A) $\frac{r^2\omega_0^2}{81\mu g}$ (B) $\frac{2}{27}\left(\frac{r^2\omega_0^2}{\mu g}\right)$

(C) $\frac{4}{81}\left(\frac{r^2\omega_0^2}{\mu g}\right)$ (D) $\frac{2}{81}\left(\frac{r^2\omega_0^2}{\mu g}\right)$

Comprehension 5

A thin uniform bar lies on a frictionless horizontal surface and is free to move in any way on the surface. Its mass is 0.16 kg and length is $\sqrt{3}$ m. Two particles, each of mass 0.08 kg are moving on the same surface and towards the bar in a direction perpendicular to the bar one with a velocity of 10 ms^{-1}, and the other with 6 ms^{-1}, as shown in figure. The first particle strikes the bar at points A and the other point B. Points A and B are at a distance of 0.5 m from the centre of the bar. The particles strike the bar at the same instant of time and stick to the bar on collision. Based on above information, answer the following questions.



11. The velocity of the centre of mass of system, just after impact is

(A) 1 ms^{-1} (B) 2 ms^{-1}

(C) 3 ms^{-1} (D) 4 ms^{-1}

12. The angular velocity of the system, just after impact is

(A) 8 $rads^{-1}$ (B) 4 $rads^{-1}$

(C) 2 $rads^{-1}$ (D) 1 $rads^{-1}$

13. The loss of kinetic energy of the system in the above collision process is

(A) 1.72 J (B) 2.72 J

(C) 3.36 J (D) 4.36 J

Comprehension 6

A uniform disk of mass M and radius R is pivoted so that it can rotate freely about a horizontal axis through its centre and perpendicular to the plane of the disk. A small particle of mass m, the half of the mass of the disc, is attached to the rim of the disk at the top directly above the pivot. The system is given a gentle start and the disk begins to rotate about O. Based on above information, answer the following questions.

14. The angular velocity of the disk when the particle is at its lowest point is

(A) $\sqrt{\frac{g}{R}}$ (B) $\sqrt{\frac{3g}{R}}$

(C) $\sqrt{\frac{2g}{R}}$ (D) $\sqrt{\frac{g}{2R}}$

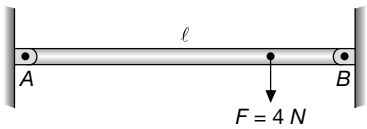
15. At the lowest point, the force that must be exerted by the disc on the particle to keep it on the disk is

(A) $\frac{Mg}{2}$ (B) $\frac{2Mg}{3}$

(C) $\frac{Mg}{3}$ (D) $\frac{3Mg}{2}$

Comprehension 7

A uniform rod of mass 2 kg and length l is suspended by two smooth hinges A and B as shown in figure. A force $F = 4$ N is applied downwards at a distance $\frac{l}{4}$ from hinge B. Due to the application of this force F, the hinge B breaks. At this instant, applied force F is also removed. The rod starts to rotate downward about hinge A. Take $g = 10\ \text{ms}^{-2}$. Based on above information, answer the following questions.



16. The reaction at hinge A, before hinge B breaks, is
(A) 10 N (B) 11 N
(C) 12 N (D) 24 N

17. The reaction at hinge A, just after hinge B breaks is
(A) 35 N (B) 25 N
(C) 15 N (D) 5 N

18. The acceleration of the end point of rod having small mass dm when the rod becomes vertical is
(A) $100\ \text{ms}^{-2}$ (B) $20\ \text{ms}^{-2}$
(C) $30\ \text{ms}^{-2}$ (D) $40\ \text{ms}^{-2}$

Comprehension 8

A vertically oriented uniform heavy rod of mass M and length L can rotate about its upper end hinged to a rigid support. A fast moving light ball of mass m strikes the lower end of the rod with velocity u and sticks to it, as a result of which the rod swings through an angle θ. Based on above information, answer the following questions.

19. The angular velocity of the system just after impact is
(A) $\frac{mu}{3ML}$ (B) $\frac{Mu}{3mL}$
(C) $\frac{3mu}{ML}$ (D) $\frac{3Mu}{mL}$

20. The velocity, u of the ball is
(A) $\frac{M}{m}\sqrt{\frac{gL}{3}}\sin\left(\frac{\theta}{2}\right)$ (B) $\frac{m}{M}\sqrt{\frac{2}{3}gL}\sin\left(\frac{\theta}{2}\right)$
(C) $\frac{M}{m}\sqrt{\frac{2}{3}gL}\sin\left(\frac{\theta}{2}\right)$ (D) $\frac{m}{M}\sqrt{gL}\sin\left(\frac{\theta}{2}\right)$

21. The momentum change in the ball-rod system during the impact
(A) $m\sqrt{\frac{gL}{6}}\sin\left(\frac{\theta}{2}\right)$ (B) $m\sqrt{gL}\sin\left(\frac{\theta}{2}\right)$
(C) $m\sqrt{gL}\cos\left(\frac{\theta}{2}\right)$ (D) $M\sqrt{\frac{gL}{6}}\sin\left(\frac{\theta}{2}\right)$

22. The distance x from the upper end of the rod where the ball must strike the rod such that the momentum of the ball + rod system remains constant during the impact is
(A) $\frac{L}{3}$ (B) $\frac{L}{6}$
(C) $\frac{2L}{3}$ (D) $\frac{L}{2}$

Comprehension 9

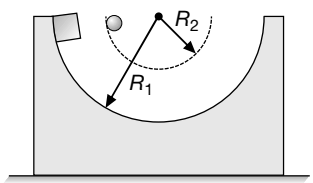
Moment of inertia of a straight wire about an axis perpendicular to the wire and passing through one of its end is I. This wire is now framed into a circle (a ring) of single turn. The moment of inertia of this ring about an axis passing through centre and normal to its plane is I_1. Now the same wire is bent into a ring of two turns and the moment of inertia about an axis passing through the ring normal to its plane is I_2. Based on above information, answer the following questions.

23. The value of I_1 is
(A) $\left(\frac{3}{\pi^2}\right)I$ (B) $\left(\frac{3}{4\pi^2}\right)I$
(C) $\left(\frac{\pi^2}{3}\right)I$ (D) $\left(\frac{4\pi^2}{3}\right)I$

24. The value of I_2 is
(A) $\left(\frac{\pi^2}{3}\right)I$ (B) $\left(\frac{\pi^2}{12}\right)I$
(C) $\left(\frac{3}{16\pi^2}\right)I$ (D) $\left(\frac{3}{8\pi^2}\right)I$

Comprehension 10

A fixed wedge has two semi-circular tracks of radius R_1 and R_2, such that $R_1 = 2R_2$. The track of radius R_1 is frictionless. A small block of mass m is held on the top of the larger track. A spherical ball of radius $r(\ll R_2)$ is held on the top of the other track. Both the block and the ball are released simultaneously. Friction in the track containing the ball is large enough to avoid slipping of the ball. Based on above information, answer the following questions.



25. Out of the block and the ball,
(A) the block reaches the bottom of the track first.
(B) the ball reaches the bottom of the track first.

(C) both reach the bottom of the track simultaneously.
(D) it is difficult to arrive at a conclusion, because some vital information is not provided in the paragraph.

26. Assuming the block and the ball to reach the bottom of their respective tracks simultaneously, then the ratio $\frac{R_1}{R_2}$ is

(A) $\frac{1}{5}$ (B) $\frac{5}{7}$
(C) $\frac{7}{5}$ (D) $\frac{9}{5}$

Comprehension 11

The angular momentum of a flywheel having a moment of inertia of $0.4\ \text{kgm}^2$ decreases from $3\ \text{kgm}^2\text{s}^{-1}$ to $2\ \text{kgm}^2\text{s}^{-1}$ in a period of 2 second. Based on above information, answer the following questions.

27. The average torque acting on the flywheel during this period is
(A) 10 Nm (B) 2.5 Nm
(C) 0.5 Nm (D) 5 Nm

28. If the angular acceleration is uniform, the number of revolutions the flywheel would have turned in that interval is
(A) 2 (B) 4
(C) 12 (D) 6

29. The work done is
(A) 3.125 Nm
(B) 12.5 Nm
(C) 62.5 Nm
(D) 6.25 Nm

Comprehension 12

A rod of mass m and length l in placed on a smooth horizontal table. A particle of same mass m strikes the rod with velocity v_0 in a direction perpendicular to the rod at distance $x\left(<\frac{l}{2}\right)$ from its centre and sticks to the rod. Let ω be the angular speed of system just after collision, then based on above information, answer the following questions.

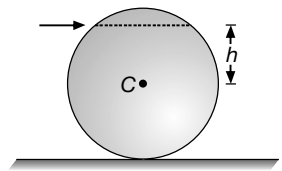
30. When x is increased from 0 to $\frac{l}{2}$, the angular speed ω
(A) will first decrease and then increase
(B) will first increase and then decrease
(C) will continuously increase
(D) will continuously decrease

31. Assume that the particle is still sticking to the rod, the maximum possible value of impulse obtained by varying x, that can be imparted to the particle during collision is

(A) $\frac{mv_0}{2}$ (B) $\frac{2mv_0}{3}$
(C) $\frac{3mv_0}{4}$ (D) $\frac{4mv_0}{5}$

Comprehension 13

A solid sphere is kept over a smooth surface as shown in figure. It is hit by a cue at height h above the centre C. In case 1, $h=\frac{R}{4}$ and in case 2, $h=\frac{R}{2}$. Suppose in case 1 the sphere acquires a total kinetic energy K_1 and in case 2 total kinetic energy is K_2.



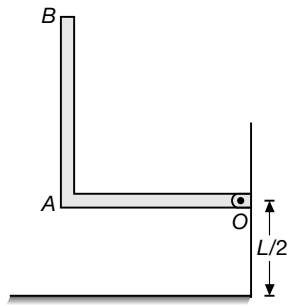
Assuming that in both the cases the sphere is hit by the same impulse, answer the following questions.

32. The appropriate comparison relation between K_1 and K_2 is
(A) $K_1 = K_2$ (B) $K_1 > K_2$
(C) $K_1 < K_2$ (D) data insufficient

33. If the surface is rough, then after hitting the sphere, in which case the force of friction is in forward direction.
(A) In case 1 (B) In case 2
(C) In both the cases (D) In none of the cases

Comprehension 14

An L shaped frame is free to rotate in a vertical plane about a horizontal axis passing through a smooth hinge O. Each side of the frame has a length L and mass m. The frame is let to fall with one side horizontal and the other vertical. Based on above information, answer the following questions.



34. The angular acceleration of the frame just after it is allowed to fall is

(A) $\frac{4g}{3L}$ (B) $\frac{9g}{8L}$

(C) $\frac{g}{2L}$ (D) $\frac{3g}{2L}$

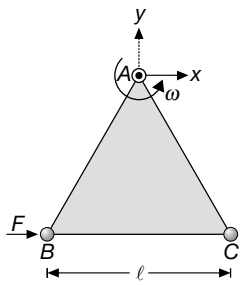
35. The speed of the end A with which it strikes the ground is

(A) $1.1\sqrt{gL}$ (B) $2\sqrt{gL}$

(C) $3.2\sqrt{gL}$ (D) $\sqrt{gL}$

Comprehension 15

Three particles A, B and C, each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side l. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω. At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown). Based on above information, answer the following questions.



36. The magnitude of the horizontal force exerted by the hinge on the body is

(A) $ml\omega^2$ (B) $\frac{ml\omega^2}{\sqrt{3}}$

(C) $\sqrt{3}ml\omega^2$ (D) $\frac{ml\omega^2}{3}$

37. The x-component of force exerted by the hinge on the body immediately after time T is

(A) $\frac{F}{2}$, along x-axis

(B) $\frac{F}{2}$, along negative x-axis

(C) $\frac{F}{4}$, along x-axis

(D) $\frac{F}{4}$, along negative x-axis

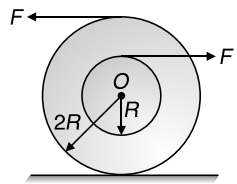
38. The y-component of the force exerted by the hinge on the body immediately after time T is

(A) $ml\omega^2$ (B) $\frac{ml\omega^2}{\sqrt{3}}$

(C) $\sqrt{3}ml\omega^2$ (D) $\frac{ml\omega^2}{3}$

Comprehension 16

In the arrangement shown, $F = 10$ N, $R = 1$ m, $M = 2$ kg and moment of inertia of the body about an axis passing through O and perpendicular to plane of body is $I = 4$ kgm^2. O is the centre of mass of the body. If the ground is smooth, the kinetic energy of the body at $t = 2$ s is K_1. However, when the ground is sufficiently rough the kinetic energy of the body at $t = 6$ s is K_2. Based on above information, answer the following questions.



39. The value K_1 equals

(A) 25 J (B) 50 J

(C) 75 J (D) 100 J

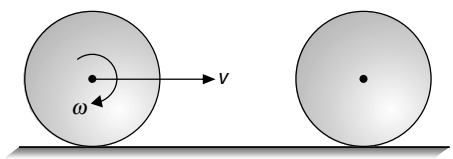
40. The value K_2 equals

(A) 10.33 J (B) 250 J

(C) 16.67 J (D) 150 J

Comprehension 17

A solid sphere of mass m, radius R is rolling without slipping on rough horizontal surface as shown in figure. It collides elastically with another identical sphere at rest. There is no friction between the two spheres.



Based on above information, answer the following questions.

41. Linear velocity of first sphere after it again starts rolling without slipping is

(A) $\frac{2}{5}R\omega$ (B) $\frac{2}{7}R\omega$

(C) $\frac{7}{10}R\omega$ (D) $\frac{7}{5}R\omega$

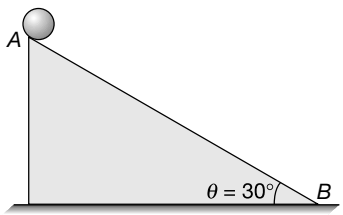
42. The net angular impulse imparted to the second sphere by the external forces is

(A) $\frac{2}{7}mvR$ (B) $\frac{5}{7}mvR$

(C) $\frac{2}{5}mvR$ (D) $\frac{7}{10}mvR$

Comprehension 18

On an inclined plane AB of length 5 m, a body is released from the point A. Assuming friction to be sufficient enough for pure rolling to take place, answer the following questions.



43. The maximum time which a body (that is capable of rolling) can take to reach the bottom is

(A) 8 s (B) 6 s

(C) 4 s (D) 2 s

44. If four bodies like ring, disc, solid sphere and hollow sphere (all having identical mass, radius and coefficient of friction) are taken and the angle θ is now gradually increased, then the body that will be the first to start slipping is

(A) Ring (B) Disc

(C) Solid sphere (D) Hollow sphere

Comprehension 19

A thin rod AB of mass M, length a has a variable mass per unit length $\frac{\rho_0(a+x)}{a}$, where x is the distance measured from the end A of the rod and ρ_0 is a constant. The rod is freely pivoted at A and is hanging in equilibrium when it is struck by a horizontal impulse of magnitude J at the point B. Based on above information, answer the following questions.

45. The total mass M of the rod is

(A) $2a\rho_0$ (B) $a\rho_0$

(C) $\frac{a\rho_0}{3}$ (D) $\frac{3a\rho_0}{2}$

46. The centre of mass of the rod is at the point at distance from the end A given by

(A) $\frac{a}{9}$ (B) $\frac{a}{3}$

(C) $\frac{5a}{9}$ (D) $\frac{2a}{3}$

47. The moment of inertia of the rod about an axis passing through A and perpendicular to AB is

(A) $\frac{7Ma^2}{18}$ (B) $\frac{5Ma^2}{18}$

(C) $\frac{1Ma^2}{12}$ (D) $\frac{1Ma^2}{18}$

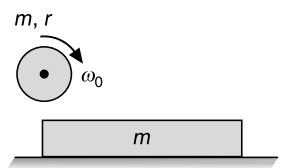
48. If on the application of impulse, the end B passes through a point vertically above A, then the minimum value of J is

(A) $\frac{M}{7}\sqrt{90ag}$ (B) $\frac{M}{9}\sqrt{70ag}$

(C) $\frac{M}{7}\sqrt{30ag}$ (D) $\frac{M}{5}\sqrt{80ag}$

Comprehension 20

In the diagram shown, a plank of mass m is lying at rest on a smooth horizontal surface. A disc of same mass m and radius r is rotated to an angular speed ω_0 and then gently placed on the plank. It is found that finally slipping ceases and 50% of the total kinetic energy of the system is lost. Assume that the plank is long enough and the coefficient of friction between disc and plank is μ. Based on above information, answer the following questions.



49. The final velocity of the plank is

(A) $\frac{r\omega_0}{4}$ (B) $\frac{r\omega_0}{\sqrt{10}}$

(C) $\frac{r\omega_0}{2}$ (D) $\frac{r\omega_0}{2\sqrt{10}}$

50. Time when slipping ceases is

(A) $\frac{r\omega_0}{2\mu g}$ (B) $\frac{r\omega_0}{\sqrt{10}\mu g}$

(C) $\frac{r\omega_0}{4\mu g}$ (D) $\frac{r\omega_0}{2\sqrt{10}\mu g}$

51. Magnitude of the change in angular momentum of the disc about center of mass of the disc is

(A) $\frac{3}{4}mr^2\omega_0$ (B) $\frac{3}{8}mr^2\omega_0$

(C) ZERO (D) $\frac{1}{2}mr^2\omega_0$

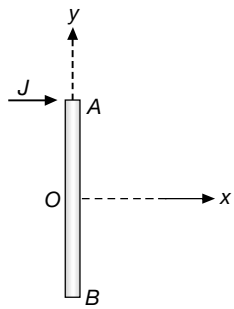
52. Distance moved by the plank from the placing of disc on the plank till the slipping ceases between disc and plank is

(A) $\frac{r^2\omega_0^2}{16\mu g}$ (B) $\frac{r^2\omega_0^2}{8\mu g}$

(C) $\frac{r^2\omega_0^2}{32\mu g}$ (D) $\frac{r^2\omega_0^2}{200\mu g}$

Comprehension 21

A rod AB of length 2 m and mass 2 kg is lying on smooth horizontal x-y plane with its centre at origin O as shown figure. An impulse J of magnitude 10 Ns is applied perpendicular to AB at A. Based on above information, answer the following questions.



53. The distance of point P from centre of the rod which is at rest just after the impact is

(A) $\frac{1}{4}$ m (B) $\frac{1}{2}$ m

(C) $\frac{1}{3}$ m (D) $\frac{2}{3}$ m

54. Co-ordinates of point A of the rod after time $t = \frac{\pi}{45}$ s will be

(A) $\left[\frac{1}{2}m, \frac{1}{2}m\right]$

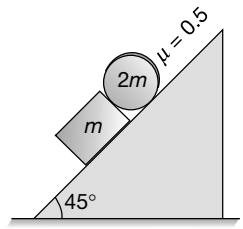
(B) $\left[\left(\frac{3}{4}m, \frac{3}{2}m\right)\right]$

(C) $\left[\left(\frac{\pi}{6}+\frac{1}{2}\right)m, \frac{1}{2}m\right]$

(D) $\left[\left(\frac{\pi}{9}+\frac{\sqrt{3}}{2}\right)m, \frac{1}{2}m\right]$

Comprehension 22

On a rough inclined plane of inclination 45°, a block of mass m and a cylinder of mass $2m$ are released. The coefficient of friction between all the surfaces of contact is 0.5. Based on above information, answer the following questions.



55. The acceleration of the block and the cylinder when both move separately is

(A) $\frac{g}{2\sqrt{2}}, \frac{g}{3\sqrt{3}}$ (B) $\frac{g}{\sqrt{2}}, \frac{g}{\sqrt{3}}$

(C) $\frac{\sqrt{2}g}{3}, \frac{g}{2\sqrt{2}}$ (D) $\frac{g}{2\sqrt{2}}, \frac{\sqrt{2}g}{3}$

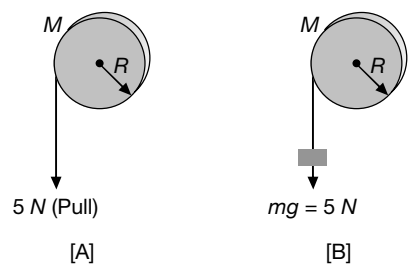
56. The acceleration of the block and the cylinder when both move together is

(A) $\frac{5g}{3\sqrt{2}}$ (B) $\frac{3g}{5\sqrt{2}}$

(C) $\frac{5g}{\sqrt{2}}$ (D) $\frac{3g}{\sqrt{2}}$

Comprehension 23

A uniform disc of mass $M = 2.5$ kg and radius $R = 0.2$ m is mounted on an axle supported on fixed frictionless bearings. A light cord wrapped around the rim is pulled with a force 5 N. On the same system of pulley and string instead of pulling it down, a body of weight 5 N is suspended. If the first process is termed A and the second B.



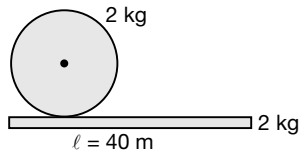
Based on above information, answer the following questions.

57. The tangential acceleration will be
(A) equal in the processes A and B
(B) greater in process A than in B
(C) greater in process B than in A
(D) independent of the two processes

58. The mechanical energy is conserved
(A) in both A and B
(B) in B only
(C) in A only
(D) neither in A nor in B

Comprehension 24

A plank of length 40 m and mass 2 kg is kept on a horizontal frictionless surface. A cylinder of mass 2 kg is placed on the plank. The coefficient of friction between the two surfaces is $\mu = 0.1$. Plank is suddenly given a velocity $v = 10\ \text{ms}^{-1}$ towards left. Based on above information, answer the following questions.



59. Initially the acceleration of point of contact with respect to the plank is

(A) $2\ \text{ms}^{-2}$ (B) $1\ \text{ms}^{-2}$

(C) $4\ \text{ms}^{-2}$ (D) $6\ \text{ms}^{-2}$

60. The time after which pure rolling starts is

(A) 2 s (B) 2.5 s

(C) 3.5 s (D) 4.5 s

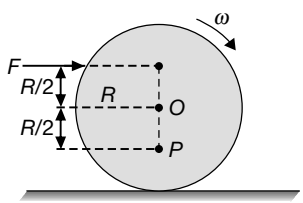
61. The time after which the plank and cylinder separates

(A) 6.75 s (B) 4 s

(C) 3.25 s (D) 5.25 s

Comprehension 25

A disc of mass 10 kg and radius 1 m is set into pure rolling on a rough horizontal surface as shown in the figure. At a particular instant, point P (figure) which is at a distance $\frac{R}{2}$ from the centre O is found to have a speed of $2\ \text{ms}^{-1}$ and acceleration of $10\ \text{ms}^{-2}$. Based on above information, answer the following questions.



62. Acceleration of the centre of mass of the disc is

(A) $4\ \text{ms}^{-2}$ (B) $8\ \text{ms}^{-2}$

(C) $12\ \text{ms}^{-2}$ (D) $20\ \text{ms}^{-2}$

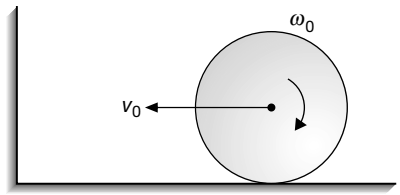
63. If a horizontal force F, applied at $\frac{R}{2}$ from the centre (as shown in the figure) causes this motion, the value of F is

(A) ZERO (B) 60 N

(C) 120 N (D) 200 N

Comprehension 26

A solid sphere of radius 1 m and mass 2 kg has linear velocity $v_0 = 4\ \text{ms}^{-1}$ and angular velocity $\omega_0 = 9\ \text{rads}^{-1}$ as shown. The ground on which it is moving, is smooth. It collides elastically with a rough wall of coefficient of friction μ. Based on above information, answer the following questions.



64. If the sphere after colliding with the wall rolls without slipping in opposite direction, then

(A) $\mu = \frac{1}{4}$ (B) $\mu = \frac{1}{3}$

(C) $\mu = \frac{1}{2}$ (D) $\mu = \frac{2}{3}$

65. The net linear impulse imparted by the wall on the sphere during impact is

(A) $\sqrt{32}$ Ns (B) $4\sqrt{5}$ Ns

(C) $4\sqrt{17}$ Ns (D) $15\sqrt{2}$ Ns

Comprehension 27

The radius of a wheel is R and its radius of gyration about its axis passing through its center and perpendicular to its plane is K. If the wheel is rolling without slipping. Based on above information, answer the following questions.

66. The ratio of its rotational kinetic energy to its translational kinetic energy is

(A) $\frac{K^2}{R^2}$ (B) $\frac{R^2}{K^2}$

(C) $\frac{R^2}{R^2+K^2}$ (D) $\frac{K^2}{R^2+K^2}$

67. The ratio of the rotational kinetic energy to total kinetic energy is

(A) $\frac{K^2}{R^2+K^2}$ (B) $\frac{R^2}{R^2+K^2}$

(C) $\frac{1}{R^2+K^2}$ (D) None of these

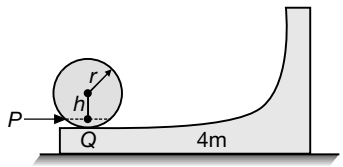
68. The ratio of the translational kinetic energy to the total kinetic energy is

(A) $\frac{K^2}{R^2+K^2}$ (B) $\frac{R^2}{R^2+K^2}$

(C) $\frac{1}{R^2+K^2}$ (D) None of these

Comprehension 28

A wedge of mass $4m$ is placed at rest on a smooth horizontal surface. A uniform solid sphere of mass m and radius r is placed at rest on the flat portion of the wedge at the point Q as shown in the figure. A sharp horizontal impulse P is given to the sphere at a point below $h = 0.4r$ from the center of the sphere. The radius of curvature of the curved portion of the wedge is R. Coefficient of friction to the left side of point Q is μ and to the right side of point Q is zero. Based on above information, answer the following questions.



69. The maximum height to which the center of mass of the sphere will climb on the curved portion of the wedge is

(A) $\frac{2P^2}{5m^2g}$ (B) $\frac{P^2}{5m^2g}$

(C) $\frac{P^2}{2m^2g}$ (D) None of these

70. Kinetic energy of the system when sphere is at the highest point is

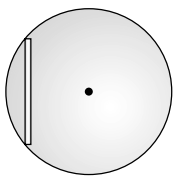
(A) $\frac{P^2}{10m}$ (B) $\frac{P^2}{5m}$

(C) $\frac{3P^2}{10m}$ (D) $\frac{3P^2}{5m}$

Comprehension 29

A rod of mass 4 kg, length 2 m is kept vertically inside a smooth spherical shell of radius 2 m. The rod starts slipping inside the shell. Based on above information, answer the following questions.

71. Angular speed of the rod (in rads^{-1}) in the position when it becomes horizontal is



(A) 3.2 (B) 4.6

(C) 6.8 (D) 7.2

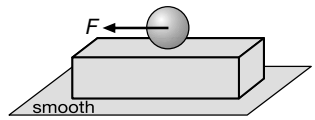
72. Velocity of centre of mass of the rod (in ms^{-1}) at this instant is approximately

(A) 4.7 (B) 5.5

(C) 6.2 (D) 10.2

Comprehension 30

A disc of mass m and radius R is placed over a plank of same mass m. There is sufficient friction between disc and plank to prevent slipping. A force F is applied at the centre of the disc. Based on above information, answer the following questions.



73. Acceleration of the plank is

(A) $\frac{F}{4m}$ (B) $\frac{3F}{4m}$

(C) $\frac{F}{2m}$ (D) $\frac{3F}{2m}$

74. Acceleration of the disc is

(A) $\frac{F}{4m}$ (B) $\frac{F}{2m}$

(C) $\frac{3F}{4m}$ (D) F

75. The angular acceleration of the disc is

(A) $\frac{F}{mR}$ (B) $\frac{F}{2mR}$

(C) $\frac{3F}{2mR}$ (D) $\frac{3F}{mR}$

76. Force of friction between the disc and the plank is

(A) $\frac{F}{4}$ (B) $\frac{F}{3}$

(C) $\frac{F}{2}$ (D) $\frac{2F}{3}$

Comprehension 31

A uniform rod AB of mass $3m$ and length $2l$ is free to turn in a vertical plane about a smooth horizontal axis through A. A particle of mass m is attached to the rod at B. When the rod is hanging in equilibrium, it is set moving with an angular velocity $\omega_0 = \sqrt{\frac{kg}{l}}$, where k is a positive real number. Based on above information, answer the following questions.

77. For $k = 2$, the height of B above the level of A when the rod first comes to instantaneous rest is

(A) $\frac{3l}{5}$ (B) $\frac{4l}{5}$

(C) $\frac{6l}{5}$ (D) $\frac{8l}{5}$

78. The minimum value of k for which the particle describes complete circle about A is

(A) $\frac{3}{2}$ (B) $\frac{5}{2}$

(C) $\frac{7}{2}$ (D) $\frac{9}{2}$

Comprehension 32

A uniform disc of radius 1 m and mass 2 kg is mounted on an axle supported on fixed frictionless bearings. A light cord is wrapped around the rim of the disc and a mass of 1 kg is tied to the free end. If it is released from rest, the tension in the cord is T. If the applied torque acts for 4 second, the total angular displacement in this interval of time is θ. The work done by the applied torque is W and the increase in rotational kinetic energy is ΔK. Based on above information, answer the following questions.

79. The value of T is

(A) 10 N (B) 5 N

(C) 40 N (D) 15 N

80. θ equals

(A) 40 rad (B) 80 rad

(C) 20 rad (D) 10 rad

81. W has a value given by

(A) 400 J (B) 50 J

(C) 100 J (D) 200 J

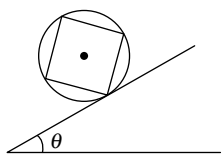
82. ΔK equals

(A) 400 J (B) 100 J

(C) 200 J (D) 80 J

Comprehension 33

Four identical rods of mass $M = 6$ kg each are welded at their ends to form a square and then welded to a massive ring having mass $m = 4$ kg having radius $R = 1$ m. If the system is allowed to roll down the incline of inclination $\theta = 30°$. Based on above information, answer the following questions.



83. The moment of inertia of the system about the centre of ring will be

(A) 10 kgm^2

(B) 20 kgm^2

(C) 40 kgm^2

(D) 60 kgm^2

84. The acceleration of the system will be

(A) $\frac{g}{8}$ (B) $\frac{g}{4}$

(C) $\frac{7g}{24}$ (D) $\frac{g}{2}$

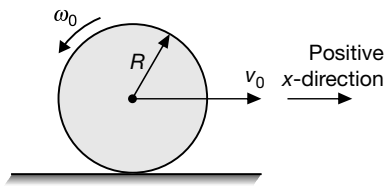
85. The minimum value of friction coefficient to prevent slipping is

(A) $\frac{5}{7}$ (B) $\frac{7}{5\sqrt{3}}$

(C) $\frac{5\sqrt{3}}{7}$ (D) $\frac{5}{12\sqrt{3}}$

Comprehension 34

A disc of mass M and radius R is given a velocity v_0 and an angular velocity ω_0 as shown in the figure. Then it is kept over a rough horizontal surface where coefficient of friction is μ. After some time slipping ceases, based on above information, answer the following questions.



86. If $v_0 = R\omega_0$, then time after which pure rolling starts is

(A) $\frac{v_0}{\mu g}$

(B) $\frac{3v_0}{\mu g}$

(C) $\frac{2}{3}\frac{v_0}{\mu g}$

(D) pure rolling will never start

87. Velocity of centre of disc at the moment when angular velocity of disc becomes zero is

(A) v_0 (B) $\frac{v_0}{2}$

(C) $\frac{v_0}{3}$ (D) $\frac{v_0}{4}$

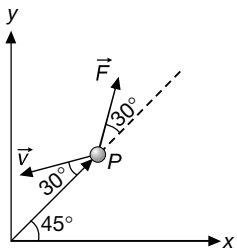
88. Velocity of centre of mass of disc when pure rolling begins is

(A) v_0 (B) $\frac{v_0}{2}$

(C) $\frac{v_0}{3}$ (D) $\frac{v_0}{4}$

Comprehension 35

A particle P with a mass 2 kg has position vector $r = 3$ m and velocity $V = 4$ ms^{-1} shown. It is accelerated by the force $F = 2$ N. All three vectors lie in the common plane. Based on above information, answer the following questions.



89. The angular momentum vector is
(A) 12 kgm^{2}s^{-1}, out of the plane of the figure
(B) 12 kgm^{2}s^{-1}, into the plane of the figure
(C) ZERO
(D) 24 kgm^{2}s^{-1}, out of the plane of the figure

90. The torque is
(A) 6 Nm, out of the page (B) 3 Nm, out of the page
(C) 6 Nm, into the page (D) 3 Nm, into the page

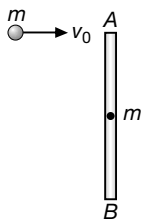
MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:

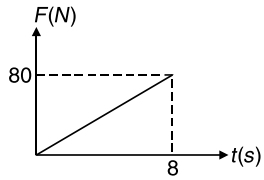
	p	q	r	s	t
A	●			●	●
B		●	●		
C	●	●			
D				●	

1. A smooth ball of mass m moving with a uniform velocity v_0 strikes a smooth uniform rod AB of equal mass m, lying on a frictionless horizontal table. The ball strikes the rod at one end A, perpendicular to the rod, as shown in Figure. The collision is perfectly elastic. If K_i, K_f, K_T, J, p_i, L_{cm}, L_i denote the initial kinetic energy, final kinetic energy, final kinetic energy of translation of rod, impulse delivered to rod, initial momentum of ball, angular momentum of centre of mass and initial angular momentum of ball about CM of rod, then match the physical quantities pertaining to this situation given in **COLUMN-I** with their values given in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) $\frac{K_f}{K_i}$	(p) $\frac{2}{5}$
(B) $\frac{J}{p_i}$	(q) $\frac{3}{5}$
(C) $\frac{L_{cm}}{L_i}$	(r) $\frac{9}{25}$
(D) $\frac{K_f}{K_T}$	(s) 3

2. A uniform disc of mass 10 kg, radius 1 m is placed on a rough horizontal surface. The coefficient of friction between the disc and the surface is 0.2. A horizontal time varying force is applied at the centre of the disc whose variation with time is shown in graph. Match the contents of **COLUMN-I** with their respective time values in **COLUMN-II**.

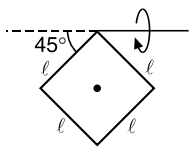
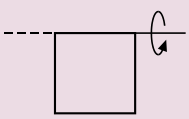


COLUMN-I	COLUMN-II
(A) Disc rolls without slipping	(p) at $t = 7$ s
(B) Disc rolls with slipping	(q) at $t = 3$ s
(C) Disc starts slipping at	(r) at $t = 4$ s
(D) Friction force is 10 N at	(s) at $t = 6$ s

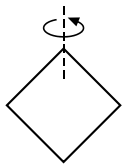
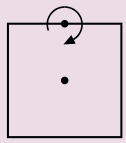
3. Match the following quantities of **COLUMN-I** to their respective match(es) in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Axial vector	(p) Rotational K.E.
(B) Scalar	(q) Translational K.E.
(C) Turning ability of force	(r) Angular momentum
(D) A rolling body can have	(s) Torque

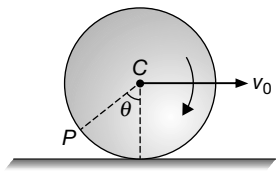
4. A square frame is made by using four uniform rods each of mass m, length l. Match the moment of inertia for the frame about axis specified in **COLUMN-I** to their respective values given in **COLUMN-II**.

COLUMN-I (Axis of rotation)	COLUMN-II (Moment of Inertia about axis of rotation)
(A) 	(p) $\frac{2}{3}ml^2$
(B) 	(q) $\frac{5}{3}ml^2$

(*Continued*)

COLUMN-I (Axis of rotation)	COLUMN-II (Moment of Inertia about axis of rotation)
(C) 	(r) $\frac{7}{3}ml^2$
(D) 	(s) $\frac{8}{3}ml^2$

5. A disc rolls without slipping on the ground such that velocity of centre of mass of the disc is v_0. A point P on circumference of disc at angle θ with the vertical have a speed v_p. Assuming that initially the point P is in contact with the ground, match the following.



COLUMN-I	COLUMN-II
(A) If $\theta = 60°$	(p) $v_P = \sqrt{2}v$
(B) If $\theta = 90°$	(q) $v_P = v$
(C) If $\theta = 120°$	(r) $v_P = 2v$
(D) If $\theta = 180°$	(s) $v_P = \sqrt{3}v$

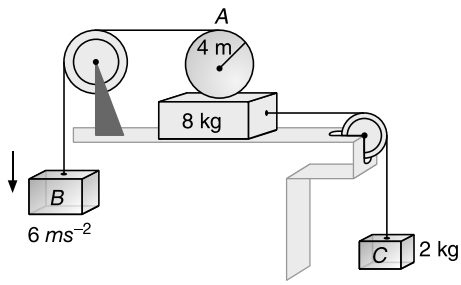
6. Match the moment of inertia of bodies about specified axis with their respective values.

COLUMN-I	COLUMN-II
(A) Moment of inertia of a circular disc of mass M and radius R about a tangent parallel to plane of disc	(p) $\frac{1}{2}MR^2$
(B) Moment of inertia of a solid sphere of mass M and radius R about a tangent.	(q) $\frac{7}{5}MR^2$

(*Continued*)

COLUMN-I	COLUMN-II
(C) Moment of inertia of a circular disc of mass M and radius R about a tangent perpendicular to plane of disc	(r) $\frac{5}{4}MR^2$
(D) Moment of inertia of a cylinder of mass M and radius R about its axis	(s) $\frac{1}{4}MR^2$
	(t) $\frac{3}{2}MR^2$

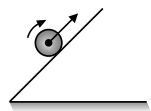
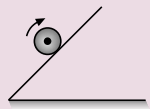
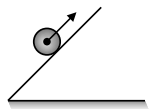
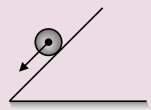
7. The diagram shows a uniform smooth solid cylinder A of radius 4 m rolling without slipping on the 8 kg plank, which in turn is supported by a fixed smooth surface. The blocks B and C both accelerate down with 6 ms^{-2} and 2 ms^{-2} respectively.



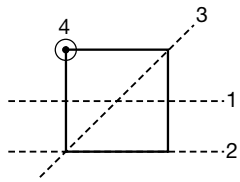
The angular acceleration of the cylinder A is α, the acceleration of centre of mass of cylinder is a, the ratio of mass of cylinder A to the mass of block B is $\frac{m_A}{m_B}$. The length of the unwrapped thread between the cylinder and the block at $t = 0$ is $l_0 = 20$ m and at $t = 2$ s is l (assume the system to be released from rest), match the following

COLUMN-I	COLUMN-II
(A) α(in $rads^{-2}$)	(p) 8
(B) $2a$(in ms^{-2})	(q) 2
(C) $\frac{m_A}{m_B}$	(r) 4
(D) $(l - l_0)$	(s) 1

8. The inclined surfaces shown in **COLUMN-I** are sufficiently rough and **COLUMN-II** gives direction of frictional forces for situations in **COLUMN-I**. Match the two columns.

COLUMN-I	COLUMN-II
(A)  Rolling upwards	(p) up the incline
(B)  Kept in rotating position	(q) down the incline
(C)  Kept in translational position	(r) maximum friction will act
(D)  Kept in translational position	(s) required value of friction will act

9. Four thin uniform rods of equal length l and mass m each form a square as shown in Figure. Moment of inertia about axes 1, 2, 3 and 4 are say I_1, I_2, I_3 and I_4. Then, match the following



COLUMN-I	COLUMN-II
(A) I_1	(p) $\frac{4}{3}ml^2$
(B) I_2	(q) $\frac{2}{3}ml^2$

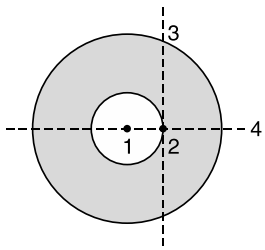
(*Continued*)

COLUMN-I	COLUMN-II
(C) I_3	(r) $\frac{10}{3}ml^2$
(D) I_4	(s) $\frac{5}{3}ml^2$

10. A uniform disc is acted upon by some forces and it rolls on a horizontal plank without slipping from north to south. The plank, in turn lies on a smooth horizontal surface. Match the following regarding this situation.

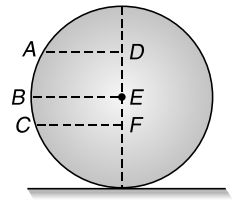
COLUMN-I	COLUMN-II
(A) Frictional force on the disc by the surface	(p) May be directed towards north
(B) Velocity of the lowermost point of the disc	(q) May be directed towards south
(C) Acceleration of centre of mass of the disc	(r) May be zero
(D) Vertical component of the acceleration of centre of mass	(s) Must be zero

11. From a uniform disc of mass M and radius R, a concentric disc of half the radius is cut out. For the remaining annular disc the moment of inertia about axis 1 is I_1 about 2 is I_2 about 3 is I_3 and about 4 is I_4. The axes 1, 2 are perpendicular to the disc and axes 3, 4 are in the plane of the disc. Axes 2, 3 and 4 intersect at a common point. Match the contents of **COLUMN-I** with those in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) I_1	(p) $\frac{21}{32}MR^2$
(B) I_2	(q) $\frac{I_1}{2}$
(C) $(I_3 + I_4)$	(r) $\frac{15}{32}MR^2$
(D) $(I_2 - I_3)$	(s) $\frac{31}{22}MR^2$

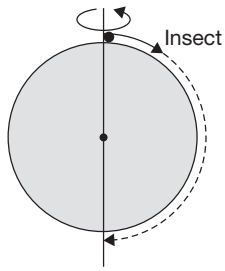
12. A solid sphere is placed on a rough horizontal ground as shown in Figure.



If E is the centre of sphere such that $DE > EF$ and a linear impulse can be applied either at point A, B or C then match the following two columns.

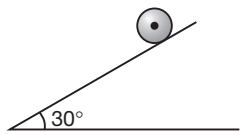
COLUMN-I	COLUMN-II
(A) Sphere will acquire maximum angular speed if impulse is applied at	(p) A
(B) Sphere will acquire maximum linear speed if impulse is applied at	(q) B
(C) Sphere can roll without slipping if impulse is applied at	(r) C
(D) Sphere can roll with forward slipping if impulse is applied at	(s) at any point A, B or C

13. A solid sphere is rotating about an axis as shown in Figure. An insect follows the dotted path on the circumference of sphere as shown. Match the following



COLUMN-I	COLUMN-II
(A) Moment of inertia	(p) will remain constant
(B) Angular velocity	(q) will first increase then decrease
(C) Angular momentum	(r) will first decrease then increase
(D) Rotational kinetic energy	(s) will continuously decrease
	(t) will continuously increase

14. A thin walled cylindrical shell of mass m and radius R is placed on a rough inclined plane so that it rolls without slipping. Match the following table

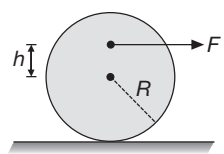
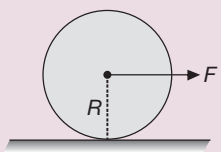
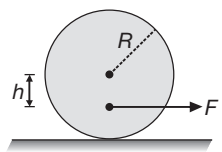
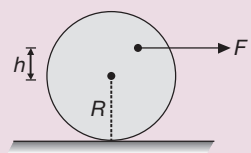


COLUMN-I	COLUMN-II
(A) Linear acceleration of centre of mass	(p) is directly proportional to m
(B) Angular acceleration	(q) is inversely proportional to m
(C) Rotational kinetic energy at any instant	(r) is directly proportional to R^2
(D) Translational kinetic energy at any instant	(s) is inversely proportional to R
	(t) constant

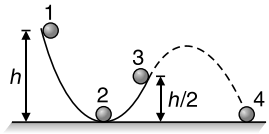
15. A solid sphere, a hollow sphere, a cylinder all of same mass and radius are released from a rough inclined plane on which all of them roll down without slipping. On reaching the bottom of the plane, match the contents of **COLUMN-I** with those of **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) Time taken to reach the bottom	(p) Maximum for solid sphere
(B) Total kinetic energy	(q) Maximum for hollow sphere
(C) Rotational kinetic energy	(r) Maximum for cylinder
(D) Translational kinetic energy	(s) Same for all

16. In each case, there is sufficient friction for regular rigid uniform disc to undergo pure rolling on a rigid horizontal surface. Match the situations in **COLUMN-I** to the quantities in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) 	(p) The direction of static friction may be forward or may be backward or static friction may be zero
(B) 	(q) The direction of static friction is towards backward
(C) 	(r) The angular acceleration will be clockwise
(D) 	(s) Acceleration of the centre mass will be along direction F

17. A small solid ball rolls down along sufficiently rough surface from point 1 to point 3 as shown in Figure.



From point 3 onwards it moves under the influence of gravity. Match the following two columns.

COLUMN-I	COLUMN-II
(A) Rotational kinetic energy of ball at point 2	(p) $\frac{1}{7}mgh$
(B) Translational kinetic energy of ball at point 3	(q) $\frac{2}{7}mgh$
(C) Rotational kinetic energy of ball at point 4	(r) $\frac{5}{7}mgh$
(D) Translational kinetic energy of ball at point 4	(s) $\frac{5}{14}mgh$

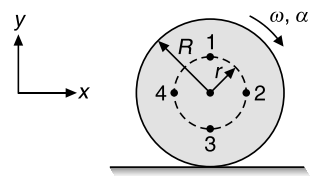
18. A particle moving on the smooth horizontal table strikes the rod AB at an end A perpendicularly and then stops. Match the following.

COLUMN-I	COLUMN-II
(A) Angular momentum of system (particle + rod) about any point is	(p) Conserved for elastic collision
(B) Linear momentum of system (particle + rod) is	(q) Not conserved for elastic collision
(C) Angular momentum of rod about its centre of mass is	(r) Conserved for inelastic collision
(D) Kinetic energy of system (particle + rod), except during collision is	(s) Not conserved for inelastic collision

19. For the following statements, except the gravity and contact force between the contact surfaces, no other force is acting on the body. Match the quantities in **COLUMN-I** to their respective match(es) in **COLUMN-II**.

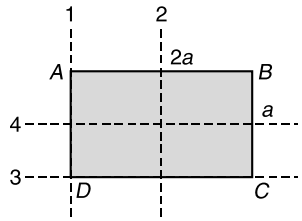
COLUMN-I	COLUMN-II
(A) When a sphere is in pure–rolling on a fixed horizontal surface.	(p) Upward direction
(B) When a cylinder is in pure rolling on a fixed inclined plane in upward direction then friction force acts in	(q) $v_{cm} > R\omega$
(C) When a cylinder is in pure rolling down a fixed incline plane, friction force acts is	(r) $v_{cm} < R\omega$
(D) When a sphere of radius R is rolling with slipping on a fixed horizontal surface, the relation between v_{cm} and ω is	(s) No frictional force acts.

20. A disc of radius R is rolling without slipping with an angular acceleration α on a horizontal plane. Four points are marked at the end of horizontal and vertical diameter of a circle of radius $r(<R)$ on the disc. Let horizontal and vertical directions be chosen as x and y axis as shown in Figure. When angular velocity of the disc is ω, then acceleration of points 1, 2, 3 and 4 are $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$ and $\vec{a}_4$ respectively. Match the accelerations given in **COLUMN-I** with their respective vectors given in **COLUMN-II**.



COLUMN-I	COLUMN-II
(A) $\vec{a}_1$	(p) $(R-r)\alpha\hat{i}+(r\omega^2)\hat{j}$
(B) $\vec{a}_2$	(q) $(R+r)\alpha\hat{i}-(r\omega^2)\hat{j}$
(C) $\vec{a}_3$	(r) $(R\alpha-r\omega^2)\hat{i}-(r\alpha)\hat{j}$
(D) $\vec{a}_4$	(s) $(R\alpha+r\omega^2)\hat{i}+(r\alpha)\hat{j}$

21. A rectangular slab $ABCD$ has dimensions $a \times 2a$ as shown in Figure.



Match the radius of gyration about the axis specified in **COLUMN-I** with their respective values in **COLUMN-II**.

COLUMN-I	COLUMN-II
(A) About axis 1	(p) $\frac{a}{\sqrt{12}}$
(B) About axis 2	(q) $\frac{2a}{\sqrt{3}}$
(C) About axis 3	(r) $\frac{a}{\sqrt{2}}$
(D) About axis 4	(s) $\frac{a}{\sqrt{3}}$

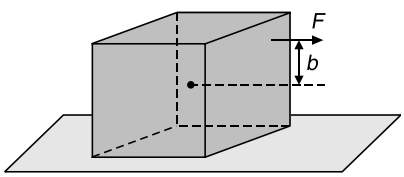
INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

1. A small ball of mass $1\ \text{kg}$ is attached to one end of a $1\ \text{m}$ long light string and the other end of the string is hung from a point. When the resulting pendulum is $30°$ from the vertical, calculate the magnitude of torque, in Nm, about the point of suspension. Take $g = 10\ \text{ms}^{-2}$.

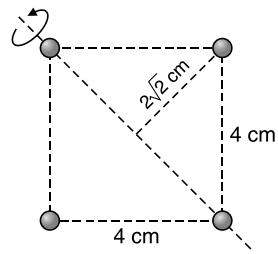
2. A uniform rod is falling without rotation on a smooth horizontal plane. Assuming the collision to be perfectly elastic, the angular velocity of the rod after striking the table is maximum when the rod makes an angle $\cos^{-1}\left(\frac{1}{\sqrt{*}}\right)$ with the horizontal just before striking where $*$ is not readable. Find $*$.

3. Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its center of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for a box not to topple before moving is ……



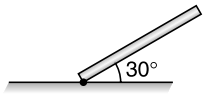
4. A uniform sphere of radius $= \frac{R}{16}$ starts rolling down without slipping from the top of another sphere of radius $R = 1\ \text{m}$. Find the angular velocity of the sphere, in rads^{-1}, after it leaves the surface of the larger sphere.

5. Four solid spheres each of diameter $\sqrt{5}\ \text{cm}$ and mass $0.5\ \text{kg}$ are placed with their centers at the corners of a square of side $4\ \text{cm}$. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}\ \text{kgm}^2$, find N.

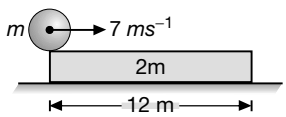


6. A uniform disc of mass $40\ \text{kg}$ and radius $0.5\ \text{m}$ can turn about a smooth axis through its centre and perpendicular to the disc. A constant torque is applied to the disc for $2\ \text{s}$ from rest and the angular velocity at the end of that time is $\frac{240}{\pi}$ revolutions per minute. Find the magnitude of the torque, in Nm. If the torque is then removed and the disc is brought to rest in t second by a constant force of $10\ \text{N}$ applied tangentially at a point on the rim of the disc, find t, in seconds.

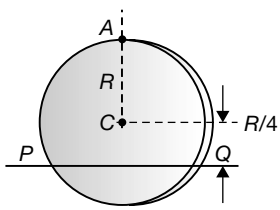
7. One end of a straight uniform $1\ \text{m}$ long bar is pivoted on horizontal surface. It is released from rest when it makes an angle $30°$ from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}\ \text{s}^{-1}$, where n is an integer. The value of n is ……



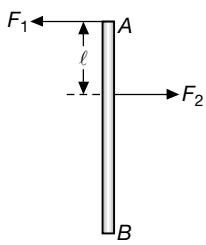
8. A cylinder of mass m is kept on the edge of a plank of mass $2m$ and length 12 metre, which in turn is kept on smooth ground. Coefficient of friction between the plank and the cylinder is 0.1. The cylinder is given an impulse, which imparts it a velocity $7\ \text{ms}^{-1}$ but no angular velocity. Find the time, in second, after which the cylinder falls off the plank. $\left(g = 10\ \text{ms}^{-2}\right)$.



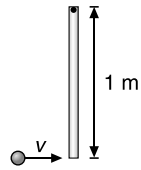
9. A uniform circular disc has radius R and mass m, A particle, also of mass m, is fixed at a point A on the edge of the disc as shown in Figure. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance $\frac{R}{4}$ from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. The linear speed of the particle as it reaches its lowest position is $\sqrt{xgR}$. Find x.



10. A thin uniform rod AB of mass $m = 1\ \text{kg}$ moves translationally with acceleration $a = 2\ \text{ms}^{-2}$ due to two anti-parallel forces F_1 and F_2. The distance between the points at which these forces are applied is equal to $l = 20\ \text{cm}$. Besides, it is known that $F_2 = 5\ \text{N}$. Find the length of the rod, in metre.



11. A thin rod of mass $0.9\ \text{kg}$ and length $1\ \text{m}$ is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of mass $0.1\ \text{kg}$ moving in a straight line with velocity $80\ \text{ms}^{-1}$ hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rads^{-1}) of the rod immediately after the collision will be ……

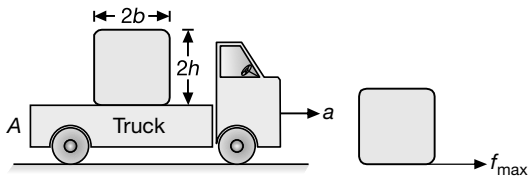


12. A flywheel whose moment of inertia about its axis of rotation is $16\ \text{kgm}^2$ is rotating freely in its own plane about a smooth axis through its centre. Its angular velocity is $9\ \text{rads}^{-1}$ when a torque is applied to bring it to rest in t_0 second. Find t_0, in seconds, if

(a) the torque is constant and of magnitude of $4\ \text{Nm}$

(b) the magnitude of the torque after t second is given by $8t$

13. A block of mass m height $2h = 2.4\ \text{m}$ and width $2b = 1.2\ \text{m}$ rests on a flat truck which moves horizontally with constant acceleration a as shown in figure. Determine



(a) the value of the acceleration, in ms^{-2}, at which slipping of the block on the truck starts if the coefficient of friction is $\mu = 0.4$.

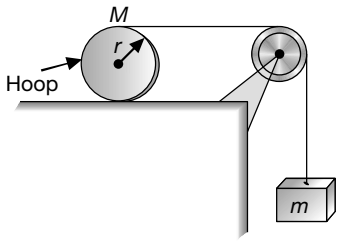
(b) the value of the acceleration a, in ms^{-2}, at which block topples about A, assuming sufficient friction to prevent slipping.

(c) the shortest distance, in metre, in which the truck can be stopped from a speed of $72\ \text{kmhr}^{-1}$ with constant deceleration so that the block is not disturbed.

Given that $g = 10\ \text{ms}^{-2}$

14. A person of $80\ \text{kg}$ mass is standing on the rim of a circular platform of mass $200\ \text{kg}$ rotating about its axis as 5 revolutions per minute (rpm). The person now starts moving towards the center of the platform. The rotational speed (in rpm) of the platform when the person reaches its centre is ……

15. For the system shown in figure a hoop of mass $M = 1$ kg radius $r = 0.2$ m is attached to a block of mass $m = 0.5$ kg. Calculate.

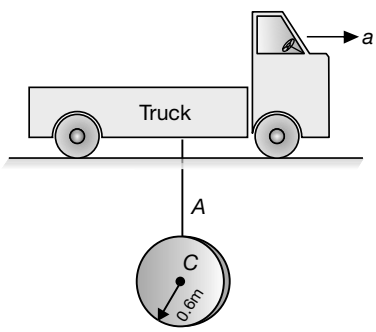


(a) the linear acceleration of hoop, in ms^{-2}.
(b) the angular acceleration of the hoop in $rads^{-2}$.
(c) the tension in the rope, in newton.

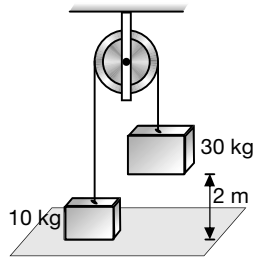
Neglect the mass of small pulley and the friction between the hoop and the horizontal surface. Masses of the pulley and string and friction between the hoop and the horizontal surface are negligible.

16. A ball rolls without sliding over a rough horizontal floor with velocity $v_0 = 7$ ms^{-1} towards a smooth vertical wall. If coefficient of restitution between the wall and the ball is $e = 0.7$, calculate velocity v of the ball, in ms^{-1}, along after the collision.

17. A 100 N circular plate of radius 0.6 m is suspended from a pin at A. If the pin is connected to a truck which is given an acceleration $a = 0.9$ ms^{-2}, determine the horizontal and vertical components of reaction, in newton, at A and the acceleration of the centre of mass (C) of the plate in cms^{-2}. Assume, the plate to be originally at rest. Also find the angular acceleration in $rads^{-2}$ of the plate.



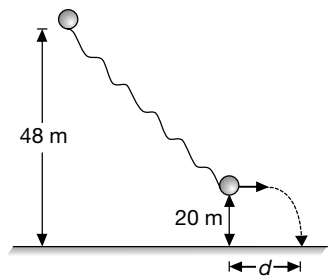
18. The system in figure is released from rest. The 30 kg body is 2 m above the floor and is connected through an ideal string passing over the pulley (a uniform disk with a radius of 10 cm and mass 20 kg) to another body of mass 10 kg. Find



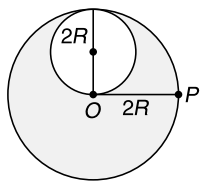
(a) the speed of the 30 kg body, in ms^{-1}, just before it hits the floor and the angular speed of the pulley, in $rads^{-1}$, at that time
(b) the tension, in newton, in the strings and
(c) the time, in second, it takes for the 30 kg body to reach the floor.

19. A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1. If another stationary disc having radius $\frac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_2. The energy lost in the process is $p\%$ of the initial energy. Value of p is ……

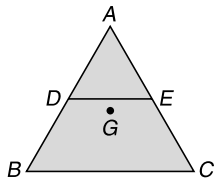
20. A sphere starts from rest at the upper end of the track shown in the accompanying figure. It rolls until it leaves the track at the right end, where the track is horizontal. Calculate the distance d to the right, in metre, of this point at which the sphere strikes the ground. Take $g = 10$ ms^{-2}.



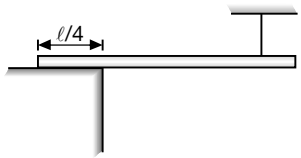
21. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. Find the ratio $\frac{I_P}{I_O}$ to the nearest integer.



22. ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AV and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0. If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is ……

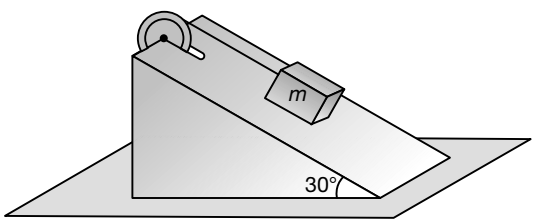


23. One fourth length of a uniform rod of mass m and length l is placed on a rough horizontal surface and it is held stationary in horizontal position by means of a light thread as shown in the figure. The thread is then burnt and the rod starts rotating about the edge. The angle between the rod and the horizontal when it is about to slide on the edge is $\tan\theta = \frac{*\mu}{13}$, where $*$ is not readable and μ is the coefficient of friction between the rod and the surface. Find $*$.

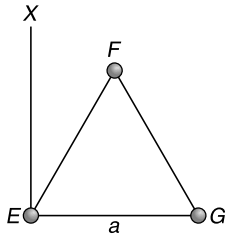


24. A bowling ball is thrown straight down the rough alley. The coefficient of friction between the ball and the alley is 0.75. When the ball starts, its centre of mass has a speed 12 ms^{-1} and it is sliding without rotating. Calculate the distance moved by the ball (in metre) down the alley before it starts rolling without slipping.

25. A block of mass $m = 1$ kg slides down the surface of a smooth incline as shown in figure. The block is tied to a string which is wrapped around a disk capable of rotating about a horizontal axis. The disk has a mass $M = 3$ kg and a radius $R = 0.2$ m. Initially the string is taut. If the mass is released, calculate its acceleration, in ms^{-2}, if $g = 10$ ms^{-2}.



26. A massless equilateral triangle EFG of side a (as shown in Figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20}ma^2$ where N is an integer. The value of N is ……

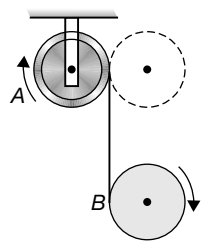


27. A uniform disk of mass $M = 40$ g and radius $R = 0.5$ cm is pivoted so that it can rotate freely about a horizontal axis through its centre and normal to the plane of the disk. A small particle of mass $m = 5$ g is attached to the rim of the disk at the top directly above the pivot. The system is given a gentle start and the disk begins to rotate.

(a) What is the angular velocity of the disk, in $rads^{-1}$, when the particle is at its lowest point?

(b) At this point, what force, in newton, must be exerted on the particle by the disk to keep it on the disk? Take $g = 10$ ms^{-2}

28. Two identical uniform discs A and B each of mass 2 kg and radius 0.5 m are held, as shown in figure with the help of a long massless string which is wrapped around the discs in opposite directions. Disc A is attached to the ceiling in such a way that it can rotate freely about its axis. The disc, B, initially held at same height as A, is then released to fall so that string unwinds from both the discs. Find the angular acceleration, in $rads^{-2}$ linear acceleration of falling disc in ms^{-2} and tension in the string, in newton. Assume that string does not slip and motion is confined in the same vertical plane and take $g = 10$ ms^{-2}.

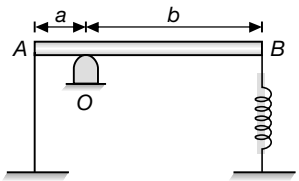


29. A grindstone in the form of a solid cylinder has a radius of 0.2 m and a mass of 30 kg.

(a) What constant torque, in newton metre, will bring it from rest to an angular velocity of 250 $revmin^{-1}$ in 10 s?

(b) Through what angle, in radian, has it turned during that time?
(c) Calculate the work done, in joule, by the torque.

30. A uniform rod AB of mass $m = 2$ kg and length $\ell = 1$ m is placed on a sharp support O such that $a = 0.4$ m and $b = 0.6$ m. A spring of force constant $k = 600\ \text{Nm}^{-1}$ is attached to end B as shown in Figure.



To keep the rod horizontal, its end A is tied with a thread such that the spring is elongated by 1 cm. Calculate reaction of support O, in newton, at the instant thread is burnt.

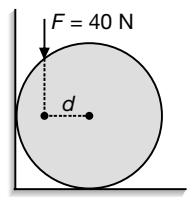
31. A solid fly wheel of 20 kg mass and 120 mm radius revolves at 600 rpm. Calculate the force (in newton) with which the brake lining should be pressed against the flywheel to stop the flywheel in 3 s, if the coefficient of friction is 0.1.

32. The polar ice caps contain about 3×10^{19} kg of ice. This contributes essentially nothing to the moment of inertia of the earth because it is located at the poles, close to the axis of rotation. Estimate the expected change in the length of the day (in second) if the polar ice caps melt, distributing the water uniformly over the entire surface of the earth if mass and radius of earth are 6×10^{24} kg and 6.4×10^{6} m.

33. A force $\vec{F} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ N acts at a point $\left(4\hat{i} + 3\hat{j} - k\right)$ m. Then the magnitude of torque about the point $\left(\hat{i} + 2\hat{j} + \hat{k}\right)$ m will be $\sqrt{x}$ Nm. The value of x is ……

34. A man stands at the centre of a circular platform holding his arms extended horizontally with 4 kg block in each hand. He is set rotating about a vertical axis at $0.5\ \text{revs}^{-1}$. The moment of inertia of the man plus platform is $1.6\ \text{kgm}^2$, assumed constant. The blocks are 90 cm from the axis of rotation. He now pulls the blocks inwards towards his body until they are 15 cm from the axis of rotation. Calculate
(a) his new angular velocity, in rads^{-1}.
(b) the initial and final kinetic energy of the man and platform (in joule).
(c) the work done by the man (in joule) to pull in the blocks.

35. A cylinder of mass $r = 0.1$ m and mass $M = 2$ kg is placed such that it is in contact simultaneously with a vertical and a horizontal surface as shown in Figure. The coefficient of static friction $\mu = \left(\frac{1}{3}\right)$ for both the surfaces. Find the distance d, in cm, from the centre of the cylinder at which a force $F = 40$ N should be applied so that the cylinder just starts rotating in the anticlockwise direction. Take $g = 10\ \text{ms}^{-2}$.



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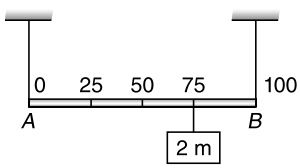
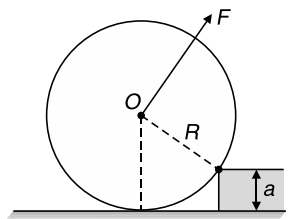
1. **[Online September 2020]**
Shown in Figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass m and has another weight of mass 2 m hung at a distance of 75 cm from A. The tension in the string at A is



(A) $2mg$ (B) $0.5mg$
(C) $0.75mg$ (D) $1mg$

2. **[Online September 2020]**
A uniform cylinder of mass M and radius R is to be pulled over a step of height $a(a > R)$ by applying a force F at its center O perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is



(A) $Mg\sqrt{1-\frac{a^2}{R^2}}$ (B) $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

(C) $Mg\frac{a}{R}$ (D) $Mg\sqrt{1\left(\frac{R-a}{R}\right)^2}$

3. [Online September 2020]

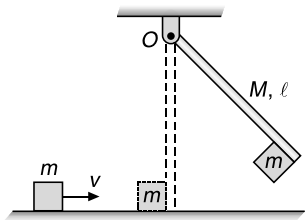
Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centers. The moment of inertia and angular velocity of the first disc are $0.1\ \text{kgm}^2$ and $10\ \text{rads}^{-1}$ respectively while those for the second one are $0.2\ \text{kgm}^2$ and $5\ \text{rads}^{-1}$ respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is

(A) $\frac{10}{3}$ J (B) $\frac{2}{3}$ J

(C) $\frac{5}{3}$ J (D) $\frac{20}{3}$ J

4. [Online September 2020]

A block of mass $m = 1$ kg slides with velocity $v = 6\ \text{ms}^{-1}$ on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass $M = 2$ kg, and length $l = 1$ m, the value of θ is approximately (Take $g = 10\ \text{ms}^{-2}$)

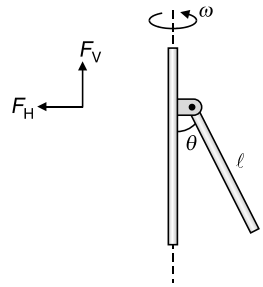


(A) 69° (B) 63°

(C) 55° (D) 49°

5. [Online September 2020]

A uniform rod of mass m, length l is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ it (see figure). To find θ, equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2\sin\theta\cos\theta$ about the center of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that



(A) $\cos\theta = \frac{g}{2l\omega^2}$ (B) $\cos\theta = \frac{3g}{2l\omega^2}$

(C) $\cos\theta = \frac{2g}{3l\omega^2}$ (D) $\cos\theta = \frac{g}{l\omega^2}$

6. [Online September 2020]

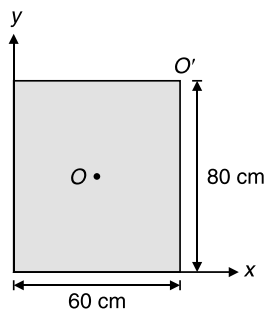
Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is

(A) $\sqrt{2}$ (B) 2

(C) 4 (D) $2\sqrt{2}$

7. [Online September 2020]

For a uniform rectangular sheet shown in Figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the center of mass) and O' (corner point) is



(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{1}{8}$ (D) $\frac{1}{4}$

8. [Online September 2020]

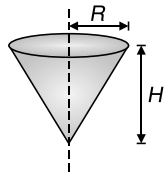
A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia $3I$ initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is

(A) 0 (B) $\frac{1}{4}$

(C) $\frac{3}{4}$ (D) $\frac{5}{6}$

9. [Online September 2020]

Shown in Figure is a hollow ice cream cone (it is open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is

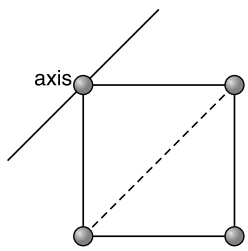


(A) $\frac{MR^2}{2}$ (B) $\frac{MH^2}{3}$

(C) $\frac{MR^2}{3}$ (D) $\frac{M(R^2+H^2)}{4}$

10. [Online September 2020]

Four point masses, each of mass m, are fixed at the corners of a square of side l. The square is rotating with angular frequency ω, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in Figure. The angular momentum of the square about this axis is



(A) $2ml^2\omega$ (B) $3ml^2\omega$

(C) $ml^2\omega$ (D) $4ml^2\omega$

11. [Online September 2020]

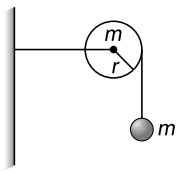
The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x)=\lambda_0\left(1+\frac{x}{L}\right)$, where x is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is

(A) $\frac{5}{12}ML^2$ (B) $\frac{3}{7}ML^2$

(C) $\frac{2}{5}ML^2$ (D) $\frac{7}{18}ML^2$

12. [Online January 2020]

As shown in Figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be



(A) $\frac{1}{r}\sqrt{\frac{2gh}{3}}$ (B) $r\sqrt{\frac{3}{4gh}}$

(C) $\frac{1}{r}\sqrt{\frac{4gh}{3}}$ (D) $r\sqrt{\frac{3}{2gh}}$

13. [Online January 2020]

The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the center of the rod, and perpendicular to it, is

(A) $\frac{l}{8}$ (B) $l\sqrt{\frac{7}{48}}$

(C) $l\sqrt{\frac{3}{8}}$ (D) $\frac{l}{4}$

14. [Online January 2020]

Mass per unit area of a circular disc of radius a depends on the distance r from its center as $\sigma(r)=A+Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its center is

(A) $2\pi a^2\left(\frac{A}{4}+\frac{aB}{5}\right)$ (B) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

(C) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$ (D) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$

15. [Online January 2020]

A uniform sphere of mass $500g$ rolls without slipping on a plane horizontal surface with its center moving at a speed of $5.00\ \text{cms}^{-1}$. Its kinetic energy is

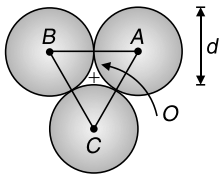
(A) 8.75×10^{-4} J (B) 8.75×10^{-3} J

(C) 6.25×10^{-4} J (D) 1.13×10^{-3} J

16. [Online January 2020]

Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centers from an equilateral triangle of side of length d. The ratio $\frac{I_0}{I_A}$ of moment of inertia I_0 of the system

about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is

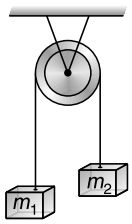


(A) $\frac{13}{23}$ (B) $\frac{15}{13}$

(C) $\frac{23}{13}$ (D) $\frac{13}{15}$

17. [Online January 2020]

A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its center of mass (see figure). A massless string is wrapped over its rim and two blocks of masses m_1 and $m_2\,(m_1 > m_2)$ are attached to the ends of the string. The system is released from rest. The angular speed of the wheel m_1 descents by a distance h is



(A) $\left[\frac{m_1+m_2}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}} gh$

(B) $\left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$

(C) $\left[\frac{2(m_1+m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$

(D) $\left[\frac{(m_1-m_2)}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}} gh$

18. [Online April 2019]

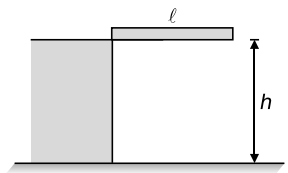
A thin circular plate of mass M and radius R has its density varying as $\rho(r)=\rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I=a(MR^2)$. The value of the coefficient a is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$

(C) $\frac{3}{5}$ (D) $\frac{8}{5}$

19. [Online April 2019]

A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slips off the table in a very short time $\tau = 0.01$ s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to

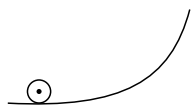


(A) 0.3 (B) 0.02

(C) 0.28 (D) 0.5

20. [Online April 2019]

A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by



(A) $\frac{4}{5}$ (B) $\frac{2}{\sqrt{5}}$

(C) 1 (D) $\frac{14}{15}$

21. [Online April 2019]

A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I, then the angular acceleration of the disc is

(A) $\frac{2k}{I}\theta$ (B) $\frac{k}{I}\theta$

(C) $\frac{k}{2I}\theta$ (D) $\frac{k}{4I}\theta$

22. [Online April 2019]

The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane

(1) a ring of radius R,

(2) a solid cylinder of radius $\frac{R}{2}$ and

(3) a solid sphere of radius $\frac{R}{4}$.

If, in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is

(A) 14 : 15 : 20 (B) 10 : 15 : 7

(C) 4 : 3 : 2 (D) 2 : 3 : 4

23. [Online April 2019]

A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be

(A) $\frac{M\omega_0}{M+3m}$ (B) $\frac{M\omega_0}{M+m}$

(C) $\frac{M\omega_0}{M+6m}$ (D) $\frac{M\omega_0}{M+2m}$

24. [Online April 2019]

Moment of inertia of a body about a given axis is 1.5 kgm^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rads^{-2} must be applied about the axis for a duration of

(A) 2.5 s (B) 2 s

(C) 5 s (D) 3 s

25. [Online April 2019]

Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$ are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$ about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $\left(E_f - E_i\right)$ is

(A) $-\frac{I_1\omega_1^2}{12}$ (B) $\frac{3}{8}I_1\omega_1^2$

(C) $\frac{I_1\omega_1^2}{6}$ (D) $-\frac{I_1\omega_1^2}{24}$

26. [Online April 2019]

A particle of mass m is moving along a trajectory given by $x = x_0 + a\cos\omega_1 t$ and $y = y_0 + b\sin\omega_2 t$.

The torque, acting on the particle about the origin, at $t = 0$ is

(A) $-m\left(x_0 b\omega_2^2 - y_0 a\omega_1^2\right)\hat{k}$ (B) $m\left(-x_0 b + y_0 a\right)\omega_1^2\hat{k}$

(C) $+my_0 a\omega_1^2\hat{k}$ (D) ZERO

27. [Online April 2019]

A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is

(A) $\frac{MR^2}{3}$ (B) $\frac{MR^2}{6}$

(C) $\frac{MR^2}{2}$ (D) $\frac{2MR^2}{3}$

28. [Online April 2019]

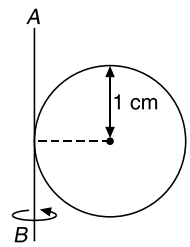
The time dependence of the position of a particle of mass $m = 2$ is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time $t = 2$ is

(A) $-34\left(\hat{k} - \hat{i}\right)$ (B) $48\left(\hat{i} - \hat{j}\right)$

(C) $36\hat{k}$ (D) $-48\hat{k}$

29. [Online April 2019]

A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in Figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5 s, is closed to



(A) 4.0×10^{-6} Nm (B) 7.9×10^{-6} Nm

(C) 2.0×10^{-5} Nm (D) 1.6×10^{-5} Nm

30. [Online April 2019]

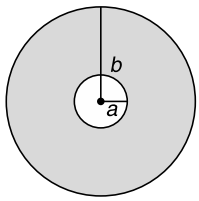
A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius $2R$. The second part is converted into a uniform solid sphere.

Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio $\frac{I_1}{I_2}$ is given by

(A) 140 (B) 185
(C) 285 (D) 65

31. [Online April 2019]

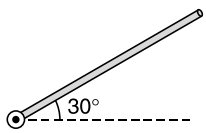
A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is



(A) $\frac{a+b}{2}$ (B) $\sqrt{\frac{a^2+b^2+ab}{2}}$
(C) $\frac{a+b}{3}$ (D) $\sqrt{\frac{a^2+b^2+ab}{3}}$

32. [Online January 2019]

A rod of length 50 cm is pivoted at one end. It is raised such that it makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rads^{-1}) will be $\left(g = 10\ \text{ms}^{-2}\right)$



(A) $\frac{\sqrt{20}}{3}$ (B) $\sqrt{30}$
(C) $\sqrt{\frac{30}{2}}$ (D) $\frac{\sqrt{30}}{2}$

33. [Online January 2019]

To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is

(A) $\frac{\mu FR}{2}$ (B) $\frac{\mu FR}{3}$
(C) $\frac{\mu FR}{6}$ (D) $\frac{2}{3}\mu FR$

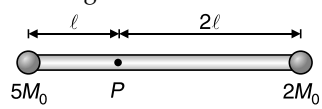
34. [Online January 2019]

A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is

(A) $\frac{F}{2MR}$ (B) $\frac{2F}{3MR}$
(C) $\frac{F}{3MR}$ (D) $\frac{3F}{2MR}$

35. [Online January 2019]

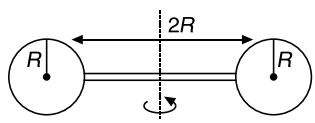
A rigid massless rod of length $3l$ has two masses attached at each end as shown in Figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be



(A) $\frac{g}{2l}$ (B) $\frac{g}{3l}$
(C) $\frac{g}{13l}$ (D) $\frac{7g}{3l}$

36. [Online January 2019]

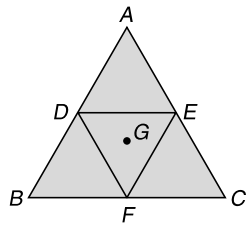
Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is



(A) $\frac{152}{15}MR^2$
(B) $\frac{17}{15}MR^2$
(C) $\frac{209}{15}MR^2$
(D) $\frac{137}{15}MR^2$

37. [Online January 2019]

An equilateral triangle ABC is cut from a thin solid sheet of wood (see figure). D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then

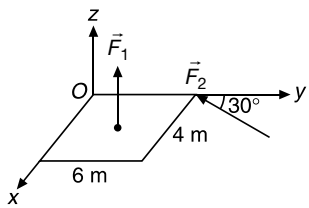


(A) $I = \frac{3}{4} I_0$ (B) $I = \frac{15}{16} I_0$

(C) $I = \frac{I_0}{4}$ (D) $I = \frac{9}{16} I_0$

38. [Online January 2019]

A slab is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$ of same magnitude F as shown in Figure. Force $\vec{F}_2$ is in xy plane while force F_1 acts along z-axis at the point $\left(2\vec{i} + 3\vec{j}\right)$. The moment of these forces about point O will be

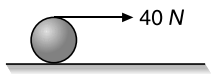


(A) $\left(3\hat{i} - 2\hat{j} + 3\hat{k}\right)F$ (B) $\left(3\hat{i} + 2\hat{j} - 3\hat{k}\right)F$

(C) $\left(3\hat{i} + 2\hat{j} + 3\hat{k}\right)F$ (D) $\left(3\hat{i} - 2\hat{j} - 3\hat{k}\right)F$

39. [Online January 2019]

A string is wound around a hollow cylinder of mass 5 kgand radius 0.5 m. If the string is now pulled with a horizontal force of 40 N and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string)



(A) 16 rads^{-2} (B) 20 rads^{-1}

(C) 12 rads^{-2} (D) 10 rads^{-2}

40. [Online January 2019]

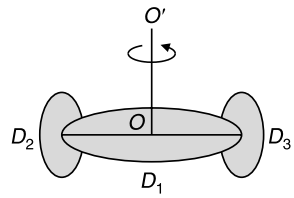
The magnitude of torque on a particle of mass 1 kg is 2.5 Nm about the origin. If the force acting on it is 1 N and the distance of the particle from the origin is 5 m, the angle between the force and the position vector is (in radians)

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

41. [Online January 2019]

A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1 as shown in Figure, will be



(A) $3MR^2$ (B) $\frac{4}{5}MR^2$

(C) MR^2 (D) $\frac{2}{3}MR^2$

42. [Online January 2019]

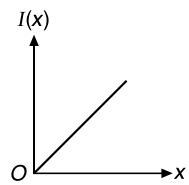
Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is

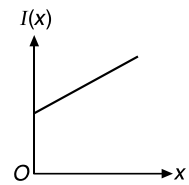
(A) 14 cm (B) 12 cm

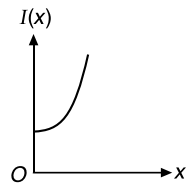
(C) 16 cm (D) 18 cm

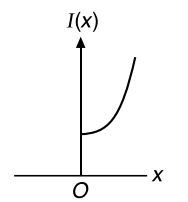
43. [Online January 2019]

The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is $I(x)$. Which one of the graphs represents the variation of $I(x)$ with x correctly?

(A) 

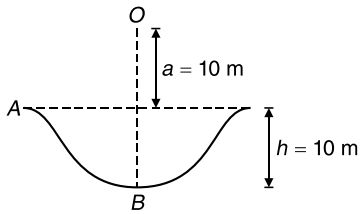
(B) 

(C) 

(D) 

44. [Online January 2019]

A particle of mass 20 g is released with an initial velocity 5 ms^{-1} along the curve from the point A, as shown in Figure. The point A is a height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be (Take $g = 10$ ms^{-2})

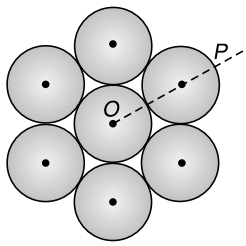


(A) $2\ \text{kgm}^2\text{s}^{-1}$ (B) $3\ \text{kgm}^2\text{s}^{-1}$

(C) $8\ \text{kgm}^2\text{s}^{-1}$ (D) $6\ \text{kgm}^2\text{s}^{-1}$

45. [2018]

Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is

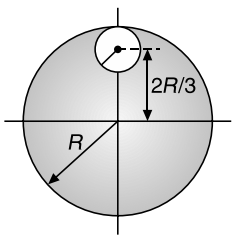


(A) $\frac{19}{2}MR^2$ (B) $\frac{55}{2}MR^2$

(C) $\frac{73}{2}MR^2$ (D) $\frac{181}{2}MR^2$

46. [2018]

From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in Figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is

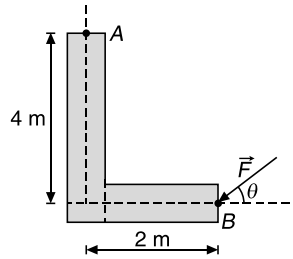


(A) $4MR^2$ (B) $\frac{40}{9}MR^2$

(C) $10MR^2$ (D) $\frac{37}{9}MR^2$

47. [Online 2018]

A force of $40\ \text{N}$ acts on a point B at the end of an L-shaped object, as shown in Figure. The angle θ that will produce maximum moment of the force about point A is given by

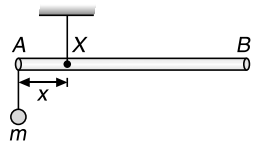


(A) $\tan\theta = \frac{1}{2}$ (B) $\tan\theta = 4$

(C) $\tan\theta = 2$ (D) $\tan\theta = \frac{1}{4}$

48. [Online 2018]

A uniform rod AB is suspended from a point X, at a variable distance x from A, as shown. To make the rod horizontal, a mass m is suspended from its end A. A set of $(m,\ x)$ values is recorded. The appropriate variables that give a straight line, when plotted are



(A) $m, \frac{1}{x^2}$ (B) m, x^2

(C) m, x (D) $m, \frac{1}{x}$

49. [Online 2018]

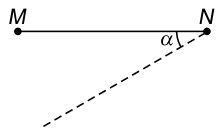
A disc rotates about its axis of symmetry in a horizontal plane at a steady rate of 3.5 revolutions per second. A coin placed at a distance of $1.25\ \text{cm}$ from the axis of rotation remains at rest on the disc. The coefficient of friction between the coin and the disc is $\left(g = 10\ \text{ms}^{-2}\right)$

(A) 0.7 (B) 0.5

(C) 0.3 (D) 0.6

50. [Online 2018]

A thin rod MN, free to rotate in the vertical plane about the fixed end N, is held horizontal. When the end M is released the speed of this end, when the rod makes an angle α with the horizontal, will be proportional to (see figure)

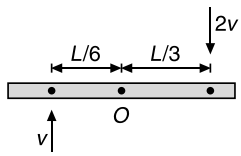


(A) $\cos\alpha$ (B) $\sin\alpha$

(C) $\sqrt{\cos\alpha}$ (D) $\sqrt{\sin\alpha}$

51. [Online 2018]

A thin uniform bar of length L and mass $8m$ lies on a smooth horizontal table. Two point masses m and $2m$ are moving in the same horizontal plane from opposite sides of the bar with speeds $2v$ and v respectively. The masses stick to the bar after collision at a distance $\frac{L}{3}$ and $\frac{L}{6}$ respectively from the centre of the bar. If the bar starts rotating about its center of mass as a result of collision, the angular speed of the bar will be

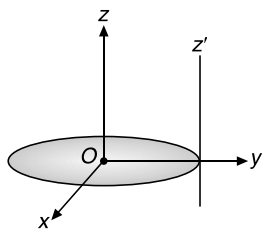


(A) $\frac{6v}{5L}$ (B) $\frac{v}{6L}$

(C) $\frac{v}{5L}$ (D) $\frac{3v}{5L}$

52. [Online 2018]

A thin circular disk is in the xy plane as shown in Figure. The ratio of its moment of inertia about z and z' axes will be



(A) 1 : 4 (B) 1 : 3

(C) 1 : 2 (D) 1 : 5

53. [2017]

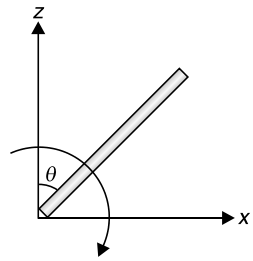
The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio $\frac{l}{R}$ such that the moment of inertia is minimum?

(A) $\sqrt{\frac{3}{2}}$ (B) $\frac{\sqrt{3}}{2}$

(C) 1 (D) $\frac{3}{\sqrt{2}}$

54. [2017]

A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (shown in Figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is

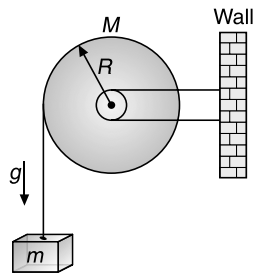


(A) $\frac{3g}{2l}\sin\theta$ (B) $\frac{2g}{3l}\sin\theta$

(C) $\frac{3g}{2l}\cos\theta$ (D) $\frac{2g}{3l}\cos\theta$

55. [Online 2017]

A uniform disc of radius R and mass M is free to rotate only about its axis. A string is wrapped over its rim and a body of mass m is tied to the free end of the string as shown in Figure. The body is released from rest. Then the acceleration of the body is



(A) $\frac{2mg}{2m+M}$ (B) $\frac{2Mg}{2m+M}$

(C) $\frac{2Mg}{2M+m}$ (D) $\frac{2mg}{2M+m}$

56. [Online 2017]

In a physical balance working on the principle of moments, when 5 mg weight is placed on the left pan, the beam becomes horizontal. Both the empty pans of the balance are of equal mass. Which of the following statements is correct?

(A) Left arm is shorter than the right arm

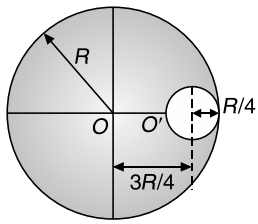
(B) Left arm is longer than the right arm

(C) Every object that is weighed using this balance appears lighter than its actual weight

(D) Both the arms are of same length

57. [Online 2017]

A circular hole of radius $\frac{R}{4}$ is made in a thin uniform disc having mass M and radius R, as shown in Figure. The moment of inertia of the remaining portion of the disc about an axis passing through the point O and perpendicular to the plane of the disc is

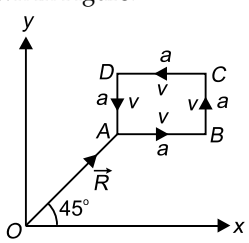


(A) $\frac{219MR^2}{256}$ (B) $\frac{197MR^2}{256}$

(C) $\frac{19MR^2}{512}$ (D) $\frac{237MR^2}{512}$

58. [2016]

A particle of mass m is moving along the side of a square of side a, with a uniform speed v in the x-y plane as shown in Figure.



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

(A) $\vec{L} = \frac{mvR}{\sqrt{2}}\hat{k}$ when the particle is moving from D to A

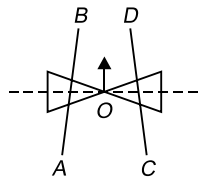
(B) $\vec{L} = -\frac{mvR}{\sqrt{2}}\hat{k}$ when the particle is moving from A to B

(C) $\vec{L} = mv\left(\frac{R}{\sqrt{2}} - a\right)\hat{k}$ when the particle is moving from C to D

(D) $\vec{L} = mv\left(\frac{R}{\sqrt{2}} + a\right)\hat{k}$ when the particle is moving from B to C

59. [2016]

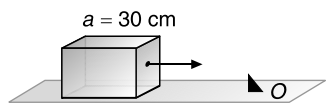
A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to



(A) turn left and right alternately
(B) turn left
(C) turn right
(D) go straight

60. [Online 2016]

A cubical block of side 30 cm is moving with velocity 2 ms^{-1} on a smooth horizontal surface. The surface has a bump at a point O as shown in Figure. The angular velocity (in $rads^{-1}$) of the block immediately after it hits the bump, is



(A) 13.3 (B) 5.0
(C) 9.4 (D) 6.7

61. [Online 2016]

Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute (rpm) to ensure proper mixing is close to (Take the radius of the drum to be 1.25 m and its axle to be horizontal)

(A) 27.0 (B) 0.4
(C) 1.3 (D) 8.0

62. [2015]

From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is

(A) $\frac{MR^2}{32\sqrt{2}\pi}$ (B) $\frac{MR^2}{16\sqrt{2}\pi}$

(C) $\frac{4MR^2}{9\sqrt{3}\pi}$ (D) $\frac{4MR^2}{3\sqrt{3}\pi}$

63. [Online 2015]
A uniform solid cylindrical roller of mass m is being pulled on a horizontal surface with force F parallel to the surface and applied at its centre. If the acceleration of the cylinder is a and it is rolling without slipping then the value of F is

(A) ma (B) $2ma$

(C) $\frac{3}{2}ma$ (D) $\frac{5}{3}ma$

64. [Online 2015]
Consider a thin uniform square sheet made of a rigid material. If its side is a, mass m and moment of inertia I about one of its diagonals, then

(A) $I > \frac{ma^2}{12}$ (B) $\frac{ma^2}{24} < I < \frac{ma^2}{12}$

(C) $I = \frac{ma^2}{12}$ (D) $I = \frac{ma^2}{24}$

65. [Online 2015]
A particle of mass 2 kg is on a smooth horizontal table and moves in a circular path of radius 0.6 m . The height of the table from the ground is 0.8 m . If the angular speed of the particle is 12 rads^{-1}, the magnitude of its angular momentum about a point on the ground right under the centre of the circle is

(A) 8.64 kgm^{2}s^{-1}

(B) 11.52 kgm^{2}s^{-1}

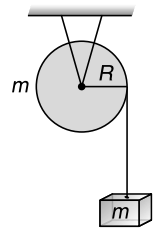
(C) 14.4 kgm^{2}s^{-1}

(D) 20.16 kgm^{2}s^{-1}

66. [2014]
A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rads^{-1} about the vertical. About the point of suspension

(A) angular momentum is conserved.
(B) angular momentum changes in magnitude but not in direction.
(C) angular momentum changes in direction but not in magnitude
(D) angular momentum changes both in direction and magnitude

67. [2014]
A mass m is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release?



(A) $\frac{2g}{3}$ (B) $\frac{g}{2}$

(C) $\frac{5g}{6}$ (D) g

68. [2013]
A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?

(A) $\frac{r\omega_0}{4}$ (B) $\frac{r\omega_0}{3}$

(C) $\frac{r\omega_0}{2}$ (D) $r\omega_0$

69. [2011]
A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kgm^2, the number of rotations made by the pulley before its direction of motion if reversed, is

(A) less than 3
(B) more than 3 but less than 6
(C) more than 6 but less than 9
(D) more than 9

70. [2011]
A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is

(A) $\frac{3}{2}g$ (B) g

(C) $\frac{2}{3}g$ (D) $\frac{g}{3}$

71. [2011]
A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves

along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc
(A) remains unchanged
(B) continuously decreases
(C) continuously increases
(D) first increases and then decreases

72. [2009]
A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of

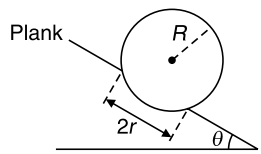
(A) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (B) $\frac{1}{6}\frac{l\omega}{g}$
(C) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (D) $\frac{1}{6}\frac{l^2\omega^2}{g}$

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

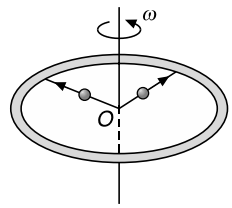
1. [JEE (Advanced) 2020]
A football of radius R is kept on a hole of radius $r(r<R)$ made on a plank kept horizontally. One end of the plank is now lifted so that it gets tilted making an angle θ from the horizontal as shown in Figure.



The maximum value of θ so that the football does not start rolling down the plank satisfies (figure is schematic and not drawn to scale)

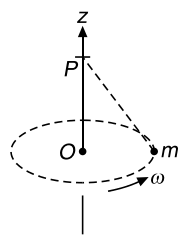
(A) $\sin\theta=\frac{r}{R}$ (B) $\tan\theta=\frac{r}{R}$
(C) $\sin\theta=\frac{r}{2R}$ (D) $\cos\theta=\frac{r}{2R}$

2. [JEE (Advanced) 2015]
A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in Figure. At some instant, the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant, the distance of the other mass from O is



(A) $\frac{2}{3}R$ (B) $\frac{1}{3}R$
(C) $\frac{3}{5}R$ (D) $\frac{4}{5}R$

3. [IIT-JEE 2012]
A small mass m is attached to a massless string whose other end is fixed at P as shown in Figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time
(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant
(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time
(D) L_O and $\vec{L}_P$ both vary with time

4. [IIT-JEE 2012]

Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?

(A) Both cylinders P and Q reach the ground at the same time

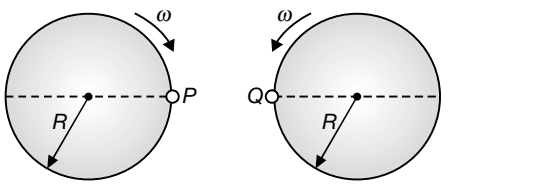
(B) Cylinder P has larger linear acceleration than cylinder Q

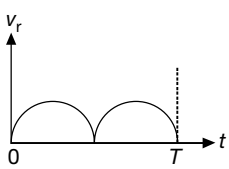
(C) Both cylinders reach the ground with same translational kinetic energy

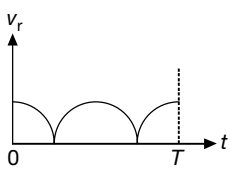
(D) Cylinder Q reaches the ground with larger angular speed

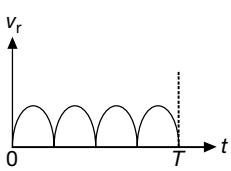
5. [IIT-JEE 2012]

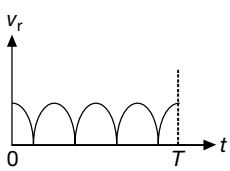
Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The disc are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in Figure. The magnitude of the relative speed between the two points P and Q is v_r which, as a function of time is best represented by



(A) 

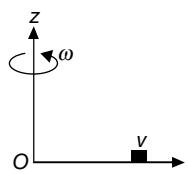
(B) 

(C) 

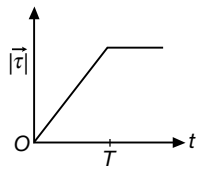
(D) 

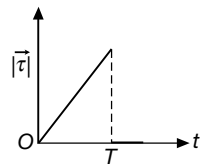
6. [IIT-JEE 2012]

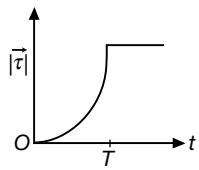
A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in Figure.

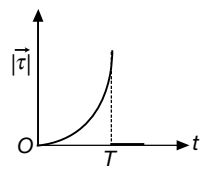


At time $t = 0$, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\vec{\tau}|)$ on the system about O, as a function of time is best represented by which plot?

(A) 

(B) 

(C) 

(D) 

7. [IIT-JEE 2009]

A block of base $10\text{ cm} \times 10\text{ cm}$ and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from $0°$. Then,

(A) at $\theta = 30°$, the block will start sliding down the plane

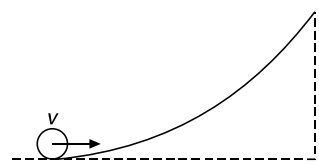
(B) the block will remain at rest on the plane up to certain θ and then it will topple

(C) at $\theta = 60°$, the block will start sliding down the plane and continue to do so at higher angles

(D) at $\theta = 60°$, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

8. [IIT-JEE 2007]

A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is



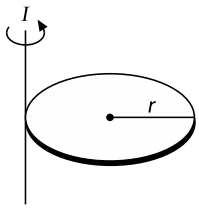
(A) ring (B) solid sphere

(C) hollow sphere (D) disc

9. [IIT-JEE 2006]

A solid sphere of radius R has moment of inertia I about its geometrical axis. It is melted into a disc of radius r and thickness t. If it's moment of inertia about

the tangential axis (which is perpendicular to plane of the disc), is also equal to I, then the value of r is equal to

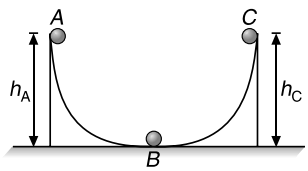


(A) $\frac{2}{\sqrt{15}}R$ (B) $\frac{2}{\sqrt{5}}R$

(C) $\frac{3}{\sqrt{15}}R$ (D) $\frac{\sqrt{3}}{\sqrt{15}}R$

10. [IIT-JEE 2006]

A ball moves over a fixed track as shown in Figure. From A to B the ball rolls without slipping. If surface BC is frictionless and K_A, K_B and K_C are kinetic energies of the ball at A, B and C respectively, then

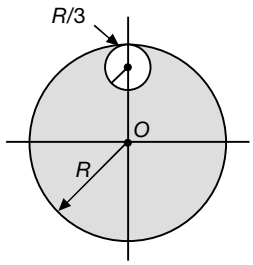


(A) $h_A > h_C$; $K_B > K_C$ (B) $h_A > h_C$; $K_C > K_A$

(C) $h_A = h_C$; $K_B = K_C$ (D) $h_A < h_C$; $K_B > K_C$

11. [IIT-JEE 2005]

A small disc of radius $\frac{R}{3}$ is removed from a circular disc of radius R and mass $9M$. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is



(A) $4MR^2$ (B) $\frac{40}{9}MR^2$

(C) $10MR^2$ (D) $\frac{37}{9}MR^2$

12. [IIT-JEE 2005]

A particle moves in a circular path with decreasing speed. Choose the correct statement

(A) Angular momentum remains constant

(B) Acceleration $(\vec{a})$ is towards the centre

(C) Particle moves in a spiral path with decreasing radius

(D) The direction of angular momentum remains constant

13. [IIT-JEE 2004]

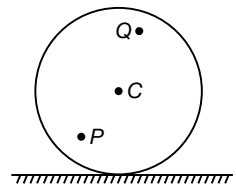
A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system doubles. The kinetic energy of the system now is

(A) $2K$ (B) $\frac{K}{2}$

(C) $\frac{K}{4}$ (D) $4K$

14. [IIT-JEE 2004]

A disc is rolling (without slipping) on a horizontal surface. C is its centre and Q and P are two points equidistant from C. Let v_P, v_Q and v_C be the magnitude of velocities of points P, Q and C respectively, then



(A) $v_Q > v_C > v_P$

(B) $v_Q < v_C < v_P$

(C) $v_Q = v_P$, $v_C = \frac{1}{2}v_P$

(D) $v_Q < v_C > v_P$

15. [IIT-JEE 2003]

A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved?

(A) Centre of circle

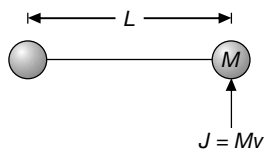
(B) On the circumference of the circle

(C) Inside the circle

(D) Outside the circle

16. [IIT-JEE 2003]

Consider a body, shown in Figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse $J = Mv$ is imparted to the body at one of its end, then its angular velocity is

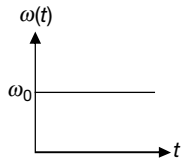


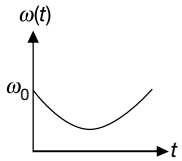
(A) $\frac{v}{L}$ (B) $\frac{2v}{L}$

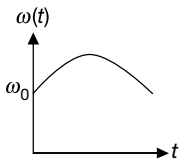
(C) $\frac{v}{3L}$ (D) $\frac{v}{4L}$

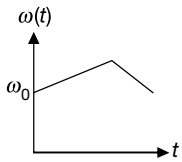
17. [IIT-JEE 2002]

A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now the platform is given an angular velocity ω_0. When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform). The angular velocity of the platform $\omega(t)$ will vary with time t as

(A) 

(B) 

(C) 

(D) 

18. [IIT-JEE 2002]

A cylinder rolls up an inclined plane, reaches some height and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are

(A) up the incline while ascending and down the incline while descending

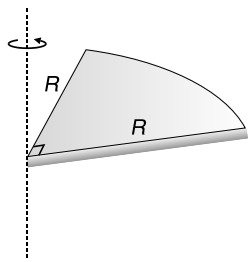
(B) up the incline while ascending as well as descending

(C) down the incline while ascending and up the incline while descending

(D) down the incline while ascending as well as descending

19. [IIT-JEE 2001]

One quarter sector is cut from a uniform disc of radius R. This sector has mass$M$. It is made to rotate about a line perpendicular to its plane and passing through the center of the original disc. Its moment of inertia about the axis of rotation is

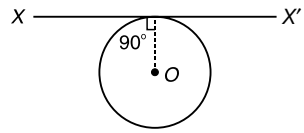


(A) $\frac{MR^2}{2}$ (B) $\frac{MR^2}{4}$

(C) $\frac{MR^2}{8}$ (D) $\sqrt{2}MR^2$

20. [IIT-JEE 2000]

A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is

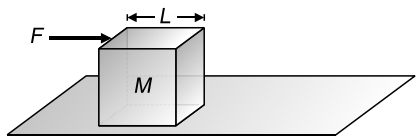


(A) $\frac{\rho L^3}{8\pi^2}$ (B) $\frac{\rho L^3}{16\pi^2}$

(C) $\frac{5\rho L^3}{16\pi^2}$ (D) $\frac{3\rho L^3}{8\pi^2}$

21. [IIT-JEE 2000]

A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is

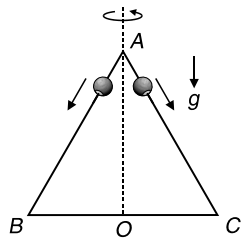


(A) infinitesimal (B) $\frac{mg}{4}$

(C) $\frac{mg}{2}$ (D) $mg(1-\mu)$

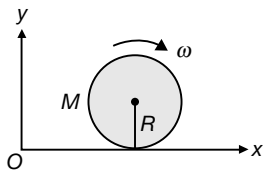
22. [IIT-JEE 2000]

An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as beads slide down are



(A) angular velocity and total energy (kinetic and potential).
(B) total angular momentum and total energy.
(C) angular velocity and moment of inertia about the axis of rotation.
(D) total angular momentum and moment of inertia about the axis of rotation.

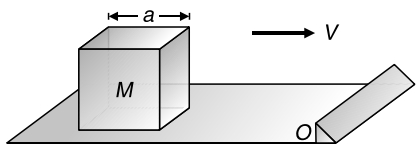
23. [IIT-JEE 1999]
A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in Figure. The magnitude of angular momentum of the disc about the origin O is



(A) $\left(\frac{1}{2}\right)MR^2\omega$ (B) $MR^2\omega$
(C) $\left(\frac{3}{2}\right)MR^2\omega$ (D) $2MR^2\omega$

24. [IIT-JEE 1999]
A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A and ω_B. Then
(A) $\omega_A < \omega_B$ (B) $\omega_A = \omega_B$
(C) $\omega_A = \omega$ (D) $\omega = \omega_B$

25. [IIT-JEE 1999]
A cubical block of side a is moving with velocity v on a horizontal smooth plane as shown in Figure. It hits a ridge at point O. If the moment of inertia of the block about an axis passing through centre of gravity is $\frac{1}{6}ma^2$, then the angular speed of the block after it hits O is



(A) $\frac{3v}{4a}$ (B) $\frac{3v}{2a}$
(C) $\sqrt{\frac{3}{2}}\frac{v}{a}$ (D) ZERO

26. [IIT-JEE 1998]
Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to
(A) I (B) $I\sin^2\theta$
(C) $I\cos^2\theta$ (D) $I\cos^2\left(\frac{\theta}{2}\right)$

27. [IIT-JEE 1997]
A mass M is moving with a constant velocity parallel to the x-axis. Its angular momentum with respect to the origin
(A) is zero
(B) remains constant
(C) increases
(D) decreases

28. [IIT-JEE 1995]
Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of
(A) 0.42 m from mass of 0.3 kg
(B) 0.70 m from mass of 0.7 kg
(C) 0.98 m from mass of 0.3 kg
(D) 0.98 m from mass of 0.7 kg

29. [IIT-JEE 1992]
A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is
(A) $\frac{M\omega^2L}{2}$ (B) $M\omega^2L$
(C) $\frac{M\omega^2L}{4}$ (D) $\frac{M\omega^2L^2}{2}$

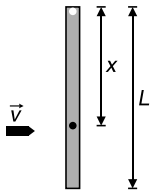
30. [IIT-JEE 1983]
A thin circular ring of mass M is rotating about its axis with a constant angular velocity ω. The two objects, each of mass m, are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity
(A) $\frac{\omega M}{M+m}$ (B) $\frac{\omega(M-2m)}{M+2m}$
(C) $\frac{\omega M}{M+2m}$ (D) $\frac{\omega(M+2m)}{M}$

Multiple Correct Choice Type Problems

In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. [JEE (Advanced) 2020]

A rod of mass m and length L, pivoted at one of its ends, is hanging vertically. A bullet of the same mass moving at speed v strikes the rod horizontally at a distance x from its pivoted end and gets embedded in it. The combined system now rotates with angular speed ω about the pivot. The maximum angular speed ω_M is achieved for $x = x_M$. Then



(A) $\omega = \dfrac{3vx}{L^2+3x^2}$ (B) $\omega = \dfrac{12vx}{L^2+12x^2}$

(C) $x_M = \dfrac{L}{\sqrt{3}}$ (D) $\omega_M = \dfrac{v}{2L}\sqrt{3}$

2. [JEE (Advanced) 2019]

A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle $60°$ with vertical? (g is the acceleration due to gravity)

(A) The angular acceleration of the rod will be $\dfrac{2g}{L}$

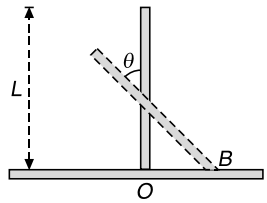
(B) The radial acceleration of the rod's centre of mass will be $\dfrac{3g}{4}$

(C) The angular speed of the rod will be $\sqrt{\dfrac{3g}{2L}}$

(D) The normal reaction force from the floor on the rod will be $\dfrac{Mg}{16}$

3. [JEE (Advanced) 2017]

A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in Figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct?



(A) Instantaneous torque about the point in contact with the floor is proportional to $\sin\theta$

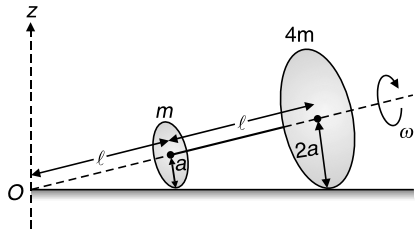
(B) The trajectory of the point A is parabola

(C) The mid-point of the bar will fall vertically downward

(D) When the bar makes an angle θ with the vertical, the displacement of its mid-point from the initial position is proportional to $(1-\cos\theta)$

4. [JEE (Advanced) 2016]

Two thin circular discs of mass m and $4m$, having radii of a and $2a$, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}a$ through their centers. This assembly is laid on a firm and flat surface and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point O is $\vec{L}$ (see the figure). Which of the following statement(s) is(are) true?



(A) The center of mass of the assembly rotates about the z-axis with an angular speed of $\dfrac{\omega}{5}$

(B) The magnitude of angular momentum of center of mass of the assembly about the point O is $81ma^2\omega$.

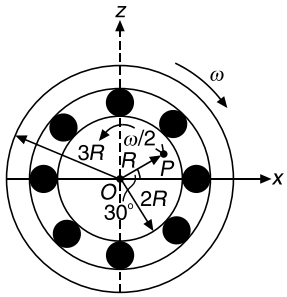
(C) The magnitude of angular momentum of the assembly about its center of mass is $17\ ma^2\dfrac{\omega}{2}$

(D) The magnitude of the z-component of $\vec{L}$ is $55ma^2\omega$.

5. [JEE (Advanced) 2012]

The figure shows a system consisting of (i) a ring of outer radius $3R$ rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius $2R$ rotating anti-clockwise with

angular speed $\frac{\omega}{2}$. The ring and disc are separated by frictionless ball bearings. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of $30°$ with the horizontal. Then with respect to the horizontal surface,



(A) the point O has a linear velocity $3R\omega\hat{i}$

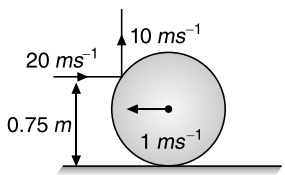
(B) the point P has a linear velocity $\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

(C) the point P has a linear velocity $\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) the point P has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$

6. [IIT-JEE 2011]

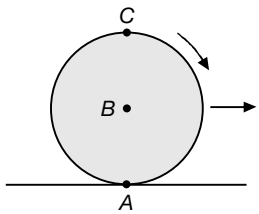
A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 ms^{-1}. A small ball of mass 0.1 kg, moving with velocity 20 ms^{-1} in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 ms^{-1}. Immediately after the collision,



(A) the ring has pure rotation about its stationary CM
(B) the ring comes to a complete stop
(C) friction between the ring and the ground is to the left
(D) there is no friction between the ring and the ground

7. [IIT-JEE 2009]

A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,

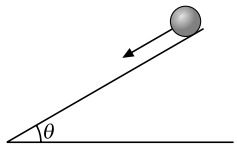


(A) $\vec{V}_C-\vec{V}_A=2\left(\vec{V}_B-\vec{V}_C\right)$ (B) $\vec{V}_C-\vec{V}_B=\vec{V}_B-\vec{V}_A$

(C) $\left|\vec{V}_C-\vec{V}_A\right|=2\left|\vec{V}_B-\vec{V}_C\right|$ (D) $\left|\vec{V}_C-\vec{V}_A\right|=4\left|\vec{V}_B\right|$

8. [IIT-JEE 2006]

A solid sphere is in pure rolling motion on an inclined surface having inclination θ



(A) frictional force acting on sphere is $f=\mu mg\cos\theta$
(B) f is dissipative force
(C) friction will increase its angular velocity and decrease its linear velocity
(D) If θ decreases, friction will decrease

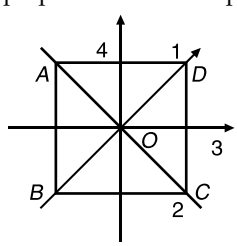
9. [IIT-JEE 1998]

The torque $\vec{\tau}$ on a body about a given point is found to be $\vec{A}\times\vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is angular momentum of the body about that point. From this it follows that

(A) $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instants of time.
(B) the component of $\vec{L}$ in the direction of $\vec{A}$ does not change with time.
(C) the magnitude of $\vec{L}$ does not change with time.
(D) $\vec{L}$ does not change with time.

10. [IIT-JEE 1992]

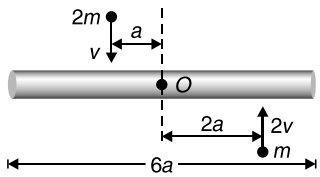
The moment of inertia of a thin square plate $ABCD$ of uniform thickness about an axis passing through the centre O and perpendicular to the plate is



(A) I_1+I_2 (B) I_3+I_4
(C) I_1+I_3 (D) $I_1+I_2+I_3+I_4$

11. [IIT-JEE 1991]

A uniform bar of length $6a$ and mass $8m$ lies on a smooth horizontal table. Two point masses m and $2m$ moving in the same horizontal plane with speeds $2v$ and v, respectively, strike the bar as shown in Figure and stick to it after collision. Denoting angular velocity (about the centre of mass), total energy and the centre of mass velocity by ω, E and V_c respectively, we have after collision



(A) $V_c = 0$ (B) $\omega = \dfrac{3v}{5a}$

(C) $\omega = \dfrac{v}{5a}$ (D) $E = \dfrac{3mv^2}{5}$

12. [IIT-JEE 1990]

A particle of mass m is projected with velocity v making an angle of 45° with the horizontal. The magnitude of angular momentum of the projectile about point of projection when the particle is at maximum height h is

(A) ZERO (B) $\dfrac{mv^3}{4\sqrt{2}g}$

(C) $\dfrac{mv^3}{\sqrt{2}g}$ (D) $m\sqrt{2gh^3}$

Reasoning Based Questions

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.

Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.

Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.

Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. [IIT-JEE 2008]

Statement-1: Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

Statement-2: By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

Linked Comprehension Type Questions

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension I

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating rings rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

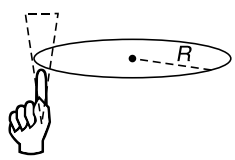


Figure 1

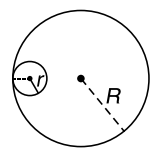


Figure 2

1. [JEE (Advanced) 2017]

The total kinetic energy of the ring is

(A) $M\omega_0^2(R-r)^2$ (B) $\dfrac{1}{2}M\omega_0^2(R-r)^2$

(C) $M\omega_0^2R^2$ (D) $\dfrac{3}{2}M\omega_0^2(R-r^2)$

2. [JEE (Advanced) 2017]

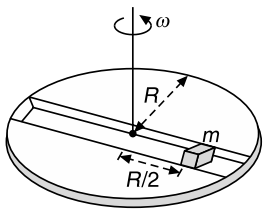
The minimum value of ω_0 below which the ring will drop down is

(A) $\sqrt{\dfrac{g}{2\mu(R-r)}}$ (B) $\sqrt{\dfrac{3g}{2\mu(R-r)}}$

(C) $\sqrt{\dfrac{g}{\mu(R-r)}}$ (D) $\sqrt{\dfrac{2g}{\mu(R-r)}}$

Comprehension 2

A frame of the reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is, $\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{v}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega}$, where, $\vec{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc



Now, consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its centre. We assign a coordinate system with the origin at the centre of the disc, the X-axis along the slot, the Y-axis perpendicular to the slot and the z-axis along the rotation axis $(\omega = \omega\hat{k})$. A small block of mass m is gently placed in the slot at $\vec{r} = \left(\frac{R}{2}\right)\hat{i}$ at $t = 0$ and is constrained to move only along the slot.

3. [JEE (Advanced) 2016]

The distance r of the block at time t is

(A) $\frac{R}{2}\cos 2\omega t$ (B) $\frac{R}{2}\cos \omega t$

(C) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$ (D) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$

4. [JEE (Advanced) 2016]

The net reaction of the disc on the block is

(A) $m\omega^2 R \sin \omega t\hat{j} - mg\hat{k}$

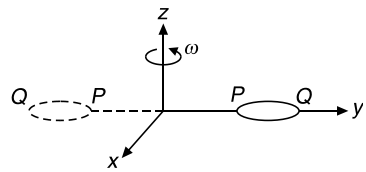
(B) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$

(C) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\hat{k}$

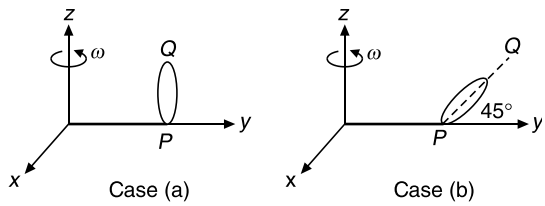
(D) $-m\omega^2 R \cos \omega t\hat{j} - mg\hat{k}$

Comprehension 3

The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in Figure.



When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case. Now, consider two similar systems as shown in Figure : Case (a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P and the systems are rotated with constant angular speed ω about the z-axis.



5. [IIT-JEE 2012]

Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is $\sqrt{2}\omega$ for both the cases

(B) It is ω for case (a) and $\frac{\omega}{\sqrt{2}}$ for case (b)

(C) It is ω for case (a) and $\sqrt{2}\omega$ for case (b)

(D) It is ω for both the cases

6. [IIT-JEE 2012]

Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is vertical for both the cases (a) and (b)

(B) It is vertical for case (a) and is at 45° to the x-z plane and lies in the plane of the disc for case (b)

(C) It is horizontal for case (a) and is at 45° to the x-z plane and is normal to the plane of the disc for case (b)

(D) It is vertical for case (a) and is at 45° to the x-z plane and is normal to the plane of the disc for case (b)

Comprehension 4

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

7. [IIT-JEE 2007]

The ratio $\frac{x_1}{x_2}$ is

(A) 2 (B) $\frac{1}{2}$

(C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

8. [IIT-JEE 2007]

When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

(A) $\frac{2I\omega}{3t}$ (B) $\frac{9I\omega}{2t}$

(C) $\frac{9I\omega}{4t}$ (D) $\frac{3I\omega}{2t}$

9. [IIT-JEE 2007]

The loss of kinetic energy during the above process is

(A) $\frac{I\omega^2}{2}$ (B) $\frac{I\omega^2}{3}$

(C) $\frac{I\omega^2}{4}$ (D) $\frac{I\omega^2}{6}$

Integer/Numerical Answer Type Questions

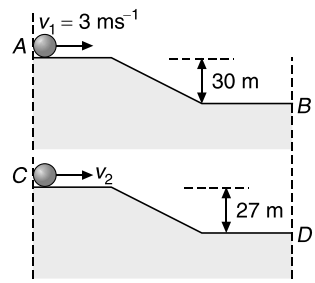
In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

1. [JEE (Advanced) 2020]

Put a uniform meter scale horizontally on your extended index fingers with the left one at 0.00 cm and the right one at 90.00 cm. When you attempt to move both the fingers slowly towards the centre, initially only the left finger slips with respect to the scale and the right finger dos not. After some distance, the left finger stops and the right one starts slipping. Then the right finger stops at a distance x_R from the centre (50.00 cm) of the scale and the left one starts slipping again. This happens because of the difference in the frictional forces on the two fingers. If the coefficients of static and dynamic friction between the fingers and the scale are 0.40 and 0.32, respectively, the value of x_R (in cm) is ……

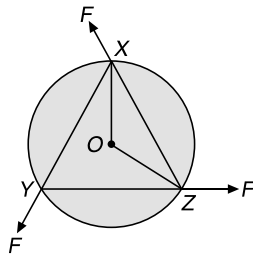
2. [JEE (Advanced) 2015]

Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ ms^{-1}, then v_2 in ms^{-1} is $\left(g = 10 \text{ ms}^{-2}\right)$



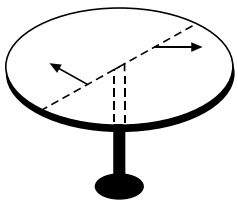
3. [IIT-JEE 2014]

A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude $F = 0.5$ N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rads^{-1} is



4. [JEE (Advanced) 2014]

A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rads^{-1} after the balls leave the platform is

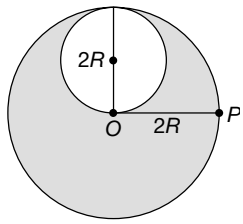


5. [JEE (Advanced) 2013]

A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rads^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rads^{-1}) of the system is

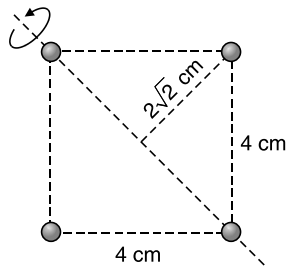
6. [IIT-JEE 2012]

A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in Figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. Find the ratio $\frac{I_P}{I_O}$ to the nearest integer.



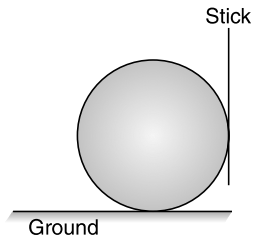
7. [IIT-JEE 2011]

Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kgm^2, find N.



8. [JEE (Advanced) 2011]

A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in Figure.



The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 ms^{-2}. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is $\left(\frac{P}{10}\right)$. The value of P is

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Moment of Inertia and Applications)

1. $23\ \text{kgm}^2, 45\ \text{kgm}^2, 27\ \text{kgm}^2, 0$
2. $34.6\ \text{kgm}^2, 23\ \text{kgm}^2$
3. $\frac{ml^2}{3}$
4. (a) $2ML^2$, (b) $\frac{1}{3}ML^2$
5. $\frac{\pi^2}{6}$
6. $m(a^2-b^2)$
7. (a) $8md^2$, (b) $4m\ell^2$
8. $\sigma\pi R^3(2h+R)$
9. $14mR^2$
10. $\frac{2}{5}M\left(\frac{a^5-b^5}{a^3-b^3}\right)$
11. $\frac{1}{2}M\left(\frac{R^4-a^4-2a^2b^2}{R^2-a^2}\right)$
12. $MR^2\left(\frac{1}{2}-\frac{16}{9\pi^2}\right)$
13. $\frac{a^2}{4}(m_1+m_2)$
14. $\frac{R}{\sqrt{2}}$
15. $\frac{2MR^2}{3}$
16. $\frac{11}{12}mL^2$
17. $\frac{m\ell^2}{2}, \frac{\ell}{\sqrt{6}}$

Test Your Concepts-II (Based on Rotational Kinematics, Combined Effect of Rotation and Translation Motion)

1. (a) $4\ \text{rads}^{-1}, 28\ \text{rads}^{-1}$ (b) $12\ \text{rads}^{-2}$ (c) $6\ \text{rads}^{-2}, 18\ \text{rads}^{-2}$
2. $\vec{a}_1=(a+R\omega^2)\hat{i}+(R\alpha)\hat{j}$, $\vec{a}_2=(a+R\alpha)\hat{i}-(R\omega^2)\hat{j}$, $\vec{a}_3=(a-R\omega^2)\hat{i}-(R\alpha)\hat{j}$, $\vec{a}_4=(a-R\alpha)\hat{i}+(R\omega^2)\hat{j}$
3. $\left(1.2\hat{i}-4.4\hat{j}\right)\ \text{ms}^{-2}$
4. (a) $4\pi\ \text{rads}^{-1}$, (b) 8π rad, (c) 4
5. $\frac{\omega l}{2}\cos\theta$
6. (a) $20.2\ \text{ms}^{-2}$, (b) 0.3 rev
7. $4\ \text{cms}^{-1}, 1.6\ \text{cms}^{-2}$, towards centre C
8. $\frac{r}{R+r}$
9. $v_0t-R\sin(\omega_0t), R-R\cos(\omega_0t)$
10. v
11. $2\hat{i}+3\hat{i}-4\hat{j}=\left(5\hat{i}-4\hat{j}\right)\ \text{ms}^{-1}$

Test Your Concepts-III (Based on Instantaneous Axis of Rotation, Pure Rolling and Conservation of Energy)

1. $2.7R, 2.5R$
2. $\frac{5}{7}h$
3. $\frac{17}{8}mv^2$
4. $\sqrt{\frac{10g(R+r)}{17r^2}}$
5. $\sqrt{\frac{8}{3}gR}$
6. $\sqrt{\frac{3g}{2\ell}}$
7. $\sqrt{\frac{4mg}{(2m+M)R}}$
8. $\frac{3v}{l}, \frac{v}{2}, \frac{1}{2}mv^2$
9. $2.5a$
10. 0.6 m
11. $R\omega_0\sqrt{\frac{3m}{2k}}$
12. $0.745\ \text{ms}^{-1}$

Test Your Concepts-IV (Based on Torque and Applications)

1. $9\sqrt{2}dm$
2. $\sqrt{\frac{6F\sin\phi}{m\ell}}$
3. $2.73\ \text{ms}^{-1}, 27.3\ \text{rads}^{-1}, 233.5\ \text{N}, 238.2\ \text{N}, 1.47\ \text{s}$

4. $\frac{mg}{4}(1-3\cos\theta)^2$, $\frac{3}{2}mg\sin\theta\left(\frac{3}{2}\cos\theta-1\right)$, Yes
5. $\frac{2(m_2-m_1)g}{(2m_1+2m_2+m)R}$, $\frac{m_1(m+4m_2)}{m_2(m+4m_1)}$
6. (a) $Mg\tan\theta$, (b) 0
7. $4\sqrt{\frac{2g}{a}}$
8. $\frac{2m_2g\sin\theta}{2m_2+m_1}$, $\frac{m_1m_2g\sin\theta}{2m_2+m_1}$, mgh, mgh, $\sqrt{\frac{2gh}{1+\frac{m_1}{2m_2}}}$
9. $\frac{3g\sin\theta}{8-3\cos\theta}$
10. (a) $\frac{3g}{2}$, (b) $\frac{3g}{4}$, (c) $\frac{mg}{4}$
11. $\frac{2g}{3}$, $\frac{Mg}{3}$
12. $\frac{32g}{a}$
13. (a) 8.16 rads^{-1}, (b) 8 rads^{-1}
14. $\frac{2g}{5R}$, $\frac{4g}{5}$, $\frac{mg}{5}$
15. $\sqrt{\frac{6g\sqrt{3}}{11r}}$, $\frac{3g}{11r}$
16. $\frac{\ell}{2\sqrt{3}}$, $\frac{g\sqrt{3}}{\ell}$

Test Your Concepts-V (Based on Uniform and Accelerated Pure Rolling)

1. $\frac{F}{M}-\mu_kg$, $\frac{2}{R}\left(\frac{F}{M}-\mu_kg\right)$
2. $\frac{g\sin\theta}{1+\frac{I}{mr^2}}$
3. $\frac{4}{11}g$, $\frac{4gt}{11}$
4. $\frac{F}{2M}$, $\frac{F}{2}$
5. $0.79Mg$
6. (a) 5.3 ms^{-1}, 5.1 ms^{-1}, 4.43 ms^{-1}
   (b) 4.2 N, 4.9 N, 7.4 N
   (c) 1.5 s, 1.6 s, 1.8 s
7. $\frac{3\mu MgR}{R+2b}$
8. $\frac{4F\cos\theta}{3M+8m}$, $\frac{3MF\cos\theta}{3M+8m}$ and $\frac{MF\cos\theta}{3M+8m}$
9. 1.4 ms^{-2}
10. $\frac{F}{3m+M}$
11. $\frac{2}{7}mg\sin\theta$, $\frac{mg}{7}(17\cos\theta-10)+\frac{mv_0^2}{(R-r)}$

Test Your Concepts-VI (Based on Angular Momentum and its Conservation)

1. $\sqrt{2g\ell\sec\theta}$
2. (a) $\frac{2v}{9L}$, (b) $\frac{1}{9}$
3. $\frac{I\omega_0}{I+mR^2}$
4. $\frac{\omega}{2}$
5. $2\sqrt{\frac{g}{r}}$, $2m\sqrt{gr}$
6. (a) $\left(1+\frac{2m}{M}\right)\omega_0$, (b) $\frac{1}{2}m\omega_0^2R^2\left(1+\frac{2m}{M}\right)$
7. $\sqrt{2gr\sec\theta_0}$
8. $\frac{4m\ell\omega}{3}$
9. $\frac{\omega_0R}{6}$, $\frac{\omega_0}{6}$
10. $\left(\frac{2m_1}{m_2+2m_1}\right)\theta$
11. $\frac{v_1}{R\left(1+\frac{I}{MR^2}\right)}$
12. $\frac{9mv_0^2}{2\ell}$
13. Linear momentum, angular momentum and kinetic energy, $\frac{M\ell^2}{12d^2+\ell^2}$
14. $\frac{3J}{7ma}$, $J>2m\sqrt{\frac{7ga}{3}}$
15. $\frac{\omega_0r}{\sqrt{1+\frac{mr^2}{I}}}$

Test Your Concepts-VII (Based on Rolling with Slipping)

1. $\frac{2\omega_0^2 r^2}{81\mu g}$
2. $\frac{2v_0}{R}$
3. $\frac{\omega_0 R}{3\mu g}, -\frac{1}{6}m\omega_0^2 R^2$
4. $\frac{12v_0^2}{49\mu_k g}$
5. $\frac{\omega_0 R}{g} + 6\sqrt{\frac{R}{g}}$
6. $\frac{5v_0}{2R}, \frac{v_0}{\mu g}, \frac{v_0^2}{2\mu g}$
7. $\frac{12v_0^2}{49\mu_k g}$
8. $\frac{3F}{3M+m}, \frac{F}{3M+m}, \frac{F}{R(3M+m)}$
9. $\frac{2}{7}\left(\frac{v_0 + R\omega_0}{\mu g}\right), \frac{1}{7}(5v_0 + 2R\omega_0)$
10. $\frac{r^2\omega_0^2(\mu\cos\alpha - \sin\alpha)}{2g(3\mu\cos\alpha - \sin\alpha)^2}, \frac{r^2\omega_0^2(\mu\cos\alpha - \sin\alpha)}{4g\sin\alpha(3\mu\cos\alpha - \sin\alpha)}$

Single Correct Choice Type Questions

1. C	**2.** A	**3.** C	**4.** C	**5.** B	**6.** B	**7.** B	**8.** B	**9.** B	**10.** D
11. D	**12.** B	**13.** C	**14.** B	**15.** D	**16.** A	**17.** C	**18.** D	**19.** C	**20.** C
21. D	**22.** C	**23.** A	**24.** B	**25.** B	**26.** D	**27.** C	**28.** B	**29.** C	**30.** B
31. C	**32.** A	**33.** B	**34.** D	**35.** C	**36.** D	**37.** B	**38.** B	**39.** D	**40.** D
41. B	**42.** D	**43.** A	**44.** D	**45.** B	**46.** B	**47.** D	**48.** D	**49.** B	**50.** C
51. A	**52.** C	**53.** D	**54.** C	**55.** A	**56.** D	**57.** A	**58.** A	**59.** D	**60.** A
61. B	**62.** A	**63.** A	**64.** A	**65.** C	**66.** D	**67.** C	**68.** A	**69.** B	**70.** D
71. A	**72.** B	**73.** C	**74.** D	**75.** D	**76.** D	**77.** C	**78.** A	**79.** C	**80.** B
81. C	**82.** C	**83.** B	**84.** B	**85.** D	**86.** C	**87.** A	**88.** D	**89.** C	**90.** A
91. B	**92.** B	**93.** D	**94.** B	**95.** B	**96.** B	**97.** C	**98.** B	**99.** C	**100.** D
101. B	**102.** B	**103.** D	**104.** C	**105.** B	**106.** A	**107.** D	**108.** D	**109.** A	**110.** C
111. C	**112.** C	**113.** A	**114.** C	**115.** B	**116.** C	**117.** B	**118.** B	**119.** D	**120.** B
121. B	**122.** C	**123.** D	**124.** C	**125.** D	**126.** A	**127.** D	**128.** B	**129.** B	**130.** B
131. B	**132.** A	**133.** B	**134.** D	**135.** B	**136.** D	**137.** A	**138.** B	**139.** B	**140.** A
141. C	**142.** B	**143.** C	**144.** A	**145.** D	**146.** D	**147.** C	**148.** A	**149.** C	**150.** B
151. C	**152.** B	**153.** D	**154.** D	**155.** B	**156.** D	**157.** A	**158.** B	**159.** B	**160.** B
161. B	**162.** D	**163.** B	**164.** B	**165.** B	**166.** C	**167.** B	**168.** B	**169.** C	**170.** D
171. B									

Multiple Correct Choice Type Questions

1. A, B	**2.** A, B, C	**3.** B	**4.** A, B, D	**5.** A, C, D
6. B, D	**7.** B, C	**8.** A, B, C	**9.** A, C	**10.** B, D
11. C, D	**12.** A, C	**13.** A, C	**14.** A, C	**15.** A, B, C, D
16. C, D	**17.** A, B, C, D	**18.** C, D	**19.** B, C	**20.** B, D
21. A, C, D	**22.** B, D	**23.** B, C, D	**24.** B, D	**25.** A, D
26. A, B, C, D	**27.** A, B	**28.** A, D	**29.** A, B, C	**30.** A, C
31. A, C	**32.** B, D	**33.** A, B, C, D	**34.** A, D	**35.** A, B, D

36. A, C	**37.** A, D	**38.** A, B	**39.** B, D	**40.** A, C
41. A, B, D	**42.** A, B, D	**43.** B, D	**44.** A, B, C	

Reasoning Based Questions

1. A	**2.** B	**3.** D	**4.** B	**5.** D	**6.** C	**7.** A	**8.** A	**9.** B	**10.** D
11. A	**12.** A	**13.** B	**14.** A	**15.** A					

Linked Comprehension Type Questions

1. C	**2.** B	**3.** D	**4.** B	**5.** C	**6.** C	**7.** B	**8.** B	**9.** A	**10.** D
11. D	**12.** C	**13.** B	**14.** C	**15.** D	**16.** B	**17.** D	**18.** C	**19.** C	**20.** C
21. D	**22.** C	**23.** B	**24.** D	**25.** B	**26.** C	**27.** C	**28.** A	**29.** D	**30.** B
31. A	**32.** C	**33.** B	**34.** B	**35.** A	**36.** C	**37.** D	**38.** C	**39.** B	**40.** D
41. B	**42.** A	**43.** D	**44.** A	**45.** D	**46.** C	**47.** A	**48.** B	**49.** A	**50.** C
51. B	**52.** C	**53.** C	**54.** D	**55.** D	**56.** B	**57.** B	**58.** A	**59.** C	**60.** B
61. A	**62.** C	**63.** C	**64.** A	**65.** C	**66.** A	**67.** A	**68.** B	**69.** A	**70.** C
71. A	**72.** B	**73.** A	**74.** C	**75.** B	**76.** A	**77.** C	**78.** B	**79.** B	**80.** A
81. D	**82.** C	**83.** B	**84.** C	**85.** D	**86.** C	**87.** B	**88.** C	**89.** A	**90.** B

Matrix Match/Column Match Type Questions

1.	A → (r)	B → (p)	C → (p)	D → (s)
2.	A → (q, r)	B → (p)	C → (s)	D → (q)
3.	A → (r, s)	B → (p, q)	C → (s)	D → (p, q, r, s)
4.	A → (s)	B → (q)	C → (p)	D → (r)
5.	A → (q)	B → (p)	C → (s)	D → (r)
6.	A → (r)	B → (q)	C → (t)	D → (p)
7.	A → (s)	B → (r)	C → (q)	D → (p)
8.	A → (p, s)	B → (p, r)	C → (q, r)	D → (p, r)
9.	A → (q)	B → (s)	C → (q)	D → (r)
10.	A → (p, q, r)	B → (p, q, r)	C → (p, q, r)	D → (s)
11.	A → (r)	B → (p)	C → (p)	D → (q)
12.	A → (p)	B → (s)	C → (p)	D → (r)
13.	A → (q)	B → (r)	C → (p)	D → (r)
14.	A → (t)	B → (s)	C → (p)	D → (p)
15.	A → (q)	B → (s)	C → (q)	D → (p)
16.	A → (p, r, s)	B → (q, r, s)	C → (q, r, s)	D → (p, r, s)
17.	A → (q)	B → (s)	C → (p)	D → (s)
18.	A → (p, r)	B → (p, r)	C → (q, s)	D → (p, s)
19.	A → (s)	B → (p)	C → (p)	D → (q, r)
20.	A → (q)	B → (r)	C → (p)	D → (s)
21.	A → (q)	B → (s)	C → (s)	D → (p)

Integer/Numerical Answer Type Questions

1. 5	**2.** 3	**3.** 75	**4.** 40	**5.** 9
6. 20, 8	**7.** 15	**8.** 2.25	**9.** 5	**10.** 1
11. 20	**12.** (a) 36, (b) 6	**13.** (a) 4, (b) 5, (c) 50	**14.** 9	**15.** (a) 1, (b) 5, (c) 1
16. 1.5	**17.** 100, 3, 30, 1	**18.** (a) 4, 40 (b) 500, 180 (c) 1	**19.** 20	**20.** 40
21. 3	**22.** 11	**23.** 4	**24.** 4, 8	**25.** 2
26. 25	**27.** (a) 4, (b) 90	**28.** 8, 8, 4	**29.** (a) 1.37, (b) 131, (c) 206	**30.** 20
31. 251.43	**32.** 0.7	**33.** 195	**34.** (a) 14.3, (b) 40, 181 (c) 141	**35.** 6

ARCHIVE: JEE MAIN

1. D	**2.** D	**3.** D	**4.** B	**5.** B	**6.** B	**7.** B	**8.** C	**9.** A	**10.** B
11. D	**12.** C	**13.** B	**14.** A	**15.** A	**16.** A	**17.** B	**18.** D	**19.** D	**20.** D
21. A	**22.** Bonus	**23.** C	**24.** B	**25.** D	**26.** C	**27.** B	**28.** D	**29.** C	**30.** A
31. D	**32.** B	**33.** D	**34.** B	**35.** C	**36.** D	**37.** B	**38.** A	**39.** A	**40.** B
41. A	**42.** C	**43.** C	**44.** D	**45.** D	**46.** A	**47.** A	**48.** D	**49.** D	**50.** D
51. A	**52.** B	**53.** A	**54.** A	**55.** A	**56.** A	**57.** D	**58.** A, C	**59.** B	**60.** B
61. A	**62.** C	**63.** C	**64.** C	**65.** C	**66.** C	**67.** B	**68.** C	**69.** B	**70.** C
71. D	**72.** D								

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. A	**2.** D	**3.** C	**4.** D	**5.** A	**6.** B	**7.** B	**8.** D	**9.** A	**10.** D
11. A	**12.** D	**13.** B	**14.** A	**15.** A	**16.** A	**17.** C	**18.** B	**19.** A	**20.** D
21. C	**22.** B	**23.** C	**24.** B	**25.** A	**26.** A	**27.** B	**28.** C	**29.** A	**30.** C

Multiple Correct Choice Type Problems

1. A, C, B	**2.** B, C, D	**3.** A, C, D	**4.** A, C	**5.** A, B
6. A, C	**7.** B, C	**8.** C, D	**9.** A, B, C	**10.** A, B, C
11. A, C, D	**12.** B, D			

Reasoning Based Questions

1. D

Linked Comprehension Type Questions

1. A	**2.** C	**3.** C	**4.** B	**5.** D	**6.** A	**7.** C	**8.** A	**9.** B

Integer/Numerical Answer Type Questions

1. 25, 6	**2.** 7	**3.** 2	**4.** 4	**5.** 8	**6.** 3	**7.** 9	**8.** 4

CHAPTER

4 Gravitation and Satellites

Learning Objectives

After reading this chapter, you will be able to understand concepts and problems based on:

(a) Newton's Laws of Gravitation
(b) Principle of Superposition
(c) Gravitational Field Strength
(d) Acceleration Due to Gravity and its Variation
(e) Gravitational Potential Energy
(f) Gravitational Self Energy
(g) Gravitational Potential
(h) Relation Between Gravitational Field and Gravitational Potential
(i) Conservation Laws for Gravitational Systems
(j) Escape Speed
(k) Satellites and Kepler's Laws
(l) Double Star System or Binary Star System

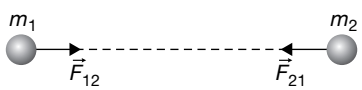
All this is followed by a variety of Exercise Sets (fully solved) which contain questions as per the latest JEE pattern. At the end of Exercise Sets, a collection of problems asked previously in JEE (Main and Advanced) are also given.

NEWTON'S LAW OF GRAVITATION

The force of gravitation between any two particles having masses m_1 and m_2 separated by a distance r is attractive and acts along the line joining the particles. The magnitude of the force is given by

$$F = \frac{Gm_1m_2}{r^2} \qquad \ldots(1)$$

where G is a universal constant called **Gravitational Constant** having the same value for all pairs of particles. Its numerical value is $6.67 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$.



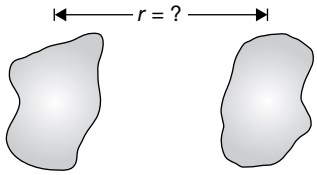
The gravitational forces between two particles always form **an action-reaction pair.** The first particle exerts a force on the second particle that is directed toward the first particle along the line joining the two.

Similarly, the second particle exerts a force on the first particle that is directed towards the second particle along the line joining the two. These forces are equal in magnitude but oppositely directed.
In the vector form, equation (1) is written as

$$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r} \qquad \ldots(2)$$

where we must remember that $\hat{r}$ **has its origin at the source of the force.**

It is important to realize that Newton's Law of Gravitation is stated only for point particles. For two arbitrary bodies, as shown in figure, there is no unique value for the separation r. So, the equation $F = \frac{Gm_1m_2}{r^2}$ cannot be used for arbitrary bodies.



To compute the force between them requires integral calculus. However, for the special case of a uniform spherical mass distribution, r may be taken as the distance to the center. Also, when the separation between two objects is very much larger than their sizes, they may be approximated as point masses and then, equation may be used.

Problem Solving Technique(s)

(a) The gravitational force between two particles is independent of the presence of other bodies or the properties of the intervening medium.

(b) Gravitational force is conservative force, therefore work done in displacing a body from one place to another is independent of path. It depends only on initial and final positions.

(c) The mass of air bubble in material medium is negative.

(d) The gravitational force obeys Newton's Third Law i.e.

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\Rightarrow \quad m_1 a_1 = m_2 a_2$$

(e) Gravitational force is a central force.

PROPERTIES OF *G*

(a) Its value does not depend upon place, time and masses of the bodies and hence it is called a **Universal Constant**.

(b) The value of G is $6.67 \times 10^{-11}\ \text{Nm}^2\,\text{kg}^{-2}$

(c) Its dimensional formula is $M^{-1}L^3T^{-2}$.

(d) Its value is extremely small. Hence, we do not experience it in our daily life.

(e) G is numerically equal to gravitational force of potential between two bodies each of mass 1 kg placed at a distance of 1 m.

ILLUSTRATION 1

Spheres of the same material and same radius r are touching each other. Show that gravitational force between them is directly proportional to r^4.

SOLUTION

Since, $m_1 = m_2 = (\text{volume})(\text{density})$

$$\Rightarrow \quad m_1 = m_2 = \left(\frac{4}{3}\pi r^3\right)\rho$$

$$\Rightarrow \quad F = \frac{Gm_1m_2}{r^2}$$

$$\Rightarrow \quad F = \frac{G\left(\frac{4}{3}\pi r^3\right)\left(\frac{4}{3}\pi r^3\right)\rho^2}{r^2}$$

$$\Rightarrow \quad F \propto r^4$$

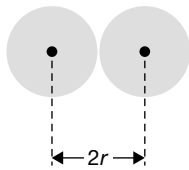
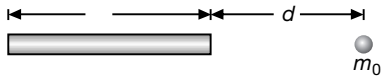


ILLUSTRATION 2

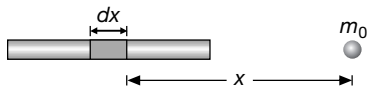
The figure shows a uniform rod of length l whose mass per unit length is λ and the total mass is M. What is the gravitational force of the rod on a particle of mass m_0 located a distance d from one end of the rod, as shown in the figure?



SOLUTION

Consider an infinitesimal element on the rod having length dx at a distance x from the point mass m_0. If dF is the force between the infinitesimal element of mass dm and m_0, then

$$dF = \frac{Gm_0\,dm}{x^2}, \text{ where } dm = \lambda dx$$



To obtain the total force on m_0 due to the rod, we integrate the above expression for force within appropriate limits. So,

$$F = \int_d^{d+l} \frac{G(\lambda dx)m_0}{x^2} = G\lambda m_0\left(\frac{1}{d} - \frac{1}{d+l}\right) = \frac{Gm_0(\lambda l)}{d(d+l)}$$

$$\Rightarrow \quad F = \frac{Gm_0M}{d(d+l)}$$

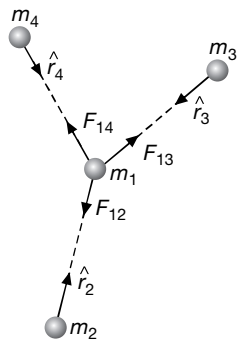
PRINCIPLE OF SUPERPOSITION

Experiments show that when several particles interact, the force between a given pair is independent of the other particles present.

The net force F_1 on the point mass m_1 due to the other particles as shown in the figure is found by first calculating its potential with each of the other particles one at a time.

The **net force** on m_1 is the vector sum of the pair-wise potentials.

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots\ldots + \vec{F}_{1n} \qquad \ldots(3)$$



The net force on m_1 is

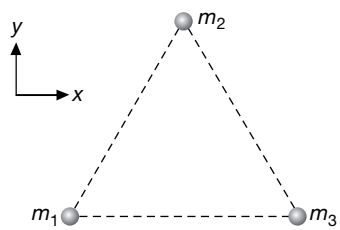
$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

Problem Solving Technique(s)

1. Set up a convenient coordinate system
2. Indicate the directions of the forces acting on the particle under consideration.
3. Calculate the (scalar) magnitudes of the forces.
4. Find the net force by using the component method.

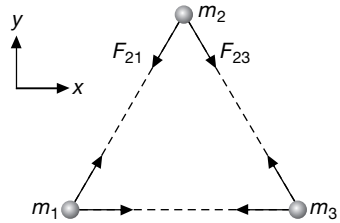
ILLUSTRATION 3

Three point particles with masses $m_1 = 4$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg are at the corners of an equilateral triangle of side $L = 2$ m, as shown in figure. Find the net force on m_2.



SOLUTION

The first two steps have already been done in the figure. The magnitudes of forces are



$$F_{21} = \frac{Gm_2m_1}{L^2} = 1.33\times10^{-10} \text{ N}$$

$$F_{23} = \frac{Gm_2m_3}{L^2} = 1.01\times10^{-10} \text{ N}$$

The net force on m_2 is $\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23}$

Its components are

$$F_{2x} = -F_{21}\cos 60° + F_{23}\cos 60° = -1.6\times10^{-11} \text{ N}$$

$$F_{2y} = -F_{21}\sin 60° - F_{23}\sin 60° = -2.03\times10^{-10} \text{ N}$$

Thus, $F_2 = -\left(1.6\hat{i} + 20.3\hat{j}\right)\times10^{-11}$ N

ILLUSTRATION 4

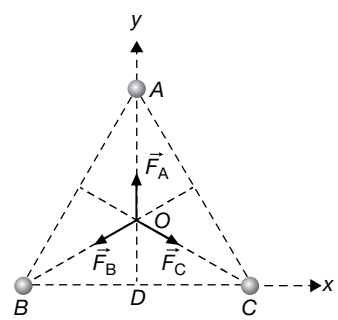
Three particles each of mass m are placed at the three corners of an equilateral triangle of side a. Find the force exerted by this system on another particle of mass m placed at

(a) the centre of the triangle and
(b) mid-point of a side.

SOLUTION

To solve the above problem, we apply the gravitational potential which follow the principle of superposition.

(a) When another mass m is placed at O, it experiences three forces $\vec{F}_A$, $\vec{F}_B$ and $\vec{F}_C$. Since AO, BO and CO are equal hence $|\vec{F}_A| = |\vec{F}_B| = |\vec{F}_C|$. Angle between any two forces is same i.e. $120°$. Therefore, the resultant force exerted by the system on particle at O is zero.



(b) In this case the particle is placed at point D, which is equidistant from B and C.

$$|\vec{F}_B| = |\vec{F}_C|$$

But they are opposite in direction. Therefore, the effective force at D will be due to mass m at A.

By geometry of the figure $AO = a\sin 60 = \dfrac{\sqrt{3}a}{2}$

Therefore, $F_A = \dfrac{4Gm^2}{3a^2}$ along DA

GRAVITATIONAL FIELD STRENGTH (E_g)

Every mass particle is surrounded by a space within which its influence can be felt. This region or space is said to be occupied with **gravitational field**.

Each point in the field is associated with a (vector) force which is experienced by a unit mass placed at that point and is called the **gravitational field strength**.

The region of space around a body in which its gravitational influence is experienced by other bodies is the gravitational field of that body.

OR

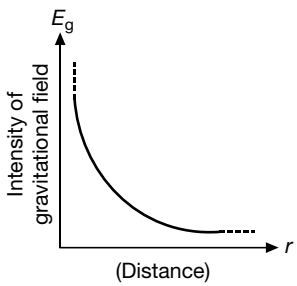
The gravitational force of attraction acting on a body of unit mass at any point in the gravitational field is defined as the intensity of gravitational field $\left(\vec{E}_g\right)$ at that point.

OR

The gravitational field is also defined as the force per unit test mass $\left(\dfrac{F}{m_0}\right)$ placed in the gravitational influence of source mass. Mathematically,

$$E_g = \frac{F}{m_0} = \frac{Gm}{r^2}$$

where m = Source Mass, m_0 = Test Mass.



Its SI unit is newton/kg $\left(\text{Nkg}^{-1}\right)$ and its dimensional formula is LT^{-2}. **It is a vector quantity and its direction is always towards the centre of the body producing the field.** The value of E_g is zero as $r \to \infty$. The graph between E_g and r.

Problem Solving Technique(s)

If a system has got a number of masses, then the total gravitational field at a point *P* due to all the masses is found in accordance with the Principle of Superposition. So,

$$\vec{E}_g = \vec{E}_{g_1} + \vec{E}_{g_2} + \vec{E}_{g_3} + \ldots + \vec{E}_{g_n}$$

i.e. net field at a point *P* is the vector sum of the fields due to individual masses (each calculated as if others were absent) at the point of consideration.

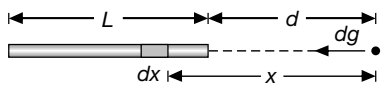
ILLUSTRATION 5

Find the field strength at a point along the axis of a thin rod of length L and mass M, at a distance d from one end.

SOLUTION

First we need to find the field due to an element of length dx. The rod must be thin if we are to assume that all points of the element are at the same distance from the field point. The mass of the element is $dm = \left(\dfrac{M}{L}\right)dx$, so contribution to the field is

$$dg = \frac{G(\lambda dx)}{x^2} = \frac{GM}{L}\frac{dx}{x^2}$$



The total field strength is

$$g = \frac{GM}{L}\int_d^{L+d}\frac{dx}{x^2} = \frac{GM}{L}\left(\frac{1}{d} - \frac{1}{L+d}\right) = \frac{GM}{d(L+d)}$$

Notice that when $d \quad L$, we find $g \to \dfrac{GM}{d^2}$, the result for a point particle.

Problem Solving Technique(s)

(a) For calculating the gravitational field at a point P at distance r due to a source mass m we proceed as follows.

STEP-1: Place a test mass of 1 kg at point P.

STEP-2: Calculate the force between m and 1 kg.

STEP-3: This force is the value of E_g due to m at the point P.

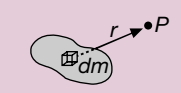
STEP-4: The direction of force experienced by the 1 kg mass is the direction of E_g due to m at the point P.

(b) If we are to find the gravitational field due to assembly of masses m_1, m_2, ..., m_n at a point P, then we apply Superposition Principle and then

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n$$

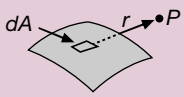
where we find $\vec{E}_1$, $\vec{E}_2$, ... $\vec{E}_n$ at point P by the technique mentioned above.

(c) If we are to find the gravitational field at a point P due to a uniform mass distribution, then we calculate the field due to an infinitesimal element of the distribution at the point P and then integrate it within appropriate limits.

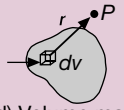


(a) Continuous distribution

(b) Linear mass density, λ



(c) Surface mass density, σ



(d) Volume mass density, ρ

$$E = \int dE = G\int \frac{dm}{r^2}$$

For mass distributed on a wire with linear mass density λ,

$$E_g = G\int_{\ell} \frac{\lambda d\ell}{r^2}$$

For mass distributed on a surface with surface mass density σ,

$$E_g = G\int_{A} \frac{\sigma dA}{r^2}$$

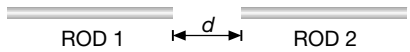
For mass distributed on a volume with volume mass density ρ,

$$E_g = G\int_{V} \frac{\rho dV}{r^2}$$

If the mass distributions are uniform, then mass densities can be taken out of the integral to get desired results.

ILLUSTRATION 6

Two identical uniform rods of length l and mass M are placed along the same line so that their closer ends are a distance d apart, as shown in the figure.



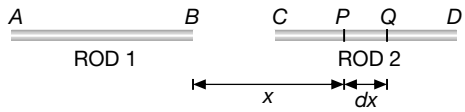
(a) Find the gravitational force of attraction between the rods.

(b) Show that $\lim_{d\to\infty} F = G\frac{M^2}{d^2}$.

SOLUTION

(a) The gravitational field due to thin rod at a distance x from one of its end is

$$E = \frac{GM}{x(x+l)}$$



Now consider an infinitesimal element of mass dm at a distance x from end B of rod 1. Then force on this element is

$$dF = (dm)E = \frac{\left(\frac{M}{l}dx\right)GM}{x(x+l)}$$

$$\Rightarrow\ dF = \frac{GM^2}{l}\frac{dx}{x(x+l)} = \frac{GM^2}{l^2}\left[\frac{1}{x} - \frac{1}{x+l}\right]dx$$

$$\Rightarrow\ F = \int_{d}^{d+l} dF = \frac{GM^2}{l^2}\left[\log_e\left(\frac{x}{x+l}\right)\right]_{d}^{d+l}$$

$$\Rightarrow \quad F = \frac{GM^2}{l^2}\log_e\left[\frac{(d+l)^2}{d(d+2l)}\right]$$

(b) The above expression can be written as,

$$F = \frac{GM^2}{l^2}\log_e\left[\frac{\left(1+\frac{l}{d}\right)^2}{\left(1+\frac{2l}{d}\right)}\right]$$

$$\Rightarrow \quad F = \frac{GM^2}{l^2}\log_e\left[\left(1+\frac{l^2}{d^2}+\frac{2l}{d}\right)\left(1-\frac{2l}{d}\right)\right]$$

$$\Rightarrow \quad F = \frac{GM^2}{l^2}\log_e\left(1+\frac{l^2}{d^2}\right)$$

$$\left\{\text{neglecting the higher terms of } \frac{l}{d}\right\}$$

$$\Rightarrow \quad F \approx \frac{GM^2}{l^2}\left(\frac{l^2}{d^2}\right) \qquad \left\{\log_e\left(1+\frac{l^2}{d^2}\right)\approx\frac{l^2}{d^2}\right\}$$

$$\Rightarrow \quad F \approx \frac{GM^2}{d^2}$$

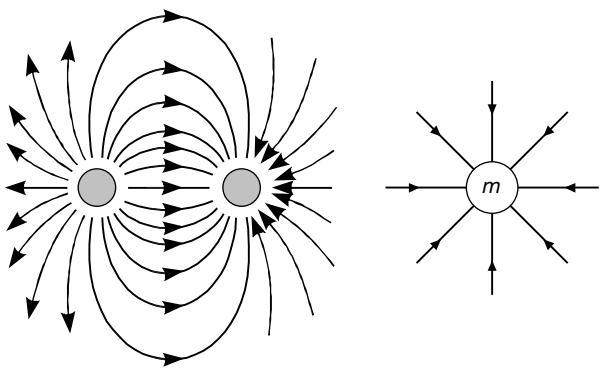
GRAVITATIONAL FIELD LINES

A gravitational field line (or a field line) is a line straight or curved such that a unit mass placed in the field of another mass would always move along this line. Field lines for an isolated mass m are radially inwards.

If two masses are placed close to each other then field lines are shown in figure.

Two field lines never cross each other.

The direction of the field can just be found by drawing a tangent to the field line at the point of consideration.



The Gravitational field lines (for two equal masses).

FIELD DUE TO SPHERE AND SHELL

For an external point $(r > R)$, a sphere (solid or hollow) behaves as if whole of its mass is concentrated at its centre, so the gravitational field outside the sphere/shell is given by

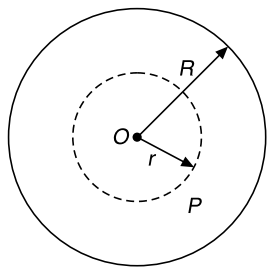
$$E_{\text{outside}} = \frac{GM}{r^2} \qquad \{\text{for } r > R\}$$

In case of a spherical shell, for an internal point, $(r < R)$ the gravitational field inside is zero. So,

$$E_{\text{inside}} = 0$$

In case of a spherical volume distribution of mass (i.e., a solid sphere) for an internal point $(r < R)$, the portion of the sphere that lies outside the radius r will not contribute to the field (because the field inside a spherical shell is zero), so

$$E_{\text{inside}} = \frac{GM'}{r^2}$$



Field inside a solid sphere

where M' = mass of sphere of radius r.

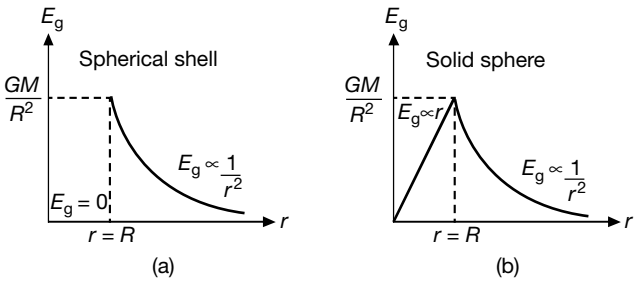
Now if M is the mass of solid sphere, then

$$\rho = \frac{M}{V} = \frac{M}{\left(\frac{4}{3}\right)\pi R^3}$$

and $\quad M' = \frac{4}{3}\pi r^3\rho = \frac{M}{R^3}r^3$

$$\Rightarrow \quad E_{\text{inside}} = \frac{G}{r^2}\left(\frac{M}{R^3}r^3\right) = \frac{GM}{R^3}r$$

i.e., intensity inside a solid sphere varies linearly with distance from the centre. So, it is minimum $(E_{\text{centre}} = 0)$ at the centre and maximum at the surface $\left(E_{\text{surface}} = \frac{GM}{R^2}\right)$. This is shown graphically in the figure.



Problem Solving Technique(s)

Gravitational Field For a Shell

$$E_g = \begin{cases} \dfrac{GM}{r^2} & r \geq R \\ & \text{(outside and at surface)} \\ 0 & r < R \\ & \text{(inside)} \end{cases}$$

Gravitational Field For a Sphere

$$E_g = \begin{cases} \dfrac{GM}{r^2} & r \geq R \\ & \text{(outside and at surface)} \\ \left(\dfrac{GM}{R^3}\right) r & r < R \\ & \text{(inside)} \end{cases}$$

ILLUSTRATION 7

There are two concentric shells of masses M_1 and M_2 and radii R_1 and R_2. Find the force on a particle of mass m when the particle is located at

(i) $r_1 > R_2$

(ii) $R_1 < r_2 < R_2$

(iii) $r_3 < R_1$

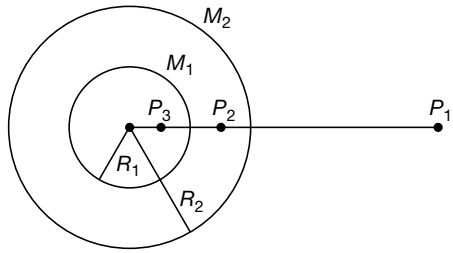
SOLUTION

(i) From the figure, it is clear that the point P_1 lies outside to both the shell. Therefore, gravitational Intensity at the point P_1 is given by

$$E_{P_1} = \frac{G(M_1 + M_2)}{r_1^2}.$$

Therefore, force on the particle of mass m is

$$F = m\left(E_{P_1}\right) = \frac{G(M_1 + M_2)m}{r_1^2}.$$



(ii) When $R_1 < r_2 < R_2$, the point P_2 lies outside the smaller shell but inside the larger shell.
Therefore, Intensity at P_2 is

$$E_{P_2} = \begin{pmatrix} \text{Intensity due to smaller shell} \end{pmatrix} + \begin{pmatrix} \text{Intensity due to larger shell} \end{pmatrix} = \frac{GM_1}{r_2^2} + 0$$

Therefore, force on mass m is $F = mE_{P_2} = \dfrac{GM_1 m}{r_2^2}$

(iii) When $r_3 < R_1$, Point P_3 lies inside to both the shells. The intensity of the field at P_3 is

$$E_{P_3} = \begin{pmatrix} \text{Intensity due to smaller shell} \end{pmatrix} + \begin{pmatrix} \text{Intensity due to larger shell} \end{pmatrix} = 0 + 0 = 0$$

ILLUSTRATION 8

A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 \text{ for } r \leq R \\ 0 \text{ for } r > R \end{cases}$, where ρ_0 is a constant. A test mass m_0 can undergo circular motion under the influence of the gravitational field of particles. Calculate its speed v as a function of distance r from the centre of the system. Also draw a plot of speed versus distance r.

SOLUTION

For $r \leq R$, $\dfrac{m_0 v^2}{r} = \dfrac{Gm_0 M}{r^2}$ …(1)

where, $M = \left(\dfrac{4}{3}\pi r^3\right)\rho_0$

Substituting in Equation (1), we get

$$\frac{v^2}{r} = \frac{G\left(\frac{4}{3}\pi r^3\right)\rho_0}{r^2}$$

$$\Rightarrow \quad v=\sqrt{\frac{4}{3}\pi G\rho_0 r^2}=r\sqrt{\frac{4}{3}\pi G\rho_0}$$

$$\Rightarrow \quad v\propto r$$

i.e. v-r graph is a straight line passing through origin.
For $r>R$, we have

$$M=M_{\text{total}}=\frac{4}{3}\pi R^3\rho_0$$

$$\Rightarrow \quad \frac{m_0v^2}{r}=\frac{Gm_0\left(\frac{4}{3}\pi R^3\right)\rho_0}{r^2}$$

$$v=\sqrt{\frac{4\pi G\rho_0R^3}{3r}}$$

$$\Rightarrow \quad v\propto\frac{1}{\sqrt{r}}$$

The corresponding v-r graph is shown in Figure.

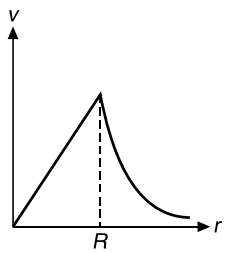
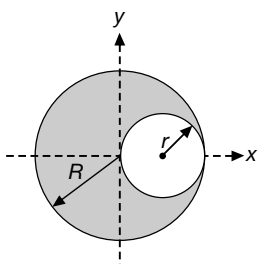


ILLUSTRATION 9

A sphere of radius R has its centre at the origin. It has uniform mass density ρ_0 except that there is a spherical hole of radius $r=\frac{1}{2}R$ whose centre is at $x=\frac{1}{2}R$, as shown in figure. Find the gravitational field at points on the x-axis for $|x|>R$.



SOLUTION

Let, E be the gravitational field at x due to the complete sphere.

If, E_1 be the field due to hole and E_2 be the field due to the remaining portion, then we have

$$E=E_1+E_2$$

$$\Rightarrow \quad E_2=E-E_1$$

$$\Rightarrow \quad E_2=\frac{GM}{x^2}-\frac{Gm}{\left(x-\frac{R}{2}\right)^2} \qquad \ldots(1)$$

where, $M=\frac{4}{3}\pi R^3\rho_0$ and

$$m=\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\rho_0$$

Substituting the values in equation (1), we get

$$E_2=-\left(\frac{\pi G\rho_0R^3}{6}\right)\left[\frac{1}{\left(x-\frac{R}{2}\right)^2}-\frac{8}{x^2}\right]$$

Problem Solving Technique(s)

For **symmetrical mass distributions,** if we are asked to calculate the gravitational field at a point which lies symmetrically with respect to the mass distribution, then we proceed as follows

STEP-1: Consider an infinitesimal element having a mass *dm* (say) that lies at a distance *r* from the point *P*. If *dE* is the gravitational field due to this element then

$$dE=\frac{Gdm}{r^2}$$

STEP-2: Now consider another identical mirror infinitesimal element having same mass and located at the same distance from *P*. Then field due to this element will also be *dE*.

STEP-3: On resolving *dE* due to both the elements along suitable axis we observe that one set of components cancels and the net gravitational field will then be calculated by taking the integral of the component of the gravitational field due to the contribution of the single element.

$$E_{\text{net}}=\int(\text{contribution due to a single element})$$

However, in the case of **unsymmetrical mass distributions**, if we have to calculate the gravitational field at any point P, then we proceed as follows:

STEP-1:
Consider an infinitesimal element having a mass *dm* (say) that lies at a distance *r* from the point *P*. If *dE* is the gravitational field due to this element then

$$dE = \frac{Gdm}{r^2}$$

STEP-2:
Resolve *dE* along the selected axes (say *x* and *y*) so as to get the components dE_x and dE_y.

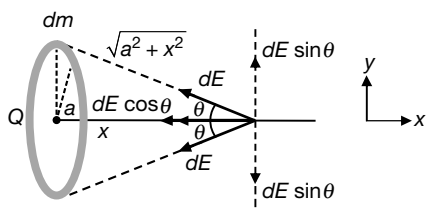
STEP-3:
Integrate dE_x and dE_y separately to get E_x and E_y.

STEP-4:
Then $\vec{E} = E_x\hat{i} + E_y\hat{j}$ such that $|\vec{E}| = E = \sqrt{E_x^2 + E_y^2}$ and if $\vec{E}$ makes an angle β with *x*-axis, then $\tan\beta = \frac{E_y}{E_x}$.

GRAVITATIONAL FIELD AT THE AXIS OF A CIRCULAR UNIFORM RING

Consider a ring of mass M, radius a and uniform linear mass density λ. Then

$$\lambda = \frac{M}{2\pi a} \qquad \ldots(1)$$



Let us calculate the gravitational field due to this ring at a point lying on the axis of the ring at a distance x from the centre of the ring. Since this point is located symmetrically with respect to the ring so let us consider two elements of length dl placed symmetrically on the two diametrically opposite ends. If dE is the field due to each such element, then $dE\sin\theta$ components will cancel out such that the net field is just due to $dE\cos\theta$ components summed up over the entire ring. So,

$$E_{net} = E_x = \int dE\cos\theta = \int \frac{Gdm}{\left(\sqrt{a^2+x^2}\right)^2}\cos\theta$$

$$\Rightarrow \quad E_x = \frac{G\lambda\cos\theta}{(a^2+x^2)}\int_0^{2\pi a} dl = \frac{G\lambda\cos\theta}{(a^2+x^2)}(2\pi a)$$

But from equation (1), we have $\lambda(2\pi a) = M$ and also, we have from the figure, $\cos\theta = \frac{x}{\sqrt{a^2+x^2}}$, so we get

$$\Rightarrow \quad E_x = E_{axis} = \frac{GMx}{(a^2+x^2)^{3/2}} \qquad \text{(along } +x \text{ axis)}$$

Remark(s)

(a) For the point P to lie at far off distance from the centre of the ring i.e., for $x >> a$ we have $(x^2+a^2)^{3/2} \cong x^3$.

$$\Rightarrow \quad E_x \simeq \frac{GM}{x^2}$$

This result under the specified condition just matches with the field due to a point mass at a distance *x* from it.

(b) For this field to be a MAXIMUM $\frac{dE}{dx} = 0$

$$\Rightarrow \quad \frac{d}{dx}\left[x(x^2+a^2)^{-3/2}\right] = 0$$

$$\Rightarrow \quad (x^2+a^2)^{-3/2} + x\left(-\frac{3}{2}\right)(x^2+a^2)^{-5/2}(2x) = 0$$

$$\Rightarrow \quad 1 = \frac{3x^2}{x^2+a^2}$$

$$\Rightarrow \quad x^2 + a^2 = 3x^2$$

$$\Rightarrow \quad x = \pm\frac{a}{\sqrt{2}}$$

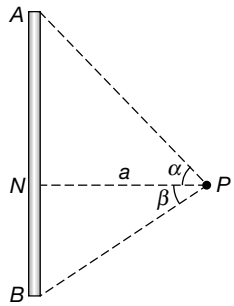
i.e., the field will attain a maximum value at $x = \pm\frac{a}{\sqrt{2}}$ along the axis of the ring and the maximum value equals E_{max} given by

$$E_{max} = \frac{GM(a/\sqrt{2})}{\left(a^2+\frac{a^2}{2}\right)^{3/2}}$$

$$\Rightarrow \quad E_{max} = \frac{2GM}{3\sqrt{3}a^2}$$

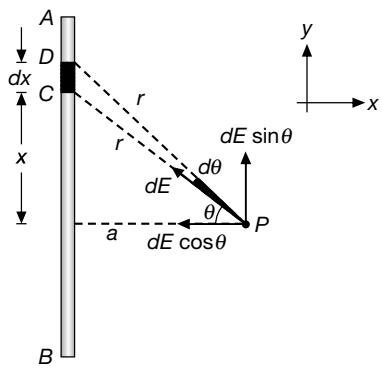
ILLUSTRATION 10

Calculate the gravitational field due to a uniform rod AB at a point P at perpendicular distance a from the rod as shown in figure. Assume that the rod has a linear mass density λ.



SOLUTION

Here it is very important to note the unsymmetrical placement of the point P and hence we must calculate the components of the gravitational fields E_x and E_y separately. For this, let us consider an infinitesimal element of length dx at a distance x as shown.



The net gravitational field at point P due to this infinitesimal element is dE. This dE is resolved into components.

(a) $dE_x = dE\cos\theta$

(b) $dE_y = dE\sin\theta$

The net field E can be calculated by finding E_x and E_y from the above expressions. So

$$E_x = \int dE\cos\theta, \text{ where } dE = \frac{G(\lambda dx)}{r^2}$$

$$\Rightarrow \quad E_x = G\lambda\int\frac{dx\cos\theta}{r^2} = G\lambda\int\frac{dx\cos\theta}{(a^2+x^2)}$$

Also, we observe that

$$\tan\theta = \frac{x}{a}$$

$$\Rightarrow \quad x = a\tan\theta$$

$$\Rightarrow \quad dx = a\sec^2\theta d\theta$$

$$\Rightarrow \quad E_x = G\lambda\int\frac{a\sec^2\theta\cos\theta d\theta}{a^2\sec^2\theta}$$

$$\Rightarrow \quad E_x = \frac{G\lambda}{a}\int_{-\beta}^{\alpha}\cos\theta d\theta$$

$$\Rightarrow \quad E_x = \frac{G\lambda}{a}(\sin\beta+\sin\alpha)$$

Similarly, let us calculate the value of E_y, given by

$$E_y = \int dE_y = \int dE\sin\theta$$

$$\Rightarrow \quad E_y = G\lambda\int\frac{dx\sin\theta}{(a^2+x^2)}$$

Since $x = a\tan\theta$

$$\Rightarrow \quad dx = a\sec^2\theta d\theta$$

$$\Rightarrow \quad E_y = G\lambda\int\frac{a\sec^2\theta\sin\theta d\theta}{a^2\sec^2\theta}$$

$$\Rightarrow \quad E_y = \frac{G\lambda}{a}\int_{-\beta}^{\alpha}\sin\theta d\theta$$

$$\Rightarrow \quad E_y = \frac{G\lambda}{a}\left(-\cos\theta|_{-\beta}^{\alpha}\right)$$

$$\Rightarrow \quad E_y = \frac{G\lambda}{a}(\cos\beta-\cos\alpha)$$

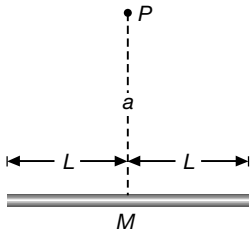
So, the gravitational field due to a rod of length having uniform mass density λ at a point P, that subtends an angle α at one end and β at the other is given by

$$E_x = \frac{G\lambda}{a}(\sin\alpha+\sin\beta) \text{ and}$$

$$E_y = \frac{G\lambda}{a}(\cos\beta-\cos\alpha)$$

ILLUSTRATION 11

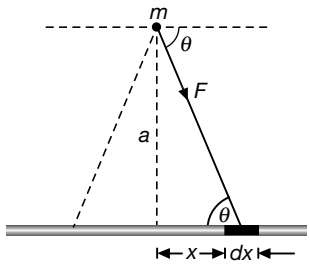
Mass M is distributed uniformly along a line of length $2L$. A particle of mass m_0 is at a point that is a distance a above the centre of the line on its perpendicular bisector (point P in figure).



For the gravitational force that the line exerts on the particle, calculate the components perpendicular and parallel to the line. What happens when $a \quad L$?

SOLUTION

Consider an infinitesimal element of mass dm, as shown in figure.



Then $dm = \left(\frac{M}{2L}\right)dx$

$$dF = \frac{Gm_0 dM}{(a^2 + x^2)}$$

Consider another identical mirror element of the already taken infinitesimal element, then we observe that the components $dF\cos\theta$ (i.e., parallel to line) due to these components cancel each other and the net force will be perpendicular to the rod.

$$F_{\text{net}} = \int \begin{pmatrix} \text{contribution due to} \\ \text{a single element} \end{pmatrix} = \int_{x=-L}^{x=L} dF\sin\theta$$

$$\Rightarrow \quad F_{\text{net}} = \int_{-L}^{L} \frac{Gm_0 M dx}{2L(a^2 + x^2)} \frac{a}{\sqrt{a^2 + x^2}}$$

$$F_{\text{net}} = \frac{GMm_0 a}{2L} \int_{-L}^{L} \frac{dx}{(a^2 + x^2)^{3/2}}$$

$$F_{\text{net}} = \frac{GMm_0}{a\sqrt{L^2 + a^2}}$$

ILLUSTRATION 12

Calculate the gravitational field at the centre of a uniform wire in the form of an arc of radius r. The wire subtends an angle ϕ at the centre.

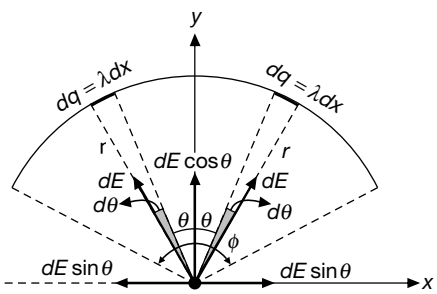
SOLUTION

Consider an element $dm(=\lambda dx)$ of the wire that makes an angle θ with y-axis and subtends an angle $d\theta$ at the centre. Then

$$dm = \lambda(rd\theta)$$

Since, $dE = \frac{Gdm}{r^2} = \frac{\lambda(rd\theta)}{r^2}$

$$\Rightarrow \quad dE = \frac{G\lambda d\theta}{r}$$



Again, take another element which is the mirror image of the element already taken. On resolution, we observe that the x components cancel, while the net field is equal to integral of contribution due to a single element. Hence

$$E = E_y = \int dE\cos\theta$$

$$\Rightarrow \quad E = \int dE\cos\theta$$

$$\Rightarrow \quad E = \frac{G\lambda}{r}\int_{-\frac{\phi}{2}}^{+\frac{\phi}{2}} \cos\theta d\theta$$

$$\Rightarrow \quad E = \frac{G\lambda}{r}\sin\theta\Big|_{-\frac{\phi}{2}}^{\frac{\phi}{2}}$$

$$\Rightarrow \quad E = \frac{G\lambda}{r}\left[\sin\left(\frac{\phi}{2}\right) - \sin\left(-\frac{\phi}{2}\right)\right]$$

$$\Rightarrow \quad E = \frac{G\lambda}{r}2\sin\left(\frac{\phi}{2}\right)$$

$$\Rightarrow \quad E = \frac{2G\lambda}{r}\sin\left(\frac{\phi}{2}\right)$$

Problem Solving Technique(s)

(a) For semi-circle, $\phi = \pi$

So, $E = \dfrac{2G\lambda}{r}$

(b) For Quarter-circle $\phi = \dfrac{\pi}{2}$

$\Rightarrow \quad E = \dfrac{2G\lambda}{r\sqrt{2}}$

(c) For circle, $\phi = 2\pi$

$\Rightarrow \quad E = 0$

ILLUSTRATION 13

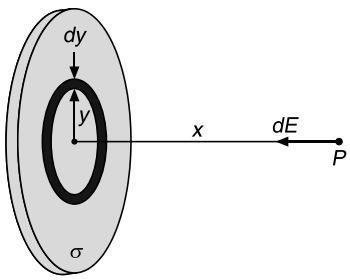
Calculate the gravitational field at point P that lies on the axis of a uniformly disc at a distance x from the centre. Assume the disc to have a radius R and uniform surface mass density σ.

SOLUTION

To find gravitational field at point P due to this disc let us consider an elemental ring of radius y and width dy. If dm and dA be the mass and area of this infinitesimal element, then

$$dm = \sigma dA = \sigma(2\pi y dy)$$

$\{\because dA = 2\pi y\, dy\}$



Now we know that gravitational field strength due to a ring of radius R, mass M, at a distance x from its centre on its axis can be given as

$$E_{\text{axis}} = \frac{GMx}{(x^2 + R^2)^{3/2}} \qquad \text{(Derived earlier)}$$

So, due to the infinitesimal elemental ring the gravitational field strength dE at point P is

$$dE = \frac{G(dm)x}{(x^2 + y^2)^{3/2}}$$

$$\Rightarrow \quad dE = \frac{G\sigma(2\pi y dy)x}{(x^2 + y^2)^{3/2}}$$

Please note that, here once point P is taken, then value of x remains fixed. So,

$$E = E_{\text{axis}} = \int_0^R dE$$

$$\Rightarrow \quad E = \int dE = 2G\sigma\pi x \int_0^R \frac{y dy}{(x^2 + y^2)^{3/2}}$$

$$\Rightarrow \quad E = G\sigma\pi x \int_0^R \frac{2y dy}{(x^2 + y^2)^{3/2}}$$

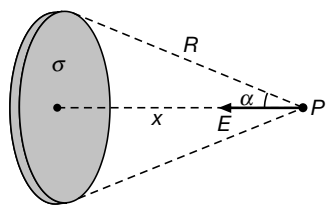
$$\Rightarrow \quad E = G\sigma\pi x \left[-\frac{2}{\sqrt{x^2 + y^2}} \Big|_{y=0}^{y=R} \right]$$

$$\Rightarrow \quad E = 2G\sigma\pi \left(1 - \frac{x}{\sqrt{x^2 + R^2}} \right)$$

Since, $\sigma = \dfrac{M}{\pi R^2}$, so the above expression becomes

$$E = \frac{2GM}{R^2} \left(1 - \frac{x}{\sqrt{x^2 + R^2}} \right)$$

If we assume that the complete disc subtends an apex angle α at P, as shown in the figure, then



$$\cos\alpha = \frac{x}{\sqrt{R^2 + x^2}}$$

So, the gravitational field can also be expressed as

$$E = \frac{2GM}{R^2} \left(1 - \frac{x}{\sqrt{R^2 + x^2}} \right) = \frac{2GM}{R^2}(1 - \cos\alpha)$$

ILLUSTRATION 14

A spherical body of radius R is made of material having constant density ρ. The body is in equilibrium under its own gravity. If $P(r)$ is the pressure

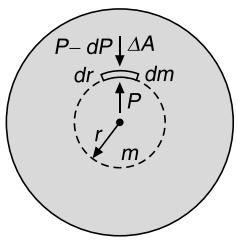
at a distance r from the centre of the body inside it, then calculate $P(r)$.

SOLUTION

Gravitational field at a distance r due to mass $m\left(=\frac{4}{3}\pi r^3\rho\right)$ is

$$E=\frac{G\rho\left(\frac{4}{3}\pi r^3\right)}{r^2}=\frac{4G\rho\pi r}{3}$$

Consider a small element of width dr and area ΔA at a distance r from the centre. Pressure force on this element is due to the gravitational force on dm from m inwards towards the centre.



$$\Rightarrow \quad (dP)\Delta A=E(dm)$$

where $dm=(\Delta A)(dr)\rho$ and $m=\left(\frac{4}{3}\pi r^3\right)\rho$

$$\Rightarrow \quad -dP\Delta A=\left(\frac{4}{3}G\pi\rho r\right)(\rho\Delta A dr)$$

$$\Rightarrow \quad -\int_0^P dP=\int_R^r\left(\frac{4G\rho^2\pi}{3}\right)rdr$$

$$\Rightarrow \quad -P=\frac{4G\rho^2\pi}{3\times 2}(r^2-R^2)$$

$$\Rightarrow \quad P=\frac{2G\rho^2\pi}{3}(R^2-r^2)$$

ACCELERATION DUE TO GRAVITY (g)

Earth attracts all bodies towards its centre. This property of the earth is called 'gravity' and the force with which it attracts a body is called the 'force of gravity' acting on that body. Thus, when a body falls freely towards the earth's surface, the force of gravity $\vec{F}$ produces an acceleration $\vec{g}$ in it given by

$$\vec{g}=\frac{\vec{F}}{m}$$

This acceleration is called acceleration due to gravity. Its magnitude g is independent of the mass, size, shape and composition of the body. It is directed radially inward to the centre of the earth.

If in Newton's law of gravitation one body (say earth) is taken as 'reference body', the force with which the reference body attracts any other body towards its centre is called force due to gravity (of reference body) and the phenomenon 'gravity'. Now if the body is free to move, force of gravity (of reference body) in accordance with Newton's Second law will produce an acceleration in it. The acceleration produced in a body by the force of gravity of reference body (usually earth) is called acceleration due to gravity and is represented by g.

If the reference body has mass M and radius R, the force on a body of mass m at the surface of reference body by Newton's law of gravitation will be

$$F=\left(\frac{GMm}{R^2}\right)$$

and if g is acceleration due to gravity of reference body on its surface, by Newton's Second law,

$$F=mg$$

$$\Rightarrow \quad g=\frac{F}{m}=\frac{GM}{R^2}$$

This is the relation between g and G showing that g depends on reference body. **Also, from this relation we observe that actually the acceleration due to gravity is also the measure of the gravitational field of the earth So,**

$$g=E_g=\frac{F}{m}=\frac{GM}{R^2}$$

It is independent of mass, shape, size, etc., of 'falling body', i.e., a given reference body produces same acceleration in a light and a heavy falling body.

VARIATION IN g

With Altitude

As for an external point a spherical distribution of mass behaves as if the whole of its mass were concentrated at the centre, i.e., $g=E_g=\left(\frac{GM}{r^2}\right)$

So, at the surface of earth

$$g = \frac{GM}{R^2}$$

and for a height h above the surface of earth

$$g_h = \left[\frac{GM}{(R+h)^2}\right] \qquad \{\text{as } r = R+h\}$$

$$\Rightarrow \quad \frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

So, with increase in height, g decreases. However, if h R, then

$$g_h = \frac{g}{\left[1+\left(\frac{h}{R}\right)\right]^2}$$

$$\Rightarrow \quad g_h = g\left(1+\frac{h}{R}\right)^{-2}$$

$$\Rightarrow \quad g_h = g\left(1-\frac{2h}{R}\right)$$

With Depth

As in case of spherical distribution of mass, for an internal point

$$g = E_g = \left(\frac{GM}{R^3}\right)r$$

So, at the surface of earth $g = \left(\frac{GM}{R^2}\right)$ and for a point at a depth d below the surface,

$$g_d = \frac{GM}{R^3}(R-d) \qquad \{\because \ r = R-d\}$$

$$\frac{g_d}{g} = \left(\frac{R-d}{R}\right)$$

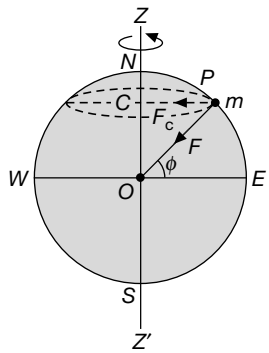
$$\Rightarrow \quad g_d = g\left(1-\frac{d}{R}\right)$$

So, with increase in depth below the surface of earth g decreases and at the centre of earth it becomes zero.

Due to Rotation of Earth

The earth rotates from west to east on its axis, due to which every object on the surface of earth experiences a centrifugal force in the reference frame of the earth. The effective value of acceleration due to gravity at a place of latitude ϕ is given by

$$g' = \sqrt{g^2 + \left(\omega^2 r\right)^2 + 2gr\omega^2 \cos(\pi - \phi)}$$



Now $r\omega^2$ g

$$\Rightarrow \quad g' \cong \sqrt{g^2 - 2gR\omega^2\cos^2\phi} \qquad \{\because \ r = R\cos\phi\}$$

$$\Rightarrow \quad g' \approx g - R\omega^2\cos^2\phi$$

At equator $\phi = 0°$ i.e. $g_E \approx g - R\omega^2$

At the poles $\phi = 90°$ i.e. $g_P = g$

Note that the vector g' is not exactly towards the center of earth. Therefore, we note that the decrease in g is maximum at equator and zero at poles. As we go from equator to the pole, value of g increases by

$$\Delta g = \left(g_p - g_e\right) = R\omega^2$$

The fractional change in the value of g is

$$\frac{\Delta g}{g} = \frac{R\omega^2}{g} \quad \frac{1}{291}$$

ILLUSTRATION 15

Find the height at which the gravitational field of the earth becomes one fourth the field at the surface.

SOLUTION

The acceleration due to gravity is the measure of the gravitational field. Let the gravitational field be one fourth the field at the surface at a height h above the earth's surface. So, we have

$$g_h = \frac{g}{4} = \frac{gR^2}{(R+h)^2}$$

$$\Rightarrow \quad \frac{1}{4} = \left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad \frac{R}{R+h} = \pm\frac{1}{2}$$

$$\Rightarrow \quad h = R$$

ILLUSTRATION 16

Assuming earth to be a sphere of uniform mass density, how much would a body weigh half way down the centre of the earth if it weighed 100 N on the surface?

SOLUTION

Given, $mg = 100$ N

$$g' = g\left(1 - \frac{h}{R}\right)$$

$$\frac{h}{R} = \frac{1}{2}$$

$$\Rightarrow \quad g' = g\left(1 - \frac{1}{2}\right) = \frac{g}{2}$$

$$\Rightarrow \quad mg' = \frac{mg}{2} = \frac{100}{2} = 50 \text{ N}$$

ILLUSTRATION 17

What is the acceleration due to gravity of earth at the surface of moon if the distance between earth and moon is 3.8×10^5 km and radius of earth is 6.4×10^3 km ?

SOLUTION

If M and R be the mass and radius of the earth then the acceleration due to gravity due to earth on the surface of earth i.e.,

$$g = \frac{GM}{R^2} \qquad \ldots(1)$$

Similarly, acceleration due to gravity at a distance $r (> R)$ of the earth i.e.,

$$g' = \frac{GM}{r^2} \qquad \ldots(2)$$

If r be the distance between earth and moon then g' will give you the value of acceleration due to gravity on the moon due to earth. Therefore, from equation (1) and (2)

$$g' = \frac{\left(6.4 \times 10^3\right)^2}{\left(3.8 \times 10^5\right)^2} g$$

$$\Rightarrow \quad g = 0.00275 \text{ ms}^{-2} \qquad \left\{\because \ g = 9.8 \text{ ms}^{-2}\right\}$$

ILLUSTRATION 18

A planet of radius R equal to one-tenth the radius of earth has the same mass density as earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere calculate the force applied at the top of the wire by a person holding it in place.
Take the radius of earth to be 6×10^6 m and the acceleration due to gravity of earth to be 10 ms^{-2}.

SOLUTION

Given, $R_{\text{planet}} = R = \frac{R_{\text{earth}}}{10}$

Since, density $\rho = \frac{M_{\text{earth}}}{\frac{4}{3}\pi R_{\text{earth}}^3}$

Also, $\rho = \frac{M_{\text{planet}}}{\frac{4}{3}\pi R_{\text{planet}}^3}$

$$\Rightarrow \quad M_{\text{planet}} = \frac{M_{\text{earth}}}{10^3} = \frac{M_e}{1000}$$

Let the acceleration due to gravity at surface of planet and at the surface of earth be g_p and g_e respectively. Then

$$g_p = \frac{GM_{\text{planet}}}{R_{\text{planet}}^2} = \frac{GM_e\,(10)^2}{(10)^3\,R_e^2} = \frac{GM_e}{10R_e^2}$$

$$\Rightarrow \quad g_p = \frac{g_e}{10}$$

The value of g inside the planet at a distance x from centre of the planet is

$$g_{\text{inside}} = g_{\text{surface of planet}}\left(\frac{x}{R}\right) = g_p\left(\frac{x}{R}\right)$$

So, total force acting on wire is

$$F = \int_{\frac{4R}{5}}^{R} (\lambda dx)\, g_p\left(\frac{x}{R}\right)$$

$$\Rightarrow \quad F = \frac{\lambda g_p}{R}\left(\frac{x^2}{2}\right)\Bigg|_{4R/5}^{R}$$

Substituting the given values, we get

$$F = 108 \text{ N}$$

ILLUSTRATION 19

Two equal masses m and m are hung from a balance whose scale pans differ in vertical height by h. Calculate the error in weighing, if any, in terms of density of earth ρ.

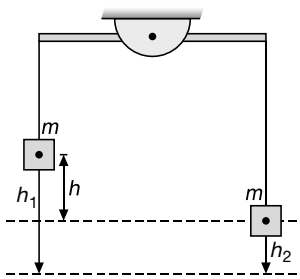
SOLUTION

Since g varies with height as $g' = \dfrac{gR^2}{(R+h)^2}$

For $h \quad R$, we have

$$g' = g\left(1 - \frac{2h}{R}\right)$$

From the figure we see that $h_1 > h_2$, so W_1 will be lesser than W_2 and hence



$$W_2 - W_1 = mg_2 - mg_1 = 2mg\left(\frac{h_1}{R} - \frac{h_2}{R}\right)$$

Since $g = \dfrac{GM}{R^2}$ and $h_1 - h_2 = h$, so we get

$$W_2 - W_1 = 2m\left(\frac{GM}{R^2}\right)\frac{h}{R} = \frac{2GMmh}{R^3}$$

Since $M = \left(\dfrac{4}{3}\pi R^3\right)\rho$

$$\Rightarrow \quad W_2 - W_1 = \frac{2mhG}{R^3}\left(\frac{4}{3}\pi R^3\rho\right) = \frac{8}{3}\pi\rho Gmh$$

ILLUSTRATION 20

Suppose the earth increases its speed of rotation. At what new time period will the weight of a body on the equator becomes zero? Take $g = 10\ \text{ms}^{-2}$ and radius of earth $R = 6400\ \text{km}$.

SOLUTION

The weight will become zero when

$$g' = 0$$

$$\Rightarrow \quad g - R\omega^2 = 0 \qquad \{\text{on the equator } g' = g - R\omega^2\}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{R}}$$

$$\Rightarrow \quad \frac{2\pi}{T} = \sqrt{\frac{g}{R}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{R}{g}}$$

Substituting the values,

$$T = \frac{2\pi\sqrt{\dfrac{6400\times10^3}{10}}}{3600}\ \text{hr}$$

$$\Rightarrow \quad T = 1.4\ \text{hr}$$

Thus, the new time period should be 1.4 hr instead of 24 hr for the weight of a body to be zero on the equator.

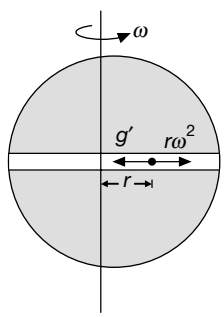
ILLUSTRATION 21

A straight smooth tunnel is dug through a spherical planet whose mass density ρ_0 is constant. The tunnel passes through the centre of the planet and is perpendicular to the planet's axis of rotation, which is fixed in space. The planet rotates with the angular velocity ω so that objects in the tunnel have no acceleration relative to the tunnel. Find ω.

SOLUTION

For no acceleration, we have

$$mg' - mr\omega^2 = 0$$



$$\Rightarrow \quad g' = r\omega^2$$

$$\Rightarrow \quad \left(\frac{GM}{R^3}\right)r = r\omega^2$$

$$\Rightarrow \quad \frac{G\left(\dfrac{4}{3}\pi R^3\rho_0\right)}{R^3} = \omega^2$$

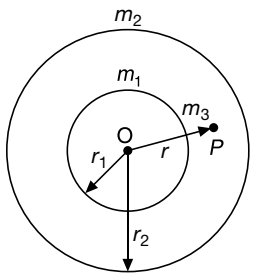
$$\Rightarrow \quad \omega = \sqrt{\frac{4}{3}\pi G\rho_0}$$

ILLUSTRATION 22

Two concentric spherical shells have masses m_1 and m_2 and radii r_1 and $r_2\,(r_2 > r_1)$. What is the force exerted by this system on a particle of mass m_3 if it is placed at a distance $r\,(r_1 < r < r_2)$ from the centre?

SOLUTION

The outer shell will have no contribution in the gravitational field at point P, because P lies inside the outer shell. So, at P, the field will only be due to the inner shell of mass m_1.



$$\Rightarrow \quad E_P = \frac{Gm_1}{r^2}$$

Thus, force on mass m_3 placed at P is,

$$F = (m_3 E_P)$$

$$\Rightarrow \quad F = \frac{Gm_1 m_3}{r^2}$$

The field $\vec{E}_P$ and the force $\vec{F}$ both are towards centre O.

ILLUSTRATION 23

A body is suspended on a spring balance in a ship sailing along the equator with a speed v. Show that the scale reading will be very close to $W_0\left(1 \pm \frac{2v\omega}{g}\right)$ where ω is the angular speed of the earth and W_0 is the scale reading when the ship is at rest. Also explain the significance of plus and minus sign.

SOLUTION

Let R be the radius of earth and ω be its angular speed.

When the ship is at rest, we have

$$W_0 = mg - m\omega^2 R \qquad \ldots(1)$$

When the ship moves with a speed in the sense of rotation of earth, then its effective speed is $v + R\omega$, otherwise it is $v - R\omega$. So, we have

$$W = mg - \frac{m(v \pm R\omega)^2}{R}$$

$$\Rightarrow \quad W = mg - \left(\frac{mv^2}{R} + m\omega^2 R \pm 2mv\omega\right)$$

$$\Rightarrow \quad W = W_0 - \left(\frac{mv^2}{R} \pm 2mv\omega\right) \qquad \ldots(2)$$

From (1), we get

$$m = \frac{W_0}{g - R\omega^2} = \frac{W_0}{g\left(1 - \frac{R\omega^2}{g}\right)}$$

Since $\frac{R\omega^2}{g} \approx \frac{1}{291}$ 1

$$\Rightarrow \quad m = \frac{W_0}{g}$$

$$\Rightarrow \quad W \approx W_0\left(1 \pm \frac{2v\omega}{g}\right)$$

Test Your Concepts-I

Based on Acceleration Due to Gravity, Gravitational Field and Applications

(Solutions on page H.255)

1. Calculate the escape velocity from the surface of moon. The mass of the moon is 7.4×10^{22} kg and radius is 1.74×10^6 m.

2. A planet of mass m_1 revolves round the sun of mass m_2. The distance between the sun and the planet is r. Considering the motion of the sun, find the total energy of the system assuming the orbits to be circular.

3. Consider the ring-shaped body of mass M and radius a. A particle of mass m is placed a distance x from the centre of the ring, along the line through the centre of the ring and perpendicular to its plane.

(a) Calculate the gravitational potential energy of this system. Take the potential energy to be zero when the two objects are far apart.

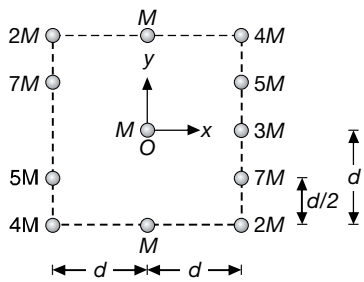
(b) Show that your answer to part (a) reduces to the expected result when x is much larger than the radius a of the ring.

(c) Use $F_x = -\frac{dU}{dx}$ to calculate the force between the objects.

(d) Show that your answer to part (c) reduces to the expected result when x is much larger than a.

(e) What is the force when $x = 0$? Explain why this result makes sense.

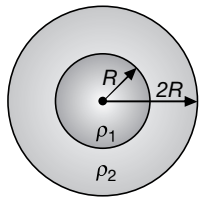
4. In the arrangement shown, find the magnitude and direction of the net gravitational force acting on the central particle at O.



5. The distance between two bodies A and B is r. Taking the gravitational force according to the law of inverse square of r, the acceleration of the body A is a. If the gravitational force follows an inverse fourth power law, then what will be the acceleration of the body A?

6. A particle of mass 20 g experiences a gravitational force of 4 N along positive x-direction. Find the gravitational field at that point.

7. The density of the core of a planet is ρ_1 and that of the outer shell is ρ_2. The radii of the core and that of the planet are R and $2R$ respectively. If the gravitational acceleration at the surface of the planet is same as at a depth R, find the ratio $\frac{\rho_1}{\rho_2}$.



8. Calculate the gravitational force of attraction on a particle of mass m placed at the centre of a semicircular wire of length L and mass M.

9. Inside a solid sphere of radius R, the density ρ is given by $\rho = \frac{\rho_0 R}{r}$, where ρ_0 is the density at the surface and r is the distance from the centre. Find the gravitational field due to this sphere at a distance $2R$ from its centre.

10. A system consists of a thin ring of radius R and a very long uniform wire oriented along axis of the ring with one of its ends coinciding with the centre of the ring. If mass of ring be M and the linear mass density of the wire be λ, then calculate the interaction force between the ring and the wire.

11. Determine the speed with which the earth would have to rotate on its axis so that a person on the equator would weigh 60% of his weight at the pole. Take $R = 6400$ km.

12. A thick spherical shell has an inner radius R_1 and an outer radius R_2. It has mass M and uniform density. Find the gravitational field E_r as a function of r.

13. At what height from the surface of earth will the value of g be reduced by 36% in comparison to the value at the surface? Given $R = 6400$ km.

Gravitational Potential Energy (U)

Gravitational potential energy of a system of particles is defined as the external work required to assemble the particles from infinity to a given configuration (as that required in system).

When particles/masses are lying at infinity, their potential energy is taken to be zero, because no potential exists between them. Since the gravitational force between the particles is attractive in nature, so the work will be done by the system and final potential energy of system will be negative.

In giving these arguments please keep this thing in mind that work done by a conservative force equals the decrease in potential energy of the system.

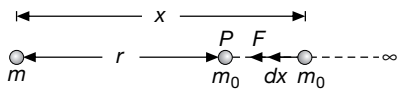
When all particles of a system are separated far apart by infinite distance, there exists no interaction between them. This state, we take as reference of Zero Potential Energy (ZPE).

Gravitational potential energy is categorised in two ways.

(a) Gravitational interaction energy (U) of a system of particles.
(b) Gravitational self energy (U_s) of a body.

GRAVITATIONAL POTENTIAL ENERGY OF A SYSTEM OF TWO PARTICLES

Figure shows two masses, a source mass m and a test mass m_0 separated by a distance r. The gravitational potential energy of this system is found by calculating the work done in bringing test mass m_0 from infinity to the given point P at a distance r from the source mass m. Let, at any instant the test mass m_0 be at a distance x from the source mass m. Let m_0 be displaced through dx towards m. The gravitational force of attraction F acts on m_0 towards m.



If dW be the work done, then

$$dW = \vec{F}\cdot d\vec{x}$$

$$\Rightarrow \quad dW = \frac{Gmm_0}{x^2}dx\cos(0°)$$

$$\Rightarrow \quad dW = Gmm_0x^{-2}dx$$

$$\Rightarrow \quad W = Gmm_0\int_0^r x^{-2}dx$$

$$\Rightarrow \quad W = Gmm_0\left(\frac{x^{-2+1}}{-2+1}\Big|_\infty^r\right)$$

$$\Rightarrow \quad W = -Gmm_0\left(\frac{1}{r}-\frac{1}{\infty}\right)$$

$$\Rightarrow \quad W_{\infty\to P} = -\frac{Gmm_0}{r} = W$$

So, work done by the external force in bringing the test mass m_0 from ∞ to the point P under the influence of source mass m is

$$W = W_{\infty\to P} = -\frac{Gmm_0}{r}$$

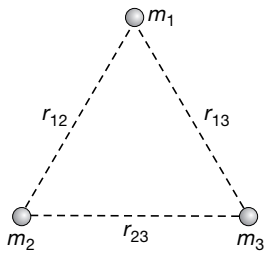
This work done is stored in the form of gravitational potential energy (U) of the system. So,

$$U = -\frac{Gmm_0}{r}$$

GRAVITATIONAL POTENTIAL ENERGY FOR A SYSTEM OF PARTICLES

When more than two particles are there in a system, the potential energy can be given by sum of potential energy of all the pairs of particles with no pair repeated.

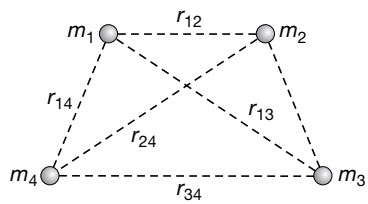
For example, if a system of three particles having masses m_1, m_2 and m_3 is given as shown in figure.



The total potential energy of this system can be given as

$$U = -G\left[\frac{m_1m_2}{r_{12}}+\frac{m_2m_3}{r_{23}}+\frac{m_1m_3}{r_{13}}\right] = U_{12}+U_{13}+U_{23}$$

Similarly, for an assembly of four masses m_1, m_2, m_3 and m_4 (having total number of six potential $= {}^4C_2$), we have



$$U = -G\left[\frac{m_1m_2}{r_{12}}+\frac{m_2m_3}{r_{23}}+\frac{m_3m_4}{r_{34}}+\right.$$
$$\left.\frac{m_4m_1}{r_{14}}+\frac{m_1m_3}{r_{13}}+\frac{m_2m_4}{r_{24}}\right]$$

$$\Rightarrow \quad U = U_{12}+U_{23}+U_{34}+U_{41}+U_{13}+U_{24}$$

From above we see that the total potential energy is simply the sum of contributions due to distinct pairs. Generalising to N masses, we get

$$U = -G\sum_{i=1}^{N}\sum_{\substack{j=1\\(j>i)}}^{N}\frac{m_im_j}{r_{ij}}$$

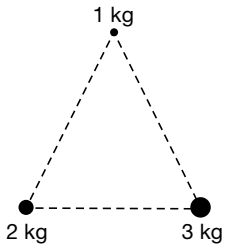
where $j > i$ assures that the pairs are not repeated. Other way round, we can count every pair twice and multiply the result by $\frac{1}{2}$. So,

$$U = -G\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{m_i m_j}{r_{ij}} = \frac{1}{2}\sum_{i=1}^{N} m_i\left(-G\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{m_j}{r_{ij}}\right)$$

ILLUSTRATION 24

Three masses of 1 kg, 2 kg and 3 kg are placed at the vertices of an equilateral triangle of side 1 m. Find the gravitational potential energy of this system. Take $G = 6.67\times10^{-11}\ \text{Nm}^2\text{kg}^{-2}$.

SOLUTION



$$U = -G\left(\frac{m_3 m_2}{r_{32}} + \frac{m_3 m_1}{r_{31}} + \frac{m_2 m_1}{r_{21}}\right)$$

where, $r_{32} = r_{31} = r_{21} = 1.0\ \text{m}$, $m_1 = 1\ \text{kg}$

$m_2 = 2\ \text{kg}$ and $m_3 = 3\ \text{kg}$

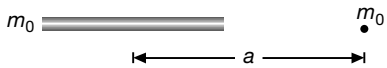
Substituting in above, we get

$$U = -(6.67\times10^{-11})\left(\frac{3\times2}{1} + \frac{3\times1}{1} + \frac{2\times1}{1}\right)$$

$$\Rightarrow \quad U = -7.337\times10^{-10}\ \text{J}$$

ILLUSTRATION 25

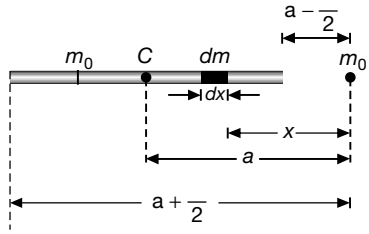
Find the gravitational potential energy of a point mass m_0 and a thin uniform rod of mass m_0 and length l, if they are located along a straight line at a distance a measured from the centre of the rod.



SOLUTION

Let us consider an infinitesimal element of length dx, mass dm at a distance x from the point mass m_0. Then,

$$dm = \left(\frac{m_0}{l}\right)dx$$



If dU be the gravitational potential energy between dm and m_0, then

$$dU = -\frac{Gm_0 dm}{x} = -\frac{Gm_0^2}{l}\frac{dx}{x}$$

$$\Rightarrow \quad U = \int dU = -\frac{Gm_0^2}{l}\int_{\left(a-\frac{l}{2}\right)}^{\left(a+\frac{l}{2}\right)}\frac{dx}{x}$$

$$\Rightarrow \quad U = -\frac{Gm_0^2}{l}\log_e x\Big|_{\left(a-\frac{l}{2}\right)}^{\left(a+\frac{l}{2}\right)}$$

$$\Rightarrow \quad U = -\frac{Gm_0^2}{l}\left[\log_e\left(a+\frac{l}{2}\right) - \log_e\left(a-\frac{l}{2}\right)\right]$$

$$\Rightarrow \quad U = -\frac{Gm_0^2}{l}\log_e\left(\frac{a+\frac{l}{2}}{a-\frac{l}{2}}\right)$$

$$\Rightarrow \quad U = -\frac{Gm_0^2}{l}\log_e\left(\frac{2a+l}{2a-l}\right)$$

GRAVITATIONAL SELF ENERGY FOR A THIN UNIFORM SHELL

To calculate the gravitational self energy of a body, it is supposed that initially the particles of the body are scattered at infinite distance from each other. Therefore, in the formation of a body some external agent has to do some work in assembling the body. This energy is stored in the body as gravitational potential energy and is known as gravitational self energy of the body.

Let us consider a thin uniform shell of mass M and radius R whose gravitational self energy is to be calculated. Let the shell at any instant have a mass m.

Since the shell is uniform and thin, so this mass is lying at its surface. Now let us bring additional mass dm to the surface of this shell, then the gravitational potential energy between m and dm is

$$dU = -\frac{Gmdm}{R}$$

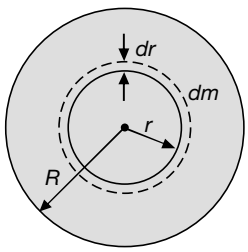
On integrating, the gravitational self energy is obtained as

$$U_s = -\int_0^M \frac{Gmdm}{R} = -\frac{GM^2}{2R}$$

GRAVITATIONAL SELF ENERGY FOR A UNIFORM SPHERE

Let us consider a uniform sphere of mass M and radius R whose gravitational self energy is to be calculated. Let the sphere at any instant have a mass m and radius r. Since the sphere is uniform, so this mass m is distributed uniformly on the sphere. If the sphere has a uniform mass density ρ, then

$$m = \left(\frac{4}{3}\pi r^3\right)\rho$$



Now, let us bring an additional layer of mass dm in the form of a thin spherical shell of inner radius r and outer radius $r+dr$ to be placed on the sphere and repeat the process until it becomes a full fledged solid sphere of radius R. Then

$$dm = \left(4\pi r^2 dr\right)\rho$$

The gravitational potential energy between the sphere and this infinitesimal shell is given by

$$dU = -\frac{Gmdm}{r} = -\frac{G\left(\frac{4}{3}\pi r^3\rho\right)\left(4\pi r^2\rho dr\right)}{r}$$

On integrating, the gravitational self energy is obtained as

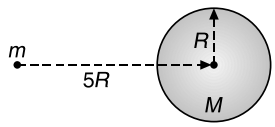
$$U_s = -\frac{16\pi^2\rho^2 G}{3}\int_0^R r^4 dr = -\frac{16\pi^2\rho^2 G}{3}\left(\frac{r^5}{5}\Big|_0^R\right)$$

But $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$, so on substituting the value of ρ in the above equation, we get

$$U_s = -\frac{3}{5}\frac{GM^2}{R}$$

ILLUSTRATION 26

A solid sphere of mass M and radius R is initially placed at a distance $5R$ from the centre of a point mass m as shown in figure.



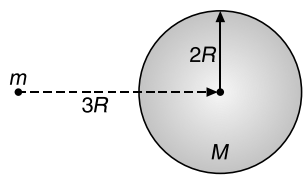
Now it is shifted to a position at a distance $3R$ from the point mass. During displacement, it is also uniformly expanded to a radius $2R$ so that its density decreases uniformly throughout its volume. Find the work required in this process.

SOLUTION

In this process during displacement the size of the sphere is also changing. So, we have to also take into account the self energy of the sphere. Initial total energy of system is the sum of self energy of M and the interaction energy of M and m. If E_m be the self energy of m, then

$$E_i = E_m + \left(-\frac{3}{5}\frac{GM^2}{R}\right) + \left(-\frac{GMm}{5R}\right)$$

Finally the radius of M becomes $2R$ and it is situated at a distance $3R$ from m as shown in figure.



So, final total energy of system is given by

$$E_f = E_m + \left(-\frac{3}{5}\frac{GM^2}{2R}\right) + \left(-\frac{GMm}{3R}\right)$$

External work W required in the process is given by

$$W = E_f - E_i$$

$$\Rightarrow \quad W = \frac{3}{10}\frac{GM^2}{R} - \frac{GMm}{R}\left(\frac{1}{3} - \frac{1}{5}\right)$$

$$\Rightarrow \quad W = \frac{GM}{30R}(9M - 4m)$$

POTENTIAL ENERGY OF A PARTICLE ON EARTH'S SURFACE

If M be the mass of earth, R be the radius of earth and m is the mass of particle on earth's surface, then potential energy of particle on earth's surface is

$$U = -\frac{GMm}{R} = -mgR \qquad \left\{\because g = \frac{GM}{R^2}\right\}$$

At height h,

$$U_h = -\frac{GMm}{R+h}$$

If a particle is taken from the surface of the earth to a height h close to the surface then $h \quad R$. So,

$$U_{\text{initial}} = -\frac{GMm}{R} = U_i \text{ and}$$

$$U_{\text{final}} = -\frac{GMm}{R+h} = U_f$$

Hence, work done is equal to change in energy.

$$\Rightarrow \quad W = U_f - U_i = -\frac{GMm}{R+h} + \frac{GMm}{R}$$

$$\Rightarrow \quad W = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

$$\Rightarrow \quad W = \frac{GMm}{R}\left[1 - \left(1 + \frac{h}{R}\right)^{-1}\right]$$

$$\Rightarrow \quad W \quad \frac{GMm}{R}\left[1 - \left(1 - \frac{h}{R}\right)\right]$$

$$\left\{\because \text{ for } x \quad 1;\ (1+x)^n \quad 1+nx\right\}$$

$$\Rightarrow \quad W \quad \frac{GMm}{R}\left(\frac{h}{R}\right)$$

$$\Rightarrow \quad W \quad m\left(\frac{GM}{R^2}\right)h = mgh$$

Conceptual Note(s)

Let us find the difference in potential energy of a mass m in two positions shown in figure. The potential energy of the mass on the surface of earth (at B) is,

$$U_B = -\frac{GMm}{R}$$

and potential energy of mass m at height h above the surface of earth (at A) is,

$$U_A = -\frac{GMm}{R+h} \qquad (U_A > U_B)$$

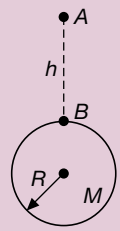
$$\Rightarrow \quad \Delta U = U_A - U_B = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$$

$$\Rightarrow \quad \Delta U = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right) = \frac{GMmh}{R(R+h)}$$

$$\Rightarrow \quad \Delta U = \frac{GMmh}{R^2\left(1 + \frac{h}{R}\right)} \qquad \left\{\frac{GM}{R^2} = g\right\}$$

$$\Rightarrow \quad \Delta U = \frac{mgh}{1 + \frac{h}{R}}$$

For $h \ll R$, $\Delta U \approx mgh$



Thus, what we read the mgh is actually the difference in potential energy (not the absolute potential energy), that too for $h \ll R$.

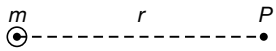
GRAVITATIONAL POTENTIAL (V)

The gravitational potential at a point P due to a source mass is the work done per unit test mass in bringing the test mass (m_0) from infinity to the point P under the gravitational influence of source mass m. So,

$$V_P = \text{Potential at point } P = \frac{W_{\infty \to P}}{m_0}$$

Since $W_{\infty \to P} = -\dfrac{Gmm_0}{r}$

$$\Rightarrow \quad V_P = \frac{W_{\infty \to P}}{m_0} = -\frac{Gm}{r}$$



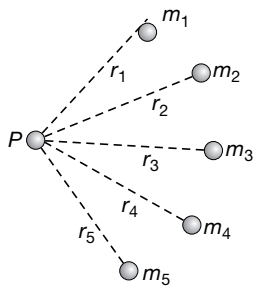
It is a scalar quantity. Its SI unit is joule/kg $\left(\text{Jkg}^{-1}\right)$ and its dimensional formula is $M^0L^2T^{-2}$

The gravitational potential due to a source mass m at a distance r from it is

$$V = -\frac{Gm}{r}$$

GRAVITATIONAL POTENTIAL DUE TO AN ASSEMBLY OF MASSES

Before moving further with this discussion, let us know this and keep in mind that gravitational potential is a scalar quantity. Let us now calculate the gravitational potential due to an assembly of masses at point P shown.



The gravitational potential V at point P due to the assembly of masses shown is the algebraic sum of the potential due to each of the mass in the assembly. So,

$$V = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3} + \frac{m_4}{r_4} + \frac{m_5}{r_5}\right)$$

Problem Solving Technique(s)

(a) The gravitational potential at earth's surface is

$$V = \frac{-GM_e}{R_e} = -gR_e$$

(b) For an assembly of masses m_1, m_2, m_3, ..., at distances r_1, r_2, r_3,... from point P, net gravitational potential equals the algebraic sum of gravitational potential due to each of the mass at point P. So,

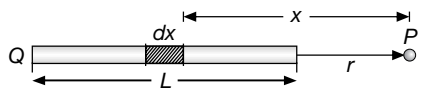
$$V = V_1 + V_2 + V_3 + \ldots$$

$$\Rightarrow \quad V = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2} + \frac{m_3}{r_3} + \ldots\right)$$

$$\Rightarrow \quad V = -G\sum_i^N \frac{m_i}{r_i}$$

GRAVITATIONAL POTENTIAL DUE TO A THIN ROD

Consider a thin rod of length L, having a uniform mass M. Let us find the gravitational potential at a point P due to the rod at a distance r from one end of the rod.



For this we consider an infinitesimal element of length dx at a distance x from the point P. Mass on this element is

$$dm = \frac{M}{L}dx$$

The potential dV due to this element at point P is

$$dV = -\frac{Gdm}{x}$$

$$\Rightarrow \quad V = -\frac{G\left(\dfrac{M}{L}\right)dx}{x}$$

Net gravitational potential at point P is obtained by integrating this expression. So

$$\Rightarrow \quad V = \int dV = -\frac{GM}{L}\int_r^{r+L}\frac{dx}{x}$$

$$\Rightarrow \quad V = -\frac{GM}{L}\log_e x\Big|_r^{r+L}$$

$$\Rightarrow \quad V = -\frac{GM}{L}\log_e\left(\frac{r+L}{r}\right)$$

If mass density of the rod is λ, then $\lambda = \dfrac{M}{L}$. So,

$$V = -G\lambda\log_e\left(\frac{r+L}{r}\right)$$

GRAVITATIONAL POTENTIAL DUE TO A RING AT ITS CENTRE

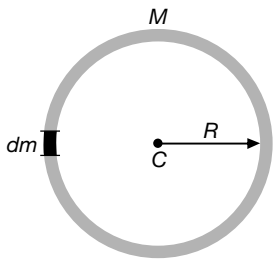
Let us first find potential dV at centre C due to an infinitesimal mass dm on ring which is

$$dV = -\frac{Gdm}{R}$$

Total potential at C is $V = \int dV$

$$\Rightarrow \quad V = -G\int \frac{dm}{R} = -\frac{GM}{R}$$

Since all the infinitesimal dm's of the ring are situated at same distance R from the ring centre C so, we can directly say that the total gravitational potential at centre of ring is $V_C = -\frac{GM}{R}$.



Here we must see that even if mass M is non-uniformly distributed on ring, the gravitational potential at C will remain same (we shall discuss this with an example below) because in that case too

$$V = -\frac{G}{R}\int dm = -\frac{G\int dm}{R}$$

$$\Rightarrow \quad V_{\text{centre}} = -\frac{GM_{\text{total on ring}}}{R}$$

GRAVITATIONAL POTENTIAL DIFFERENCE (V)

Gravitational potential difference (ΔV) is the work done per unit test mass m_0 in bringing it from point A to point B under the gravitational influence of a source mass m.

$$\Rightarrow \quad \Delta V = V_B - V_A = \frac{W_{A\to B}}{m_0} = -Gm\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$\Rightarrow \quad W_{A\to B} = m_0\left(V_B - V_A\right) = -Gmm_0\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

THE GRAVITATIONAL POTENTIAL AND FIELD STRENGTH DUE TO A THIN SPHERICAL SHELL

Let M be the mass and R, the radius of thin spherical shell.

$$V = \begin{cases} -\dfrac{GM}{r} & r \geq R \\ & (\text{outside and at surface}) \\ -\dfrac{GM}{R} & r < R \\ & (\text{inside}) \end{cases}$$

Also, as discussed earlier, we know that the gravitational field due to a thin spherical shell is

$$E_g = \begin{cases} \dfrac{GM}{r^2} & r \geq R \\ & (\text{outside and at surface}) \\ 0 & r < R \\ & (\text{inside}) \end{cases}$$

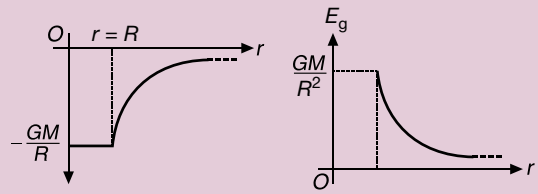
Problem Solving Technique(s)

(a) Gravitational field E_g exhibits discontinuity at the surface of shell (at $r = R$). Because inside it is zero whereas outside and at surface it suddenly shoots up to a value $\frac{GM}{R^2}$.

(b) Gravitational potential exhibits continuity at the surface of shell.

(c) Gravitational potential inside the shell is equal to the potential at the surface of shell i.e.

$$V_{\text{inside}} = V_{\text{surface}} = -\frac{GM}{R}$$



THE GRAVITATIONAL POTENTIAL AND FIELD STRENGTH DUE TO A SOLID SPHERE

Let M be the mass and R, the radius of solid sphere

$$V = \begin{cases} -\dfrac{GM}{r} & r \geq R \\ & (\text{outside and at surface}) \\ -\dfrac{GM}{2R^3}\left(3R^2 - r^2\right) & r < R \\ & (\text{inside}) \end{cases}$$

Also, we observe that at the centre of the sphere, where $r = 0$, the gravitational potential is

$$V_{\text{centre}} = -\frac{3}{2}\frac{GM}{R}$$

Also, as discussed earlier, we know that the gravitational field due to a sphere is

$$E_g = \begin{cases} \dfrac{GM}{r^2} & r \geq R \\ & (\text{outside and at surface}) \\ \left(\dfrac{GM}{R^3}\right) r & r < R \\ & (\text{inside}) \end{cases}$$

Problem Solving Technique(s)

Both field and potential exhibit continuity at the surface. The graphs also show the continuous nature.

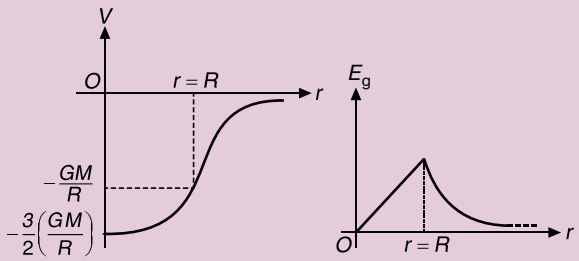


ILLUSTRATION 27

A particle of mass 1 kg is kept on the surface of a uniform sphere of mass 20 kg and radius 1.0 m. Find the work to be done against the gravitational force between them to take the particle away from the sphere.

SOLUTION

Potential at the surface of sphere,

$$V = -\frac{GM}{R} = -\frac{(6.67\times10^{-11})(20)}{1}\ \text{Jkg}^{-1}$$

$$V = -1.334\times10^{-9}\ \text{Jkg}^{-1}$$

i.e., 1.334×10^{-9} J work is obtained to bring a mass of 1 kg from infinity to the surface of sphere. Hence, the same amount of work will have to be done to take the particle away from the surface of sphere. Thus,

$$W = 1.334\times10^{-9}\ \text{J}$$

Problem Solving Technique(s)

To find the gravitational potential due to a shell of radius *R*, mass *m* we should keep in mind the following two points:

(a) Gravitational potential on the surface and at any point inside the shell is

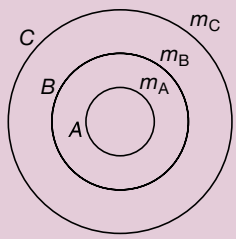
$$V = -\frac{Gm}{R}$$

(b) Gravitational potential at any point outside the sphere/shell at a distance *r* from the centre is

$$V = -\frac{Gm}{r}$$

For example, in the figure shown, potential at *A* is

$$V_A = -G\left(\frac{m_A}{r_A} + \frac{m_B}{r_B} + \frac{m_C}{r_C}\right)$$



Similarly, potential at *B* is

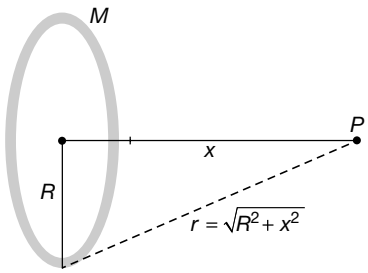
$$V_B = -G\left(\frac{m_A}{r_B} + \frac{m_B}{r_B} + \frac{m_C}{r_C}\right)$$

and potential at *C* is

$$V_C = -G\left(\frac{m_A}{r_C} + \frac{m_B}{r_C} + \frac{m_C}{r_C}\right)$$

GRAVITATIONAL POTENTIAL AT A POINT *P* ON THE AXIS OF THE RING AT DISTANCE *X* FROM ITS CENTRE

If we wish to find the gravitational potential at a point P lying on the axis of ring, we can directly calculate the result because here too, all points of ring are at same distance $r = \sqrt{R^2 + x^2}$ from the point P.



So, potential at P is

$$V_P = -\frac{GM}{r} = -\frac{GM}{\sqrt{R^2 + x^2}}$$

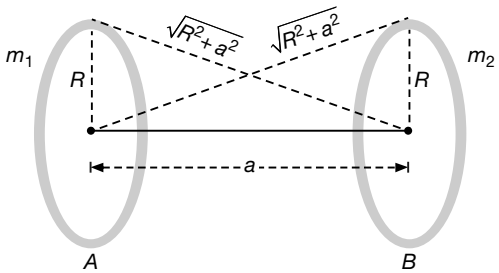
ILLUSTRATION 28

There are two thin wire rings, each of radius R, whose axes coincide. The masses of the rings are m_1 and m_2. Find the gravitational potential difference between the centres of the rings separated by a distance a. Also calculate the work done to move a mass m_0 from centre of first ring to the centre of other ring.

SOLUTION

Net potential at centre of ring A is

$$V_A = \begin{pmatrix}\text{Potential at } A\\ \text{due to itself}\end{pmatrix} + \begin{pmatrix}\text{Potential at } A\\ \text{due to } B\end{pmatrix}$$



$$\Rightarrow \quad V_A = -G\left(\frac{m_1}{R} + \frac{m_2}{\sqrt{R^2 + a^2}}\right)$$

Similarly, net potential at centre of ring B is

$$V_B = \begin{pmatrix}\text{Potential at } B\\ \text{due to itself}\end{pmatrix} + \begin{pmatrix}\text{Potential at } B\\ \text{due to } A\end{pmatrix}$$

$$\Rightarrow \quad V_B = -G\left(\frac{m_2}{R} + \frac{m_1}{\sqrt{R^2 + a^2}}\right)$$

Thus, potential difference,

$$\Delta V = V_B - V_A$$

$$\Rightarrow \quad \Delta V = G\left(-\frac{m_2}{R} - \frac{m_1}{\sqrt{R^2 + a^2}} + \frac{m_1}{R} + \frac{m_2}{\sqrt{R^2 + a^2}}\right)$$

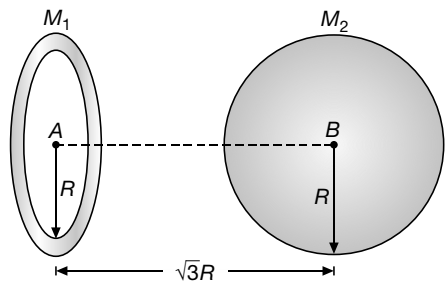
$$\Rightarrow \quad \Delta V = G(m_1 - m_2)\left(\frac{1}{R} - \frac{1}{\sqrt{R^2 + a^2}}\right)$$

Since, $W_{A\to B} = m_0(V_B - V_A)$

$$\Rightarrow \quad W_{A\to B} = Gm_0(m_1 - m_2)\left(\frac{1}{R} - \frac{1}{\sqrt{R^2 + a^2}}\right)$$

ILLUSTRATION 29

Figure shows a ring of mass M_1 and a sphere of mass M_2 separated by a distance $\sqrt{3}R$. A small object of mass m is displaced from A to B. Find the work done by gravitational forces.



SOLUTION

In this case as shifting is from A to B so, work done by the gravitational forces is given by

$$W_g = m(V_A - V_B)$$

where V_A and V_B are gravitational potentials at points A and B respectively, given by

$$V_A = -\frac{GM_1}{R} - \frac{GM_2}{2R}$$

and $$V_B = -\frac{3GM_2}{2R} - \frac{GM_1}{R}$$

So, work done by gravitational force is given by

$$W_g = m\left[\left(-\frac{GM_1}{R} - \frac{GM_2}{2R}\right) - \left(\frac{3}{2}\frac{GM_2}{R} - \frac{GM_1}{2R}\right)\right]$$

$$\Rightarrow \quad W_g = \frac{GM_2}{R} - \frac{GM_1}{2R}$$

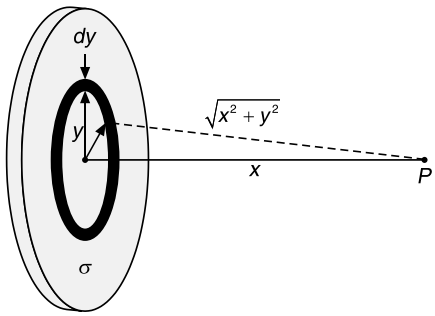
$$\Rightarrow \quad W_g = \frac{GM}{2R}(2M_2 - M_1)$$

GRAVITATIONAL POTENTIAL DUE TO A UNIFORM DISC AT A POINT *P* ON ITS AXIS

Consider a uniform disc of radius R with surface mass density σ. We wish to find gravitational potential at point P lying on its axis at distance x from the centre. For this we consider an infinitesimal elemental ring of radius y and thickness dy concentric with the disc. Then mass on this infinitesimal element is

$$dm = \sigma \text{ (Area of element)}$$

$$\Rightarrow \quad dm = \sigma(2\pi y dy)$$



The gravitational potential at point P due to this infinitesimal element is

$$dV = -\frac{Gdm}{r} = -\frac{Gdm}{\sqrt{x^2+y^2}}$$

(Please note here that once the point P is taken, then x becomes a fixed value)

$$\Rightarrow \quad dV = -\frac{G\sigma(2\pi y dy)}{\sqrt{x^2+y^2}}, \text{ where } \sigma = \frac{M}{\pi R^2}$$

Net gravitational potential at point P due to entire disc is calculated by integrating this expression.

$$\Rightarrow \quad V = \int dV = -2\pi\sigma G\int_0^R \frac{ydy}{\sqrt{x^2+y^2}}$$

$$\Rightarrow \quad V = -2\pi\sigma G\int_0^R \frac{ydy}{\sqrt{x^2+y^2}}$$

$$\Rightarrow \quad V = -2\pi\sigma G\int_0^R \left(x^2+y^2\right)^{-\frac{1}{2}} 2ydy \qquad \ldots(1)$$

Again using $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1}, n \neq -1$

$$\Rightarrow \quad V = -2\pi\sigma G\left(\frac{\left(x^2+y^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Bigg|_0^R\right)$$

$$\Rightarrow \quad V = -2\pi\sigma G\left(\sqrt{x^2+y^2}\Big|_0^R\right)$$

$$\Rightarrow \quad V_P = -2\pi\sigma G\left(\sqrt{R^2+x^2}-x\right)$$

Since $\sigma = \frac{M}{\pi R^2}$

$$\Rightarrow \quad V_P = -\frac{2GM}{R^2}\left(\sqrt{R^2+x^2}-x\right)$$

Also, **at the centre of the disc**, we have $x = 0$, so

$$V_{\text{centre}} = V_C = -2\pi\sigma GR$$

$$\Rightarrow \quad V_{\text{centre}} = V_C = -2\pi\left(\frac{M}{\pi R^2}\right)GR$$

$$\Rightarrow \quad V_{\text{centre}} = V_C = -\frac{2GM}{R}$$

If we are asked to find the potential due to an annular disc of inner radius R_1 and outer radius R_2, then we shall be integrating equation (1), within the new limits as

$$V = -2\pi\sigma G\int_{R_1}^{R_2}\left(x^2+y^2\right)^{-\frac{1}{2}} 2ydy \text{, where}$$

$$\sigma = \frac{M}{\pi\left(R_2^2-R_1^2\right)}$$

$$\Rightarrow \quad V = -2\pi\sigma G\left(\frac{\left(x^2+y^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Bigg|_{R_1}^{R_2}\right)$$

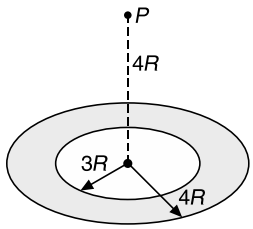
$$\Rightarrow \quad V = -2\pi\sigma G\left(\sqrt{x^2+y^2}\Big|_{R_1}^{R_2}\right)$$

$$\Rightarrow \quad V_P = -2\pi\sigma G\left(\sqrt{R_2^2+x^2}-\sqrt{R_1^2+x^2}\right)$$

$$\Rightarrow \quad V_P = -\frac{2GM}{\left(R_2^2-R_1^2\right)}\left(\sqrt{R_2^2+x^2}-\sqrt{R_1^2+x^2}\right)$$

ILLUSTRATION 30

A thin uniform annular disc (shown in figure) of mass M has outer radius $4R$ and inner radius $3R$. Calculate the work required to take a unit mass from point P lying on its axis to infinity.



SOLUTION

For annular disc, gravitational potential at the point P lying on the axis at a distance x from centre is

$$V_P = -\frac{2GM}{\left(R_2^2 - R_1^2\right)}\left(\sqrt{R_2^2 + x^2} - \sqrt{R_1^2 + x^2}\right)$$

where $R_2 = 4R$, $R_1 = 3R$ and $x = 4R$

$$\Rightarrow \quad V_P = -\frac{2GM}{7R^2}\left(4\sqrt{2}R - 5R\right)$$

Also, $V_\infty = 0$

Since $W_{P\to\infty} = m_0\left(V_\infty - V_P\right)$, where $m_0 = 1$ unit

$$\Rightarrow \quad W_{P\to\infty} = \frac{2GM}{7R}\left(4\sqrt{2} - 5\right)$$

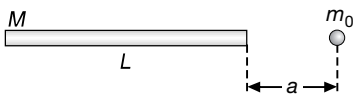
Test Your Concepts-II

Based on Gravitational Potential, Potential Energy and Applications

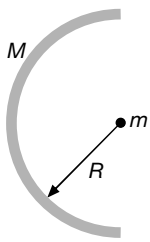
(Solutions on page H.257)

1. Find the gravitational potential energy of a system of eight identical masses each of mass m placed at the corners of a cube of side a.

2. Calculate the gravitational potential energy of a point mass m_0 and a uniform thin rod of mass M, length L. Assume that m_0 is at a distance a from one end of the rod on the extended portion of the rod, as shown in figure.



3. A particle of mass m is placed at the centre of a fixed, uniform semi-circular ring of radius R and mass M as shown in figure. Calculate the work required to displace the particle from centre of the ring to infinity.



4. A particle of mass m is located at a distance r from the centre of a sphere of radius $R(<r)$ and mass M. Find the gravitational potential energy of the particle-sphere system. What happens when the sphere is replaced by a thin shell?

5. A particle of mass m is transferred from the centre of the base of a uniform solid hemisphere of mass M and radius R to infinity. Find the work performed in the process by the gravitational force exerted on the particle by the hemisphere.

6. Two bodies of masses m and M are placed a distance d apart. Find the gravitational potential at the position where the gravitational field due to them is zero.

7. A uniform spherical shell of mass M and radius R has a particle of mass M placed at its centre. Find the gravitational potential at a distance $\frac{R}{2}$ from the centre.

8. Find the gravitational self energy of a uniform
- **(a)** thin spherical shell of mass M and radius R.
- **(b)** solid sphere of mass M and radius R.

9. A thin uniform rod of length 2ℓ has mass per unit length λ. Calculate the gravitational potential as a function of distance a from centre of the rod along the straight line
- **(a)** perpendicular to the rod and passing through the centre,
- **(b)** coinciding with the rod's axis (at points lying outside the rod).

RELATION BETWEEN GRAVITATIONAL FIELD AND GRAVITATIONAL POTENTIAL

Since $dV = -\vec{E}_g \cdot d\vec{l}$

In cartesian coordinates, we have

$\vec{E}_g = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$ and $d\vec{l} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$,

so we get

$$dV = -\left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$\Rightarrow \quad dV = -\left(E_x dx + E_y dy + E_z dz\right)$$

$$\Rightarrow \quad E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y} \text{ and } E_z = -\frac{\partial V}{\partial z}$$

Now, by introducing a new differential quantity called the "del (gradient) operator $(\vec{\nabla})$", we get Gravitational field as the negative gradient of potential. So,

$$\vec{E}_g = -\vec{\nabla}V \qquad \ldots(1)$$

where $\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

$$\Rightarrow \quad \vec{E}_g = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)$$

where

$$\frac{\partial V}{\partial x} = \left(\begin{array}{c}\text{Partial Derivative of } V \text{ w.r.t. } x \quad \text{OR}\\ \text{Derivative of } V \text{ w.r.t. } x \text{ keeping}\\ y \text{ and } z \text{ constant}\end{array}\right)$$

$$\frac{\partial V}{\partial y} = \left(\begin{array}{c}\text{Partial Derivative of } V \text{ w.r.t. } y \quad \text{OR}\\ \text{Derivative of } V \text{ w.r.t. } y \text{ keeping}\\ x \text{ and } z \text{ constant}\end{array}\right)$$

$$\frac{\partial V}{\partial z} = \left(\begin{array}{c}\text{Partial Derivative of } V \text{ w.r.t. } z \quad \text{OR}\\ \text{Derivative of } V \text{ w.r.t. } z \text{ keeping}\\ x \text{ and } y \text{ constant}\end{array}\right)$$

Just to make you understand the concept discussed here, I have a small problem discussed here for your comfort.

PROBLEM: **Find the gravitational field $\vec{E}_g$ for the potential $V = kxy$.**

SOLUTION: Since

$$\vec{E}_g = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right),$$

where

$$\frac{\partial V}{\partial x} = ky\left(\frac{\partial x}{\partial x}\right) = ky,$$

$$\frac{\partial V}{\partial y} = kx\left(\frac{\partial y}{\partial y}\right) = kx \text{ and}$$

$$\frac{\partial V}{\partial z} = 0$$

$$\Rightarrow \quad \vec{E} = -k\left(y\hat{i} + x\hat{j}\right)$$

Notice that $\vec{\nabla}$ operates on a scalar quantity (such as gravitational potential) and results in a vector quantity (such as gravitational field). Mathematically, we can think of $\vec{E}_g$ as the negative of the gradient of the gravitational potential V. Physically, the negative sign implies that if $\vec{E}_g$ goes from higher V to lower V.

ILLUSTRATION 31

Suppose that the gravitational potential in some region of space is given by

$$V(x,\ y,\ z) = V_0\exp(-az)\cos ax,$$

where a is a positive constant. Find the gravitational field everywhere.

SOLUTION

$$\vec{E}_g = -\vec{\nabla}V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

$$E_x = -\frac{\partial V}{\partial x} = -V_0\exp(-az)\frac{\partial}{\partial x}[\cos(ax)]$$

$$\Rightarrow \quad E_x = \frac{\partial V}{\partial x} = +aV_0\sin(ax)\exp(-az)$$

Since the given expression has no dependence on y, so

$$E_y = \frac{\partial V}{\partial y} = 0$$

$$E_z = -\frac{\partial V}{\partial z} = -V_0\cos(ax)\frac{\partial}{\partial z}[\exp(-az)]$$

$$\Rightarrow \quad E_z = -\frac{\partial V}{\partial z} = aV_0\cos(ax)\exp(-az)$$

$$\Rightarrow \quad \vec{E}_g = E_x\hat{i} + E_z\hat{k} \qquad \{\because\ E_y = 0\}$$

$$\Rightarrow \quad \vec{E}_g = aV_0\exp(-az)[\sin(ax)\hat{i} + \cos(ax)\hat{k}]$$

Problem Solving Technique(s)

If we are given $\vec{E}_g(x, y, z) = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$, then

$$V = -\int \vec{E}_g \cdot d\vec{\ell}$$

where, $d\vec{\ell} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$\Rightarrow \quad V = -\left(\int E_x dx + \int E_y dy + \int E_z dx\right)$$

The integrals are to be calculated within specified limits.

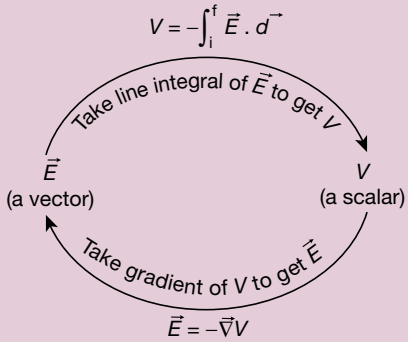


ILLUSTRATION 32

A gravitational field is given by $\vec{E}_g = -k\left(y\hat{i} + x\hat{j}\right)$. Find the gravitational potential generating such a field.

SOLUTION

Since $V = -\int \vec{E}_g \cdot d\vec{l}$

Let $d\vec{l} = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz$

$$\Rightarrow \quad V = \int k\left(y\hat{i} + x\hat{j}\right)\cdot\left(\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz\right)$$

$$\Rightarrow \quad V = k\int y\,dx + x\,dy$$

$$\Rightarrow \quad V = k\int d(xy) \qquad \{\because d(xy) = x\,dy + y\,dx\}$$

$$\Rightarrow \quad V = k(xy) + \text{constant}$$

ILLUSTRATION 33

Find the potential function $V(x, y)$ of a gravitational field $\vec{E}_g = 2axy\,\hat{i} + a\left(x^2 - y^2\right)\hat{j}$ where a is a constant.

SOLUTION

Let V_0 be the potential at origin and V be the potential at the point (x, y), then we have

$$dV = -\vec{E}_g \cdot d\vec{\ell}$$

$$\Rightarrow \quad dV = -\left(2axydx + a\left(x^2 - y^2\right)dy\right)$$

$$\Rightarrow \quad \int_{(0,0)}^{(x,y)} dV = -\int_{(0,0)}^{(x,y)} 2axydx + a\left(x^2 - y^2\right)dy$$

$$\Rightarrow \quad V - V_0 = -\int_{(0,0)}^{(x,y)} \left(2axy\,dx + ax^2\,dy\right) - ay^2 dy$$

$$\Rightarrow \quad V - V_0 = -\int_{(0,0)}^{(x,y)} \left\{d\left(ax^2y\right) - ay^2 dy\right\}$$

$$\Rightarrow \quad V - V_0 = \left[\left(-ax^2y + \frac{ay^3}{3}\right)\right]_{(0,0)}^{(x,y)}$$

$$\Rightarrow \quad V = V_0 - ax^2y + \frac{ay^3}{3}$$

Remark(s)

(a) For an attractive system U is always **NEGATIVE**.
(b) For a repulsive system U is always **POSITIVE**.
(c) For a stable system U must be **MINIMUM**

i.e., $\dfrac{dU}{dx} = 0$

Since $F = -\dfrac{dU}{dx}$

$$\Rightarrow \quad F = -\frac{dU}{dx} = 0 \qquad \text{(FOR A STABLE SYSTEM)}$$

(d) If two points A and B are at potentials V_A and V_B, then work done in taking a test mass m_0 from A to B is $W_{A\to B} = m_0(V_B - V_A)$

(e) Consider two points A and B situated in a uniform gravitational field at a distance d such that the line joining A and B is parallel to the field. If V be the potential difference between them, then

$$V = Ed\,(\text{in magnitude})$$

Test Your Concepts-III

Based on Relation Between Gravitational Field and Potential

(Solutions on page H.259)

1. In a region of space the gravitational field is given by $\vec{E} = \left(x\hat{i} - 2y\hat{j} + z\hat{k}\right)\ \text{Nkg}^{-1}$. Calculate the potential difference V_{AB} between $A(2,1,0)\,\text{m}$ and $B(0,2,4)\,\text{m}$.
2. Determine the potential $V(x,y,z)$ of a gravitational field $\vec{E}_g = ay\hat{i} + (ax + bz)\hat{j} + by\hat{k}$, where a and b are constants, $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors of the axis x, y, z.
3. Find potential difference V_{AB} between $A(0,0,0)\,\text{m}$ and $B(1,1,1)\,\text{m}$ in a gravitational field given by
   (a) $\vec{E} = y\hat{i} + x\hat{j}$
   (b) $\vec{E} = 3x^2y\hat{i} + x^3\hat{j}$.
   What can you say about the nature of the fields? Explain.
4. Find the gravitational field due to the gravitational potential given by $V(x,y,z) = 3x^2y + y^3z$.
5. Determine the gravitational field strength vector if the potential of this field depends on x, y coordinates as
   (a) $V = a\left(x^2 - y^2\right)$
   (b) $V = axy$
   where a is a positive constant.
6. Over a certain region of space, the gravitational potential is $V = 5x - 3x^2y + 2yz^2$. Find the expressions for the x, y and z components of the gravitational field over this region. Also calculate the magnitude of the field at the point P that has coordinates $(1,0,-2)\,\text{m}$?
7. In a region of space, the gravitational potential is given by, $V = 20(x+y)\ \text{Jkg}^{-1}$. Find the magnitude of the gravitational force on a particle of mass 0.5 kg placed at the origin.
8. The potential of a certain gravitational field has the form $V = a\left(x^2 + y^2\right) + bz^2$, where a and $b$are constants. Find the magnitude and direction of the gravitational field strength vector.
9. In a region of space, the gravitational potential is represented by $V = 2x + 3y - z$. Obtain an expression for the gravitational field strength.

CONSERVATION LAWS FOR GRAVITATIONAL SYSTEMS

For a variety of problems, we have to understand the Conservation Laws that can be applied to the gravitational system of particles.

LAW I: Law of conservation of Linear Momentum

For a gravitational system of masses, when no external force acts on the system, then we observe that no net internal forces will be acting on the system because gravitational forces always form an action reaction pair. So, by a suitable selection of a system or subsystem we can apply Law of Conservation of Linear Momentum.

$$\begin{pmatrix}\text{Total Initial}\\ \text{Momentum}\end{pmatrix} = \begin{pmatrix}\text{Total Final}\\ \text{Momentum}\end{pmatrix}$$

LAW II: Law of Conservation of Mechanical Energy

Since gravitational forces are conservative in nature, so, in the absence of an external force and a dissipative force (such as forces of friction) we can easily use this law as

$$\begin{pmatrix}\text{Total Initial Energy}\\ \text{of the system}\end{pmatrix} = \begin{pmatrix}\text{Total Final Energy}\\ \text{of the system}\end{pmatrix}$$

Here we need to read the problem carefully for the energies possessed by the system (as selected) initially and finally.

Another point which should be kept in mind is that gravitational field is a conservative field. Work done by gravitational field on a moving mass only depends on the positions of the mass. If a mass is moving freely in the gravitational field, work done by gravitational field on mass is equal to the change in kinetic energy of the mass i.e.,

$$W = \Delta K$$

In the presence of an external force and the dissipative non- conservative force, the law will be suitably modified as below i.e.,

$$W_{\text{ext}} + W_{nc} = \Delta U + \Delta K$$

LAW III: Law of Conservation of Angular Momentum

Since gravitational forces are central in nature, so torque due to gravitational force will be zero. So, we can easily use Law of Conservation of Angular Momentum too.

ILLUSTRATION 34

Two masses m_1 and m_2 at an infinite distance from each other are initially at rest, start interacting gravitationally. Find their velocity of approach when they are at a distance r apart. Assume air drag and other dissipative forces to be absent.

SOLUTION

Since both the particles, initially at rest and lying at infinity, start interacting gravitationally, i.e. the gravitational force between them is an internal force and also no external force is acting on them, so according to Law of Conservation of Linear Momentum we have

$$\begin{pmatrix}\text{Total Initial}\\ \text{Momentum}\end{pmatrix}_{\text{at }\infty} = \begin{pmatrix}\text{Total Final}\\ \text{Momentum}\end{pmatrix}_{\text{at }r}$$

$$\Rightarrow \quad 0+0 = m_1v_1 + m_2(-v_2)$$

$$\Rightarrow \quad m_1v_1 = m_2v_2 \qquad \ldots(1)$$

Further, applying the Law of Conservation of Energy, we get

$$(U+K)_{at\ \infty} = (U+K)_{at\ r}$$

$$\Rightarrow \quad 0+0 = -\frac{Gm_1m_2}{r} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad \ldots(2)$$

From equation (1), we have $v_2 = \frac{m_1v_1}{m_2}$. So, from (2), we get

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2\left(\frac{m_1v_1}{m_2}\right)^2 = \frac{Gm_1m_2}{r}$$

$$\Rightarrow \quad \frac{1}{2}m_1v_1^2\left(1+\frac{m_1}{m_2}\right) = \frac{Gm_1m_2}{r}$$

$$\Rightarrow \quad v_1 = m_2\sqrt{\frac{2G}{(m_1+m_2)r}}$$

Similarly using the above equations, we get

$$v_2 = m_1\sqrt{\frac{2G}{(m_1+m_2)r}}$$

If v_r be their velocity of approach, then we have

$$v_r = v_1 + v_2$$

$$\Rightarrow \quad v_r = m_2\sqrt{\frac{2G}{(m_1+m_2)r}} + m_1\sqrt{\frac{2G}{(m_1+m_2)r}}$$

$$\Rightarrow \quad v_r = (m_1+m_2)\sqrt{\frac{2G}{(m_1+m_2)r}}$$

$$\Rightarrow \quad v_r = \sqrt{\frac{2G(m_1+m_2)}{r}}$$

Problem Solving Technique(s)

This problem can also be solved quickly by using the concept of reduced mass $\mu = \frac{m_1m_2}{m_1+m_2}$ and the Law of Conservation of Energy according to which, we have

$$(U+K)_{\text{at }\infty} = (U+K)_{\text{at }r}$$

$$0+0 = \frac{1}{2}\mu v_r^2 - \frac{Gm_1m_2}{r} \qquad \ldots(1)$$

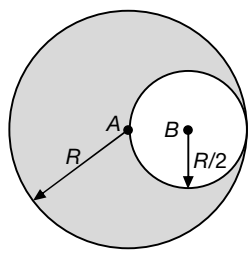
where, μ = Reduced Mass $= \frac{m_1m_2}{m_1+m_2}$

Substituting in equation (1), we get

$$v_r = \sqrt{\frac{2G(m_1+m_2)}{r}}$$

ILLUSTRATION 35

Inside a fixed sphere of radius R and uniform density ρ, there is spherical cavity of radius $\frac{R}{2}$ such that surface of the cavity passes through the centre of the sphere as shown in figure. A particle of mass m is released from rest at centre B of the cavity. Calculate velocity with which particle strikes the centre A of the sphere. Neglect earth's gravity. Initially sphere and particle are at rest.



SOLUTION

Applying conservation of mechanical energy, we have

$$(U+K)_{\text{at } B} = (U+K)_{\text{at } A}$$

$$\Rightarrow \quad U_B + K_B = U_A + K_A$$

$$\Rightarrow \quad U_B + 0 = U_A + \frac{1}{2}mv_A^2$$

where $U_B = mV_B$ and $U_A = mV_A$, so we have

$$\frac{1}{2}mv_A^2 = U_B - U_A = m(V_B - V_A)$$

$$\Rightarrow \quad v_A = \sqrt{2(V_B - V_A)} \qquad \ldots(1)$$

Potential at A

$$V_A = \begin{pmatrix}\text{Potential Due}\\ \text{to the Complete}\\ \text{Sphere at A}\end{pmatrix} - \begin{pmatrix}\text{Potential Due}\\ \text{to the Spherical}\\ \text{Cavity at A}\end{pmatrix}$$

$$V_A = -\frac{3}{2}\frac{GM}{R} - \left(-\frac{GM'}{r}\right) = \frac{GM'}{r} - \frac{3}{2}\frac{GM}{R}$$

where

$$M = \frac{4}{3}\pi R^3\rho,\ r = \frac{R}{2} \text{ and } M' = \frac{4}{3}\pi r^3\rho = \frac{\pi\rho R^3}{6}$$

Substituting the values, we get

$$V_A = \frac{G}{R}\left(\frac{\pi\rho R^3}{3} - 2\pi\rho R^3\right) = -\frac{5}{3}\pi G\rho R^2$$

Potential at B

$$V_B = \begin{pmatrix}\text{Potential Due}\\ \text{to the Complete}\\ \text{Sphere at } B\end{pmatrix} - \begin{pmatrix}\text{Potential Due}\\ \text{to the Spherical}\\ \text{Cavity at } B\end{pmatrix}$$

$$\Rightarrow \quad V_B = -\frac{GM}{2R^3}\left(3R^2 - r^2\right) - \left(-\frac{3}{2}\frac{GM'}{r}\right)$$

where

$$M = \frac{4}{3}\pi R^3\rho,\ r = \frac{R}{2} \text{ and } M' = \frac{4}{3}\pi r^3\rho = \frac{\pi\rho R^3}{6}$$

$$\Rightarrow \quad V_B = -\frac{11}{8}\frac{GM}{R} + \frac{3GM'}{R}$$

$$\Rightarrow \quad V_B = \frac{G}{R}\left(\frac{\pi\rho R^3}{2} - \frac{11\pi\rho R^3}{6}\right) = -\frac{4}{3}\pi G\rho R^2$$

$$\Rightarrow \quad V_B - V_A = \frac{1}{3}\pi G\rho R^2$$

So, from equation (1)

$$v = \sqrt{\frac{2}{3}\pi G\rho R^2}$$

LAUNCHING SPEED OF A PROJECTILE

Let us suppose we have to launch a projectile having mass m to reach a height h. The gravitational potential energy of the projectile on the surface of Earth is

$$U(R) = -\frac{GMm}{R} \qquad \ldots(1)$$

Its gravitational potential energy at height h from the surface of earth is

$$U(R+h) = -\frac{GMm}{(R+h)} \qquad \ldots(2)$$

Therefore, the change in potential energy is

$$\Delta U = U(R+h) - U(R) = -GMm\left\{\frac{1}{R+h} - \frac{1}{R}\right\}$$

$$\Rightarrow \quad \Delta U = \frac{GM}{R^2}\frac{mh}{\left(1+\frac{h}{R}\right)} = \frac{mgh}{\left(1+\frac{h}{R}\right)}$$

If h R, $\Delta U = mgh$, which we have already used in the **Chapter Work, Energy and Power.**

This difference in the potential energy is fulfilled by providing an initial kinetic energy. If v be the velocity then

$$\frac{1}{2}mv^2 = \frac{mgh}{\left(1+\frac{h}{R}\right)}$$

$$\Rightarrow \quad v = \sqrt{\frac{2gh}{\left(1+\frac{h}{R}\right)}}$$

MAXIMUM HEIGHT ATTAINED BY A PROJECTILE

Let a projectile of mass m be projected vertically upward with velocity v, so that it attains a maximum height h. At maximum height h, the velocity of particle is zero, so kinetic energy is zero. When a body is thrown vertically upwards with a velocity v then by Law of Conservation of Energy,

$$(U+K)_{\text{at surface}} = (U+K)_{\text{at height } h}$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mv^2=-\frac{GMm}{R+h}+0$$

$$\Rightarrow \quad \frac{v^2}{2}=GM\left(\frac{1}{R}-\frac{1}{R+h}\right)$$

$$\Rightarrow \quad \frac{v^2}{2}=\frac{GMh}{R(R+h)}$$

$$\Rightarrow \quad \frac{2GM}{v^2R}=\frac{R+h}{h}$$

$$\Rightarrow \quad \frac{2GM}{v^2R}=1+\frac{R}{h}$$

Since, $v_e=\sqrt{2gR}=\sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad h=\frac{R}{\left(\frac{2GM}{v^2R}-1\right)}=\frac{Rv^2}{2gR-v^2}$$

$$\Rightarrow \quad h=R\left(\frac{v^2}{v_e^2-v^2}\right)$$

From this we can see that

(i) if $v=v_e$ or $v^2=v_e^2=2gR$, $h\to\infty$ and if we have

(ii) v to be small, then we have $h=\frac{v^2}{2g}$

IMPORTANT

If $h\ll R$ then

$$v^2=\frac{2GM}{R}\left[1-\left(1+\frac{h}{R}\right)^{-1}\right]$$

$$\Rightarrow \quad v^2=\frac{2GM}{R}\left[1-\left(1-\frac{h}{R}\right)\right]$$

$$\Rightarrow \quad v^2=\frac{2GM}{R}\left(\frac{h}{R}\right)$$

$$\Rightarrow \quad v^2=2\left(\frac{GM}{R^2}\right)h$$

$$\Rightarrow \quad v^2=2gh$$

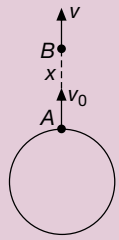
$$\Rightarrow \quad v=\sqrt{2gh}$$

So, the formula $v=\sqrt{2gh}$ only holds good for heights close to surface of the earth.

Problem Solving Technique(s)

Time taken by the particle to reach a height *h*

Suppose a particle of mass m is projected vertically upwards with a speed v_0. We want to find the time taken by the particle to reach a height h. First of all we find the speed of the particle at a height x by applying Law of Conservation of Mechanical Energy, i.e.,



$$\frac{1}{2}m\left(v_0^2-v^2\right)=\frac{mgx}{1+\frac{x}{R}}$$

or $$v=\sqrt{v_0^2-\frac{2gx}{1+\frac{x}{R}}} \qquad ...(1)$$

Now, v can be written as $-\frac{dx}{dt}$

Hence, equation (1) reduces to

$$\frac{-dx}{\sqrt{v_0^2-\frac{2gx}{1+\frac{x}{R}}}}=dt$$

By integrating with limits from 0 to h on left hand side and from 0 to t on right hand side we get the desired time.

ILLUSTRATION 36

A particle is projected from the surface of earth with an initial speed of $4\ \text{kms}^{-1}$. Find the maximum height attained by the particle. Radius of earth is $R=6400\ \text{km}$ and $g=9.8\ \text{ms}^{-2}$.

SOLUTION

The maximum height attained by the particle is,

$$h=\frac{v^2}{2g-\frac{v^2}{R}}$$

Substituting the values, we have

$$h = \frac{(4\times10^3)^2}{2\times9.8 - \frac{(4\times10^3)^2}{6.4\times10^6}} = 9.35\times10^5 \text{ m}$$

$$\Rightarrow \quad h \approx 935 \text{ km}$$

ILLUSTRATION 37

A spaceship is launched into a circular orbit close to the earth's surface. What additional velocity has now to be imparted to the spaceship in the orbit to overcome the gravitational pull. Given that, radius of the earth is 6400 km and $g = 9.8 \text{ ms}^{-2}$.

SOLUTION

The speed of the spaceship in a circular orbit close to the earth's surface is given by,

$$v_0 = \sqrt{gR}$$

and escape velocity is given by,

$$v_e = \sqrt{2gR}$$

So, the additional velocity required to escape is given by

$$v_e - v_0 = \sqrt{2gR} - \sqrt{gR}$$

$$\Rightarrow \quad v_e - v_0 = (\sqrt{2}-1)\sqrt{gR}$$

Substituting the values of g and R, we get

$$v_e - v_0 = 3.278\times10^3 \text{ ms}^{-1}$$

ESCAPE SPEED

The minimum velocity to be imparted to a body from the surface of the earth (or planet) such that it just escapes the gravitational pull of the earth **(i.e. reaches infinity and stops there)** is called the escape velocity.

When a body is launched from the surface of the earth with the minimum speed, say **escape speed** v_e then it will reach infinity with zero speed. Then by Law of Conservation of Energy, we have

$$(U+K)_{\text{at surface}} = (U+K)_{\text{at }\infty}$$

$$\Rightarrow \quad -\frac{GmM}{R} + \frac{1}{2}mv_e^2 = 0 + 0$$

$$\Rightarrow \quad v_e = \sqrt{\frac{2GM}{R}}$$

Substituting $M = 6\times10^{24}$ kg,

$G = 6.67\times10^{-11} \text{ Nm}^2kg^{-2}$ and

$R = 6.4\times10^6$ m, we get

$v_{\text{esc}} = 11.2 \text{ kms}^{-1}$

Note that the escape speed does not depend on the mass of the body launched from the surface of the earth.

Also, escape velocity is independent of the angle of launch or the angle of projection.

Conceptual Note(s)

(a) If the particle is launched from the surface of the earth with a velocity $u > v_e$, then the body will reach infinity with a non zero velocity v which can be calculated by using the Law of Conservation of Energy, so

$$(U+K)_{\text{at surface}} = (U+K)_{\text{at }\infty}$$

$$\Rightarrow \quad -\frac{GmM}{R} + \frac{1}{2}mu^2 = 0 + \frac{1}{2}mv^2$$

(b) If the particle is launched from the surface of the earth with a velocity $u < v_e$, then the body will attain a maximum height h (say) and then return to the surface of the earth. The maximum height h can be calculated by using the Law of Conservation of Energy, so

$$(U+K)_{\text{at surface}} = (U+K)_{\text{at } h}$$

$$\Rightarrow \quad -\frac{GmM}{R} + \frac{1}{2}mu^2 = -\frac{GmM}{R+h} + 0$$

ILLUSTRATION 38

A rocket is fired vertically with half the escape speed. What is its maximum altitude in terms of the radius of the earth R? Ignore the earth's rotation.

SOLUTION

This Illustration can be solved easily by using the Law of Conservation of Energy, according to which

$$(U+K)_{\text{at surface}} = (U+K)_{\text{at height } h}$$

$$\Rightarrow \quad -\frac{GmM}{R} + \frac{1}{2}m\left(\frac{v_e}{2}\right)^2 = -\frac{GmM}{R+h} + \frac{1}{2}m(0)^2$$

However, we know that $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad -\frac{GmM}{R}+\frac{GmM}{4R}=-\frac{GmM}{R+h}$$

$$\Rightarrow \quad -\frac{3}{4}\frac{GmM}{R}=-\frac{GmM}{R+h}$$

$$\Rightarrow \quad 3R+3h=4R$$

$$\Rightarrow \quad h=\frac{R}{3}$$

Test Your Concepts-IV

Based on Conservation Laws, Escape Velocity and Applications

(Solutions on page H.261)

1. If a body of mass m is projected vertically upward from the surface of the earth (of radius R) so as to reach a height nR above the surface. Then find
(a) the increase in its potential energy.
(b) the velocity with which it must be projected.

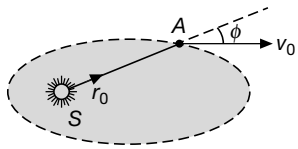
2. If a body is released from a great distance from the centre of the earth, find its velocity when it strikes the surface of the earth. Given, $R = 6400$ km and $g = 9.8\ \text{ms}^{-2}$.

3. Two particles of masses 20 kg and 10 kg are initially at a distance of 1 m. Find the speeds of the particles when the separation between them decreases to 0.5 m, if only gravitational forces are acting.

4. Two bodies, each of mass M, are kept fixed with a separation $2L$. A particle of mass m is projected from the mid-point of the line joining their centres, perpendicular to the line joining them. If the universal gravitational constant is denoted by G, then calculate the minimum initial velocity of the mass m so that it escapes the gravitational field of the two bodies.

5. Distance between the centres of two stars is $10a$. The masses of these stars are M and $16M$ and their radii a and $2a$ respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G, M and a.

6. Find the maximum and minimum distances of the planet A from the sun S if at a certain moment of time it was at a distance r_0 and travelling with the velocity v_0, with the angle between the radius vector and velocity vector being equal to ϕ.



7. Find the total mechanical energy of a body launched from the surface of earth with a velocity equal to escape velocity.

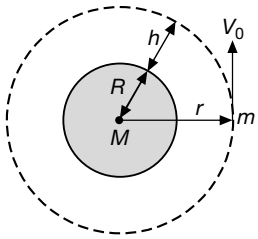
8. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2\ \text{kms}^{-1}$. Calculate the minimum initial velocity (v_s) required for the rocket to be able to leave the Sun-Earth system. Ignore the rotation and revolution of the Earth and the presence of any other planet.

9. Given a thin homogeneous disc of radius a and mass m_1. A particle of mass m_2 is placed at a distance ℓ from the disk on its axis of symmetry. Initially both are motionless in free space but they ultimately collide because of gravitational attraction. Find the relative velocity at the time of collision. Assume $\ell \gg a$.

10. Two spherical planets 1 and 2 have the same uniform density ρ, masses M_1 and M_2, surface areas A and $4A$ respectively. The escape velocity from the surface of planets 1 and 2 are v_1 and v_2 respectively. The escape velocity from the surface of a spherical planet 3 that has same uniform density and mass equal to the sum of masses of planets 1 and 2 is v_3. Calculate $\frac{v_3^3}{v_1^3}$ and $\frac{v_2}{v_1}$.

SATELLITES

Consider a satellite of mass m revolving around the earth in circular orbit of radius r.



The gravitational pull between the satellite and earth $\left(=\frac{GMm}{r^2}\right)$ provides the necessary centripetal force to satellite to revolve in the orbit.

$$\Rightarrow \quad \frac{mv_0^2}{r}=\frac{GMm}{r^2}$$

So, orbital speed is given by

$$v_0=\sqrt{\frac{GM}{r}}$$

If h is height of satellite from earth's surface and R is the radius of earth, then

$$r=R+h$$

$$\Rightarrow \quad v_0=\sqrt{\frac{GM}{R+h}}=\sqrt{\frac{R^2g}{R+h}}$$

$$\Rightarrow \quad v_0=R\sqrt{\frac{g}{R+h}}$$

If T is the time period of satellite, then

$$T=\frac{2\pi r}{v_0}=\frac{2\pi r}{\sqrt{\frac{GM}{r}}}=2\pi\sqrt{\frac{r^3}{GM}}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{(R+h)^3}{GM}}=2\pi\sqrt{\frac{r^3}{GM}}$$

Problem Solving Technique(s)

(a) The square of the time period is directly proportional to the cube of the distance of separation.

(b) We observe that the time period and the orbital velocity of a satellite of mass *m* is independent of mass of satellite and depend upon mass of planet (M) and the radius of orbit (r).

(c) If satellite revolves close to the surface of planet, then $r\simeq R$ (= the radius of planet). So

$$T=2\pi\sqrt{\frac{R^3}{G\frac{4}{3}\pi R^3\rho}}$$

$$\Rightarrow \quad T=\sqrt{\frac{3\pi}{G\rho}}$$

i.e., Time period for two satellites revolving about two different planets close to their surface is proportional to $\frac{1}{\sqrt{\rho}}$. Hence

$$\frac{T_1}{T_2}=\sqrt{\frac{\rho_2}{\rho_1}}$$

(d) If a satellite is close to earth's surface such that $h\ll R$, then orbital speed

$$v_0=\sqrt{\frac{GM}{R}}=\sqrt{\frac{R^2g}{R}}=\sqrt{Rg}$$

The periodic time *T* and the angular velocity ω are given by

$$T=2\pi\sqrt{\frac{R^3}{GM}}=2\pi\sqrt{\frac{R^3}{R^2g}}=2\pi\sqrt{\frac{R}{g}}$$

$$\omega=\sqrt{\frac{GM}{R^3}}$$

For earth $R=6400\text{ km}=6.4\times10^6\text{ m}$ and $g=9.8\text{ ms}^{-2}$

orbital speed, $v_0=\sqrt{6.4\times10^6\times9.8}$

$$v_0=8\times10^3\text{ ms}^{-1}=8\text{ kms}^{-1}$$

and period of revolution i.e.

$$T=2\pi\sqrt{\frac{6.4\times10^6}{9.8}}=5\times10^6\text{ sec}=84.6\text{ min}$$

(e) The bodies are weightless in the satellite because the centrifugal force on body in rotating satellite is balanced by gravitational pull of earth.

ILLUSTRATION 39

A satellite revolving in a circular equatorial orbit of radius $r=2.00\times10^4$ km from west to east appears over a certain point at the equator every 11.6 hr. Using this data, calculate the mass of the earth. The gravitational constant is supposed to be known.

SOLUTION

Here, the absolute angular velocity of satellite is given by

$$\omega = \omega_s + \omega_E$$

where ω_E is the angular velocity of earth, which is also from west to east.

$$\Rightarrow \quad \omega = \frac{2\pi}{t} + \frac{2\pi}{T}$$

where, $t = 11.6$ hr and $T = 24$ hr

Since, $\omega = \sqrt{\frac{GM}{r^3}}$

$$\Rightarrow \quad \sqrt{\frac{GM}{r^3}} = \frac{2\pi}{t} + \frac{2\pi}{T}$$

$$\Rightarrow \quad M = \frac{4\pi^2 r^3}{G}\left(\frac{1}{t} + \frac{1}{T}\right)^2$$

$$\Rightarrow \quad M = \frac{4\pi^2 (2\times 10^7)^3}{(6.67\times 10^{-11})}\left(\frac{1}{11.6\times 3600} + \frac{1}{24\times 3600}\right)^2$$

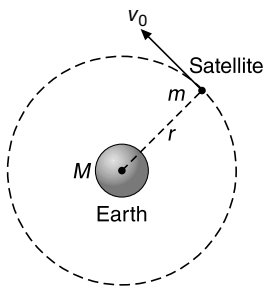
$$\Rightarrow \quad M \approx 6.0\times 10^{24} \text{ kg}$$

ILLUSTRATION 40

A satellite of mass m is moving in a circular orbit of radius r. Calculate its angular momentum with respect to the centre of the orbit in terms of the mass of the earth.

SOLUTION

The situation in which a satellite is revolving around the earth is shown in figure.



The angular momentum of the satellite with respect to the centre of orbit is given by

$$L = mv_0 r \sin(90°) = mv_0 r$$

The direction of $\vec{L}$ is perpendicular to the plane of the orbit.

Since the orbital speed v_0 of satellite is given by

$$v_0 = \sqrt{\frac{GM}{r}}$$

$$\Rightarrow \quad L = mv_0 r = m\sqrt{\frac{GM}{r}}r$$

$$\Rightarrow \quad L = \sqrt{GMm^2 r}$$

KINETIC ENERGY OF A SATELLITE

The kinetic energy of a satellite having mass m and orbital speed v_0 is given by

$$K = \frac{1}{2}mv_0^2 = \frac{GMm}{2r} \qquad \left\{\because v_0 = \sqrt{\frac{GM}{r}}\right\}$$

POTENTIAL ENERGY OF A SATELLITE

The potential energy of a satellite of mass m orbiting around a planet of mass M in an orbit of radius r is given by

$$U = -\frac{GMm}{r}$$

TOTAL MECHANICAL ENERGY OF A SATELLITE

Total mechanical energy (E) is

$$E = K + U = -\frac{GMm}{2r}$$

This is negative, indicating that the satellite is bound to the planet.

Problem Solving Technique(s)

$$\text{T.E.} = -\text{K.E.} = \frac{1}{2}(\text{P.E.})$$

$$\Rightarrow \quad \text{P.E.} = -2(\text{K.E.}) = 2(\text{T.E.})$$

BINDING ENERGY OF A SATELLITE

Binding energy is the minimum energy required to make a satellite free from gravitational attraction of the planet so, binding energy (B) is given by

$$B = -E = \frac{GMm}{2r}$$

COMMUNICATION SATELLITE

It is also called Geostationary or Geosynchronous or Parking or Relay or Transmission Satellite. It is an artificial satellite of earth and appears stationary to any observer on the surface of earth. A satellite is said to be Geostationary when it fulfills the following conditions.

(a) The orbit of the satellite must be circular and in the equatorial plane of the earth. (Equatorial plane divides the earth into two equal hemispheres).

(b) The sense of revolution of satellite must be same as the sense of rotation of the earth i.e. from west to east.

(c) The period of revolution of the satellite must be equal to the period of rotation of earth about its axis. Hence,

$$T = 24 \text{ hour} = 86400 \text{ s}$$

Since

$$T = 2\pi\sqrt{\frac{r^3}{GM}} = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

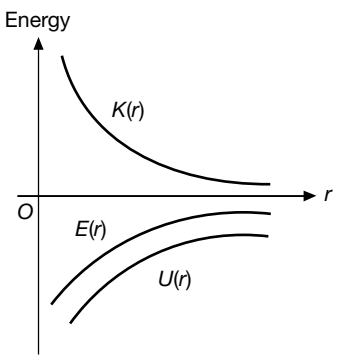
$$\Rightarrow \quad R + h = r \quad 42400 \text{ km}$$

$$\Rightarrow \quad h \quad 36000 \text{ km}$$

(d) A geostationary satellite must revolve at a height of 36000 km above equator or in an orbit of radius 42400 km.

ENERGY GRAPH

The variations of kinetic energy K have been shown by the graph as shown, potential energy U and total energy E with radius for a satellite in a circular orbit. From following graph it is clear that the value of U and E are negative and that of K is positive. As the r increases three curves have the tendency to approach the value of zero.



IMPORTANT

1. Total energy of a closed system is always negative. For example energy of planet-sun, satellite-earth or electron-nucleus system are always negative.

2. Since the Law of Force obeys the inverse square law i.e., $F \propto \frac{1}{r^2}$. Also, $F = \frac{-dU}{dr}$.

$$\Rightarrow \quad K = \frac{|U|}{2} = |E|$$

The same is true for electron-nucleus system because there also, the gravitational force

$$F_e \propto \frac{1}{r^2}$$

ILLUSTRATION 41

A satellite of mass m is moving in a circular orbit of radius r. Due to atmospheric drag if it loses energy at a constant rate W. Find the time in which satellite will fall to the surface of earth. The mass of the earth is M and radius R.

SOLUTION

$$E_i = -\frac{GMm}{2r}, \; E_f = -\frac{GMm}{2R}, \; E_i > E_f$$

$$\Rightarrow \quad \Delta E = E_i - E_f = \frac{GMm}{2}\left(\frac{r-R}{Rr}\right)$$

$$\Rightarrow \quad t = \frac{\Delta E}{W}$$

$$\Rightarrow \quad t = \frac{GMm}{2W}\left(\frac{1}{R} - \frac{1}{r}\right)$$

ILLUSTRATION 42

A rocket with a payload of mass m is at rest at the surface of the earth. Calculate the work needed to raise the payload to the following states:

(a) at rest at an altitude equal to radius of earth (R)

(b) in circular orbit at an altitude R

SOLUTION

In both cases the initial energy of the payload is purely potential energy.

$$E_1 = K_1 + U_1 = 0 - \frac{GmM}{R}$$

(a) At the given altitude the distance to the center of the earth is $r = 2R$.

$$E_2 = K_2 + U_2 = 0 - \frac{GmM}{2R}$$

The work needed is $E_2 - E_1 = +\frac{GmM}{2R}$

(b) Since, the total energy in orbit is

$$E_3 = K_3 + U_3 = -\frac{GmM}{2(2R)} = -\frac{GmM}{4R}$$

The work needed is $E_3 - E_1 = +\frac{3GmM}{4R}$.

Naturally, it takes more work to put the satellite into orbit than merely to raise it to the same altitude.

ILLUSTRATION 43

A sky lab of mass 2×10^3 kg is first launched from the surface of earth in a circular orbit of the radius $2R$ (from the centre of earth) and then it is shifted from this circular orbit to another circular orbit of radius $3R$. Calculate the minimum energy required

(a) to place the lab in the first orbit
(b) to shift the lab from first orbit to the second orbit,
Given $R = 6400$ km and $g = 10\ \text{ms}^{-2}$.

SOLUTION

According to the problem sky lab exists in three energy levels, our task is to calculate the total energy of the three level. i.e. on the surface, first orbit and second orbit. Energy difference between first orbit and surface of the earth is the answer of (a) and that between first orbit and second orbit is the answer of (b).

Total mechanical energy of the sky lab on the surface of earth

$$E_1 = KE + PE = 0 + -\frac{GMm}{R} = -\frac{GMm}{R} \qquad \ldots(1)$$

Total mechanical energy of the sky lab in first orbit i.e.

$$E_2 = -\frac{GMm}{4R} \qquad \ldots(2)$$

Total mechanical energy of the skylab in the second orbit i.e.

$$E_3 = -\frac{GMm}{6R}$$

(a) Required energy

$$\Delta E_1 = -\frac{GMm}{4R} + \frac{GMm}{R} = \frac{3}{4}\frac{GMm}{R}$$

$$\Rightarrow \quad \Delta E_1 = \frac{3}{4}mgR = 9.6\times10^{10}\ \text{J}$$

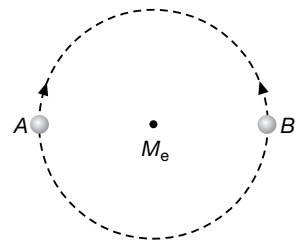
(b) In this case required energy

$$\Delta E_2 = -\frac{GMm}{6R} + \frac{GMm}{4R} = \frac{1}{12}\frac{GMm}{R}$$

$$\Rightarrow \quad \Delta E_2 = \frac{1}{12}mgR = 1.1\times10^{10}\ \text{J}$$

ILLUSTRATION 44

Consider two satellite A and B of equal mass moving in the same circular orbit of radius r around the earth but in the opposite sense and therefore, on a collision course as shown in the figure.



(a) Find the total mechanical energy $E_A + E_B$ of the two satellite plus-earth system before collision.
(b) If the collision is completely inelastic find the total mechanical energy immediately after collision. Describe the subsequent motion of the combined satellite.

SOLUTION

(a) $E_A = -\frac{1}{2}\frac{GM_em}{r}$; $E_B = -\frac{1}{2}\frac{GM_em}{r}$

$$E_A + E_B = -\frac{GM_em}{r}$$

(b) Applying Law of Conservation of Momentum along the axis perpendicular to the gravitational force i.e., along the tangent, we get

$$mv + (-mv) = 2mv'$$

$$\Rightarrow \quad v' = 0$$

Energy of the system = Gravitational energy

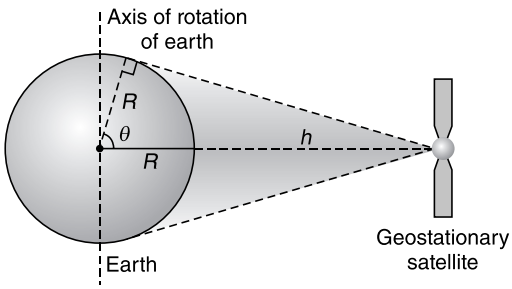
$$E = -\frac{2GM_em}{r}$$

Since velocity = 0, satellite will fall vertically down.

BROADCASTING REGION OF A SATELLITE

If we know the height of a geostationary satellite, then we can easily find the area of earth exposed to the satellite or area of the region in which the communication can be made using this satellite. Figure shows earth (of radius R) and its area exposed to a geostationary satellite. The angle θ is given by

$$\theta = \cos^{-1}\left(\frac{R}{R+h}\right)$$



The solid angle Ω which the exposed area subtends at the earth's centre is given by

$$\Omega = 2\pi(1-\cos\theta)$$

$$\Rightarrow \quad \Omega = 2\pi\left(1-\frac{R}{R+h}\right) = \frac{2\pi h}{R+h}$$

Since the solid angle Ω subtended at the centre of earth is

$$\Omega = \frac{\text{Area exposed to the satellite}}{R^2} = \frac{A}{R^2}$$

Thus, the area of earth's surface exposed to the geostationary satellite is

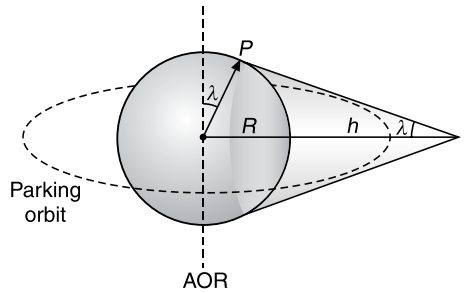
$$A = \Omega R^2 = \frac{2\pi h R^2}{R+h}$$

ILLUSTRATION 45

Find the minimum colatitude angle which can directly receive a signed from a geostationary satellite. Assume that the radius of the geostationary satellite is seven times the radius of earth.

SOLUTION

The farthest point on earth, which can receive signals from the parking orbit is the point where a length is drawn on earth surface from satellite as shown in figure.



The colatitude angle λ of a point P is given by

$$\sin\lambda = \frac{R}{R+h}$$

For a parking orbit, we have $R+h=7R$

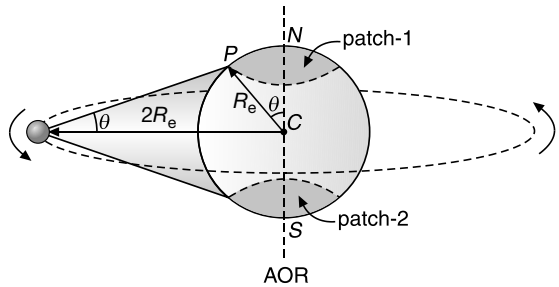
$$\Rightarrow \quad \lambda = \sin^{-1}\left(\frac{1}{7}\right)$$

ILLUSTRATION 46

A satellite is orbiting around the earth in an orbit in equatorial plane of radius $2R_e$ where R_e is the radius of earth. Find the area on earth, this satellite covers for communication purpose in its complete revolution.

SOLUTION

When satellite S revolves around the earth, it covers a complete circular belt on earth's surface for communication.



If the colatitude of the farthest point on surface upto which signals can be received (point P) is θ then we have

$$\sin\theta = \frac{R_e}{2R_e} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{6}$$

During revolution satellite leaves two spherical patches 1 and 2 on earth's surface at north and south poles where no signals can be transmitted due to the

curvature of earth. The areas of these patches can be obtained by using the concept of solid angle.
The solid angle Ω subtended by a patch on earth's centre is

$$\Omega = 2\pi(1-\cos\theta) = \pi\left(2-\sqrt{3}\right)$$

So, area of each patch is

$$A_P = \Omega R_e^2 = \pi\left(2-\sqrt{3}\right)R_e^2$$

Hence total area on earth's surface to which communication can be made is

$$A_C = 4\pi R_e^2 - 2A_P$$

$$\Rightarrow \quad A_C = 4\pi R_e^2 - 2\pi\left(2-\sqrt{3}\right)R_e^2$$

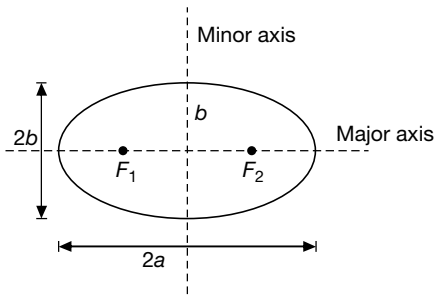
$$\Rightarrow \quad A_C = 2\pi R_e^2\left(2-2+\sqrt{3}\right)$$

$$\Rightarrow \quad A_C = 2\sqrt{3}R_e^2$$

KEPLER'S LAWS

First Law

The planets move around the sun in elliptical orbits with the sun at one focus.
An ellipse with two foci F_1 and F_2 is shown in the figure.

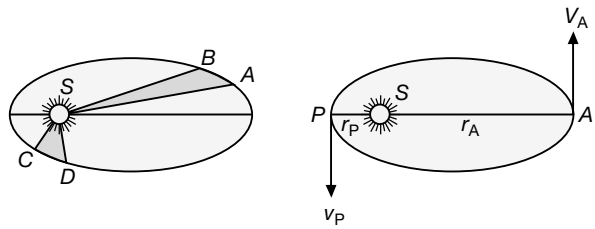


A circle is a special case in which the foci coincide at the center. The short dimension is called the minor axis and has length $2b$; the long dimension is called the major axis and has length $2a$. The closest point P, in the orbit to the sun is called the perihelion; the farthest point A, is called the aphelion.

Second Law

The line joining the sun to a planet sweeps out equal areas in equal time.

Suppose that in a given time interval a planet moves from A to B as shown in the figure, and from C to D during another time interval. According to the second law the areas SAB and SCD are equal. The speed of the planet, therefore, must vary during its orbit. It is greatest at the perihelion, and least at the aphelion. This law is a consequence of the Law of Conservation of Angular Momentum.



Third Law

The square of the period of planet is proportional to the cube of its mean distance from the sun.
The mean distance turns out to be the semi-major axis, a.
Mathematically, $T^2 \propto a^3$ or $T^2 = \kappa a^3$
where κ is a constant that applies to all planets.

ILLUSTRATION 47

According to Kepler's second law of planetary motion, the line joining the sun to a planet sweeps out equal areas in equal time intervals. Show that this is a consequence of the Law of Conservation of Angular Momentum. The path of the planet is an ellipse.

SOLUTION

The gravitational force exerted by the sun on a planet is a central force – it acts along the line joining the two bodies, as shown in figure. The torque on the planet is

$$\vec{\tau} = \vec{r}\times\vec{F} = rF\sin 180^\circ\,\hat{n} = 0$$

which means that its angular momentum is constant. The constancy in direction implies that the plane of the orbit does not change. The constancy in magnitude leads to the "law of areas."

In a time interval Δt, the planet moves from A to B by a distance $AB = v\Delta t$. The height of the triangle SAB is

$$h = AB\sin(180^\circ-\theta)^\circ = v\Delta t\sin\theta$$

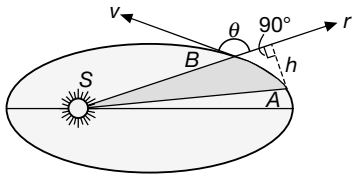
Thus, its area is

$$\Delta A = \frac{1}{2}rh = \frac{1}{2}rv\Delta t\sin\theta$$

so, $$\frac{\Delta A}{\Delta t} = \frac{1}{2}rv\sin\theta \qquad \ldots(1)$$

The angular momentum of the planet is

$$L = rp\sin\theta = mrv\sin\theta \qquad \ldots(2)$$



Combining (1) and (2) we see that

$$\frac{\Delta A}{\Delta t} = \frac{L}{2m} = \text{constant}$$

The rate at which the radial line sweeps out area is constant. Or, equivalently, the radial line sweeps out equal areas in equal time intervals.

Problem Solving Technique(s)

1. The areal velocity of a planet is constant (Kepler's second law) and is given by

$$\frac{dA}{dt} = \frac{L}{2m}$$

Here, L is the angular momentum of the planet about sun.

2. Most of the problems of gravitation are solved by two conservation laws:

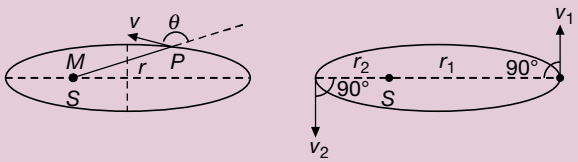
(i) Law of Conservation of Angular Momentum about sun and

(ii) Law of Conservation of Mechanical (potential + kinetic) Energy

Hence, the following two equations are used in most of the cases,

$$mvr\sin\theta = \text{constant} \qquad \ldots(1)$$

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = \text{constant} \qquad \ldots(2)$$



At aphelion and perihelion positions $\theta = 90°$

Hence, equation (1) can be written as,

$$mvr\sin 90° = \text{constant}$$

$$\Rightarrow \quad mvr = \text{constant} \qquad \ldots(3)$$

Further, since mass of the planet (m) also remains constant, equation (1) can also be written as

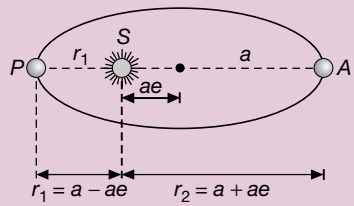
$$vr\sin\theta = \text{constant} \qquad \ldots(4)$$

$$\Rightarrow \quad v_1 r_1 = v_2 r_2 \qquad \{\theta = 90°\}$$

Since, $r_1 > r_2$

$$\Rightarrow \quad v_1 < v_2$$

3. Applying the above mentioned Conservation Laws at the aphelion and perihelion positions along with the knowledge that



$$r_1 = r_{aphelion} = a(1+e) \text{ and } r_2 = r_{perhilion} = a(1-e)$$

where e is the eccentricity of the elliptical orbit. Then, we can show that

$$v_{\min} = v_{aphelion} = v_1 = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)}$$

$$v_{\max} = v_{perhilion} = v_2 = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)}$$

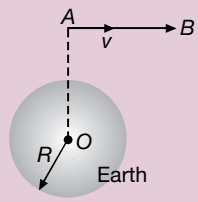
and total energy of the planet is calculated by

$$E = U + K = -\frac{GmM}{r_1} + \frac{1}{2}mv_1^2 = -\frac{GmM}{r_2} + \frac{1}{2}mv_2^2$$

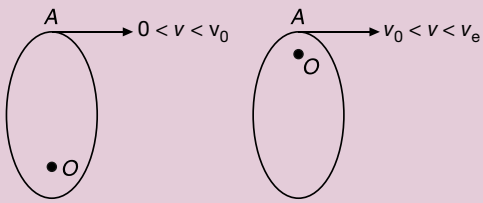
$$\Rightarrow \quad E = -\frac{GMm}{2a}$$

It is interesting to note that the total energy is same for both the circular and elliptical orbit.

4. **Trajectory of a body projected from point *A* in the direction *AB* with different initial velocities:** Let a body be projected from point *A* with velocity *v* in the direction *AB*. For different values of *v* the paths are different. Here are the possible cases.

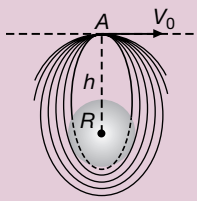


(i) If $v = 0$, path is a straight line from A to O.
(ii) If $0 < v < v_0$, path is an ellipse with centre O of the earth as a focus.
(iii) If $v = v_0$, path is a circle with O as the centre.
(iv) If $v_0 < v < v_e$, path is again an ellipse with O as a focus.
(v) If $v = v_e$, body escapes from the gravitational pull of the earth and path is a parabola.
(vi) If $v > v_e$, body again escapes but now the path is a hyperbola. Here, v_0 = orbital speed $\left(\sqrt{\frac{GM}{r}}\right)$ at A and v_e = escape velocity at A.



NOTE:

(a) From case (i) to (iv) total energy of the body is negative. Hence, these are the closed orbits and are also called **Bound Trajectories.** For case (v) total energy is zero and for case (vi) total energy is positive. In these two cases orbits are open and are called **Unbound Trajectories**.



(b) If v is not very large the elliptical orbit will intersect the earth and the body will fall back to earth.

ILLUSTRATION 48

Two satellites S_1 and S_2 revolve around a planet in coplanar circular orbits in the opposite sense. The periods of revolutions are T and ηT respectively. Find the angular speed of S_2 as observed by an astronaut in S_1, when they are closest to each other.

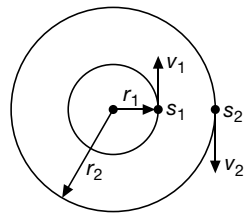
SOLUTION

Since $T^2 \propto r^3$

$$\Rightarrow \quad r \propto T^{\frac{2}{3}}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \left(\frac{T_1}{T_2}\right)^{2/3} = \left(\frac{T}{\eta T}\right)^{2/3} = \frac{1}{\eta^{2/3}}$$

Now when they are closest to each other, then, the separation between them is minimum i.e., as shown in figure.



So, $\omega_{21} = \frac{v_{\text{rel}}}{r_\perp} = \frac{v_2 + v_1}{r_2 - r_1}$

$$\Rightarrow \quad \omega_{21} = \frac{\frac{2\pi r_2}{T_2} + \frac{2\pi r_1}{T_1}}{r_2 - r_1} = \frac{2\pi\left(\frac{1}{T_2} + \frac{1}{T_1}\frac{r_1}{r_2}\right)}{1 - \frac{r_1}{r_2}}$$

$$\Rightarrow \quad \omega_{21} = \frac{2\pi\left(\frac{1}{\eta T} + \frac{1}{T}\frac{1}{\eta^{2/3}}\right)}{1 - \frac{1}{\eta^{2/3}}}$$

$$\Rightarrow \quad \omega_{21} = \frac{\frac{2\pi}{T}\left(\eta^{-1/3} + 1\right)}{\left(\eta^{2/3} - 1\right)}$$

ILLUSTRATION 49

A planet of mass m moves along an ellipse around the sun so that its maximum and minimum distances from the sun are equal to r_1 and r_2 respectively. Find the angular momentum of this planet relative to the centre of the sun. Mass of the sun is M.

SOLUTION

Applying the Law of Conservation of Angular Momentum and Mechanical Energy at P and A, we get

$$mv_1 r_1 \sin 90° = mv_2 r_2 \sin 90°$$

$$\Rightarrow \quad v_2 = \left(\frac{r_1}{r_2}\right)v_1 \qquad \ldots(1)$$

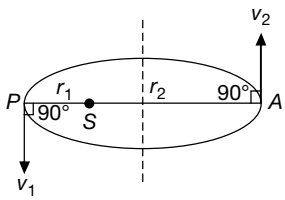
Also, $\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2}$ $\qquad \ldots(2)$

Substituting value of v_2 from equation (1) in equation (2), we get

$$\frac{1}{2}m\left(\frac{r_1^2}{r_2^2}-1\right)v_1^2 = GMm\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$

$$\Rightarrow \quad v_1^2\left(\frac{r_1+r_2}{r_2}\right) = \frac{2GM}{r_1}$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{2GMr_2}{r_1(r_1+r_2)}}$$



Since, angular momentum of the planet about the sun is

$$L = mv_1r_1$$

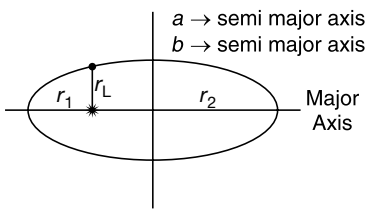
$$\Rightarrow \quad L = m\sqrt{\frac{2GMr_1r_2}{(r_1+r_2)}}$$

ILLUSTRATION 50

If a planet revolves around the sun in an elliptical orbit such that its minimum distance from sun is r_1 and maximum distance is r_2. Calculate the distance of planet from sun when it is at a position where the line joining the planet and sun is perpendicular to the major axis of the ellipse.

SOLUTION

The situation is shown in Figure.



Since, $r_L = \frac{1}{2}$ (latus rectum)

$$\Rightarrow \quad r_L = \frac{1}{2}\left(\frac{2b^2}{a}\right) = \frac{b^2}{a} \qquad \ldots(1)$$

Here semi major axis is given by

$$a = \frac{r_1+r_2}{2}$$

Also, we know that $b^2 = a^2(1-e^2)$ and $a - r_1 = ae$

$$\Rightarrow \quad b^2 = a^2 - (a-r_1)^2 = 2ar_1 - r_1^2$$

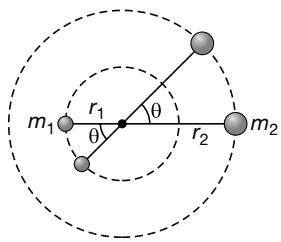
$$\Rightarrow \quad b^2 = 2r_1\left(\frac{r_1+r_2}{2}\right) - r_1^2 = r_2r_1$$

So, from equation (1), we get

$$r_L = \frac{b^2}{a} = \frac{2r_1r_2}{r_1+r_2}$$

DOUBLE STAR SYSTEM OR BINARY STAR SYSTEM

In motion of a planet round the sun we have assumed the mass of the sun to be too large in comparison to the mass of the planet. Under such situation the sun remains stationary and the planet revolves round the sun. If however masses of sun and planet are comparable and motion of sun is also to be considered, then both of them revolve around their centre of mass with same angular velocity but different linear speeds in the circles of different radii. The centre of mass remains stationary.



We use following equations under this condition.

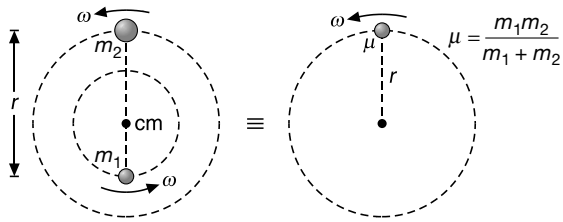
$$m_1r_1 = m_2r_2 \qquad \ldots(1)$$

$$m_1r_1\omega^2 = m_2r_2\omega^2 = \frac{Gm_1m_2}{(r_1+r_2)^2} \qquad \ldots(2)$$

Solving these two equations, we can find that

$$\omega = \sqrt{\frac{G(m_1+m_2)}{r^3}} = \sqrt{\frac{GM}{r^3}} \quad \text{or} \quad T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

Here, $M = m_1 + m_2$ and $r = r_1 + r_2$



Further, angular momentum of the system about centre of mass

$$L = (I_1 + I_2)\omega = \left(\frac{m_1 m_2}{m_1 + m_2}\right) r^2 \omega = \mu r^2 \omega$$

Kinetic energy of system,

$$K = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right) r^2 \omega^2 = \frac{1}{2}\mu r^2 \omega^2$$

and moment of inertia of system,

$$I = \left(\frac{m_1 m_2}{m_1 + m_2}\right) r^2 = \mu r^2$$

Here, $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ = reduced mass.

Thus, the two bodies can be replaced by a single body whose mass is equal to reduced mass. This single body revolve in a circular orbit whose radius is equal to the distance between two bodies and centripetal force of circular motion is equal to force of potential between two bodies for actual separation.

ILLUSTRATION 51

A double star is a system of two stars moving around the centre of mass of the system due to gravitation. Find the distance between the components of the double star if the total mass equals M and time period is T.

SOLUTION

Let d be the distance between the stars and let d_1 and d_2 be the distances of stars from centre of mass

Therefore, $d_1 = \left(\dfrac{d}{m_1 + m_2}\right) m_2$ and $d_2 = \left(\dfrac{d}{m_1 + m_2}\right) m_1$

Since, we have $\dfrac{G m_1 m_2}{d^2} = m_1 \omega^2 d_1 = m_2 \omega^2 d_2$

Also, $m_1 + m_2 = M$ {given}

$$\Rightarrow \quad d_1 = \frac{d}{M} m_2 \text{ and } d_2 = \frac{d}{M} m_1$$

$$\Rightarrow \quad \frac{G m_1 (M - m_1)}{d^2} = m_1 \omega^2 \left[\frac{d}{M}(M - m_1)\right]$$

$$\Rightarrow \quad GM = \omega^2 d^2$$

$$\Rightarrow \quad d^3 = \frac{GMT^2}{4\pi^2}$$

$$\Rightarrow \quad d = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$$

Test Your Concepts-V

Based on Satellites, Kepler's Laws and Applications

(Solutions on page H.263)

1. Two satellites *A* and *B* of the same mass are orbiting the earth at altitudes *R* and 3*R* respectively, where *R* is the radius of the earth. Taking their orbits to be circular obtain the ratios of their kinetic and potential energies.

2. A satellite of mass *m* is orbiting a planet of mass *M* at a radial distance *R* from the centre of a planet. The satellite explodes by expelling very rapidly a small amount of its mass Δm in an opposite direction to its orbital velocity. The immediate recoil velocity of the satellite is v_r (additional to the velocity already possessed by the satellite).

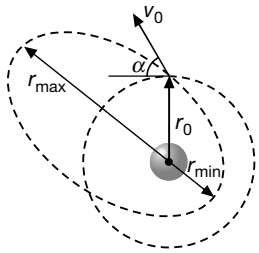
(a) Show that the largest possible value of v_r for which the satellite remains within the gravitational field assuming $\Delta m \ll m$ is,

$$v_r = (\sqrt{2} - 1)\sqrt{\frac{GM}{R}}$$

(b) Deduce the result for v_r in case of Δm not being negligible.

3. If a satellite is revolving close to a planet of density ρ with period *T*, show that the quantity ρT^2 is a universal constant.

4. A satellite is projected into space with a velocity v_0 at a distance r_0 from the centre of the earth by the last stage of its launching rocket. The velocity v_0 was designed to send the satellite into a circular orbit of radius r_0. However, owing to a malfunction of control, the satellite is not projected horizontally but at an angle α with the horizontal and, as a result, is propelled into an elliptic orbit. Determine the maximum and minimum values of the distance from the center of the earth to the satellite.



5. Two small dense stars rotate about their common centre of mass as a binary system with the period of 1 year for each. One star is of double the mass of the other and the mass of the lighter one is of $\left(\frac{1}{3}\right)$ the mass of the sun. Given the distance between the earth and the sun is R. If the distance between the two stars is r, then obtain the relation between r and R.

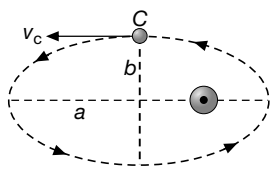
6. In a double star, two stars one of mass m_1 and another of mass m_2, with a separation d, rotate about their common centre of mass. Find:
- **(a)** an expression for their time period of revolution
- **(b)** the ratio of their kinetic energies
- **(c)** the ratio of their angular momenta about the centre of mass and
- **(d)** the total angular momentum of the system
- **(e)** the kinetic energy of the system.

7. Two satellites A and B revolve around a planet in two coplanar circular orbits in the same sense with radii 10^4 km and 2×10^4 km respectively. Time period of A is 28 hours. What is time period of another satellite. Find the speed of B with respect to A when A and B are at farthest distance from each other.

8. A satellite is revolving round the earth in a circular orbit of radius r and velocity v_0. A particle is projected from the satellite in forward direction with relative velocity $v=\left(\sqrt{\frac{5}{4}}-1\right)v_0$. Calculate its minimum and maximum distances from earth's centre during subsequent motion of the particle.

9. Two identical stars each of mass M orbit around their centre of mass. Each orbit is circular and has radius R, so that the two stars are always on opposite sides of the circle.
- **(a)** Find the gravitational force of one star on the other.
- **(b)** Find the orbital speed of each star and the period of the orbit.
- **(c)** What minimum energy would be required to separate the two stars to infinity?

10. Prove that the velocity of a satellite travelling in an elliptical orbit when it reaches point C on the end of the semi-minor axis is $R\sqrt{\frac{g}{a}}$. Also show that this velocity is the same as that for a circular orbit of radius a.

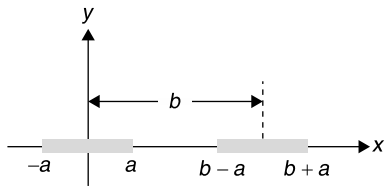


11. Binary stars of comparable masses m_1 and m_2 rotate under the influence of each other's gravity with a time period T. If they are stopped suddenly in their motions, find their relative velocity when they collide with each other. The radii of the stars are R_1 and R_2 respectively. G is the universal constant of gravitation.

SOLVED PROBLEMS

PROBLEM 1

Two identical thin rods of length $2a$ have equal mass M that is distributed uniformly along their lengths. The rods lie along the x-axis with their centres separated by a distance $b > 2a$.



Show that the magnitude of the force exerted by the left rod on the right one is given by

$$F = \frac{GM^2}{4a^2}\log_e\left(\frac{b^2}{b^2-4a^2}\right)$$

SOLUTION

This problem is a very special one where we shall learn to calculate the force between two extended bodies using the concept of gravitational field. For this let us divide the problem in two parts.

PART I

Here we shall imagine the rod to the right to be absent and then let us calculate the field due to the rod on the left at a hypothetical point P (say) at a distance d from one end of the rod. Let us consider an element of length dx at a distance x from the point P. If dE_g be the gravitational field due to the rod at P, then

$$dE_g = \frac{Gdm}{x^2} = \left(\frac{GM}{2a}\right)x^{-2}dx$$

$$\Rightarrow \quad E_g = \frac{GM}{2a}\left(\int_d^{d+2a} x^{-2}dx\right) = \frac{GM}{2a}\left(\frac{x^{-2+1}}{-2+1}\right)\Bigg|_d^{d+2a}$$

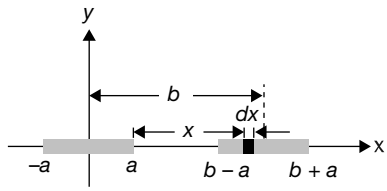
$$\Rightarrow \quad E_g = -\frac{GM}{2a}\left(\frac{1}{x}\right)\Bigg|_d^{d+2a} = -\frac{GM}{2a}\left(\frac{1}{d+2a}-\frac{1}{d}\right)$$

$$\Rightarrow \quad E_g = \frac{GM}{d(d+2a)}$$

PART II

Now after we have calculated the field due to one rod, we actually observe that the second rod will be lying in the field of the first rod. For finding the force on this rod let us again consider an infinitesimal element of length dx at a distance x from the nearest end of the left rod. If dF be the force due to this rod then

$$dF = E_g dm = \left[\frac{GM}{x(x+2a)}\right]\lambda dx$$



Since $\lambda = \frac{M}{2a}$

$$\Rightarrow \quad dF = \frac{GM^2}{2a}\left[\frac{dx}{x(x+2a)}\right]$$

$$\Rightarrow \quad F = \int dF = \frac{GM^2}{2a}\int_{b-2a}^{b}\frac{dx}{x(x+2a)}$$

$$\Rightarrow \quad F = \frac{GM^2}{2a}\left[-\frac{1}{2a}\log_e\left(\frac{2a+x}{x}\right)\right]\Bigg|_{b-2a}^{b}$$

$$\Rightarrow \quad F = \frac{GM^2}{4a^2}\left[-\log_e\left(\frac{2a+b}{b}\right)+\log_e\left(\frac{b}{b-2a}\right)\right]$$

$$\Rightarrow \quad F = \frac{GM^2}{4a^2}\log_e\left[\frac{b^2}{(b-2a)(b+2a)}\right]$$

$$\Rightarrow \quad F = \frac{GM^2}{4a^2}\log_e\left(\frac{b^2}{b^2-4a^2}\right)$$

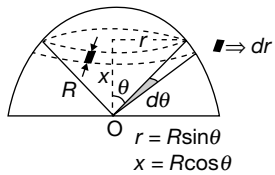
PROBLEM 2

Calculate the gravitational field due to a thin uniform very large hemispherical shell at its centre. Assume the shell to have a radius R and a mass M.

SOLUTION

Let us consider an infinitesimal elemental ring on its surface having angular width $d\theta$ at an angle θ from its axis as shown. The surface area dA of this ring is

$$dA = (2\pi R\sin\theta)(Rd\theta)$$



Mass on this elemental ring is

$$dm = \sigma dA = \sigma\left(2\pi R^2 \sin\theta d\theta\right)$$

where σ is the surface mass density of the shell given by

$$\sigma = \frac{M}{2\pi R^2}$$

Since the gravitational field strength due to a thin ring of mass dm, radius r at a point lying on its axis at a distance x from its centre is

Now due to this ring, gravitational field strength at centre C is

$$dE = \frac{G(dm)x}{\left(r^2 + x^2\right)^{3/2}} \qquad \ldots(1)$$

Also, from the figure, we observe that

$$x = R\cos\theta \text{ and } r = R\sin\theta$$

Substituting these values in equation (1), we get

$$dE = \frac{Gdm(R\cos\theta)}{\left(R^2\sin^2\theta + R^2\cos^2\theta\right)^{3/2}}$$

$$\Rightarrow \quad dE = \frac{G\sigma\left(2\pi R^2\sin\theta d\theta\right)R\cos\theta}{R^3}$$

$$\Rightarrow \quad dE = G\sigma\pi\left(2\sin\theta\cos\theta d\theta\right) = G\sigma\pi\sin\left(2\theta\right)d\theta$$

Net gravitational field at center is obtained by integrating the above expression for dE between the limits zero to $\frac{\pi}{2}$. So,

$$E = \int dE = G\sigma\pi\int_0^{\pi/2}\sin\left(2\theta\right)d\theta$$

$$\Rightarrow \quad E = G\sigma\pi\left(-\frac{\cos\left(2\theta\right)}{2}\Big|_0^{\pi/2}\right)$$

$$\Rightarrow \quad E = G\sigma\pi\left(\frac{1}{2} + \frac{1}{2}\right)$$

$$\Rightarrow \quad E = G\sigma\pi = G\left(\frac{M}{2\pi R^2}\right)\pi$$

$$\Rightarrow \quad E = \frac{GM}{2R^2}$$

PROBLEM 3

Calculate the self energy of the sun, taking its mass to be equal to 2×10^{30} kg and its radius to be very nearly 7×10^{8} metre. If its radius contracts by 1 km per year, without affecting its mass, calculate the rate at which it radiates out energy.

SOLUTION

The gravitational self energy of the sun is given by

$$U_s = -\frac{3}{5}\frac{GM^2}{R}$$

This is actually the self energy of the solid sphere. So,

$$U_s = -\frac{3}{5}\frac{\left(6.67\times10^{-11}\right)\left(2\times10^{30}\right)^2}{\left(7\times10^{8}\right)}$$

$$\Rightarrow \quad U_s = -2.29\times10^{41}\ \text{J}$$

Now, rate of change of energy of the sun, is

$$\frac{dU_s}{dt} = \frac{3}{5}\frac{M^2G}{R^2}\frac{dR}{dt}$$

where, $\left|\frac{dR}{dt}\right| = \frac{1000}{365\times24\times3600}\ \text{ms}^{-1}$

So, the rate of energy radiated out by the sun,

$$\frac{dU_s}{dt} = \frac{3}{5}\frac{M^2G}{R^2}\frac{dR}{dt}$$

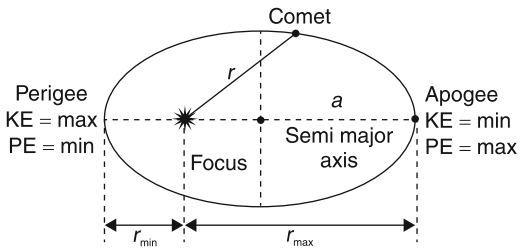
$$\Rightarrow \quad \frac{dU_s}{dt} = \frac{3}{5}\frac{\left(2\times10^{30}\right)^2\left(6.67\times10^{-11}\right)}{\left(7\times10^{8}\right)^2}\left(\frac{1000}{365\times24\times3600}\right)$$

$$\Rightarrow \quad \frac{dU_s}{dt} = 1.03\times10^{28}\ \text{Js}^{-1}$$

PROBLEM 4

Halley's comet has a period of 76 years and in the year 1986 the distance of closest approach of the comet to the sun was 8.9×10^{10} m. Calculate the comet's farthest distance from the sun if the mass of sun is 2×10^{30} kg and $G = 6.67\times10^{-11}\ \text{Nm}^2\text{kg}^{-2}$ and $1\ \text{yr} = 3.15\times10^{7}$ s

SOLUTION



From the problem it is self-evident that the orbit of the comet is elliptical with sun being at one focus. For elliptical orbits, according to Kepler's third law, we have

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3$$

$$\Rightarrow \quad a = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$$

where $T = 76 \times 365 \times 86400$

$$\Rightarrow \quad a = \left(\frac{6.67\times10^{-11}\times2\times10^{30}\times(76\times3.15\times10^{7})^2}{4\pi^2}\right)^{\frac{1}{3}}$$

$$\Rightarrow \quad a \approx 2.7\times10^{12} \text{ m}$$

For an ellipse, we know that

$$2a = r_{min} + r_{max}$$

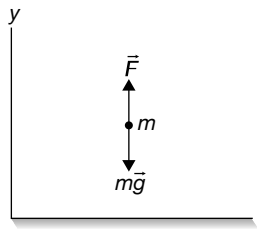
$$\Rightarrow \quad r_{max} = 2a - r_{min}$$

$$\Rightarrow \quad r_{max} = 2(2.7\times10^{12}) - 8.9\times10^{10}$$

$$\Rightarrow \quad r_{max} \approx 5.3\times10^{12} \text{ m}$$

PROBLEM 5

A body of mass m ascends from the earth's surface with zero initial velocity due to the action of two forces as shown in figure.



The force $\vec{F}$ varying with height y as $\vec{F} = -2m\vec{g}(1-ay)$ where a is a positive constant. Find the work performed by the force $\vec{F}$ over the first half of the ascent and the corresponding increase in the potential energy of the body in the earth's gravity field, which is assumed to be uniform.

SOLUTION

First let us find the total height of ascent. At the beginning and the end of the path, the velocity of the body is happens to be equal to zero and therefore, the change in kinetic energy of the body is also equal to zero. Therefore, from Work-Energy Theorem, we have

$$\text{Work Done by All the Forces} = 0$$

$$\Rightarrow \quad \int_0^h (\vec{F} + m\vec{g})\,dy = 0$$

$$\Rightarrow \quad \int_0^h mg(1-2ay)\,dy = 0$$

$$\Rightarrow \quad mgh(1-ah) = 0$$

$$\Rightarrow \quad h = \frac{1}{a}$$

The work performed by the force $\vec{F}$ over the first half of the ascent is,

$$W = \int_0^{h/2} F\,dy = 2mg\int_0^{1/2a}(1-ay)\,dy = \frac{3mg}{4a}$$

The corresponding increase in the potential energy is

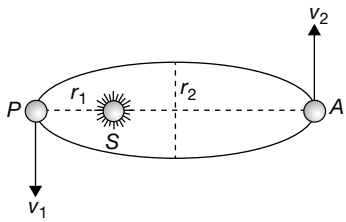
$$\Delta U = \frac{mgh}{2} = \frac{mg}{2a}$$

PROBLEM 6

Find the speeds of a planet of mass m in its perihelion and aphelion positions. The semi-major axis of its orbit is a, eccentricity is e and the mass of the sun is M. Also find the total energy of the planet in terms of the given parameters.

SOLUTION

Let v_1 and v_2 be the speeds of the planet at perihelion



and aphelion positions, then

$$r_1 = a(1-e) \text{ and } r_2 = a(1+e) \qquad \ldots(1)$$

Applying the Law of Conservation of Angular momentum of the planet at P (perihelion) and A (aphelion) about the sun, we get

$$mv_1 r_1 \sin 90° = mv_2 r_2 \sin 90°$$

$$\Rightarrow \quad v_1 r_1 = v_2 r_2 \qquad \ldots(2)$$

Applying the Law of Conservation of Mechanical Energy in these two positions, we get

$$(U+K)_{at\ P} = (U+K)_{at\ A}$$

$$\Rightarrow \quad \frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2} \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$v_1 = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)} \text{ and } v_2 = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)}$$

Further, total energy of the planet is

$$E = \frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2}$$

$$\Rightarrow \quad E = \frac{1}{2}m\left[\frac{GM}{a}\left(\frac{1+e}{1-e}\right)\right] - \frac{GMm}{a(1-e)}$$

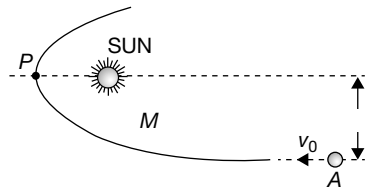
$$\Rightarrow \quad E = \frac{GMm}{a(1-e)}\left[\left(\frac{1+e}{2}\right) - 1\right]$$

$$\Rightarrow \quad E = \frac{GMm}{a(1-e)}\left(\frac{e-1}{2}\right)$$

$$\Rightarrow \quad E = -\frac{GMm}{2a}$$

PROBLEM 7

A cosmic body A coming from infinity with a velocity v_0 is approaching the Sun of mass M, with its line of motion at a distance l from the Sun, as shown in the figure. When it gets closest to the Sun i.e. at P, what will be its distance from the sun?



SOLUTION

By Law of Conservation of Mechanical Energy, we have

$$(U+K)_{at\ \infty} = (U+K)_{at\ P}$$

$$\Rightarrow \quad 0 + \frac{1}{2}mv_0^2 = -\frac{GMm}{r} + \frac{1}{2}mv^2 \qquad \ldots(1)$$

where

M is the mass of the sun
m is the mass of the body
r is the distance of closest approach and
v is the velocity of the body at the point P

Since the torque due to the gravitational force on the body about the sun is zero, so the angular momentum of the body about the sun will remain conserved, therefore by Law of Conservation of Angular Momentum, we have

$$(mv_0)l = (mv)r$$

$$\Rightarrow \quad v_0 l = vr \qquad \ldots(2)$$

$$\Rightarrow \quad v = \frac{v_0 l}{r}$$

Substituting this value of v in equation (1), we get

$$\Rightarrow \quad \frac{1}{2}mv_0^2 = -\frac{GMm}{r} + \frac{1}{2}m\left(\frac{v_0 l}{r}\right)^2$$

$$\Rightarrow \quad v_0^2 = -\frac{2GMr + \left(v_0^2 l^2\right)}{r^2}$$

$$\Rightarrow \quad v_0^2 r^2 + 2GMr - v_0^2 l^2 = 0$$

Solving this quadratic in r, we get

$$r = -\frac{-2GM \pm \sqrt{4G^2M^2 + 4v_0^4 l^2}}{2v_0^2}$$

Since we are here calculating the distance of closest approach, i.e. the least distance between the sun and the body, so we will reject the negative sign. Hence, we have

$$r = -\frac{-2GM + \sqrt{4G^2M^2 + 4v_0^4 l^2}}{2v_0^2}$$

$$\Rightarrow \quad r_{\min} = \frac{GM}{v_0^2}\left(-1 + \sqrt{1 + \left(\frac{v_0^2 l^2}{GM}\right)^2}\right)$$

PROBLEM 8

A body is projected vertically upwards from the surface of earth with a velocity sufficient to carry it to infinity. Calculate the time taken by it to reach a height h.

SOLUTION

Let v be the velocity of the body at a distance r from the centre of earth. Applying Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mv_e^2 - \frac{GMm}{R} \qquad \ldots(1)$$

where, $v_e = \sqrt{2gR}$ and $g = \frac{GM}{R^2}$

Substituting in equation (1), we get

$$v = \frac{R\sqrt{2g}}{\sqrt{r}}$$

Since $v = \frac{dr}{dt}$, so we have

$$\frac{dr}{dt} = \frac{R\sqrt{2g}}{\sqrt{r}}$$

$$\Rightarrow \quad \int_0^t dt = \frac{1}{R\sqrt{2g}}\int_R^{R+h} r^{1/2}dr$$

$$\Rightarrow \quad t = \frac{2}{3}\frac{1}{R\sqrt{2g}}\left[(R+h)^{3/2} - R^{3/2}\right]$$

$$\Rightarrow \quad t = \frac{1}{3}\sqrt{\frac{2R}{g}}\left[\left(1 + \frac{h}{R}\right)^{3/2} - 1\right]$$

PROBLEM 9

An artificial satellite of the earth (radius R and mass M) moves in an orbit whose radius is n times the radius of the earth. Assuming resistance to the motion to be proportional to the square of velocity that is $F = av^2$. Find how long the satellite will take to fall on to the earth.

SOLUTION

Tangential force, $F_t = m\frac{dv}{dt}$

$$\Rightarrow \quad -av^2 = m\frac{dv}{dt}$$

Rearranging, we get

$$\int_0^t dt = -\frac{m}{a}\int_{v_i}^{v_f}\frac{dv}{v^2} = -\frac{m}{a}\int_{v_i}^{v_f} v^{-2}dv$$

$$t = -\frac{m}{a}\left(\frac{v^{-2+1}}{-2+1}\right)\Bigg|_{v_i}^{v_f} = -\frac{m}{a}\left(-\frac{1}{v}\right)\Bigg|_{v_i}^{v_f}$$

$$\Rightarrow \quad t = \frac{m}{a}\left(\frac{1}{v_f} - \frac{1}{v_t}\right)$$

Also, $v_i = \sqrt{\frac{GM}{nR}}$ and $v_f = \sqrt{\frac{GM}{R}}$

$$t = \frac{m}{a}\left(\sqrt{\frac{nR}{GM}} - \sqrt{\frac{R}{GM}}\right)$$

Since $g = \frac{GM}{R^2}$, so we get

$$t = \frac{m\sqrt{R}}{a\sqrt{GM}}\left(\sqrt{n} - 1\right)$$

PROBLEM 10

A satellite is to be placed in an orbit just above the earth's atmosphere with a speed $\sqrt{\frac{3}{2}}$ times the speed for a circular orbit at that height. The initial velocity imparted is horizontal. Calculate the maximum distance of the satellite from the earth, when it is in the orbit.

SOLUTION

Since the satellite is to be placed in an orbit just above the atmosphere i.e. it is to be placed in an orbit close to surface of earth. Also, it is given that

$$v_i = \sqrt{\frac{3}{2}}v_{\text{orbit}}$$

Close to surface of earth

$$v_0 \approx \sqrt{gR}$$

$$\Rightarrow \quad v_i = \sqrt{\frac{3}{2}}\sqrt{gR}$$

By Law of Conservation of Angular Momentum, we have

$$mv_i r_i = mv_f r_f$$

$$\Rightarrow \quad m\left(\sqrt{\frac{3}{2}}\sqrt{gR}\right)R = mv_f r_{\max} \qquad \ldots(1)$$

By Laws of Conservation of Energy, we have

$$\frac{1}{2}m\left(\frac{3}{2}gR\right) - \frac{GMm}{R} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_{\max}} \qquad \ldots(2)$$

$$\Rightarrow \quad \frac{3}{4}gR - gR = \frac{1}{2}v_f^2 - \frac{gR^2}{r_{\max}}$$

From equation (1), we have

$$v_f = \sqrt{\frac{\frac{3}{2}gR^3}{r_{\max}}}$$

So, equation (2) becomes

$$-\frac{1}{4}gR = \frac{1}{2}\left(\frac{\sqrt{\frac{3}{2}}\sqrt{gR^3}}{r_{\max}}\right)^2 - \frac{gR^2}{r_{\max}}$$

$$\Rightarrow \quad \frac{R^2}{r_{\max^2}} - \frac{4}{3}\frac{R}{r_{\max}} + \frac{1}{3} = 0$$

$$\Rightarrow \quad \frac{R}{r_{\max}} = \frac{\frac{4}{3} \pm \sqrt{\frac{16}{9} - \frac{4}{3}}}{2} = \frac{\frac{4}{3} \pm \frac{2}{3}}{2} = \frac{2}{3} \pm \frac{1}{3}$$

$$\Rightarrow \quad \frac{R}{r_{\max}} = 1 \text{ or } \frac{1}{3}$$

Since $r_{\max} \neq R$

$$\Rightarrow \quad r_{\max} = 3R$$

So, maximum distance of satellite from the earth surface is

$$h_{\max} = r_{\max} - R$$

$$\Rightarrow \quad h_{\max} = 3R - R = 2R$$

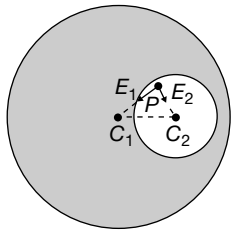
PROBLEM 11

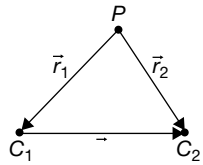
Inside a uniform sphere of density ρ there is a spherical cavity whose centre is at a distance l from the centre of the sphere. Find the strength of the gravitational field inside the cavity.

SOLUTION

Let us consider a point P inside the cavity. Let $\vec{E}_1$ be gravitational field due to sphere at P, then

$$\vec{E}_1 = \left(\frac{4}{3}\pi G\rho\right)\vec{r}_1$$





If E_2 be the gravitational field due to cavity at P, then

$$\vec{E}_2 = \frac{4}{3}\pi G(-\rho)\vec{r}_2$$

$$\Rightarrow \quad \vec{E}_2 = -\frac{4}{3}\pi G\rho\vec{r}_2$$

So, net field at P, due to sphere and the cavity is

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\Rightarrow \quad \vec{E} = \frac{4}{3}\pi G\rho(\vec{r}_1 - \vec{r}_2) \qquad \ldots(1)$$

However, in triangle PC_1C_2, we have

$$\vec{r}_1 + \vec{l} = \vec{r}_2$$

$$\Rightarrow \quad \vec{r}_1 - \vec{r}_2 = -\vec{l}$$

Substituting in (1), we get

$$\vec{E} = -\frac{4}{3}\pi G\rho\vec{l}$$

$$\Rightarrow \quad |\vec{E}| = \frac{4}{3}\pi G\rho l$$

So, we observe that the gravitational field inside the cavity is independent of the location of the point P and just depends upon the distance of the cavity's centre from the centre of sphere.

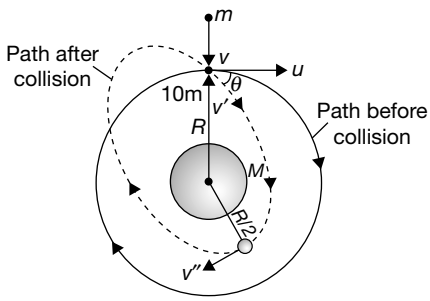
PROBLEM 12

A meteorite of mass m collides perpendicularly with a satellite which was orbiting around a planet of mass M in a circular path of radius R. Due to collision, the meteorite sticks to the satellite of mass $10m$ and the satellite is seen to have gone into an orbit whose

minimum distance from the planet is $\frac{R}{2}$. Determine the velocity v of the meteorite before collision.

SOLUTION

The situation before collision and after collision is shown in the figure.



Before collision, the speed of satellite is the orbital speed given by

$$u_0 = \sqrt{\frac{GM}{R}} \qquad \ldots(1)$$

After collision with meteorite, let the combined mass $11m$ moves at an angle θ with the orbit as shown in figure. If finally, $11m$ moves at speed v' then applying the Law of Conservation of Momentum along horizontal and vertical directions, we get

$$mv = 11mv' \sin\theta$$

$$\Rightarrow \quad v' \sin\theta = \left(\frac{v}{11}\right) \qquad \ldots(2)$$

$$10mu_0 = 11mv' \cos\theta$$

$$\Rightarrow \quad v' \cos\theta = \left(\frac{10v_0}{11}\right) \qquad \ldots(3)$$

After collision applying the Law of Conservation of Angular Momentum, we get

$$11m(v' \cos\theta)R = 11m(v'')\frac{R}{2} \qquad \ldots(4)$$

where v'' is the speed at perigee.

Applying the Law of Conservation of Energy, we have

$$-\frac{GM(11m)}{R} + \frac{1}{2}(11m)v'^2 = -\frac{GM(11m)}{(R/2)} + \frac{1}{2}(11m)v''^2$$

$$\Rightarrow \quad -\frac{GM}{R} + \frac{v'^2}{2} = -\frac{2GM}{R} + \frac{v''^2}{2} \qquad \ldots(5)$$

From equation (2) and (3), we get

$$v'^2 = \left(\frac{v}{11}\right)^2 + \left(\frac{10u_0}{11}\right)^2 = \frac{v^2 + 100u_0^2}{121} \qquad \ldots(6)$$

From equation (4), we get

$$v'' = 2v' \cos\theta = \left(\frac{20}{11}\right)v_0 \qquad \ldots(7)$$

Substituting the values of v'^2 from (6) and v'' from (7) in equation (5), we get

$$-\frac{GM}{R} + \frac{1}{2}\left(\frac{v^2 + 100u_0^2}{121}\right) = -\frac{2GM}{R} + \frac{1}{2}\left(\frac{20}{11}u_0\right)^2$$

$$\Rightarrow \quad \frac{v^2}{242} = -\frac{GM}{R} + \left(\frac{1}{2}\right)\left(\frac{1}{121}\right)\left(400u_0^2 - 100u_0^2\right)$$

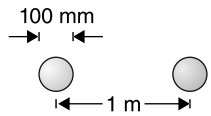
$$\Rightarrow \quad \frac{v^2}{242} = -\frac{GM}{R} + \frac{300}{242}\left(\frac{GM}{R}\right) \qquad \left\{\because u_0 = \sqrt{\frac{GM}{R}}\right\}$$

$$\Rightarrow \quad \frac{v^2}{242} = \frac{58GM}{242R}$$

$$\Rightarrow \quad v = \sqrt{\frac{58GM}{R}}$$

PROBLEM 13

Two iron spheres, each of mass 1 kg and 100 mm in diameter, are released from rest with a centre-to-centre separation of 1 m . Assume an environment in space with no forces other than the force of mutual gravitational attraction and calculate the time t required for the spheres to contact each other and the absolute speed v of each sphere upon contact.



SOLUTION

Let v_r be the relative speed at separation r, then by Law of Conservation of Mechanical Energy, we have

$$(U+K)_{\text{at 1 m}} = (U+K)_r$$

$$-\frac{GmM}{1} + \frac{1}{2}\mu(0)^2 = -\frac{GmM}{r} + \frac{1}{2}\mu v_r^2$$

$$\Rightarrow \quad \frac{1}{2}\mu v_r^2 = Gmm\left(\frac{1}{r} - 1\right) \qquad \ldots(1)$$

where, μ is the reduced mass given by

$$\mu = \frac{m(m)}{m+m} = \frac{m}{2}$$

Substituting in equation (1) with $m = 1$ kg, we get

$$v_r = 2\sqrt{G\left(\frac{1}{r}-1\right)} \qquad \ldots(2)$$

$$\Rightarrow \quad -\frac{dr}{dt} = 2\sqrt{G\left(\frac{1}{r}-1\right)}$$

$$\Rightarrow \quad dt = -\frac{dr}{2\sqrt{G\left(\frac{1}{r}-1\right)}}$$

$$\Rightarrow \quad \int_0^t dt = -\frac{1}{2\sqrt{G}}\int_{1.0}^{0.1}\sqrt{\frac{r}{1-r}}dr$$

Let the integral be

$$I = \int\sqrt{\frac{r}{1-r}}dr$$

Substituting $r = \sin^2\theta$, we get

$$dr = 2\sin\theta\cos\theta d\theta$$

$$\Rightarrow \quad I = \int\sqrt{\frac{\sin^2\theta}{\cos^2\theta}}2\sin\theta\cos\theta d\theta$$

$$\Rightarrow \quad I = \int 2\sin^2\theta d\theta = \int(1-\cos 2\theta)d\theta$$

$$\Rightarrow \quad I = \theta - \frac{\sin(2\theta)}{2}$$

$$\Rightarrow \quad I = \theta - \sin\theta\cos\theta$$

But $\sin^2\theta = r$

$$\Rightarrow \quad I = \sin^{-1}\sqrt{r} - \sqrt{r(1-r)}$$

$$\Rightarrow \quad t = -\frac{1}{2\sqrt{G}}\left(\sin^{-1}\sqrt{r} - \sqrt{r(1-r)}\right)\Big|_1^{0.1}$$

$$\Rightarrow \quad t = \frac{1}{2\sqrt{G}}\left[\sin^{-1}(1) - 0 - \sin^{-1}\sqrt{0.1} + \sqrt{(0.1)(0.9)}\right]$$

$$\Rightarrow \quad t = \frac{1}{2\sqrt{G}}\left(\frac{\pi}{2} - 0.32 + 0.3\right)$$

$$\Rightarrow \quad t = \frac{1}{2\sqrt{G}}(1.55)$$

$$\Rightarrow \quad t = 95\times10^3 \text{ s}$$

From equation (1), relative speed at the time of collision,

$$v_r = 2\sqrt{(6.67\times10^{-11})\left(\frac{1}{0.1}-1\right)} = 4.9\times10^{-5} \text{ ms}^{-1}$$

So, the speed of each sphere is $\frac{v_r}{2} = 2.45\times10^{-5}$ ms^{-1}

PROBLEM 14

Two satellites S_1 and S_2 are to be set into the orbits of $\frac{R}{4}$ and $\frac{R}{6}$ above the earth's surface respectively. They revolve around the earth in coplanar circular orbit in the opposite sense. Determine

(a) the ratio of the speeds of projection from the surface of the earth.

(b) the angular speed of S_2 as observed by an astronaut in S_1 when their radius vectors are making an angle $120°$ with each other.

SOLUTION

(a) By Law of Conservation of Energy

$$(U+K)_{\text{surface}} = (U+K)_{\text{orbit}}$$

$$\Rightarrow \quad -\frac{GMm_1}{R} + \frac{1}{2}m_1v_1^2 = -\frac{GMm_1}{2\left(R+\frac{R}{4}\right)}$$

$$\Rightarrow \quad \frac{1}{2}m_1v_1^2 = -\frac{GMm_1}{2\left(R+\frac{R}{4}\right)} - \left(-\frac{GMm_1}{R}\right)$$

$$\Rightarrow \quad v_1^2 = \frac{6}{5}\frac{GM}{R} \qquad \ldots(1)$$

Similarly, $-\frac{GMm_2}{R} + \frac{1}{2}m_2v_2^2 = -\frac{GMm_2}{2\left(R+\frac{R}{6}\right)}$

$$\Rightarrow \quad \frac{1}{2}m_2v_2^2 = -\frac{GMm_2}{2\left(R+\frac{R}{6}\right)} - \left(-\frac{GMm_2}{R}\right)$$

$$\Rightarrow \quad v_2^2 = \frac{8}{7}\frac{GM}{R} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{v_1^2}{v_2^2} = \frac{21}{20}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{21}{20}}$$

(b) $r_1 = R + \frac{R}{4} = \frac{5R}{4}$ and $r_2 = R + \frac{R}{6} = \frac{7R}{6}$

$P = (0, r_2)$

and $Q = (r_2 \cos 30°, -r_1 \sin 30°) = \left(\frac{\sqrt{3}r_1}{2}, -\frac{r_1}{2}\right)$

$\Rightarrow PQ = \sqrt{\frac{3r_1^2}{4} + \left(r_2 + \frac{r_1}{2}\right)^2} = 2.1R$

Applying Lami's Theorem, we get

$$\frac{r_2}{\sin(90° - \beta)} = \frac{r_1}{\sin(90° - \alpha)} = \frac{PQ}{\sin(120°)}$$

$\Rightarrow \cos\alpha = \frac{r_1}{PQ}\sin(120°) = 0.5$ and

$\cos\beta = \frac{r_2}{PQ}\sin(120°) = 0.48$

Now, $\omega_r = \frac{v_2\cos\alpha + v_1\cos\beta}{PQ}$

$\Rightarrow \omega_r = \frac{\sqrt{\frac{GM}{r_2}}\cos\alpha + \sqrt{\frac{GM}{r_1}}\cos\beta}{PQ}$

$\Rightarrow \omega_r = \frac{\sqrt{\frac{gR^2}{r_2}}\cos\alpha + \sqrt{\frac{gR^2}{r_1}}\cos\beta}{2.1R}$ …(3)

Since, $r_1 = \frac{5R}{4}$ and $r_2 = \frac{7R}{6}$

$$\sqrt{\frac{gR^2}{r_2}}\cos\alpha = \left(\sqrt{\frac{9.81\times6400\times10^3}{7/6}}\right)0.5$$

$\Rightarrow \sqrt{\frac{gR^2}{r_2}}\cos\alpha = 3666$

$$\sqrt{\frac{gR^2}{r_1}}\cos\beta = \left(\sqrt{\frac{9.81\times6400\times10^3}{5/4}}\right)0.48$$

$\Rightarrow \sqrt{\frac{gR^2}{r_1}}\cos\beta = 3402$

Substituting these values in equation (3), we get

$$\omega_r = \frac{3666 + 3402}{2.1\times6400\times10^3} = 5.25\times10^{-4} \text{ rads}^{-1}$$

PROBLEM 15

An earth satellite is revolving in a circular orbit of radius a with velocity v_0. A gun is in the satellite and is aimed directly towards the earth. A bullet is fired from the gun with muzzle velocity $\frac{v_0}{2}$. Neglecting resistance offered by cosmic dust and recoil of gun, calculate maximum and minimum distance of bullet from the centre of earth during its subsequent motion.

SOLUTION

Orbital speed of satellite is

$$v_0 = \sqrt{\frac{GM}{a}} \quad \ldots(1)$$

Applying the Law of Conservation of Angular Momentum at P and Q, we get

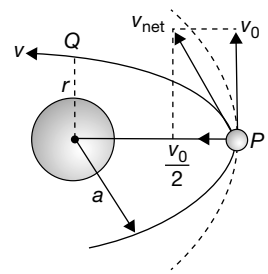
$$mav_0 = mvr$$

$\Rightarrow v = \frac{av_0}{r}$ …(2)

Applying the Law of Conservation of Mechanical Energy at P and Q, we get

$$\frac{1}{2}m\left(v_0^2 + \frac{v_0^2}{4}\right) - \frac{GMm}{a} = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$\Rightarrow \frac{5}{8}v_0^2 - \frac{GM}{a} = \frac{v^2}{2} - \frac{GM}{r}$



Substituting values of v and v_0 from equations (1) and (2), we get

$$\frac{5}{8}\frac{GM}{a} - \frac{GM}{a} = \frac{a^2}{r^2}\cdot\left(\frac{GM}{2a}\right) - \frac{GM}{r}$$

$\Rightarrow -\frac{3}{8a} = \frac{a}{2r^2} - \frac{1}{r}$

$\Rightarrow -3r^2 = 4a^2 - 8ar$

$\Rightarrow 3r^2 - 8ar + 4a^2 = 0$

$$\Rightarrow \quad r = \frac{8a \pm \sqrt{64a^2 - 48a^2}}{6}$$

$$\Rightarrow \quad r = \frac{8a \pm 4a}{6}$$

$$\Rightarrow \quad r = 2a \text{ and } \frac{2a}{3}$$

Hence, the maximum and minimum distances are $2a$ and $\frac{2a}{3}$ respectively.

PRACTICE EXERCISES

SINGLE CORRECT CHOICE TYPE QUESTIONS

This section contains Single Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. The gravitational force between a point like mass M and an infinitely long, thin rod of linear mass density λ at perpendicular distance L from M is

(A) $\frac{MG\lambda}{L}$ (B) $\frac{1}{2}\frac{MG\lambda}{L}$

(C) $\frac{2MG\lambda}{L}$ (D) infinite

2. The radius of a planet is R. A satellite revolves around it in a circle of radius r with angular speed ω. The acceleration due to gravity on planet's surface will be

(A) $\frac{r^3\omega}{R}$ (B) $\frac{r^2\omega^3}{R}$

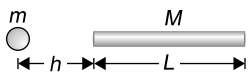
(C) $\frac{r^3\omega^2}{R^2}$ (D) $\frac{r^2\omega^2}{R}$

3. Consider a particle of mass m released from a point lying on the axis of a ring of mass M and radius r and at a distance r from the centre of ring. Assuming that the particle is moving under the gravitational attraction of the ring to reach the centre of the ring, then the velocity of the particle at the centre of the ring is

(A) $\sqrt{\frac{GM}{r}}$ (B) $\sqrt{\frac{2GM}{r}}$

(C) $\sqrt{\frac{2GM}{r(\sqrt{2}+1)}}$ (D) $\sqrt{\frac{2GM}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$

4. A homogeneous bar of length L and mass M is at a distance h from a point mass m as shown. The force on m is F.



(A) $F=\frac{GMm}{(h+L)^2}$ (B) $F=\frac{GMm}{h^2}$

(C) $F=\frac{GMm}{h(h+L)}$ (D) $F=\frac{GMm}{L^2}$

5. A uniform ring of mass M and radius R is placed directly above a uniform sphere of mass $8M$ and of same radius R. The centre of the ring is at a distance of $d=\sqrt{3}R$ from the centre of the sphere. The gravitational attraction between the sphere and the ring is

(A) $\frac{8GM^2}{R^2}$ (B) $\frac{2GM^2}{\sqrt{3}R^2}$

(C) $\frac{3GM^2}{2R^2}$ (D) $\frac{\sqrt{3}GM^2}{R^2}$

6. A hole is drilled from the surface of earth to its centre. A particle is dropped from the surface of the earth. The speed of the particle when it reaches the centre of the earth in terms of its escape velocity (v_e) at the surface of earth is

(A) $\frac{v_e}{2}$ (B) v_e

(C) $\sqrt{2}v_e$ (D) $\frac{v_e}{\sqrt{2}}$

7. Two identical solid spheres of radius R placed in contact with each other, the gravitational attraction between them is proportional to

(A) R^2 (B) R^{-2}

(C) R^4 (D) R^{-4}

8. A planet has a mass and density both equal to eight times the mass and average density of the earth. If g be the acceleration due to gravity at the earth's surface, then acceleration due to gravity at the planet's surface is

(A) $2g$ (B) $4g$

(C) $8g$ (D) $16g$

9. Gravitational force between two charged bodies is F_1 and electrostatic force between them is F_2. They are at distance r from each other. Now the air between them is sucked out and distance is increased to $2r$. Then the new forces F_1' and F_2' are such that

(A) $F_1'=\frac{F_1}{4}$ (B) $F_2'=\frac{F_2}{4}$

(C) $F_2'>\frac{F_2}{4}$ (D) $F_1'>\frac{F_1}{4}$

10. If g_h and g_d be the accelerations due to gravity at height h and at depth d, above and below the surface of earth respectively. Assuming $h \quad R$ and $d \quad R$ and if $g_h=g_d$ then,

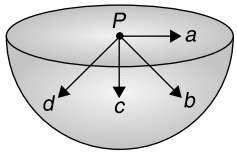
(A) $d=h$

(B) $d=2h$

(C) $h=2d$

(D) data is insufficient to arrive at a conclusion

11. A hemispherical shell having uniform mass density is shown in Figure. The direction of gravitational field intensity at point P will be along



(A) a (B) b
(C) c (D) d

12. What should be the velocity of a body thrown vertically upwards from the surface of earth (mass M and radius R) so that it may reach at a height of $7R$ from the surface ?

(A) $\sqrt{\frac{20GM}{11R}}$ (B) $\sqrt{\frac{20GM}{9R}}$
(C) $\sqrt{\frac{17GM}{9R}}$ (D) $\sqrt{\frac{7GM}{4R}}$

13. Rate of change of weight near the earth's surface varies with height (h) as

(A) h (B) h^o
(C) h^{-1} (D) $h^{\frac{1}{2}}$

14. A uniform spherical planet of radius R has acceleration due to gravity at its surface g. Points P and Q located inside and outside the planet respectively have same acceleration due to gravity equal to $\frac{g}{4}$. Maximum possible separation between P and Q is

(A) $\frac{7R}{4}$ (B) $\frac{9R}{4}$
(C) $\frac{3R}{2}$ (D) None of these

15. The radius of the earth is 6400 km and $g = 10\ \text{ms}^{-2}$. In order that the body of 5 kg weighs zero at the equator, the angular speed of the earth should be

(A) $\frac{1}{80}\ \text{rads}^{-1}$ (B) $\frac{1}{400}\ \text{rads}^{-1}$
(C) $\frac{1}{800}\ \text{rads}^{-1}$ (D) $\frac{1}{1600}\ \text{rads}^{-1}$

16. If the radius of the earth were increased by a factor of 2, keeping the mass constant, then the factor by which its density has to be changed to keep g the same is

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$
(C) $\frac{1}{2}$ (D) 4

17. If radius of a solid sphere is decreased to half, keeping density of sphere unchanged, the slope of E-r graph will

(A) remain unchanged
(B) become two times
(C) become four times
(D) remain $\frac{1}{8}$th

18. The potential energy of a body of mass m is $U = ax + by$ the magnitude of acceleration of the body will be

(A) $\frac{ab}{m}$ (B) $\left(\frac{a+b}{m}\right)$
(C) $\frac{\sqrt{a^2+b^2}}{m}$ (D) $\frac{a^2+b^2}{m}$

19. The ratio of acceleration due to gravity at a depth h below the surface of earth and at a height h above the surface of earth for h R

(A) is constant
(B) increases parabolically with h
(C) increases linearly with h
(D) decreases parabolically with h

20. The gravitational field at a distance r from the centre of the sphere (having radius R, density ρ) inside it is

(A) $\frac{4\rho G\pi r}{3}$ (B) $\frac{4\rho Gr^2}{3}$
(C) $\frac{4\rho GR^3}{3r^2}$ (D) $\frac{\rho GR^3}{\pi r}$

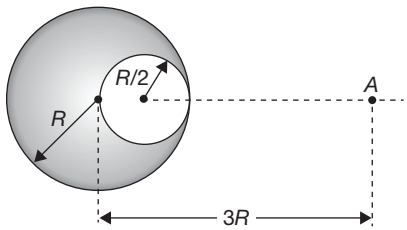
21. The ratio of the energy required to raise a satellite upto a height h above the earth (of radius R) to the kinetic energy of the satellite into the orbit there is

(A) $h : R$ (B) $R : 2h$
(C) $2h : R$ (D) $R : h$

22. A mass m is raised Slowly from a distance $2R$ from surface of earth to $3R$. Work done to do so against gravity will

(A) $\frac{mgR}{10}$ (B) $\frac{mgR}{11}$
(C) $\frac{mgR}{12}$ (D) $\frac{mgR}{14}$

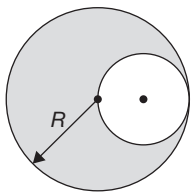
23. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at a distance $3R$ from the centre of the sphere. A spherical cavity of radius $\frac{R}{2}$ is now made in the sphere as shown in the figure. The sphere with cavity now applies a gravitational force F_2 on the same particle. The ratio $\frac{F_2}{F_1}$ is



(A) $\frac{9}{50}$ (B) $\frac{41}{50}$

(C) $\frac{3}{50}$ (D) $\frac{11}{25}$

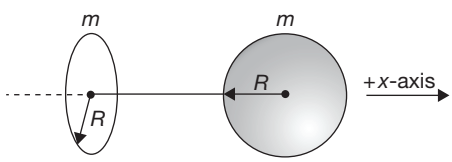
24. A spherical hole of radius $\frac{R}{2}$ is made in a solid sphere of radius R, mass M. The gravitational field at the centre of the hole due to the remaining mass is

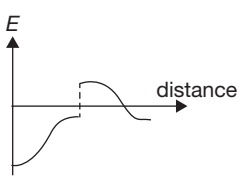


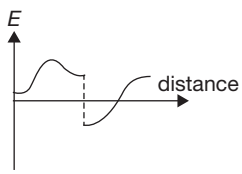
(A) zero (B) $\frac{GM}{8R^2}$

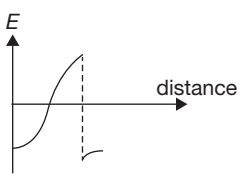
(C) $\frac{GM}{2R^2}$ (D) $\frac{GM}{R^2}$

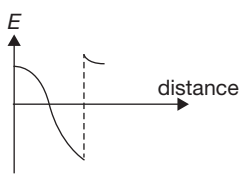
25. A ring of radius R, mass m and a solid sphere of same mass m and same radius R are placed with their centres on positive x-axis. The observer is moving from some finite distance on negative x-axis towards positive x-axis and the plane of the ring is perpendicular to x-axis. If the observer moves only upto the surface of the solid sphere, then the net gravitational field varies with the distance moved along x-axis as



(A) 

(B) 

(C) 

(D) 

26. A particle of mass M is placed at the centre of a uniform spherical shell of mass $3M$ and radius R. The gravitational potential at the surface of the shell is

(A) $-\frac{GM}{R}$ (B) $-\frac{3GM}{R}$

(C) $-\frac{2GM}{R}$ (D) $-\frac{4GM}{R}$

27. Two identical thin rings each of radius R are coaxially placed at a distance R. If the rings have a uniform mass distribution and each has mass m_1 and m_2 respectively, then the work done in moving a mass m from centre of one ring to that of the other is

(A) zero (B) $\frac{Gm(m_1-m_2)(\sqrt{2}-1)}{\sqrt{2}R}$

(C) $\frac{Gm\sqrt{2}(m_1+m_2)}{R}$ (D) $\frac{Gmm_1(\sqrt{2}+1)}{m_2R}$

28. An infinite number of masses, each of 1 kg, are placed on the positive x-axis at 1 m, 2 m, 4 m, 8 m, … from the origin. The value of the gravitational field at the origin due to this distribution is

(A) $2G$ (B) $\frac{4G}{3}$

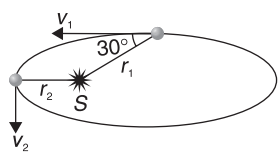
(C) $\frac{3G}{4}$ (D) ∞

29. For a given density of planet, the orbital period of a satellite near the surface of planet of radius R is proportional to

(A) $R^{\frac{1}{2}}$ (B) $R^{\frac{3}{2}}$

(C) $R^{-\frac{1}{2}}$ (D) R^0

30. For a planet revolving around sun in an elliptical path as shown



(A) $v_1r_1 = v_2r_2$ (B) $2v_1r_1 = v_2r_2$

(C) $v_1r_1 = 2v_2r_2$ (D) $2v_1r_1 = \sqrt{3}v_2r_2$

31. The magnitude of gravitational potential energy of the earth-satellite system is U with zero potential energy at infinite separation. If mass of satellite $\ll$ mass of earth and the kinetic energy of satellite is K, then

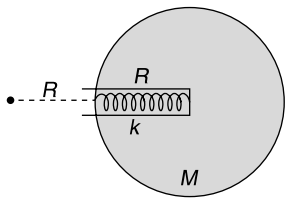
(A) $K = 2U$ (B) $K = \frac{U}{2}$

(C) $K = U$ (D) $K = 4U$

32. A particle on earth's surface is given a velocity equal to its escape velocity. The total mechanical energy of the particle is

(A) infinite (B) negative
(C) positive (D) zero

33. A small ball of mass m is released at a height R above the earth surface, as shown in the given figure. The maximum depth of the ball to which it goes inside the earth through a narrow groove before coming to rest momentarily is $\frac{R}{2}$. The groove, contains an ideal spring of spring constant k and natural length R. The value of k, if R is the radius of earth and M mass of earth, is



(A) $\frac{3GMm}{R^3}$ (B) $\frac{6GMm}{R^3}$
(C) $\frac{9GMm}{R^3}$ (D) $\frac{12GMm}{R^3}$

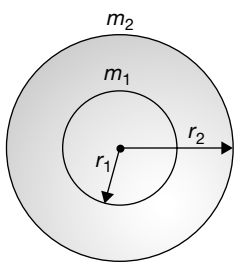
34. A projectile is fired upwards from the surface of the earth with a velocity kv_e where v_e is the escape velocity and $k<1$. If r is the maximum distance from the centre of the earth to which it rises and R is the radius of the earth, then r equals

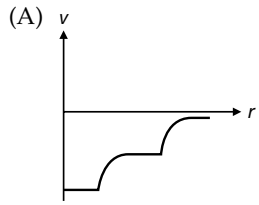
(A) $\frac{R}{k^2}$ (B) $\frac{R}{1-k^2}$
(C) $\frac{2R}{k^2}$ (D) $\frac{2R}{1-k^2}$

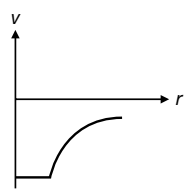
35. The binding energy of a particle at the surface of earth is E. If a kinetic energy greater than E is given to this particle, then the total energy of particle will be

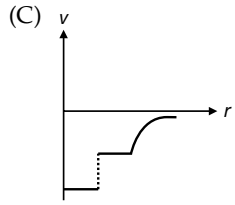
(A) zero (B) infinite
(C) >0 (D) <0

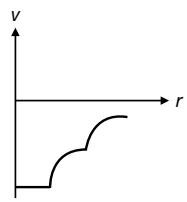
36. Two concentric spherical shells are as shown in figure. The V-r graph will be



(A) 

(B) 

(C) 

(D) 

37. A star with a mass of more than three times that of the sun contracts so much upon cooling that it loses its capacity to radiate; neither material particles nor light are able to overcome its gravitational field (such an object is called a "black hole"). Taking mass of sun to be 2×10^{30} kg, the radius of such an object could be (approximately)

(A) 6 km (B) 7 km
(C) 8 km (D) 9 km

38. Four particles of equal mass M move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(A) $\frac{GM}{R}$ (B) $\sqrt{\left[2\sqrt{2}\frac{GM}{R}\right]}$
(C) $\sqrt{\left[\frac{GM}{R}\left(2\sqrt{2}+1\right)\right]}$ (D) $\sqrt{\left[\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)\right]}$

39. The gravitational field in a region of space is given by $\vec{E}=\left(4\hat{i}+\hat{j}\right)$ Nkg^{-1}. The work done by this field is zero when a particle is moved along the line

(A) $y+4x=2$ (B) $4y+x=6$
(C) $x+y=5$ (D) $y-4x=2$

40. The speed of a planet in an elliptical orbit with semi-major axis a about sun of mass M at a distance r from sun is

(A) $\sqrt{GM\left(\frac{2}{r}-\frac{1}{a}\right)}$ (B) $\sqrt{GM\left(\frac{1}{r}-\frac{1}{a}\right)}$
(C) $\sqrt{GM\left(\frac{1}{r}-\frac{2}{a}\right)}$ (D) $\sqrt{\frac{GMr}{2a^2}}$

41. A body is projected up with a velocity equal to three fourth of the escape velocity from the surface of the earth. The maximum distance it reaches from the centre of the earth is (Radius of the earth is R)

(A) $\frac{10R}{9}$ (B) $\frac{16R}{7}$

(C) $\frac{9R}{8}$ (D) $\frac{10R}{3}$

42. The work done in slowly lifting a body from earth's surface to a height R (radius of earth) is equal to two times the work done in lifting the same body from earth's surface to a height h, then

(A) $h=\frac{R}{6}$ (B) $h=\frac{R}{4}$

(C) $h=\frac{R}{3}$ (D) $h=\frac{R}{2}$

43. A particle of mass m starts moving from rest along the line $y=b$ with constant acceleration a. The areal velocity of the position vector of the particle at time t is

(A) constant (B) $\frac{abt}{2}$

(C) $\frac{abt}{2m}$ (D) $\frac{a^2bt}{2m}$

44. A particle is placed in a field characterised by a value of gravitational potential given by $V=-kxy$, where k is a constant. If $\vec{E}_g$ is the gravitational field then,

(A) $\vec{E}_g=k\left(x\hat{i}+y\hat{j}\right)$ and is conservative in nature.

(B) $\vec{E}_g=k\left(y\hat{i}+x\hat{j}\right)$ and is conservative in nature.

(C) $\vec{E}_g=k\left(x\hat{i}+y\hat{j}\right)$ and is non-conservative in nature.

(D) $\vec{E}_g=k\left(y\hat{i}+x\hat{j}\right)$ and is non-conservative in nature.

45. The gravitational potential of two homogeneous spherical shells A and B of same surface density at their respective centres are in the ratio $3:4$. If the two shells coalesce into single one such that surface charge density remains same, then the ratio of potential at an internal point of the new shell to the potential inside shell A is

(A) $3:2$ (B) $4:3$

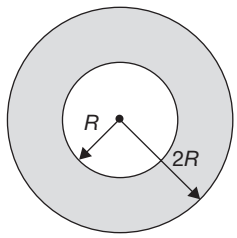
(C) $5:3$ (D) $5:6$

46. A satellite is revolving round the earth in an orbit of radius r with time period T. If the satellite is revolving round the earth in an orbit of radius $r+\Delta r$ ($\Delta r \ll r$) with time period $T+\Delta T$ ($\Delta T \ll T$) then,

(A) $\frac{\Delta T}{T}=\frac{3}{2}\frac{\Delta r}{r}$ (B) $\frac{\Delta T}{T}=\frac{2}{3}\frac{\Delta r}{r}$

(C) $\frac{\Delta T}{T}=\frac{\Delta r}{r}$ (D) $\frac{\Delta T}{T}=-\frac{\Delta r}{r}$

47. The gravitational potential at the centre of a sphere of radius $2R$ having a concentric spherical cavity of radius R and net mass M is



(A) $-\frac{3GM}{7R}$ (B) $-\frac{9GM}{14R}$

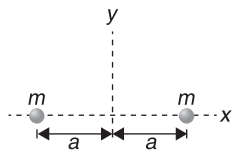
(C) $-\frac{5GM}{7R}$ (D) $-\frac{7GM}{14R}$

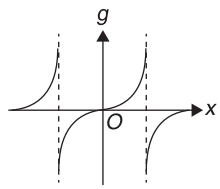
48. A particle is projected vertically upwards with a velocity $\sqrt{gR}$, where R denotes the radius of the earth and g the acceleration due to gravity on the surface of the earth. Then the maximum height ascended by the particle is

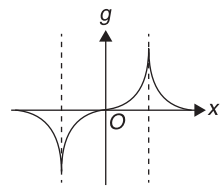
(A) $\frac{R}{2}$ (B) R

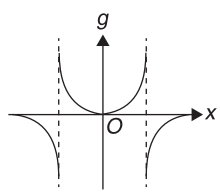
(C) $2R$ (D) $\frac{5R}{4}$

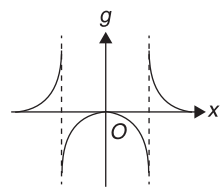
49. Two identical spherical balls each of mass m are placed as shown in figure. The variation of gravitational intensity g along the x-axis is best represented in



(A) 

(B) 

(C) 

(D) 

50. A mass is taken from surface to a height h. The change in potential energy in this process is equal to the change in potential energy if it is now taken from that point to infinity. Then,

(A) $h=R$ (B) $h=2R$

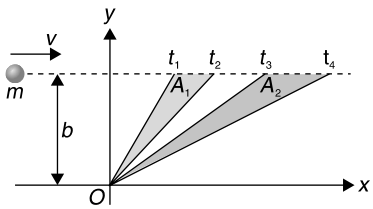
(C) $h=\frac{3R}{2}$ (D) $h=4R$

51. The time period of an earth satellite close to the surface of the earth is 83 minute. The time period of another earth satellite in an orbit at a distance of three earth radii from its surface will be

(A) 83 minute (B) $83\sqrt{8}$ minute

(C) 664 minute (D) 249 minute

52. A particle of mass m moves along a straight line with constant speed in the x direction at a distance b from the x-axis as shown and sweeps areas A_1 and A_2, then



(A) $A_1 > A_2$ if $t_4 - t_3 > t_2 - t_1$

(B) $A_1 > A_2$ if $t_4 - t_3 < t_2 - t_1$

(C) $A_1 = A_2$ if $t_4 - t_3 < t_2 - t_1$

(D) A_1 is always equal to A_2

53. Two particles each of mass m are revolving in circular orbits of radius $r = 5R$ in opposite directions. They collide perfectly inelastically and fall to the ground. The speed of combined mass on striking the ground is

(A) $2\sqrt{2}v_0$ (B) $\sqrt{2}v_0$

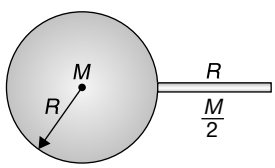
(C) $2v_0$ (D) v_0

54. The ratio of Earth's orbital angular momentum (about the Sun) to its mass is $4.4\times10^{15}\ \text{m}^2\text{s}^{-1}$. The area enclosed by the earth's orbit is approximately

(A) $1\times10^{22}\ \text{m}^2$ (B) $3\times10^{22}\ \text{m}^2$

(C) $5\times10^{22}\ \text{m}^2$ (D) $7\times10^{22}\ \text{m}^2$

55. A uniform thin rod of mass $\frac{M}{2}$ and length R is placed normally on surface of earth as shown. The mass of earth is M and its radius is R. Then the magnitude of gravitational force exerted by earth on the rod is



(A) $\frac{GM^2}{4R^2}$ (B) $\frac{GM^2}{2R^2}$

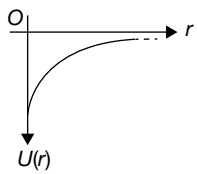
(C) $\frac{4GM^2}{9R^2}$ (D) $\frac{GM^2}{8R^2}$

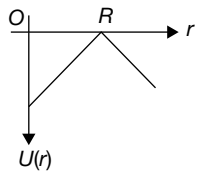
56. A particle is projected from the surface of earth with velocity equal to its escape velocity at $45°$ with horizontal. The angle of its velocity with horizontal at height $h = R$, where horizontal at some point means a line parallel to tangent on earth just below that point, is

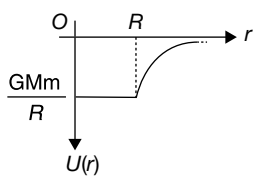
(A) $30°$ (B) $\cos^{-1}\left(\frac{1}{3}\right)$

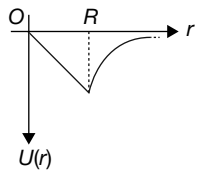
(C) $60°$ (D) $\cos^{-1}\left(\frac{1}{4}\right)$

57. A shell of mass M and radius R has a point mass m placed at a distance r from its centre. The gravitational potential energy $U(r)$ vs r will be

(A) 

(B) 

(C) 

(D) 

58. In a double star system, the masses of the two stars are M and $3M$. The orbit radius of the lighter star is R. The time period of each star is

(A) $8\pi\sqrt{\frac{R^3}{GM}}$ (B) $16\pi\sqrt{\frac{R^3}{GM}}$

(C) $8\pi\sqrt{\frac{R^3}{27GM}}$ (D) None of these

59. A particle of mass m is projected upwards with velocity half the escape velocity. At the highest point, the potential energy of the particle is

(A) $-\frac{GMm}{2R}$ (B) $-\frac{GMm}{4R}$

(C) $-\frac{3GMm}{4R}$ (D) $-\frac{2GMm}{3R}$

60. A satellite of mass m is moving around the earth in a circular orbit of radius r. If the mass of the earth is M, then the energy required to take the satellite to an orbit of radius $3r$ is

(A) $\frac{GMm}{r}$ (B) $\frac{GMm}{2r}$

(C) $\frac{GMm}{3r}$ (D) $\frac{GMm}{6r}$

61. Three particles each having a mass of $100\ g$ are placed on the vertices of an equilateral triangle of side $20\ cm$. The work that has to be done to increase the side of this triangle to $40\ cm$ is $\left(G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}\right)$

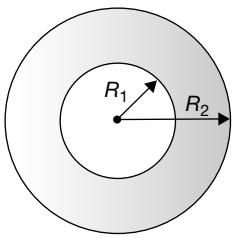
(A) $5 \times 10^{-8}\ J$

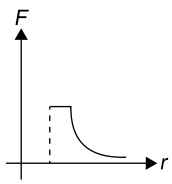
(B) $4 \times 10^{-10}\ J$

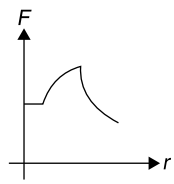
(C) $5 \times 10^{-12}\ J$

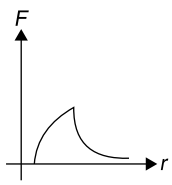
(D) $5 \times 10^{-15}\ J$

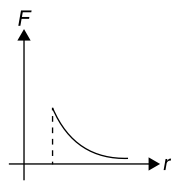
62. A sphere of mass M and radius R_2 has a concentric cavity of radius R_1 as shown in figure. The force F exerted by the sphere on a particle of mass m located at a distance r from the centre of sphere varies as $(0 \le r \le \infty)$



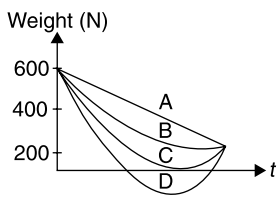
(A) 

(B) 

(C) 

(D) 

63. Imagine the acceleration due to gravity on earth is $10\ ms^{-2}$ and on mars is $4\ ms^{-2}$. A traveler of mass $60\ kg$ goes from earth to mars by a rocket moving with constant velocity. If effect of other planets is assumed to be negligible, which one of the following graphs shown the variation of weight $W(\text{in N})$ of traveler with time t (in second)



(A) A
(B) B
(C) C
(D) D

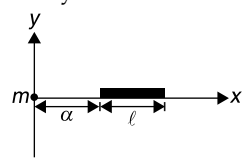
64. The gravitational field due to a mass distribution is $E = \frac{K}{x^3}$ in the x-direction, where K is a constant. Taking the gravitational potential to be zero infinity, its value at a distance x is

(A) $\frac{K}{x}$
(B) $\frac{K}{2x}$
(C) $\frac{K}{x^2}$
(D) $\frac{K}{2x^2}$

65. A system consists of n identical particles each of mass m. The total number of interactions possible is

(A) $n(n+1)$
(B) $\frac{n(n+1)}{2}$
(C) $n(n-1)$
(D) $\frac{n(n-1)}{2}$

66. A straight rod of length l extends from $x = \alpha$ to $x = l + \alpha$. If the mass per unit length is $(a + bx^2)$. The gravitational force it exerts on a point mass m placed at $x = 0$ is given by



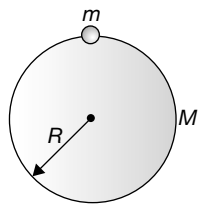
(A) $Gm\left(a\left(\frac{1}{\alpha} - \frac{1}{\alpha + l}\right) + bl\right)$

(B) $\frac{Gm(a + bx^2)}{l^2}$

(C) $Gm\left(\alpha\left(\frac{1}{a} - \frac{1}{a + l}\right) + bl\right)$

(D) $Gm\left(a\left(\frac{1}{\alpha + l} - \frac{1}{\alpha}\right) + bl\right)$

67. A thin spherical shell of mass M and radius R has a small hole. A particle of mass m is released at the mouth of the hole. Then the particle



(A) will execute simple harmonic motion inside the shell.

(B) will oscillate inside the shell, but the oscillations are not simple harmonic.

(C) will not oscillate, but the speed of the particle will go on increasing.

(D) stays at rest in its equilibrium position.

68. A small meteorite of mass m travelling towards the centre of earth strikes the earth at the equator. The earth is a uniform sphere of mass M and radius R. The length of the day was T before the meteorite struck. After the meteorite strikes the earth, the length of day increases by

(A) $\frac{5mT}{2M}$ (B) $\frac{mT}{M}$

(C) $\frac{4mT}{5M}$ (D) $\frac{M}{3mT}$

69. A ring of radius R has a total mass M distributed non-uniformly over its circumference. A point mass m is placed at the centre (C) of the ring. The work done in taking away this point mass from centre to infinity is

(A) $-\frac{GMm}{R}$ (B) $\frac{GMm}{R}$

(C) $\frac{GMm}{2R}$ (D) $-\frac{GMm}{2R}$

70. The gravitational self energy of a spherical shell of mass M and radius R is

(A) $-\frac{GM^2}{4R}$ (B) $-\frac{GM^2}{R}$

(C) $-\frac{3}{5}\frac{GM^2}{R}$ (D) $-\frac{GM^2}{2R}$

71. An artificial satellite is moving in a circular orbit around the earth at some height with a speed equal to half the magnitude of the escape velocity from the surface of earth. Suppose the satellite is stopped suddenly in its orbit and allowed to fall freely. On reaching earth its speed will be

(A) $\sqrt{gR}$ (B) $2\sqrt{gR}$

(C) $3\sqrt{gR}$ (D) $5\sqrt{gR}$

72. The height above the surface of earth where the total energy of a satellite is equal to its potential energy at a height of $2R$ from surface of earth (if R is the radius of the earth) is

(A) $\frac{R}{4}$ (B) $\frac{R}{2}$

(C) $2R$ (D) $4R$

73. Two particles having masses m_1 and m_2 start moving towards each other from the state of rest from infinite separation. Their relative velocity of approach when they are interacting gravitationally at a separation r will be

(A) $\sqrt{\frac{G(m_1+m_2)}{r}}$ (B) $\sqrt{\frac{2G(m_1+m_2)}{r}}$

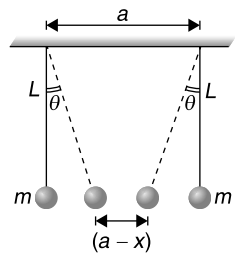
(C) $\sqrt{\frac{3G(m_1+m_2)}{r}}$ (D) $\sqrt{\frac{4G(m_1+m_2)}{r}}$

74. A particle is projected upward from the surface of earth with a speed equal to the orbital speed of a satellite near the earth's surface. If R be the radius of earth, then the height to which it would rise is

(A) $\sqrt{2}R$ (B) $\frac{R}{\sqrt{2}}$

(C) R (D) $2R$

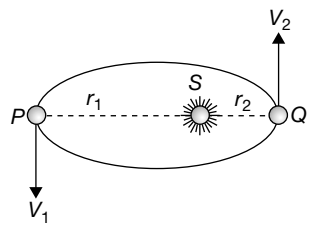
75. Two small balls of mass m each are suspended side by side by two equal threads of length L. If the distance between the upper ends of the threads be a, the angle θ that the threads will make with the vertical due to attraction between the balls is



(A) $\tan^{-1}\left[\frac{(a-x)g}{Gm}\right]$ (B) $\tan^{-1}\left[\frac{Gm}{(a-x)^2 g}\right]$

(C) $\tan^{-1}\left[\frac{(a-x)^2 g}{Gm}\right]$ (D) $\tan^{-1}\left[\frac{(a^2-x^2)g}{Gm}\right]$

76. A planet is moving in an elliptical orbit around the sun as shown in figure. At positions P and Q it has respective speeds v_1 and v_2 and respective distances r_1 and r_2 from the sun, then $\frac{v_1}{v_2}$ equals



(A) $\frac{r_1}{r_2}$ (B) $\frac{r_2}{r_1}$

(C) $\sqrt{\frac{r_1}{r_2}}$ (D) $\left(\frac{r_1}{r_2}\right)^2$

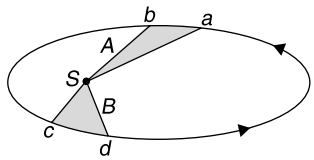
77. A body is projected away from the earth with a speed $3v_e$ where v_e is the escape velocity. The speed of the body at infinity will be

(A) v_e (B) $\sqrt{2}v_e$

(C) $2v_e$ (D) $2\sqrt{2}v_e$

78. Two air bubbles in water
(A) repel each other
(B) attract each other
(C) do not exert any force on each other
(D) may attract or repel depending upon the distance between them

79. The figure shows the motion of a planet around the sun in an elliptic orbit with the sun at one focus. The shaded areas A and B can be assumed to be equal. If t_1 and t_2 represent the times taken by the planet to move from a to b and from c to d respectively, then



(A) $t_1 < t_2$ (B) $t_1 > t_2$
(C) $t_1 = t_2$ (D) Data Insufficient

80. A satellite of mass m orbits the earth in an elliptical orbit having aphelion distance r_a and perihelion distance r_p. The period of the orbit is T. The semi-major and semi-minor axes of the ellipse are $\frac{r_a + r_p}{2}$ and $\sqrt{r_p r_a}$ respectively. The angular momentum of the satellite is

(A) $\frac{m\pi\left(r_a + r_p\right)\sqrt{r_a r_p}}{T}$

(B) $\frac{2m\pi\left(r_a + r_p\right)\sqrt{r_a r_p}}{T}$

(C) $\frac{m\pi\left(r_a + r_p\right)\sqrt{r_a r_p}}{2T}$

(D) $\frac{m\pi\left(r_a + r_p\right)\sqrt{r_a r_p}}{4T}$

81. A satellite of mass m moves along an elliptical path around the earth. The areal velocity of the satellite is proportional to
(A) m^{-1} (B) m
(C) $m^{\frac{1}{2}}$ (D) m^0

82. The time period of an artificial satellite in a circular orbit of radius R is 2 days and its orbital velocity is v_0. If time period of another satellite in a circular orbit is 16 days, then
(A) its radius of orbit is $4R$ and orbit velocity is v_0
(B) its radius of orbit is $2R$ and orbital velocity is v_0
(C) its radius of orbit is $2R$ and orbital velocity is $\frac{v_0}{2}$
(D) its radius of orbit is $4R$ and orbital velocity is $\frac{v_0}{2}$

83. The period of revolution of an earth satellite close to the surface of earth is 90 minutes. The time period of another satellite in an orbit at an altitude of three times the radius of earth is
(A) $90\sqrt{8}$ min (B) 270 min
(C) 360 min (D) 720 min

84. Two particles of equal mass m go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(A) $V = \frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (B) $V = \sqrt{\frac{Gm}{2R}}$

(C) $V = \frac{1}{2}\sqrt{\frac{Gm}{R}}$ (D) $V = \sqrt{\frac{4Gm}{R}}$

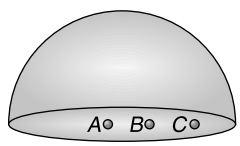
85. The minimum energy required to launch a satellite of mass m from the surface of earth (of mass M, radius R) in a circular orbit at an altitude $2R$ is

(A) $\frac{GmM}{2R}$ (B) $\frac{GmM}{3R}$

(C) $\frac{5GmM}{6R}$ (D) $\frac{2GmM}{3R}$

86. Consider a circular disc of mass M, radius R with surface mass density σ. The gravitational potential due to the disc at a point P lying on its axis at distance r from the centre is
(A) $-2\pi\sigma Gr$
(B) $-4\pi\sigma Gr$
(C) $-2\pi\sigma Gr\left(1 - \frac{r}{\sqrt{r^2 + R^2}}\right)$
(D) $-4\pi\sigma Gr\left(1 - \frac{r}{\sqrt{r^2 + R^2}}\right)$

87. Mass M is uniformly distributed only on curved surface of a thin hemispherical shell. A, B and C are three points on the circular base of hemisphere, such that A is the centre. Let the gravitational potential at points A, B and C be V_A, V_B, V_C respectively. Then



(A) $V_A > V_B > V_C$
(B) $V_C > V_B > V_A$
(C) $V_B > V_A$ and $V_B > V_C$
(D) $V_A = V_B = V_C$

88. An electron moves in a circular orbit at a distance from a proton with kinetic energy E. To escape to infinity, the energy which must be supplied to the electron is

(A) $0.5E$ (B) $2E$

(C) E (D) $\sqrt{2}E$

89. A body of mass m is kept at a small height h above the ground. If the radius of the earth is R and its mass is M, then the potential energy of the body and earth system, with the reference position being taken at infinity, is

(A) $\frac{GMm}{R}+mgh$ (B) $\frac{-GMm}{R}+mgh$

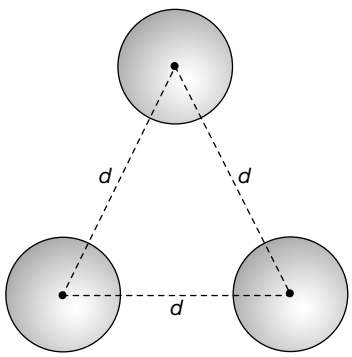
(C) $\frac{GMm}{R}-mgh$ (D) $\frac{-GMm}{R}-mgh$

90. The gravitational field in a region is given by $\vec{I}=\left(2\hat{i}+3\hat{j}\right)\text{Nkg}^{-1}$. The work done in moving a particle from $(1,\,1)$ m to $\left(2,\,\frac{1}{3}\right)$ m along the line $3y+2x=5$ is

(A) Zero (B) +20 J

(C) –15 J (D) +18 J

91. Three solid spheres each of mass m and radius R are released from the position shown in figure. The speed of any one sphere at the time of collision would be



(A) $\sqrt{Gm\left(\frac{1}{d}-\frac{3}{R}\right)}$ (B) $\sqrt{Gm\left(\frac{3}{d}-\frac{1}{R}\right)}$

(C) $\sqrt{Gm\left(\frac{2}{R}-\frac{1}{d}\right)}$ (D) $\sqrt{Gm\left(\frac{1}{R}-\frac{2}{d}\right)}$

92. The masses and the radii of earth and moon are respectively M_e, R_e and M_m, R_m. The distance between the centres is r. At what minimum velocity a particle of mass m be projected from the mid point of the distance between their centres so that it may escape into space?

(A) $\sqrt{\frac{4G}{r}(M_e+M_m)}$ (B) $\frac{4G}{r}\sqrt{(M_e+M_m)}$

(C) $\sqrt{\frac{2G}{r}(M_e+M_m)}$ (D) $\frac{2G}{r}\sqrt{(M_e+M_m)}$

93. The potential energy of gravitational interaction of a point mass m and a thin uniform rod of mass M and length l, if they are located along a straight line at a distance a from each other is

(A) $U=-\frac{GMm}{a}\log_e\left(\frac{a+l}{a}\right)$

(B) $U=GMm\left(\frac{1}{a}-\frac{1}{a+l}\right)$

(C) $U=-\frac{GMm}{l}\log_e\left(\frac{a+l}{a}\right)$

(D) $U=-\frac{GMm}{a}$

94. An object of mass 100 kg starts to fall from a height equal to radius of earth (R). Its speed when it has lost half the initial height is [M is mass of earth]

(A) $\sqrt{\frac{100G}{3R}}$ (B) $\sqrt{\frac{GM}{6R}}$

(C) $\sqrt{\frac{50G}{3R}}$ (D) $\sqrt{\frac{GM}{3R}}$

95. The magnitude of the potential energy per unit mass of the object at the surface of earth is E. Then the escape velocity of the object is

(A) $\sqrt{2E}$ (B) $2\sqrt{E}$

(C) $\sqrt{E}$ (D) $\sqrt{\frac{E}{2}}$

96. If g is the acceleration due to gravity at the surface of the earth, then the energy required to launch a satellite of mass m from the surface of the earth into a circular orbit at an altitude of $2R$, R being the radius of the earth, is

(A) $\frac{mgR}{6}$ (B) $\frac{mgR}{3}$

(C) $\frac{2mgR}{3}$ (D) $\frac{5}{6}mgR$

97. If the period of revolution of an artificial satellite just above the earth's surface is T and the density of earth is ρ, then ρT^2 equals

(A) $\frac{3\pi}{G}$ (B) $\frac{3\pi}{2G}$

(C) $\frac{2\pi}{G}$ (D) $\frac{\pi}{G}$

98. Consider a mass m_0 enclosed by a closed imaginary surface S. Let $\vec{E}$ be the gravitational field intensity due to m_0 at the surface element $d\vec{S}$ directed as outward normal to it. The surface integral of the gravitational field over S is

(A) $-m_0G$ (B) $-4\pi m_0G$

(C) $-\dfrac{m_0G}{4\pi}$ (D) None of above

99. If M_e is the mass of earth and M_m is the mass of moon $(M_e = 81M_m)$. The potential energy of an object of mass m situated at a distance R from the centre of earth and r from the centre of moon, will be

(A) $-GmM_m\left(\dfrac{R}{81}+r\right)\dfrac{1}{R^2}$ (B) $-GmM_e\left(\dfrac{81}{r}+\dfrac{1}{R}\right)$

(C) $-GmM_m\left(\dfrac{81}{R}+\dfrac{1}{r}\right)$ (D) $GmM_m\left(\dfrac{81}{R}-\dfrac{1}{r}\right)$

100. A person brings a mass of 2 kg from A to B. The increase in kinetic energy of the mass is 4 J and the work done by the person on the mass is -10 J. The potential difference between A and B is ΔV, given by

(A) 4 Jkg^{-1} (B) 7 Jkg^{-1}

(C) -3 Jkg^{-1} (D) -7 Jkg^{-1}

101. A research satellite of mass 200 kg circles the Earth in an orbit of average radius $\dfrac{3R}{2}$, where R is the radius of the Earth. Assuming gravitational pull on a mass 1 kg on Earth's surface to be 10 N, the pull on the satellite will be

(A) 880 N (B) 889 N

(C) 890 N (D) 892 N

102. A satellite is revolving round the earth with orbital speed v_0. If it stops suddenly, the speed with which it will strike the surface of earth (if v_e is the escape speed of the particle from the earth's surface), would be

(A) $\dfrac{v_e^2}{v_0}$ (B) v_0

(C) $\sqrt{v_e^2 - v_0^2}$ (D) $\sqrt{v_e^2 - 2v_0^2}$

103. A planet is moving in an elliptic orbit. If T, U, E and L stand respectively, for its kinetic energy, gravitational potential energy, total energy and angular momentum about the centre of force, then

(A) T is conserved

(B) U is always positive

(C) E is always negative

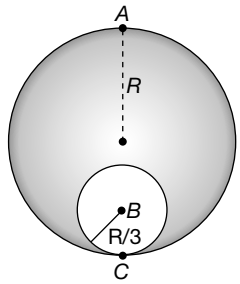
(D) magnitude of L is conserved but its direction changes continuously

104. A planet revolves around the sun in an elliptical orbit of eccentricity e. If T is the time period of the planet, then the time spent by the planet between the ends of the minor axis close to the sun is

(A) $\dfrac{T}{2}\left(1-\dfrac{e}{\pi}\right)$ (B) $T\left(\dfrac{1}{4}-\dfrac{e}{2\pi}\right)$

(C) $\left(\dfrac{e}{\pi}-1\right)T$ (D) $\dfrac{T\pi}{e}$

105. Inside a uniform sphere of mass M and radius R, a cavity of radius $\dfrac{R}{3}$ is made in the sphere as shown. Select the correct statement.



(A) Gravitational field inside the cavity is uniform

(B) Gravitational field inside the cavity is non-uniform

(C) The escape velocity of a particle projected from point A is $\sqrt{\dfrac{88GM}{15R}}$

(D) Escape velocity is defined for earth and particle system only

106. The angular momentum (L) of earth revolving round the sun in circular orbit is proportional to r^n, where r is the orbital radius of the earth. The value of n is

(A) 1 (B) 2

(C) $-\dfrac{1}{2}$ (D) $\dfrac{1}{2}$

107. Consider a thin uniform spherical layer of mass M and radius R. The potential energy of gravitational interaction of matter forming this shell is

(A) $-\dfrac{GM^2}{R}$ (B) $-\dfrac{1}{2}\dfrac{GM^2}{R}$

(C) $-\dfrac{3}{2}\dfrac{GM^2}{R}$ (D) $-\dfrac{2}{3}\dfrac{GM^2}{R}$

108. By what percent the energy of a satellite has to be increased to shift it from an orbit of radius r to $\dfrac{3}{2}r$?

(A) 15% (B) 20.3%

(C) 33.3% (D) 66.7%

109. For a spherical shell of mass M and radius R, the energy required to double its radius is

(A) $\frac{GM^2}{R}$ (B) $\frac{3GM^2}{2R}$

(C) $\frac{GM^2}{4R}$ (D) $\frac{GM^2}{2R}$

110. A satellite is seen after every 6 hours over the equator. If it revolves from east to west, the angular velocity of the satellite about the centre of earth is

(A) $\frac{\pi}{2}$ radhr^{-1} (B) $\frac{\pi}{3}$ radhr^{-1}

(C) $\frac{\pi}{4}$ radhr^{-1} (D) $\frac{\pi}{8}$ radhr^{-1}

111. Consider a solid sphere of mass M and radius R, then the potential energy of gravitational interaction of matter forming this solid sphere is

(A) $-\frac{GM^2}{R}$ (B) $-\frac{1}{2}\frac{GM^2}{R}$

(C) $-\frac{3}{5}\frac{GM^2}{R}$ (D) $-\frac{3}{2}\frac{GM^2}{R}$

112. A planet of mass m is revolving in an elliptical orbit about the sun with an orbital period T. If $m \quad M_{\text{sun}}$ and A be the area of orbit, then the angular momentum of the planet is

(A) $\frac{mA}{2T}$ (B) $\frac{mA}{4T}$

(C) $\frac{2mA}{T}$ (D) $\frac{4mA}{T}$

113. An artificial satellite moving in circular orbit around the earth has a total $(\text{kinetic} + \text{potential})$ energy E_0. Its potential energy and kinetic energy respectively are

(A) $2E_0$ and $-2E_0$

(B) $-2E_0$ and $3E_0$

(C) $2E_0$ and $-E_0$

(D) $-2E_0$ and $-E_0$

114. A satellite revolving in a circular equatorial orbit from west to east appears over a certain point on the equator every 8 hours. Its time period is

(A) 16 hr (B) 8 hr

(C) 6 hr (D) 32 hr

115. A "double star" is a composite system of two stars rotating about their centre of mass under their mutual gravitational attraction. Let us consider such a "double star" which has two stars of masses m and $2m$ at separation l. If T is the time period of rotation about their centre of mass then,

(A) $T = 2\pi\sqrt{\frac{l^3}{mG}}$ (B) $T = 2\pi\sqrt{\frac{l^3}{2mG}}$

(C) $T = 2\pi\sqrt{\frac{l^3}{3mG}}$ (D) $T = 2\pi\sqrt{\frac{l^3}{4mG}}$

116. What should be the angular velocity of earth about its own axis so that the weight of the body at equator would become $\frac{3}{4}$th of its present value

(A) $\frac{1}{400}$ rads^{-1} (B) $\frac{1}{800}$ rads^{-1}

(C) $\frac{1}{1600}$ rads^{-1} (D) $\frac{1}{3200}$ rads^{-1}

117. Two bodies of masses m and M are placed a distance d apart. The gravitational potential at the position where the gravitational field due to them is zero is V.

(A) $V = -\frac{G}{d}(m+M)$

(B) $V = -\frac{Gm}{d}$

(C) $V = -\frac{GM}{d}$

(D) $V = -\frac{G}{d}\left(\sqrt{m}+\sqrt{M}\right)^2$

118. A point $P\left(R\sqrt{3}, 0, 0\right)$ lies on the axis of a ring of mass M and radius R. The ring is located in y-z plane with its centre at origin O. A small particle of mass m starts from P and reaches O under gravitational attraction only. Its speed at O will be

(A) $\sqrt{\frac{GM}{R}}$ (B) $\sqrt{\frac{Gm}{R}}$

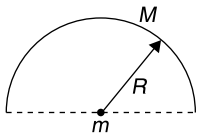
(C) $\sqrt{\frac{GM}{\sqrt{2}R}}$ (D) $\sqrt{\frac{Gm}{\sqrt{2}R}}$

119. A point particle is held on the axis of a ring of mass m and radius r at a distance r from its center C. When released, it reaches C under the gravitational attraction of the ring. Its speed at C will be

(A) $\sqrt{\frac{2Gm}{r}}$ (B) $\sqrt{\frac{2Gm}{r}}\left(\sqrt{2}-1\right)$

(C) $\sqrt{\frac{2Gm}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$ (D) $\sqrt{\frac{Gm}{r}}$

120. A particle of mass m is placed on the centre of a fixed uniform semi-circular ring of radius R and mass M as shown. Then work required to displace the particle slowly from centre of ring to infinity is (Assume only gravitational interaction of ring and particle)



(A) $\frac{GMm}{R}$ (B) $-\frac{GMm}{R}$

(C) $\frac{GMm}{\pi R}$ (D) $-\frac{GMm}{\pi R}$

121. Consider an infinite plane sheet of mass with surface mass density σ. The gravitational field intensity at a point P at perpendicular distance r from such a sheet is

(A) zero (B) $-\sigma G$

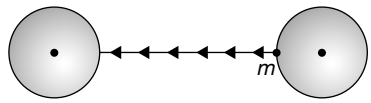
(C) $-2\pi\sigma G$ (D) $-4\pi\sigma G$

122. In PROBLEM 121, the gravitational potential at the point P is

(A) zero (B) $-\sigma Gr$

(C) $-2\pi\sigma Gr$ (D) $-4\pi\sigma Gr$

123. Two identical spherical masses are kept at some distance as shown. The potential energy when a mass m is taken from the surface of one sphere to the other



(A) increases continuously
(B) decreases continuously
(C) first increases then decreases
(D) first decreases then increases

124. The radius and mass of earth are increased by 0.5%. Which of the following statements are true at the surface of the earth?

(a) g will decrease

(b) Escape velocity will remain unchanged

(c) Potential energy will remain unchanged

(A) (a) only (B) both (a) and (b)

(C) (a), (b) and (c) (D) both (b) and (c)

MULTIPLE CORRECT CHOICE TYPE QUESTIONS

This section contains Multiple Correct Choice Type Questions. Each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. An object is revolving around the earth of radius R_e at height h from earth's surface. Select the correct statement(s).

(A) Its time period is independent of h.
(B) Its time period depends on h.
(C) Orbital velocity of an object depends on h.
(D) If h R_e then the time period of this object revolving around the earth is 24 hrs.

2. If, at the surface of earth, the potential energy of a particle is U and potential is V, then the change in potential energy and the change in potential at a height $h = R$ are ΔU and ΔV, then

(A) $\Delta U = -\frac{U}{2}$ (B) $\Delta U = \frac{U}{2}$

(C) $\Delta V = \frac{V}{2}$ (D) $\Delta V = -\frac{V}{2}$

3. A shell of mass m_2, radius r_2 lies inside and is concentric with a larger uniform shell of mass m_1, radius r_1. If E_P is the gravitational field at point P at distance r from the common centre then,

(A) $E_P = G\left(\frac{m_1 + m_2}{r^2}\right)$ for $r > r_1$ & $r > r_2$

(B) $E_P = G\frac{m_2}{r^2}$ for $r < r_1$ and $r > r_2$

(C) $E_P = 0$ for $r < r_2$

(D) $E_P \neq 0$ for $r < r_2$

4. In PROBLEM 3, if V_P is the gravitational potential at the point P then,

(A) $V_P = -G\left(\frac{m_1 + m_2}{r}\right)$ for $r > r_1$ and $r > r_2$

(B) $V_P = 0$ for $r < r_1$

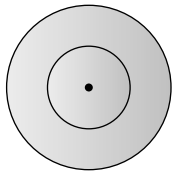
(C) $V_P = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r}\right)$ for $r < r_1$ and $r > r_2$

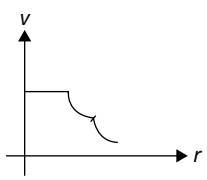
(D) $V_P = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right)$ for $r < r_2$

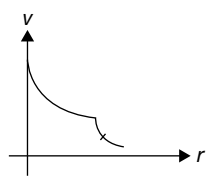
5. A satellite is to be stationed in an orbit such that it can be used for relay purposes (such a satellite is called a Geostationary satellite). The condition(s) to be fulfilled by such a satellite is/are

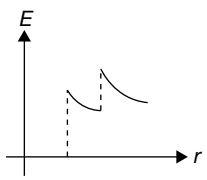
(A) Its orbit must lie in equatorial plane
(B) Its sense of revolution must be from west to east
(C) Its orbital radius must be 42400 km
(D) Its orbit must not be elliptical and should be circular

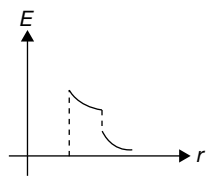
6. Two concentric spherical shells are as shown in figure. The magnitude of gravitational potential (V) and field strength (E) vary with distance (r) from centre as



(A) 

(B) 

(C) 

(D) 

7. A body is imparted a velocity v from the surface of the earth. If v_o is orbital velocity and v_e be the escape velocity then for

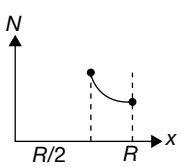
(A) $v = v_o$, the body follows a circular track around the earth.

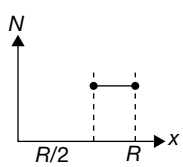
(B) $v > v_o$ but $< v_e$, the body follows elliptical path around the earth.

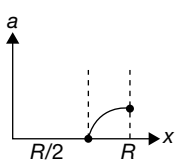
(C) $v < v_o$, the body follows elliptical path and returns to surface of earth.

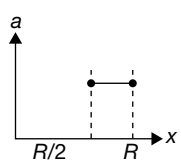
(D) $v > v_e$, the body follows hyperbolic path and escapes the gravitational pull of the earth.

8. A tunnel is dug along a chord of the earth at a perpendicular distance $\frac{R}{2}$ from the earth's centre. The wall of the tunnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The force N exerted by the particle on the wall and the acceleration a of the particle varies with x (distance of the particle from the centre) according to

(A) 

(B) 

(C) 

(D) 

9. The gravitational potential on the surface of a planet of radius R mass M is

(A) g (B) $\frac{gM}{R}$

(C) $\frac{-GM}{R}$ (D) $-gR$

10. Let V and E be the gravitational potential and gravitational field. Then select the correct alternative(s)

(A) the plot of V vs r is continuous for a spherical shell

(B) the plot of E vs r (distance from centre) is discontinuous for a spherical shell

(C) the plot of V vs r is continuous for a solid sphere

(D) the plot of E vs r is discontinuous for a solid sphere

11. Total energy of planet of mass m moving around the sun of mass M_s along an elliptical orbit depends on

(A) Mass of the sun

(B) Mass of the planet

(C) Semi major axis

(D) Independent of mass of the sun

12. A planet is revolving round the sun. Its distance from the sun at Apogee is r_A and that at Perigee is r_P. The mass of planet and sun is m and M respectively, v_A and v_P is the velocity of planet at Apogee and Perigee respectively and T is the time period of revolution of planet round the sun.

(A) $T^2 = \frac{\pi^2}{2Gm}(r_A + r_P)^3$

(B) $T^2 = \frac{\pi^2}{2GM}(r_A + r_P)^3$

(C) $v_A r_A = v_P r_P$

(D) $v_A < v_P$; $r_A > r_P$

13. Two tunnels are dug from one side of the earth's surface to the other side, one along a diameter and the other along a chord. Now two particles are dropped from one end of each of the tunnels. Both the particles oscillate simple harmonically along the tunnels. Let T_1 and T_2 be the time periods and v_1 and v_2 be the maximum speed of the particles in the two tunnels. Then

(A) $T_1 > T_2$ (B) $T_1 = T_2$

(C) $v_1 = v_2$ (D) $v_1 > v_2$

14. In some region, the gravitational field is zero. The gravitational potential in this region

(A) may be zero (B) cannot be zero

(C) must be constant (D) may be variable

15. A planet is revolving round the sun in an elliptical orbit. The work done on the planet by the gravitational force of sun is zero

(A) in some parts of the orbit
(B) in any part of the orbit
(C) in no part of the orbit
(D) in one complete revolution

16. A point P is lying at a distance $r(<a)$ from the centre of shell of radius a. If E and V be the gravitational field and potential at the point P then,

(A) $E=0$ (B) $E=-\frac{GM}{r^2}$

(C) $V=0$ (D) $V=-\frac{GM}{a}$

17. Select the correct statement(s).
(A) Acceleration due to gravity increases with increasing altitude.
(B) Acceleration due to gravity decreases with increasing depth (assume the earth to be a sphere of uniform density)
(C) Acceleration due to gravity is independent of mass of the body
(D) The formula $-GMm\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$ is more accurate than the formula mg (r_2-r_1) for the difference of potential energy between two points r_2 and r_1 distance away from the centre of earth

18. Two objects of masses m and $4m$ are at rest at an infinite separation. They move towards each other under mutual gravitational attraction. If G is the universal gravitational constant. Then, at separation r
(A) the total energy of the two objects is zero
(B) their relative velocity of approach is $\sqrt{\frac{10Gm}{r}}$ in magnitude
(C) the total kinetic energy of the objects is $\frac{4Gm^2}{r}$
(D) net angular momentum of both the particles is zero about any point

19. Which of the following statement(s) is/are correct
(A) An astronaut going from Earth to Moon will experience weightlessness once
(B) When a thin uniform spherical shell gradually shrinks maintaining its shape, the gravitational potential at the centre decreases
(C) In the case of spherical shell, the plot of potential versus distance from centre is continuous
(D) In the case of spherical shell, the plot of gravitational field intensity I versus distance from centre is continuous

20. Two identical satellites are orbiting at distances R and $7R$ from the surface of the earth, R being the radius of the earth. The ratio of their
(A) kinetic energies is 4
(B) potential energies is 4
(C) total energies is 4
(D) total energies is 1

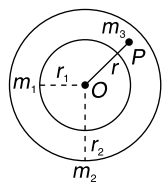
21. In circular orbit of a satellite of mass m, revolving around a planet of mass M in an orbit of radius r and having the period of revolution T, we have
(A) orbital speed equals $\sqrt{\frac{GM}{r}}$
(B) $T^2 \propto r^3$
(C) kinetic energy equals $\frac{GMm}{2r}$
(D) potential energy equals $-\frac{GMm}{2r}$

22. Which (one or more) of the following quantities remain constant during planetary motion in an elliptic orbit?
(A) angular speed
(B) areal speed
(C) angular momentum
(D) energy

23. A planet of mass m is revolving round the sun (of mass M_s) in an elliptical orbit. If $\vec{v}$ is the velocity of the planet when its position vector from the sun is $\vec{r}$ and the planet rotates in counter clockwise direction, then areal velocity has a direction
(A) given by "Right Hand Thumb Rule".
(B) given by "Left Hand Thumb Rule".
(C) normal to the plane of orbit upwards.
(D) normal to the plane of orbit downwards.

24. For a solid sphere, the magnitude of
(A) gravitational potential is maximum at centre
(B) gravitational potential is minimum at centre
(C) field strength is maximum at centre
(D) field strength is minimum at centre

25. Two concentric spherical shells have masses m_1 and m_2 and radii r_1 and r_2. Select the correct statement(s).



(A) Outer shell will have no contribution in gravitational field at point P
(B) Force on P is directed towards O
(C) Force on P is $\frac{Gm_1m_2}{r^2}$
(D) Force on P is $\frac{Gm_1m_3}{r^2}$

26. For a satellite revolving in a circular orbit, let v_0 be the orbital speed, T be the time period of revolution, U be its potential energy and K the kinetic energy. Now if the value of G is decreased, then

(A) v_0 will decrease (B) T will decrease

(C) U will decrease (D) K will decrease

27. A double star in a system of two stars of masses m and $2m$, rotating about their centre of mass only under their mutual gravitational attraction. If r is the separation between these two stars then their time period of rotation about their centre of mass will be proportional to

(A) $r^{\frac{3}{2}}$ (B) r

(C) $m^{\frac{1}{2}}$ (D) $m^{-\frac{1}{2}}$

28. The acceleration due to earth's gravity decreases if

(A) we go down from the surface of the earth towards its centre

(B) we go up from the surface of the earth

(C) we go from the equator towards of the poles

(D) the rotational speed of the earth is increased

29. For a planet revolving round the sun in an elliptical orbit, the

(A) potential energy is constant

(B) kinetic energy is constant

(C) total mechanical energy is constant

(D) angular momentum about centre of sun is constant

30. A particle of mass m is moved from the surface of the earth to a height h. The work done by an external agency to do this is

(A) mgh for $h \quad R$ (B) mgh for all h

(C) $-\frac{1}{2}mgR$ for $h = R$ (D) $\frac{1}{2}mgR$ for $h = R$

31. A satellite is orbiting round the earth's surface in an orbit as close as possible to the surface of the earth. Then

(A) The time period of revolution of satellite is independent of its mass and is minimum

(B) The orbital speed of satellite is maximum

(C) The total energy of the "earth plus satellite" system is minimum

(D) The total energy of the "earth plus satellite" system is maximum

32. A student uses a simple pendulum of exactly 1 m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 s for this and records 40 s for 20 oscillations. For this observation, which of the following statement(s) is (are) true?

(A) Error ΔT in measuring T, the time period, is 0.05 s

(B) Error ΔT in measuring T, the time period, is 1 s

(C) Percentage error in the determination of g is 5%

(D) Percentage error in the determination of g is 2.5%

33. Three point masses each of mass m are at the corners of an equilateral triangle of side l. The system rotates about the centre of the triangle with the separation of masses not changing during rotation. If T is the time period of rotation then,

(A) $T \propto l^{\frac{3}{2}}$ (B) $T \propto l^{\frac{1}{2}}$

(C) $T \propto m^{\frac{1}{2}}$ (D) $T \propto m^{-\frac{1}{2}}$

34. Three masses each of mass m are kept at corner of an equilateral triangle and are revolving under the effect of mutual gravitational force. Select the correct statement(s).

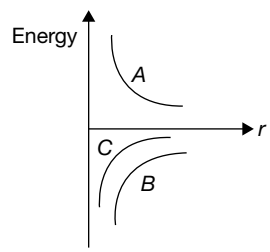
(A) Radius of circular path followed by mass is $\frac{a}{2}$

(B) Velocity of mass is $\sqrt{\frac{Gm}{a}}$

(C) Binding energy of system is $\frac{3Gm^2}{2a}$

(D) Time period of mass is $\sqrt{\frac{\pi a^3}{2Gm}}$

35. Figure shows the variation of energy with the orbit radius r of a satellite in a circular motion. Select the incorrect statement(s).



(A) C shows the total energy, B the kinetic energy and A the potential energy of the satellite

(B) A shows the kinetic energy, B the total energy and C the potential energy of the satellite

(C) A and B are the kinetic and potential energies and C the total energy of the satellite

(D) C and A are the kinetic and potential energies respectively and B the total energy of the satellite

36. The potential at the surface of a planet of mass M and radius R is assumed to be zero. Choose the most appropriate option

(A) The potential at infinity is zero
(B) The potential at the centre of the planet is $-\frac{3GM}{2R}$
(C) The potential at infinity is $\frac{GM}{R}$
(D) The potential at the centre of planet is $-\frac{GM}{2R}$

37. A body of mass m is projected vertically upwards with velocity v from earth's surface and attains height h. If v_e is the escape velocity from the surface of earth then for
(A) $v = v_e$ the body reaches infinity.
(B) small v, maximum height h attained by the body is $\frac{v^2}{2g}$.
(C) $h = R$, the projection kinetic energy is $\frac{mgR}{2}$.
(D) $h = \frac{R}{2}$, total energy is negative.

38. A double star consists of two stars having masses m and $2m$ separated by a distance r. Which of the following statement(s) is/are incorrect?
(A) Radius of circular path of star of mass $2m$ is $\frac{2r}{3}$
(B) Kinetic energy of $2m$ mass star is double that of lighter star
(C) Time period of revolution of both are not same
(D) Angular momentum of lighter star is more

REASONING BASED QUESTIONS

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.
Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.
Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.
Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. **Statement-1:** Kepler's Second Law can be understood by conservation of angular momentum principle.
Statement-2: Kepler's Second Law is related with areal velocity, which can further be proved to be based on conservation of angular momentum as $\frac{dA}{dt} = \frac{1}{2}r^2\omega$.

2. **Statement-1:** For the planets orbiting around the sun, angular speed, linear speed, kinetic energy changes with time, but angular momentum remains constant.
Statement-2: No torque is acting on the rotating planet, so its angular momentum is constant.

3. **Statement-1:** Escape velocity is independent of the angle of projection.
Statement-2: Escape velocity from the surface of earth is $\sqrt{2gR}$, where R is radius of earth.

4. **Statement-1:** A spherically symmetric shell produces no gravitational field anywhere.
Statement-2: The field due to various mass elements cancels out, everywhere inside the shell.

5. **Statement-1:** Gravitational potential is zero inside a shell.
Statement-2: Gravitational potential is equal to the work done in bringing a unit mass from infinity to a point inside gravitational field.

6. **Statement-1:** Rate of change of weight near the earth's surface with height h is independent of h.
Statement-2: Since gravitational potential is given by $V = -\frac{GM}{r}$.

7. **Statement-1:** The magnitude of gravitational potential at the surface of solid sphere is less than that of the centre of sphere.
Statement-2: Due to solid sphere, gravitational potential is same within the sphere.

8. **Statement-1:** For a satellite revolving very near to earth's surface the time period of revolution is given by 1 hour 24 minutes.
Statement-2: The period of revolution of a satellite depends only upon its height above the earth's surface.

9. **Statement-1:** Work done in gravitational field in cyclic process is zero.
Statement-2: Work done in conservative field does not depend upon path.

10. **Statement-1:** The value of acceleration due to gravity does not depend upon mass of the body.
Statement-2: Acceleration due to gravity is a constant quantity.

LINKED COMPREHENSION TYPE QUESTIONS

This section contains Linked Comprehension Type Questions or Paragraph based Questions. Each set consists of a Paragraph followed by questions. Each question has four choices (A), (B), (C) and (D), out of which only one is correct. (For the sake of competitiveness there may be a few questions that may have more than one correct options)

Comprehension 1

The mean radius of the earth's orbit around the sun is 1.5×10^{11} m and that of the orbit of mercury is 6×10^{10} m. Based on above information, answer the following questions.

1. The mercury will revolve around the sun in nearly

(A) $\sqrt{\frac{2}{5}}$ year (B) $\frac{2}{5}$ year

(C) $\left(\frac{2}{5}\right)^2$ year (D) $\left(\frac{2}{5}\right)^{3/2}$ year

2. The ratio of the orbital velocity of mercury to that of the earth is

(A) $\sqrt{\frac{5}{2}}$ (B) $\frac{2}{\sqrt{10}}$

(C) $2\sqrt{10}$ (D) $\frac{1}{2}\sqrt{10}$

Comprehension 2

The radius of the earth is R and acceleration due to gravity at its surface is g. A body of mass m is sent to a height of $\frac{R}{4}$ from the earth's surface. Based on above information, answer the following questions.

3. The potential energy increases by

(A) $\frac{mgR}{3}$ (B) $\frac{mgR}{4}$

(C) $\frac{mgR}{5}$ (D) $\frac{mgR}{16}$

4. The minimum speed with which the body must be thrown from the surface of the earth so as to reach the specified height is

(A) $\sqrt{\frac{1}{2}gR}$ (B) $\sqrt{gR}$

(C) $\sqrt{\frac{1}{5}gR}$ (D) $\sqrt{\frac{2}{5}gR}$

Comprehension 3

Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hour, respectively. The radius of the orbit of S_1 is 10^4 km. Based on above information, answer the following questions.

5. The speed of S_2 relative to S_1, when they are closest, in kmh^{-1}, is

(A) $10^4\pi$ (B) $2\times10^4\pi$

(C) $\frac{1}{2}\times10^4\pi$ (D) $4\times10^4\pi$

6. The angular speed of S_2 as observed by an astronaut is S_1 is

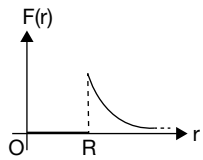
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

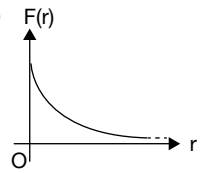
Comprehension 4

A particle of mass m is located at a distance r from the centre of a shell/sphere of mass M and radius R. The following plots (I), (II), (III) and (IV) show the variation of force $F(r)$ between the shell and mass or alternatively the variation of force between the sphere and the mass. Based on above information, answer the following questions.

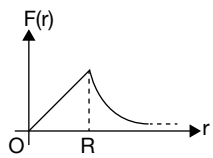
(I)



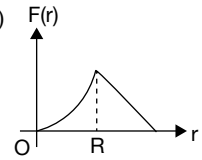
(II)



(III)



(IV)



7. The appropriate variation of the force between the shell and the point mass is represented by

(A) I (B) II

(C) III (D) IV

8. The appropriate variation of the force between the sphere and the point mass is represented by

(A) I (B) II

(C) III (D) IV

Comprehension 5

An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the escape speed from the surface of earth. Now the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth. Assuming

the radius of the earth to be R, the acceleration due to gravity on earth's surface to be g. Based on above information, answer the following questions.

9. The height of the satellite above the surface of the earth must be

(A) $\frac{R}{2}$ (B) $\frac{2R}{3}$

(C) R (D) $2R$

10. The speed with which it hits the surface of the earth is

(A) $\sqrt{Rg}$ (B) $\sqrt{2Rg}$

(C) $\sqrt{\frac{Rg}{2}}$ (D) $\frac{\sqrt{Rg}}{2}$

Comprehension 6

A smooth tunnel is dug across a diameter of earth and a particle is released from the surface of earth, the particle oscillates simple harmonically along it. Based on above information, answer the following questions.

11. Time period of the particle is equal to

(A) $2\pi\sqrt{\frac{R^3}{GM}}$ (B) $2\pi\sqrt{\frac{R}{g}}$

(C) 84.6 min (D) 24 hr

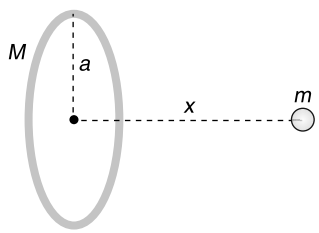
12. Maximum speed attained by the particle is

(A) $\sqrt{\frac{2GM}{R}}$ (B) $\sqrt{\frac{GM}{R}}$

(C) $\sqrt{\frac{3GM}{2R}}$ (D) $\sqrt{\frac{GM}{2R}}$

Comprehension 7

A particle of mass m is placed at a distance x from the centre of ring along the line through the centre of the ring and perpendicular to its plane.



Based on above information, answer the following questions.

13. Gravitational potential energy of this system

(A) $-\frac{GMm}{\sqrt{a^2+x^2}}$ (B) $\frac{GMm}{a}$

(C) $\frac{GMm}{x}$ (D) None of these

14. The force, when $x = 0$ is

(A) $\frac{GMm}{x^2}$ (B) $\frac{GMmx}{\sqrt{a^2+x^2}}$

(C) $\frac{GMmx}{(a^2+x^2)^{3/2}}$ (D) zero

15. When given a small displacement x a from the centre of the ring, the particle of mass m will

(A) move away from the centre

(B) perform oscillations about the centre of ring

(C) move to infinity and never return

(D) None of these

Comprehension 8

A satellite of mass 5000 kg is projected in space with an initial speed of 4000 ms^{-1} making an angle of $30°$ with the radial direction from a distance 3.6×10^7 m away from the centre of the earth.

16. The angular momentum of satellite

(A) 3.6×10^7 Js (B) 4.9×10^7 Js

(C) 9.2×10^7 Js (D) 3.6×10^{14} Js

17. The energy of satellite is

(A) 1.6×10^7 joule (B) 4.9×10^7 joule

(C) 0.2×10^7 joule (D) -1.5×10^{10} joule

18. The semi-major axis of the orbit of satellite

(A) 6.6×10^7 m (B) 14.9×10^7 m

(C) 19.2×10^7 m (D) 1.6×10^4 m

19. Semi-minor axis of the orbit of satellite

(A) 16.6×10^7 m (B) 3.92×10^7 m

(C) 10.2×10^{17} m (D) 2.6×10^4 m

20. The minimum distance of satellite from earth

(A) 66.6×10^7 m (B) 14.9×10^7 m

(C) 1.29×10^7 m (D) 1.6×10^4 m

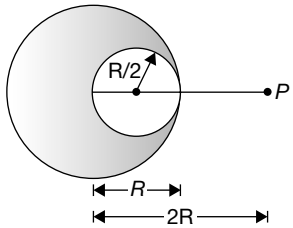
21. The maximum distance of satellite from earth.

(A) 6.6×10^7 m (B) 24.9×10^7 m

(C) 11.9×10^7 m (D) 1.6×10^4 m

Comprehension 9

A solid sphere having uniform density and radius R exerts a gravitational force of attraction F_1 on a particle placed at P. The distance of P from the centre of the sphere is $2R$. A spherical cavity of radius $\frac{R}{2}$ is now made in the sphere. The sphere (with cavity) exerts a gravitational force F_2 on the same particle at P.



Based on the above facts, answer the following questions.

22. F_1 is

(A) $\frac{GMm}{R^2}$ (B) $\frac{GMm}{2R^2}$

(C) $\frac{GMm}{3R^2}$ (D) $\frac{GMm}{4R^2}$

23. F_2 is

(A) $\frac{7}{13}\frac{GMm}{R^2}$ (B) $\frac{7}{16}\frac{GMm}{R^2}$

(C) $\frac{7}{32}\frac{GMm}{R^2}$ (D) $\frac{7}{36}\frac{GMm}{R^2}$

24. The ratio $\frac{F_1}{F_2}$ is

(A) $\frac{9}{7}$ (B) $\frac{7}{9}$

(C) $\frac{1}{2}$ (D) $\frac{4}{3}$

Comprehension 10

A binary star system consists of two stars of masses m and $2m$. The separation between their centres is l. They revolve two about a point under their mutual gravitational interaction force. Based on the above facts, answer the following questions.

25. The system revolves about

(A) the centre (B) the centre of mass

(C) both (D) None of these

26. Heavier star revolves in an orbit of radius

(A) l (B) $\frac{l}{2}$

(C) $\frac{l}{3}$ (D) $\frac{l}{4}$

27. The system revolves with angular velocity ω (about an axis specified above) given by

(A) $\sqrt{\frac{Gm}{l^3}}$ (B) $\sqrt{\frac{2Gm}{l^3}}$

(C) $\sqrt{\frac{3Gm}{l^3}}$ (D) $2\sqrt{\frac{Gm}{l^3}}$

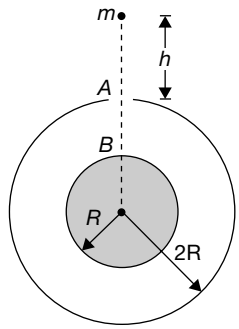
28. If K_L be the kinetic energy of light body and K_H kinetic energy of heavy body then

(A) $K_L + K_H = \frac{ml^2\omega^2}{3}$ (B) $K_L + K_H = \frac{2ml^2\omega^2}{9}$

(C) $K_L + K_H = \frac{2ml^2\omega^2}{3}$ (D) $K_L + K_H = \frac{ml^2\omega^2}{9}$

Comprehension 11

A solid sphere of mass M and radius R is surrounded by a spherical shell of same mass M and radius $2R$ as shown. A small particle of mass m is released from rest from a height $h(\quad R)$ above the shell. There is a hole in the shell.



Based on above information, answer the following questions.

29. The time taken by the particle just to enter the hole at A is

(A) $2\sqrt{\frac{hR^2}{GM}}$ (B) $\sqrt{\frac{2hR^2}{GM}}$

(C) $\sqrt{\frac{hR^2}{GM}}$ (D) None of these

30. The time taken by the particle to move from A to B is

(A) $= \frac{R^2}{\sqrt{GMh}}$ (B) $> \frac{R^2}{\sqrt{GMh}}$

(C) $< \frac{R^2}{\sqrt{GMh}}$ (D) None of these

31. The approximate speed with which the particle hits B is

(A) $\sqrt{\frac{2GM}{R}}$ (B) $\sqrt{\frac{GM}{2R}}$

(C) $\sqrt{\frac{3GM}{2R}}$ (D) $\sqrt{\frac{GM}{R}}$

Comprehension 12

The minimum and maximum distances of a satellite from the centre of the earth are $2R$ and $4R$, respectively, where R is the radius of earth and M is the mass of the

earth. Based on above information, answer the following questions.

32. The minimum speed of the satellite is

(A) $\sqrt{\frac{GM}{9R}}$ (B) $\sqrt{\frac{GM}{5R}}$

(C) $\sqrt{\frac{GM}{6R}}$ (D) $\sqrt{\frac{GM}{3R}}$

33. The maximum speed of the satellite is

(A) $\sqrt{\frac{3GM}{2R}}$ (B) $\sqrt{\frac{2GM}{3R}}$

(C) $\sqrt{\frac{2GM}{R}}$ (D) $\sqrt{\frac{5GM}{2R}}$

34. Radius of curvature at the point of minimum distance is

(A) $\frac{8R}{3}$ (B) $\frac{5R}{3}$

(C) $\frac{4R}{3}$ (D) $\frac{7R}{3}$

Comprehension 13

An Earth station receives data transmitted back in time from a future intergalactic expedition. The table summarizes the data for the moons of a planet what will be discovered in a distance galaxy.

	Radius	Gravitational Acceleration at the Surface	Orbital Radius	Orbital Period
Moon I			5×10^7 m	
Moon II			9×10^7 m	3×10^5 s
Moon III	2×10^5 m	0.20 ms^{-2}		2×10^6 s

Based on above information, answer the following questions.

35. In table, the mass of the planet is closest to

(A) 1.2×10^{24} kg (B) 1.7×10^{24} kg

(C) 2.4×10^{24} kg (D) 4.8×10^{24} kg

36. In table, the mass of Moon III is closest to

(A) 2.4×10^{19} kg (B) 4.8×10^{19} kg

(C) 1.2×10^{20} kg (D) 2.4×10^{20} kg

37. In table, the centripetal acceleration of Moon II due to orbital motion is closest to

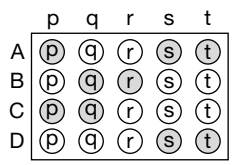
(A) 0.02 ms^{-2} (B) 0.04 ms^{-2}

(C) 0.06 ms^{-2} (D) 0.08 ms^{-2}

MATRIX MATCH/COLUMN MATCH TYPE QUESTIONS

Each question in this section contains statements given in two columns, which have to be matched. The statements in **COLUMN-I** are labelled A, B, C and D, while the statements in **COLUMN-II** are labelled p, q, r, s (and t). Any given statement in **COLUMN-I** can have correct matching with **ONE OR MORE** statement(s) in **COLUMN-II**. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following examples:

If the correct matches are A → p, s and t; B → q and r; C → p and q; and D → s and t; then the correct darkening of bubbles will look like the following:



1. Match the gravitational field and potential in COLUMN-I to the respective quantities in COLUMN-II.

COLUMN-I	COLUMN-II
(A) $E_g = 0$	(p) Inside a spherical shell
(B) $E_g \neq 0$	(q) At centre of a solid sphere
(C) $V = 0$	(r) Inside the solid sphere (excluding centre)
(D) $V \neq 0$	(s) At centre of a circular ring
	(t) None of these

(*Continued*)

2. Density of a planet is two times the density of earth and radius of this planet is half the radius of earth. Match, the following. (As compared to earth).

COLUMN-I	COLUMN-II
(A) Gravitational strength at the surface of this planet	(p) Half
(B) Gravitational potential on the surface	(q) Same
(C) Gravitational potential at centre	(r) Two times
(D) Gravitational strength at centre	(s) Four times

3. In elliptical orbit of a planet, as the planet moves from aphelion position to perihelion position, match the following table:

COLUMN-I	COLUMN-II
(A) distance of planet from centre of sun	(p) remains same
(B) speed of planet	(q) decreases
(C) potential energy	(r) increases
(D) angular momentum about centre of sun	(s) zero
(E) change in mechanical energy	

4. Match the following

COLUMN-I	COLUMN-II
(A) Time period of a body orbiting the planet in circular orbit	(p) Independent of mass of body
(B) Orbital velocity of body	(q) Dependent of radius of orbit
(C) Mechanical energy of body	(r) Dependent of mass of planet
(D) Escape velocity of a body from surface of planet	(s) Constant in an orbit

5. If the angular speed of rotation of earth about its own axis decreases, then match the following

COLUMN-I	COLUMN-II
(A) Value of g at pole	(p) remains same
(B) Value of g at equator	(q) increases
(C) Distance of geostationary satellite	(r) decreases
(D) Energy of geostationary satellite	(s) None of these

6. Match the following

COLUMN-I	COLUMN-II
(A) Gravitational force	(p) Law of Conservation of Angular Momentum
(B) Kepler's First Law	(q) $T^2 \propto r^{3/2}$
(C) Kepler's Second Law	(r) Inverse square law
(D) Kepler's Third Law	(s) Orbit of planet is elliptical
	(t) Conservative

7. On the surface of earth the gravitational field is E and gravitational potential is V. Match the following

COLUMN-I	COLUMN-II
(A) At height $h = R$, value of E	(p) decreases by a factor $\frac{1}{4}$
(B) At depth $d = \frac{R}{2}$, value of E	(q) decreases by a factor $\frac{1}{2}$
(C) At height $h = R$, value of V	(r) increases by a factor $\frac{11}{8}$
(D) At depth $d = \frac{R}{2}$, value of V	(s) increases by a factor 2
	(t) decreases by a factor $\frac{11}{8}$

8. Consider A planet of mass M and radius R and a satellite is launched with velocity V from its surface. Match the following

COLUMN-I	COLUMN-II
(A) $V > \sqrt{\frac{2GM}{R}}$	(p) Ellipse
(B) $V = \sqrt{\frac{2GM}{R}}$	(q) Hyperbola
(C) $V < \sqrt{\frac{2GM}{R}}$	(r) Circle
(D) $V = \sqrt{\frac{GM}{R}}$	(s) Parabola

9. If the velocity of launch (v), escape velocity (v_e) and orbital velocity v_0 are related as shown in COLUMN-I, then match them with the corresponding quantities of COLUMN-II.

COLUMN-I	COLUMN-II
(A) When $v < v_0$	(p) The path of satellite is hyperbolic
(B) When $v = v_e = \sqrt{2}v_0$	(q) The satellite will strike the earth
(C) When $v > v_e$	(r) The orbit of satellite is elliptical
(D) $v_e > v > v_0$	(s) The path of satellite is parabolic

10. Let a satellite of mass m is revolving in a circular orbit of radius r around a planet of mass M such that system is bound. If E, U, K and B denotes total energy, potential energy, kinetic energy and binding energy respectively then match the following

COLUMN-I	COLUMN-II
(A) E	(p) $+\frac{GMm}{2r}$
(B) U	(q) $+\frac{GMm}{r}$
(C) K	(r) $-\frac{GMm}{r}$
(D) B	(s) $-\frac{GMm}{2r}$

11. Match the following for a planet revolving around sun in an elliptical orbit of semi-major axis a and semi-minor axis b.

COLUMN-I	COLUMN-II
(A) $T^2 \propto r^3$	(p) Kepler's First Law
(B) Areal velocity is constant	(q) Kepler's Second Law
(C) Orbital of planet is elliptical	(r) Kepler's Third Law
(D) $\frac{2\pi mab}{T} = \text{constant}$	(s) None of these

INTEGER/NUMERICAL ANSWER TYPE QUESTIONS

In this section, the answer to each question is a numerical value obtained after doing series of calculations based on the data given in the question(s).

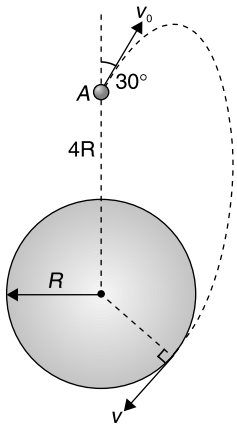
1. Calculate the ratio of the orbital radius of a communication satellite to the radius of earth so that it can cover 75% of the surface area of earth during its revolution. Take $(2.65)^2 \approx 7$.

2. Calculate the work done, in newton, by the gravitational field when a particle is moved along the line $3y + 2x = 5$ in the region of gravitational field $\vec{E}_g = \left(2\hat{i} + 3\hat{j}\right)\ \text{Nkg}^{-1}$, from $(1,\ 1)$ m to $(-2,\ 3)$ m.

3. A circular ring of mass M and radius R is placed in YZ plane with centre at origin. A particle of mass m is released from rest at a point $x = 2R$. The speed with which it will crosses the centre of ring is $v = \left[x\left(1 - \frac{1}{y}\right)\frac{GM}{R}\right]^{\frac{1}{z}}$. Find $\frac{y^2}{xz}$.

4. A particle is fired vertically upward with a speed of $15\ \text{kms}^{-1}$. Find the speed of the particle in kms^{-1}, when it goes out of the earth's gravitational pull.

5. A particle is projected from point A (which lies at a distance $4R$ from the centre of the Earth), with a speed v_0 in a direction making $30°$ with the line joining the centre of the Earth and point A, as shown.

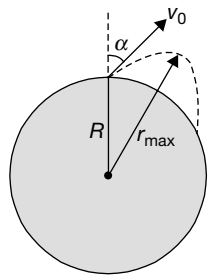


Calculate the speed v_0 of particle, in ms^{-1}, if particle passes grazing the surface of the earth. Consider that only the gravitational interaction between these two exists.
Given that $\frac{GM}{R} = 6.4 \times 10^7 \text{ m}^2\text{s}^{-2}$

6. A ring of radius $R = 4$ m made of a highly dense material has a mass $m_1 = 5.4 \times 10^9$ kg distributed uniformly over its circumference. A highly dense particle of mass $m_2 = 6 \times 10^8$ kg is placed on the axis of the ring at a distance $x_0 = 3$ m from the centre of the ring. Neglecting all other forces, except mutual gravitational interaction of the two, calculate
(a) displacement of the ring, in cm, when particle is at the centre of ring and
(b) speed of the particle, in cms^{-1}, at that instant.

7. Two particles each of mass M are fixed at positions $(0, a)$ and $(0, -a)$. Another particle of mass $\frac{M}{2}$ is thrown from origin along $+z$ axis so that it is just able to reach a point $\left(0, 0, 2\sqrt{3}a\right)$. The speed with which the particle was projected from the origin is calculated as $v = \sqrt{\frac{\alpha GM}{a}\left(1 - \frac{1}{\sqrt{\beta}}\right)}$. Find α and β.

8. A projectile of mass m is fired form the surface of the earth at an angle $\alpha = 60°$ from the vertical. The initial speed of the projectile is $v_0 = \sqrt{\frac{GM}{R}}$, where M and R are mass and radius of earth. The projectile rises to a height $\frac{R}{*}$, where $*$ is not readable. Find $*$. Neglect air resistance and the earth's rotation.



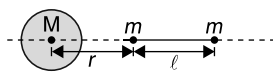
9. If a satellite is revolving around the earth in a circular orbit in a plane containing earth's axis of rotation. If the angular speed of satellite is equal to that of earth, calculate the time (in hour) taken by it to move from a point above north pole to a point above the equator.

10. A smooth tunnel is dug along the radius of earth that ends at centre. A ball is released from the surface of earth along tunnel. Coefficient of restitution for collision between the centre of the earth and ball is 0.5. Calculate the distance travelled by ball just before second collision at centre is xR. Given mass of the earth is M and radius of the earth is R, find x.

11. If the distance between the centres of Earth and Moon is D and mass of Earth is 81 times that of Moon. The distance from the centre of earth where the gravitational field will be zero is $\frac{xD}{y}$, then calculate the value of $(y - x)$.

12. If a planet was suddenly stopped in its orbit supposed to be circular, show that it would fall onto the sun in a time $\frac{1}{\sqrt{2^x}}$ times the period of the planet's revolution. Find x.

13. A large spherical mass M is fixed at one position and two identical masses m are kept on a line passing through the centre of M as shown in figure.



The point masses are connected by a rigid massless rod of length l and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3l$ from M the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. Find the value of k.

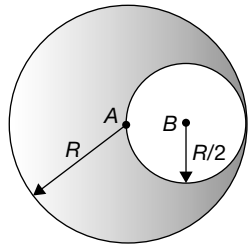
14. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{12}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{4}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be $11\ \text{kms}^{-1}$, find the escape speed on the surface of the planet in kms^{-1}.

15. A small point mass m is to be thrown with such a speed at a distance x from the axis of a long cylinder of radius R and density ρ, so that m starts revolving around the cylinder in a circular orbit of radius x with centre at the axis of cylinder. The speed with which point mass is thrown is found to be $v = R(*G\rho\pi)^{\frac{1}{*}}$, where $*$ is not readable. Calculate $*$.

16. A binary star consists of two stars A (mass $2.2M_S$) and B (mass $11M_S$), where M_S is the same mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. Find the ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass.

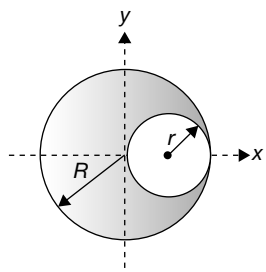
17. An artificial satellite is describing an equatorial orbit at $1600\ \text{km}$ above the surface of the earth. Calculate its orbital speed, in ms^{-1} and the period of revolution, in seconds.
If the satellite is travelling in the same direction as the rotation of the earth (i.e., from west to east), calculate the interval (in seconds) between two successive times at which it will appear vertically overhead to an observer at a fixed point on the equator. Radius of earth is $6400\ \text{km}$ and $g = \pi^2\text{ms}^{-2}$ at the surface of earth.

18. Inside a fixed sphere of radius R and uniform density ρ, there is spherical cavity of radius $\frac{R}{2}$ such that surface of the cavity passes through the centre of the sphere as shown in figure. A particle of mass m is released from rest at centre B of the cavity. The velocity with which particle strikes the centre A of the sphere is v. Neglect earth's gravity. Initially sphere and particle are at rest. If the escape speed of the particle from surface of the outer bigger sphere is v_e. Then find $\frac{2v_e^2}{v^2}$.



19. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $\frac{1}{9}$th of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere)

20. A sphere of radius R has its centre at the origin. It has uniform mass density ρ_0 except that there is a spherical hole of radius $r = \frac{1}{2}R$ whose centre is at $x = \frac{1}{2}R$, as shown in figure. The gravitational field at $x = 2R$ is $\left(\frac{a}{a+20}\right)\pi G\rho_0 R$. Find a.



21. A spaceship approaches the moon (of mass M and radius R) along a parabolic path which is almost tangential to its surface. At the moment of maximum approach, the brake rocket is fired to convert the spaceship into a satellite of the moon. The change in speed is found to be $(\sqrt{x}-1)\sqrt{\frac{GM}{R}}$. Find x.

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1. [Online September 2020]
The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance r from its center. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as

(A) $T \propto R$ (B) $T^2 \propto \frac{1}{R^3}$

(C) $T^2 \propto R$ (D) $T^2 \propto R^3$

2. [Online September 2020]
The height h at which the weight of a body will be the same as that at the same depth h from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected)

(A) $\frac{\sqrt{5}R-R}{2}$ (B) $\frac{\sqrt{5}}{2}R-R$

(C) $\frac{R}{2}$ (D) $\frac{\sqrt{3}R-R}{2}$

3. [Online September 2020]
A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that is become $\sqrt{\frac{3}{2}}$ times larger. Due to this, the farthest distance from the center of the earth that the satellite reaches is R, value of R is

(A) $4R_e$ (B) $3R_e$

(C) $2R_e$ (D) $2.5R_e$

4. [Online September 2020]
The mass density of a planet of radius R varies with the distance r from its center as $\rho(r)=\rho_0\left(1-\frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at

(A) $r=\frac{1}{\sqrt{3}}R$ (B) $r=\sqrt{\frac{5}{9}}R$

(C) $r=\sqrt{\frac{3}{4}}R$ (D) $r=R$

5. [Online September 2020]
On the x-axis and a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x. Taking its value to be zero at infinity, is

(A) $\frac{A}{(x^2+a^2)^{1/2}}$ (B) $\frac{A}{(x^2+a^2)^{3/2}}$

(C) $A(x^2+a^2)^{3/2}$ (D) $A(x^2+a^2)^{1/2}$

6. [Online September 2020]
A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is

(A) 1 (B) 2

(C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

7. [Online September 2020]
The value of the acceleration due to gravity is g_1 at a height $h=\frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals

(A) $\frac{7}{9}$ (B) $\frac{4}{9}$

(C) $\frac{1}{3}$ (D) $\frac{5}{9}$

8. [Online September 2020]
The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is: (h R, where R is the radius of the earth)

(A) $\frac{R^2\omega^2}{8g}$ (B) $\frac{R^2\omega^2}{4g}$

(C) $\frac{R^2\omega^2}{g}$ (D) $\frac{R^2\omega^2}{2g}$

9. [Online September 2020]
A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is

(A) 1 : 6 (B) 3 : 4

(C) 1 : 3 (D) 1 : 2

10. [Online September 2020]

Two planets have masses M and $16M$ and their radii are a and $2a$, respectively. The separation between the centers of the planets is $10a$. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centers. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is

(A) $\sqrt{\frac{GM^2}{ma}}$ (B) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

(C) $4\sqrt{\frac{GM}{a}}$ (D) $2\sqrt{\frac{GM}{a}}$

11. [Online January 2020]

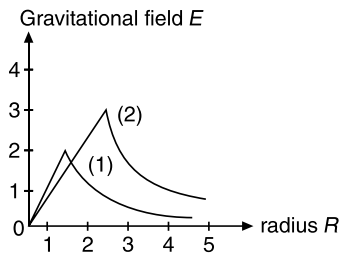
A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g = 10\ \text{ms}^{-2}$ at the north pole and the radius of the earth = 6400 km)

(A) 195.66 N (B) 194.66 N

(C) 194.32 N (D) 195.32 N

12. [Online January 2020]

Consider two solid spheres of radii $R_1 = 1$ m, $R_2 = 2$ m and masses M_1 and M_2, respectively. The gravitational field due to sphere (1) and (2) are shown. The value of $\frac{M_1}{M_2}$ is



(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{1}{3}$ (D) $\frac{1}{6}$

13. [Online January 2020]

A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ of A. The collision is completely inelastic. Then, the combined body

(A) starts moving in an elliptical orbit around the planet.

(B) continues to move in a circular orbit

(C) Falls vertically downwards towards the planet

(D) Escapes from the Planet's Gravitational field.

14. [Online January 2020]

Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is

(A) 4 (B) 1

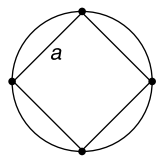
(C) 2 (D) 3

15. [Online January 2020]

An asteroid is moving directly towards the center of the earth. When at a distance of $10R$ (R is the radius of the earth) from the earths center, it has a speed of $12\ \text{kms}^{-1}$. Neglecting the effect of earth's atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is $11.2\ \text{kms}^{-1}$)? Give your answer to the nearest integer in kms^{-1}.

16. [Online April 2019]

Four identical particles of mass M are located at the corners of a square of side a. What should be their speed if each of them revolves under the influence of others' gravitational field in a circular orbit circumscribing the square?



(A) $1.41\sqrt{\frac{GM}{a}}$ (B) $1.16\sqrt{\frac{GM}{a}}$

(C) $1.21\sqrt{\frac{GM}{a}}$ (D) $1.35\sqrt{\frac{GM}{a}}$

17. [Online April 2019]

A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon.

(A) $\frac{E}{64}$ (B) $\frac{E}{4}$

(C) $\frac{E}{16}$ (D) $\frac{E}{32}$

18. [Online April 2019]

A solid sphere of mass M and radius a is surrounded by a uniform concentric spherical shell of thickness $2a$

and mass $2M$. The gravitational field at distance $3a$ from the centre will be

(A) $\frac{GM}{9a^2}$ (B) $\frac{2GM}{9a^2}$

(C) $\frac{GM}{3a^2}$ (D) $\frac{2GM}{3a^2}$

19. [Online April 2019]

A test particle is moving in a circular orbit in the gravitational field produced by a mass density $\rho(r)=\frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T

(A) $\frac{T}{R}$ is a constant (B) TR is a constant

(C) $\frac{T}{R^2}$ is a constant (D) $\frac{T^2}{R^3}$ is a constant

20. [Online April 2019]

The value of acceleration due to gravity at Earth's surface is $9.8\ \text{ms}^{-2}$. The altitude above its surface at which the acceleration due to gravity decreases to $4.9\ \text{ms}^{-2}$, is close to (Radius of earth $=6.4\times10^6$ m)

(A) 9.0×10^6 m (B) 6.4×10^6 m

(C) 1.6×10^6 m (D) 2.6×10^6 m

21. [Online April 2019]

A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet?

[Given: Mass of planet $=8\times10^{22}$ kg, Radius of planet $=2\times10^6$ m, Gravitational constant $G=6.67\times10^{-11}\ \text{Nm}^2\text{kg}^{-2}$]

(A) 17 (B) 13

(C) 11 (D) 9

22. [Online April 2019]

The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is $9:4$. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If R is the radius of the Earth, what is the radius of the planet? (Take the planets to have the same mass density)

(A) $\frac{R}{4}$ (B) $\frac{R}{3}$

(C) $\frac{R}{9}$ (D) $\frac{R}{2}$

23. [Online January 2019]

If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is

(A) $\frac{L}{m}$ (B) $\frac{4L}{m}$

(C) $\frac{L}{2m}$ (D) $\frac{2L}{m}$

24. [Online January 2019]

The energy required to take a satellite to a height h above Earth surface (radius of Earth $=6.4\times10^3$ km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is

(A) 3.2×10^3 km (B) 1.6×10^3 km

(C) 1.28×10^4 km (D) 6.4×10^3 km

25. [Online January 2019]

A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is

(A) $2mv^2$ (B) mv^2

(C) $\frac{1}{2}mv^2$ (D) $\frac{3}{2}mv^2$

26. [Online January 2019]

Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is: (Take Gravitational constant $G=6.67\times10^{-11}\ \text{Nm}^2\text{kg}^{-2}$)

(A) $2.8\times10^5\ \text{ms}^{-1}$ (B) $1.4\times10^5\ \text{ms}^{-1}$

(C) $2.4\times10^4\ \text{ms}^{-1}$ (D) $3.8\times10^4\ \text{ms}^{-1}$

27. [Online January 2019]

A satellite is revolving in a circular orbit at a height h from the earth surface, such that h R where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is

(A) $\sqrt{2gR}$ (B) $\sqrt{gR}\left(\sqrt{2}-1\right)$

(C) $\sqrt{gR}$ (D) $\sqrt{\frac{gR}{2}}$

28. [Online January 2019]

A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force it exerts on a point mass m at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by

(A) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) + BL\right]$

(B) $Gm\left[A\left(\frac{1}{a+L} - \frac{1}{a}\right) - BL\right]$

(C) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

(D) $Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) - BL\right]$

29. [Online January 2019]

A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth, collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same, just before the collision. The subsequent motion of the combined body will be

(A) such that it escapes to infinity
(B) in a circular orbit of a different radius
(C) in an elliptical orbit
(D) in the same circular orbit of radius R

30. [Online January 2019]

Two satellites, A and B, have masses m and $2m$ respectively. A is in a circular orbit of radius R and B is in a circular orbit of radius $2R$ around the earth. The ratio of their kinetic energies, $\frac{T_A}{T_B}$ is

(A) 1 (B) $\frac{1}{2}$

(C) 2 (D) $\sqrt{\frac{1}{2}}$

31. [2018]

A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is

(A) $-\frac{k}{4a^2}$ (B) $\frac{k}{2a^2}$

(C) zero (D) $-\frac{3}{2}\frac{k}{a^2}$

32. [2018]

A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then

(A) $T \propto R^{\frac{3}{2}}$ for any n (B) $T \propto R^{\frac{n}{2}+1}$

(C) $T \propto R^{\frac{(n+1)}{2}}$ (D) $T \propto R^{\frac{n}{2}}$

33. [Online 2018]

A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$ and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is

(A) $+\frac{GMm}{6R}$ (B) $\frac{GMm}{2R}$

(C) $-\frac{GMm}{2R}$ (D) $-\frac{GMm}{6R}$

34. [Online 2018]

Take the mean distance of the moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22} kg and 2×10^{30} kg respectively. The radius of the earth is 6400 km. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest points on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. Then, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is

(A) 2 (B) 0.6

(C) 6 (D) 10^{-2}

35. [Online 2018]

Suppose that the angular velocity of rotation of earth is increased. Then, as a consequence

(A) there will be no change in weight anywhere on the earth.
(B) weight of the object, everywhere on the earth, will increase.
(C) except at poles, weight of the object on the earth will decrease.
(D) weight of the object, everywhere on the earth, will decrease.

36. [Online 2018]

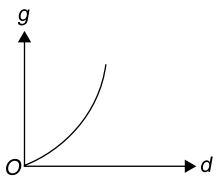
Two particles of the same mass m are moving in circular orbits because of force, given by $F(r) = \frac{-16}{r} - r^3$. The first particle is at a distance $r = 1$ and the second, at $r = 4$. The best estimate for the ratio of kinetic energies of the first and the second particle is closest to

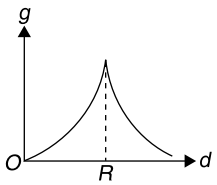
(A) 6×10^{-2} (B) 10^{-1}

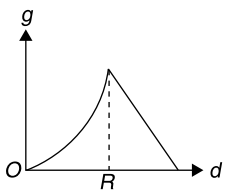
(C) 3×10^{-3} (D) 6×10^{2}

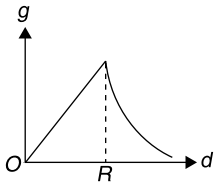
37. [2017]

The variation of acceleration due to gravity g with distance d from centre of the Earth is best represented by (R = Earth's radius)

(A) 

(B) 

(C) 

(D) 

38. [Online 2017]

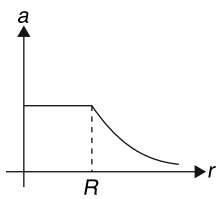
If the Earth has no rotational motion, the weight of a person on the equator is W. Determine the speed with which the earth would have to rotate about its axis so that the person at the equator will weigh $\frac{3}{4}W$. Radius of the Earth is 6400 km and $g = 10\ \text{ms}^{-2}$.

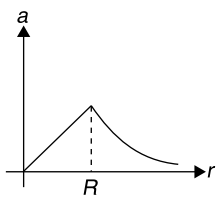
(A) 0.63×10^{-3} rads^{-1} (B) 0.83×10^{-3} rads^{-1}

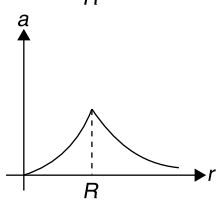
(C) 0.28×10^{-3} rads^{-1} (D) 1.1×10^{-3} rads^{-1}

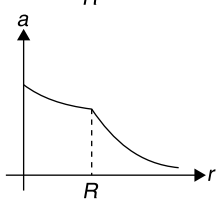
39. [Online 2017]

The mass density of a spherical body is given by $\rho(r)=\frac{k}{r}$ for $r\le R$ and $\rho(r)=0$ for $r>R$, where r is the distance from the centre. The correct graph that describes qualitatively the acceleration, a of a test particle as a function of r is

(A) 

(B) 

(C) 

(D) 

40. [2016]

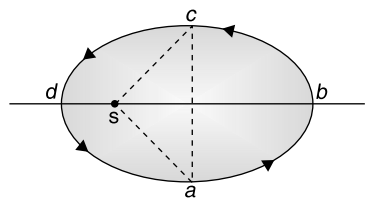
A satellite is revolving in a circular orbit at a height h from the earth's surface (radius of earth R; h R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to (Neglect the effect of atmosphere)

(A) $\sqrt{gR}(\sqrt{2}-1)$ (B) $\sqrt{2gR}$

(C) $\sqrt{gR}$ (D) $\sqrt{\frac{gR}{2}}$

41. [Online 2016]

Figure shows elliptical path $abcd$ of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse. (See figure) With bd as the major axis, and ca as the minor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then



(A) $t_1 = 4t_2$ (B) $t_1 = 2t_2$

(C) $t_1 = 3t_2$ (D) $t_1 = t_2$

42. [Online 2016]

An astronaut of mass m is working on a satellite orbiting the earth at a distance h from the earth's surface. The radius of the earth is R, while its mass is M. The gravitational pull F_G on the astronaut is

(A) Zero since astronaut feels weightless

(B) $\frac{GMm}{(R+h)^2} < F_G < \frac{GMm}{R^2}$

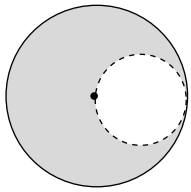
(C) $F_G = \frac{GMm}{(R+h)^2}$

(D) $0 < F_G < \frac{GMm}{R^2}$

43. [2015]

From a solid sphere of mass M and radius R, a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre P of the cavity thus formed is

(G = gravitational constant)



(A) $\frac{-GM}{2R}$ (B) $\frac{-GM}{R}$

(C) $\frac{-2GM}{3R}$ (D) $\frac{-2GM}{R}$

44. [Online 2015]

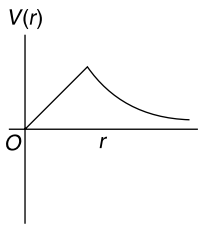
A very long (length L) cylindrical galaxy is made of uniformly distributed mass and has radius $R(R \quad L)$. A star outside the galaxy is orbiting the galaxy in a plane perpendicular to the galaxy and passing through its centre. If the time period of star is T and its distance from the galaxy's axis is r, then

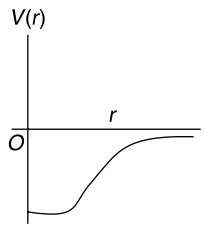
(A) $T^2 \propto r^3$ (B) $T \propto r^2$

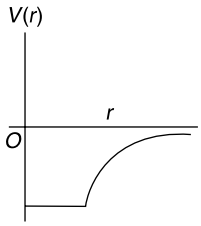
(C) $T \propto r$ (D) $T \propto \sqrt{r}$

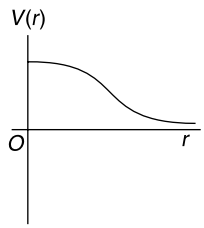
45. [Online 2015]

Which of the following most closely depicts the correct variation of the gravitational potential $V(r)$ due to a large planet of radius R and uniform mass density? (figures are not drawn to scale)

(A) 

(B) 

(C) 

(D) 

46. [2014]

Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(A) $\sqrt{\frac{GM}{R}}$ (B) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

(C) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (D) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

47. [2013]

What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$?

(A) $\frac{5GmM}{6R}$ (B) $\frac{2GmM}{3R}$

(C) $\frac{GmM}{2R}$ (D) $\frac{GmM}{3R}$

48. [2012]

The mass of a spaceship is $1000\ \text{kg}$. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are $10\ \text{ms}^{-2}$ and $6400\ \text{km}$ respectively. The required energy for this work will be

(A) 6.4×10^{8} Joules

(B) 6.4×10^{9} Joules

(C) 6.4×10^{10} Joules

(D) 6.4×10^{11} Joules

49. [2011]

Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(A) zero (B) $-\frac{4Gm}{r}$

(C) $-\frac{6Gm}{r}$ (D) $-\frac{9Gm}{r}$

50. [2009]

The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where $g=$ the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth is

(A) $2R$ (B) $\frac{R}{\sqrt{2}}$

(C) $\frac{R}{2}$ (D) $\sqrt{2}R$

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Single Correct Choice Type Problems

In this section each question has four choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

1. [JEE (Advanced) 2019]

Consider a spherical gaseous cloud of mass density $\rho(r)$ in free space where r is the radial distance from its centre. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common centre with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time, the particle number density $n(r)=\dfrac{\rho(r)}{m}$ is [G is universal gravitational constant]

(A) $\dfrac{3K}{\pi r^2 m^2 G}$ (B) $\dfrac{K}{2\pi r^2 m^2 G}$

(C) $\dfrac{K}{6\pi r^2 m^2 G}$ (D) $\dfrac{K}{\pi r^2 m^2 G}$

2. [JEE (Advanced) 2017]

A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of Earth. The escape velocity from Earth's gravitational field is $v_e=11.2\ \text{kms}^{-1}$. The minimum initial velocity (v_s) required for the rocket to be able to leave the Sun-Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet)

(A) $v_s=72\ \text{kms}^{-1}$ (B) $v_s=22\ \text{kms}^{-1}$

(C) $v_s=42\ \text{kms}^{-1}$ (D) $v_s=62\ \text{kms}^{-1}$

3. [JEE (Advanced) 2014]

A planet of radius $R=\dfrac{1}{10}$ (Radius of Earth) has the same mass density as earth. Scientists dig a well of depth $\dfrac{R}{5}$ on it and lower a wire of the same length and of linear mass density $10^{-3}\ \text{kgm}^{-1}$ into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of earth to be 6×10^6 m and the acceleration due to gravity of earth is $10\ \text{ms}^{-2}$)

(A) 96 N (B) 108 N

(C) 120 N (D) 150 N

4. [IIT-JEE 2011]

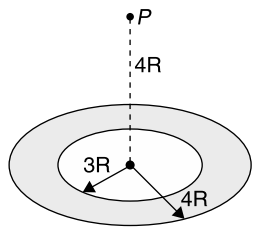
A satellite is moving with a constant speed v in a circular orbit about the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

(A) $\dfrac{1}{2}mv^2$ (B) mv^2

(C) $\dfrac{3}{2}mv^2$ (D) $2mv^2$

5. [IIT-JEE 2010]

A thin uniform annular disc (shown in figure) of mass M has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is

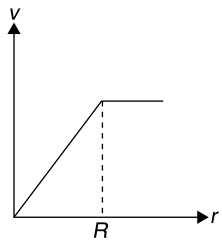


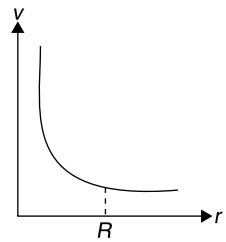
(A) $\dfrac{2GM}{7R}(4\sqrt{2}-5)$ (B) $-\dfrac{2GM}{7R}(4\sqrt{2}-5)$

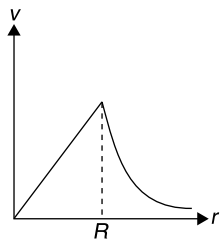
(C) $\dfrac{GM}{4R}$ (D) $\dfrac{2GM}{5R}(\sqrt{2}-1)$

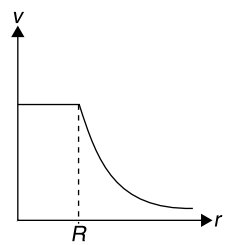
6. [IIT-JEE 2008]

A spherically symmetric gravitational system of particles has a mass density $\rho=\begin{cases}\rho_0 & \text{for } r\le R\\ 0 & \text{for } r>R\end{cases}$, where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r from the centre of the system is represented by

(A)


(B)


(C)


(D)


7. [IIT-JEE 2006]
A double star system consists of two stars A and B which have time periods T_A and T_B. Radius R_A and R_B and mass M_A and M_B. Choose the correct option.
(A) If $T_A > T_B$ then $R_A > R_B$
(B) If $T_A > T_B$ then $M_A > M_B$
(C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$
(D) $T_A = T_B$

8. [IIT-JEE 2002]
A geostationary satellite orbits around the earth in a circular orbit of radius 36000 km. Then, the time period of a spy satellite orbiting a few 100 km above the earth's surface $(R_e = 6400 \text{ km})$ will approximately be
(A) $\frac{1}{2}$ h
(B) 1 h
(C) 2 h
(D) 4 h

9. [IIT-JEE 2001]
A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of $\frac{T_2}{T_1}$ is
(A) 1 (B) $\sqrt{2}$
(C) 4 (D) 2

10. [IIT-JEE 1998]
A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth
(A) the acceleration of S is always directed towards the centre of the earth
(B) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
(C) the total mechanical energy of S varies periodically with time
(D) the linear momentum of S remains constant in magnitude

11. [IIT-JEE 1996]
If the distance between the earth and the sun were half its present value, the number of days in a year would have been
(A) 64.5 (B) 129
(C) 182.5 (D) 730

12. [IIT-JEE 1989]
Imagine a light planet revolving around a very massive star in a circular orbit of radius R with period T. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$, the T^2 is proportional to
(A) R^3 (B) $R^{7/2}$
(C) $R^{3/2}$ (D) $R^{3.75}$

13. [IIT-JEE 1983]
If g is the acceleration due to gravity at the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is
(A) $\frac{1}{2}mgR$ (B) $2mgR$
(C) mgR (D) $\frac{1}{4}mgR$

14. [IIT-JEE 1981]
If the radius of the earth were to shrink by one percent, its mass remaining the same, the acceleration due to gravity on the earth's surface would
(A) decrease by 1%
(B) remain unchanged
(C) increase by 2%
(D) decrease by 2%

Multiple Correct Choice Type Problems

In this section each question has four choices (A), (B), (C) and (D), out of which ONE OR MORE is/are correct.

1. [JEE (Advanced) 2015]
A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If $P(r)$ is the pressure at $r(r < R)$, then the correct options is/are
(A) $P(r=0) = 0$
(B) $\frac{P\left(r=\frac{3R}{4}\right)}{P\left(r=\frac{2R}{3}\right)} = \frac{63}{80}$
(C) $\frac{P\left(r=\frac{3R}{5}\right)}{P\left(r=\frac{2R}{5}\right)} = \frac{16}{21}$
(D) $\frac{P\left(r=\frac{R}{2}\right)}{P\left(r=\frac{R}{3}\right)} = \frac{20}{27}$

2. [JEE (Advanced) 2013]
Two bodies, each of mass M, are kept fixed with a separation $2L$. A particle of mass m is projected from the mid-point of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are)

(A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

(B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$

(C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

(D) The energy of the mass m remains constant

3. [IIT-JEE 2012]

Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q and surface areas A and $4A$, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R are V_P, V_Q and V_R respectively. Then

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$

(C) $\frac{V_R}{V_P} = 3$ (D) $\frac{V_P}{V_Q} = \frac{1}{2}$

4. [IIT-JEE 1994]

The magnitudes of gravitational field at distances r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2, respectively. Then

(A) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

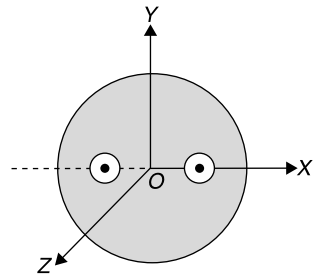
(B) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

(C) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

(D) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

5. [IIT-JEE 1993]

A solid sphere of uniform density and radius 4 unit is located with its centre at the origin of coordinates, O. Two spheres of equal radii of 1 unit, with their centres at $A(-2, 0, 0)$ and $B(2, 0, 0)$ respectively, are taken out of solid sphere, leaving behind spherical cavities as shown in figure.



(A) The gravitational force due to this object at the origin is ZERO.

(B) The gravitational force at the point $B(2, 0, 0)$ is ZERO.

(C) The gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$.

(D) The gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.

Reasoning Based Questions

This section contains Reasoning type questions, each having four choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Each question contains STATEMENT 1 and STATEMENT 2. You have to mark your answer as

Bubble (A) If both statements are TRUE and STATEMENT 2 is the correct explanation of STATEMENT 1.

Bubble (B) If both statements are TRUE but STATEMENT 2 is not the correct explanation of STATEMENT 1.

Bubble (C) If STATEMENT 1 is TRUE and STATEMENT 2 is FALSE.

Bubble (D) If STATEMENT 1 is FALSE but STATEMENT 2 is TRUE.

1. [IIT-JEE 2008]

Statement-1: An astronaut in an orbiting space station above the earth experiences weightlessness.

Statement-2: An object moving around the earth under the influence of earth's gravitational force is in a state of free-fall.

Integer/Numerical Answer Type Questions

In this section, the answer to each question is a numerical value obtained after series of calculations based on the data provided in the question(s).

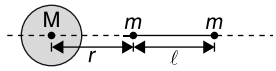
1. [JEE (Advanced) 2015]

A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to

the planet's gravity is $\frac{1}{4}$th of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere)

2. [JEE (Advanced) 2015]

A large spherical mass M is fixed at one position and two identical masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length l and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3l$ from M the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is



3. [IIT-JEE 2010]

Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be $11\ \text{kms}^{-1}$, find the escape speed on the surface of the planet in kms^{-1}.

ANSWER KEYS–TEST YOUR CONCEPTS AND PRACTICE EXERCISES

Test Your Concepts-I (Based on Acceleration Due to Gravity, Gravitational Field and Applications)

1. $2.4\ \text{kms}^{-1}$
2. $-\dfrac{Gm_1m_2}{2r}$
3. (a) $-\dfrac{GMm}{\sqrt{a^2+x^2}}$ (c) $-\dfrac{GMm\cdot x}{(x^2+a^2)^{3/2}}$ (e) 0
4. $\dfrac{3GM^2}{d^2}$, along $+x$ axis
5. $\dfrac{a}{r^2}$
6. $200\ \text{Nkg}^{-1}$, along $+x$ direction
7. $\dfrac{7}{3}$
8. $\dfrac{2\pi GMm}{L^2}$
9. $\dfrac{\pi RG\rho_0}{2}$
10. $\dfrac{GM\lambda}{R}$
11. $7.8\times10^{-4}\ \text{rads}^{-1}$
12. $\begin{cases} 0 & \text{for} \quad r<R_1 \\ \dfrac{GM}{r^2}\left(\dfrac{r^3-R_1^3}{R_2^3-R_1^3}\right) & \text{for} \quad R_1<r<R_2 \\ \dfrac{GM}{r^2} & \text{for} \quad r>R_2 \end{cases}$
13. 1600 km

Test Your Concepts-II (Based on Gravitational Potential, Potential Energy and Applications)

1. $-\dfrac{Gm^2}{a}\left(12+\dfrac{12}{\sqrt{2}}+\dfrac{4}{\sqrt{3}}\right)$
2. $-\dfrac{GMm_0}{L}\log_e\left(\dfrac{a+L}{a}\right)$
3. $\dfrac{GMm}{R}$
4. $-\dfrac{GMm}{r}$, remain the same
5. $-\dfrac{3}{2}\dfrac{GMm}{R}$
6. $-\dfrac{G}{d}\left(\sqrt{m}+\sqrt{M}\right)^2$
7. $-\dfrac{3M}{R}$
8. (a) $-\dfrac{GM^2}{2R}$ (b) $-\dfrac{3}{5}\dfrac{GM^2}{R}$
9. (a) $-2G\lambda\log_e\left(\dfrac{\ell+\sqrt{\ell^2+a^2}}{a}\right)$

   (b) $-G\lambda\log_e\left(\dfrac{a+L}{a-L}\right)$

Test Your Concepts-III (Based on Relation Between Gravitational Field and Potential)

1. $3\ \text{Jkg}^{-1}$
2. $-(axy+byz)+C$
3. (a) $1\ \text{Jkg}^{-1}$ (b) $1\ \text{Jkg}^{-1}$, conservative
4. $-(6xy)\hat{i}-3\left(x^2+y^2z\right)\hat{j}-y^3\hat{k}$
5. (a) $-2a\left(x\hat{i}-y\hat{j}\right)$ (b) $-ay\hat{i}-ax\hat{j}$
6. $5\sqrt{2}\ \text{Nkg}^{-1}$
7. $10\sqrt{2}$ N
8. $-\left(2ax\hat{i}+2ay\hat{j}+2bz\hat{k}\right)$, $2\sqrt{a^2\left(x^2+y^2\right)+b^2z^2}$
9. $-2\hat{i}-3\hat{j}+\hat{k}$

Test Your Concepts-IV (Based on Conservation Laws, Escape Velocity and Applications)

1. (a) $\left(\dfrac{n}{n+1}\right)mgR$ (b) $\sqrt{\dfrac{2ngR}{n+1}}$
2. $11.2\ \text{kms}^{-1}$
3. $2.1\times10^{-5}\ \text{ms}^{-1}$, $4.2\times10^{-5}\ \text{ms}^{-1}$
4. $2\sqrt{\dfrac{GM}{L}}$
5. $\dfrac{3}{2}\sqrt{\dfrac{5GM}{a}}$
6. $\dfrac{r_0}{2-K}\left(1+\sqrt{1-K(2-K)\sin^2\phi}\right)$,
   $\dfrac{r_0}{2-K}\left(1-\sqrt{1-K(2-K)\sin^2\phi}\right)$

7. 0

8. $42\ \text{kms}^{-1}$

9. $\sqrt{2G(m_1+m_2)\left(\frac{2}{a}-\frac{1}{\ell}\right)}$

10. 9, 2

Test Your Concepts-V (Based on Satellites, Kepler's Laws and Applications)

1. 2, 2

4. $(1+\sin\alpha)r_0,\ (1-\sin\alpha)r_0$

5. $r = R$

6. (a) $2\pi\sqrt{\frac{d^3}{G(m_1+m_2)}}$ (b) $\frac{m_2}{m_1}$ (c) $\frac{m_2}{m_1}$ (d) $\mu\omega d^2$ (e) $\frac{1}{2}\mu\omega^2 d^2$

7. $56\sqrt{2}$ hour, $3829\ \text{kmhr}^{-1}$

8. $r,\ \frac{5r}{3}$

9. (a) $\frac{GM^2}{4R^2}$ (b) $\sqrt{\frac{GM}{4R^3}},\ 4\pi\sqrt{\frac{R^3}{GM}}$ (c) $\frac{GM^2}{4R}$

11. $\sqrt{2G(m_1+m_2)\left[\frac{1}{R_1+R_2}-\left\{\frac{4\pi^2}{G(m_1+m_2)T^2}\right\}^{1/3}\right]}$

Single Correct Choice Type Questions

1. C	**2.** C	**3.** D	**4.** C	**5.** D	**6.** D	**7.** C	**8.** C	**9.** C	**10.** B
11. C	**12.** D	**13.** B	**14.** B	**15.** C	**16.** C	**17.** A	**18.** C	**19.** C	**20.** A
21. C	**22.** C	**23.** B	**24.** C	**25.** B	**26.** D	**27.** B	**28.** B	**29.** D	**30.** C
31. B	**32.** D	**33.** D	**34.** B	**35.** C	**36.** D	**37.** D	**38.** D	**39.** A	**40.** A
41. B	**42.** C	**43.** B	**44.** B	**45.** C	**46.** A	**47.** B	**48.** B	**49.** A	**50.** A
51. C	**52.** B	**53.** A	**54.** D	**55.** A	**56.** C	**57.** C	**58.** C	**59.** C	**60.** C
61. C	**62.** C	**63.** C	**64.** D	**65.** D	**66.** A	**67.** D	**68.** A	**69.** B	**70.** D
71. A	**72.** B	**73.** B	**74.** C	**75.** B	**76.** B	**77.** D	**78.** B	**79.** C	**80.** A
81. D	**82.** D	**83.** D	**84.** C	**85.** C	**86.** C	**87.** D	**88.** C	**89.** B	**90.** A
91. D	**92.** A	**93.** C	**94.** D	**95.** A	**96.** D	**97.** A	**98.** B	**99.** C	**100.** D
101. B	**102.** D	**103.** C	**104.** B	**105.** A	**106.** D	**107.** B	**108.** C	**109.** C	**110.** C
111. C	**112.** C	**113.** C	**114.** C	**115.** C	**116.** C	**117.** D	**118.** A	**119.** C	**120.** A
121. C	**122.** C	**123.** C	**124.** C						

Multiple Correct Choice Type Questions

1. B, C	**2.** A, D	**3.** A, B, C	**4.** A, C, D	**5.** A, B, C, D
6. A, C	**7.** A, B, C, D	**8.** B, C	**9.** C, D	**10.** A, B, C
11. A, B, C	**12.** B, C, D	**13.** B, D	**14.** A, C	**15.** A, D
16. A, D	**17.** B, D	**18.** A, B, C, D	**19.** A, B, C	**20.** A, B, C
21. A, B, C	**22.** B, C	**23.** A, C	**24.** A, D	**25.** A, B, D
26. A, D	**27.** A, D	**28.** A, B, D	**29.** C, D	**30.** A, D
31. A, B, C	**32.** A, C	**33.** A, D	**34.** B, C	**35.** A, B, D
36. C, D	**37.** A, B, C, D	**38.** A, B, C		

Reasoning Based Questions

1. A	**2.** A	**3.** A	**4.** D	**5.** D	**6.** B	**7.** C	**8.** A	**9.** A	**10.** C

Linked Comprehension Type Questions

1. D	**2.** A	**3.** C	**4.** D	**5.** A	**6.** B	**7.** A	**8.** C	**9.** C	**10.** A
11. A, B, C	**12.** B	**13.** A	**14.** D	**15.** B	**16.** D	**17.** D	**18.** A	**19.** B	**20.** C
21. C	**22.** D	**23.** D	**24.** A	**25.** B	**26.** C	**27.** C	**28.** A	**29.** A	**30.** C
31. D	**32.** C	**33.** B	**34.** A	**35.** D	**36.** C	**37.** B			

Matrix Match/Column Match Type Questions

1. A → (p, q, r)	B → (r)	C → (t)	D → (p, q, r, s)	
2. A → (q)	B → (p)	C → (p)	D → (q)	
3. A → (q)	B → (r)	C → (q)	D → (p)	E → (s)
4. A → (p, q, r, s)	B → (p, q, r, s)	C → (q, r, s)	D → (p, r)	
5. A → (p)	B → (q)	C → (q)	D → (q)	
6. A → (r, t)	B → (s)	C → (p)	D → (q)	
7. A → (p)	B → (q)	C → (s)	D → (t)	
8. A → (q)	B → (s)	C → (p)	D → (r)	
9. A → (r, q)	B → (s)	C → (p)	D → (r)	
10. A → (s)	B → (r)	C → (p)	D → (p)	
11. A → (r)	B → (q)	C → (p)	D → (r)	

Integer/Numerical Answer Type Questions

1. 1.51	**2.** 0	**3.** 1.25	**4.** 10	**5.** 5656
6. (a) 30 (b) 18	**7.** $\alpha = 2$ and $\beta = 13$	**8.** 2	**9.** 6	**10.** 2
11. 1	**12.** 5	**13.** 7	**14.** 3	**15.** 2
16. 6	**17.** 7155, 7071, 7702	**18.** 7	**19.** 1.5	**20.** 7
21. 2				

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1. C	**2.** A	**3.** B	**4.** B	**5.** A	**6.** C	**7.** D	**8.** D	**9.** A	**10.** B
11. D	**12.** D	**13.** A	**14.** A	**15.** 16	**16.** B	**17.** C	**18.** C	**19.** A	**20.** D
21. C	**22.** D	**23.** C	**24.** A	**25.** B	**26.** A	**27.** B	**28.** C	**29.** C	**30.** A
31. C	**32.** C	**33.** D	**34.** A	**35.** C	**36.** A	**37.** D	**38.** A	**39.** A	**40.** A
41. C	**42.** C	**43.** B	**44.** C	**45.** B	**46.** D	**47.** A	**48.** C	**49.** D	**50.** A

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Single Correct Choice Type Problems

1. B	**2.** C	**3.** B	**4.** B	**5.** A	**6.** C	**7.** D	**8.** C	**9.** D	**10.** A
11. B	**12.** B	**13.** A	**14.** C						

Multiple Correct Choice Type Problems

1. B, C	**2.** B, D	**3.** B, D	**4.** A, B	**5.** A, C, D

Reasoning Based Questions

1. A

Matrix Match/Column Match Type Questions

1. 2	**2.** 7	**3.** 3

HINTS AND EXPLANATIONS

Test Your Concepts-I (Based on Work Done by a Constant and Variable Force)

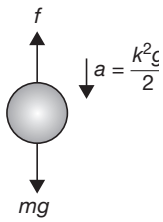
1. **Method I:**

Let a be the net downward acceleration of the ball. Then

$$v^2 - u^2 = 2ah$$

$$\Rightarrow \quad k^2(gh) - 0^2 = 2ah$$

$$\Rightarrow \quad a = \frac{k^2 g}{2}$$



If f be the air drag, then using Newton's Second Law, we get

$$mg - f = ma$$

$$\Rightarrow \quad f = mg - ma$$

$$\Rightarrow \quad f = mg\left(1 - \frac{k^2}{2}\right)$$

Now $W_f = fh\cos(180°)$

$$\Rightarrow \quad W_f = -mgh\left(1 - \frac{k^2}{2}\right)$$

Method II:

Since we know that

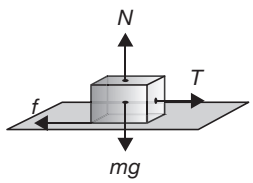
$$W_{nc} = \Delta U + \Delta k$$

$$\Rightarrow \quad W_{\text{air drag}} = (0 - mgh) + \left(\frac{1}{2}mv^2 - 0\right)$$

$$\Rightarrow \quad W_{\text{air drag}} = -mgh + \frac{1}{2}mk^2(gh)$$

$$\Rightarrow \quad W_{\text{air drag}} = -mgh\left(1 - \frac{k^2}{2}\right)$$

2. $W_T = T\ell_0$



$W_N = 0 \qquad \{\because \theta = 90°\}$

$W_{mg} = 0 \qquad \{\because \theta = 90°\}$

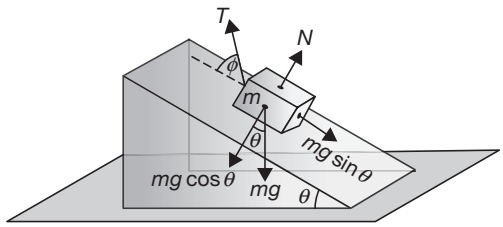
$W_f = f\ell_0\cos(180°) = -f\ell_0$

3. $W_T = Tl\cos\phi$

$W_N = 0 \qquad \left\{\because \theta = \frac{\pi}{2}\right\}$

$F_f = -fl \qquad \{\because \theta = \pi\}$

$W_{mg} = -mgl\sin\theta$



4. (a) $W_1 = \int_{x_1}^{x_2}(-kx)dx = \frac{k}{2}\left(x_1^2 - x_2^2\right)$

$$W_1 = \frac{500}{2}\left[(0.15)^2 - (0.08)^2\right] = 4.03 \text{ J}$$

(b) $W_2 = -(mg\sin\theta)(x_2 - x_1)$

$\Rightarrow \quad W_2 = -(6)(9.8)(\sin 15°)(0.23) = -3.5 \text{ J}$

5. (a) $W_1 = \int_{(0.1,\,0)}^{(0.1,\,0.4)} \vec{F}\cdot d\vec{r}$

where $\vec{F} = -\alpha x^2\hat{i}$ and $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$\Rightarrow \quad W_1 = \int_{(0.1,\,0)}^{(0.1,\,0.4\ )} -\alpha x^2 dx = 0$$

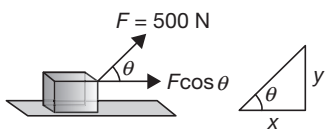
(b) $W_2 = \int_{(0.1,\,0)}^{(0.3,\,0)} -\alpha x^2 dx = -\frac{\alpha}{3}(x^3)\Big|_{(0.1,\,0)}^{(0.3,\,0)} = -\frac{\alpha}{3}(0.026)$

(c) $W_3 = \int_{(0.3,\,0)}^{(0.1,\,0)} -\alpha x^2 dx = -\frac{\alpha}{3}(x^3)\Big|_{(0.3,\,0)}^{(0.1,\,0)} = \frac{\alpha}{3}(0.026)$

6. Using our knowledge of constraints we have

$$x = 8\cot\theta$$

$$\Rightarrow \quad dx = -8\text{cosec}^2\theta\, d\theta$$



Since, $W = \int_{\theta=30^\circ}^{60^\circ} (F\cos\theta)\,dx$

$\Rightarrow \quad W = \int_{30^\circ}^{60^\circ} (500\cos\theta)\left(-8\text{cosec}^2\theta\right)d\theta$

$\Rightarrow \quad W = -4000\int_{30^\circ}^{60^\circ} \cos\theta\,\text{cosec}^2\,\theta d\theta$

But $\int \cos\theta\,\text{cosec}^2\,\theta d\theta = \int \text{cosec}\,\theta\cot\theta d\theta$

$\Rightarrow \quad \int \text{cosec}\,\theta\cot\theta d\theta = -\text{cosec}\,\theta$

$\Rightarrow \quad W = -4000(-\text{cosec}\,\theta)\big|_{30^\circ}^{60^\circ}$

$\Rightarrow \quad W = 4000[\text{cosec}(60^\circ) - \text{cosec}(30^\circ)]$

$\Rightarrow \quad W = 4000[1.155 - 2]$

$\Rightarrow \quad W = 4000(-0.845)$

$\Rightarrow \quad W = -3380$ J

7. $W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$

where $d\vec{r} = \hat{i}\,dx + \hat{j}dy + \hat{k}dx$

$\Rightarrow \quad W = \int_{\vec{r}_1}^{\vec{r}_2} \left(3x^2\hat{i} + 2y\hat{j}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$

$\Rightarrow \quad W = \int_{\vec{r}_1}^{\vec{r}_2} \left(3x^2dx + 2ydy\right) = \left(x^3 + y^2\right)\Big|_{(2,3)}^{(4,6)}$

$\Rightarrow \quad W = 83$ J

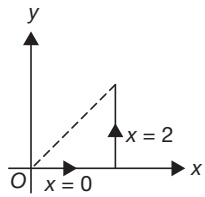
8. (a) $W_1 = \int_{(0,0)}^{(2,2)} -\alpha y^2 dy = \int_{(0,0)}^{(2,2)} -\alpha y^3 dy$

(Along the path $x = y$)

$\Rightarrow \quad W_1 = -\frac{\alpha}{4}\left(y^4\right)\Big|_{(0,0)}^{(2,2)} = -4\alpha \quad \{\because\ \alpha = 1.5\ \text{Nm}^{-3}\}$

$\Rightarrow \quad W_1 = -6$ J

(b) Initially when we move along x-axis, the work done is zero because the force is acting along y-axis. Then along the path parallel to y-axis, we can put $x = 2$ to get



$W_2 = \int_{(2,0)}^{(2,2)} -2\alpha y^2 dy$

$\Rightarrow \quad W_2 = -\frac{2\alpha}{3}\left(y^3\right)\Big|_{(2,0)}^{(2,2)}$

$\Rightarrow \quad W_2 = -8$ J $\qquad \{\because\ \alpha = 1.5\ \text{Nm}^{-3}\}$

(c) As $W_1 \neq W_2$, force is non-conservative in nature.

9. Since work done is equal to the area under a curve in F-x graph. So

$W = \frac{1}{2}(-2)(-10) + \frac{1}{2}(-2)(10) + (10)(2) + \frac{1}{2}(2)(10)$

$\Rightarrow \quad W = 10 - 10 + 20 + 10$

$\Rightarrow \quad W = 30$ J

10. The work from $x = 0$ to $x = 8$ m is the area under the curve.

$W = (10\times 2) + \frac{1}{2}(10)(4-2) + 0 + \frac{1}{2}(-5)(8-6)$

$\Rightarrow \quad W = 25$ J

11. (a) W = Area under F-x graph

$\Rightarrow \quad W_{0\to 3} = \frac{1}{2}\times 2\times 2 + 1\times 2 = 4$ J

(b) $W_{3\to 4} = 0$

(c) $W_{4\to 7} = -\frac{1}{2}\times 2\times 1 = -1$ J

$W_{4\to 7}$ is negative, because force is in negative x-direction while displacement is towards positive x-axis.

(d) $W_{0\to 7} = 4 + 0 - 1 = 3$ J

12. Work done = Area under F-s graph

$\Rightarrow \quad W = \frac{1}{2}\times 4\times 10 + (8-4)\times 10$

$\Rightarrow \quad W = 20 + 40$

$\Rightarrow \quad W = 60$ J

13. Since, $dW = \vec{F} \cdot d\vec{r}$, where

$$d\vec{r} = (dx)\hat{i} + (dy)\hat{j}$$

$$\Rightarrow \quad dW = -k(ydx + xdy)$$

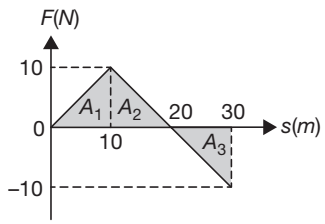
Since, $d(xy) = x\,dy + y\,dx$

$$\Rightarrow \quad dW = -kd(xy)$$

$$\Rightarrow \quad W = \int dW = -k \int_{(0,0)}^{(a,a)} d(xy)$$

$$\Rightarrow \quad W = -k(xy)\Big|_{(0,0)}^{(a,a)} = -ka^2 - 0 = -ka^2$$

14. Since we know that work done is equal to the area under F-s graph.



(a) Work done at the end of 10 m is the area under the curve from 0 to 10 m.

$$\Rightarrow \quad W = A_1 = \frac{1}{2}(10)(10)\ \text{J} = 50\ \text{J}$$

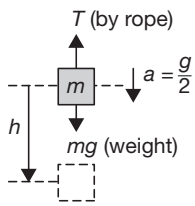
(b) Work done at the end of 20 m is the area under the curve from 0 to 20 m.

$$\Rightarrow \quad W = A_1 + A_2 = 50 + 50 = 100\ \text{J}$$

(c) Work done at the end of 30 m is the area under the curve from 0 to 30 m.

$$\Rightarrow \quad W = A_1 + A_2 + A_3 = 50 + 50 - 50 = 50\ \text{J}$$

15. The FBD of the block placed in the lift is shown in Figure.



According to Newton's Second Law, we have

$$mg - T = ma = \frac{mg}{2}$$

$$\Rightarrow \quad T = \frac{mg}{2}$$

Work done by the gravitational force is

$$W_{mg} = (mg)(h)\cos(0°) = mgh$$

Work done by the tension force is

$$W_T = Th\cos(180°) = -\frac{mgh}{2}$$

Test Your Concepts-II (Based on Kinetic Energy, Potential Energy and Power)

1. $P = Fv$

$$\Rightarrow \quad F = \frac{P}{v} \qquad \ldots(1)$$

Now, we have

$$F - f = ma = m\left(\frac{dv}{dt}\right)$$

At maximum speed, $\frac{dv}{dt} = 0$

$$\Rightarrow \quad F = f$$

So, from (1), we get $\frac{P}{v_{\max}} = f$

$$\Rightarrow \quad v_{\max} = \frac{P}{f}$$

Further, since $F = \frac{P}{v}$

$$\Rightarrow \quad m\left(\frac{dv}{dt}\right) = \frac{P}{v}$$

$$\Rightarrow \quad \int_0^{P/2r} v\,dv = \left(\frac{P}{m}\right)\int_0^t dt$$

$$\Rightarrow \quad \left(\frac{P}{2r}\right)^2 = \left(\frac{2P}{m}\right)t$$

$$\Rightarrow \quad t = \frac{Pm}{8r^2}$$

2. (a) P = constant

$$\Rightarrow \quad Fv = P$$

$$\Rightarrow \quad mv\frac{dv}{dt} = P \qquad \left\{\because F = \frac{dv}{dt}\right\}$$

$$\Rightarrow \quad mv\,dv = P\,dt$$

$$\Rightarrow \quad m\int_0^v v\,dv = P\int_0^t dt$$

$$\Rightarrow \quad \frac{mv^2}{2} = Pt$$

$$\Rightarrow \quad v = \sqrt{\frac{2Pt}{m}}$$

CHAPTER 1

(b) Since, $v=\sqrt{\frac{2Pt}{m}}$

$\Rightarrow \quad \frac{dx}{dt}=\sqrt{\frac{2P}{m}}t^{1/2} \qquad \left\{\because v=\frac{dx}{dt}\right\}$

$\Rightarrow \quad dx=\sqrt{\frac{2P}{m}}t^{1/2}dt$

$\Rightarrow \quad \int_0^x dx=\sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt$

$\Rightarrow \quad x=\left(\frac{2}{3}\sqrt{\frac{2P}{m}}\right)t^{3/2} \qquad \left\{\because \int t^{1/2}dt=\frac{2}{3}t^{3/2}\right\}$

$\frac{ds}{dt}=\left(\frac{2Pt}{m}\right)^{1/2}$ i.e., $\int_0^s ds=\int_0^t\left(\frac{2Pt}{m}\right)^{1/2}dt$

which on integration gives

$$s=\left(\frac{2P}{m}\right)^{1/2}\frac{2}{3}t^{3/2} \text{ or } s=\left(\frac{8P}{9m}\right)^{1/2}t^{3/2}$$

3. From Work-Energy Theorem, we have

$W=\Delta KE$

$\Rightarrow \quad Pt=\frac{1}{2}m(v^2-u^2) \qquad \{\because W=Pt\}$

$\Rightarrow \quad t=\frac{m}{2P}(v^2-u^2) \qquad \ldots(1)$

Further, $Fv=P$

$\Rightarrow \quad m\left(\frac{dv}{ds}\right)v=P$

$\Rightarrow \quad kv^2\frac{dx}{dt}=P \qquad \left\{\because \frac{dv}{dt}=v\frac{dv}{dx}\right\}$

$\Rightarrow \quad \int_u^v v^2dv=\frac{P}{m}\int_0^x dx$

$\Rightarrow \quad (v^3-u^3)=\frac{3P}{m}x$

$\Rightarrow \quad \frac{m}{P}=\frac{3x}{v^3-u^3}$

Substituting in Equation (1), we get

$$t=\frac{3x(u+v)}{2(u^2+v^2+uv)}$$

4. **For** $t\le 0.2$ s

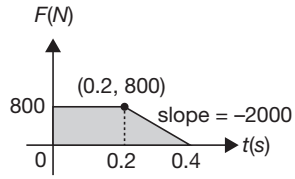
$F=800$ N and $v=\left(\frac{20}{0.4}\right)t=50$ t

$\Rightarrow \quad P=Fv=(40000\text{ t})\text{watt}=40\text{ t kW}$

For $t>0.2$ s

A straight line with slope $m=-\frac{800}{0.4}=-2000$ passing through the point $(x_1y_1)=(0.2,\ 800)$ is

$y-y_1=m(x-x_1)$



Here x-axis is time axis and y-axis is force axis

So, we get

$F-800=-2000(t-0.2)$

$\Rightarrow \quad F=800-2000t+400$

$\Rightarrow \quad F=-2000t+1200$

Further $v=\left(\frac{20}{0.4}\right)t=50$ t

$\Rightarrow \quad P=Fv=(-2000t+1200)(50t)$ watt

$\Rightarrow \quad P=Fv=(-20t+12)5t$ kW

So, $P=\begin{cases}40t & \text{for} \quad t\le 0.2\text{ s}\\ -100t^2+60t & \text{for} \quad t>0.2\text{ s}\end{cases}$

Now, $P=\frac{dW}{dt}$

$\Rightarrow \quad W=\int Pdt$

$\Rightarrow \quad W=\int_0^{0.2}40tdt+\int_{0.2}^{0.4}(-100t^2+60t)dt$

$\Rightarrow \quad W=\left[40\left(\frac{t^2}{2}\Big|_0^{0.2}\right)-100\left(\frac{t^3}{3}\Big|_{0.2}^{0.4}\right)+60\left(\frac{t^2}{2}\Big|_{0.2}^{0.4}\right)\right]$ kJ

$\Rightarrow \quad W=\left[20(0.04)-\frac{100}{3}(0.056)+30(0.12)\right]\times 1000$ J

$\Rightarrow \quad W=(800-1867+3600)$ J

$\Rightarrow \quad W=2533$ J

5. Since the collar is lifted with a constant speed of 0.6 ms^{-1}. So

$F\cos\theta-mg=0$

$\Rightarrow \quad F\cos\theta=5(10)=50$

$\Rightarrow \quad F=\frac{50}{\cos\theta} \qquad \ldots(1)$

Now $P = \vec{F} \cdot \vec{v}$

$\Rightarrow \quad P = \left(\frac{50}{\cos\theta}\right)(v\cos\theta)$

$\Rightarrow \quad P = 50v$

$\Rightarrow \quad P = 30 \text{ W} \qquad \{\because v = 0.6 \text{ ms}^{-1}\}$

6. At constant speed, there is no acceleration, so the forces acting on the train are in equilibrium.

$\Rightarrow \quad F = R = 3\times10^4 \text{ N}$

Since we know that $P = Fv$

$\Rightarrow \quad P = 3\times10^4 \times 40 = 1.2\times10^6 \text{ watt}$

7. Since, $F \propto x^{-1/2}$

$\Rightarrow \quad a = v\frac{dv}{dx} = kx^{-1/2}$

$\Rightarrow \quad vdv = kx^{-1/2}dx$

Integrating both sides, we get

$$\int_0^v vdv = \int_0^x x^{-1/2}dx$$

$\Rightarrow \quad \frac{v^2}{2} = 2x^{\frac{1}{2}}$

$\Rightarrow \quad v \propto x^{\frac{1}{4}}$

Since power is defined as

$P = Fv$

$\Rightarrow \quad P \propto \frac{x^{\frac{1}{4}}}{x^{\frac{1}{2}}} = x^{\frac{1}{4}-\frac{1}{2}} = x^{-\frac{1}{4}}$ i.e., $P \propto x^{-\frac{1}{4}}$

$\Rightarrow \quad n = -\frac{1}{4}$

8. Here the only force acting is $\vec{F} = (2+3x)\hat{i}$. So, body starts moving along x-axis. According to Work-Energy Theorem,

$$W = \int \vec{F}\cdot d\vec{x} = \Delta K$$

$\Rightarrow \quad \int_0^5 (2+3x)dx = \frac{1}{2}(5)v^2 - 0$

$\Rightarrow \quad \left(2x + \frac{3x^2}{2}\right)\Big|_0^5 = \frac{5}{2}v^2$

$\Rightarrow \quad 10 + \frac{75}{2} - 0 = \frac{5}{2}v^2$

$\Rightarrow \quad v = \sqrt{19} \text{ ms}^{-1}$

9. According to Work-Energy Theorem, we have

$$W = \Delta K = \frac{1}{2}m(v^2 - u^2)$$

Given that $m = 0.5$ kg and $v = ax\sqrt{x} = 5x^{3/2}$

Initial velocity u i.e. velocity at $x = 0$ is

$u = a(0) = 0$

Final velocity v i.e. velocity at $x = 2$ is

$v = 5\left(2^{\frac{3}{2}}\right) = 10\sqrt{2} \text{ ms}^{-1}$

$\Rightarrow \quad W = \Delta K = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2}(0.5)(200)$

$\Rightarrow \quad W = 50 \text{ J}$

10. Since $P = Fv$, where $F = v\frac{dm}{dt}$

If ρ be the density of wind, then $\frac{dm}{dt} = \rho\left(\frac{dV}{dt}\right)$

where, $\frac{dV}{dt}$ is the volume of wind flowing per second through the generator.

Since $\frac{dV}{dt} = A\left(\frac{dx}{dt}\right) = Av$

where A is the area of the generator through which wind flows

$\Rightarrow \quad \frac{dm}{dt} = \rho\left(\frac{dV}{dt}\right) = Av\rho$

$\Rightarrow \quad F = v\left(\frac{dm}{dt}\right) = v(Av\rho)$

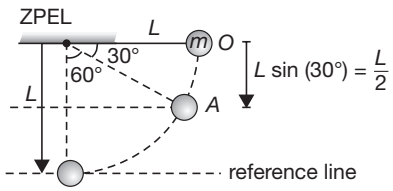
$\Rightarrow \quad F = Av^2\rho$

$\Rightarrow \quad P = Fv = Av^3\rho$

$\Rightarrow \quad P \propto v^3$

So, $n = 3$

11.



Let the reference line pass through point of support as shown.

$\Rightarrow \quad U_O = 0$

$\Rightarrow \quad U_A = -mg\left(\frac{L}{2}\right)$ and $U_B = -mgL$

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$$\Rightarrow \quad U_O - U_A = \frac{mgL}{2} = U_A - U_B$$

$$\Rightarrow \quad \frac{U_O - U_A}{U_A - U_B} = 1$$

12. Potential energy (PE) for a stretch of 20 cm is

$$U_1 = \frac{1}{2}k(0.2)^2 = 30 \text{ J}$$

$$\Rightarrow \quad k = \frac{30 \times 2}{0.04} = 1500 \text{ Nm}^{-1}$$

PE acquired for a stretch of $20 + 40 = 60$ cm is

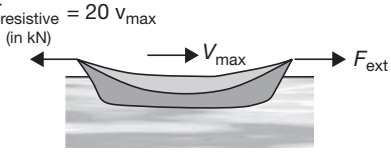
$$U_2 = \frac{1}{2} \times 1500 \times (0.6)^2 = 1500 \times 0.18$$

$$\Rightarrow \quad U_2 = 270 \text{ J}$$

Additional work required is

$$W = U_2 - U_1 = 270 - 30 = 240 \text{ J}$$

13. $F_{\text{resistive}} = 20\ v_{\max}$ (in kN)



Power of engine is

$$P = 50 \text{ HP} = 50 \times 746 = 37300 \text{ W}$$

Let $v_{\max}$ be the maximum speed attained by the boat.

At $v = v_{\max}$, acceleration of boat is zero, so net force acting on boat is also zero.

$$\Rightarrow \quad F_{\text{ext}} - F_{\text{resistive}} = 0$$

$$\Rightarrow \quad F_{\text{ext}} = (20v_{\max})1000 \text{ N}$$

Since, $P = Fv$

$$\Rightarrow \quad 37300 = (20000v_{\max})v_{\max}$$

$$\Rightarrow \quad v_{\max}^2 = \frac{37300}{20000}$$

$$\Rightarrow \quad v_{\max} = \sqrt{\frac{373}{200}} = 1.36 \text{ ms}^{-1} = 4.9 \text{ kmh}^{-1}$$

14. When mass is suspended from the spring, then in equilibrium position, let extension in spring be x_1.

$$\Rightarrow \quad mg = kx_1$$

$$\Rightarrow \quad k = \frac{mg}{x_1} = \frac{4 \times 10}{(0.5 - 0.3)} = 200 \text{ Nm}^{-1}$$

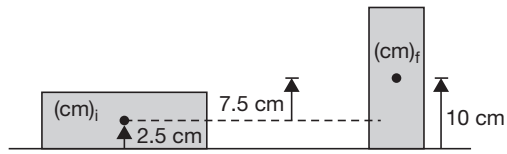
Total stretch in the spring in the final state is

$$x_2 = 0.6 - 0.3 = 0.3 \text{ m}$$

So, elastic energy stored in the final position is

$$U = \frac{1}{2}kx_2^2 = \frac{1}{2} \times 200 \times (0.3)^2 = 9 \text{ J}$$

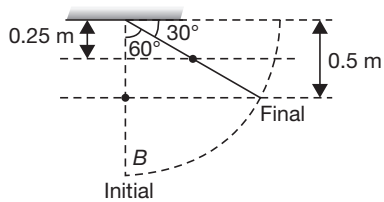
15.



Since centre of mass of brick rises by 7.5 cm, so

$$\Delta U = mgh = (5)(10)\left(\frac{7.5}{100}\right) = 3.75 \text{ J}$$

16.



Initially, the centre of mass of the metre scale lies 0.5 m below the hinge and finally 0.25 m below the hinge when the scale is turned to a position making $30°$ with horizontal. Therefore, centre of mass of the scale rises by

$$h = 0.5 - 0.25 = 0.25 \text{ m}$$

Hence work done by gravity is $W = mgh\cos 180°$

$$\Rightarrow \quad W = 0.1 \times 10 \times 0.25 \times (-1) = -0.25 \text{ J}$$

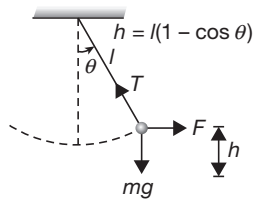
Test Your Concepts-III (Based on Conservation of Energy and Work Energy Theorem)

1. In this case three forces are acting on the object

(i) tension (T)

(ii) weight (mg) and

(iii) applied force (F)



Using Work-Energy Theorem

$$W_{\text{net}} = \Delta K.E.$$

$$\Rightarrow \quad W_T + W_{mg} + W_F = 0 \qquad \ldots(1)$$

as $\Delta K.E. = 0$ because $K_i = K_f = 0$

Further, $W_T = 0$, as tension is always perpendicular to displacement.

$$W_{mg} = -mgh$$

$$\Rightarrow \quad W_{mg} = -mg\ell(1-\cos\theta)$$

Substituting these values in Equation (1), we get

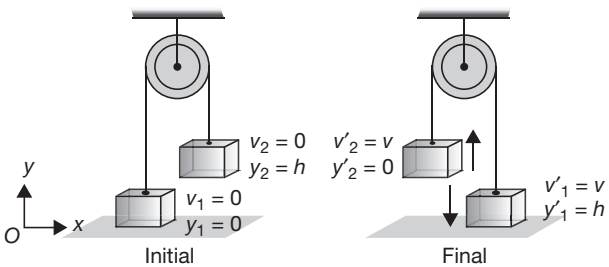
$$W_F = mg\ell(1-\cos\theta)$$

Conceptual Note(s)

Here the applied force F is variable. So, if we do not apply the work energy theorem, we will first find the magnitude of F at different locations and then integrate $dW\left(=\vec{F}\cdot d\vec{r}\right)$ with proper limits.

2. The initial and final configurations are shown in the figure.

It is convenient to set $U_g = 0$ at the floor. Initially, only m_2 has potential energy. As it falls, it loses potential energy and gains kinetic energy. At the same time, m_1 gains potential energy and kinetic energy. Just before m_2 lands, it has only kinetic energy. Let v the final speed of each mass. Then, using the Law of Conservation of Mechanical Energy.



$$K_f + U_f = K_i + U_i$$

$$\Rightarrow \quad \frac{1}{2}(m_1+m_2)v^2 + m_1gh = 0 + m_2gh$$

$$\Rightarrow \quad v^2 = \frac{2(m_2-m_1)gh}{m_1+m_2}$$

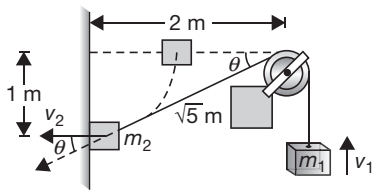
Substitute $m_1 = 3$ kg, $m_2 = 5$ kg, $h = 5$ m and $g = 10$ ms^{-2}.

$$\Rightarrow \quad v^2 = \frac{2(5-3)(10)(5)}{5+3}$$

$$\Rightarrow \quad v = 5 \text{ ms}^{-1}$$

3. From conservation of energy,

$$m_2gh_2 = m_1gh_1 + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$



Using our knowledge of constraint relations, we get

$$v_1 = v_2\cos\theta$$

$$\Rightarrow \quad 2\times10\times1 = (0.5)(10)\left(\sqrt{5}-2\right) + \frac{1}{2}\times0.5\times v_2^2\times\left(\frac{2}{\sqrt{5}}\right)^2 + \frac{1}{2}\times2\times v_2^2$$

$$\Rightarrow \quad 20 = 1.18 + 0.2v_2^2 + v_2^2$$

$$\Rightarrow \quad v_2 = 3.96 \text{ ms}^{-1} \text{ and } v_1 = v_2\cos\theta = \frac{2}{\sqrt{5}}\times3.96$$

$$\Rightarrow \quad v_1 = 3.54 \text{ ms}^{-1}$$

4. According to Work Energy Theorem, applied on A to B, we have

$$\begin{pmatrix}\text{Work done by force}\\ \text{from A to B}\end{pmatrix} = \begin{pmatrix}\text{Change in KE}\\ \text{from A to B}\end{pmatrix}$$

$$\Rightarrow \quad W_{AB} = \frac{1}{2}m\left(v_B^2 - v_A^2\right)$$

$$\Rightarrow \quad (2T - mg\sin\theta)(AB) = \frac{1}{2}\text{ m}\left(v_B^2 - v_A^2\right)$$

$$\Rightarrow \quad \left(1150 - 130\times10\times\frac{5}{13}\right)(3) = \frac{1}{2}\times130\left(v_B^2 - 9\right)$$

$$\Rightarrow \quad (650)(3) = 65\left(v_B^2 - 9\right)$$

$$\Rightarrow \quad v_B^2 - 9 = 30$$

$$\Rightarrow \quad v_B = \sqrt{39} = 6.25 \text{ ms}^{-1}$$

While using work-energy theorem we are concerned with speed not the velocity.

5. (a) For spring 1, $X_{10} = \dfrac{(m_1+m_2)g}{k_1} = \dfrac{30}{1500} = 0.02$ m

For spring 2, $X_{20} = \dfrac{m_2g}{k_2} = \dfrac{10}{500} = 0.02$ m

$$\Rightarrow \quad U = \frac{1}{2}k_1x_{01}^2 + \frac{1}{2}k_2x_{02}^2 = (750+250)(0.02)^2$$

$$\Rightarrow \quad U = 0.4 \text{ J}$$

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(b) Let Δx_1 and Δx_2 be the additional elongation due to pulling m_2 downwards by $\ell = 0.08$ m. Additional forces on m_2 are equal in magnitude and opposite in direction.

$$\Rightarrow \quad k_1\Delta x_1 = k_2\Delta x_2$$

$$\Rightarrow \quad \frac{\Delta x_1}{\Delta x_2} = \frac{1}{3} \qquad \ldots(1)$$

Also, $\Delta x_1 + \Delta x_2 = 0.08$ $\ldots(2)$

Solving Equation (1) and (2), we get

$$\Delta x_1 = 0.02 \text{ m} \text{ and } \Delta x_2 = 0.06 \text{ m}$$

According to Work Energy Theorem

$$W_{\text{total}} = \Delta K$$

Since, m_2 is pulled down slowly, so

$$\Delta K = 0$$

$$\Rightarrow \quad W_{mg} + W_{\text{spring}} + W_F = \Delta KE = 0$$

$$\Rightarrow \quad W_F = -\left(W_{mg} + W_{\text{spring}}\right) \qquad \ldots(3)$$

Now, $W_{mg} = m_1 g\Delta x_1 + m_2 g\left(\Delta x_2 + \Delta x_1\right) = 1.2$ J

Also, $W_{\text{spring}} = -\left(U_f - U_i\right) = U_i - U_f$

where, $U_i = 0.4$ J and

$$U_f = \frac{1}{2}k_1\left(x_{01} + \Delta x_1^2\right) + \frac{1}{2}k_2\left(x_{02} + \Delta x_2\right)^2 = 2.8 \text{ J}$$

$$\Rightarrow \quad W_{\text{spring}} = -2.4 \text{ J}$$

Since, from (3), we have

$$W_F = -\left(W_{mg} + W_{\text{spring}}\right) = -(1.2 - 2.4) \text{ J} = 1.2 \text{ J}$$

6. The lower block will bounce when extension in spring,

$$kx = Mg$$

$$\Rightarrow \quad x = \frac{Mg}{k} \qquad \ldots(1)$$

By Law of Conservation of Energy, loss in gravitational potential energy (GPE) equals gain in elastic potential energy (EPE) of spring, so

$$mgh - mgx = \frac{1}{2}kx^2$$

$$\Rightarrow \quad mg\left(h - \frac{Mg}{k}\right) = \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$\Rightarrow \quad h = \frac{\frac{M^2g^2}{2k} + \frac{Mmg^2}{k}}{mg} = \frac{M^2g}{2km} + \frac{Mg}{k}$$

7. Let downward velocity of sphere be v_s and horizontal velocity of wedge be v_w. From our knowledge of constraint relations, the constraint equation for velocity can be written as,

$$\frac{v_s}{v_w} = \tan\alpha$$

$$\Rightarrow \quad v_s = v_w \tan\alpha \qquad \ldots(1)$$

When the sphere hits the ground, its centre will still be at a height R above the ground. Using Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in PE}\\ \text{of sphere}\end{pmatrix} = \begin{pmatrix}\text{Gain in KE}\\ \text{of sphere}\end{pmatrix} + \begin{pmatrix}\text{Gain in KE}\\ \text{of wedge}\end{pmatrix}$$

$$\Rightarrow \quad mg(2R) - mgR = \frac{1}{2}mv_s^2 + \frac{1}{2}mv_w^2$$

$$\Rightarrow \quad 2mgR = m\left(1 + \tan^2\alpha\right)v_w^2 = mv_w^2\sec^2\alpha$$

$$\Rightarrow \quad v_w = \sqrt{2gR}\cos\alpha \text{ and } v_s = v_w\tan\alpha = \sqrt{2gR}\sin\alpha$$

"Please note that the centre of mass of wedge stays at same distance from the ground both in the initial and final situations, so we have not accounted for change in potential energy of wedge, because that happens to be zero".

8. (a) According to Work Energy Theorem, we have

$$W_{\text{total}} = \Delta K$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = W_F + W_{\text{spring}} = Fx - \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}\times 0.5 \times v^2 = 20 \times 0.25 - \frac{1}{2}\times 40 \times (0.25)^2$$

$$\Rightarrow \quad v = 3.87 \text{ ms}^{-1}$$

(b) Total mechanical energy at point B is also equal to work done by the external force F. So, we have

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 = Fx = 20 \times 0.25 = 5 \text{ J}$$

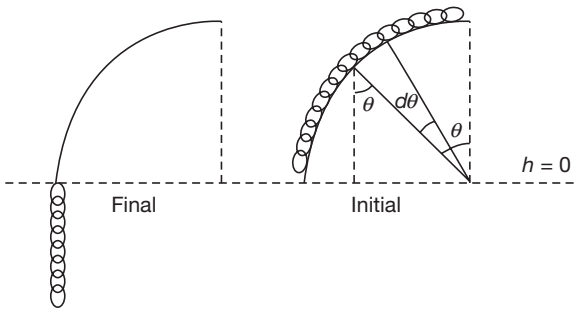
If x_f be the final maximum extension in the spring, then

$$\frac{1}{2}kx_f^2 = 5 \text{ J}$$

$$\Rightarrow \quad x_f = \sqrt{\frac{10}{k}} = \sqrt{\frac{10}{40}} = 0.5 \text{ m}$$

Hence, closest distance from wall is $d = 0.6 - x_f = 0.1$ m

9.



Initial potential energy is

$$U_i = \int_{\theta=0^\circ}^{\theta=\pi/2} (rd\theta)(\lambda)(g)(r\cos\theta)$$

$$U_i = (\lambda g r^2)\sin\theta\Big|_0^{\pi/2} = \lambda g r^2$$

Finally, the centre of mass of chain is $\frac{\pi(r/2)}{2}$ below the last link or the slot. So, final potential energy is given by

$$U_f = \left(\frac{\pi r}{2}\times\lambda\right)(g)\left(-\frac{\pi r/2}{2}\right) = -\frac{\pi^2 r^2 \lambda g}{8}$$

$$\Rightarrow \quad \Delta U = U_f - U_i = -r^2\lambda g\left(1+\frac{\pi^2}{8}\right)$$

By Law of Conservation of Energy

Loss in PE = Gain in KE

$$\Rightarrow \quad -\Delta U = \Delta K$$

$$\Rightarrow \quad r^2\lambda g\left(1+\frac{\pi^2}{8}\right) = \frac{1}{2}\left(\frac{\pi r}{2}\right)(\lambda)v^2$$

$$\Rightarrow \quad v = \sqrt{4rg\left(\frac{1}{\pi}+\frac{\pi}{8}\right)} = \sqrt{rg\left(\frac{4}{\pi}+\frac{\pi}{2}\right)}$$

10. Let T be the tension in the cable, then

$$T - mg = ma$$

$$T - 80g = 80a$$

$$\Rightarrow \quad T = 800 + 80 = 880 \text{ N}$$

(a) $W_T = Tx\cos(0^\circ) = (880)(15) = 13.2$ kJ

(b) $W_{mg} = mgx\cos(180^\circ) = -(880)(15) = -12$ kJ

(c) $KE = W_{\text{total}} = 13.2 - 12 = 1.2$ kJ

(d) $\frac{1}{2}mv^2 = 1200$

$$\Rightarrow \quad 40v^2 = 1200$$

$$\Rightarrow \quad v = 5.47 \text{ ms}^{-1}$$

11. When the block descends 12 mm, spring will further stretch 24 m. So total extension in the spring is $(76+24)$ mm i.e., 100 mm

$$\begin{pmatrix}\text{Decrease}\\ \text{in PE}\\ \text{of Block}\end{pmatrix} = \begin{pmatrix}\text{Increase}\\ \text{in KE}\\ \text{of Block}\end{pmatrix} + \begin{pmatrix}\text{Increase in}\\ \text{Elastic PE}\\ \text{of spring}\end{pmatrix}$$

$$\Rightarrow \quad (50)(10)(0.012) = \frac{1}{2}(1000)\left[(0.1)^2 - (0.076)^2\right] + \frac{1}{2}(50)v^2$$

$$\Rightarrow \quad 6 = 25v^2 + 2.5$$

$$\Rightarrow \quad v = 0.374 \text{ ms}^{-1}$$

12. (a) From Work Energy Theorem, we have

$$\Delta K = W = \int_0^2 \vec{F}\cdot d\vec{r}$$

where $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$\Rightarrow \quad W = \int_0^2 (2.5 - x^2)dx = 2.33 \text{ J}$$

$$\Rightarrow \quad W = \Delta K = 2.33 \text{ J}$$

(b) Speed of block at x is given by

$$\frac{1}{2}\times 1.5 \times v^2 = \int_0^x (2.5 - x^2)dx$$

$$\Rightarrow \quad v^2 = 1.33\left(2.5x - \frac{x^3}{3}\right)$$

For maximum K.E., $\frac{dv^2}{dx} = 0$

$$\Rightarrow \quad \frac{d}{dx}\left(2.5x - \frac{x^3}{3}\right) = 0$$

$$\Rightarrow \quad 2.5 - x^2 = 0$$

$$\Rightarrow \quad x^2 = 2.5$$

$$\Rightarrow \quad x = 1.58 \text{ m}$$

$$\Rightarrow \quad K_{\max} = \frac{1}{2}\times 1.5 \times 1.33\left(2.5\times 1.58 - \frac{(1.58)^3}{3}\right)$$

$$\Rightarrow \quad K_{\max} = 2.635 \text{ J}$$

13. (a) $U(x) = -W_{\text{spring}} = \int_0^x (\alpha x + \beta x^2)dx = \frac{\alpha x^2}{2} + \frac{\beta x^3}{3}$

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(b) By Law of Conservation of Energy, we get

$$\begin{pmatrix}\text{Gain in KE}\\ \text{of Mass}\end{pmatrix}=\begin{pmatrix}\text{Loss in PE}\\ \text{of Spring}\end{pmatrix}$$

$$\Rightarrow \frac{1}{2}mv^2 = U_i - U_f = U(1\text{ m}) - U(0.5\text{ m})$$

Since $U(x)=\frac{\alpha x^2}{2}+\frac{\beta x^3}{3}$

$$\Rightarrow U(1\text{ m})=\frac{80(1)}{2}+\frac{24(1)}{3}=48\text{ J}$$

$$\Rightarrow U(0.5\text{ m})=\frac{80(0.5)^2}{2}+\frac{24(0.5)^3}{3}=11\text{ J}$$

$$\Rightarrow \frac{1}{2}(2)v^2 = 48-11$$

$$\Rightarrow v^2 = 48-11$$

$$\Rightarrow v=\sqrt{37}\text{ ms}^{-1}\approx 6\text{ ms}^{-1}$$

14. Block will begin to move when,

$$kx = mg\sin\theta + \mu_s gm\cos\theta$$

$$\Rightarrow x=\frac{mg(\sin\theta+\mu_s\cos\theta)}{k}$$

$$\Rightarrow U=\frac{1}{2}kx^2=\frac{\left[mg(\sin\theta+\mu_s\cos\theta)\right]^2}{2k}$$

15. Using our knowledge of constrained motion of connected bodies, we observe that if $x_A = h$ (say), then $x_B = 2h$. Also, $v_A = \dot{x}_A = 2\text{ ms}^{-1}$

$$\Rightarrow v_B = \dot{x}_B = 2v_A = 4\text{ ms}^{-1}$$

Using Law of Conservation of Energy, we get

$$\begin{pmatrix}\text{Decrease}\\ \text{in PE of A}\end{pmatrix}=\begin{pmatrix}\text{Increase}\\ \text{in PE of B}\end{pmatrix}+\begin{pmatrix}\text{Increase in KE}\\ \text{of both A and B}\end{pmatrix}$$

$$\Rightarrow (30)(10)h=(5)(10)(2h)+\frac{1}{2}(30)(2)^2+\frac{1}{2}(5)(4)^2$$

$$\Rightarrow 300h = 100h+60+40$$

$$\Rightarrow 200h = 100$$

$$\Rightarrow h = 0.5\text{ m}$$

16. To use $E_f = E_i$ we would need to assign the initial heights of the blocks arbitrary values h_1 and h_2. The corresponding potential energies, m_1gh_1 and m_2gh_2 would appear in both E_i and E_f and hence would cancel.

We avoid this process by using the form $\Delta K + \Delta U = 0$ instead, since it does not require $U = 0$ reference level.

(a) At the maximum extension $x_{\max}$, the blocks come to rest, and thus $\Delta K = 0$. Next, we must find the changes in U_g and U_s. When m_2 falls by $x_{\max}$, the spring extends by $x_{\max}$ and m_1 rises by $x_{\max}\sin\theta$

$$\Rightarrow \Delta K + \Delta U_g + \Delta U_s = 0$$

$$\Rightarrow 0+\left(-m_2gx_{\max}+m_1gx_{\max}\sin\theta\right)+\frac{1}{2}kx_{\max}^2=0$$

Thus, $x_{\max}=\frac{2g}{k}(m_2-m_1\sin\theta)=0.98\text{ m}$

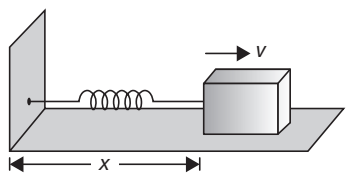
(b) In this case the change in kinetic energy is $\Delta K=\frac{1}{2}(m_1+m_2)v^2$.

The change in potential energy has the same form as in part (a), but with $x_{\max}$ replaced by $x = 0.5$ m, so

$$\frac{1}{2}(m_1+m_2)v^2+\left(-m_2gx+m_1gx\sin\theta\right)+\frac{1}{2}kx^2=0$$

Substituting values we get $v = 1.39\text{ ms}^{-1}$.

17. When the block is released, the spring pushes it towards right. The velocity of the block increases till the spring acquires its natural length. Thereafter, the block loses contact with the spring and moves with constant velocity.



Initially, the compression in the spring is $\frac{L_0}{2}$. When the distance of the block from the wall becomes x, where $x < L_0$, then compression is $(L_0 - x)$. Using the Law of Conservation of Energy,

$$\frac{1}{2}k\left(\frac{L_0}{2}\right)^2=\frac{1}{2}k(L_0-x)^2+\frac{1}{2}mv^2$$

$$\Rightarrow v=\sqrt{\frac{k}{m}\left[\frac{L_0^2}{4}-(L_0-x)^2\right]}$$

When the spring acquires its natural length i.e., $x = L_0$, then velocity becomes $v=\left(\sqrt{\frac{k}{m}}\right)\frac{L_0}{2}$. Thereafter, the block continues to move with this velocity.

18. Since all surfaces are smooth. According to Law of Conservation of Mechanical Energy, we have

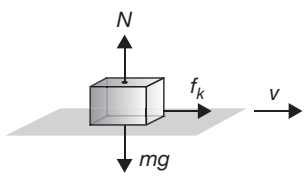
$$\begin{pmatrix}\text{Decrease in P.E. of}\\ \text{both the blocks}\end{pmatrix}=\begin{pmatrix}\text{Increase in K.E. of}\\ \text{both the blocks}\end{pmatrix}$$

$$\Rightarrow \quad (mgr)+(2mg)\left(\frac{\pi r}{2}\right)=\frac{1}{2}(m+2m)v^2$$

$$\Rightarrow \quad v=\sqrt{\frac{2}{3}(1+\pi)gr}$$

Test Your Concepts-IV (Based on Work Energy Theorem for Non-conservative Systems)

1. (a) When the box is first placed on the belt there will be slipping between the two. But the force of friction on the box and its displacement are in the same direction. Consequently, the work done by kinetic friction is positive. Since the final speed of the box is v,



$$W_f=\Delta K=+\frac{1}{2}mv^2 \qquad \ldots(1)$$

(b) The force of friction is $f=\mu_k N=\mu_k mg$ and $W_f=+fd$, so from equation (1), we get

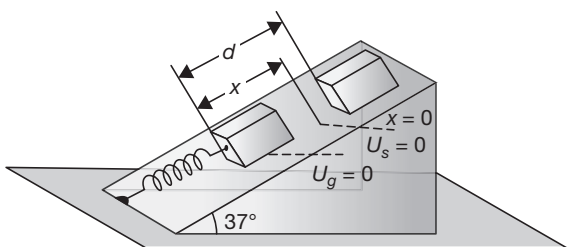
$$+\mu_k mgd=+\frac{1}{2}mv^2 \qquad \ldots(2)$$

$$\Rightarrow \quad d=\frac{v^2}{2\mu_k g}$$

(c) If the box takes a time t to reach speed v, then $v=at$ where a is the acceleration of box. In this time it will move $d=\frac{1}{2}at^2=\frac{1}{2}vt$. Since the belt's speed is fixed, in time t it moves a distance $vt=2d$.

The belt moves twice as far as the box while the box is accelerating.

2.



(a) K_i and K_f both are zero, so $E_i=\frac{1}{2}kx_o^2$ and $E_f=mgd\sin\theta$. From Work Energy Theorem,

$$W_{nc}=E_f-E_i=\Delta U+\Delta K$$

$$\Rightarrow \quad mgd\sin\theta-\frac{1}{2}kx_0^2=-fd$$

$$\Rightarrow \quad f=0.82\ \text{N}$$

(b) Initial energy mechanical E_i is still the same as above, but final mechanical energy at $x=0$ is

$$E_f=\frac{1}{2}mv^2+mgx_o\sin\theta$$

From Work Energy Theorem, we have

$$W_{nc}=E_f-E_i=\Delta U+\Delta K$$

$$\Rightarrow \quad \frac{1}{2}mv^2+mgx_0\sin\theta-\frac{1}{2}kx_0^2=-fx_0$$

$$\Rightarrow \quad v=2.45\ \text{ms}^{-1}$$

3. (a) Work done by friction $=-\mu Mg\ell$

(b) Work done by the spring force $=-\frac{1}{2}k\ell^2$

(c) Gravitational force and normal reaction of the table do not work as they act in a direction perpendicular to displacement.

(d) Total work done on the block $=-\left(\mu Mg\ell+\frac{1}{2}k\ell^2\right)$

(e) According to Work Energy Theorem

$$W_{\text{total}}=\Delta K$$

$$\Rightarrow \quad W_C+W_{nC}=\Delta K$$

$$\Rightarrow \quad -\Delta U+W_{nC}=\Delta K$$

$$\Rightarrow \quad 0-\frac{1}{2}Mv_0^2=-\left(\mu Mg\ell+\frac{1}{2}k\ell^2\right)$$

$$\Rightarrow \quad \ell^2+\frac{2\mu Mg\ell}{k}-\frac{Mv_0^2}{k}=0$$

$$\Rightarrow \quad \ell=\frac{1}{k}\left[\sqrt{\mu^2M^2g^2+Mkv_0^2}-\mu Mg\right]$$

4. Net retarding force $F=kx+\mu N$

$$\Rightarrow \quad F=kx+Mgbx$$

If a be the net retardation, then

$$a=-\frac{vdv}{dx}=\frac{F}{m}=\left(\frac{k+Mgb}{M}\right)x$$

$$\Rightarrow \quad \int_{v_0}^{0}vdv=-\left(\frac{k+Mgb}{M}\right)\int_0^x xdx$$

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$$\Rightarrow \quad -\frac{v_0^2}{2} = -\left(\frac{k+Mgb}{M}\right)\frac{x^2}{2}$$

$$\Rightarrow \quad x = v_0\sqrt{\frac{M}{k+Mgb}}$$

Loss in Mechanical Energy is $-\Delta E = E_i - E_f$

$$\Rightarrow \quad -\Delta E = (U+K)_{\text{initial}} - (U+K)_{\text{final}}$$

$$\Rightarrow \quad \text{Loss} = \left(0+\frac{1}{2}Mv_0^2\right) - \left(\frac{1}{2}kx^2+0\right)$$

$$\Rightarrow \quad \text{Loss} = \frac{1}{2}Mv_0^2 - \frac{1}{2}kx^2, \text{ where } x = v_0\sqrt{\frac{M}{k+Mgb}}$$

$$\Rightarrow \quad \text{Loss} = \left(\frac{M^2v_0^2}{2}\right)\left(\frac{gb}{k+Mgb}\right)$$

5. **From B to C**

$$0 = (4)^2 - 2(\mu_K g)(4)$$

$$\Rightarrow \quad \mu_K = \frac{16}{(2)(10)(4)} = 0.2$$

From A to B

Since we know that

$$W_{nc} = \Delta U + \Delta K$$

$$\Rightarrow \quad W_{\text{friction}} = -mgR + \frac{1}{2}mv_B^2$$

$$\Rightarrow \quad W_{\text{friction}} = -(0.2)(10)(2) + \frac{1}{2}(0.2)(16)$$

$$\Rightarrow \quad W_{\text{friction}} = -4 + 1.6 = -2.4 \text{ J}$$

6. (a) $100 = kx = 2000x$

$$\Rightarrow \quad x = 0.05 \text{ m}$$

Since, $\frac{1}{2}mv^2 + \mu_k mgx = \frac{1}{2}kx^2$

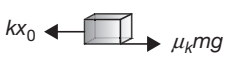
$$\Rightarrow \quad v = \sqrt{\frac{k}{m}x^2 - 2\mu_K gx}$$

$$\Rightarrow \quad v = \sqrt{\left(\frac{2000}{5}\right)(0.05)^2 - 2(0.36)(10)(0.05)}$$

$$\Rightarrow \quad v = 0.8 \text{ ms}^{-1}$$

(b) Velocity is maximum where

$$kx_0 = \mu_K mg$$



$$\Rightarrow \quad x_0 = \frac{\mu_K mg}{k} = \frac{0.36\times 5\times 10}{2000} = 0.009 \text{ m}$$

Since $\frac{1}{2}mv_{\max}^2 + \mu_K mg(x-x_0) = \frac{1}{2}k(x^2-x_0^2)$

$$\Rightarrow \quad v_{\max} = \sqrt{\frac{k}{m}(x^2-x_0^2) - 2\mu_K g(x-x_0)}$$

Substituting, $k = 2000 \text{ Nm}^{-1}$, $m = 5$ kg, $x = 0.05$ m, $x_0 = 0.009$ m, $\mu_k = 0.36$

$$\Rightarrow \quad v_{\max} = 0.82 \text{ ms}^{-1}$$

7. From Work Energy Theorem, we have

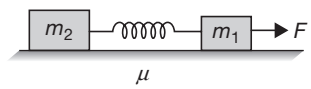
$$W_{mg} + W_{\text{air}} = \Delta KE$$

$$\Rightarrow \quad W_{\text{air}} = \Delta KE - W_{mg}$$

$$\Rightarrow \quad W_{\text{air}} = \frac{1}{2}\times 5\times (10)^2 - (5)(10)(20) = -750 \text{ J}$$

Negative sign indicates that work is done against the push of the air.

8. Block m_2 will shift when $kx = \mu m_2 g$ …(1)



For m_1, using Work Energy Theorem, we get

$$Fx = \frac{1}{2}kx^2 + \mu m_1 xg$$

$$\Rightarrow \quad kx = 2F - 2\mu m_1 g \qquad \ldots(2)$$

Equating Equation (1) and (2), we get

$$F = \left(m_1 + \frac{m_2}{2}\right)\mu g$$

9. For horizontal portion, retardation $a_1 = \mu g = 1.5 \text{ ms}^{-2}$

$$\Rightarrow \quad v^2 = 2a_1 s_1 = 2\times 1.5\times 0.5 = 1.5 \text{ m}^2\text{s}^{-2}$$

$$\Rightarrow \quad v = 1.22 \text{ ms}^{-1}$$

For inclined portion, acceleration is given by

$$a_2 = g\sin\theta - \mu g\cos\theta = \left(\frac{1}{2} - 0.15\times\frac{\sqrt{3}}{2}\right)\times 10$$

$$\Rightarrow \quad a_2 = 3.7 \text{ ms}^{-2}$$

$$\Rightarrow \quad s_2 = \frac{v^2}{2a_2} = \frac{1.5}{2\times 3.7} = 0.2 \text{ m}$$

Work performed by friction forces is

$$W_{\text{friction}} = W_{\text{case I}} + W_{\text{case II}}$$

$$\Rightarrow \quad W_f = -\mu mgs_1 - (\mu mg\cos\theta)s_2$$

$$\Rightarrow \quad W_f = -0.15\times 0.05\times 10\left(0.5 + 0.2\times\frac{\sqrt{3}}{2}\right) = -0.044 \text{ J}$$

10. From constraint relations, we can see that

$v_A = 2v_B$

Therefore, $v_A = 2(0.3) = 0.6 \text{ ms}^{-1}$

as $v_B = 0.3 \text{ ms}^{-1}$ {given}

Since $W_{nc} = \Delta U + \Delta K$

$\Rightarrow \quad -\mu m_A g x_A = -m_B g x_B + \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

$\Rightarrow \quad x_A = 2x_B = 2 \text{ m}, \quad$ since $\quad x_B = 1 \text{ m}, \quad m_A = 4 \text{ kg},$ $m_B = 1 \text{ kg}$, so we get

$-80\mu = -10 + 0.72 + 0.045$

$\Rightarrow \quad 80\mu = 9.235$

$\Rightarrow \quad \mu = 0.115$

11. Since the applied force is always parallel to the track, normal reaction will always be $mg\cos\theta$. In this case, work done against friction is

$W = \mu mg \times$ horizontal distance covered

$\Rightarrow \quad W = \mu mgL$

$\Rightarrow \quad W = 0.5 \times 1 \times 10 \times 1 = 5 \text{ J}$

Test Your Concepts-V (Based on Relation Between Conservative Force and Potential Energy and Types of Equilibrium)

1. We use $\vec{F} = -\nabla U = -\left(2y\hat{i} + (2x + z)\hat{j} + y\hat{k}\right)$

2. Since, $\vec{F} = -\vec{\nabla} U = -\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right)$

$$F_x = -\frac{\partial U}{\partial x} = -U_0 \exp(-az)\frac{\partial}{\partial x}[\cos(ax)]$$

$$\Rightarrow \quad F_x = \frac{\partial U}{\partial x} = +aU_0 \sin(ax)\exp(-az)$$

Since, U has no dependence on y, so

$$F_y = \frac{\partial U}{\partial y} = 0$$

$$F_z = -\frac{\partial U}{\partial z} = -U_0 \cos(ax)\frac{\partial}{\partial z}[\exp(-az)]$$

$$\Rightarrow \quad F_z = -\frac{\partial U}{\partial z} = aU_0 \cos(ax)\exp(-az)$$

$$\Rightarrow \quad F = F_x\hat{i} + F_z\hat{k} \qquad \{\because F_y = 0\}$$

$$\Rightarrow \quad \vec{F} = aU_0 \exp(-az)\left[\sin(ax)\hat{i} + \cos(ax)\hat{k}\right]$$

3. Since we know that

$dU = -\vec{F} \cdot d\vec{l}$

$\Rightarrow \quad dU = -\left[2axy\hat{i} + a\left(x^2 - y^2\right)\hat{j}\right] \cdot \left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$

$\Rightarrow \quad dU = -\left[2axydx + a\left(x^2 - y^2\right)dy\right]$

$$\Rightarrow \quad \int_{(0,0)}^{(x,y)} dU = -\int_{(0,0)}^{(x,y)} 2axydx + a\left(x^2 - y^2\right)dy$$

$$\Rightarrow \quad U_{(x,y)} - U_{(0,0)} = -\int_{(0,0)}^{(x,y)} \left(2axy\,dx + ax^2\,dy\right) - ay^2 dy$$

Let $U_{(x,y)} = U$ and $U_{(0,0)} = U_0$, then we have

$$U - U_0 = -\int_{(0,0)}^{(x,y)} \left\{d\left(ax^2y\right) - d\left(\frac{y^3}{3}\right)\right\}$$

$$\Rightarrow \quad U - U_0 = -\int_{(0,0)}^{(x,y)} d\left(ax^2y - \frac{y^3}{3}\right)$$

$$\Rightarrow \quad U - U_0 = \left[\left(-ax^2y + \frac{ay^3}{3}\right)\Bigg|_{(0,0)}^{(x,y)}\right]$$

$$\Rightarrow \quad U = U_0 - ax^2y + \frac{ay^3}{3}$$

4. Since $\Delta U = -\int \vec{F} \cdot d\vec{l}$

where, $dl = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz$

$$\Rightarrow \quad \Delta U = \int_{(0,0)}^{(2,3)} 4\left(y\hat{i} + x\hat{j}\right) \cdot \left(\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz\right)$$

$$\Rightarrow \quad U - (-4) = 4\int_{(0,0)}^{(2,3)} ydx + xdy$$

Since, we know that

$d(xy) = xdy + ydx$

$$\Rightarrow \quad U - (-4) = 4\int_{(0,0)}^{(2,3)} d(xy)$$

$$\Rightarrow \quad U - (-4) = 4(xy)\Big|_{(0,0)}^{(2,3)}$$

$\Rightarrow \quad U - (-4) = 4[(2)(3) - (0)(0)] = 24$

$\Rightarrow \quad U = 24 - 4 = 20 \text{ J}$

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5. Potential energy of the particle is given as

$$U = 2 - 20x + 5x^2 \qquad \ldots(1)$$

Therefore, its potential energy at $x = -3$ is

$$U_1 = 2 - 20 \times (-3) + 5(-3)^2 = 2 + 60 + 45 = 107 \text{ J}$$

Since the particle is released from $x = -3$ m, its kinetic energy at this position is zero and therefore its total energy is $107 + 0 = 107$ J.

Particle will again come to rest at a position where its potential energy once again becomes 107 J.

Substituting $U = 107$ J in equation (1), we get

$$2 - 20x + 5x^2 = 107$$

$$\Rightarrow \quad 5x^2 - 20x - 105 = 0$$

$$\Rightarrow \quad x^2 - 4x - 21 = 0$$

$$\Rightarrow \quad x = -3 \text{ m} \text{ and } x = 7 \text{ m}$$

Hence the particle released from $x = -3$ m will come to rest again at $x = 7$ m

6. Since $F = -\dfrac{dU}{dx}$

$$\Rightarrow \quad dU = -F dx$$

$$\Rightarrow \quad U(x) = -\int_0^x (-kx + ax^3)dx = \frac{kx^2}{2} - \frac{ax^4}{4}$$

$$\Rightarrow \quad U(x) = \frac{kx^2}{2}\left(1 + \sqrt{\frac{a}{2k}}x\right)\left(1 - \sqrt{\frac{a}{2k}}x\right)$$

Now, $U(x) = 0$ at $x = 0$ and $x = \sqrt{\dfrac{2k}{a}}$

Also, $U(x)$ is negative for $x > \sqrt{\dfrac{2k}{a}}$

For $0 < x < \sqrt{\dfrac{2k}{a}}$, $U(x)$ first increases and then decreases.

From the given function we can see that $F = 0$ at $x = 0$ i.e., slope of U-x graph is zero at $x = 0$. Therefore, the most appropriate OPTION is (D).

7. Since, $F = -\dfrac{dU}{dx}$

$$\Rightarrow \quad \int_0^{U(x)} dU = -\int_0^x F dx = -\int_0^x (kx)dx$$

$$\Rightarrow \quad U(x) = -\frac{kx^2}{2} \qquad \{\because U(0) = 0\}$$

Test Your Concepts-VI (Based on Vertical Circle)

1. For the body to be able to successfully complete the loop, its velocity at the bottom of the loop must be greater than or equal to $\sqrt{5gr}$.

Now from Law of Conservation of Energy, we get

$$mgh = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v^2 = 2gh \geq 5gR$$

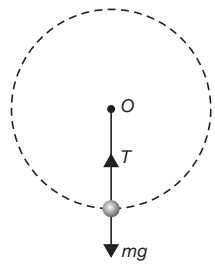
$$\Rightarrow \quad h \geq \frac{5}{2}R$$

So, the minimum height required is $\dfrac{5}{2}R$

2. Let T be the tension in the string at the lowest point of circle. Then the net force on the stone is $(T - mg)$, which is directed towards the centre of the circle and provides necessary centripetal force to the particle to revolve in a circle of radius r. So

$$T - mg = \frac{mv^2}{r}$$

where v is the speed of the stone at the lowest point of the circle of radius r.



$$\Rightarrow \quad T = \frac{mv^2}{r} + mg = m\left(\frac{v^2}{r} + g\right)$$

$$\Rightarrow \quad T = 0.5\left(\frac{3^2}{0.40} + 9.8\right) = 16.15 \text{ N}$$

3. (a) Speed of the particle at the lowest point is first calculated by the Law of Conservation of Energy, so

$$(U + K)_A = (U + K)_B$$

$$\Rightarrow \quad mgL(1 - \cos\theta) + \frac{1}{2}mu^2 = 0 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v^2 = u^2 + 2gL(1 - \cos\theta) \qquad \ldots(1)$$

Now, tension at the lowest point is

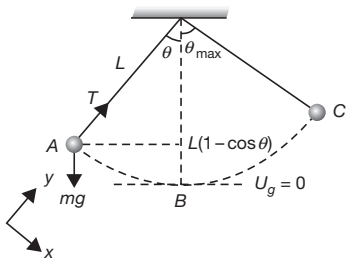
$$T = \frac{m}{L}(v_B^2 + gL), \text{ where } v_B = v$$

$$\Rightarrow \quad T = \frac{m}{L}\left(u^2 + 3gL - 2gL\cos\theta\right)$$

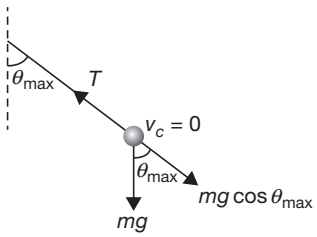
Now, according to question, we have

$$u = 1 \text{ ms}^{-1},\ g = 10 \text{ ms}^{-2},\ L = 2 \text{ m},\ m = 2 \text{ kg}$$

$$\Rightarrow \quad T = \left(\frac{2}{2}\right)(1 + 60 - 32) = 29 \text{ N}$$



(b) At the highest point, we have $T_C = mg\cos\theta_{max}$



Now, please do not confuse θ_{max} with θ as both may be different. So, let us first calculate θ_{max}. For this purpose, we shall try to calculate the total mechanical energy of the particle at A, which is

$$E_A = U_A + K_A$$

$$\Rightarrow \quad E_A = mgL(1 - \cos\theta) + \frac{1}{2}mu^2$$

$$\Rightarrow \quad E_A = (2)(10)(2)\left(1 - \frac{4}{5}\right) + \frac{1}{2}(2)(1)^2$$

$$\Rightarrow \quad E_A = 8 + 1 = 9 \text{ J}$$

So, now at maximum height, we have $v_C = 0$ and by Law of Conservation of Energy, total energy at C should also be 9 J

$$\Rightarrow \quad mgL(1 - \cos\theta_{max}) = 9$$

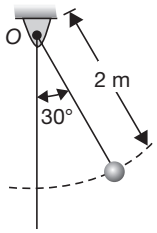
$$\Rightarrow \quad (2)(10)(2)(1 - \cos\theta_{max}) = 9$$

$$\Rightarrow \quad 1 - \cos\theta_{max} = \frac{9}{40}$$

$$\Rightarrow \quad \cos\theta_{max} = \frac{31}{40}$$

$$\Rightarrow \quad T_C = (2)(10)\left(\frac{31}{40}\right) = 15.5 \text{ N}$$

4. Since, $T - mg\cos\theta = \dfrac{mv^2}{\ell}$



$$\Rightarrow \quad 2.5mg - mg\cos 30° = \frac{mv^2}{2}$$

$$\Rightarrow \quad v = 5.66 \text{ ms}^{-1}$$

$$\Rightarrow \quad a = \sqrt{\left(\frac{v^2}{\ell}\right)^2 + (g\sin\theta)^2}$$

$$\Rightarrow \quad a = \sqrt{\left(\frac{5.66 \times 5.66}{2}\right)^2 + (9.8 \times \sin 30°)^2}$$

$$\Rightarrow \quad a = 16.75 \text{ ms}^{-2}$$

5. (a) $T_1 = \dfrac{mv_1^2}{R_1} = \dfrac{(0.8)(1)^2}{0.4} = 2 \text{ N}$

(b) $T_2 = \dfrac{mv_2^2}{R} = \dfrac{(0.8)(4)^2}{0.2} = 64 \text{ N}$

(c) $W = \dfrac{1}{2}m\left(v_2^2 - v_1^2\right) = \dfrac{1}{2} \times 0.8(4 - 1) = 1.2 \text{ J}$

6. Using Work Energy Theorem, for the rough track, we get

$$W_{nc} = \Delta U + \Delta K$$

$$\Rightarrow \quad -\mu mg\ell = \left(0 - \frac{1}{2}k\delta^2\right) + \left(\frac{1}{2}mv^2 - 0\right)$$

For looping the loop, we have $v^2 = 5gR$

$$\Rightarrow \quad \frac{1}{2}k\delta^2 = \frac{1}{2}m(5gR) + \mu_k mgR$$

$$\Rightarrow \quad \delta = \sqrt{\frac{mgR}{k}}\sqrt{5 + 2\mu_k}$$

7. Since, $h = r(1 + \sin\theta)$ ...(1)

Also, by Law of Conservation of Energy, we get

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgh$$

$$\Rightarrow \quad v^2 = u^2 - 2gh$$

$$\Rightarrow \quad v^2 = u^2 - 2gr(1 + \sin\theta) \qquad \ldots(2)$$

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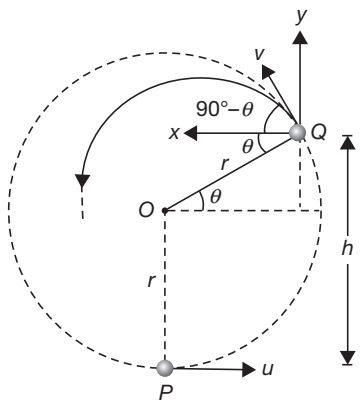
Coordinates of point P with Q as origin (x and y axes as shown), are $P(r\cos\theta, -r-r\sin\theta)$



At Q, we have, $T=0$ and $v\neq 0$, so motion of particle ceases to be circular and the particle becomes a projectile, launched with initial velocity v and launch angle α such that its x and y coordinates are related to each other as

$$y = x\tan\alpha - \frac{gx^2}{2v^2\cos^2\alpha}, \text{ where } \alpha = 90-\theta$$

$$\Rightarrow \quad -r(1+\sin\theta) = (r\cos\theta)\tan(90^\circ-\theta) - \frac{gr^2\cos^2\theta}{2v^2\cos^2(90-\theta)}$$

$$\Rightarrow \quad (1+\sin\theta) = \frac{gr\cot^2\theta}{2v^2} - \frac{\cos^2\theta}{\sin\theta} \qquad \ldots(3)$$

Substituting value of v from equation (2) in (3), we get

$$\theta = 30^\circ \text{ and } u = \sqrt{\frac{7rg}{2}}$$

8. (a) Applying conservation of energy

$$mgh = \frac{1}{2}m\left(\sqrt{3gL}\right)^2$$

$$\Rightarrow \quad h = \frac{3L}{2}$$

(b) Since $\sqrt{3gL}$ lies between $\sqrt{2gL}$ and $\sqrt{5gL}$, the string will slack in upper half of the circle. Assuming that string slacks when it makes an angle θ with horizontal, then we get

$$mg\sin\theta = \frac{mv^2}{L} \qquad \ldots(1)$$

$$\Rightarrow \quad v^2 = \left(\sqrt{3gL}\right)^2 - 2gL(1+\sin\theta) \qquad \ldots(2)$$

Solving Equation (1) and (2), we get

$$\sin\theta = \frac{1}{3} \text{ and } v^2 = \frac{gL}{3}$$

Maximum height of the bob from starting point, is

$$H = L(1+\sin\theta) + \frac{v^2\sin^2(90^\circ-\theta)}{2g}$$

$$\Rightarrow \quad H = L\left(1+\frac{1}{3}\right) + \left(\frac{gL}{6g}\right)\left(\frac{8}{9}\right) = \frac{4L}{3} + \frac{4L}{27}$$

$$\Rightarrow \quad H = \frac{40}{27}L$$

9. Here tension in the rod at the top most point of circle can be zero or negative for completing the loop. So, velocity at the top most point is zero.
From energy conservation

$$\frac{1}{2}mv^2 + \frac{1}{2}m\frac{v^2}{4} = mg(2\ell) + mg(4\ell) + 0$$

$$\Rightarrow \quad v = \sqrt{\frac{48g\ell}{5}}$$

10. When the motorcyclist is at the highest point of the death-well, the normal reaction N on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also act downwards.

$$F_{\text{net}} = ma_C$$

$$\Rightarrow \quad N + mg = \frac{mv^2}{r} \qquad \ldots(1)$$

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass of the motor cycle). Because of the balancing of the forces, the motorcyclist does not fall down.

The minimum speed required to perform a vertical loop is given by equation (1), when $N=0$

$$\Rightarrow \quad mg = \frac{mv_{\min}^2}{r}$$

$$\Rightarrow \quad v_{\min}^2 = gr$$

$$\Rightarrow \quad v_{\min} = \sqrt{gr} = \sqrt{9.8\times 25}\ \text{ms}^{-1} = 15.65\ \text{ms}^{-1}$$

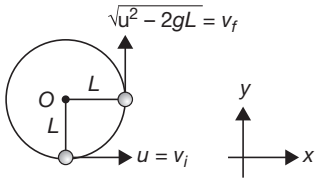
11. Let v_i be the initial velocity i.e., velocity at lowest point

$$\Rightarrow \quad \vec{v}_i = u\hat{i}\,.$$

Let v_f be the final velocity i.e., velocity when the string becomes vertical

$$v_f^2 - v_i^2 = 2(-g)L$$

$$\Rightarrow \quad v_f = \sqrt{u^2 - 2gL} \text{ (in magnitude)}$$



$\Rightarrow \quad \vec{v}_f=\left(\sqrt{u^2-2gL}\right)\hat{j}$

So, $\Delta\vec{v}=\vec{v}_f-\vec{v}_i=\left(\sqrt{u^2-2gL}\right)\hat{j}-u\hat{i}$

$\Rightarrow \quad |\Delta\vec{v}|=\sqrt{u^2-2gL+u^2}=\sqrt{2\left(u^2-gL\right)}$

$\Rightarrow \quad |\Delta\vec{v}|=\sqrt{2\left[(10)^2-(10)(5)\right]}$

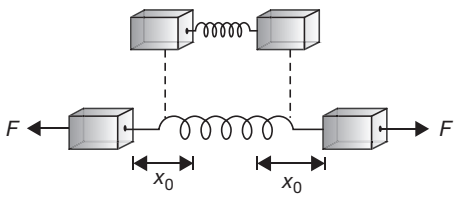
$\Rightarrow \quad |\Delta\vec{v}|=\sqrt{2(100-50)}=\sqrt{100}\ =10\ \text{ms}^{-1}$

Single Correct Choice Type Questions

1. $U_i=0$ and $U_f=\frac{1}{2}k(2x_0)^2$

$\Rightarrow \quad U_f=2kx_0^2$

Since we know that, $W_C=-\Delta U$



If W be the work done by the spring on earth mass, then

$W_C=2W=0-2kx_0^2$

$\Rightarrow \quad W=-kx_0^2$

Hence, the correct answer is (D).

2. Force needed to pull the block down by a distance y from equilibrium position is

$F=k(x+y)-mg=ky$

Therefore, work done to extend the spring is

$W=\frac{1}{2}ky^2$

At equilibrium, $mg=kx$

$\Rightarrow \quad k=\frac{mg}{x}=\frac{2\times10}{0.06}=\frac{1000}{3}\ \text{Nm}^{-1}$

When pulled down by additional distance $10\ \text{cm}$, external work done $W=\frac{1}{2}\times k\times(0.1)^2$

$\Rightarrow \quad W=\frac{1}{2}\times\frac{1000}{3}\times0.01=1.67\ \text{J}$

Hence, the correct answer is (D).

3. By Law of Conservation of Energy

$mg(h+x)=\frac{1}{2}kx^2$

$\Rightarrow \quad x=\frac{mg+\sqrt{m^2g^2+2mghk}}{k}$

Hence, the correct answer is (A).

4. From Work Energy Theorem $W=\Delta\text{KE}$

$\Rightarrow \quad W=K_f-K_i=\frac{1}{2}m\left(v_f^2-v_i^2\right)$

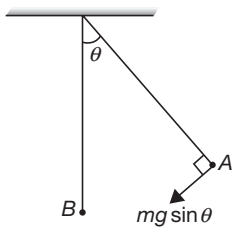
$\Rightarrow \quad W=\frac{1}{2}(2)(400-100)=300\ \text{J}$

Hence, the correct answer is (A).

5. $P_{\text{inst}}=\vec{F}\cdot\vec{v}\ =50-30+120\ =140\ \text{W}$

Hence, the correct answer is (C).

6.



At extreme position A, since the ball is at rest so, net acceleration is equal to the tangential acceleration.

$\Rightarrow \quad a_A=g\sin\theta$

At lowermost position B, net acceleration is centripetal acceleration, i.e.,

$a_B=\frac{v^2}{L}$ where $v=\sqrt{2gL(1-\cos\theta)}$

$\Rightarrow \quad a_B=2g(1-\cos\theta)$

Since, $a_A=a_B$

$\Rightarrow \quad g\sin\theta=2g(1-\cos\theta)$

$\Rightarrow \quad 2g\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)=2g\left(1-\left(1-2\sin^2\frac{\theta}{2}\right)\right)$

Since $\sin\theta=2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$ and

$1-\cos\theta=2\sin^2\left(\frac{\theta}{2}\right)$

CHAPTER 1

$$\Rightarrow \quad 2g\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) = 2g \times 2\sin^2\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \tan\frac{\theta}{2} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{\theta}{2} = 26.5^\circ$$

$$\Rightarrow \quad \theta = 53^\circ$$

Hence, the correct answer is (C).

7. $v = \dfrac{dx}{dt} = 3 - 8t + 3t^2$

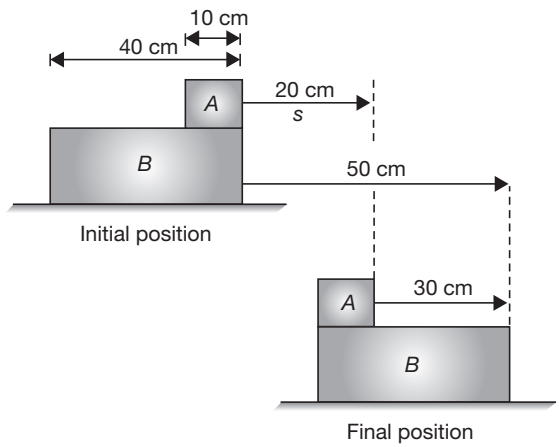
Initial velocity (velocity at $t = 0$) is $u = 3\ \text{ms}^{-1}$. Final velocity (velocity at $t = 4s$) is $v = 19\ \text{ms}^{-1}$

Since, $W = \dfrac{1}{2}m\left(v^2 - u^2\right) = \dfrac{1}{2}\left(\dfrac{3}{1000}\right)(361 - 9)$

$$\Rightarrow \quad W = \frac{528}{1000}\ \text{J} = 528\ \text{mJ}$$

Hence, the correct answer is (B).

8.



The displacement of block A in ground frame towards right is s (as shown in figure).

$$\Rightarrow \quad s = 0.5 - 0.3 = 0.2\ \text{m}$$

Kinetic friction acting on block A acting towards right is

$$f_k = \mu mg = 0.2 \times 45 \times 10 = 90\ \text{N}$$

So, work done by friction force in the ground frame is

$$W = fs\cos(0^\circ) = 90 \times 0.2 \times 1 = 18\ \text{J}$$

Hence, the correct answer is (B).

9. $a = \left(\dfrac{m_2 - \mu m_1}{m_1 + m_2}\right)g$

$$\Rightarrow \quad a = \left[\frac{4 - (0.2)(10)}{4 + 10}\right]9.8$$

$$\Rightarrow \quad a = 1.4\ \text{ms}^{-2}$$

Since, $v^2 - 0^2 = 2as$

$$\Rightarrow \quad v^2 = 2(1.4)(4)$$

$$\Rightarrow \quad v^2 = 11.2$$

$$\Rightarrow \quad v \cong 3.5\ \text{ms}^{-1}$$

Hence, the correct answer is (D).

10. Since, $\vec{F}$ is a constant force, so work done will not depend on path. So,

$$W = \int_{(0,0,0)}^{(2,2,2)} \vec{F} \cdot d\vec{\ell}$$

$$\Rightarrow \quad W = \int_{(0,0,0)}^{(2,2,2)} \left(2\hat{i} + 5\hat{j} + \hat{k}\right) \cdot \left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$\Rightarrow \quad W = \int_{(0,0,0)}^{(2,2,2)} \left(2dx + 5dy + dz\right)$$

$$\Rightarrow \quad W = \left(2x + 5y + z\right)\Big|_{(0,0,0)}^{(2,2,2)} = 16\ \text{J}$$

Hence, the correct answer is (D).

11. Given force is a constant force and work done is path independent under a constant force. Hence, $W_1 = W_2$.

Hence, the correct answer is (A).

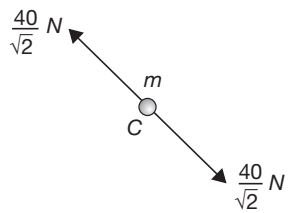
12. Since $\Delta U = -\int \vec{F} \cdot d\vec{l}$

$$\Rightarrow \quad U - U_0 = \int 40\left(ydx + xdy\right) = 40\int d\left(xy\right)$$

$$\Rightarrow \quad U - U_0 = 40xy$$

$$\Rightarrow \quad U = U_0 + 40xy$$

At the mid point (C) of the rod, the particle is in equilibrium.



Also, we must note that from A till mid point C, the force is towards A, so the particle faces opposition from force to reach the mid point C. However after C i.e., between C and B, the force is directed towards B. This makes us conclude that if we give the particle

a sufficient speed just to reach the mid point C, then after crossing C it will automatically reach B. Hence by Law of Conservation of Energy applied between A and C we have

$$\begin{pmatrix}\text{Loss in KE}\\ \text{of particle}\end{pmatrix}=\begin{pmatrix}\text{Gain in PE}\\ \text{of particle}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}mv_0^2 = 40\left(\frac{2}{2}\right)\left(\frac{2}{2}\right)-40(0)(2)$$

$$\Rightarrow \quad \frac{1}{2}(5)v_0^2 = 40$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{80}{5}} = \sqrt{16} = 4 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

13. **Method-I:**

$$dW = \vec{F}\cdot d\vec{\ell} = 2dx + 3dy$$

Along the path $3y + kx = 5$, we have

$$3dy + kdx = 0 \qquad \ldots(1)$$

Since $W = 0$ i.e., $\int dW = 0$

$$\Rightarrow \quad \int 2dx - kdx = 0 \qquad \{\because \text{ from (1), } 3dy = -kdx\}$$

$$\Rightarrow \quad k = 2$$

Method-II:

Force is parallel to a line $y = \frac{3}{2}x + c$

The equation of given line can be written as

$$y = -\frac{k}{3}x + \frac{5}{3}$$

Work done will be zero, when force is perpendicular to the displacement i.e., the above two lines are perpendicular or

$$m_1m_2 = -1$$

$$\Rightarrow \quad \left(\frac{3}{2}\right)\left(-\frac{k}{3}\right) = -1$$

$$\Rightarrow \quad k = 2$$

Hence, the correct answer is (A).

14. In the first case, speed imparted to the marble by the spring is

$$\frac{1}{2}kd^2 = \frac{1}{2}mu^2$$

$$\Rightarrow \quad u = \sqrt{\frac{k}{m}}d$$

Since horizontal range of marble is uT, where T is the time of flight. So, we get

$$R - r = uT = \left(\sqrt{\frac{k}{m}}d\right)T \qquad \ldots(1)$$

Let the required compression in the spring for landing the ball in the box be x', then

$$R = \left(\sqrt{\frac{k}{m}}x'\right)T \qquad \ldots(2)$$

Please note that time of flight is independent of the velocity of the ball and therefore same in both the cases. From (1) and (2), we get

$$\frac{R}{R-r} = \frac{x'}{d}$$

$$\Rightarrow \quad x' = \frac{Rd}{R-r}$$

Hence, the correct answer is (D).

15. Since $W_{nc} = \Delta U + \Delta K = mgh - (-mgh) + 0 = 2mgh$

Hence, the correct answer is (B).

16. According to Work Energy Theorem, $W = \Delta K$, so kinetic energy of block at $x = x$ is

$$K = W = \int_0^x F dx = \int_0^x (4 - x^2)dx$$

$$\Rightarrow \quad K = 4x - \frac{x^3}{3}$$

For K to be maximum $\frac{dK}{dx} = 0$

$$\Rightarrow \quad 4 - x^2 = 0$$

$$\Rightarrow \quad x = \pm 2 \text{ m}$$

At $x = +2$ m, we have $\frac{d^2K}{dx^2}$ to be negative

So, kinetic energy (K) is maximum at $x = +2$ m

$$\Rightarrow \quad K_{\max} = (4)(2) - \frac{(2)^3}{3} = \frac{16}{3} \text{ J} \approx 5.33 \text{ J}$$

Hence, the correct answer is (C).

17. Kinetic energy of particle at $x = x$ is

$$K = 4x - \frac{x^3}{3}$$

At maximum x-displacement i.e., at point of reversal of motion, we have

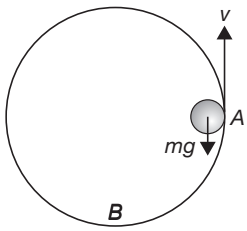
$$v = 0, \text{ so } K = 0$$

$$\Rightarrow \quad x = 2\sqrt{3} \text{ m}$$

Hence, the correct answer is (A).

CHAPTER 1

18.



Velocity will be vertical at position A as shown in Figure. At this position, tangential acceleration is due to weight. Therefore,

$$ma_t = mg$$

$$\Rightarrow \quad a_t = g$$

As the particle is just able to complete the circle, speed at position A is

$$v = \sqrt{3gr}$$

Therefore, centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{3gr}{r} = 3g$$

Hence, total acceleration of the particle is

$$a_{\text{total}} = \sqrt{a_c^2 + a_t^2} = \sqrt{(3g)^2 + g^2}$$

$$\Rightarrow \quad a_{\text{total}} = \sqrt{10}\text{g}$$

Hence, the correct answer is (C).

19. Since,

$$\vec{F} = -\vec{\nabla}U$$

$$\Rightarrow \quad \vec{F} = -\lambda\left(\hat{i} + \hat{j}\right)$$

Further, $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$$\Rightarrow \quad \Delta\vec{r} = \left(2\hat{i} + 3\hat{j}\right) - \left(\hat{i} + \hat{j}\right) = \hat{i} + 2\hat{j}$$

So, $W = \vec{F} \cdot \Delta\vec{r} = -\lambda - 2\lambda$

$$\Rightarrow \quad W = -3\lambda$$

Hence, the correct answer is (B).

20. Work done = Area under F-s graph

Area $= 36 + (-6) = 30$ J

Hence, the correct answer is (B).

21. Let the maximum extension in the spring be $x_{\max} = x_m$, then by Applying Work-Energy Theorem from the frame attached to point A, we get

$$(ma)x_m - \frac{1}{2}kx_m^2 = 0$$

$$\Rightarrow \quad x_m = \frac{2ma}{k}$$

Hence, the correct answer is (C).

22. $\vec{v} = \alpha\hat{i} + \beta t\,\hat{j}$

$$\Rightarrow \quad \vec{a} = \frac{d\vec{v}}{dt} = \beta\hat{j}$$

So, total acceleration $|\vec{a}| = \beta$

The speed, $v = \sqrt{\alpha^2 + \beta^2 t^2}$

So, tangential acceleration, a_T is

$$a_T = \frac{dv}{dt} = \frac{1}{2}\left(\alpha^2 + \beta^2 t^2\right)^{1/2}\left(2\beta^2 t\right)$$

At $\quad t = \dfrac{\sqrt{3}\alpha}{\beta}$

Tangential acceleration, $a_T = \dfrac{\sqrt{3}\beta}{2}$

and normal acceleration, $a_N = \sqrt{\beta^2 - \dfrac{3\beta^2}{4}} = \dfrac{\beta}{2}$

Hence, the correct answer is (A).

23. $\sin\theta = \dfrac{5}{10} = \dfrac{1}{2}$

$$\Rightarrow \quad \theta = 30^\circ$$

$$W = fs\cos 180 = -fs$$

$$\Rightarrow \quad W = -\left(\mu mg\cos 30\right)s$$

$$\Rightarrow \quad W = -(0.1)(0.5)(10)\left(\frac{\sqrt{3}}{2}\right)(10+10)$$

$$\Rightarrow \quad W = -5\sqrt{3}\text{ J}$$

Hence, the correct answer is (D).

24. Let x be the extension in the spring when 2 kg block leaves the contact with ground. Then,

$$kx = 2g$$

$$\Rightarrow \quad x = \frac{2g}{k} = \frac{2\times 10}{40} = \frac{1}{2}\text{ m}$$

Now by Law of Conservation of Energy, we have

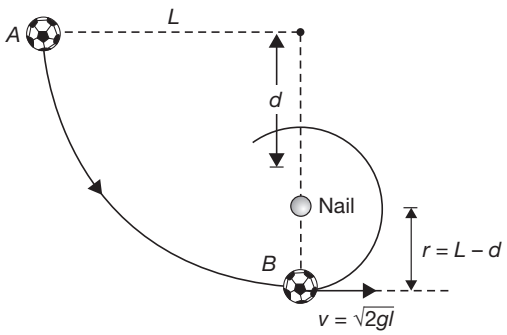
$$mgx = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 \qquad \{\text{where } m = 5\text{ kg}\}$$

$$\Rightarrow \quad v = \sqrt{2gx - \frac{kx^2}{m}}$$

$$\Rightarrow \quad v = \sqrt{2\times 10\times\frac{1}{2} - \frac{40}{4\times 5}} = 2\sqrt{2}\text{ ms}^{-1}$$

Hence, the correct answer is (B).

25.



The ball rotates in a circle around the nail after it goes past the point B.

At point B, velocity of the ball is $v = \sqrt{2gL}$ ...(1)

Since the ball just completes the vertical circle about the nail, its speed at the lowest point should be

$$v = \sqrt{5gr} = \sqrt{5g(L-d)} \quad \ldots(2)$$

Equating (1) and (2), we get

$$\sqrt{2gL} = \sqrt{5g(L-d)}$$

$$\Rightarrow \quad 2L = 5L - 5d$$

$$\Rightarrow \quad d = 0.6L$$

Hence, the correct answer is (A).

26. $-fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

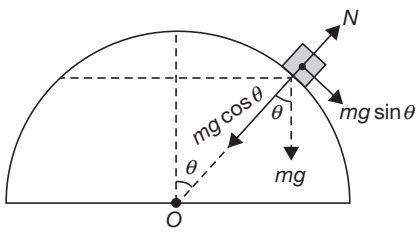
$$\Rightarrow \quad -f\left(\frac{20}{100}\right) = \frac{1}{2}\left(\frac{60}{1000}\right)\left[(300)^2 - (600)^2\right]$$

$$\Rightarrow \quad f\left(\frac{20}{100}\right) = \frac{1}{2}\left(\frac{60}{1000}\right)(10^4)(36-9)$$

$$\Rightarrow \quad f = 1.5\times10^3\times27 = 40.5\times10^3 \text{ N} = 40.5 \text{ kN}$$

Hence, the correct answer is (A).

27.



When the particle looses its contact with the sphere, then we know that

$$\cos\theta = \frac{2}{3}$$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{5}}{3}$$

So, tangential acceleration at this position is

$$a_T = g\sin\theta = \frac{\sqrt{5}g}{3}$$

Hence, the correct answer is (B).

28. The cable does not work on the bob.
So, the bob's energy is conserved

$$\Rightarrow \quad \frac{1}{2}mv_0^2 = mgh \quad \ldots(1)$$

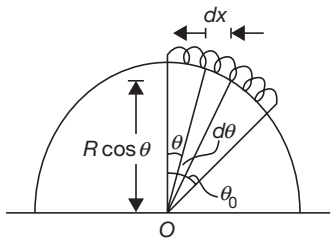
Where $h = \ell(1-\cos\theta)$ ...(2)

From equations (1) and (2), we get

$$v_0 = 7 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

29. Reference level at the centre of the sphere means gravitational potential energy is zero at centre.
Consider an element of length dx subtending an angle $d\theta$ at an angle θ with the vertical. The vertical height of this element is $R\cos\theta$.



If dU be the potential energy of the element, then $dU = (dm)gh$, where dm is mass of element.

$$\Rightarrow \quad dm = \left(\frac{m}{\ell}\right)dx = \frac{m}{\ell}(Rd\theta) \qquad \{\because dx = Rd\theta\}$$

So, $dU = \frac{m}{\ell}(Rd\theta)g(R\cos\theta) = \frac{m}{\ell}R^2g\cos\theta\,d\theta$

$$\Rightarrow \quad U = \int dU = \frac{mgR^2}{\ell}\int_0^{\theta_0}\cos\theta\,d\theta = \frac{mgR^2}{\ell}\left[\sin\theta|_0^{\theta_0}\right]$$

$$\Rightarrow \quad U = \frac{mgR^2}{\ell}\left[\sin\theta_0\right] = \frac{mgR^2}{\ell}\sin\left(\frac{\ell}{R}\right) \quad \left\{\because \theta_0 = \frac{\ell}{R}\right\}$$

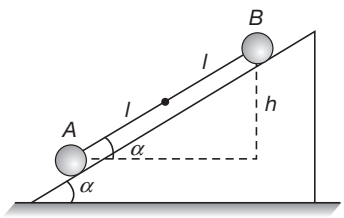
Hence, the correct answer is (A).

30. $h = 2\ell\sin\alpha$

A is the lowest point and B the highest point. At B, in critical case tension is zero. Let velocity of particle at B at this instant be v_B. Then

$$mg\sin\alpha = \frac{mv_B^2}{\ell}$$

$\Rightarrow \quad v_B^2 = g\ell\sin\alpha$



Now by Law of Conservation of Energy, we have

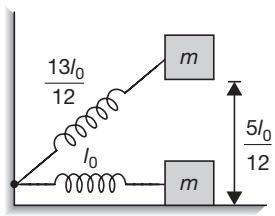
$$v_A^2 = v_B^2 + 2gh$$

$\Rightarrow \quad v_A^2 = (g\ell\sin\alpha) + 2g(2\ell\sin\alpha)$

$\Rightarrow \quad v_A = \sqrt{5g\ell\sin\alpha}$

Hence, the correct answer is (D).

31.



Elongation in spring is

$$x = \frac{13l_0}{12} - l_0 = \frac{l_0}{12}$$

Work done by lifting force is

$$W_{ext} = \Delta U + \Delta K = \Delta U_g + \Delta U_e + 0$$

$\Rightarrow \quad W_{ext} = \Delta U_g + \Delta U_e$

$\Rightarrow \quad W_{ext} = mg\left(\frac{5l_0}{12}\right) + \frac{1}{2}k\left(\frac{l_0}{12}\right)^2$

$\Rightarrow \quad W = \frac{5mgl_0}{12} + \frac{kl_0^2}{288}$

Hence, the correct answer is (A).

32. If R be the radius of the circumference, then

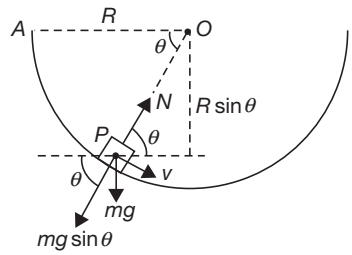
$$OP = OQ\cos 60° = (2R)\left(\frac{1}{2}\right) = R$$

$\Rightarrow \quad h_1 = OP\cos 60° = \frac{R}{2}$ and $h_2 = 2R$

$\Rightarrow \quad \frac{v_1}{v_2} = \frac{\sqrt{2gh_1}}{\sqrt{2gh_2}} = \sqrt{\frac{h_1}{h_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

Hence, the correct answer is (C).

33.



By Law of Conservation of Energy applied between A and P, we get

$$\frac{1}{2}mv^2 = mgR\sin\theta$$

$\Rightarrow \quad v = \sqrt{2gR\sin\theta}$

At point P, $N = mg\sin\theta + \frac{mv^2}{R}$

$\Rightarrow \quad N = mg\sin\theta + 2mg\sin\theta = 3mg\sin\theta$

Centripetal force is given by

$$F_C = \frac{mv^2}{R} = 2mg\sin\theta$$

So, the required ratio is

$$\frac{F_C}{N} = \frac{2}{3}$$

Hence, the correct answer is (C).

34. $a = \left(\frac{m_2 - m_1\sin\theta - \mu m_1\cos\theta}{m_1 + m_2}\right)g$

$\Rightarrow \quad a = \left\{\frac{10 - 4\sin 30 - (0.2)4\cos 30}{14}\right\}10$

$\Rightarrow \quad a = 5.2 \text{ ms}^{-2}$

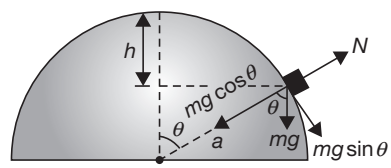
Since, $v^2 - 0^2 = 2as$

$\Rightarrow \quad v^2 = 2(5.2)(4)$

$\Rightarrow \quad v \approx 6.5 \text{ ms}^{-1}$

Hence, the correct answer is (B).

35.



$$mg\cos\theta - N = \frac{mv^2}{a}$$

The body breaks off the sphere when $N = 0$

$$\Rightarrow \quad mg\cos\theta = \frac{mv^2}{a}$$

$$\Rightarrow \quad v^2 = ga\cos\theta \qquad \ldots(1)$$

Applying Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}mv^2 = mgh$$

$$\Rightarrow \quad v^2 = 2gh \qquad \ldots(2)$$

From (1) and (2), we get

$$ga\cos\theta = 2gh$$

$$\Rightarrow \quad \cos\theta = 2h/a$$

Hence, the correct answer is (B).

36. $W = \int_{(0,\,0)}^{(1,\,1)} \vec{F}\cdot d\vec{\ell}$

where $d\vec{\ell} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$$\Rightarrow \quad W = \int_{(0,\,0)}^{(1,\,1)} \left(x^2dy + ydx\right)$$

$$\Rightarrow \quad W = \int_{(0,\,0)}^{(1,\,1)} \left(y^2dy + xdx\right) \qquad \{\because\ x = y\}$$

$$\Rightarrow \quad W = \left(\frac{y^3}{3} + \frac{x^2}{2}\right)\Bigg|_{(0,\,0)}^{(1,\,1)} = \frac{5}{6}\ \text{J}$$

Hence, the correct answer is (B).

37. $P_{\text{input}} = \dfrac{mg\left(h_2 - h_1\right)}{t} = \dfrac{(2000)(10)(500)}{1}$

$$\Rightarrow \quad P_{\text{input}} = 10\ \text{MW}$$

Since $\eta = \dfrac{P_{\text{output}}}{P_{\text{input}}}$

$$\Rightarrow \quad 0.8 = \frac{x}{10}$$

$$\Rightarrow \quad x = 8\ \text{MW}$$

Hence, the correct answer is (A).

38. For the particle just to complete the vertical circle, speed at topmost point is

$$v_0 = \sqrt{gr}$$

Speed when the string becomes horizontal is

$$v = \sqrt{u^2 + 2gr} = \sqrt{gr + 2gr} = \sqrt{3gr}$$

$$\Rightarrow \quad v = \sqrt{3}v_0$$

Tension, when string becomes horizontal, is

$$T = \frac{mv^2}{r} = \frac{m\left(\sqrt{3}v_0\right)^2}{r}$$

$$\Rightarrow \quad T = \frac{3mv_0^2}{r} = \frac{3m(gr)}{r} = 3mg$$

Hence, the correct answer is (D).

39. From Work Energy Theorem, $W = \Delta K$

$$\Rightarrow \quad Pt = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2Pt}{m}} \qquad \ldots(1)$$

$$\Rightarrow \quad v = \frac{ds}{dt} = \sqrt{\frac{2P}{m}}t^{1/2}$$

$$\Rightarrow \quad \int_0^s ds = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt$$

$$\Rightarrow \quad s = \frac{2}{3}\sqrt{\frac{2P}{m}}t^{3/2} \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{s}{v} = \frac{2}{3}t$$

$$\Rightarrow \quad \frac{s}{v} \propto t$$

i.e., graph between $\frac{s}{v}$ and t is a straight line passing through origin.

Hence, the correct answer is (B).

40. $u^2 = gL(2 + 3\sin\phi) = \dfrac{7}{2}gL$

$$\Rightarrow \quad \sin\phi = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 90° + \phi = 120°$$

Hence, the correct answer is (C).

41. The centre of mass of the rope is at a height $\frac{h}{2}$. So, total work done is the sum of work done to pull the bucket of mass M to a height h and the work done to pull the CM of the rope of mass m to a height $\frac{h}{2}$. So,

CHAPTER 1

$$W = Mgh + mg\frac{h}{2}$$

$$\Rightarrow \quad W = \left(M + \frac{m}{2}\right)gh$$

Hence, the correct answer is (B).

42. Initial velocity = final velocity $= 0$

But displacement $\Delta x \neq 0$

$\{\because \ \Delta x =$ Area under v-t graph$\}$

From Work Energy Theorem, $W = \Delta KE = 0$

Hence, the correct answer is (B).

43. $a = \dfrac{|\vec{F}|}{m} = \dfrac{5}{2} = 2.5 \text{ ms}^{-2}$

Velocity at $t = 20$ s is $v = at$

$$\Rightarrow \quad v = (2.5)(20) = 50 \text{ ms}^{-1}$$

$$\Rightarrow \quad \text{K.E.} = \frac{1}{2}(2)(2500)$$

$$\Rightarrow \quad \text{K.E.} = 2500 \text{ J}$$

Hence, the correct answer is (C).

44. From Work Energy Theorem

$$\Delta KE = W_{\text{net}}$$

$$\Rightarrow \quad K_f - K_i = \int P dt$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \int_0^2 \left(\frac{3}{2}t^2\right)dt \qquad \{\because \ m = 2 \text{ kg}\}$$

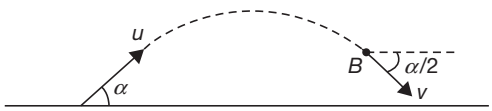
$$\Rightarrow \quad v^2 = \left(\frac{t^3}{2}\right)\Bigg|_0^2$$

$$\Rightarrow \quad v = 2 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

45. Work done by gravitational force is equal to the change in kinetic energy.

Let v be the velocity of particle at point B is v



The horizontal component of velocity remains constant, so

$$u\cos\alpha = v\cos\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \quad v = \frac{u\cos\alpha}{\cos(\alpha/2)}$$

$$\Rightarrow \quad W = \frac{1}{2}mv^2 - \frac{1}{2}m(u\cos\alpha)^2$$

$$\Rightarrow \quad W = \frac{1}{2}m\left[\frac{u^2\cos^2\alpha}{\cos^2(\alpha/2)} - u^2\cos^2\alpha\right]$$

$$\Rightarrow \quad W = \frac{1}{2}mu^2\cos^2\alpha\left[\sec^2\left(\frac{\alpha}{2}\right) - 1\right]$$

Since $1 + \tan^2\theta = \sec^2\theta$

$$\Rightarrow \quad W = \frac{1}{2}mu^2\cos^2\alpha\tan^2\left(\frac{\alpha}{2}\right)$$

Hence, the correct answer is (C).

46. Increase in P.E. is the increase in P.E. of C.M.

$$\Delta U = mg\ell_{CM}(1 - \cos\theta)$$

$$\Rightarrow \quad \Delta U = \left(\frac{100}{1000}\right)(10)\left(\frac{1}{2}\right)\left(1 - \frac{1}{2}\right)$$

$$\Rightarrow \quad \Delta U = 0.25 \text{ J}$$

Hence, the correct answer is (A).

47. $(K.E.)_{\text{man}} = \dfrac{1}{2}(\text{K.E.})_{\text{boy}}$

$$\Rightarrow \quad \frac{1}{2}Mv_m^2 = \frac{1}{2}\left[\frac{1}{2}\left(\frac{M}{2}\right)v_b^2\right]$$

$$\Rightarrow \quad v_m = \frac{v_b}{2} \qquad \ldots(1)$$

Further,

$$\frac{1}{2}M(v_m + 1)^2 = \frac{1}{2}\left(\frac{M}{2}\right)v_b^2$$

$$\Rightarrow \quad v_m + 1 = \frac{v_b}{\sqrt{2}} \qquad \ldots(2)$$

Solving (1) & (2), we get

$$v_b = 2\left(\sqrt{2} + 1\right) = 4.82 \text{ ms}^{-1}$$

$$v_m = \sqrt{2} + 1 = 2.41 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

48. Acceleration $= a = \dfrac{v_0}{t_0}$

$$P_{\text{inst}} = Fv_t = (ma)v_t = m\left(\frac{v_0}{t_0}\right)(at)$$

$$\Rightarrow \quad P_{\text{inst}} = m\left(\frac{v_0}{t_0}\right)^2 t$$

Hence, the correct answer is (D).

49. In the initial position, the length of the stretched spring is

$$L = \frac{h}{\cos 37^\circ} = \frac{5h}{4} = 1.25h$$

So, extension in the spring is

$$x = L - h = 0.25h$$

PE stored in the spring is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}k(0.25h)^2 = \frac{kh^2}{32}$$

When the spring becomes vertical, it acquires natural length and hence the PE gets converted into kinetic energy of the ring

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{kh^2}{32}$$

$$\Rightarrow \quad v = \frac{h}{4}\sqrt{\frac{k}{m}}$$

Hence, the correct answer is (D).

50. When the particle at x_1 is displaced slightly towards left or right, the force is in the same direction. Hence, the equilibrium is unstable, whereas at x_2 the force is directed opposite to displacement.

Hence, the correct answer is (B).

51. Since, $a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)g$ and $v^2 - u^2 = 2as$

$$\Rightarrow \quad v^2 - 0^2 = 2\left(\frac{m_2 - m_1}{m_2 + m_1}\right)gh$$

$$\Rightarrow \quad v = \sqrt{2\left(\frac{m_2 - m_1}{m_2 + m_1}\right)gh}$$

Hence, the correct answer is (D).

52. Acceleration down the plane is $a = g(\sin\theta - \mu\cos\theta)$

velocity of block at time t is $v = at = gt(\sin\theta - \mu\cos\theta)$

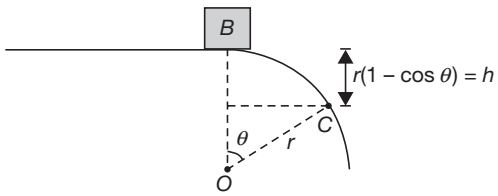
Force of friction is $f = \mu mg\cos\theta$

So, rate of work done by force of friction will be

$$P = fv = \mu mg^2 t\cos\theta(\sin\theta - \mu\cos\theta)$$

Hence, the correct answer is (C).

53.



Let velocity of block at C be v

By Law of Conservation of Energy, we get

$$\frac{1}{2}mv_0^2 + mgr(1 - \cos\theta) = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{v_0^2 + 2gr(1 - \cos\theta)}$$

When block leaves the surface, we have

$$N = 0$$

$$\Rightarrow \quad mg\cos\theta = \frac{mv^2}{r}$$

$$\Rightarrow \quad rg\cos\theta = v_0^2 + 2gr(1 - \cos\theta)$$

$$\Rightarrow \quad 3rg\cos\theta = \frac{rg}{4} + 2rg$$

$$\Rightarrow \quad 3\cos\theta = \frac{9}{4}$$

$$\Rightarrow \quad \cos\theta = \frac{3}{4}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{3}{4}\right)$$

Hence, the correct answer is (B).

54. x = Elongation in spring due to mass 10 kg

$$x = \frac{mg}{k} = \frac{10 \times 10}{100} = 1 \text{ m}$$

$$W_F = \frac{1}{2} \times 100 \times [(3)^2 - (1)^2] - 10 \times 10 \times 2 = 200 \text{ J}$$

Hence, the correct answer is (A).

55. By Law of Conservation of Energy

$$mgh = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v^2 = 2gh = 2(10)(5)$$

$$\Rightarrow \quad v = 10 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

56. $F \propto x^{-1/3}$

i.e., acceleration $a \propto x^{-1/3}$

$$\Rightarrow \quad v\frac{dv}{dx} = Kx^{-1/3}$$

$$\Rightarrow \quad v^2 \propto x^{2/3}$$

$$\Rightarrow \quad v \propto x^{1/3}$$

Now $P = Fv$

$$\Rightarrow \quad P \propto x^{-1/3} \times x^{1/3}$$

$$\Rightarrow \quad P \propto x^0$$

So, power is independent of x

Hence, the correct answer is (D).

57. Since forces are conservative in nature, so the line integral of their sum or their difference in a closed path should also be zero. Hence $W_1 = W_2 = 0$

Hence, the correct answer is (C).

CHAPTER 1

58. Three forces are acting on the particle with respect to the sphere

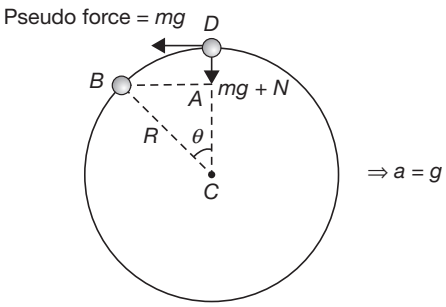
(i) pseudo force $= ma = mg$ $\{\text{as } a = g\}$

(ii) weight $= mg$

(iii) normal reaction $= N$

Of these first two forces are constant and do work. The third force is not constant but it does not perform any work. Applying Work Energy Theorem, in non-inertial frame, we get

$$\frac{1}{2}mv_r^2 = \begin{pmatrix}\text{work done by}\\ \text{pseudo force}\end{pmatrix} + \begin{pmatrix}\text{work done by}\\ \text{force of gravity}\end{pmatrix}$$



Where v_r = speed of particle relative to sphere

$$\Rightarrow \quad \frac{1}{2}mv_r^2 = mg(AB) + mg(AD)$$

$$\Rightarrow \quad \frac{1}{2}v_r^2 = g[R\sin\theta + R(1-\cos\theta)]$$

$$\Rightarrow \quad \frac{1}{2}v_r^2 = gR(1+\sin\theta-\cos\theta)$$

$$\Rightarrow \quad v_r = \sqrt{2gR(1+\sin\theta-\cos\theta)}$$

Hence, the correct answer is (D).

59. $P_{\text{output}} = \dfrac{(100)(10)(9)}{12}$

$$\Rightarrow \quad P_{\text{output}} = 750 \text{ W}$$

Since, $\eta = 0.8$

$$\Rightarrow \quad 0.8 = \frac{750}{P_{\text{input}}}$$

$$\Rightarrow \quad P_{\text{input}} = \frac{750}{0.8} = 937.5 \text{ W}$$

$$\Rightarrow \quad P_{\text{input}} \simeq 0.94 \text{ kW}$$

Hence, the correct answer is (C).

60. Let the initial amplitude decreases to a_1 to the other side i.e., after the first sweep;

$$\begin{pmatrix}\text{Decrease in Elastic}\\ \text{Potential Energy}\end{pmatrix} = \begin{pmatrix}\text{Work Done Against}\\ \text{Friction}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}ka^2 - \frac{1}{2}ka_1^2 = \mu mg(a+a_1)$$

$$\Rightarrow \quad \frac{1}{2}k(a+a_1)(a-a_1) = \mu mg(a+a_1)$$

$$\Rightarrow \quad a - a_1 = \frac{2\mu mg}{k} \qquad \ldots(1)$$

Similarly $a_1 - a_2 = \dfrac{2\mu mg}{k}$ …(2)

$$a_{n-1} - a_n = \frac{2\mu mg}{k} \qquad \ldots(3)$$

Adding all the above equations, we get

$$a - a_n = \frac{2n\mu mg}{k} \qquad \ldots(4)$$

Now, the block stops when

$$\mu mg = ka_n$$

$$\Rightarrow \quad a_n = \frac{\mu mg}{k}$$

Substituting in the above equation (4), we get

$$(2n+1)\left(\frac{\mu mg}{k}\right) = a$$

$$\Rightarrow \quad (2n+1) = \frac{ka}{\mu mg} = \frac{(20)(0.3)}{(0.04)(1)(10)} = 15$$

$$\Rightarrow \quad n = 7$$

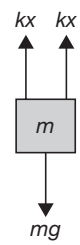
Hence, the correct answer is (C).

61. $\eta = \dfrac{P_{\text{output}}}{P_{\text{input}}} \times 100 = \dfrac{100 \times 10 \times 25}{30,000} \times 100$

$$\Rightarrow \quad \eta = 83.3\%$$

Hence, the correct answer is (C).

62. Speed is maximum at equilibrium, so when the block has descended by x, then both springs apply a force $2kx$ (upwards) on the block as shown in figure.



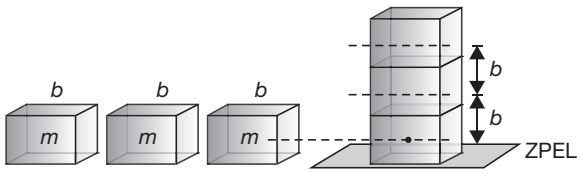
For equilibrium, we have

$$2kx = mg$$

$$x = \frac{mg}{2k}$$

Hence, the correct answer is (A).

63.



Work done = change in potential energy

$$W = mgb + mg(2b) = 3mgb$$

Hence, the correct answer is (C).

64. At break up, $F = 0$

$$\Rightarrow \quad -\frac{dU}{dx} = 0 \text{ at } x = x_0 \qquad \text{(say)}$$

$$\Rightarrow \quad \frac{d}{dx}\left(ax^{-12} - bx^{-6}\right) = 0$$

$$\Rightarrow \quad a(-12)x_0^{-13} - (b)(-6)x_0^{-7} = 0$$

$$\Rightarrow \quad x_0^6 = \frac{2a}{b}$$

$$\Rightarrow \quad U(x_0) = \frac{a}{x_0^{12}} - \frac{b}{x_0^6}$$

$$\Rightarrow \quad U(x_0) = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)}$$

$$\Rightarrow \quad U(x_0) = \frac{b^2}{4a} - \frac{b^2}{2a}$$

$$\Rightarrow \quad U(x_0) = -\frac{b^2}{4a}$$

This is the energy with which two atoms are associated. So, dissociation energy equals $\frac{b^2}{4a}$.

Hence, the correct answer is (C).

65. $a = \frac{10g - 5g - 25}{15} = \frac{100 - 50 - 25}{15} = \frac{5}{3}$ ms^{-2}

Since, $v^2 - 0^2 = 2as$

$$\Rightarrow \quad v^2 = 2\left(\frac{5}{3}\right)(2) = \frac{20}{3}$$

$$\Rightarrow \quad v = 2.6 \text{ ms}^{-1}$$

Hence, the correct answer is (A).

66. Power $P = Fv = \frac{K}{v}v = K = \text{constant}$

$$\Rightarrow \quad W = Pt = Kt$$

Hence, the correct answer is (C).

67. For month of June (having 30 days), each day motor is operated for 8 hrs. So total time (t) for which motor is operational in June is

$$t = (8)(30) = 240 \text{ hrs}$$

So, total energy consumed in the month of June is equal to work done $(1 \text{ HP} = 746 \text{ W})$

$$W = Pt = (2 \times 746)(240) \text{ watt-hour}$$

Since 1 unit of electricity $(= 1 \text{ kw-hr})$ costs ₹6,

So, bill for the month of June is

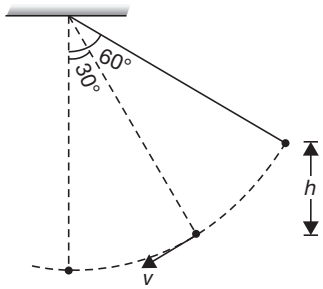
$$\frac{2 \times 746 \times 240}{1000} \times 6 = ₹2148$$

Hence, the correct answer is (A).

68. Given $\ell = 1$ m

$$\Rightarrow \quad h = \ell(\cos 30° - \cos 60°)$$

$$\Rightarrow \quad h = 1(0.86 - 0.5) = 0.36 \text{ m}$$



Since, $v = \sqrt{2gh} = \sqrt{2(10)(0.36)}$

$$\Rightarrow \quad v = 2.68 \text{ ms}^{-1}$$

Also, $P = Fv = (mg\sin\theta)v$

$$\Rightarrow \quad P = (mg\sin 30°)v = (1)(10)\left(\frac{1}{2}\right)(2.68)$$

$$\Rightarrow \quad P = 13.4 \text{ W}$$

Hence, the correct answer is (B).

69. Velocity of ball to just reach the top of the tube should be given by Law of Conservation of Energy, according to which

$$(U + K)_i = (U + K)_f$$

$$\Rightarrow \quad mgh + \frac{1}{2}mv^2 = mg(2R) + \frac{1}{2}m(0)^2$$

$$\Rightarrow \quad v = \sqrt{2g(2R - h)}$$

Hence, the correct answer is (D).

70. Since velocity is constant so the kinetic energy does not change. If W_F be the work done by external force and W_f be the work done by friction, then

CHAPTER 1

$$W_F + W_f = \Delta K = 0$$

$$\Rightarrow \quad W_F = -W_f$$

$$\Rightarrow \quad W_F = 0.5\mu_k Mgx + \mu_k Mgx + 0.5\mu_k Mgx$$

$$\Rightarrow \quad W_F = 2\mu_k Mgx$$

Hence, the correct answer is (B).

71. According to Work Energy Theorem, we have

$$W_{\text{total}} = \Delta K$$

$$\Rightarrow \quad W_{\text{ext}} + W_{nc} = \Delta U + \Delta K$$

$$\Rightarrow \quad Fx - \mu m_1 gx = \frac{1}{2}kx^2 + 0$$

$$\Rightarrow \quad Fx - \mu m_1 gx - \frac{1}{2}kx^2 = 0$$

But $kx = \mu m_2 g$, for just shifting m_2

$$\Rightarrow \quad Fx - \mu m_1 gx = \frac{1}{2}(\mu m_2 g)x$$

$$\Rightarrow \quad F = \mu\left(m_1 + \frac{m_2}{2}\right)g$$

$$\Rightarrow \quad F = 0.4\left(1 + \frac{2}{2}\right)(10) = 8 \text{ N}$$

Hence, minimum constant force is 8 N.

Hence, the correct answer is (A).

72. When the brick just reaches at the required height, the man saves the excess kinetic energy possessed by the brick.

So, percentage energy saved is

$$\frac{KE}{KE + PE} \times 100 = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2 + mgh} \times 100\%$$

$$\% \text{ saved} = \frac{v^2}{v^2 + 2gh} \times 100\%$$

$$\% \text{ saved} = \frac{(10)^2}{(10)^2 + (2)(10)(5)} \times 100 = 50\%$$

Hence, the correct answer is (D).

73. Work done = Change in potential energy $= \Delta U$

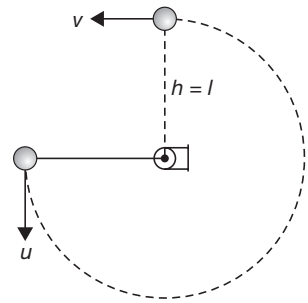
$$\Rightarrow \quad \Delta U = mgL - mgL\cos\theta$$

$$\Rightarrow \quad \Delta U = mgL(1 - \cos\theta)$$

Hence, the correct answer is (A).

74. In the critical case, velocity at topmost point should be zero i.e.,

$$v = 0$$



Using Law of Conservation of Energy, we get

$$\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgh$$

$$\Rightarrow \quad v^2 = u^2 - 2gh$$

$$\Rightarrow \quad 0 = u^2 - 2g\ell \qquad \{\because h = \ell\}$$

$$\Rightarrow \quad u = \sqrt{2g\ell}$$

Hence, the correct answer is (B).

75. For a simple pendulum, when string is vertical, tension in the string is

$$T_1 = mg + \frac{mv^2}{L}$$

In extreme position $(v = 0)$, so tension is

$$T_2 = mg\cos\theta$$

Given that, $T_1 = 2T_2$

$$\Rightarrow \quad mg + \frac{mv^2}{L} = 2mg\cos\theta$$

The speed (v) at the lowermost point is given by

$$v = \sqrt{2gL(1 - \cos\theta)}$$

$$\Rightarrow \quad mg + \frac{m}{L}\left[2gL(1 - \cos\theta)\right] = 2mg\cos\theta$$

$$\Rightarrow \quad 1 + 2 - 2\cos\theta = 2\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{3}{4}$$

Hence, the correct answer is (D).

76. Total mechanical energy = mgH

Given that $\frac{PE}{KE} = \frac{1}{2}$

$$\Rightarrow \quad PE = \frac{1}{3}mgH \text{ and } KE = \frac{2}{3}mgH$$

So, height from the ground at this instant is $h = \frac{H}{3}$ and speed of particle is

$$v = \sqrt{2g\left(\frac{2H}{3}\right)} = 2\sqrt{\frac{gH}{3}}$$

Hence, the correct answer is (C).

77. Decrease in potential energy = Increase in kinetic energy

$$\Rightarrow \quad \frac{1}{2}mv^2 = mg\left(\ell - \frac{\ell}{2}\right)$$

$$\Rightarrow \quad v = \sqrt{g\ell}$$

Hence, the correct answer is (B).

78. If x_i be the compression in the spring initially, then, in equilibrium,

$$F + m_1 g = kx_i$$

$$\Rightarrow \quad x_i = \frac{F + m_1 g}{k} \qquad \ldots(1)$$

Let x_f be the final elongation in spring when lower block just lifts. For this to happen, we have

$$kx_f = m_2 g$$

$$\Rightarrow \quad x_f = \frac{m_2 g}{k} \qquad \ldots(2)$$

Now by Law of Conservation of Energy

$$\begin{pmatrix}\text{Increase in}\\ \text{Gravitational}\\ \text{Potential}\\ \text{Energy of } m_1\end{pmatrix} = \begin{pmatrix}\text{Decrease in}\\ \text{Elastic Potential}\\ \text{Energy of Spring}\end{pmatrix}$$

$$\Rightarrow \quad m_1 g\left(x_i + x_f\right) = \frac{1}{2}k\left(x_i^2 - x_f^2\right)$$

Substituting the values of x_i and x_f from equations (1) and (2), we get

$$F = \left(m_1 + m_2\right)g$$

Hence, the correct answer is (C).

79. $W_{nc} = W_{\text{friction}} = \Delta U + \Delta K$

$$\Delta U = (\Delta U)_{A\to B} + (\Delta U)_{B\to C}$$

$$\Rightarrow \quad \Delta U = U_B - U_A + U_C - U_B$$

$$\Rightarrow \quad \Delta U = U_C - U_A = 0 - (2)(10)(1)$$

$$\Rightarrow \quad \Delta U = -20 \text{ J}$$

$$\Rightarrow \quad \Delta K = (\Delta K)_{A\to B} + (\Delta K)_{B\to C}$$

$$\Rightarrow \quad \Delta K = K_B - K_A + K_C - K_B$$

$$\Rightarrow \quad \Delta K = K_C - K_A = 0 - 0$$

$$\Rightarrow \quad \Delta K = 0$$

So, $W_{nc} = -20$ J

Hence, the correct answer is (B).

80. Since, $\frac{3}{4}$ th energy is lost i.e., $\frac{1}{4}$ th kinetic energy is left with the particle. Hence, its velocity becomes $\frac{v_0}{2}$ under a retardation of μg in time t_0.

$$\Rightarrow \quad \frac{v_0}{2} = v_0 - \mu g t_0$$

$$\Rightarrow \quad \mu g t_0 = \frac{v_0}{2}$$

$$\Rightarrow \quad \mu = \frac{v_0}{2gt_0}$$

Hence, the correct answer is (B).

81. $F_{down} = mg\sin\theta = \frac{3}{5}mg$

$$f_{\max} = \mu mg\cos\theta = \left(\frac{3}{4}\right)\left(\frac{4}{5}\right)mg = \frac{3}{5}mg$$

$$\Rightarrow \quad f_{\max} = mg\sin\theta = \frac{3}{5}mg$$

Block will move upwards when

$$kx_0 = f_{\max} + mg\sin\theta = \frac{6}{5}mg \qquad \ldots(1)$$

By Law of Conservation of Energy, we have

$$Mgx_0 = \frac{1}{2}kx_0^2$$

$$\Rightarrow \quad M = \frac{kx_0}{2g} = \frac{\frac{6}{5}mg}{2g} = \frac{3\text{ m}}{5}$$

Hence, the correct answer is (A).

82. Retardation $\propto x$

$$\Rightarrow \quad a = -kx$$

$$\Rightarrow \quad v\frac{dv}{dx} = -kx$$

$$\Rightarrow \quad vdv = -kx\,dx$$

$$\Rightarrow \quad \int v\,dv = -k\int x\,dx$$

$$\Rightarrow \quad \frac{v^2}{2} = -\frac{kx^2}{2}$$

$$\Rightarrow \quad \frac{mv^2}{2} = K.E. = -km\frac{x^2}{2}$$

$$\Rightarrow \quad K.E. \propto x^2$$

Hence, the correct answer is (B).

CHAPTER 1

83. Potential energy of particle at $x=\sqrt{\frac{2E}{k}}$ i.e., for $x>0$ is zero

So, Total Energy = Kinetic Energy $=E$

$\Rightarrow \quad \frac{1}{2}mv^2=E$

$\Rightarrow \quad v=\sqrt{\frac{2E}{m}}$

Hence, the correct answer is (B).

84. $P=(F_{\text{net}})v$

where

$F_{\text{net}}=mg+0.1mg+0.02mg$

$\Rightarrow \quad F_{\text{net}}=1.12\text{mg}$

$\Rightarrow \quad P=(1.12)(1000)(10)(10)$

$\Rightarrow \quad P=112\text{ kW}$

Hence, the correct answer is (A).

85. Unit vector corresponding to $\hat{i}+\hat{j}+\hat{k}$ is $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

So, $\vec{F}=\frac{30}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)=10\sqrt{3}\left(\hat{i}+\hat{j}+\hat{k}\right)$

Since, $W=\vec{F}\cdot\Delta\vec{r}=\vec{F}\cdot\left(\vec{r}_2-\vec{r}_1\right)$

$\Rightarrow \quad W=10\sqrt{3}\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\left(2\hat{i}+\hat{k}\right)$

$\Rightarrow \quad W=10\sqrt{3}(2+1)=30\sqrt{3}\ J$

Hence, the correct answer is (B).

86. By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in Gravitational}\\ \text{Potential Energy}\end{pmatrix}=\begin{pmatrix}\text{Gain in Elastic}\\ \text{Potential Energy}\end{pmatrix}$$

$\Rightarrow \quad mg(h+x)=\frac{1}{2}kx^2$

Putting values to solve the Quadratic Equation, we get

$x=0.1$ m

Hence, the correct answer is (A).

87. Tension is maximum at the lowest point, so

$T_L=\frac{m}{r}\left(u^2+gr\right)$

Tension is minimum at the highest point, so

$T_H=\frac{m}{r}\left(u^2-5gr\right)$

Given that $\frac{T_L}{T_H}=4$

$\Rightarrow \quad \frac{T_L}{T_H}=\frac{\frac{m}{r}\left(u^2+gr\right)}{\frac{m}{r}\left(u^2-5gr\right)}=4$

$\Rightarrow \quad 4u^2-20gr=u^2+gr$

$\Rightarrow \quad 3u^2=21gr$

$\Rightarrow \quad u=\sqrt{7gr}$

Since $v^2=u^2-2gh$

$\Rightarrow \quad v=\sqrt{7gr-2g(2r)}=\sqrt{3gr}$

$\Rightarrow \quad v=\sqrt{3(10)\left(\frac{10}{3}\right)}=10\text{ ms}^{-1}$

Hence, the correct answer is (D).

88. Velocity of bob at the lowest point just before hitting the pin is

$v^2=2gL(1-\cos\alpha)$

It will rise further to a height h (say), then by Law of Conservation of Energy, we get

$mgh=\frac{1}{2}mv^2$

$\Rightarrow \quad h=\frac{v^2}{2g}=L(1-\cos\alpha)$

Now $h=(L-\ell)(1-\cos\theta)$

$\Rightarrow \quad (L-\ell)(1-\cos\theta)=L(1-\cos\alpha)$

$\Rightarrow \quad \theta=\cos^{-1}\left(\frac{L\cos\alpha-\ell}{L-\ell}\right)$

Hence, the correct answer is (C).

89. $W_{ext}=W_{F_0}=\int_0^{\frac{\pi}{2}}\vec{F}_0\cdot d\vec{r}$

$\Rightarrow \quad W_{F_0}=\frac{\pi F_0 l}{2}$

Applying Law of Conservation of Mechanical Energy, we get

$\frac{\pi F_0 l}{2}=\frac{1}{2}mv_1^2+mgl$

$\Rightarrow \quad v_1=\sqrt{\frac{l}{m}\left(\pi F_0-2mg\right)}$

Hence, the correct answer is (A).

90. In this case, the particle has both tangential and centripetal accelerations. Given that

$a_T=\alpha t$

$$\Rightarrow \quad \frac{dv}{dt} = \alpha t$$

$$\Rightarrow \quad \int_0^v dv = \alpha \int_0^t t\,dt$$

$$\Rightarrow \quad v = \frac{\alpha t^2}{2}$$

Since, $a_{\text{total}} = \sqrt{a_C^2 + a_T^2}$

Since a_{total} is equally inclined to a_c and a_T, so $\beta = 45°$

$$\Rightarrow \quad \tan\beta = \tan 45° = \frac{a_T}{a_C} = 1$$

$$\Rightarrow \quad \frac{\alpha t}{\left(\frac{v^2}{r}\right)} = \frac{\alpha t \times 4 \times 2}{\alpha^2 t^4} = 1$$

At $t = 2$ s, we have

$$\frac{\alpha(2) \times 4 \times 2}{\alpha^2 (2)^4} = 1$$

$$\Rightarrow \quad \alpha = 1 \text{ ms}^{-3}$$

Hence, the correct answer is (B).

91. Since the car is moving with a constant velocity, so Applied Force = Force of Friction

$$\Rightarrow \quad F = 50 \text{ N}$$

$$\Rightarrow \quad W = Fs$$

$$\Rightarrow \quad W = (50)(1000)$$

$$\Rightarrow \quad W = 5 \times 10^4 \text{ J}$$

Hence, the correct answer is (A).

92. When it is lowered gradually then to attain the equilibrium, we have

$$mg = kd \qquad \ldots(1)$$

Further when it is allowed to fall suddenly, and if x is the maximum stretching in the spring, then by Law of Conservation of Energy

$$\begin{pmatrix}\text{Loss in Gravitational}\\ \text{Potential Energy}\\ \text{of Body}\end{pmatrix} = \begin{pmatrix}\text{Gain in Elastic}\\ \text{Potential Energy}\\ \text{of Spring}\end{pmatrix}$$

$$\Rightarrow \quad mgx = \frac{1}{2}kx^2$$

$$\Rightarrow \quad mg = \frac{1}{2}kx \qquad \ldots(2)$$

Equating (1) and (2), we get

$$x = 2d$$

Hence, the correct answer is (B).

93. By Law of Conservation of Energy

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}(4)(1.6)^2 = \frac{1}{2}(2)x^2$$

$$\Rightarrow \quad x = 2.26 \text{ m}$$

Hence, the correct answer is (C).

94. $a = g(\sin 30 - \mu\cos\theta)$

$$\Rightarrow \quad a = 9.8\left(\frac{1}{2} - 0.2\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad a = 9.8(0.3268)$$

$$\Rightarrow \quad a = 3.2 \text{ ms}^{-2}$$

Since $v^2 - (10)^2 = 2(3.2)(10)$

$$\Rightarrow \quad v^2 = 164$$

$$\Rightarrow \quad v \cong 13 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

95. Speed will be maximum at equilibrium, i.e., when

$$F = kx$$

$$\Rightarrow \quad x = \frac{F}{k}$$

Applying $MWET$, we get

$$W_{ext} = \Delta U + \Delta K$$

$$\Rightarrow \quad Fx = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad F\left(\frac{F}{k}\right) = \frac{1}{2}k\left(\frac{F}{k}\right)^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{F^2}{2k}$$

$$\Rightarrow \quad v = \frac{F}{\sqrt{mk}}$$

Hence, the correct answer is (A).

96. $\begin{pmatrix}\text{Loss in Kinetic}\\ \text{Energy}\end{pmatrix} = \begin{pmatrix}\text{Gain in Gravitational}\\ \text{Potential Energy}\end{pmatrix}$

$$\Rightarrow \quad \frac{1}{2}mV_0^2 - \frac{1}{2}mV^2 = mgh$$

$$\Rightarrow \quad V^2 = V_0^2 - 2gh$$

$$\Rightarrow \quad V = \sqrt{V_0^2 - 2gh}$$

Hence, the correct answer is (C).

CHAPTER 1

97. Potential energy of cube at position 1,

$$U_1 = Mg(4R) = 4MgR$$

Applying Law of Conservation of Energy between 1 and 2, we get

$$K_1 + U_1 = K_2 + U_2$$

$$\Rightarrow \quad 4MgR = 2MgR + \frac{1}{2}Mv^2$$

$$\Rightarrow \quad 4MgR = Mv^2$$

$$\Rightarrow \quad v^2 = 4gR$$

Since, $N = \frac{Mv^2}{R} - Mg$

$$\Rightarrow \quad N = \frac{M}{R}(4Rg) - Mg = 3Mg$$

Hence, the correct answer is (A).

98. Let l be the length of spring at the situation shown in Figure.

$$\Rightarrow \quad \cos(37°) = \frac{h}{l}$$

$$\Rightarrow \quad l = \frac{h}{\cos(37°)} = \frac{5h}{4}$$

Extension x in spring is

$$x = l - h = \frac{5h}{4} - h = \frac{h}{4}$$

By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in EPE}\\ \text{of springs}\end{pmatrix} = \begin{pmatrix}\text{Gain in KE}\\ \text{of Bead}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}kx^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad 2\left(\frac{1}{2}(1000)\frac{h^2}{16}\right) = \frac{1}{2}(5)v^2$$

$$\Rightarrow \quad v^2 = 25h^2$$

$$\Rightarrow \quad v = 5h$$

Hence, the correct answer is (A).

99. Speed of ball is maximum at equilibrium, so

$$x_{eq} = \frac{mg}{k}$$

Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. According to Work-Energy Theorem for a non-conservative system, we have

$$W_c + W_{nc} = \Delta k \qquad \{\text{OPTION } \textbf{(C)}\}$$

Also, we know that the work done by a conservative force equals the decrease in potential energy, so

$$W_c = -\Delta U \qquad \{\text{OPTION } \textbf{(B)}\}$$

$$\Rightarrow \quad W_{nc} - \Delta U = \Delta k \qquad \{\text{OPTION } \textbf{(A)}\}$$

Hence, (A), (B) and (C) are correct.

2. Tension in the string will be maximum when mass m is at point B, so

$$T = mg + \frac{mv^2}{\ell}$$

By Law of Conservation of Energy,

$$\frac{1}{2}mv^2 = mgh = mg\ell(1 - \cos\theta)$$

$$\Rightarrow \quad v^2 = 2g\ell(1 - \cos(60°))$$

$$\Rightarrow \quad T = mg + \frac{2mg\ell}{\ell}\left(1 - \frac{1}{2}\right)$$

$$\Rightarrow \quad T = 2mg$$

For mass $4m$ to be stationary, $f \geq T$

$$\Rightarrow \quad \mu(4mg) \geq 2mg$$

$$\Rightarrow \quad \mu \geq 0.5$$

Hence, **(A)** is correct

When m moves from A to B it has vertical displacement downwards, i.e., along the direction of gravitational force.

So, **(B)** is correct.

Tension is always perpendicular to the velocity, so power delivered by the tension is zero.

So, **(C)** is correct.

According to Work Energy Theorem,

$$W_{\text{total}} = \Delta K$$

Since work done by tension is zero therefore work is done only by the gravitational force. So, **(D)** is also correct.

Hence, (A), (B), (C) and (D) are correct.

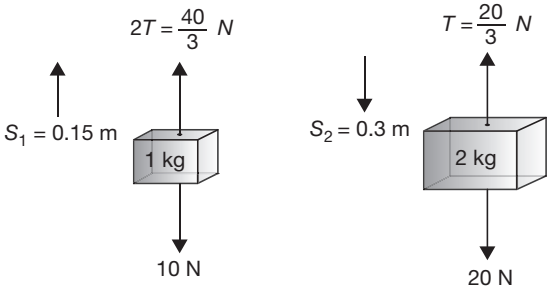
3. P = (Total Frictional Force) (velocity)

$$\Rightarrow \quad P = (mf)v$$

$$\Rightarrow \quad P_{extra} = mg(\sin\theta)v = \frac{mghv}{s}$$

Hence, (A), (B) and (C) are correct.

4. Using the concepts of Laws of Motion and Kinematics we can draw following diagrams and then using the definition of work, we can find corresponding work done.



Hence, (A), (B), (C) and (D) are correct.

5. For a light string pulled by a constant

Force $F = 40$ N, tension (T) in the string is also 40 N.
So, $T = 40$ N

Since $W = 40$ J $= Fs$ where $F = 40 - 20$ N $= 20$ N

$\Rightarrow \quad 40 = 20\ s$

$\Rightarrow \quad s = 2$ m

Since acceleration is constant, so $v \propto t$ and hence $P = Fv$

$\Rightarrow \quad P \propto t$

Also, $v = \sqrt{2ax}$

$\Rightarrow \quad P \propto \sqrt{x}$.

So, all options (A), (B), (C) and (D) are correct.
Hence, (A), (B), (C) and (D) are correct.

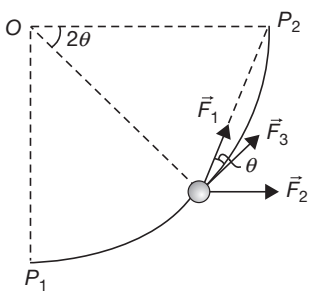
6. Work done by $\vec{F}_1$ is

$$W_1 = \int_{P_1}^{P_2} F_1 \cos\theta ds$$

Where, $ds = (6)d(-2\theta) = -12d\theta$ and $F_1 = 20$ N

$$\Rightarrow \quad W_1 = -240 \int_{\pi/4}^{0} \cos\theta d\theta$$

$$\Rightarrow \quad W_1 = 240 \sin\left(\frac{\pi}{4}\right) = 120\sqrt{2} \text{ J}$$



$\vec{F}_1$ is conservative because it is always directed towards a fixed point P_2. Therefore, W_1 can be directly calculated as

$$W_1 = F_1(P_1P_2) = (20)(6\sqrt{2}) = 120\sqrt{2} \text{ J}$$

Similarly, $W_2 = F_2(OP_2) = (30)(6) = 180$ J

and $W_3 = \int_0^{6(\pi/2)} F_3 ds = \int_0^{3\pi} 15 ds$

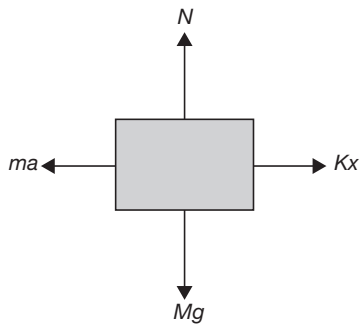
$$W_3 = 15s\big|_0^{3\pi} = 45\pi \text{ J}$$

Hence, (A), (C) and (D) are correct.

7. According to Work Energy Theorem, work done by all the forces is equal to change in kinetic energy, so

$$W_{\text{total}} = \Delta K$$

$$\Rightarrow \quad W_{\text{ext}} + W_{nc} = \Delta U + \Delta K$$



Since, $W_{nc} = 0$, because friction is absent

So, $W_{\text{ext}} = \Delta U + \Delta K$

$$\Rightarrow \quad (Ma)x_{\max} = \frac{1}{2}kx_{\max}^2$$

$$\Rightarrow \quad x_{\max} = \frac{2Ma}{k}$$

The block will execute SHM and its initial position will be the one of the extreme positions i.e., block returns to its initial position after maximum elongation in the spring and compression in the spring is zero.
So, **(D)** is also correct.
Hence, (B) and (D) are correct.

8. $$W = \int \vec{F} \cdot d\vec{r} = \int \left(\hat{i}F_x + \hat{j}F_y\right) \cdot \left(\hat{i}dx + \hat{j}dy\right)$$

$$W = \int \left(7\hat{i} - 6\hat{j}\right) \cdot \left(\hat{i}dx + \hat{j}dy\right)$$

$$\Rightarrow \quad W = 7\int_0^{-3} dx - 6\int_0^{4} dy = -21 - 24 = -45 \text{ J}$$

CHAPTER 1

Displacements along the direction of X and Y axes are opposite to the direction of force acting along the respective directions, hence the particle must have some velocity at $(0, 0)$.

Hence, (B) and (D) are correct.

9. In region AB, $s \propto t$

$\Rightarrow$ $v =$ constant

$\Rightarrow$ $\Delta K = 0$

$\Rightarrow$ $W_{A\to B} = \Delta K = 0$

Similarly, from B to C, slope of s-t graph is decreasing, so v is also decreasing and hence

$W = \Delta K =$ NEGATIVE

Hence, (B) and (C) are correct.

10. Let m be the mass of the block
Initial elongation of the spring will be

$$x_i = \frac{mg}{k} \qquad \ldots(1)$$

On application of F work done by F and gravity is used to increase the elastic potential energy of spring. So,

$$(F + mg)x_0 = \frac{1}{2}k(x_i + x_0)^2 - \frac{1}{2}kx_i^2 \qquad \ldots(2)$$

From (1) and (2) we get,

$$x_0 = \frac{2F}{k}$$

Work done by applied force F is $W = Fx_0$

Hence, (C) and (D) are correct.

11. Since $P = \vec{F}\cdot\vec{v}$. At 1 s and 3 s, P is positive. Hence, angle is acute. At 7 s power is negative. Hence, angle is obtuse.
Area under P-t graph is the work done which equals the change in kinetic energy.

Hence, (A), (C) and (D) are correct.

12. In a conservative field, the work done W_C by the conservative force is always equal to the decrease in potential energy.

$\Rightarrow$ $W_c = U_{\text{initial}} - U_{\text{final}} = -\Delta U$

$\Rightarrow$ $W_c = U_A - U_B$

Decrease in potential energy implies $U_A > U_B$

Hence, (B) and (C) are correct.

13. **Conceptual (B) and (D) are correct.**

14. By Work-Energy Theorem

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \vec{F}\cdot\vec{s}$$

K.E. increasing with time means $v > u$

$\Rightarrow$ Either $\theta = 0°$

OR $0 < \theta \le 90°$ i.e. θ is acute

OR $P = \dfrac{W}{t} \neq 0$, because

$W \neq 0$

Since, K.E. is increasing, so P.E. must be decreasing and hence height above the ground must be decreasing.

Hence, (A), (C), (D) and (E) are correct.

15. Force on the particle is given by

$$\vec{F} = -\nabla U$$

$\Rightarrow$ $\vec{F} = -\dfrac{\partial U}{\partial x}\hat{i} - \dfrac{\partial U}{\partial y}\hat{j} = -\left(6xy^2 + 6\right)\hat{i}$

Now, for initial acceleration i.e. $t = 0$ and $x = 1$, $y = 1$, we have

$$\vec{a} = \frac{\vec{F}}{M} = 12\hat{i} - 6\hat{j}$$

$\Rightarrow$ $|\vec{a}| = 6\sqrt{5}$ ms^{-2}

Since particle is at rest at $x = 1$, $y = 1$ so, the total mechanical energy (E) is

$E = U + K$

$\Rightarrow$ $U(1, 1) + K(1, 1) = E$

$\Rightarrow$ $E = 9$ J

According to modified work-Energy Theorem, we have

$W_{\text{ext}} = \Delta U + \Delta K$

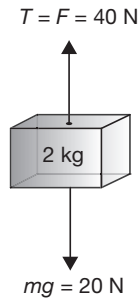
Since the particle is moving slowly, so $\Delta K = 0$

$\Rightarrow$ $W_{\text{ext}} = \Delta U = U(0, 0) - U(1, 1)$

$\Rightarrow$ $W_{\text{ext}} = -9$ J

Hence, (A) and (C) are correct.

16. Free body diagram of block is as shown in figure.



From Work Energy Theorem

$W_{\text{net}} = \Delta$KE

$\Rightarrow \quad (40-20)s = 40$

$\Rightarrow \quad s = 2 \text{ m}$

Work done by gravity is

$W_g = -20 \times 2 = -40 \text{ J}$

and work done by tension is

$W_T = 40 \times 2 = 80 \text{ J}$

Hence, (A), (B) and (D) are correct.

17. From 0 to t_1, slope is increasing, so speed is increasing. Hence, work done is positive.

From t_1 to t_2, slope is decreasing, so speed is decreasing. Hence, work done is negative.

From t_2 to t_3, slope of s-t graph is zero, so speed is zero. Hence, work done is zero.

From t_3 to t_4, slope is constant, so speed is constant. Hence, work done is zero.

Hence, (A), (B) and (C) are correct.

18. As soon as the block hits the wall, the suspension point B comes to a stop, whereas the particle C keeps moving with a velocity v_0 towards left. In order that it complete a full circle, it must have kinetic energy enough to take it to the top of the circle.

$\Rightarrow \quad \frac{1}{2}mv_0^2 = mg(2l)$

$\Rightarrow \quad v_0 = \sqrt{4gl}$

Hence, (A) and (C) are correct.

19. **Conceptual (B), (C) and (D) are correct.**

20. $K.E. \propto t$

$\Rightarrow \quad \frac{1}{2}mv^2 = kt$

$\Rightarrow \quad v = \sqrt{\frac{2k}{m}}t^{1/2} \qquad \ldots(1)$

$\Rightarrow \quad a = \frac{dv}{dt} = \sqrt{\frac{2k}{m}}\left(\frac{1}{2\sqrt{t}}\right)$

Since $F = ma$

$\Rightarrow \quad F \propto \frac{1}{\sqrt{t}}$

From (1), we get

$\frac{1}{\sqrt{t}} = \sqrt{\frac{2k}{m}}\frac{1}{v}$

$\Rightarrow \quad a = \sqrt{\frac{2k}{m}}\left(\frac{1}{2}\sqrt{\frac{2k}{m}}\frac{1}{v}\right) = \frac{2k}{m}\left(\frac{1}{2}\right)\left(\frac{1}{v}\right)$

$\Rightarrow \quad a = \frac{k}{mv}$

$\Rightarrow \quad F \propto \frac{1}{v}$

Hence, (B) and (D) are correct.

21. Motion of block is non-uniform circular motion, so, acceleration is not constant throughout.

At B:

$v^2 = 2gR$

$\Rightarrow \quad a = \frac{v^2}{R} = 2g$

Hence, (A) and (C) are correct.

22. $\vec{F} = -\vec{\nabla}U$

$\Rightarrow \quad \vec{F} = 7\hat{i} - 24\hat{j}$

$\Rightarrow \quad m\vec{a} = 7\hat{i} - 24\hat{j}$

$\Rightarrow \quad \vec{a} = \frac{7}{5}\hat{i} - \frac{24}{5}\hat{j}$

$\Rightarrow \quad |\vec{a}| = 5 \text{ ms}^{-2}$

Also, $\vec{a} \cdot \vec{u} = 0$

$\Rightarrow \quad \vec{a} \perp \vec{u}$

Further we have

$\vec{v} = \vec{u} + \vec{a}t$

$\Rightarrow \quad \vec{v} = \left(14.4\hat{i} + 4.2\hat{j}\right) + \left(\frac{7}{5}\hat{i} - \frac{24}{5}\hat{j}\right)4$

$\Rightarrow \quad \vec{v} = \left(14.4\hat{i} + 4.2\hat{j}\right) + \left(\frac{28}{5}\hat{i} - \frac{96}{5}\hat{j}\right)$

$\Rightarrow \quad \vec{v} = \left(14.4\hat{i} + 4.2\hat{j}\right) + \left(5.6\hat{i} - 19.2\hat{j}\right)$

$\Rightarrow \quad \vec{v} = 20\hat{i} - 15\hat{j}$

$\Rightarrow \quad |\vec{v}| = \sqrt{20^2 + 15^2} = 25 \text{ ms}^{-1}$

Hence, (A), (B) and (C) are correct.

23. By Law of Conservation of Energy, we have

$\frac{1}{2}mu^2 + 0 = 0 + mg\ell(1 - \cos\theta)$

Since $1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$

$\Rightarrow \quad u^2 = 2g\ell\left[2\sin^2\left(\frac{\theta}{2}\right)\right]$

$\Rightarrow \quad \frac{\theta}{2} = \sin^{-1}\left(\frac{u}{2\sqrt{g\ell}}\right)$

CHAPTER 1

$$\Rightarrow \quad \theta = 2\sin^{-1}\left(\frac{u}{2\sqrt{g\ell}}\right)$$

Hence, (B) and (C) are correct.

24. At the highest point, if T' and v' be the respective tension and velocity, then

$$mg + T' = \frac{mv'^2}{\ell}$$

Since $T' = 2mg$

$$\Rightarrow \quad \frac{mv'^2}{\ell} = mg + 2mg$$

$$v' = \sqrt{3g\ell}$$

By Law of Conservation of Energy

$$\frac{1}{2}mv'^2 + mg(2\ell) = \frac{1}{2}mv^2$$

where v is the velocity at the lowest point

$$\Rightarrow \quad v = \sqrt{7g\ell}$$

Hence, (B) and (D) are correct.

25. $W_{Sp} = U_i - U_f = -\frac{1}{2}k\left(x_2^2 - x_1^2\right)$

Since $W_{Sp} = \frac{1}{2}kx^2$

So, $x_2 = 0$ and $x_1 = x$ i.e., either the spring was initially stretched by a distance x and finally was in its natural length or the spring was initially compressed by a distance x and finally was in its natural length.

Hence, (A) and (C) are correct.

26. In equilibrium $F = 0$

Since, $F = -\frac{dU}{dr}$

$$\Rightarrow \quad \frac{dU}{dr} = 0$$

Either U is zero or it is a non-zero constant.
So, for equilibrium, we have

Either $F = 0$, $U = 0$

or $F = 0$, U is a constant (but not zero).

Hence, (A) and (C) are correct.

27. In simple harmonic motion, the body is accelerated at all points except at mean position.
At the starting point, spring force is zero. The only force is mg (downwards). Hence, acceleration is maximum, i.e., g (downwards). From mean position to initial position, retardation is due to forces, spring force and force of gravity. When spring is detached only gravity is left i.e., retardation has reduced and so the block rises to a height greater than from where it was released more height.

Hence, (A), (C) and (D) are correct.

28. $\vec{a} = \frac{\vec{F}}{m} = -\frac{1}{m}\left(\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j}\right)$ $\{m = 1 \text{ kg}\}$

$$\vec{a} = \frac{\vec{F}}{m} = -\left(3\hat{i} + 4\hat{j}\right) = \text{constant}$$

$$\Rightarrow \quad a_x = -3 \text{ ms}^{-2} \text{ and } a_y = -4 \text{ ms}^{-2}$$

$$\Rightarrow \quad a = \sqrt{a_x^2 + a_y^2} = 5 \text{ ms}^{-2} = \text{constant}$$

Hence, **(A)** is correct

$$v_x = -3t \text{ and } v_y = -4t$$

$$x = 6 + \frac{1}{2}a_x t^2 = 6 - \frac{3}{2}t^2$$

and $y = 4 + \frac{1}{2}a_y t^2 = 4 - 2t^2$

Particle crosses the y-axis when

$$x = 0 \text{ or } t = 2 \text{ s}$$

speed of particle at this moment

$$v = at = 5 \times 2 = 10 \text{ ms}^{-1}$$

co-ordinates of particle at $t = 1$ s are

$$(x, y) = (4.5, 2)$$

Hence **(C)** and **(D)** are also correct.

Hence, (A), (C) and (D) are correct.

29. Work done against friction on ice is zero and work done against friction on the road is $(\mu mg)\ell$. So, average work done is

$$\frac{0 + (\mu mg)\ell}{2} = (\mu mg)\frac{\ell}{2}$$

Thus, indicating that the effective length of the sledge that has to be dragged so that it just gets completely on the road is $\frac{\ell}{2}$.

Distance covered by the sledge on the road before coming to rest is $\frac{v_0^2}{2\mu g}$. $\left\{\because 0^2 - v_0^2 = 2(-\mu g)s\right\}$

So total distance moved by the sledge is

$$\left(\frac{v_0^2}{2\mu g} + \frac{\ell}{2}\right)$$

Distance covered by the sledge on the road is

$$\ell-\left(\frac{v_0^2}{2\mu g}+\frac{\ell}{2}\right)=\left(\frac{v_0^2}{2\mu g}-\frac{\ell}{2}\right)$$

Hence, (B), (C) and (D) are correct.

30. In uniformly accelerated motion

$$v=u+at$$

and $v^2=u^2+2as$

$$\Rightarrow \quad v=\sqrt{u^2+2as}$$

So, power, $P=Fv=F(u+at)$

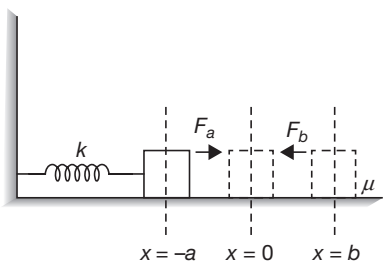
also $P=F\sqrt{u^2+2as}$

i.e., power varies linearly with time and parabolically with displacement.

So, **(B)** and **(C)** are incorrect

Hence, (B) and (C) are correct.

31.



Work done by spring on block is

$$W=\frac{1}{2}ka^2-\frac{1}{2}kb^2=\frac{1}{2}k(a^2-b^2)$$

According to *MWET*, we have $W_{nc}=\Delta U+\Delta K$

Since $\Delta K=0$

$$\Rightarrow \quad -\mu mg(a+b)=\frac{1}{2}k(b^2-a^2)$$

$$\Rightarrow \quad \mu mg(a+b)=\frac{1}{2}k(a^2-b^2)$$

$$\Rightarrow \quad \mu=\frac{k(a-b)}{2mg}$$

Hence, (B) and (C) are correct.

32. Work done by spring in parts (A) and (C) is $\frac{1}{2}kx^2$ and work done by spring in parts (B) and (D) is $-\frac{1}{2}kx^2$.

Hence, (A) and (C) are correct.

33. At maximum extension, x, we have

$$mgx=\frac{1}{2}kx^2$$

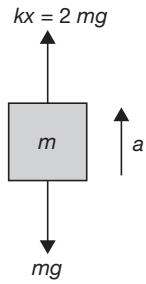
$$\Rightarrow \quad x=\frac{2mg}{k}$$

When $x'=\frac{x}{2}$, $F_{net}=0$

When $x'=x$, then

$$kx-mg=ma$$

$$\Rightarrow \quad a=g \text{ (upwards)}$$



Hence, (B), (C) and (D) are correct.

34. $\vec{F}=-\vec{\nabla}U=-\left(\hat{i}\frac{\partial U}{\partial x}+\hat{j}\frac{\partial U}{\partial y}+\hat{k}\frac{\partial U}{\partial z}\right)$

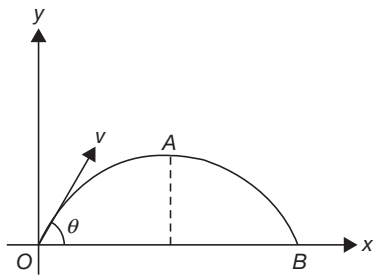
$$\Rightarrow \quad \vec{F}=k\left(y\hat{i}+x\hat{j}\right)$$

Work done by this force field is independent of the path followed between any two points and hence this force field is conservative in nature.

Hence, (A) and (C) are correct.

35. Power, $P=\vec{F}\cdot\vec{v}$

Between O and A angle between $\vec{F}(=m\vec{g})$ and $\vec{v}$ is greater than $90°$. Hence, power is negative. At A angle is $90°$. Hence, power is zero. From A to B angle is less than $90°$ therefore, power is positive.



At any instant, t

$$\vec{F}=-mg\,\hat{j}$$

$$\vec{v}=(u\cos\theta)\hat{i}+(u\sin\theta-gt)\hat{j}$$

$$\Rightarrow \quad P=\vec{F}\cdot\vec{v}=mg(gt-u\cos\theta)$$

$$\Rightarrow \quad P=mg(gt-u\cos\theta)$$

i.e., P varies linearly with t

Therefore, OPTIONS **(B)** and **(D)** are correct

Hence, (B) and (D) are correct.

36. Momentum, $p=\sqrt{2mK}$

Therefore, without knowing the mass we cannot compare the momentum.

From Work Energy Theorem, more work will be done to stop B.

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Distance travelled before stopping is $s=\frac{v^2}{2a}$, where $a=\mu g$.

Again, distance cannot be compared unless v is known and for v mass should be known to us.

Hence, (B) and (D) are correct.

37. In region OA particle is accelerated, in region AB particle has uniform velocity while in region BC particle is decelerating. Therefore, work done is positive in region OA, zero in region AB and negative in region BC.

Hence, (B) and (C) are correct.

38. For $x>0$, $F=-\text{ve}$

For $x<0$, $F=+\text{ve}$

From $x=3$ to $x=1$, displacement is in negative direction.

From $x=-1$ to $x=-3$ also displacement is in negative direction.

Now work done is positive if force and displacement are parallel to each other and work done is negative if they are antiparallel.

Hence, (A) and (D) are correct.

39. $\text{Power Loss}=\frac{\frac{1}{2}(dm)v^2}{dt}=\frac{1}{2}\frac{(A\,dx\,\rho)v^2}{dt}=\frac{1}{2}\rho A v^3$

Further, $\text{Power}=Fv=\frac{1}{2}\rho A v^3$

$\Rightarrow \quad F=\frac{1}{2}\rho A v^2$

Hence, (A) and (C) are correct.

40. $Fx_0=\frac{1}{2}kx_0^2$

$\Rightarrow \quad x_0=\frac{2F}{k}$

Work done by applied force is

$W=Fx_0$

Hence, (C) and (D) are correct.

Reasoning Based Questions

1. Work done by force of gravity is path independent near the surface of Earth, for one reason that the force of gravity is constant near the surface of Earth. Also work done by force of gravity is path independent at greater height as the force of gravity is a conservative force.

Hence, the correct answer is (D).

2. According to Work-Energy Theorem, work done equals change in kinetic energy, so

$$W=Fx=\frac{1}{2}mv^2=\Delta K$$

Since for both the cars of different mass, ΔK will be different, so W is also different.

Hence, the correct answer is (D).

3. Since $m_{\text{car}}<m_{\text{truck}}$, so retardation on the car is more than the truck and hence the car will stop earlier in lesser distance.

Hence, the correct answer is (D).

4. $F=-\frac{dU}{dx}$

$F=0$ at points B and C.

Hence, the correct answer is (D).

5. At the topmost point, the velocity is $\sqrt{gR}(\neq 0)$, because to just complete the circle, velocity at lowest point is $\sqrt{5gR}$. At the highest point velocity is zero, however acceleration is non-zero equal to g.

Hence, the correct answer is (D).

6. $\frac{1}{2}mv^2=mgh_1$

$\Rightarrow \quad h_1=\frac{v^2}{2g}$

Also, $\frac{1}{2}(2m)v^2=(2m)gh_2$

$\Rightarrow \quad h_2=\frac{v^2}{2g}$

Hence, the correct answer is (A).

7. Work done and power developed is zero in uniform circular motion, because $\vec{F}\perp\vec{r}$. Also $P=\frac{W}{t}$, so Statement-2 is the correct explanation to Statement-1.

Hence, the correct answer is (A).

8. $W(mg)+W(sp)+W(Man)=0$

$\Rightarrow \quad -\Delta U+E+W=0$

$\Rightarrow \quad \Delta U=E+W$

Work done by spring force is positive when a compressed spring is released or stretched spring released.

Hence, the correct answer is (C).

9. By Law of Conservation of Momentum, we have

$$O = m_1 v_1 - m_2 v_2$$

and $K_1 + K_2 = E$ the energy released in explosion

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{m_2}{m_1}$$

$$\Rightarrow \quad P = \sqrt{2mE}$$

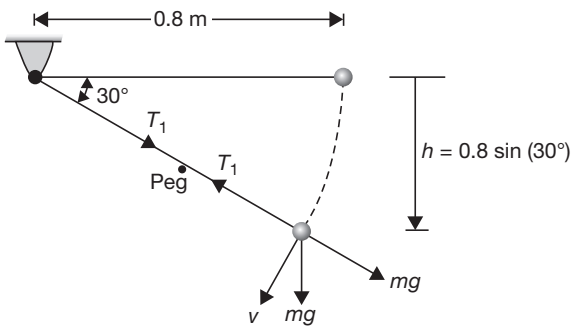
Hence, the correct answer is (C).

10. Statement-1 is false because normal force cannot do any work in the absence of any displacement. The Statement-2 is correct because an external force alone can shift the position of centre of mass.

Hence, the correct answer is (D).

Linked Comprehension Type Questions

1. Just before the cord comes in contact with the peg, we have



$$h = 0.8 \sin 30° = 0.4 \text{ m}$$

$$\Rightarrow \quad v^2 = 2gh$$

So, $T_1 - mg\sin(30°) = \frac{mv^2}{R_1}$ $\qquad \{R_1 = 0.8 \text{ m}\}$

$$\Rightarrow \quad T_1 = \frac{mg}{2} + \frac{m(2g)(0.4)}{0.8} = \frac{3mg}{2}$$

Hence, the correct answer is (C).

2. Just after the cord comes in contact with the peg, we have

$$h = 0.8\sin(30°) = 0.4$$

$$\Rightarrow \quad v^2 = 2gh$$

So, $T_2 - mg\sin(30°) = \frac{mv^2}{R_2}$

$$\Rightarrow \quad T_2 = \frac{mg}{2} + \frac{m(2g)(0.4)}{0.4}$$

$$\Rightarrow \quad T_2 = \frac{5mg}{2}$$

Hence, the correct answer is (A).

3. At maximum extension (situation of momentary rest),

$$v_A = v_B = zero$$

By Law of Conservation of Energy

$$\begin{pmatrix}\text{Loss of Gravitational}\\ \text{Potential Energy}\\ \text{of block B}\end{pmatrix} = \begin{pmatrix}\text{Gain in Elastic}\\ \text{Potential Energy}\\ \text{of the spring}\end{pmatrix}$$

$$\Rightarrow \quad (2m)gx_{\max} = \frac{1}{2}kx_{\max}^2$$

$$\Rightarrow \quad x_{\max} = \frac{4mg}{k}$$

Hence, the correct answer is (D).

4. At $x = \frac{x_{\max}}{2} = \frac{2\text{ mg}}{\text{k}}$, we have

$v_A = v_B = v$ (say)

{Because A and B are connected by a rigid string}

Then by Law of Conservation of Energy

$$\begin{pmatrix}\text{Decrease}\\ \text{in GPE}\\ \text{of B}\end{pmatrix} = \begin{pmatrix}\text{Increase in}\\ \text{EPE of}\\ \text{the spring}\end{pmatrix} + \begin{pmatrix}\text{Increase}\\ \text{in KE of}\\ \text{both blocks}\end{pmatrix}$$

$$\Rightarrow \quad (2m)gx = \frac{1}{2}kx^2 + \frac{1}{2}(m+2m)v^2$$

$$\Rightarrow \quad (2mg)\left(\frac{2mg}{k}\right) = \frac{1}{2}k\left(\frac{2mg}{k}\right)^2 + \frac{1}{2}(3m)v^2$$

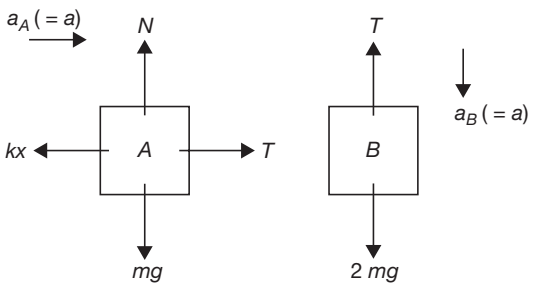
$$\Rightarrow \quad \frac{8m^2g^2}{k} = \frac{4m^2g^2}{k} + 3mv^2$$

$$\Rightarrow \quad 3mv^2 = \frac{4m^2g^2}{k}$$

$$\Rightarrow \quad v = 2g\sqrt{\frac{m}{3k}}$$

Hence, the correct answer is (D).

5. At $x = \frac{x_{\max}}{4} = \frac{mg}{k}$, we draw the free body diagrams for the blocks A and B, below



Since A and B are connected with a rigid thread, so at any instant $a_A = a_B = a$ (say)

$\Rightarrow \quad T - kx = ma$

$\Rightarrow \quad T - mg = ma \qquad \ldots(1) \qquad \left\{\because x = \frac{mg}{k}\right\}$

Also, $2mg - T = 2ma \qquad \ldots(2)$

Solving these two equations, we get

$mg = 3ma$

$\Rightarrow \quad a = \frac{g}{3}$

Hence, the correct answer is (C).

6. By Law of Conservation of Energy

$\frac{1}{2}mv^2 = mgH - mgh$

$\Rightarrow \quad v = \sqrt{2g(H-h)}$

Hence, the correct answer is (D).

7. From concepts of projectile motion, we have

$h = \frac{1}{2}gt^2$

$\Rightarrow \quad t = \sqrt{\frac{2h}{g}}$

Hence, the correct answer is (B).

8. $s = vt$

$\Rightarrow \quad s = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}}$

$\Rightarrow \quad s = 2\sqrt{h(H-h)} \qquad \ldots(1)$

For s to be MAXIMUM, s^2 has to be MAXIMUM.

So, $\frac{d(s^2)}{dh} = 0$

$\Rightarrow \quad \frac{d}{dh}[4h(H-h)] = 0$

$\Rightarrow \quad H - 2h = 0$

$\Rightarrow \quad h = \frac{H}{2}$

Hence, the correct answer is (B).

9. $s_{\max} = 2\sqrt{\left(\frac{H}{2}\right)\left(\frac{H}{2}\right)}$

$\Rightarrow \quad s_{\max} = H$

Hence, the correct answer is (D).

10. Total height $h = 2 + \frac{u_B^2 \sin^2\theta}{2g}$

$\Rightarrow \quad h = 2 + \frac{2g(h_A - h_B)\sin^2\theta}{2g}$

$\Rightarrow \quad h = 2 + 4\sin^2\theta$

We can see that $h = 6$ m only when $\theta = 90°$

Hence, the correct answer is (B).

11. Since, $h = 2 + 4\sin^2\theta$, so at $\theta = 30°$, $h = 3$ m

Hence, the correct answer is (A).

12. $dW = \vec{F} \cdot d\vec{\ell}$

$\Rightarrow \quad dW = k\left(x^2y^3dx + x^3y^2dy\right)$

$\Rightarrow \quad dW = k\left[y^3 d\left(\frac{x^3}{3}\right) + x^3 d\left(\frac{y^3}{3}\right)\right]$

$\Rightarrow \quad dW = k\left[d\left(\frac{x^3y^3}{3}\right)\right]$

$\Rightarrow \quad W = \int dW = \frac{k}{3}\left(x^2y^3\right)$

$W_{(0,\,0)\to(a,\,0)} = \frac{k}{3}\left(x^3y^3\right)\Big|_{(0,\,0)}^{(a,\,0)} = \frac{24}{3}(0-0) = \text{zero}$

Hence, the correct answer is (A).

13. $W_{(a,\,0)\to(a,\,a)} = \frac{k}{3}\left(x^3y^3\right)\Big|_{(a,\,0)}^{(a,\,a)} = \frac{9}{3}\left(a^6 - 0\right) = 3a^6$

Hence, the correct answer is (C).

14. $W_{(0,\,0)\to(a,\,a)} = \frac{k}{3}\left(x^3y^3\right)\Big|_{(0,\,0)}^{(a,\,a)} = \frac{k}{3}\left(a^6 - 0\right) = \frac{ka^6}{3}$

Hence, the correct answer is (C).

15. Since we observe that the work done in all the three paths is same i.e., $\frac{ka^6}{3}$. So, work done due to this force is independent of the path followed between $(0,\,0)$ to $(a,\,a)$. Hence $\vec{F}$ is a conservative force.

Hence, the correct answer is (A).

16. $W_{mg} = mgh\cos 180°$

$\Rightarrow \quad W_{mg} = (2)(10)(2)(-1) = -40$ J

From Work Energy Theorem, we get

$W_{mg} + W_{\text{upthrust}} = \Delta KE$

$\Rightarrow \quad -40 \text{ J} + W_{\text{upthrust}} = 16$

$\Rightarrow \quad W_{\text{upthrust}} = 56$ J

Hence, the correct answer is (D).

17. Let ρ be the density of ball, then

$$56 = (\text{upthrust force}) \times (\text{displacement}) \times \cos 0°$$

$$\Rightarrow \quad 56 = \left(\frac{m}{\rho}\right)\rho_w g \times 2 = \frac{2}{\rho} \times 1000 \times 10 \times 2$$

$$\Rightarrow \quad \rho = \left(\frac{40}{56}\right) \times 10^3 \text{ kgm}^{-3}$$

$$\Rightarrow \quad \rho = \frac{5}{7} \times 10^3 \text{ kgm}^{-3}$$

Hence, the correct answer is (A).

18. At equilibrium r_0

$$F = \frac{dU}{dr} = 0$$

$$\Rightarrow \quad F|_{r=r_0} = \left.\frac{dU}{dr}\right|_{r=r_0} = -\frac{2A}{r_0^3} + \frac{B}{r_0^2} = 0$$

$$\Rightarrow \quad r_0 = \frac{2A}{B}$$

Hence, the correct answer is (B).

19. Since, $\frac{dU}{dr} = -\frac{2A}{r^3} + \frac{B}{r^2}$

$$\Rightarrow \quad \frac{d^2U}{dr^2} = \frac{6A}{r^4} - \frac{2B}{r^3}$$

$$\Rightarrow \quad \left.\frac{d^2U}{dr^2}\right|_{r_0=\frac{2A}{B}} = \frac{6A}{\left(\frac{2A}{B}\right)^4} - \frac{2B}{\left(\frac{2A}{B}\right)^3}$$

$$\Rightarrow \quad \left.\frac{d^2U}{dr^2}\right|_{r_0=\frac{2A}{B}} = \text{A POSITIVE VALUE}$$

So, U is minimum at equilibrium position r_0. So, the equilibrium is stable.

Hence, the correct answer is (A).

20. Work done by a conservative force equals the decrease in potential energy of the particle. So,

$$W = U_i - U_f = U_{\text{at equilibrium}} - U_{\text{at }\infty}$$

Now $U_i = U_{\text{at } r_0} = \frac{A}{r_0^2} - \frac{B}{r_0} = -\frac{B^2}{4A}$

and $U_\infty = 0$

So, work done by external agent will be $\frac{B^2}{4A}$, so that the particle is taken from equilibrium position to infinity.

Hence, the correct answer is (B).

21. For velocity to be zero, i.e., kinetic energy has to be zero, so the total energy must be purely potential.

$$\Rightarrow \quad E = -\frac{3B^2}{16A} = \frac{A}{r^2} - \frac{B}{r}$$

Also, we observe that at $r = \frac{2r_0}{3} = \frac{4A}{B}$, we have

$$E = \frac{A}{\left(\frac{4A}{B}\right)^2} - \frac{B}{\left(\frac{2A}{B}\right)}$$

$$\Rightarrow \quad E = \frac{B^2}{16A} - \frac{B^2}{2A}$$

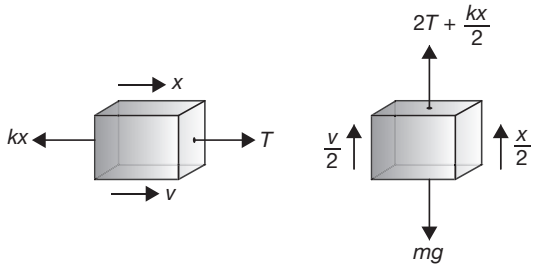
$$\Rightarrow \quad E = -\frac{3B^2}{16A}$$

Hence, the correct answer is (C).

22. In equilibrium $(F_{\text{net}} = 0)$

$$T = kx \qquad \ldots(1)$$

$$2T + \frac{kx}{2} = mg \qquad \ldots(2)$$



Solving these equations, we have

$$x = \frac{2mg}{5k} = \frac{2 \times 1 \times 10}{5 \times 10} = 0.4 \text{ m}$$

Hence, the correct answer is (B).

23. Let v be the speed of block placed horizontally, then by Law of Conservation of Energy, we have

$$\frac{1}{2}mv^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2 + \frac{1}{2}k\left(\frac{x}{2}\right)^2 = mg\left(\frac{x}{2}\right)$$

$$\Rightarrow \quad \frac{3}{8}mv^2 = \frac{mgx}{2} - \frac{3}{8}kx^2$$

$$\Rightarrow \quad v = \sqrt{\frac{8}{3}\left(\frac{gx}{2} - \frac{3kx^2}{8m}\right)}$$

Substituting the values, we have

$$v = \sqrt{\frac{8}{3}\left(\frac{10 \times 0.4}{2} - \frac{3 \times 10 \times 0.16}{8 \times 1}\right)}$$

$$\Rightarrow \quad v = 1.93 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

CHAPTER 1

24. $KE = \frac{1}{2}(4)(20)^2$

$\Rightarrow \quad KE = 800 \text{ J}$

Hence, the correct answer is (C).

25. Work done from OAB

$\text{Area} = \frac{1}{2}(4)(12) = 24 \text{ J}$

Since the force is non-conservative from 0 to A, so KE must decrease and hence KE at $x = 4$ m is $(800 - 24)$ J

$\Rightarrow \quad KE|_{x=4 \text{ m}} = 776 \text{ J}$

Hence, the correct answer is (C).

26. Gain in KE from BCDE is 60 J

So, new KE is $(776 + 60)$ J $= 836$ J

Hence, the correct answer is (B).

27. $A \to O$ is a non conservative force, so again negative work will be done while going back.

Hence, the correct answer is (C).

28. $a_t = g \sin 60° = 5\sqrt{3} \text{ ms}^{-2}$

$a_n = \frac{v^2}{R} = 16 \text{ ms}^{-2}$

$\Rightarrow \quad a = \sqrt{a_t^2 + a_n^2} = 18.2 \text{ ms}^{-2}$

Hence, the correct answer is (C).

29. Height from bottom at when $\theta = 60°$ is

$h_1 = R(1 - \cos 60°) = \frac{R}{2} = 0.5 \text{ m}$

Further height raised,

$h_2 = \frac{v^2}{2g} = \frac{16}{20} = 0.8 \text{ m}$

Now since $h_1 + h_2 = 1.3 \text{ m} > R$

The bob will rise to $\theta = 90°$, but there string will slack and total height will be less than 1.3 m.

Hence, the correct answer is (C).

30. $a = g \sin\theta - \mu g \cos\theta$

$a = 10 \times \frac{3}{5} - 0.5 \times 10 \times \frac{4}{5} = 2 \text{ ms}^{-2}$

Velocity of block at B or at C is

$v = \sqrt{2as} = \sqrt{2 \times 2 \times \frac{h}{\sin\theta}} = \sqrt{\frac{20h}{3}}$

For the block, not to leave contact anywhere

$v \le \sqrt{2gR}$

$\Rightarrow \quad \sqrt{\frac{20h}{3}} \le \sqrt{2gR}$

$\Rightarrow \quad h \le 3R$

$\Rightarrow \quad h \le 1.5 \text{ m}$

Hence, the correct answer is (D).

31. Since, $g \sin\theta - \mu g \cos\theta = 2 \text{ ms}^{-2}$

$\Rightarrow \quad g \sin\theta > \mu g \cos\theta$

Block will never stop on rough inclined surface. So, it will cross point C infinite number of times.

Hence, the correct answer is (A).

32. Potential energy of the projectile at any instant will be

$U = \frac{1}{2}mv_0^2 - \frac{1}{2}mv^2$

$\Rightarrow \quad \frac{dU}{dt} = 0 - mv\frac{dv}{dt}$

Since, $m = 1$ kg, $\frac{dv}{dt} = a_t = \frac{\vec{a} \cdot \vec{v}}{v} = \text{tangential acceleration}$

$\Rightarrow \quad \left|\frac{dU}{dt}\right| = \vec{a} \cdot \vec{v}$

Hence, the correct answer is (B).

33. At height h, we have

$K = \frac{1}{2}mv_0^2 - mgh$

and $U = mgh$

$\Rightarrow \quad \frac{K}{U} = \frac{v_0^2}{2gh} - 1 = \text{constant}$

Hence, $\frac{K}{U}$ versus $\frac{1}{h}$ graph is a straight line not passing through the origin.

Hence, the correct answer is (B).

34. $\text{KE}|_{x=5} = \frac{1}{2}(2)(4)^2 = 16 \text{ J}$

$\text{PE}|_{x=5} = 20 + (5 - 2)^2 = 29 \text{ J}$

$\Rightarrow$ Total Energy = Mechanical Energy

So, $E = U + K = 16 + 29 = 45 \text{ J}$

$\Rightarrow \quad E = 45 \text{ J}$

Hence, the correct answer is (C).

35. KE possessed by the particle will be MAXIMUM, when PE is MINIMUM i.e. at $x = 2$ m

So, $\text{PE}|_{\text{MIN}} = 20 \text{ J}$

$\Rightarrow \quad \text{KE}|_{\text{MAX}} = 45 - 20 = 25 \text{ J}$

Hence, the correct answer is (B).

36. The particle will be in equilibrium i.e. $F = 0$

$\Rightarrow \quad \dfrac{dU}{dx} = 0$

$\Rightarrow \quad x_0 - 2 = 0$

$\Rightarrow \quad x_0 = 2 \text{ m}$

Hence, the correct answer is (B).

37. Let lower spring compresses maximum by x metre, then by Law of Conservation of Mechanical Energy, we have

$$\begin{pmatrix}\text{Decreases in}\\ \text{potential energy}\\ \text{of block}\end{pmatrix} = \begin{pmatrix}\text{Increases in elastic}\\ \text{potential energy of}\\ \text{both the springs}\end{pmatrix}$$

$\Rightarrow \quad (2)(10)(x+1) = \dfrac{1}{2}\times 10 \times x^2 + \dfrac{1}{2}\times 10 \times (x+1)^2$

$\Rightarrow \quad 20x + 20 = 5x^2 + 5x^2 + 5 + 10x$

$\Rightarrow \quad 10x^2 - 10x - 15 = 0$

$\Rightarrow \quad 2x^2 - 2x - 3 = 0$

$\Rightarrow \quad x = \dfrac{2 \pm \sqrt{4+24}}{4} = 1.82 \text{ m}$

So, maximum extension in the upper spring is

$x_{\text{max}} = 1 + x = 2.82 \text{ m}$

Hence, the correct answer is (C).

38. At equilibrium position net force on the block should be zero. So, let it is at distance y from where it was released.

Then,

$mg = Ky + K(y-1)$

$\Rightarrow \quad 20 = 10y + 10(y-1)$

$\Rightarrow \quad 20y = 30$

$\Rightarrow \quad y = 1.5 \text{ m}$

Hence, the correct answer is (D).

39. Equation can be written as,

$v^2 = (9 + 4s)$

Comparing this with $v^2 = u^2 + 2as$, we get

$u = 3 \text{ ms}^{-1}$ and $a = 2 \text{ ms}^{-1}$

At $t = 0$, velocity is 3 ms^{-1} and at $t = 2$ s, velocity is 7 ms^{-1}. From Work Energy Theorem, we have

$W = K_f - K_i = \dfrac{1}{2}m\left(v_f^2 - v_i^2\right) = \dfrac{1}{2}(2)(49-9)$

$\Rightarrow \quad W = 40 \text{ J}$

Hence, the correct answer is (D).

40. $P_{\text{av}} = P_{\text{inst}}$

$\Rightarrow \quad \dfrac{W}{t} = Fv_{\text{inst}}$

$\Rightarrow \quad \dfrac{1}{2}m\dfrac{\left(v_f^2 - v_i^2\right)}{t} = \dfrac{3}{4}(mav)$

$\Rightarrow \quad (3+2t)^2 - (3)^2 = \dfrac{3}{2}atv$

$\Rightarrow \quad 9 + 4t^2 + 12t - 9 = \dfrac{3}{2}(2)(t)(3+2t)$

$\Rightarrow \quad 4t^2 + 12t = 9t + 6t^2$

$\Rightarrow \quad 2t^2 = 3t$

$\Rightarrow \quad t = 1.5 \text{ s}$

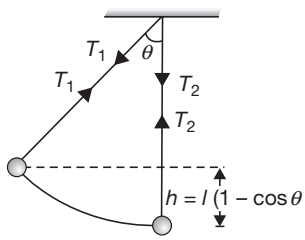
Hence, the correct answer is (A).

41. $T_1 = mg\cos\theta$ …(1)

$T_2 - mg = \dfrac{mv^2}{\ell} = \dfrac{m}{\ell}(2gh)$

$\Rightarrow \quad T_2 - mg = \dfrac{2mg}{\ell}(1-\cos\theta)\ell$

$\Rightarrow \quad T_2 - mg = 2mg(1-\cos\theta)$



$\Rightarrow \quad T_2 = mg + 2mg(1-\cos\theta)$ …(2)

Given $T_2 = 2T_1$

$\Rightarrow \quad mg + 2mg(1-\cos\theta) = 2mg\cos\theta$

$\Rightarrow \quad \cos\theta = \dfrac{3}{4}$

$\Rightarrow \quad \theta = \cos^{-1}\left(\dfrac{3}{4}\right)$

Hence, the correct answer is (D).

42. Tension is maximum at bottommost point, so

$a_{\text{max}} = \dfrac{2mg - mg}{m} = g$

CHAPTER 1

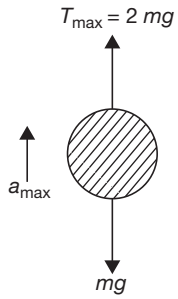
$$\Rightarrow \quad \frac{v^2}{\ell} = g$$

$$\Rightarrow \quad \frac{2\ell(1-\cos\theta)}{\ell} = 1$$

$$\Rightarrow \quad 2 - 2\cos\theta = 1$$

$$\Rightarrow \quad 2\cos\theta = 1$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$



Hence, the correct answer is (D).

43. Let t be the time taken by the ball from B to C.

$$R = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{2R}{g}}$$

Since, $x = vt$

$$\Rightarrow \quad 2R = v\sqrt{\frac{2R}{g}}$$

$$\Rightarrow \quad v = \sqrt{2gR}$$

Applying Energy Conservation at points A and B, we get

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + mg(2R)$$

$$\Rightarrow \quad v_0 = \sqrt{6gR}$$

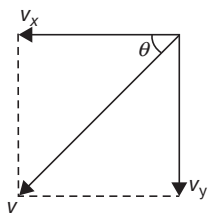
Hence, the correct answer is (D).

44. $v_y = gt$

$$\Rightarrow \quad v_y = g\sqrt{\frac{2R}{g}} = \sqrt{2gR}$$

and $v_x = \sqrt{2gR}$

$$\tan\theta = \frac{v_y}{v_x} = 1$$



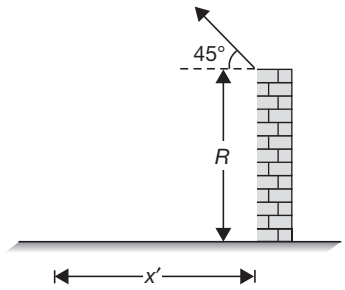
$$\Rightarrow \quad \theta = 45°$$

Hence, the correct answer is (B).

45. Since all surfaces are frictionless, so collision is elastic, hence

$$v' = \sqrt{v_x^2 + v_y^2} = \sqrt{2gR + 2gR}$$

$$\Rightarrow \quad v' = 2\sqrt{gR}$$



Since, $-R = v'\sin(45°)t - \frac{1}{2}gt^2$

$$\Rightarrow \quad t = \sqrt{\frac{2R}{g}}(1+\sqrt{2})$$

$$x' = (v'\cos(45°))t$$

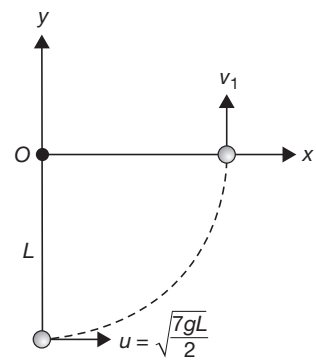
$$\Rightarrow \quad x' = 2(1+\sqrt{2})R$$

Hence, the correct answer is (C).

46. By Law of Conservation of Energy, we get

$$\frac{1}{2}m\frac{7gL}{2} = mgL + \frac{1}{2}mv_1^2$$

$$\Rightarrow \quad 7gL = 4gL + 2v_1^2$$



$$\Rightarrow \quad v_1^2 = \frac{3}{2}gL$$

$$\Delta\vec{v} = \vec{v}_1 - \vec{u}$$

$$\Rightarrow \quad \Delta\vec{v} = \left(\sqrt{\frac{3}{2}gL}\right)\hat{j} - \left(\sqrt{\frac{7}{2}gL}\right)\hat{i}$$

$$\Rightarrow \quad |\Delta\vec{v}| = \sqrt{\frac{3}{2}gL + \frac{2gL}{2}}$$

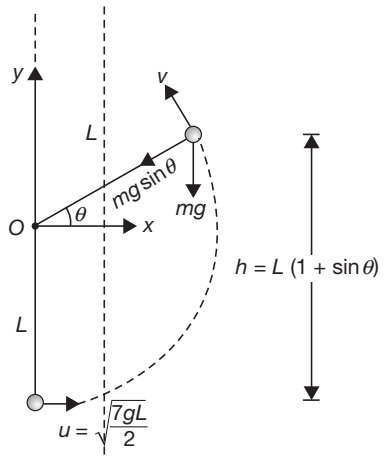
$$\Rightarrow \quad |\Delta\vec{v}| = \sqrt{5gL}$$

Hence, the correct answer is (D).

47. Since $\frac{1}{2}m\left(\frac{7gL}{2}\right)=mgL(1+\sin\theta)+\frac{1}{2}mv^2$

$\Rightarrow \quad 7gL=4gL(1+\sin\theta)+2v^2$

$\Rightarrow \quad 2v^2=gL(3-4\sin\theta)$ ...(1)



Also, when the motion ceases to be circular, then $T=0$

$\Rightarrow \quad \frac{mv^2}{L}=mg\sin\theta$

$\Rightarrow \quad v^2=gL\sin\theta$

$\Rightarrow \quad 2gL\sin\theta=gL(3-4\sin\theta)$

$\Rightarrow \quad 6\sin\theta=3$

$\Rightarrow \quad \theta=30^\circ$

Hence, the correct answer is (A).

48. $v^2=gL\sin(30^\circ)=\frac{gL}{2}$

$\Rightarrow \quad \vec{v}=-v\cos\theta\hat{i}+v\sin\theta\hat{j}$

$\Rightarrow \quad \vec{v}=\sqrt{\frac{gL}{2}}\left(-\sqrt{3}\hat{i}+\hat{j}\right)$

Hence, the correct answer is (C).

49. Work done by all the force = change in KE

Hence, the correct answer is (D).

50. Gravitational force is constant.

Hence, the correct answer is (C).

51. Work done by kinetic friction in this case is negative because force of friction is acting in the opposite direction of motion.

Hence, the correct answer is (C).

52. Velocity at B is

$v_B=\sqrt{2gh}=\sqrt{2g(2R)}$

$\Rightarrow \quad v_B=2\sqrt{gR}$

The normal force N_B at B is

$N_B=\frac{mv_B^2}{R}=4mg$

Hence, the correct answer is (C).

53. $v_C=\sqrt{2gh'}=\sqrt{2g(3R)}$

$\Rightarrow \quad v_C=\sqrt{6gR}$

The normal force N_C at C is $N_C=mg+\frac{mv_C^2}{R}$

$\Rightarrow \quad N_C=mg+\frac{m(6gR)}{R}=7mg$

Hence, the correct answer is (D).

54. $v_D=\sqrt{2g(2R)}=2\sqrt{gR}$

Let the final velocity when its stops be $v=0$.

Retardation offered beyond D is

$a=-\left[g\sin(30^\circ)+\mu_k\cos(30^\circ)\right]$

$\Rightarrow \quad a=-\frac{g}{2}\left(1+\sqrt{3}\mu_k\right)$

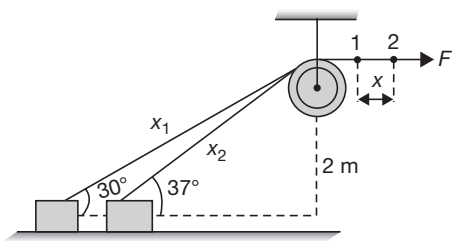
Since $v^2-v_D^2=2ax$

$\Rightarrow \quad 0^2-(4gR)=-\frac{2g}{2}\left(1+\sqrt{3}\mu_k\right)x$

$\Rightarrow \quad x=\frac{4R}{1+\sqrt{3}\mu_k}$

Hence, the correct answer is (C).

55.



Let us consider the motion of a point on the free portion of string on which F is being applied. Then

$x=x_1-x_2$

$\Rightarrow \quad x=\frac{2}{\sin(30^\circ)}-\frac{2}{\sin(37^\circ)}$

$\Rightarrow \quad x=\frac{2}{\left(\frac{1}{2}\right)}-\frac{2}{\left(\frac{3}{5}\right)}=4-\frac{10}{3}$

CHAPTER 1

$\Rightarrow \quad x = \frac{2}{3}$ m

So, work done is

$$W = Fx = 50\left(\frac{2}{3}\right) = \frac{100}{3} \text{ J}$$

Hence, the correct answer is (C).

56. From Work-Energy Theorem, we have

$$W = \Delta K$$

$$\Rightarrow \quad \frac{100}{3} = \frac{1}{2}mv^2 = 5v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{20}{3}} \text{ ms}^{-1}$$

Hence, the correct answer is (C).

57. Initial Acceleration is $a_1 = \frac{F\cos(30°)}{m}$

Final Acceleration is $a_2 = \frac{F\cos(37°)}{m}$

So, ratio $\frac{a_1}{a_2} = \frac{\cos(30°)}{\cos(37°)}$

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{5\sqrt{3}}{8}$$

Hence, the correct answer is (B).

58. As the particle moves straight from $(0, 0)$ to (a, a), then it must move along the line

$$y = x$$

$$\Rightarrow \quad \frac{dy}{dx} = 1$$

$$\Rightarrow \quad dy = dx$$

Since, work done is

$$W = \int \vec{F} \cdot d\vec{\ell} = \int \left(y\hat{i} + x\hat{j}\right)\left(dx\hat{i} + dy\hat{j}\right)$$

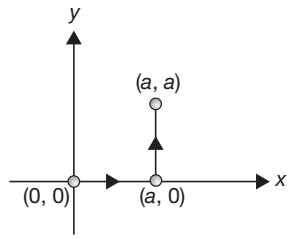
$$\Rightarrow \quad W = \int \left(ydx + xdy\right)$$

Taking $y = x$, $dy = dx$ and the limits from $(0, 0)$ to (a, a), we get

$$W = \int_0^a 2xdx = \left(x^2\right)\Big|_0^a = a^2$$

Hence, the correct answer is (A).

59. The path followed is shown in the Figure.



For the path between $(0, 0)$ to $(a, 0)$, $y = 0$, so $dy = 0$.

$$\Rightarrow \quad W_{(0,\,0)\to(a,\,0)} = W_1 = \int ydx + xdy = 0$$

For the path between $(a, 0)$ to (a, a), $x = a$, so $dx = 0$.

$$\Rightarrow \quad W_{(a,\,0)\to(a,\,a)} = W_2 = \int ydx + xdy$$

$$\Rightarrow \quad W_{(a,\,0)\to(a,\,a)} = W_2 = \int_0^a ady = a^2$$

So, the total work done is

$$W = W_1 + W_2 = a^2$$

Hence, the correct answer is (A).

60. Since, $dW = \vec{F} \cdot d\vec{\ell} = ydx + xdy$

Also, we know that $d(xy) = ydx + xdy$

$$\Rightarrow \quad dW = d(xy)$$

Integrating, we get

$$W = \int d(xy)$$

$$\Rightarrow \quad W = (xy)\Big|_{(x_1,\,y_1)}^{(x_2,\,y_2)}$$

$$\Rightarrow \quad W = x_2y_2 - x_1y_1$$

In this case, we observe that the work done by the force depends only on (x_1, y_1), the initial position and (x_2, y_2), the final position. So, the work done by the force depends only on initial and final values of x and y.

Also, when the object returns to the original position, then we have $(x_2, y_2) = (x_1, y_1)$, and hence

$$W = 0.$$

Hence, (A) and (D) are correct.

Matrix Match/Column Match Type Questions

1. A → (q)
B → (s)
C → (r)
(D) → (r)

Since $W_{\text{total}} = \Delta K$ and $W_C = -\Delta U$

and $W_{\text{ext}} = \Delta U + \Delta K = (U_f + K_f) - (U_i + K_i)$

2. A → (s)
B → (r)
C → (p)
D → (q)

$v_B^2 = u_A^2 - 2gh_{AB} = (9g\ell) - (2g\ell) = 7g\ell$

$\Rightarrow \quad v_B = \sqrt{7g\ell}$

$\Rightarrow \quad \dfrac{v_B^2}{g\ell} = 7$

Further, $T_B = \dfrac{mv_B^2}{\ell} = 7mg$

$\Rightarrow \quad \dfrac{T_B}{2mg} = \dfrac{7}{2}$

Again, $v_C^2 = v_A^2 - 2gh_{AC} = (9g\ell) - 2g(2\ell) = 5g\ell$

$\Rightarrow \quad v_C = \sqrt{5g\ell}$

$\Rightarrow \quad \dfrac{v_C^2}{g\ell} = 5$

Further, $T_C + mg = \dfrac{mv_C^2}{\ell}$

$\Rightarrow \quad T_C = 4mg$

$\Rightarrow \quad \dfrac{T_C}{mg} = 4$

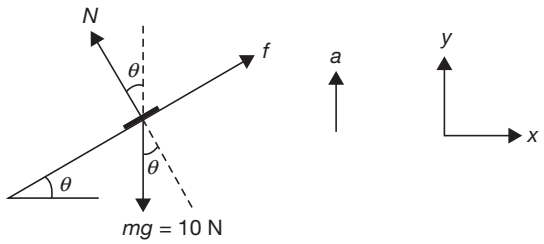
3. A → (t)
B → (p)
C → (s)
D → (q)

At $t = 4$ s,

$v = at = 8 \text{ ms}^{-1}$

and $s = \dfrac{1}{2}at^2 = 16m$

$KE = \dfrac{1}{2}mv^2 = 32 \text{ J}$



From Work-Energy Theorem, we have
Work done by all the forces $= \Delta KE = 32$ J
Work done by gravity

$= -mgh = -(1)(10)(16) = -160$ J

Writing equation of motion, we have, $\sum F_y = ma$

$N\cos 30° + f\sin 30° - 10 = ma = 2$

$\Rightarrow \quad \sqrt{3}\text{ N} + f = 24 \qquad \ldots(1)$

$\sum F_x = 0$

$\Rightarrow \quad N\sin 30° = f\cos 30°$

$\Rightarrow \quad N = \sqrt{3}f \qquad \ldots(2)$

Solving equations (1) and (2), we have

$f = 6$ N

and $N = \dfrac{18}{\sqrt{3}} = 6\sqrt{3}$ N

So, $W_N = (N\cos\theta)(s) = (6\sqrt{3})\left(\dfrac{\sqrt{3}}{2}\right)(16) = 144$ J

and $W_f = (f\sin\theta)(s) = (6)\left(\dfrac{1}{2}\right)(16) = 48$ J

4. A → (r)
B → (q)
C → (p)
D → (t)

$v_f - v_i =$ area under a-x graph $= 12 \text{ ms}^{-1}$

$\Rightarrow \quad v_f = 12 + 4 = 16 \text{ ms}^{-1}$

$\Rightarrow \quad \Delta KE = \dfrac{1}{2}m(v_f^2 - v_i^2) = 120$ J

Work done by all the forces $= \Delta KE = 120$ J

$K_f = \dfrac{1}{2}mv_f^2 = 128$ J

Work done by conservative forces $= U_i - U_f = 240$ J

CHAPTER 1

Work done by external force i.e. W_{ext} is given by

$$W_{ext} = \begin{pmatrix}\text{Total}\\ \text{work}\\ \text{done}\end{pmatrix} - \begin{pmatrix}\text{Work done by}\\ \text{conservative}\\ \text{forces}\end{pmatrix}$$

$\Rightarrow \quad W_{ext} = -112$ J

5. A → (p, q)
B → (t)
C → (r)
D → (s)

Angle between net force and the string can never be obtuse. It is $90°$ at A, $0°$ at B and acute in between.

6. A → (p, q, r)
B → (p, q, r)
C → (q, r)
D → (p, r)
Conceptual

7. A → (r)
B → (p)
C → (q, s)
D → (q, s)
Conceptual

8. A → (p, s)
B → (q, r)
C → (p, s)
D → (q, r)
Conceptual

9. A → (s)
B → (r)
C → (p)
D → (q)

$$\frac{v^2}{R} = 9t^2$$

$$\Rightarrow \quad v^2 = (16)(9t^2)$$

$$\Rightarrow \quad v = 12t$$

$$\Rightarrow \quad a_t = \frac{dv}{dt} = 12$$

$$\Rightarrow \quad F_t = ma_t = \frac{1}{2}(12) = 6 \text{ N}$$

At $t = 1$ s, $a_n = \dfrac{v^2}{R} = 9$

$$\Rightarrow \quad F_n = ma_n = \frac{9}{2} \text{ N}$$

$$\Rightarrow \quad F = \sqrt{F_t^2 + F_n^2} = \sqrt{36 + \frac{81}{4}}$$

$$\Rightarrow \quad F = \sqrt{\frac{225}{4}} = \frac{15}{2} = 7.5 \text{ N}$$

$$P = \vec{F} \cdot \vec{v}$$

$$\Rightarrow \quad P = (F_t)(v)$$

$$\Rightarrow \quad P = (6)(12) = 72 \text{ watt}$$

$$P_{av} = \frac{W}{t}$$

$$\Rightarrow \quad P_{av} = F_t (v_{av})$$

But $v_{av} = \dfrac{\int_0^1 v\,dt}{\int_0^1 dt} = \dfrac{12\int_0^1 t\,dt}{1}$

$$\Rightarrow \quad v_{av} = \frac{12t^2}{2}\bigg|_0^1 = 6 \text{ ms}^{-1}$$

$$\Rightarrow \quad P_{av} = (6)(6) = 36 \text{ watt}$$

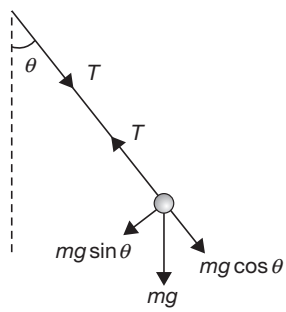
10. A → (s)
B → (r)
C → (q)
D → (p)

Just after the vehicle is stopped, the bob swings out to an angle of $60°$. So, by Law of Conservation of Mechanical Energy, we have

$$mgh = \frac{1}{2}mv_0^2$$

$$\Rightarrow \quad \frac{1}{2}mv_0^2 = mg\ell(1 - \cos\theta)$$

$$\Rightarrow \quad v_0 = \sqrt{2(10)(5)\left(1 - \frac{1}{2}\right)} = 5\sqrt{2} \text{ ms}^{-1}$$



So, net force on the bob at the lowest point is

$$F = \frac{mv_0^2}{\ell} = \frac{(2)(50)}{5} = 20 \text{ N}$$

Acceleration of the bob at lowest point is

$$a = \frac{F}{m} = \frac{v_0^2}{\ell} = 10 \text{ ms}^{-2}$$

At the highest point, net force is

$$F = mg\sin(60°) = 10\sqrt{3} \text{ N}$$

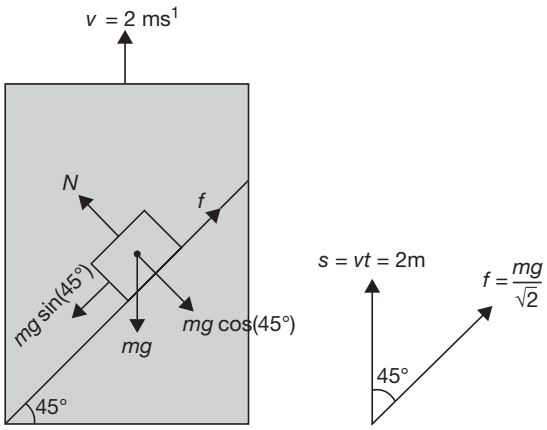
So, acceleration at highest point is

$$a = \frac{F}{m} = 5\sqrt{3} \text{ ms}^{-2}$$

Integer/Numerical Answer Type Questions

1. Since the block does not slide on the wedge. So

$$f = mg\sin(45°) = \frac{(1)(10)}{\sqrt{2}} = \frac{10}{\sqrt{2}} \text{ N}$$



Now $W_f = \vec{f} \cdot \vec{s}$

$\Rightarrow \quad W_f = (f\cos 45°)(2)$

$\Rightarrow \quad W_f = \left(\frac{10}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(2)$

$\Rightarrow \quad W_f = 10 \text{ J}$

2. Since, $P = Fv$

$\Rightarrow \quad F = \frac{P}{v} = \frac{8\times10^5}{20} = 4\times10^4 \text{ N}$

At constant speed, the forces acting on the train are in equilibrium. Resolving the forces parallel to the hill

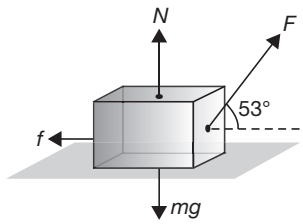
$$F = R + (2\times10^5)\times g\times\left(\frac{1}{50}\right)$$

$\Rightarrow \quad 4\times10^4 = R + 39200$

$\Rightarrow \quad R = 800 \text{ N}$

Therefore, the resistance is 800 N.

3. (a) The forces acting on the block are shown in the figure.



Clearly, $W_N = 0$ and $W_g = 0$, whereas $W_F = Fs\cos\theta$

$$W_f = -fs = -\mu_k Ns \text{ where } N = mg - F\sin\theta$$

According to Work Energy Theorem,

$$\Delta K = W_{net} = W_F + W_f$$

$\Rightarrow \quad \Delta K = Fs\cos\theta - \mu_k(mg - F\sin\theta)s$

$\Rightarrow \quad \Delta K = (30)(2)(0.6) - \frac{1}{8}(40-24)(2) = 32 \text{ J}$

(b) Now $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 32 \text{ J}$

Since $v_i = 3 \text{ ms}^{-1}$

therefore, $v_f = 5 \text{ ms}^{-1} = 500 \text{ cms}^{-1}$

4. (a) By Law of Conservation of Energy, we get

$$0 + mg(4R) = \frac{1}{2}mv^2$$

$\Rightarrow \quad 4gR = \frac{v^2}{2}$

At Q, we have

$$N = \frac{mv^2}{R} = \frac{m(8gR)}{R} = 8mg$$

So, net force at Q is

$$F_Q = \sqrt{N^2 + (mg)^2}$$

$\Rightarrow \quad F_Q = \sqrt{64m^2g^2 + m^2g^2} = \sqrt{65}mg$

$\Rightarrow \quad x = 65$

(b) For block to exert force (on the top of track) equal to weight, we have

$$N = mg$$

Since $mg + N = \frac{mv^2}{R}$

$\Rightarrow \quad \frac{mv^2}{R} = 2mg$

CHAPTER 1

$\Rightarrow \quad v^2 = 2Rg$

Applying Law of Conservation of Energy, we get

$$mg(h) = \frac{1}{2}m(2Rg) + mg(2R)$$

$\Rightarrow \quad h = 3R$

$\Rightarrow \quad * = 3$

5. Let m be the mass of B. From its free-body diagram

$$T - \mu N = m(0) = 0$$

where T is the tension in the string and $N = mg$

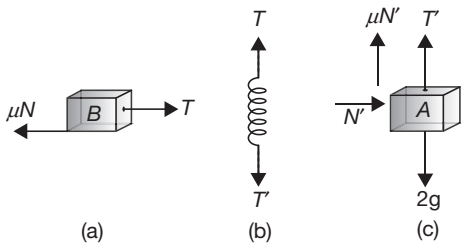
$\Rightarrow \quad T = \mu(mg)$

From the free-body diagram of the spring

$$T - T' = 0$$

where T' is the force exerted by A on the spring

$\Rightarrow \quad T = T' = \mu mg$



From the free-body diagram of A

$$2g - (T' + \mu N') = 2(0) = 0$$

where N' is the normal reaction of the vertical wall of C on A and $N' = 2(0) = 0$ (because there is no horizontal acceleration of A)

$\Rightarrow \quad 2g = T' = \mu mg$

$\Rightarrow \quad m = \dfrac{2g}{\mu g} = \dfrac{2}{0.2} = 10 \text{ kg}$

Tensile force on the spring is given by

$$T = T' = \mu mg = 0.2 \times 10 \times 9.8 = 19.6 \text{ N}$$

Now, for a spring, we have $F = kx$

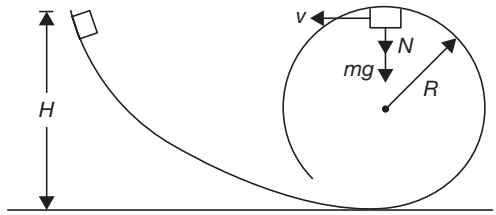
$\Rightarrow \quad 19.6 = 1960x$

$\Rightarrow \quad x = \dfrac{1}{100} \text{ m}$

$\Rightarrow$ Energy stored in the spring is given by

$$U = \frac{1}{2}kx^2 = \frac{1}{2} \times 1960 \times \left(\frac{1}{100}\right)^2 = 0.098 \text{ J}$$

6. Let v be the velocity of the particle at the highest point.



According to Newton's Second Law, the net force $F_{\text{net}} = N + mg$ provides the centripetal force to the body to revolve in a circle of radius R. So

$$N + mg = \frac{mv^2}{R}$$

The body does not fall at the uppermost point of the loop if $N \geq 0$. In the limiting case, $N = 0$. So,

$$mg = \frac{mv^2}{R}$$

$\Rightarrow \quad v^2 = gR \qquad \ldots(1)$

Applying Law of Conservation of Energy at the initial and highest point of the loop, we get

$$mgH = mg(2R) + \frac{1}{2}mv^2$$

From (1), $v^2 = gR$, so, we get

$$mgH = mg(2R) + \frac{1}{2}m(gR)$$

$\Rightarrow \quad H = \dfrac{5}{2}R = 2.5R$

Since $R = 40$ cm, we get

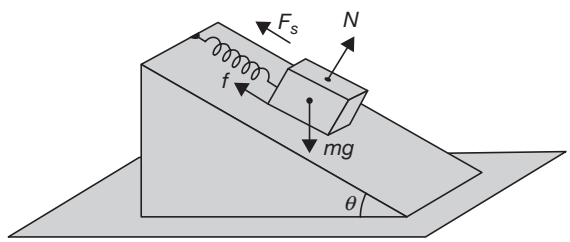
$\Rightarrow \quad H = (2.5)(40) = 100$ cm

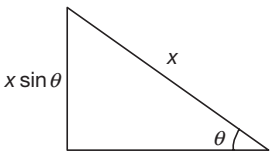
7. Since $W_{nc} + W_{\text{ext}} = \Delta U + \Delta K$

Let the block move down the incline through a distance x, then

$$fx\cos(180°) + 0 = \frac{1}{2}kx^2 + (-mgx\sin\theta) + \frac{1}{2}mv^2$$

$\Rightarrow \quad -(\mu_k mg\cos\theta)x = \dfrac{1}{2}kx^2 - mgx\sin\theta + \dfrac{1}{2}mv^2$





$\Rightarrow \quad -1 = 1 - 6 + v^2$

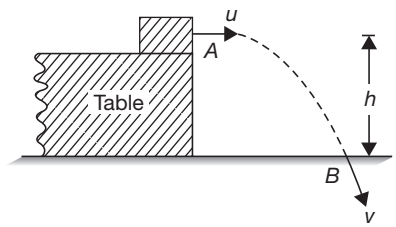
$\Rightarrow \quad v^2 = 4$

$\Rightarrow \quad v = 2 \text{ ms}^{-1}$

8. Let u be the speed of ice block when it leaves the table top. Then

$$\frac{1}{2}mu^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad u^2 = \left(\frac{k}{m}\right)x^2$$



If v be its speed when it reaches the floor, then
By Law of Conservation of Energy, we have

$$(U+K)_{\text{at A}} = (U+K)_{\text{at B}}$$

$$\Rightarrow \quad \frac{1}{2}mu^2 + mgh = \frac{1}{2}mv^2 + 0$$

$$\Rightarrow \quad v = \sqrt{u^2 + 2gh} = \sqrt{\frac{kx^2}{m} + 2gh}$$

Now, $x = 5 \text{ cm} = 0.05 \text{ m}$

$k = 2400 \text{ Nm}^{-1}$

$m = 120 \text{ g} = 0.12 \text{ kg}$

$h = 2.5 \text{ m}$

$$\Rightarrow \quad v = \sqrt{\frac{(2400)(0.05)^2}{0.12} + 2(10)(2.5)}$$

$$\Rightarrow \quad v = \sqrt{50+50} = 10 \text{ ms}^{-1}$$

9. (a) Increase in thermal energy of block-floor system is the work done against friction.

$$\Rightarrow \quad W_{\text{friction}} = \mu_K mgs = (0.25)(4)(10)(8)$$

$$\Rightarrow \quad W_{\text{friction}} \approx 80 \text{ J}$$

(b) Maximum kinetic energy of the block is the mechanical energy of spring block system which actually is the energy dissipated as heat $= 80$ J

(c) $$80 = \frac{1}{2}kx^2 = \frac{1}{2}(640)x^2$$

$$\Rightarrow \quad x = 0.5 \text{ m} = 50 \text{ cm}$$

10. Let ℓ_0 be the unstretched length of the spring, then by Law of Conservation of Energy, we have

$$(U+K)_{\text{at A}} = (U+K)_{\text{at B}}$$

$$\frac{1}{2}k(3-\ell_0)^2 + \frac{1}{2}mv_A^2 = \frac{1}{2}k(5-\ell_0)^2 + \frac{1}{2}mv_B^2$$

But $v_B = 0$

$$\Rightarrow \quad (3-\ell_0)^2 + 4 = (5-\ell_0)^2$$

$$\Rightarrow \quad \left(9 + \cancel{\ell_0^2} - 6\ell_0\right) + 4 = \left(25 + \cancel{\ell_0^2} - 10\ell_0\right)$$

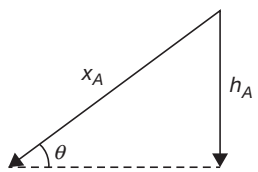
$$\Rightarrow \quad 16 - 4\ell_0 = 4$$

$$\Rightarrow \quad \ell_0 = 3 \text{ m}$$

11. Using our knowledge of constraint relations, we observe that at any instant $x_B = 2x_A$ and $v_B = 2v_A$. So, let $v_A = v$ after $x_A = 1 \text{ m}$. Using Work Energy Theorem for a non-conservative system, we get $W_{nc} = \Delta U + \Delta K$.

Before we make use of the above relation, we must observe that

(i) A move down by x_A, so its potential energy decreases by $m_A g h_A$ where $h_A = x_A \sin\theta = (1)\left(\frac{3}{5}\right) = \frac{3}{5}$ m.



(ii) B moves up by $x_B = h_B$, so its potential energy as well as kinetic energy increases.

$$\Rightarrow \quad f_K x_A \cos(180°) = \Delta U_A + \Delta K_A + \Delta U_B + \Delta K_B$$

where $f_K = \mu_K m_A g \cos\theta$, $\cos\theta = \frac{4}{5}$, $x_A = 1 \text{ m}$, $x_B = 2 \text{ m}$, $h_A = 0.6 \text{ m}$, $m_A = 30 \text{ kg}$, $m_B = 5 \text{ kg}$, $g = 10 \text{ ms}^{-2}$.

$$\Rightarrow \quad -(\mu_K m_A g \cos\theta)x_A = -m_A g h_A + m_B g h_B + \frac{1}{2}m_A v^2 + \frac{1}{2}m_B(2v)^2$$

CHAPTER 1

Substituting the values, we get

$$-\left(\frac{1}{5}\right)(300)\left(\frac{4}{5}\right)=-(300)\left(\frac{3}{5}\right)+(50)(2)+\frac{1}{2}(30)v^2+\frac{1}{2}(5)(2v)^2$$

$\Rightarrow \quad -48=-180+100+25\,v^2$

$\Rightarrow \quad v=\sqrt{\frac{32}{25}}=\frac{4\sqrt{2}}{5}$

$\Rightarrow \quad *=2$

12. $h=\ell\sin\theta=2\sin\theta$

$v^2=v_0^2+2gh=25+2\times10\times2\sin\theta$

$\Rightarrow \quad v^2=25+40\sin\theta$

Further, $T-mg\sin\theta=\frac{mv^2}{\ell}$

Given, $T=2mg$

$\Rightarrow \quad 2mg-mg\sin\theta=\frac{mv^2}{\ell}$

$\Rightarrow \quad 2(10)-10\sin\theta=\frac{25+40\sin\theta}{2}$

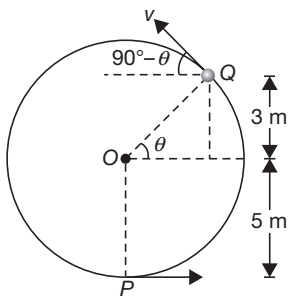
$\Rightarrow \quad \sin\theta=\frac{1}{4}$

$\Rightarrow \quad \theta=\sin^{-1}\left(\frac{1}{4}\right)$

So, $*=4$

13. $\sin\theta=\frac{3}{5}$

At Q, $T=0$ and hence the particle will become a projectile following a parabolic path.



$\Rightarrow \quad \frac{mv^2}{r}=mg\sin\theta$

$\Rightarrow \quad v=\sqrt{rg\sin\theta}=\sqrt{(5)(10)\left(\frac{3}{5}\right)}=\sqrt{30}\ \text{ms}^{-1}$

$\Rightarrow \quad *=30$

The maximum height of the projectile is

$$H=\frac{v^2\sin^2(90^\circ-\theta)}{2g}=\frac{v^2\cos^2\theta}{2g}$$

$\Rightarrow \quad H=\frac{(30)\left(\frac{16}{25}\right)}{2(10)}=\frac{24}{25}$

$\Rightarrow \quad H=0.96\ \text{m}=96\ \text{cm}$

$\Rightarrow \quad H=24x=96$

$\Rightarrow \quad x=4$

14. Speed of each particle at angle θ using Law of Conservation of Energy is

$v=\sqrt{2gh}$, where $h=R(1-\cos\theta)$

$\Rightarrow \quad v=\sqrt{2gR(1-\cos\theta)}$

Also, at angle θ, we have

$$N+mg\cos\theta=\frac{mv^2}{R}$$

$\Rightarrow \quad N+mg\cos\theta=2mg(1-\cos\theta)$

$\Rightarrow \quad N=2mg-3mg\cos\theta \qquad \ldots(1)$

When the tube just breaks its contact with the ground then

$2N\cos\theta=Mg$

Substituting the value of N from (1), we get

$4mg\cos\theta-6mg\cos^2\theta=Mg$

Substituting, $\theta=60^\circ$, we get

$2mg-\frac{3mg}{2}=Mg$

$\Rightarrow \quad \frac{m}{M}=2$

Conceptual Note(s)

Initially, the normal reaction on each ball will be radially outwards and later it will be radially inwards, so that the net normal reaction on the tube is radially outwards to break it off from the ground.

15. Let v be the speed of ball at point B. Then by Law of Conservation of Energy applied between A and B, we get

$U_A+K_A=U_B+K_B$

$\Rightarrow \quad 0+\frac{1}{2}(1)(5)^2=(1)(10)(1)+\frac{1}{2}(1)v^2$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{25}{2} - 10$$

$$\Rightarrow \quad v^2 = 5$$

Force acting on particle at B is

$$F = \sqrt{(mg)^2 + \left(\frac{mv^2}{r}\right)^2}$$

$$\Rightarrow \quad F = \sqrt{(10)^2 + \left(\frac{1 \times 5}{1}\right)^2}$$

$$\Rightarrow \quad F = \sqrt{125}$$

$$\Rightarrow \quad F = 5\sqrt{5} \text{ N}$$

$$\Rightarrow \quad x = 5$$

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1. After reading the problem, we conclude that

for $0 \le x \le 15$, $F = 200$ N and

for $15 \le x < 30$, $F = 200 - \frac{100}{15}(x - 15)$

$$\Rightarrow \quad F \text{ (in N)} = \begin{cases} 200 & \text{, for } 0 \le x \le 15 \\ 300 - \frac{100}{15}x & \text{, for } 15 \le x < 30 \end{cases}$$

Since, $W = \int F dx$

$$\Rightarrow \quad W = \int_0^{15} 200 dx + \int_{15}^{30} \left(300 - \frac{100}{15}x\right) dx$$

$$\Rightarrow \quad W = (200)(15) + (300)(15) - \left(\frac{100}{15}\right)\left(\frac{(30^2 - 15^2)}{2}\right)$$

$$\Rightarrow \quad W = 3000 + 4500 - 2250 = 5250 \text{ J}$$

Hence, the correct answer is (B).

2. Given that, $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$

$$\Rightarrow \quad F = -\frac{dU}{dr} = -A(-6r^{-7}) + B(-12r^{-13})$$

At equilibrium, $F = 0$

$$\Rightarrow \quad 0 = \frac{6A}{r^7} - \frac{12B}{r^{13}}$$

$$\Rightarrow \quad \frac{6A}{12B} = \frac{1}{r^6}$$

$$\Rightarrow \quad r = \left(\frac{2B}{A}\right)^{1/6}$$

So, $U_{eq} = U|_{r=\left(\frac{2B}{A}\right)^{1/6}} = -\frac{A}{(2B/A)} + \frac{B}{(4B^2/A^2)}$

$$\Rightarrow \quad U_{eq} = -\frac{A^2}{2B} + \frac{A^2}{4B} = -\frac{A^2}{4B}$$

Hence, the correct answer is (C).

3. Since, P = constant . From Work-Energy Theorem

$$W = \frac{1}{2}mv^2$$

Also, we know that $W = Pt$

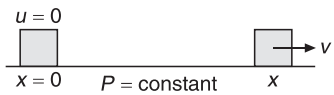
$$\Rightarrow \quad K = Pt = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \frac{ds}{dt} = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2P}{m}}t^{1/2}$$

$$\Rightarrow \quad s = \int ds = \sqrt{\frac{2P}{m}} \int_0^t t^{1/2} dt = \frac{2}{3}\sqrt{\frac{2P}{m}}t^{3/2}$$

Hence, the correct answer is (D).

4.



Since, P = constant , so from Work-Energy Theorem, we get $W = \frac{1}{2}mv^2$. Also, $W = Pt$

$$\Rightarrow \quad K = Pt = \frac{1}{2}mv^2 \text{ i.e., } v = \sqrt{\frac{2Pt}{m}}$$

$$\Rightarrow \quad v = \frac{dx}{dt} = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2P}{m}}t^{1/2}$$

$$\Rightarrow \quad x = \int dx = \sqrt{\frac{2P}{m}} \int_0^t t^{1/2} dt = \frac{2}{3}\sqrt{\frac{2P}{m}}t^{3/2}$$

$$\Rightarrow \quad x = \frac{2}{3}\sqrt{\frac{2(1)}{2}}(9)^{3/2} = \frac{2}{3}(1)(27) = 18 \text{ m}$$

Hence, the correct answer is (18).

5. Since, $W_F = \frac{1}{2}mv^2 = mgh$

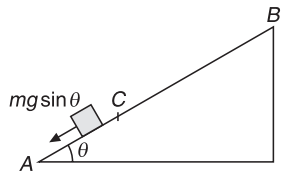
$$\Rightarrow \quad Fs = mgh$$

$$\Rightarrow \quad F(0.2) = (0.15)(10)(20)$$

$$\Rightarrow \quad F = 150 \text{ N}$$

Hence, the correct answer is (150).

6. Applying Work-Energy Theorem, we get



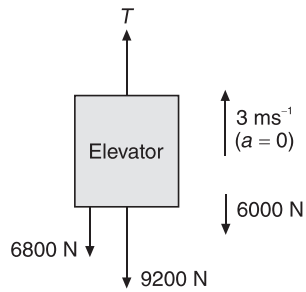
$$mg\sin\theta(AC+2AC)-(\mu mg\cos\theta)AC=0$$

$$\Rightarrow \quad \mu=3\tan\theta$$

$$\Rightarrow \quad k=3$$

Hence, the correct answer is (3).

7. Since the elevator is moving with a constant speed, so we have



$$T=6800+9200+6000=22000\ \text{N}$$

So, power delivered by the motor is

$$P=Tv=22000\times3=66000\ \text{W}$$

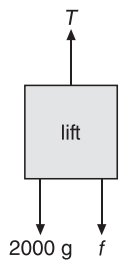
Hence, the correct answer is (C).

8. $$W=\int_{\vec{r}_i}^{\vec{r}_g}\vec{F}\cdot d\vec{r}=\int_1^0 -x\,dx+\int_0^1 y\,dy=\left(-\frac{x^2}{2}\right)\Big|_1^0+\left(\frac{y^2}{2}\right)\Big|_0^1$$

$$\Rightarrow \quad W=-\left(\frac{0^2}{2}-\frac{1^2}{2}\right)+\left(\frac{1^2}{2}-\frac{0^2}{2}\right)=1\ \text{J}$$

Hence, the correct answer is (B).

9. Let the elevator be moving up with a constant speed v. Then frictional force f will be downwards. So, tension in the cable is



$$T=2000g+f=2000+4000$$

$$\Rightarrow \quad T=24000\ \text{N}$$

Power of the electric motor of the elevator is

$$P=Fv=Tv$$

$$\Rightarrow \quad 60\times746=(24000)v$$

$$\Rightarrow \quad v=\frac{60\times746}{24000}=1.865\approx1.9\ \text{ms}^{-1}$$

Hence, the correct answer is (1.9).

10. Applying Law of Conservation of Mechanical Energy between A and B, we get

$$U_A+K_A=U_B+K_B$$

$$\Rightarrow \quad mg(2)=mg(1)+K_2$$

$$\Rightarrow \quad K_2=mg=10\ \text{J}$$

Hence, the correct answer is (10).

11. $W=\Delta K=$ Area under F-x graph

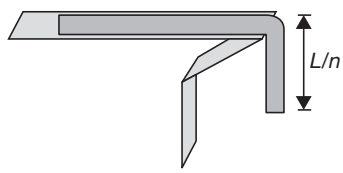
$$\Rightarrow \quad \Delta K=W=\frac{1}{2}\times(3+2)\times(3-2)+2\times2=2.5+4$$

$$\Rightarrow \quad \Delta K=6.5\ \text{J}$$

Hence, the correct answer is (D).

12. Mass of hanging portion is

$$m_1=\frac{M}{n}$$



So, to pull the complete hanging portion up, the centre of mass of hanging portion has to be pulled through $\frac{L}{2n}$.

$$\Rightarrow \quad W=\left(\frac{Mg}{n}\right)\left(\frac{L}{2n}\right)$$

$$\Rightarrow \quad W=\frac{MgL}{2n^2}$$

Hence, the correct answer is (C).

13. Initial equilibrium position of block is at $x=0$

Speed of the block will be maximum at equilibrium position i.e., when $F=kx_{\text{eq}}$

$$\Rightarrow \quad x_{\text{eq}}=\frac{F}{k}$$

Applying Law of Conservation of Energy, we get

$$(U+K)_i = (U+K)_f$$

$$\Rightarrow \quad Fx_{eq} = \frac{1}{2}kx_{eq}^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad F\left(\frac{F}{k}\right) = \frac{1}{2}k\left(\frac{F}{k}\right)^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{F^2}{2k}$$

$$\Rightarrow \quad v = \frac{F}{\sqrt{mk}}$$

Hence, the correct answer is (D).

14. $v = \dfrac{dx}{dt} = 6t$

$u = v(t=0) = 0$

$v = v(t = 5\text{ s}) = 30\text{ ms}^{-1}$

$$\Rightarrow \quad W = \Delta KE = \frac{1}{2}(2)(30)^2 = 900\text{ J}$$

Hence, the correct answer is (B).

15. $N - mg = \dfrac{mg}{2}$

$$\Rightarrow \quad N = \frac{3mg}{2}$$

Since, $s = \dfrac{1}{2}\left(\dfrac{g}{2}\right)t^2$

So, work done by normal reaction is

$$W_N = Ns\cos(0°) = Ns$$

$$\Rightarrow \quad W = \left(\frac{3mg}{2}\right)\left(\frac{g}{4}t^2\right)$$

$$\Rightarrow \quad W = \frac{3mg^2t^2}{8}$$

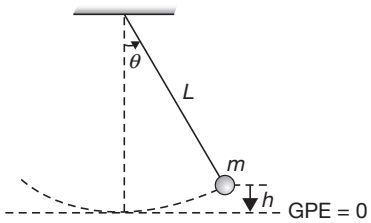
Hence, the correct answer is (A).

16. $W = K_f - K_i = \left(3\hat{i} - 12\hat{j}\right)\cdot 4\hat{i} = 12\text{ J}$

$$\Rightarrow \quad K_f = 12 + K_i = 12 + 3 = 15\text{ J}$$

Hence, the correct answer is (A).

17.



By Law of Conservation of Energy,

$$\begin{pmatrix}\text{Loss in GPE}\\ \text{of pendulum}\end{pmatrix} = \begin{pmatrix}\text{Gain in KE}\\ \text{of pendulum}\end{pmatrix}$$

$$\Rightarrow \quad mgh = \frac{1}{2}mv^2$$

$$\Rightarrow \quad K_1 = mgh = mgL(1-\cos\theta)$$

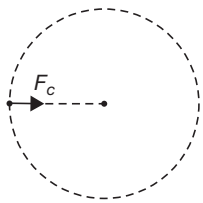
Now, when L is doubled, keeping θ same, then Kinetic energy is also doubled. So

$$K_2 = 2K_1$$

Hence, the correct answer is (A).

18. Force on particle in the given potential field is given as

$$\vec{F} = -\left(\frac{dU}{dr}\right)\hat{r} = \frac{k}{r^3}\hat{r}$$



For body to move in a circle, we have

$$F_C = \frac{k}{r^3} = \frac{mv^2}{r}$$

$$\Rightarrow \quad mv^2 = \frac{k}{r^2}$$

$\Rightarrow$ The kinetic energy of particle is

$$KE = \frac{1}{2}mv^2 = \frac{k}{2r^2}$$

Total energy of particle $TE = KE + PE$

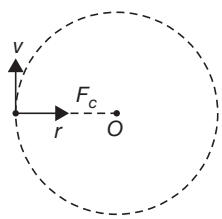
$$\Rightarrow \quad E = \frac{k}{2r^2} - \frac{k}{2r^2} = 0$$

Hence, the correct answer is (C).

19. For particle to move at speed v in a circular orbit, we have

$$F_C = \left|\frac{16}{r} + r^3\right| = \frac{mv^2}{r}$$

$$\Rightarrow \quad mv^2 = \left(\frac{16}{r} + r^3\right)(r) = 16 + r^4$$



So, kinetic energy of particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(16 + r^4)$$

Ratio of kinetic energy of the two particles is given by

$$\frac{K_1}{K_2} = \frac{\frac{1}{2}(16 + (1)^4)}{\frac{1}{2}(16 + (4)^4)} = \frac{16 + 1}{16 + 256}$$

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{17}{272} = 0.06$$

Hence, the correct answer is (B).

20. $v = a\sqrt{s}$

$$\Rightarrow \quad \frac{ds}{dt} = a\sqrt{s}$$

$$\Rightarrow \quad \int_0^s \frac{ds}{\sqrt{s}} = \int_0^t a\,dt$$

$$\Rightarrow \quad 2\sqrt{s} = at$$

$$\Rightarrow \quad s = \frac{a^2t^2}{4}$$

Velocity at any time t is given by

$$v = \frac{ds}{dt} = \frac{a^2t}{2}$$

Applying Work Energy theorem, we get

$$W = \Delta K$$

$$\Rightarrow \quad W = \frac{1}{2}mv^2 - 0 = \frac{1}{2}m\left(\frac{a^4t^2}{4}\right) = \frac{1}{8}ma^4t^2$$

Hence, the correct answer is (C).

21. Since, $\frac{K_f}{K_i} = \frac{\frac{1}{8}mv_0^2}{\frac{1}{2}mv_0^2} = \frac{1}{4}$

$$\Rightarrow \quad \frac{v_f}{v_i} = \frac{1}{2}$$

$$\Rightarrow \quad v_f = \frac{v_i}{2} = \frac{v_0}{2}$$

Since, $F = -kv^2 = \frac{mdv}{dt}$

$$\Rightarrow \quad \int_{v_0}^{\frac{v_0}{2}} \frac{dv}{v^2} = \int_0^{t_0} \frac{-k\,dt}{m}$$

$$\Rightarrow \quad \left(-\frac{1}{v}\right)\Big|_{v_0}^{\frac{v_0}{2}} = -\frac{k}{m}t_0$$

$$\Rightarrow \quad \frac{1}{v_0} - \frac{2}{v_0} = -\frac{k}{m}t_0$$

$$\Rightarrow \quad -\frac{1}{v_0} = -\frac{k}{m}t_0$$

$$\Rightarrow \quad k = \frac{m}{v_0t_0}$$

$$\Rightarrow \quad k = \frac{10^{-2}}{10 \times 10}$$

$$\Rightarrow \quad k = 10^{-4}\ \text{kgm}^{-1}$$

Hence, the correct answer is (C).

22. $F = 6t = m\frac{dv}{dt}$ where $m = 1$ kg

$$\Rightarrow \quad \int_0^v dv = \int_0^1 6t\,dt$$

$$\Rightarrow \quad v = 6\left(\frac{t^2}{2}\right)\Big|_0^1$$

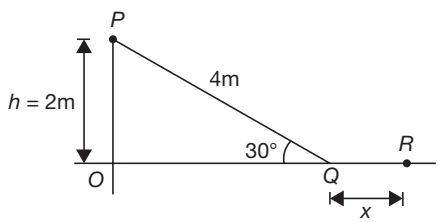
$$\Rightarrow \quad v = 3\ \text{ms}^{-1}$$

According to Work Energy Theorem, we have

$$W = \Delta KE = \frac{1}{2}(1)(9) = 4.5\ \text{J}$$

Hence, the correct answer is (A).

23.



Energy lost over path PQ is

$$E_1 = \mu mg\cos\theta \times 4$$

Energy lost over path QR is

$$E_2 = \mu mgx$$

Since $E_1 = E_2$

$$\Rightarrow \quad \mu mgx = \mu mg\cos\theta \times 4$$

$$\Rightarrow \quad x = \cos\theta \times 4 = 4\cos(30°)$$

$$\Rightarrow \quad x = 2\sqrt{3} = 3.45\ \text{m}$$

From Q to R energy loss is half of the total energy loss, so

$$\mu mgx = \frac{1}{2}(mgh)$$

$$\Rightarrow \quad \mu = 0.29$$

Hence, the correct answer is (D).

24. Work done against gravity in lifting the mass 1000 times is

$$W = (1000)(mgh)$$

$$\Rightarrow \quad W = 10 \times 9.8 \times 10^3$$

$$\Rightarrow \quad W = 9.8 \times 10^4 \text{ Joule}$$

Let M kg of fat be used up in the process. Then energy supplied by fat is

$$E = (3.8 \times 10^7 \text{ M})\frac{20}{100} \text{ J}$$

This energy supplied by the fat must equal to the work done by him against gravity. So,

$$\frac{20}{100}(3.8 \times 10^7 \text{ M}) = 9.8 \times 10^4$$

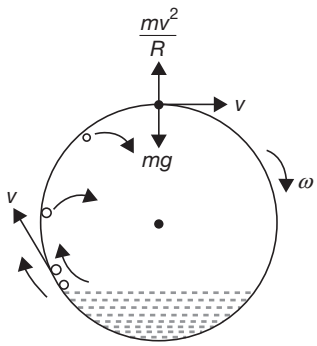
$$\Rightarrow \quad M = 12.89 \times 10^{-3} \text{ kg}$$

Hence, the correct answer is (A).

25. In the frame of rotating drum, at the topmost point the mixture will not loose contact with the drum wall if

$$\frac{mv^2}{r} \geq mg$$

$$\Rightarrow \quad v \geq \sqrt{gR}$$



If $v \geq \sqrt{gR}$, then proper mixing will not take place

Thus, for proper mixing $v < \sqrt{gR}$

$$\Rightarrow \quad \omega = \frac{v}{R} = \sqrt{\frac{g}{R}} = \sqrt{\frac{10}{1.25}} \text{ rads}^{-1}$$

ω in rpm is $\omega_{RPM} = \frac{60}{2\pi}\sqrt{\frac{10}{1.25}} \approx 27$ rpm

So, if then before reaching top, ingredients fall and mix proper.

Hence, the correct answer is (A).

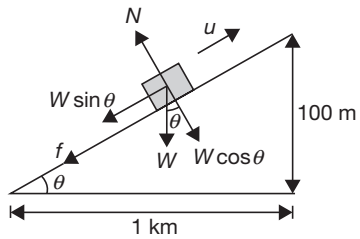
26. Inclination of road, $\theta = \tan^{-1}\left(\frac{100}{1000}\right) = \tan^{-1}\left(\frac{1}{10}\right)$

$$\Rightarrow \quad \tan\theta = \frac{1}{10}$$

$$\Rightarrow \quad \tan\theta \approx \sin\theta = \frac{1}{10} \text{ (for very small value of } \theta\text{)}$$

When car is moving uphill with a speed of u=10 ms^{-1}, then

$$F = W\sin\theta + f$$



$$\Rightarrow \quad P = Fu = (W\sin\theta + f)u$$

$$\Rightarrow \quad P = \left(\frac{W}{10} + \frac{W}{20}\right) \times 10$$

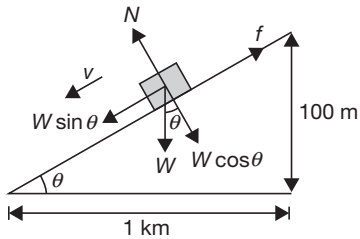
$$\Rightarrow \quad P = \frac{3W}{20} \times 10 = \frac{3W}{2}$$

When car is moving downhill with a constant speed, say v, then

$$F' = W\sin\theta - f$$

$$\Rightarrow \quad \frac{P}{2} = F'v = (W\sin\theta - f)v$$

$$\Rightarrow \quad \frac{3W}{4} = \left(\frac{W}{10} - \frac{W}{20}\right)v$$



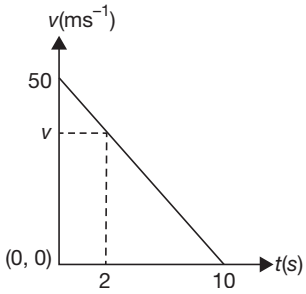
$$\Rightarrow \quad \frac{3W}{4} = \frac{W}{20}v$$

$$\Rightarrow \quad v = 15 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

CHAPTER 1

27.



Given that, $m = 10$ kg , $t = 2$ s

$u = 50$ ms^{-1} at $t = 0$ s

Since, $a = \dfrac{\Delta v}{\Delta t} = \dfrac{50-0}{0-10} = -5$ ms^{-2}

So, speed of the body at $t = 2$ s is given by

$v = u + at = 50 + (-5) \times 2 = 40$ ms^{-1}

Using Work-Energy theorem i.e. $W = \Delta K$, we get

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}(10)(40)^2 - \frac{1}{2}(10)(50)^2$$

$\Rightarrow \quad W = 5(40^2 - 50^2) = 5 \times (40-50)(40+50)$

$\Rightarrow \quad W = -4500$ J

Hence, the correct answer is (C).

28. Centripetal acceleration is $a_c = n^2Rt^2 = \dfrac{v_T^2}{R}$

$\Rightarrow \quad v_T^2 = n^2R^2t^2$

$\Rightarrow \quad v_T = nRt$

Tangential force on the particle is

$$F_T = M\frac{dv_T}{dt} = MnR$$

Power delivered to the particle is

$$P = F_T v_T = (MnR)(nRt)$$

$\Rightarrow \quad P = Mn^2R^2t$

Hence, the correct answer is (B).

29. The block comes momentarily to rest means that its velocity at that point was also 3 ms^{-1} .

$$\begin{pmatrix}\text{Loss in EPE}\\ \text{of Spring}\end{pmatrix} = \begin{pmatrix}\text{Gain in KE}\\ \text{of Block}\end{pmatrix}$$

$\Rightarrow \quad \dfrac{1}{2}k\left(x^2 - \dfrac{x^2}{4}\right) = \dfrac{1}{2}mv^2$

$\Rightarrow \quad \dfrac{1}{2}kx^2\left(1 - \dfrac{1}{4}\right) = \dfrac{1}{2}mv^2$

$\Rightarrow \quad \dfrac{1}{2}kx^2 = U_i = \dfrac{4}{6}mv^2 = \dfrac{2}{3}mv^2$

$\Rightarrow \quad U_i = \dfrac{2}{3}(0.1)(9) = 0.6$ J

Hence, the correct answer is (B).

30. Centripetal force $F = \alpha r^2$

$\Rightarrow \quad \dfrac{mv^2}{r} = \alpha r^2$

So, kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 = \frac{\alpha r^3}{2}$$

Potential energy of the particle is

$$U = \int_0^r F dr = \int_0^r \alpha r^2 dr = \frac{\alpha r^3}{3}$$

Total energy of the particle is

$$E = K + U = \frac{\alpha r^3}{2} + \frac{\alpha r^3}{3}$$

$\Rightarrow \quad E = \dfrac{5}{6}\alpha r^3$

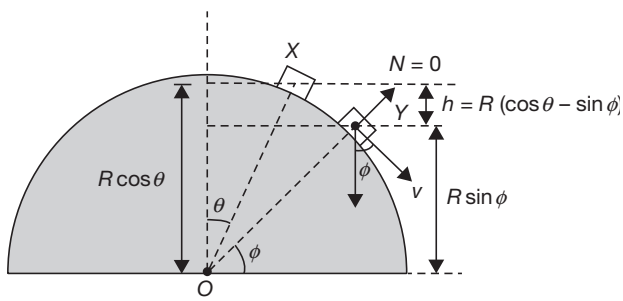
Hence, the correct answer is (D).

31. $W = \displaystyle\int_0^L F dx = \int_0^L (ax + bx^2)dx$

$\Rightarrow \quad W = \dfrac{aL^2}{2} + \dfrac{bL^3}{3}$

Hence, the correct answer is (C).

32.



Applying Energy Theorem, we get

$$mgh = \frac{1}{2}mv^2$$

$\Rightarrow \quad v = \sqrt{2gR(\cos\theta - \sin\phi)}$

At point Y, we have along radial direction in circular motion

$$\frac{mv^2}{R} = mg\sin\phi$$

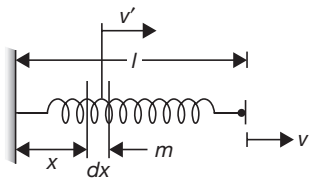
$\Rightarrow \quad 2mg(\cos\theta - \sin\phi) = mg\sin\phi$

$\Rightarrow \quad 2\cos\theta = 3\sin\phi$

Hence, the correct answer is (C).

33. To find KE of spring, we consider an elemental part of spring as shown.

Mass of element is $dm = \frac{m}{\ell}dx$



As spring is uniformly stretched, the speed v' of element is

$$v' = \left(\frac{v}{\ell}\right)x$$

Kinetic energy of element is $dK = \frac{1}{2}dmv'^2$

$\Rightarrow \quad dK = \frac{1}{2}\left(\frac{m}{\ell}dx\right)\left(\frac{v}{\ell}x\right)^2$

$\Rightarrow \quad \int dK = \frac{1}{2}\frac{mv^2}{\ell^3}\int_0^{\ell} x^2 dx$

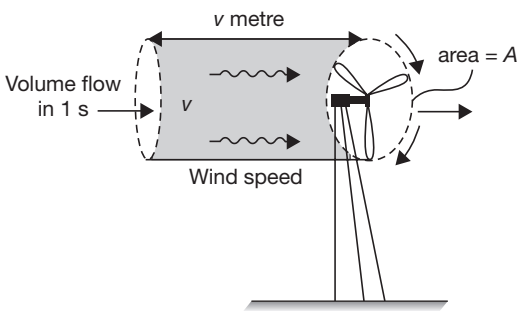
Total kinetic energy of spring is given as

$$K = \frac{1}{2}\frac{mv^2}{\ell^3}\left(\frac{\ell^3}{3}\right) = \frac{1}{6}mv^2$$

Hence, the correct answer is (D).

34. Wind volume flowing through the cross-sectional area of blades (rotating) of wind mill per unit time is given by

$$V = Av$$



If ρ be the density of air, then mass of wind intercepting the blades per second is

$$m = \rho Av$$

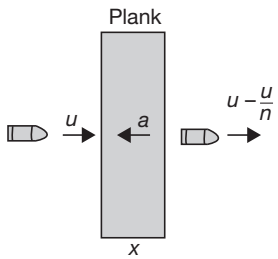
Kinetic energy of wind mass intercepting the section of blades per second is

$$\frac{dK}{dt} = \frac{1}{2}mv^2 = \frac{1}{2}\rho Av^3$$

$\Rightarrow$ Electrical power $\propto v^3$

Hence, the correct answer is (D).

35. If plank thickness is x, then, we have

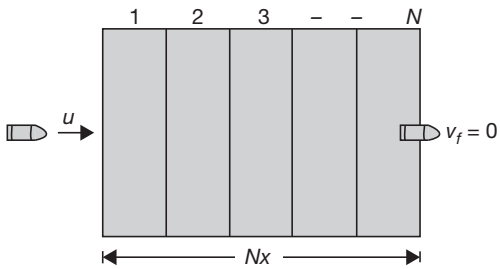


$$u^2\left(1-\frac{1}{n}\right)^2 = u^2 - 2ax$$

$\Rightarrow \quad 2ax = u^2\left[1-\left(\frac{n-1}{n}\right)^2\right]$

$\Rightarrow \quad 2ax = u^2\left[\frac{2n-1}{n^2}\right]$

Using constant retardation, we have



$$0 = u^2 - 2a(Nx)$$

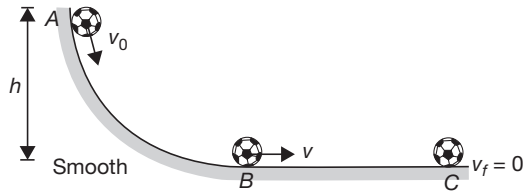
$\Rightarrow \quad u^2 = Nu^2\left[\frac{2n-1}{n^2}\right]$

$\Rightarrow \quad N = \frac{n^2}{2n-1}$

Hence, the correct answer is (A).

CHAPTER 1

36. Applying Work-Energy Theorem from point A to C, we get



$$\frac{1}{2}mv_0^2 + mgh - \mu mgL = 0$$

$$\Rightarrow \quad L = \frac{v_0^2}{2\mu g} + \frac{h}{\mu}$$

Hence, the correct answer is (B).

37. Maximum energy loss is

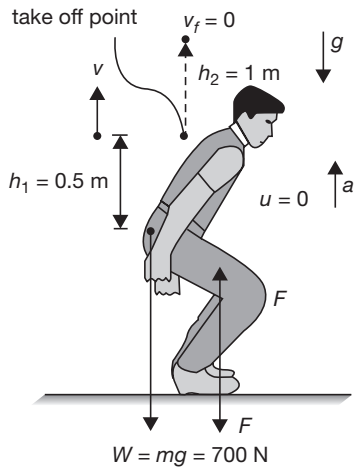
$$\frac{1}{2}\frac{Mm}{M+m}(v-0)^2 = \frac{M}{M+m}\left(\frac{1}{2}mv^2\right)$$

So, Statement-I is wrong.

Hence, the correct answer is (D).

38. If a is upward acceleration of man before leaping, then

$$v = \sqrt{2ah_1} = \sqrt{2gh_2}$$



$$\Rightarrow \quad a = \frac{gh_2}{h_1} = \frac{10\times 1}{0.5} = 20 \text{ ms}^{-2}$$

Velocity at take-off is

$$v = \sqrt{2\times 10\times 1} = \sqrt{20} \text{ ms}^{-1}$$

Power delivered by muscles is

$$P = Fv = mav$$

$$\Rightarrow \quad P = 70\times 20\times\sqrt{20}$$

$$\Rightarrow \quad P = 6.26\times 10^3 \text{ W}$$

Hence, the correct answer is (B).

39. For the same force, $F = k_1x_1 = k_2x_2$ …(1)

Work done on spring S_1 is

$$W_1 = \frac{1}{2}k_1x_1^2 = \frac{(k_1x_1)^2}{2k_1} = \frac{F^2}{2k_1} \quad \{\text{using (1)}\}$$

Work done on spring S_2 is

$$W_2 = \frac{1}{2}k_2x_2^2 = \frac{(k_2x_2)^2}{2k_2} = \frac{F^2}{2k_2} \quad \{\text{using (1)}\}$$

$$\Rightarrow \quad \frac{W_1}{W_2} = \frac{k_2}{k_1}$$

Since, $W_1 > W_2$

$$\Rightarrow \quad k_2 > k_1$$

$$\Rightarrow \quad k_1 < k_2$$

Statement 2 is true.

For the same extension, i.e. for $x_1 = x_2 = x$ …(2)

Work done on spring S_1 is $W_1 = \frac{1}{2}k_1x_1^2 = \frac{1}{2}k_1x^2$

Work done on spring S_2 is $W_2 = \frac{1}{2}k_2x_2^2 = \frac{1}{2}k_2x^2$

$$\Rightarrow \quad \frac{W_1}{W_2} = \frac{k_1}{k_2}$$

Since, $k_1 < k_2$

$$\Rightarrow \quad W_1 < W_2$$

Statement 1 is false.

Hence, the correct answer is (D).

40. By Work Energy Theorem

$$W = \vec{F}\cdot\Delta\vec{r} = \Delta K$$

$$\Rightarrow \quad \Delta K = \left(7\hat{i} + 4\hat{j} + 3\hat{k}\right)\cdot\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

$$\Rightarrow \quad \Delta K = 14 + 12 + 12 = 38 \text{ J}$$

Hence, the correct answer is (A).

41. As $x \to \infty$, $U(x) = 0$

At equilibrium, force on atoms is $F = 0$

$$\Rightarrow \quad F = -\frac{dU}{dx} = \frac{12a}{x^{13}} - \frac{6b}{x^7} = 0$$

$$\Rightarrow \quad x^6 = \frac{2a}{b}$$

$$\Rightarrow \quad x = \left(\frac{2a}{b}\right)^{\frac{1}{6}}$$

Potential energy at equilibrium is given by

$$U_{\text{eqbm}} = \frac{a}{\left(\frac{2a}{b}\right)^{\frac{12}{6}}} - \frac{b}{\left(\frac{2a}{b}\right)^{\frac{6}{6}}}$$

$$\Rightarrow \quad U_{\text{eqbm}} = \frac{a}{\left(\frac{4a^2}{b^2}\right)} - \frac{b}{\frac{2a}{b}}$$

$$\Rightarrow \quad U_{\text{eqbm}} = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$$

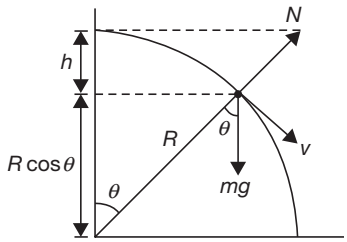
$$\Rightarrow \quad D = U_{\infty} - U_{\text{eqbm}} = 0 - \left(-\frac{b^2}{4a}\right) = \frac{b^2}{4a}$$

Hence, the correct answer is (D).

ARCHIVE: JEE ADVANCED

Single Correct Choice Type Problems

1. Let at any instant, the bead make an angle θ with the vertical, as shown in Figure.



Since $h = R - R\cos\theta$

Applying Law of Conservation of energy, we get

$$mgR(1-\cos\theta) = \frac{1}{2}mv^2$$

According to Newton's Second Law, we have

$$\Sigma F_{\text{towards the centre}} = ma_c = \frac{mv^2}{R}$$

$$\Rightarrow \quad mg\cos\theta - N = \frac{mv^2}{R}$$

where, N is the normal force on bead by the wire

$$\Rightarrow \quad N = mg\cos\theta - \frac{mv^2}{R}$$

Since $v^2 = 2gR(1-\cos\theta)$

$$\Rightarrow \quad N = mg(3\cos\theta - 2)$$

Now, $N = 0$ at $\cos\theta = \frac{2}{3}$

The normal force N will act radially outwards on bead, when

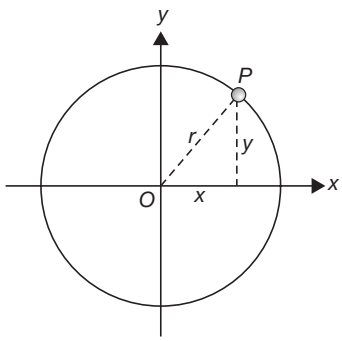
$\cos\theta > \frac{2}{3}$ and normal force N will act radially inwards on bead, when

$$\cos\theta < \frac{2}{3}$$

So, force acting on ring is opposite to normal force acting on bead.

Hence, the correct answer is (C).

2.



Since, $\vec{r} = \overrightarrow{OP} = x\hat{i} + y\hat{j}$

$$\Rightarrow \quad \vec{F} = \frac{k}{\left(x^2+y^2\right)^{\frac{3}{2}}}\left(x\hat{i} + y\hat{j}\right) = \frac{k}{r^3}(\vec{r})$$

$$\Rightarrow \quad \vec{F} \parallel \vec{r}$$

Since, $\vec{F}$ is along $\vec{r}$ or in radial direction, so work done is zero.

Hence, the correct answer is (D).

3. Area under F-t graph (A) is equal to change in momentum. So,

$$p = \sqrt{2mK}$$

$$\Rightarrow \quad K = \frac{p^2}{2m} = \frac{A^2}{2m}$$

$$\Rightarrow \quad K = \frac{A^2}{2m}$$

$$\text{Area } (A) = \frac{1}{2}(3)(4) - \frac{1}{2}(1.5)(2)$$

$$\Rightarrow \quad A = 6 - 1.5 = 4.5 \text{ kgms}^{-1}$$

$$\Rightarrow \quad K = \frac{A^2}{2m} = \frac{(4.5)^2}{2(2)}$$

$$\Rightarrow \quad K = 5.0625 \text{ J}$$

Hence, the correct answer is (C).

4. Applying Law of Conservation of Energy, we get

$$\frac{1}{2}kx^2 = \frac{1}{2}(4k)y^2$$

$$\Rightarrow \quad \frac{y}{x} = \frac{1}{2}$$

Hence, the correct answer is (C).

5. Since horizontal velocity at A is sufficient enough to make the bob reach to B, so

$$v = \sqrt{5gL} \qquad \ldots(1)$$

When speed of bob is half this speed, then

$$\left(\frac{v}{2}\right)^2 = v^2 - 2gh \qquad \ldots(2)$$

Since, $h = L(1 - \cos\theta)$ $\ldots(3)$

Solving equations (1), (2) and (3), we get

$$\cos\theta = -\frac{7}{8}$$

$$\Rightarrow \quad \theta = \cos^{-1}\left(-\frac{7}{8}\right) = 151°$$

i.e., $\frac{3\pi}{4} < \theta < \pi$

Hence, the correct answer is (D).

6. Since, $F = -\frac{dU}{dx}$

$$\Rightarrow \quad \int_0^{U(x)} dU = -\int_0^x F dx = -\int_0^x (kx)dx$$

$$\Rightarrow \quad U(x) = -\frac{kx^2}{2} \qquad \{\because U(0) = 0\}$$

Hence, the correct answer is (A).

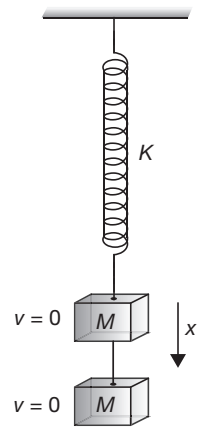
7. Since, gravitational field is a conservative force field in which the work done is independent of path followed between two points, so

$$W_1 = W_2 = W_3$$

Hence, the correct answer is (B).

8. Let x be the maximum extension of the spring. Then, by Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Decrease in}\\ \text{Gravitational}\\ \text{Potential Energy}\end{pmatrix} = \begin{pmatrix}\text{Increase in}\\ \text{Elastic}\\ \text{Potential Energy}\end{pmatrix}$$



$$\Rightarrow \quad Mgx = \frac{1}{2}kx^2$$

$$\Rightarrow \quad x = \frac{2Mg}{k}$$

Hence, the correct answer is (B).

9. $F = -\frac{dU}{dx}$

$$\Rightarrow \quad dU = -Fdx$$

$$\Rightarrow \quad U(x) = -\int_0^x (-kx + ax^3)dx$$

$$\Rightarrow \quad U(x) = \frac{kx^2}{2} - \frac{ax^4}{4}$$

Now, $U(x) = 0$ at $x = 0$ and $x = \sqrt{\frac{2k}{a}}$

$$\Rightarrow \quad U(x) \text{ is negative for } x > \sqrt{\frac{2k}{a}}$$

From the given function we can see that $F = 0$ at $x = 0$ i.e., slope of U-x graph is zero at $x = 0$. Therefore, the most appropriate OPTION is (D).

Hence, the correct answer is (D).

10. Since $P = Fv$

In this case, $F = v\dfrac{dm}{dt}$

If ρ be the density of wind, then

$$\frac{dm}{dt} = \rho\left(\frac{dV}{dt}\right)$$

where $\dfrac{dV}{dt}$ is the volume of wind flowing per second through the generator.

Since $\dfrac{dV}{dt} = A\left(\dfrac{dx}{dt}\right) = Av$

where A is the area of the generator through which wind flows

$$\frac{dm}{dt} = \rho\left(\frac{dV}{dt}\right) = Av\rho$$

$\Rightarrow \quad F = v\left(\dfrac{dm}{dt}\right) = v(Av\rho)$

$\Rightarrow \quad F = Av^2\rho$

$\Rightarrow \quad P = Fv = Av^3\rho$

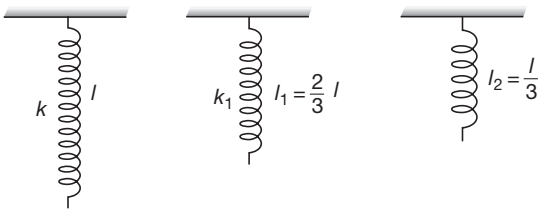
$\Rightarrow \quad P \propto v^3$

Hence, the correct answer is (C).

11. $\ell_1 = 2\ell_2$

If ℓ is the total natural length of the spring, then

$\therefore \quad \ell_1 = \dfrac{2}{3}\ell$ and $\ell_2 = \dfrac{\ell}{3}$



Since for a spring $k\ell$ = constant

$\Rightarrow \quad k_1\ell_1 = k_2\ell_2 = k\ell$

$\Rightarrow \quad k_1 = \dfrac{3}{2}k$ and $k_2 = 3k$

Hence, the correct answer is (B).

13. Since

$dW = \vec{F}\cdot d\vec{r}$, where

$d\vec{r} = (dx)\hat{i} + (dy)\hat{j}$

$\Rightarrow \quad dW = -k(ydx + xdy)$

$\Rightarrow \quad dW = -kd(xy) \qquad \{\because d(xy) = x\,dy + y\,dx\}$

$\Rightarrow \quad W = \int dW = -k\int_{(0,0)}^{(a,a)} d(xy)$

$\Rightarrow \quad W = -k(xy)\Big|_{(0,0)}^{(a,a)}$

$\Rightarrow \quad W = -ka^2 - 0$

$\Rightarrow \quad W = -ka^2$

Hence, the correct answer is (C).

14. $a_c = k^2rt^2$

$\Rightarrow \quad \dfrac{v^2}{r} = k^2rt^2$

$\Rightarrow \quad v = krt$

So, tangential acceleration, $a_T = \dfrac{dv}{dt} = kr$

$\Rightarrow$ Tangential force, $F_T = ma_T = mkr$

Since, only the tangential force does work, so

Power $= F_Tv = (mkr)(krt)$

$\Rightarrow$ Power $= mk^2r^2t$

Hence, the correct answer is (B).

15. The centre of mass of part hanging is at $\dfrac{L}{6}$. At this point a mass of $\dfrac{M}{3}$ exists.

So, work done to drag a weight $\dfrac{Mg}{3}$ up is

$$W = \left(\frac{Mg}{3}\right)\left(\frac{L}{6}\right)$$

$\Rightarrow \quad W = \dfrac{MgL}{18}$

Hence, the correct answer is (D).

17. $p = \sqrt{2Km}$

$\Rightarrow \quad p \propto \sqrt{m}$

Since, $\dfrac{m_1}{m_2} = \dfrac{1}{4}$

$\Rightarrow \quad \dfrac{p_1}{p_2} = \dfrac{1}{2}$

Hence, the correct answer is (C).

18. The correct answer is (B).

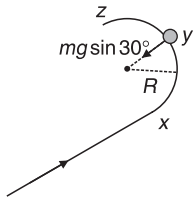
CHAPTER 1

Multiple Correct Choice Type Problems

1. At point y, we have

$$mg\sin 30° = \frac{mv^2}{R}$$

$$\Rightarrow \quad v^2 = \frac{gR}{2}$$



Applying conservation of mechanical Energy, we get

$$\frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v_0^2 - 2gh = \frac{gR}{2}$$

Along the path xyz, speed is maximum at points x and z due to which maximum centripetal force will be required.

Hence, (A) and (D) are correct.

2. The given case is of uniform circular motion. In which speed or kinetic energy remain constant. However, the direction of velocity and acceleration keep on changing although its magnitude remains constant.

Hence, (C) and (D) are correct.

Reasoning Based Questions

1. **For Statement 1**, decrease in mechanical energy in CASE-1 will be

$$\Delta U_1 = \frac{1}{2}mv^2$$

However, decrease in mechanical energy in CASE-2 will be

$$\Delta U_2 = \frac{1}{2}mv^2 - mgh$$

$$\Rightarrow \quad \Delta U_2 < \Delta U_1$$

So, Statement 1 is True.

For Statement 2, coefficient of friction will not change and hence this statement is False.

Hence, the correct answer is (C).

Matrix Match/Column Match Type Questions

1. (A) → (p, q, r, t); (B) → (q, s); (C) → (p, q, r, s); (D) → (p, r, t)

METHOD-1: ANALYTICAL METHOD

(A) $\vec{F} = -\left(\frac{dU}{dx}\right)\hat{i} = -\frac{U_0}{2}2\left(1-\left(\frac{x}{a}\right)^2\right)\times\left[-2\left(\frac{x}{a}\right)\times\frac{1}{a}\right]\hat{i}$

$$\Rightarrow \quad F = 2U_0\left[1-\left(\frac{x}{a}\right)^2\right]\left(\frac{x}{a^2}\right)$$

When $x = 0$, $\vec{F} = \frac{U_0}{2}[2(1)\times 0] = \vec{0}$ and $U = \frac{U_0}{2}$

When $x = a$, $\vec{F} = \vec{0}$ and $U = 0$

When $x = -a$, $\vec{F} = \vec{0}$ and $U = 0$

(B) $\vec{F} = -\left(\frac{dU}{dx}\right)\hat{i} - \frac{U_0}{2}\times 2\left(\frac{x}{a}\right)\times\frac{1}{a}\hat{i} = -\frac{U_0 x}{a^2}\hat{i}$

When $x = 0$, $\vec{F} = 0$ and $U = 0$

When $x = a$, $\vec{F} = -\frac{U_0}{a}\hat{i}$ and $U = \frac{U_0}{2}$

When $x = -a$, $\vec{F} = +\frac{U_0}{a}\hat{i}$ and $U = \frac{U_0}{2}$

(C) $\vec{F} = -\left(\frac{dU}{dx}\right)\hat{i}$

$$\Rightarrow \quad F = \frac{U_0 e^{-\frac{x^2}{a^2}}}{a^4}x(x^2 - a^2)$$

When $x = 0$, $F = 0$ and $U = 0$

When $x = \pm a$, $F = 0$ and $U = \frac{U_0}{2e}$

For $|x| < a$ i.e., $-a < x < a$, the force is attractive in nature.

(D) $\vec{F} = -\left(\frac{dU}{dx}\right)\hat{i}$

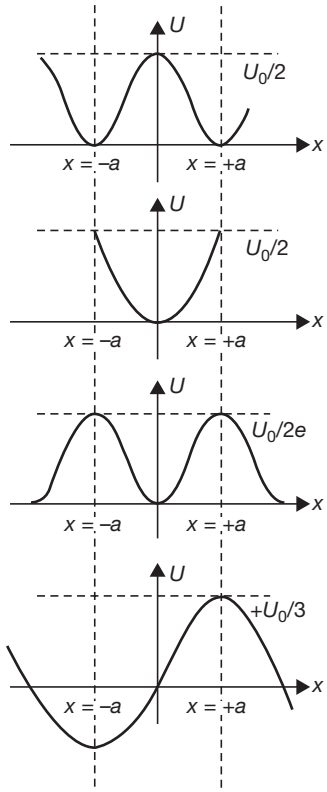
$$\Rightarrow \quad F = -\frac{U_0}{2}\left(\frac{1}{a} - \frac{x^2}{a^3}\right)$$

$$\Rightarrow \quad F = -\frac{U_0}{2a}\left(1-\frac{x^2}{a^2}\right) = \frac{U_0}{2a^3}(x^2 - a^2)$$

At $x = 0$, $F = -\frac{U_0}{2a}$ and $U = 0$

At $x = \pm a$, $F = 0$ and $U = \frac{U_0}{3}$

METHOD-2: GRAPHICAL METHOD



Linked Comprehension Type Questions

1. Height fallen up to point Q

$$h = R\sin 30° = 40 \times \frac{1}{2} = 20 \text{ m}$$

Since, $W_{nc} = \Delta U + \Delta K$, so we get

$$W_{nc} = -mgh + \frac{1}{2}mv^2$$

$$\Rightarrow \quad W_{nc} = -(1)(10)(20) + \frac{1}{2}(1)v^2$$

Given that $W_{nc} = -150$ J

$$\Rightarrow \quad -150 = -200 + \frac{v^2}{2}$$

$$\Rightarrow \quad v^2 = 100$$

$$\Rightarrow \quad v = 10 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

2. At point Q, component of weight along PQ (radially outwards) is $mg\cos 60°$ i.e., $\frac{mg}{2}$.

Normal reaction is radially inwards, so we have

$$N - \frac{mg}{2} = \frac{mv^2}{R}$$

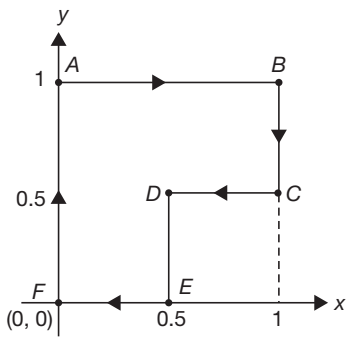
$$\Rightarrow \quad N = \frac{mg}{2} + \frac{mv^2}{R}$$

$$\Rightarrow \quad N = \frac{1\times 10}{2} + \frac{1\times (10)^2}{40} = 7.5 \text{ N}$$

Hence, the correct answer is (A).

Integer/Numerical Answer Type Questions

1. Given that, $\vec{F} = \alpha y\hat{i} + 2\alpha x\hat{j}$ where $\alpha = -1$



Since Work done is given by

$$W = \int \vec{F} \cdot d\vec{\ell}$$

$$\Rightarrow \quad dW = \alpha(ydx + 2xdy)$$

For $A \to B$, $y = 1$, so $dy = 0$

$$\Rightarrow \quad W_{A\to B} = \alpha \int ydx = \alpha \int_0^1 dx = \alpha$$

For $B \to C$, $x = 0$, so $dx = 0$

$$\Rightarrow \quad W_{B\to C} = 2\alpha \int_1^{0.5} dy = 2\alpha(-0.5) = -\alpha$$

For $C \to D$, $y = 0.5$, so $dy = 0$

$$\Rightarrow \quad W_{C\to D} = 0.5\alpha \int_1^{0.5} dx = -\frac{\alpha}{4}$$

For $D \to E$, $x = 0.5$, so $dx = 0$

$$\Rightarrow \quad W_{D\to E} = 2\alpha(0.5) \int_{0.5}^{0} dy = -\frac{\alpha}{2}$$

For $E \to F$, $x = 0$, $y = 0$, so $dx = 0$ and $dy = 0$

$\Rightarrow \quad W_{E\to F} = 0$

For $F \to A$, $x = 0$, so $dx = 0$

$\Rightarrow \quad W_{F\to A} = 0$

Hence total work done is

$$W_{total} = \alpha - \alpha - \frac{\alpha}{4} - \frac{\alpha}{2}$$

$$\Rightarrow \quad W_{total} = -\frac{3\alpha}{4} = \frac{-3(-1)}{4} = \frac{3}{4}\text{ J} = 0.75\text{ J}$$

2. From Modified Work-Energy Theorem, we have

$$W_{\text{ext}} = \Delta U + \Delta K$$

$$\Rightarrow \quad W_F = \Delta U + \Delta K$$

$$\Rightarrow \quad (18)(5) = (1)(10)(4) + \left(K_f - 0\right)$$

$$\Rightarrow \quad 90 = 40 + K_f$$

$$\Rightarrow \quad K_f = 50\text{ J}$$

$$\Rightarrow \quad K_f = 5(10\text{ J})$$

$$\Rightarrow \quad n = 5$$

$$\Rightarrow \quad W_F + W_{mg} = K_f - K_i$$

$$18 \times 5 + (1 \times 10)(-4) = K_f$$

$$90 - 40 = K_f$$

$$\Rightarrow \quad K_f = 50\text{ J} = 5 \times 10\text{ J}$$

$$\Rightarrow \quad n = 5$$

3. Velocity of first bob at highest point (when it just completes the circle) is

$$v_1 = \sqrt{g\ell_1}$$

Since collision of first bob with second bob is elastic and both have equal mass, so first bob will stop and second will move with velocity of first such that this velocity is sufficient enough for second bob to just complete the circle of radius ℓ_2. So

$$\sqrt{g\ell_1} = \sqrt{5g\ell_2}$$

$$\Rightarrow \quad \frac{\ell_1}{\ell_2} = 5$$

4. $W = \frac{1}{2}mv^2$

$$\Rightarrow \quad Pt = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2 \times 0.5 \times 5}{0.2}} = 5\text{ ms}^{-1}$$

5. According to Work-Energy Theorem, we have

$$\frac{1}{2}mV^2 = (\mu mg)(0.06) + \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}(0.18)V^2 = (0.1)(0.18)(10)(0.06) + \frac{1}{2}(2)(0.06)^2$$

$$\Rightarrow \quad V = 0.4\text{ ms}^{-1}$$

$$\Rightarrow \quad 0.4 = \frac{N}{10}$$

$$\Rightarrow \quad N = 4$$

CHAPTER 2: CENTRE OF MASS, CONSERVATION OF LINEAR MOMENTUM AND COLLISIONS

Test Your Concepts-I (Based on Centre of Mass)

1. Let the new centre of mass be at C', then its distance from C equals the displacement of the centre of mass, i.e. CC' given by

$$\Delta x = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2} = \frac{m_1\ell_1 + m_2\ell_2}{m_1 + m_2}$$

$$\Rightarrow \quad CC' = \Delta x = \frac{m_1\ell_1 + m_2\ell_2}{m_1 + m_2}$$

2. For a two particle system, the centre of mass is given by

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

Let the masses be placed on x-axis, then according to the problem, we have

$$\Delta x_{cm} = 0$$

$$\Rightarrow \quad \Delta x_{cm} = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2} = 0$$

$$\Rightarrow \quad 0 = \frac{m_1\Delta x_1 + m_2x}{m_1 + m_2}$$

$$\Rightarrow \quad \Delta x_1 = -\left(\frac{m_2x}{m_1}\right)$$

Thus m_1 should be displaced by $\frac{m_2x}{m_1}$, opposite to the displacement of m_2.

3. The position vector of cm of the three particles will be given by

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

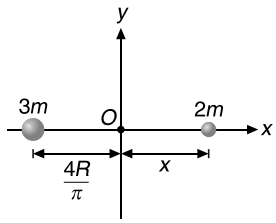
Substituting the values, we get

$$\vec{r}_{cm} = \frac{(1)\left(\hat{i} + 4\hat{j} + \hat{k}\right) + (2)\left(\hat{i} + \hat{j} + \hat{k}\right) + 3\left(2\hat{i} - \hat{j} - 2\hat{k}\right)}{1 + 2 + 3}$$

$$\Rightarrow \quad \vec{r}_{cm} = \frac{9\hat{i} + 3\hat{j} - 3\hat{k}}{6}$$

$$\Rightarrow \quad \vec{r}_{cm} = \frac{1}{2}\left(3\hat{i} + \hat{j} - \hat{k}\right) \text{ m}$$

4. The ring can be substituted by a particle of mass $3m$ at a distance $\frac{2(2R)}{\pi} = \frac{4R}{\pi}$ from the origin.

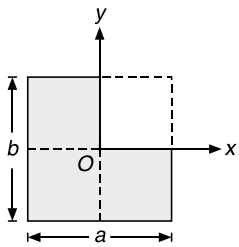


Now, the centre of mass of the two particle system is at origin, so we have

$$3m\left(-\frac{4R}{\pi}\right) + 2mx = 0$$

$$\Rightarrow \quad x = \frac{6R}{\pi}$$

5. The centre of mass of the remaining system is



$$x_{cm} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2} = \frac{(ab)(0) - (ab/4)(a/4)}{ab - (ab/4)}$$

$$\Rightarrow \quad x_{cm} = -\frac{a}{12}$$

Similarly, we have

$$y_{cm} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2} = \frac{(ab)(0) - (ab/4)(b/4)}{ab - (ab/4)}$$

$$\Rightarrow \quad y_{cm} = -\frac{b}{12}$$

$$\Rightarrow \quad (x_{cm},\ y_{cm}) = \left(-\frac{a}{12}, -\frac{b}{12}\right)$$

6. The mass distribution is symmetrical about x-axis, hence y-coordinate of centre of mass will be zero. So, we have

$$x_{cm} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

$$\Rightarrow \quad x_{cm} = \frac{(2r^2)\frac{r}{2} - \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)}{(2r^2) - \left(\frac{\pi r^2}{2}\right)} = \frac{2r}{3(4-\pi)}$$

Hence, coordinates of centre of mass of system are

$$\left(\frac{2r}{3(4-\pi)},\ 0\right)$$

7. Since we know that

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{3}{5}$$

$$\Rightarrow \quad \frac{(1)(1)+(2)(4)+m(-3)}{1+2+m} = \frac{3}{5}$$

$$\Rightarrow \quad \frac{9-3m}{3+m} = \frac{3}{5}$$

$$\Rightarrow \quad 45 - 15m = 9 + 3m$$

$$\Rightarrow \quad 18m = 36$$

$$\Rightarrow \quad m = 2 \text{ kg}$$

Also, we have

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = 2$$

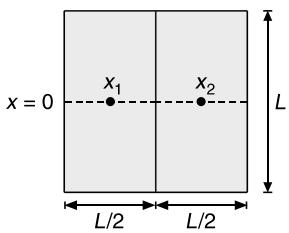
$$\Rightarrow \quad \frac{(1)(2)+(2)(6)+m(-2)}{1+2+m} = 2$$

$$\Rightarrow \quad 14 - 2m = 6 + 2m$$

$$\Rightarrow \quad 4m = 8$$

$$\Rightarrow \quad m = 2 \text{ kg}$$

8. The centre of mass of each half is located at the geometrical centre of that half.



$$\Rightarrow \quad x_{cm} = \frac{\rho_1A_1x_1 + \rho_2A_2x_2}{\rho_1A_1 + \rho_2A_2}$$

$$\Rightarrow \quad x_{cm} = \frac{\rho_1\left(\frac{L^2}{2}\right)\left(\frac{L}{4}\right) + \rho_2\left(\frac{L^2}{2}\right)\left(\frac{3L}{4}\right)}{\rho_1\left(\frac{L^2}{2}\right) + \rho_2\left(\frac{L^2}{2}\right)}$$

$$\Rightarrow \quad x_{cm} = \frac{\left(\frac{\rho_1}{8} + \frac{3\rho_2}{8}\right)L}{\left(\frac{\rho_1}{2} + \frac{\rho_2}{2}\right)} = \frac{(\rho_1 + 3\rho_2)L}{4(\rho_1 + \rho_2)}$$

9. The masses and coordinates of centre of mass of the three sheets are

Square, σl^2 at $\left(\frac{l}{2}, \frac{l}{2}\right)$

Triangle, $\sigma\left(\frac{\sqrt{3}}{4}l^2\right)$ at $\left[\frac{l}{2}, l\left(1 + \frac{1}{2\sqrt{3}}\right)\right]$

Disc, $\sigma\left(\frac{\pi l^2}{4}\right)$ at $\left(\frac{3l}{2}, \frac{l}{2}\right)$

So, coordinates of centre of mass of system are

$$x_{cm} = \frac{\sigma\left[l^2\left(\frac{l}{2}\right) + \left(\frac{\sqrt{3}l^2}{4}\right)\left(\frac{l}{2}\right) + \left(\frac{\pi l^2}{4}\right)\left(\frac{3l}{2}\right)\right]}{\sigma\left(l^2 + \frac{\sqrt{3}l^2}{4} + \frac{\pi l^2}{4}\right)}$$

$$\Rightarrow \quad x_{cm} = \frac{l}{2}\left(\frac{4 + \sqrt{3} + 3\pi}{4 + \sqrt{3} + \pi}\right)$$

Similarly, we have

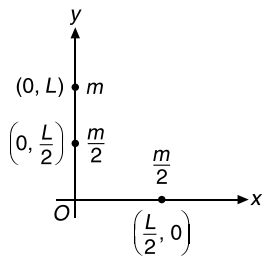
$$y_{cm} = \frac{l^2\left(\frac{l}{2}\right) + \left(\frac{\sqrt{3}l^2}{4}\right)\left(l + \frac{l}{2\sqrt{3}}\right) + \left(\frac{\pi l^2}{4}\right)\left(\frac{l}{2}\right)}{l^2 + \frac{\sqrt{3}l^2}{4} + \frac{\pi l^2}{4}}$$

$$\Rightarrow \quad y_{cm} = \frac{l}{2}\left(\frac{5 + 2\sqrt{3} + \pi}{4 + \sqrt{3} + \pi}\right)$$

10. $$x_{cm} = \frac{\int_0^\ell x\,dm}{\int_0^\ell dm} = \frac{\int_0^\ell (\lambda dx)x}{\int_0^\ell \lambda dx} = \frac{\int_0^\ell \left(\lambda_0 \frac{x^2}{\ell^2}\right)x\,dx}{\int_0^\ell \left(\lambda_0 \frac{x^2}{\ell^2}\right)dx}$$

$$\Rightarrow \quad x_{cm} = \frac{3\ell}{4}$$

11. Consider the rod to be a combination of two point masses $\frac{m}{2}, \frac{m}{2}$ placed at $x = \frac{L}{2}$ and $y = \frac{L}{2}$. Then this system consists of three particles having masses and positions given by $\frac{m}{2}$ at $\left(\frac{L}{2}, 0\right)$, $\frac{m}{2}$ at $\left(0, \frac{L}{2}\right)$ and m at $(0, L)$ as shown in Figure.

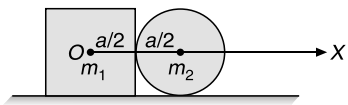


$$\Rightarrow \quad x_{cm} = \frac{\left(\frac{m}{2}\right)\left(\frac{L}{2}\right)+\left(\frac{m}{2}\right)(0)+m(0)}{\frac{m}{2}+\frac{m}{2}+m} = \frac{L}{8}$$

$$\Rightarrow \quad y_{cm} = \frac{\left(\frac{m}{2}\right)(0)+\left(\frac{m}{2}\right)\left(\frac{L}{2}\right)+mL}{\frac{m}{2}+\frac{m}{2}+m} = \frac{5L}{8}$$

So, the position of centre of mass is $\left(\frac{L}{8}, \frac{5L}{8}\right)$

12. Let σ be the surface mass density, i.e. the mass per unit area. The square can be treated as a point mass $m_1 = a^2\sigma$ placed at its geometric centre $(0,0)$ and the disc is treated as another point mass $m_2 = \frac{\pi a^2\sigma}{4}$ placed at its centre, i.e. at $(a,0)$ as shown in Figure.



The position of centre of mass is given by

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1+m_2} = \frac{(a^2\sigma)(0)+(\pi a^2\sigma/4)a}{a^2\sigma+(\pi a^2\sigma/4)}$$

$$\Rightarrow \quad x_{cm} = \frac{\pi a/4}{1+(\pi/4)} = \frac{\pi a}{\pi+4}$$

Hence, the centre of mass is at a distance $\left(\frac{\pi a}{\pi+4}\right)$ from the centre of square on the line joining the centres of the two objects.

13. For a uniform disc, its mass is proportional to area, so if σ is the surface mass density, then we have

$$m_1 = \sigma\left(\pi R^2 - \frac{R^2}{2}\right) = \left(\frac{2\pi-1}{2}\right)\sigma R^2$$

If l be the length of the square plate of diagonal R removed from the disc, then we have

$$l^2 + l^2 = R^2$$

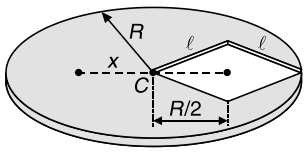
$$\Rightarrow \quad l = \frac{R}{\sqrt{2}}$$

The mass of square plate cut from the disc is

$$m_2 = \sigma l^2 = \sigma\left(\frac{R}{\sqrt{2}}\right)^2 = \frac{\sigma R^2}{2}$$

Also, we must note that the common centre of mass of m_1 and m_2 must be at the centre of the full disc. So, if centre of mass of given object is at a distance x from C, then for a two particle system, we have

$$m_1x_1 = m_2x_2$$

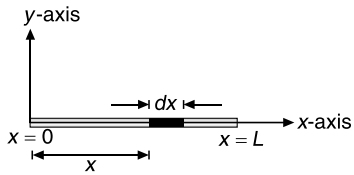


$$\Rightarrow \quad m_1x = m_2\frac{R}{2}$$

$$\Rightarrow \quad \left[\left(\frac{2\pi-1}{2}\right)(\sigma R^2)\right]x = \left(\frac{\sigma R^2}{2}\right)\left(\frac{R}{2}\right)$$

$$\Rightarrow \quad x = \frac{R}{2(2\pi-1)}$$

14. Let the rod be placed along the x axis as shown in Figure.



Consider an infinitesimal element of mass dm, length dx at a distance x from the lighter end of the rod (assuming lighter end of the rod to be placed at the origin of the axis). If linear mass density of the rod is λ, then we have

$$\lambda = \frac{dm}{dx}$$

According to the problem, the linear mass density of the rod increases linearly from λ_1 to λ_2, so we can write the mathematical expression for λ as

$$\frac{\lambda-\lambda_1}{x} = \frac{\lambda_2-\lambda_1}{L}$$

$$\Rightarrow \quad \lambda = \lambda_1 + \left(\frac{\lambda_2-\lambda_1}{L}\right)x$$

$$\Rightarrow \quad \lambda = a + bx \qquad \ldots(1)$$

where, $a = \lambda_1$ and $b = \frac{\lambda_2-\lambda_1}{L}$

The position of centre of mass is given by

$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^L x(\lambda\,dx)}{\int_0^L \lambda\,dx} = \frac{\int_0^L (ax+bx^2)\,dx}{\int_0^L (a+bx)\,dx}$$

$$\Rightarrow \quad x_{cm} = \frac{\frac{aL^2}{2}+\frac{bL^3}{3}}{aL+\frac{bL^2}{2}} = \frac{(3a+2bL)L}{(2a+bL)3}$$

$$\Rightarrow \quad x_{cm} = \left(\frac{\lambda_1+2\lambda_2}{\lambda_1+\lambda_2}\right)\frac{L}{3}$$

CHAPTER 2

Test Your Concepts-II (Based on Motion of Centre of Mass)

1. Suppose the centre of mass of the ice is a distance x_1 above the bottom and that of the tray is a distance x_2 above the bottom. The height of the centre of mass of the ice-tray system is

$$x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

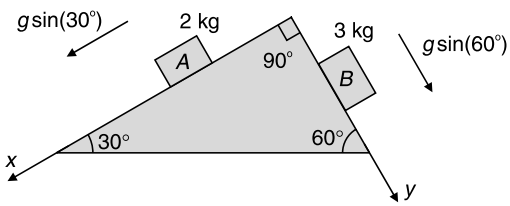
When the ice melts, the water of mass m_1 spreads on the surface of the tray. As the tray is large, the height of water is negligible. The centre of mass of the water is then on the surface of the tray and is at a distance $\left(x_1 - \frac{L}{2}\right)$ above the bottom. The new centre of mass of the ice-tray system will be at the height

$$x' = \frac{m_1\left(x_1 - \frac{L}{2}\right) + m_2x_2}{m_1 + m_2}$$

The shift in the centre of mass is

$$x - x' = \frac{m_1L}{2(m_1 + m_2)}$$

2. As shown in figure, blocks A slides with acceleration $\frac{g}{2}$ and block B slides with acceleration $\frac{\sqrt{3}g}{2}$.



Now the acceleration of centre of mass of the system of blocks A and B is

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} \quad \ldots(1)$$

where, $\vec{a}_1 = (g\sin 30°)\hat{i} = 5\hat{i}$ and

$$\vec{a}_2 = (g\sin 60°)\hat{j} = (5\sqrt{3})\hat{j}$$

Substituting these values in equation (1), we get

$$\vec{a}_{cm} = \frac{(2)(5\hat{i}) + (3)(5\sqrt{3}\hat{j})}{2+3} = 2\hat{i} + 3\sqrt{3}\hat{j}$$

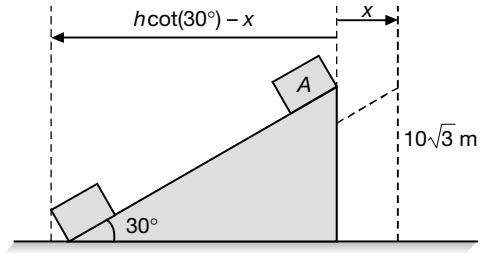
$$\Rightarrow \quad a_{cm} = \sqrt{(2)^2 + (3\sqrt{3})^2} = \sqrt{31}\ \text{ms}^{-2}$$

3. Applying $m_Rx_R = m_Lx_L$

$$\Rightarrow \quad 30x = 5(0.5 - x)$$

$$\Rightarrow \quad x = 0.0714\ \text{m} = 71.4\ \text{mm}$$

When block reaches the bottom i.e., moves to the left, then wedge moves to the right say by x. So, effective displacement of block is $\Delta x_L = h\cot\theta - x$



where $h = 10\sqrt{3}$ m and $\theta = 30°$

$$\Rightarrow \quad m_R\Delta x_R = m_L\Delta x_L$$

$$\Rightarrow \quad 25x = 5\left[10\sqrt{3}\cot(30°) - x\right]$$

$$\Rightarrow \quad 25x = 5\left[10\sqrt{3}(\sqrt{3}) - x\right]$$

$$\Rightarrow \quad 25x = 5(30 - x)$$

$$\Rightarrow \quad 30x = 150$$

$$\Rightarrow \quad x = 5\ \text{m}$$

4. Since, we know that the acceleration of the centre of mass of the system is

$$a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$

along the direction of motion of the heavier mass

$$\Rightarrow \quad a_{cm} = \left(\frac{4-2}{4+2}\right)^2(10) = \frac{10}{9}\ \text{ms}^{-2}$$

$$\Rightarrow \quad s_{cm} = \frac{1}{2}a_{cm}t^2 = \frac{1}{2}\left(\frac{10}{9}\right)(9)^2 = 45\ \text{m}$$

Similarly, we have

$$v_{cm} = u_{cm} + a_{cm}t$$

$$\Rightarrow \quad v_{cm} = 0 + \left(\frac{10}{9}\right)(9) = 10\ \text{ms}^{-1}$$

5. In the horizontal direction, there is no force on the system (dog + boat). Therefore, the centre of mass of the system does not move in the horizontal direction. Since the dog moves towards the shore, the boat moves away from the shore to keep centre of mass stationary. Let d be the distance by which the boat moves backwards and let x be the initial distance of the boat from the shore. The initial x-coordinate of the centre of mass

$$(x_{cm})_{initial} = \frac{(10)(20) + 40x}{10 + 40}$$

The final x-coordinate of the centre of mass

$$(x_{cm})_{final} = \frac{10(20 - 8 + d) + 40(x + d)}{10 + 40}$$

Equating the two, we get

$$d = \frac{8}{5} \text{ m}$$

So, from the shore, the dog is at a distance of

$$s = 20 - 8 + \frac{8}{5} = 13.6 \text{ m}$$

6. Since no external force acts on the system in the horizontal direction, i.e. along the x direction, so we have

$$\Delta x_{cm} = 0$$

Let wedge displace to the right with respect to ground through x. Taking rightwards direction as positive, we get

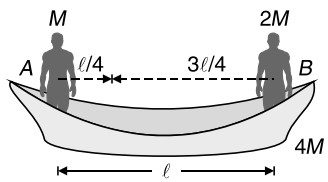
$$\Delta x_{cm} = \frac{m(l\cos 60° + x) + (4m)x - m(l - x)}{m + 4m + m}$$

$$\Rightarrow \quad 0 = \frac{ml}{2} - ml + 6mx$$

$$\Rightarrow \quad x = \frac{l}{12}$$

7. Let the boat be displaced through a distance x towards right, then the horizontal displacement of centre of mass of system must be zero. Taking rightward direction as positive, we get

$$\Delta x_{boat} = +x, \ \Delta x_A = \frac{l}{4} + x, \ \Delta x_B = -\left(\frac{3l}{4} - x\right)$$



$$0 = \frac{M(l/4 + x) - 2M(3l/4 - x) + 4Mx}{7M}$$

$$\Rightarrow \quad \frac{Ml}{4} + 5Mx = \frac{3Ml}{2} - 2Mx$$

$$\Rightarrow \quad 7x = \frac{3l}{2} - \frac{l}{4} = \frac{5l}{4}$$

$$\Rightarrow \quad x = \frac{5l}{28}$$

8. (a) The given time is of no consequence since v_{cm} is constant for all times.

From equation, in component form

$$(v_{cm})_x = \frac{m_1 v_{1x} + m_2 v_{2x}}{M}$$

$$\Rightarrow \quad (v_{cm})_x = \frac{(3)(-5\cos 37°) + (5)(0)}{8}$$

$$\Rightarrow \quad (v_{cm})_x = -1.5 \text{ ms}^{-1}$$

Also, $(v_{cm})_y = \dfrac{m_1 v_{1y} + m_2 v_{2y}}{M}$

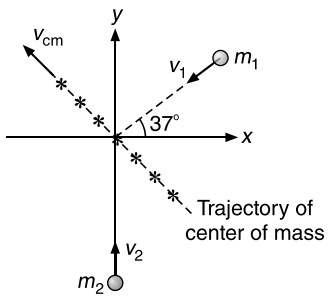
$$\Rightarrow \quad (v_{cm})_y = \frac{(3)(-5\sin 37°) + (5)(5)}{8}$$

$$\Rightarrow \quad (v_{cm})_y = 2 \text{ ms}^{-1}$$

So, $\vec{v}_{cm} = \left(1.5\hat{i} + 2\hat{j}\right) \text{ ms}^{-1}$

(b) Since the collision occurs at the origin, the position of the centre of mass 2 s later is

$$\vec{r}_{cm} = (\vec{v}_{cm})t = \left(-3\hat{i} + 4\hat{j}\right) \text{ m}$$



9. Since no external force acts on the system in the horizontal direction, i.e. along the x direction, so we have

$$\Delta x_{cm} = 0$$

Let wedge displace to the left with respect to ground through x, then the block displaces to the right through $R\sin\theta - x$, such that

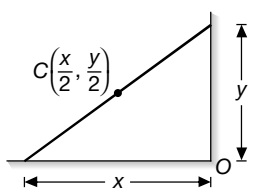
$$m(R\sin\theta - x) = Mx$$

$$\Rightarrow \quad x = \frac{mR\sin\theta}{m + M}$$

10. Since, $y = \sqrt{L^2 - x^2}$

$$\Rightarrow \quad \frac{dy}{dt} = -\frac{x}{\sqrt{L^2 - x^2}}\frac{dx}{dt}$$

$$\Rightarrow \quad v_y = \frac{dy}{dt} = -\frac{3 \times 2}{4} = -\frac{3}{2} \text{ ms}^{-1}$$



At any time, the coordinates of centre of mass of rod are $\left(\frac{x}{2}, \frac{y}{2}\right)$

$$\Rightarrow \quad (v_{cm})_x = \frac{d}{dt}\left(\frac{x}{2}\right) = \frac{1}{2}\left(\frac{dx}{dt}\right) = \frac{1}{2}v_x$$

CHAPTER 2

and $(v_{cm})_y = \frac{d}{dt}\left(\frac{y}{2}\right) = \frac{1}{2}\left(\frac{dy}{dt}\right) = \frac{1}{2}v_y$

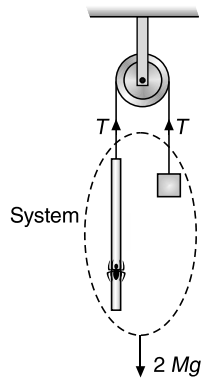
$\Rightarrow \quad \vec{v}_{cm} = (v_{cm})_x \hat{i} + (v_{cm})_y \hat{j}$

$\Rightarrow \quad \vec{v}_{cm} = \frac{1}{2}\left(v_x\hat{i} + v_y\hat{j}\right)$

$\Rightarrow \quad |\vec{v}_{cm}| = \frac{1}{2}\sqrt{v_x^2 + v_y^2} = \frac{1}{2}\sqrt{4 + \frac{9}{4}}$

$\Rightarrow \quad |\vec{v}_{cm}| = 1.25 \text{ ms}^{-1}$

11. (a) Considering insect, rod and counter weight as a system, the external forces acting on the system are tension in the string and gravitational force as shown in Figure.



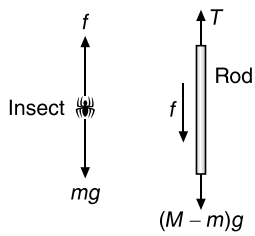
When insect is at rest, we have

$\Sigma F_{system} = 2T - 2Mg = 0$

$\Rightarrow \quad T = Mg$

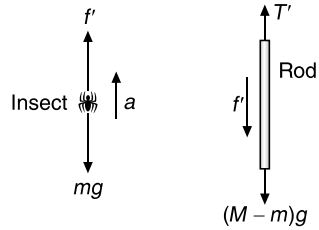
Hence, no net external force is acting on the system.

(b) When we consider free body diagram of the insect, then we observe that the friction force is balancing the weight of insect when it is at rest.



However, when the insect moves with constant velocity, then too the net force on insect is zero and hence the magnitude of friction force will remain unchanged. This simply means that the tension in the string will not change. So, no external force acts on the system. Therefore, the acceleration of the centre of mass of the system will be zero.

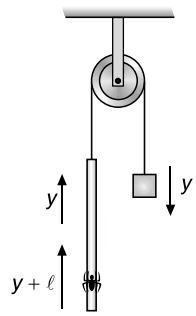
(c) When the insect moves up with acceleration, the friction force between the insect and rod increases. This increased friction increases the tension in the string $(T > Mg)$.



Now in this case, there will be net force on the system in upward direction. Hence the centre of mass of the system will accelerate in upward direction.

(d) Let displacement of the rod when the insect reaches to the other end of the rod be y upward. Hence the displacement of the counter weight M will be y downward as the rod and counter weight both are connected together by a single string. During this time the displacement of the insect will be $(y + l)$.

We know



$$\Delta y_{cm} = \frac{m_1\Delta y_1 + m_2\Delta y_2 + m_3\Delta y_3}{(m_1 + m_2 + m_3)}$$

$\Rightarrow \quad \Delta y_{cm} = \frac{m(y+l) + (M-m)y + M(-y)}{m + (M-m) + M}$

$\Rightarrow \quad \Delta y_{cm} = \frac{ml}{2M}$, upwards

12. (a) $x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$

$\Rightarrow \quad 3 = \frac{m_1(0) + (0.1)(12)}{0.1 + m_1} = \frac{1.2}{0.1 + m_1}$

$\Rightarrow \quad m_1 = 0.3 \text{ kg}$

(b) $\vec{p} = (m_1 + m_2)\vec{v}_{cm} = (0.3 + 0.1)\left(6\hat{j}\right) \text{ kgms}^{-1}$

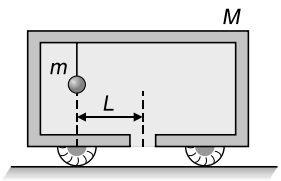
$\Rightarrow \quad \vec{p} = 2.4\hat{j} \text{ kgms}^{-1}$

(c) $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

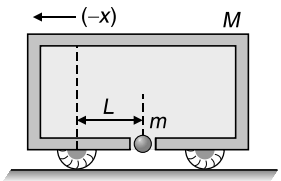
$\Rightarrow \quad 6\hat{j} = \dfrac{(0.3)\vec{v}_1 + (0.1)(0)}{0.4}$

$\Rightarrow \quad \vec{v}_1 = 8\hat{j}\ \text{ms}^{-1}$

13. Let displacement of the cart be x towards left



Then displacement of the bob will be $(L - x)$ such that



$m_1\Delta x_1 = m_2\Delta x_2$

$\Rightarrow \quad Mx = m(L - x)$

$x = \dfrac{ML}{m + M}$

14. (a) $h_1 = \dfrac{1}{2}gt_1^2 = \dfrac{1}{2}\times 10\times(0.4)^2 = 0.8\ \text{m}$

$h_2 = \dfrac{1}{2}gt_2^2 = \dfrac{1}{2}\times 10\times(0.2)^2 = 0.2\ \text{m}$

Since, $h_{cm} = \dfrac{m_1h_1 + m_2h_2}{m_1 + m_2}$

$\Rightarrow \quad h_{cm} = \dfrac{m(0.8) + 2m(0.2)}{3m} = 0.4\ \text{m}$

$\Rightarrow \quad h_{cm} = 40\ \text{cm}$

(b) $v_1 = gt_1 = (10)(0.4) = 4\ \text{ms}^{-1}$

$v_2 = gt_2 = (10)(0.2) = 2\ \text{ms}^{-1}$

Since, $v_{cm} = \dfrac{mv_1 + 2mv_2}{3m}$

Substituting the values, we get

$v_{cm} = \dfrac{8}{3}\ \text{ms}^{-1} = 2.67\ \text{ms}^{-1}$

15. When ball is to enter the hole, then the ball travels a horizontal distance l with respect to the car. During this motion, to maintain the position of centre of mass the car moves towards left, say by a distance x w.r.t. ground. So, distance travelled by the ball towards right w.r.t. ground is $(l - x)$ such that

$m(l - x) = Mx$

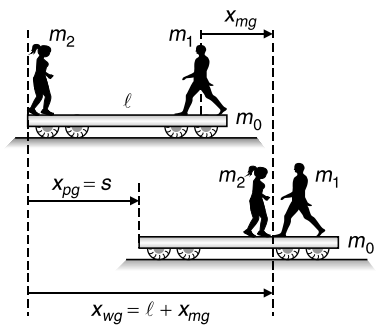
$\Rightarrow \quad x = \dfrac{ml}{M + m}$

16. Since it is given that x_1 is the displacement of the man with respect to the platform, so we have

$x_{mp} = x_{mg} - x_{pg} = -x_1$

$\Rightarrow \quad x_{mg} = x_{pg} - x_1 = s - x_1$

Also, we know that no external force is acting on the system, so the displacement of the centre of mass is zero. The displacements of the man (m), woman (w) and the platform (p) all w.r.t. ground (g) are shown in Figure.



Since, $\Delta x_{cm} = 0$

$\Rightarrow \quad m_1x_{mg} + m_2x_{wg} + m_0x_{pg} = 0$

where x_{mg}, x_{wg} and x_{pg} are the respective displacements of the man w.r.t. ground, woman w.r.t. ground and platform w.r.t. ground.

$\Rightarrow \quad m_0s + m_1(s - x_1) + m_2(l + s - x_1) = 0$

$\Rightarrow \quad s = \dfrac{(m_1 + m_2)x_1 - m_2l}{m_1 + m_2 + m_0}$

17. If we take ball 1 and ball 2 as system, there is no external force acting on the system. Hence the velocity of the centre of mass of the system will be constant. From given figure, we can calculate the velocity of the centre of mass, which will also be the velocity of centre of mass 3 s before the collision. x component of velocity of centre of mass

$(v_{cm})_x = \dfrac{m_1v_{1x} + m_2v_{2x}}{m_1 + m_2}$

$\Rightarrow \quad (v_{cm})_x = \dfrac{(3)(-5\cos 37^\circ) + (5)(0)}{8\ \text{kg}} = -1.5\ \text{ms}^{-1}$

y component of velocity of centre of mass

$(v_{cm})_y = \dfrac{m_1v_{1y} + m_2v_{2y}}{m_1 + m_2}$

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$$\Rightarrow \quad (v_{cm})_y = \frac{(3)(-5\sin 37°)+(5\times 5)}{8\text{ kg}} = +2\text{ ms}^{-1}$$

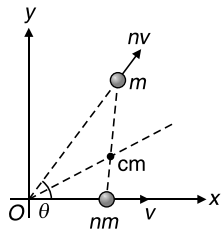
$$\Rightarrow \quad \vec{v}_{cm} = -1.5\hat{i} + 2\hat{j}\text{ ms}^{-1}$$

As collision occurs at the origin $(\vec{r}_i = 0)$, so the position of centre of mass at the time of collision will be at origin. The position of the centre of mass 2 s after the collision is

$$\vec{r}_{cm} = \vec{r}_i + \vec{v}_{cm}t = \vec{v}_{cm}t \qquad \{\because \vec{r}_i = \vec{0}\}$$

$$\Rightarrow \quad \vec{r}_{cm} = \left(-1.5\hat{i} + 2\hat{j}\right)\times 2 = \left(-3\hat{i} + 4\hat{j}\right)\text{ m}$$

18. Let the mass $m_1 = m$ and $m_2 = nm$ be moving as shown in Figure.



Since, $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_1\vec{v}_2}{m_1 + m_2}$

$$\Rightarrow \quad \vec{v}_{cm} = \frac{(nm)v\hat{i} + m(nv)\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)}{m + nm}$$

$$\Rightarrow \quad \vec{v}_{cm} = \frac{nmv(1+\cos\theta)\hat{i} + (nmv\sin\theta)\hat{j}}{m + nm}$$

$$\Rightarrow \quad |\vec{v}_{cm}| = \frac{nmv\sqrt{(1+\cos\theta)^2 + (\sin\theta)^2}}{m(1+n)}$$

$$\Rightarrow \quad |\vec{v}_{cm}| = \frac{nv\sqrt{1+\cos^2\theta + 2\cos\theta + \sin^2\theta}}{1+n}$$

$$\Rightarrow \quad |\vec{v}_{cm}| = \frac{nv\sqrt{2+2\cos\theta}}{1+n} = \frac{\sqrt{2}nv\sqrt{1+\cos\theta}}{1+n}$$

Since $1+\cos\theta = 2\cos^2\left(\dfrac{\theta}{2}\right)$

$$\Rightarrow \quad |\vec{v}_{cm}| = \frac{2nv\cos\left(\dfrac{\theta}{2}\right)}{1+n}$$

Test Your Concepts-III (Based on Work Energy Theorem for System of Particles and its Applications from Centre of Mass Reference Frame)

1. The kinetic energy of a two particle system with respect to the centroidal frame is

$$K_{cm} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)|\vec{u}_{rel}|^2$$

$$\Rightarrow \quad K_{cm} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)|\vec{u}_1 - \vec{u}_2|^2$$

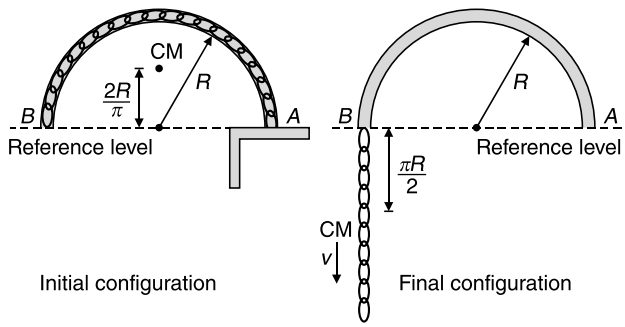
Since $\vec{u}_1$ and $\vec{u}_2$ are perpendicular to each other, so

$$|\vec{u}_1 - \vec{u}_2| = \sqrt{u_1^2 + u_2^2} = \sqrt{\left(5\sqrt{3}\right)^2 + 5^2} = 10\text{ ms}^{-1}$$

$$\Rightarrow \quad K_{cm} = \frac{1}{2}\left(\frac{0.6\times 0.3}{0.6+0.3}\right)(10)^2 = 10\text{ J}$$

2. The centre of mass of the semi-circular chain will be at a distance $\left(\dfrac{2R}{\pi}\right)$ from the centre and the mass of the chain will be $\lambda\pi R$, where λ is the linear mass density of the chain.

Let us consider the horizontal line passing through points A and B to be the Zero Potential Energy Level (ZPEL) to calculate the gravitational potential energy in the initial and final configurations of the chain as shown in Figure.



Initial potential energy of chain is

$$U_{initial} = mgh_i$$

$$\Rightarrow \quad U_{initial} = (\lambda\pi R)g\left(\frac{2R}{\pi}\right) = 2\lambda gR^2$$

Final potential energy of chain is

$$U_{final} = mg\left(-h_f\right)$$

$$\Rightarrow \quad U_{final} = mg\left(-h_f\right) = (\lambda\pi R)g\left(\frac{-\pi R}{2}\right)$$

$$\Rightarrow \quad U_{final} = -\frac{\lambda\pi^2 gR^2}{2}$$

When the chain starts slipping from the tube, then its initial kinetic energy $K_{initial} = 0$ and when it slips off completely from the tube, then all the links of the chain move with same common velocity v due to which its kinetic energy in the final configuration is

$$K_{final} = \frac{1}{2}mv^2 = \frac{1}{2}(\lambda\pi R)v^2$$

Since all surfaces are frictionless, so there will not be any loss of mechanical energy and hence by Law of Conservation of Mechanical Energy, we get

$$U_{initial} + K_{initial} = U_{final} + K_{final}$$

$$\Rightarrow \quad 2\lambda gR^2 + 0 = -\frac{\lambda\pi^2 gR^2}{2} + \frac{1}{2}(\lambda\pi R)v^2$$

$$\Rightarrow \quad \frac{1}{2}(\lambda\pi R)v^2 = 2\lambda gR^2 + \frac{\lambda\pi^2 gR^2}{2}$$

$$\Rightarrow \quad v^2 = \frac{2\lambda gR^2}{\lambda\pi R}\left(2 + \frac{\pi^2}{2}\right)$$

$$\Rightarrow \quad v = \sqrt{2gR\left(\frac{2}{\pi} + \frac{\pi}{2}\right)}$$

3. Applying Work Energy Theorem from the centre of mass reference frame, we get

$$W_{total} = \Delta K_{rel} + \Delta K_{cm}$$

As no external forces acting, the velocity of centre of mass will be constant. Hence $\Delta K_{cm} = 0$

$$\Rightarrow \quad W_{total} = W_{int} + W_{ext} = \Delta K_{rel}$$

where, $W_{int} = 0$ and

$$W_{ext} = W_{gravity} = -\Delta U = -(3m)gh$$

Also, $(\Delta K)_{rel} = \frac{1}{2}\mu\left(v_{rel}^2 - u_{rel}^2\right)$

where, μ is reduced mass of system given by

$$\mu = \frac{(6m)(3m)}{6m + 3m} = 2m$$

So, we have

$$-3mgh = \frac{1}{2}(2m)\left[(0)^2 - (3v_0)^2\right]$$

$$\Rightarrow \quad h = \frac{3v_0^2}{g}$$

4. When the extension in the spring is maximum, both the blocks move with a common velocity. Hence the final relative velocity of the blocks is zero. Applying Work Energy Theorem from the centre of mass reference frame, we get

$$W = W_{int} + W_{ext} = (\Delta K)_{cm} = \frac{1}{2}\mu\left(v_{rel}^2 - u_{rel}^2\right)$$

$$W_{int} = (\Delta K)_{cm} = \frac{1}{2}\mu v_{rel}^2 - \frac{1}{2}\mu u_{rel}^2$$

where, μ is reduced mass of system given by

$$\mu = \frac{(m)(3m)}{m + 3m} = \frac{3m}{4}$$

Since, $W_{int} = W_{spring} = -\Delta U = -\frac{1}{2}kx_{max}^2$

$$\Rightarrow \quad -\frac{1}{2}kx_{max}^2 = 0 - \frac{1}{2}\left(\frac{3m}{4}\right)(v_0 + 3v_0)^2$$

$$\Rightarrow \quad kx_{max}^2 = \left(\frac{3m}{4}\right)(16v_0^2)$$

$$\Rightarrow \quad x_{max} = 2v_0\sqrt{\frac{3m}{k}}$$

5. In the centre of mass reference frame, i.e. a reference frame travelling with the velocity of the centre of mass of the system, the initial mechanical energy of the system is equal to the elastic potential energy of the spring (because initial velocity of both blocks is zero). When the spring regains its natural length, then this energy gets converted into kinetic energy, so we have

$$\frac{1}{2}kx^2 = \frac{1}{2}\mu\left(v_r^2 - 0^2\right) \qquad \ldots(1)$$

where, μ is reduced mass of system given by

$$\mu = \frac{(6m)(3m)}{6m + 3m} = 2m$$

and v_r is the final relative velocity of the blocks when the spring comes to its natural length.

So, from equation (1), we get

$$kx^2 = \mu v_r^2$$

$$\Rightarrow \quad kx^2 = (2m)v_r^2$$

$$\Rightarrow \quad v_r = x\sqrt{\frac{k}{2m}}$$

6. At maximum extension in the spring, let the displacements of the blocks m_1 and m_2 with respect to CM be x_1 (towards left) and x_2 (towards right) respectively. So, the maximum extension in the spring is

$$x = x_1 + x_2 \qquad \ldots(1)$$

From centre of mass reference frame, we can write,

$$-m_1x_1 + m_2x_2 = 0$$

$$\Rightarrow \quad m_1x_1 = m_2x_2 \qquad \ldots(2)$$

From equation (1) and (2), we get

$$x_1 = \frac{m_2x}{m_1 + m_2} \text{ and } x_2 = \frac{m_1x}{m_1 + m_2}$$

Applying Work Energy Theorem from the centre of mass reference frame, we get

$$W = W_{int} + W_{ext} = \Delta K_{cm}$$

$$\Rightarrow \quad W = \frac{1}{2}\mu\left(v_{rel}^2 - u_{rel}^2\right) \qquad \ldots(3)$$

Since both the particles m_1 and m_2 start from rest, so $u_{rel} = 0$

At maximum extension in the spring both the masses move with a common velocity, so $v_{rel} = 0$

Hence, from equation (3), we get

$$W_{int} + W_{ext} = 0 \qquad \ldots(4)$$

In this case, we must understand that since the centre of mass frame is accelerating, so we do have pseudo force acting on the centre of mass. The work done due to pseudo force in the centre of mass frame is $W_{pseudo} = 0$. So, we have

$$W_{int} = W_{spring} + W_{pseudo} = W_{spring}$$

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Since, $W_{spring} = \Delta U = -\frac{1}{2}kx^2$, so we have

$$W_{int} = W_{spring} = -\frac{kx^2}{2} \quad \ldots(5)$$

Since F_1 and F_2 are constant external forces, so the work done by these external forces is

$$W_{ext} = F_1x_1 + F_2x_2 \quad \ldots(6)$$

So, from equation (4), we get

$$F_1x_1 + F_2x_2 - \frac{1}{2}kx^2 = 0 \quad \ldots(7)$$

Substituting the values of x_1 and x_2 in equation (7), we get

$$F_1\left(\frac{m_2x}{m_1+m_2}\right) + F_2\left(\frac{m_1x}{m_1+m_2}\right) = \frac{kx^2}{2}$$

$$\Rightarrow \quad x = \frac{2(m_1F_2 + m_2F_1)}{k(m_1+m_2)}$$

7. When the slipping stops, then both the block and plank move with same common velocity. Applying Work Energy Theorem from the centre of mass reference frame, we get

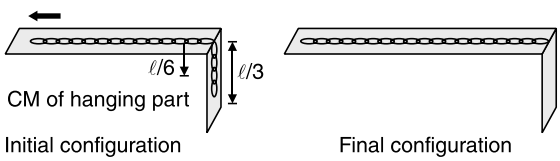
$$W = W_{int} + W_{ext} = (\Delta K)_{cm} = \frac{1}{2}\mu\left(v_{rel}^2 - u_{rel}^2\right)$$

where, μ is reduced mass of system given by

$$\mu = \frac{(m)(3m)}{m+3m} = \frac{3m}{4}$$

$$\Rightarrow \quad W_{friction} = -\frac{1}{2}\left(\frac{3m}{4}\right)v_0^2 = -\frac{3}{8}mv_0^2$$

8. The initial and final situation is shown in Figure.



The work done in slowly pulling the complete chain up the table is

$$W_{ext} = \Delta U_{cm} \quad \ldots(1)$$

Assuming ZPEL to be at the table, then the initial and final potential energy of the part of the chain hanging from the table are given by

$$U_{initial} = mg(-h_{cm}) = -\left(\frac{m}{3}\right)g\left(\frac{l}{6}\right) = -\frac{mgl}{18}$$

and $U_{final} = 0$

So, from equation (1), we get

$$W_{ext} = 0 - \left(-\frac{mgl}{18}\right) = \frac{mgl}{18}$$

9. At maximum extension in the spring, both the rings move with same common velocity. Applying Work Energy Theorem from the centre of mass reference frame, we get

$$W = W_{int} + W_{ext} = (\Delta K)_{cm} = \frac{1}{2}\mu\left(v_{rel}^2 - u_{rel}^2\right)$$

where, μ is reduced mass of system given by

$$\mu = \frac{(6m)(12m)}{6m+12m} = 4m$$

Since no external forces are there, so $W_{ext} = 0$ and due to extension of the spring, we have

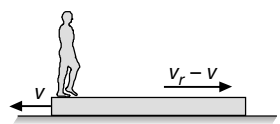
Since, $W_{int} = W_{spring} = -\Delta U = -\frac{1}{2}kx_{max}^2$

$$\Rightarrow \quad -\frac{1}{2}kx_{max}^2 = -\frac{1}{2}(4m)v_0^2 - 0$$

$$\Rightarrow \quad x_{max} = 2v_0\sqrt{\frac{m}{k}}$$

Test Your Concepts-IV (Based on Conservation of Linear Momentum)

1. Absolute velocity of man i.e., velocity of man w.r.t. ground is $v_r - v$ where v is the recoil velocity of platform. Taking the platform and the man as a system, net external force on the system in horizontal direction is zero. The linear momentum of the system remains constant. Initially both the man and the platform were at rest.

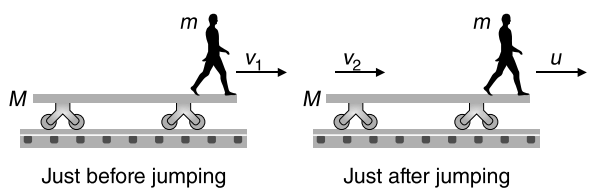


Hence, $0 = m_1(v_r - v) - m_2v$

$$\Rightarrow \quad v = \frac{m_1v_r}{m_1+m_2}$$

2. Let the velocity of the car just after jumping of man be v. The net velocity of the man with respect to the ground will be $u - v$, as u is its velocity with respect to the car.

If we take man and car as system. There is no external force act on it in horizontal direction. It means the linear momentum of the system should be conserved in horizontal direction.



Initially, the system was at rest, thus according to momentum conservation, momentum after jump must be zero. Hence,

$$m(u-v)=Mv$$

$$\Rightarrow \quad v=\frac{mu}{m+M}$$

3. At the lowest point, when m leaves the surface of M, it has a horizontal velocity v_1 (say). If v_2 be velocity of M, then by Law of Conservation of Linear Momentum, we have

$$mv_1=Mv_2 \qquad \ldots(1)$$

By Law of Conservation of Mechanical Energy, we have

$$mgR=\frac{1}{2}mv_1^2+\frac{1}{2}Mv_2^2 \qquad \ldots(2)$$

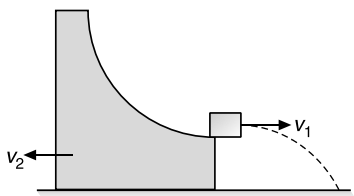
Also, using concepts of Projectile Motion, we have

$$\frac{R}{2}=\frac{1}{2}gt^2$$

$$\Rightarrow \quad t=\sqrt{\frac{R}{g}} \qquad \ldots(3)$$

The desired distance is

$$s=(v_1+v_2)t \qquad \ldots(4)$$

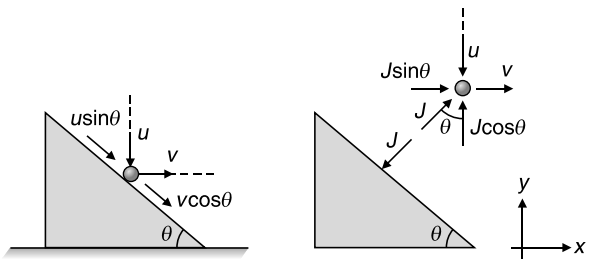


Solving equation (1) and (2) for v_1 and v_2 and substituting in equation (4), we get

$$s=R\sqrt{\frac{2(M+m)}{M}}$$

4. Velocity of the particle just before impact with the plane

$$u=\sqrt{2gh}$$



In the direction normal to impulse, the linear momentum of the particle just after impact will not change. Hence velocity of the particle parallel to inclined plane just before collision and just after collision is the same.

$$\Rightarrow \quad v\cos\theta=u\sin\theta$$

$$\Rightarrow \quad v=u\tan\theta$$

$$\Rightarrow \quad v=\sqrt{2gh}\tan\theta$$

The particle just after collision started moving in horizontal direction. Hence the component of the velocity of the ball in vertical direction after impact should be zero. Applying impulse momentum equation to the particle in vertical direction, we get

$$\vec{J}=(\vec{p}_{\text{final}})_y-(\vec{p}_{\text{initial}})_y$$

$$\Rightarrow \quad J\cos\theta=0-(-mu)=mu$$

$$\Rightarrow \quad J=\frac{mu}{\cos\theta}=mu\sec\theta$$

$$\Rightarrow \quad J=m\sqrt{2gh}\sec\theta$$

Loss in kinetic energy due to impact is

$$-\Delta K=K_{\text{initial}}-K_{\text{final}}=\frac{1}{2}mu^2-\frac{1}{2}mv^2$$

$$\Rightarrow \quad -\Delta K=\frac{1}{2}m\left[\left(\sqrt{2gh}\right)^2-\left(\sqrt{2gh}\right)^2\tan^2\theta\right]$$

$$\Rightarrow \quad -\Delta K=\frac{1}{2}m\left[2gh(1-\tan^2\theta)\right]$$

$$\Rightarrow \quad -\Delta K=mgh(1-\tan^2\theta)$$

5. Velocity of projectile at highest point is $24\cos 60°$ or $12\ \text{ms}^{-1}$ (horizontally). Applying Law of Conservation of Linear Momentum, we get

$$mv_B\hat{j}+m\vec{v}_A=2m(12\hat{i})$$

$$\Rightarrow \quad \vec{v}_A=24\hat{i}-v_B\hat{j}$$

Since they fall 45 m apart, so

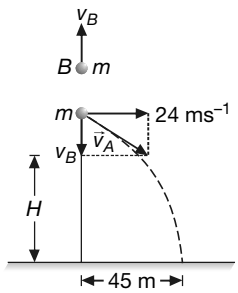
$$x=(v_A)_x t$$

$$\Rightarrow \quad 45=24t$$

$$\Rightarrow \quad t=1.875\ \text{s}$$

Since, $H=vt+\frac{1}{2}gt^2$

$$\Rightarrow \quad \frac{(24)^2\sin^2 60°}{2(10)}=v_B(1.875)+\frac{1}{2}g(1.875)^2$$



Solving this equation, we get

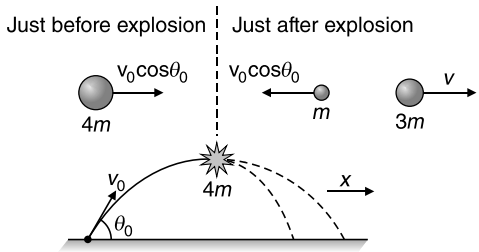
$$\Rightarrow \quad v_B=2.55\ \text{ms}^{-1}$$

$$\Rightarrow \quad \vec{v}_A = 24\hat{i} - 2.55\hat{j}$$

$$\Rightarrow \quad |\vec{v}_A| = \sqrt{(24)^2 + (2.55)^2}$$

$$\Rightarrow \quad |\vec{v}_A| = 24.1 \text{ ms}^{-1}$$

6. Let the projectile be of mass $4m$. The projectile just before explosion is at the highest point of its path and moving with horizontal velocity of $v_0 \cos\theta_0$. The lighter mass m formed after explosion retraces its path and hence its velocity at the top is just reversed due to explosion. So, m moves leftwards with a velocity $v_0 \cos\theta_0$. At this moment, the heavier part of mass $3m$ is moving rightwards with velocity v.



The linear momentum just before explosion

$$\vec{p}_{\text{initial}} = (4mv_0 \cos\theta_0)\hat{i}$$

The linear momentum just after explosion

$$\vec{p}_{\text{final}} = -(mv_0 \cos\theta_0)\hat{i} + (3m)\vec{v}$$

Applying Law of Conservation of Linear Momentum along horizontal x-axis, we get

$$(4mv_0 \cos\theta_0)\hat{i} = -(mv_0 \cos\theta_0)\hat{i} + (3m)\vec{v}$$

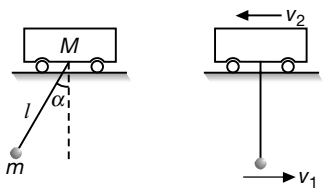
$$\Rightarrow \quad (3m)\vec{v} = (5mv_0 \cos\theta_0)\hat{i}$$

$$\Rightarrow \quad \vec{v} = \left(\frac{5v_0 \cos\theta_0}{3}\right)\hat{i}$$

Hence the velocity of the heavier part just after the explosion is $v = \frac{5}{3}v_0 \cos\theta_0$, along x direction.

7. Applying Law of Conservation of Linear Momentum, we get

$$mv_1 = Mv_2 \qquad \ldots(1)$$



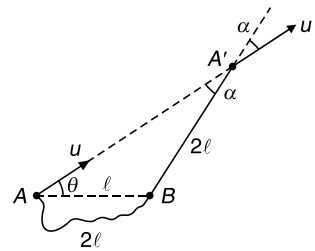
By Law of Conservation of Energy, we get

$$mg\ell(1 - \cos\alpha) = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2 \qquad \ldots(2)$$

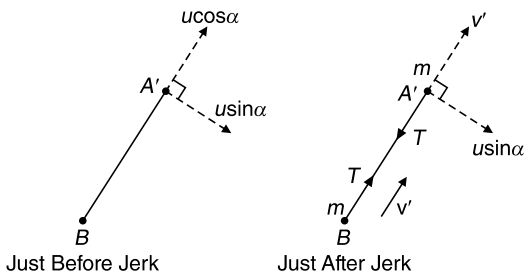
Solving these two equations, we get velocity of wagon

$$v_2 = 2m\sin\left(\frac{\alpha}{2}\right)\sqrt{\frac{g\ell}{(M+m)M}}$$

8. Let us solve the problem for a general case when direction of u makes an angle θ to the line AB as shown in Figure.



At position A', the string gets taut and the components of velocities along and perpendicular to the string just before and after the jerk are shown in Figure.



Applying sine rule to $\Delta AA'B$, we get

$$\frac{\sin\theta}{2l} = \frac{\sin\alpha}{l}$$

$$\Rightarrow \quad \sin\alpha = \frac{1}{2}\sin\theta = \frac{1}{2}\sqrt{1 - \cos^2\theta} \qquad \ldots(1)$$

Since, velocity of A perpendicular to the string remains constant during jerk, so applying the Impulse Momentum theorem, we get

For A, $\int -T dt = mv' - mu\cos\alpha$ $\qquad \ldots(2)$

For B, $\int T dt = mv' - 0$ $\qquad \ldots(3)$

Adding equations (2) and (3), we get

$$0 = mv' + mv' - mu\cos\alpha$$

$$\Rightarrow \quad v' = \frac{u\cos\alpha}{2} \qquad \ldots(4)$$

So, Impulse $= \int T dt = mv' = \frac{mu\cos\alpha}{2}$ $\qquad \ldots(5)$

For (a), u is along BA, i.e. $\theta = \pi$ and $\alpha = 0°$

$$\Rightarrow \quad v' = \frac{u}{2} \text{ and } \int T dt = \frac{mu}{2}$$

For (b), u makes an angle of 120° with AB, i.e. $\theta = \frac{2\pi}{3}$ and $\alpha = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

$\Rightarrow \quad v' = \frac{u\sqrt{13}}{8}$ and $\int Tdt = \frac{mu\sqrt{13}}{8}$

For (c), u makes an angle of 90° with AB, i.e. $\theta = \frac{\pi}{2}$ and $\alpha = \frac{\pi}{6}$

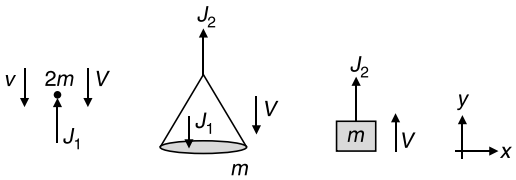
$\Rightarrow \quad v' = \frac{u\sqrt{3}}{4}$ and $\int Tdt = \frac{mu\sqrt{3}}{4}$

9. When the smaller block reaches A, then both the blocks will have same horizontal velocity. Applying Law of Conservation of Linear Momentum (along axis perpendicular to gravity), we get

$$mv = (m+M)v_x$$

$$\Rightarrow \quad v_x = \frac{mv}{m+M}$$

10. When the particle strikes the pan, the normal reaction between the pan and the particle provides the impulse. Hence tension in the string is impulsive in nature. Let the speed of the system just after collision be V. The impulse diagram is shown in Figure.



Impulse on the particle | Impulse on the pan | Impulse on the block

From impulse momentum equation, we have

$$\vec{J} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}}$$

Taking downward direction as positive, then for the particle, we have

$$-J_1 = 2m(V - v) \qquad \ldots(1)$$

for the pan, we have

$$J_1 - J_2 = mV \qquad \ldots(2)$$

and for the block, we have

$$-J_2 = -mV$$

$$\Rightarrow \quad J_2 = mV \qquad \ldots(3)$$

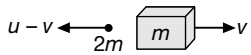
From equations (1), (2) and (3), we get

$$V = \frac{v}{2}$$

11. (a) By Law of Conservation of Linear Momentum, we get

$$Mv = 2m(u - v)$$

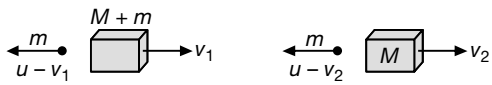
$$\Rightarrow \quad v = \frac{2mu}{M+2m}$$



(b) Again, applying Law of Conservation of Linear Momentum, but one by one, we get

$$(M+m)v_1 = m(u - v_1)$$

$$\Rightarrow \quad v_1 = \frac{mu}{M+2m}$$



Further, $Mv_2 = m(u - v_1) + m(u - v_2)$

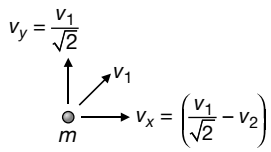
$$\Rightarrow \quad (M+m)v_2 = (M+m)\left(\frac{mu}{M+2m}\right) + mu$$

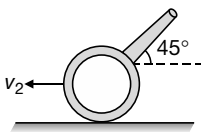
$$\Rightarrow \quad v_2 = \frac{mu}{M+2m} + \frac{mu}{M+m}$$

12. If shell is fired at muzzle at speed v_1, then by conservation of linear momentum, we have

$$m\left(\frac{v_1}{\sqrt{2}} - v_2\right) = kmv_2$$

$$\Rightarrow \quad v_2 = \frac{v_1}{\sqrt{2}(k+1)}$$





The ratio of kinetic energies is given by

$$\frac{K_{\text{shell}}}{K_{\text{gun}}} = \frac{\frac{1}{2}m\left[\frac{v_1^2}{2} + \left(\frac{v_1}{\sqrt{2}} - v_2\right)^2\right]}{\frac{1}{2}kmv_2^2}$$

$$\Rightarrow \quad \frac{K_{\text{shell}}}{K_{\text{gun}}} = \frac{\frac{v_1^2}{2} + \frac{k^2v_1^2}{2(k+1)^2}}{\frac{kv_1^2}{2(k+1)^2}}$$

$$\Rightarrow \quad \frac{K_{\text{shell}}}{K_{\text{gun}}} = \frac{2k^2 + 2k + 1}{k}$$

CHAPTER 2

13. Applying Law of Conservation of Linear Momentum and Mechanical Energy we get,

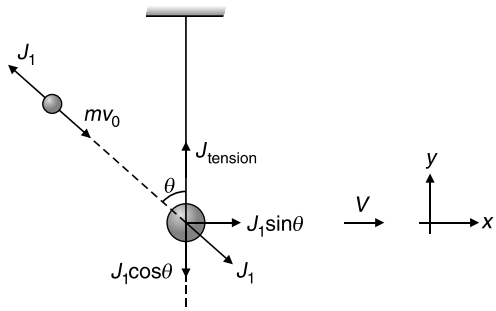
$$mv = MV \quad \ldots(1)$$

and $$mgR = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 \quad \ldots(2)$$

Solving (1) and (2), we get velocity of cube to be

$$v = \sqrt{\frac{2MgR}{M+m}}$$

14. Since tension in the string will be impulsive in nature. The impulse diagram of the situation is shown in Figure.



Applying impulse momentum equation to the particle, we get

$$-J_1 = 0 - mv_0$$

$$\Rightarrow \quad J_1 = mv_0$$

As the string is inextensible, so the bob cannot move in vertical direction. Hence,

$$J_{tension} = J_1\cos\theta$$

$$\Rightarrow \quad J_{tension} = mv_0\cos\theta$$

The horizontal component of impulse $(J_1\sin\theta)$ will be responsible for motion of the bob in horizontal direction, hence

$$J_1\sin\theta = MV$$

$$\Rightarrow \quad V = \frac{J_1\sin\theta}{M}$$

$$\Rightarrow \quad V = \frac{mv_0\sin\theta}{M}$$

So, we observe that after collision, the bob starts moving in the circular path.

15. Just after the string becomes taut, velocity of M (upwards) is given by

$$(M+m)v = m\sqrt{2gh}$$

$$\Rightarrow \quad v = \frac{m\sqrt{2gh}}{(M+m)}$$

Retardation,

$$a = \left(\frac{M-m}{M+m}\right)g$$

$$\Rightarrow \quad t = \frac{2v}{a} = \frac{2m\sqrt{2gh}}{(M+m)} \times \frac{M+m}{(M-m)g}$$

$$\Rightarrow \quad t = \frac{2m}{M-m}\sqrt{\frac{2h}{g}}$$

Loss in kinetic energy is given by

$$-\Delta K = K_i - K_f$$

$$\Rightarrow \quad -\Delta K = mgh - \frac{1}{2}(M+m)v^2$$

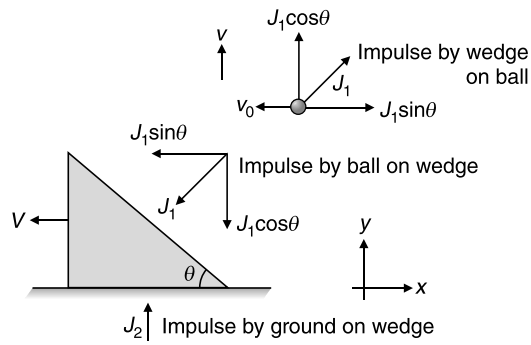
$$\Rightarrow \quad -\Delta K = mgh - \frac{1}{2}(M+m)\frac{m^2(2gh)}{(M+m)^2}$$

$$\Rightarrow \quad -\Delta K = \frac{Mmgh}{M+m}$$

16. In this case both the normal reactions, i.e. between the ball and wedge and between wedge and ground will be impulsive in nature and since impulse is

$$\vec{J} = \int_0^{\Delta t} \vec{F}dt$$

Hence the direction of impulse will be same as the direction of impulsive force which in this case is the normal reaction. Making impulsive diagram of the situation (impulsive diagram will be same as FBD in laws of motion) is shown in Figure.



Applying impulse momentum equation for ball along the x-direction, we get

$$(J_1)_x = (\vec{p}_{final})_x - (\vec{p}_{initial})_x$$

Since the final linear momentum of the ball in x-direction will be zero because the ball starts moving in upward direction, so we get

$$J_1\sin\theta = 0 - (-mv_0) = mv_0 \quad \ldots(1)$$

From equation (1), we get

$$J_1 = \frac{mv_0}{\sin\theta} = mv_0\,\text{cosec}\,\theta \quad \ldots(2)$$

Since the wedge is not moving in the vertical direction, so from impulse diagram for the wedge, we see that

$$J_2 = J_1\cos\theta = mv_0\cot\theta$$

Since the ball just after collision starts moving up, so we can apply impulse momentum equation along the y-direction and get

$$(\vec{J}_1)_y = (\vec{p}_{\text{final}})_y - (\vec{p}_{\text{initial}})_y$$

$$\Rightarrow \quad J_1 \cos\theta = 0 - (-mv) = mv$$

$$\Rightarrow \quad v = \frac{J_1 \cos\theta}{m} = \frac{mv_0 \cot\theta}{m} \qquad \{\because \text{ of (2)}\}$$

$$\Rightarrow \quad v = v_0 \cot\theta$$

Just after collision the wedge starts moving towards left. Applying impulse-momentum equation to the wedge along the horizontal direction, we get

$$(\vec{J}_1)_x = (\vec{p}_{\text{final}})_x - (\vec{p}_{\text{initial}})_x$$

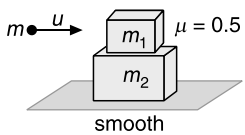
$$-J_1 \sin\theta = -MV - 0$$

$$\Rightarrow \quad V = \frac{J_1 \sin\theta}{M} = \frac{mv_0}{M}$$

Since the normal reaction between ground and wedge is impulsive, hence this means that the friction force between the wedge and ground will also be impulsive. So, impulse due to frictional force will be

$$J_{\text{friction}} = \mu J_2 = \mu(mv_0 \cot\theta)$$

17. Given $m = 0.25$ kg, $m_1 = 37.5$ kg, $m_2 = 12.5$ kg and $u = 302$ ms^{-1}



Let v be the common velocity then applying Conservation of Linear Momentum, we get

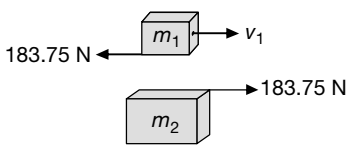
$$mu = (m + m_1 + m_2)v$$

$$\Rightarrow \quad v = \frac{mu}{m + m_1 + m_2} = \frac{0.25 \times 302}{0.25 + 37.5 + 12.5}$$

$$\Rightarrow \quad v = 1.5 \text{ ms}^{-1}$$

Let v_1 be the velocity of $(m + m_1)$ just after collision. Applying Conservation of Linear Momentum, we get

$$v_1 = \frac{mu}{m + m_1} = \frac{0.25 \times 302}{0.25 \times 37.5} = 2 \text{ ms}^{-1}$$



Force of friction between m_1 and m_2 till it stops is

$$f_1 = \mu m_1 g = 183.75 \text{ N}$$

$$\Rightarrow \quad a_r = \frac{183.75}{m_1} + \frac{183.75}{m_2} = 19.6 \text{ ms}^{-2}$$

Since, $v_r = v_1 = 2$ ms^{-1}

So, displacement of m_1 or m_2 is

$$s_r = \frac{v_r^2}{2a_r} = \frac{(2)^2}{2 \times 19.6} = 0.1 \text{ m}$$

18. When m leaves M, both would be moving at same speed horizontally, so we have

$$mu = (m + M)v$$

$$\Rightarrow \quad v = \frac{mu}{m + M}$$

The mass m will also have a vertical speed v_y due to which it rises to a maximum height H. Applying the Work Energy Theorem, we get

$$\frac{1}{2}mu^2 - mgH = \frac{1}{2}(m + M)\left(\frac{mu}{m + M}\right)^2$$

$$\Rightarrow \quad u^2 - 2gH = \frac{mu^2}{m + M}$$

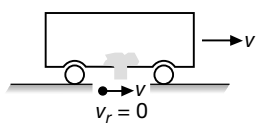
$$\Rightarrow \quad 2gH = u^2 - \frac{mu^2}{m + M} = \frac{Mu^2}{m + M}$$

$$\Rightarrow \quad H = \frac{Mu^2}{2g(M + m)}$$

Test Your Concepts-V (Based on Variable Mass Systems)

1. Since the sand spills through a hole in the bottom of the cart, so the relative velocity of the sand v_r will be zero because it will acquire the same velocity as that of the cart at all the instants. So thrust force is

$$F_t = 0 \qquad \left\{\text{as } F_t = v_r \frac{dm}{dt}\right\}$$



Hence the net force will only be F

$$\Rightarrow \quad F_{\text{net}} = F$$

$$\Rightarrow \quad m\left(\frac{dv}{dt}\right) = F \qquad \ldots(1)$$

But $m = m_0 - \mu t$

$$\Rightarrow \quad (m_0 - \mu t)\frac{dv}{dt} = F$$

$$\Rightarrow \quad \int_0^v dv = \int_0^t \frac{F dt}{m_0 - \mu t}$$

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$$\Rightarrow \quad v = \frac{F}{-\mu}\left[\log_e(m_0 - \mu t)\right]\Big|_0^t$$

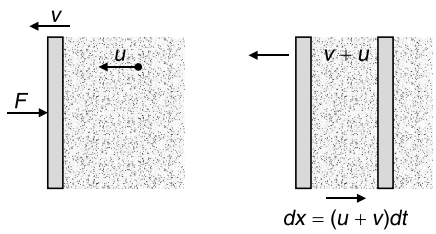
$$\Rightarrow \quad v = \frac{F}{\mu}\log_e\left(\frac{m_0}{m_0 - \mu t}\right)$$

From equation (1), we get acceleration of the cart to be

$$a = \frac{dv}{dt} = \frac{F}{m}$$

$$\Rightarrow \quad a = \frac{F}{m_0 - \mu t}$$

2. Velocity of plate relative to the dust particles is $(u + v)$, rightwards as shown in Figure.



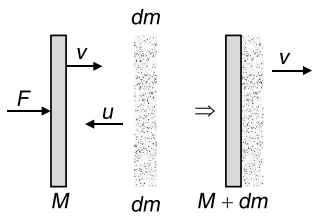
In time dt, the plate comes in contact with dust particles stored in volume given by

$$dV = Adx = A(u + v)dt$$

So, mass of dust striking the plate in time dt is

$$dm = \rho dV = \rho A(v + u)dt \qquad \ldots(1)$$

It we consider a plate of dust of mass dm as a system, and apply Impulse Momentum Theorem over time dt, then we get



$$Fdt = (M + dm)v - (Mv - udm)$$

$$\Rightarrow \quad Fdt = Mv + vdm - Mv + udm$$

$$\Rightarrow \quad F = (u + v)\left(\frac{dm}{dt}\right) \qquad \ldots(2)$$

Substituting equation (1) in (2), we get

$$F = \rho A(v + u)^2$$

3. (a) (i) To just lift the rocket off the launching pad, we have

Weight = Thrust Force

$$\Rightarrow \quad mg = v_r\left(\frac{-dm}{dt}\right)$$

$$\Rightarrow \quad \left(\frac{-dm}{dt}\right) = \frac{mg}{v_r}$$

Substituting the values, we get

$$\left(\frac{-dm}{dt}\right) = \frac{(450 + 50)(10)}{2 \times 10^3} = 2.5 \text{ kgs}^{-1}$$

(ii) Net acceleration $a = 20 \text{ ms}^{-2}$

Since we have $ma = F_t - mg$

$$\Rightarrow \quad a = \frac{F_t}{m} - g$$

$$\Rightarrow \quad a = \frac{v_r}{m}\left(\frac{-dm}{dt}\right) - g$$

$$\Rightarrow \quad \left(-\frac{dm}{dt}\right) = \frac{m(g + a)}{v_r}$$

$$\Rightarrow \quad -\frac{dm}{dt} = \frac{(450 + 50)(10 + 20)}{2 \times 10^3}$$

$$\Rightarrow \quad -\frac{dm}{dt} = 7.5 \text{ kgs}^{-1}$$

(b) The rate of fuel consumption is 10 kgs^{-1}. So, the time for the consumption of entire fuel is

$$t = \frac{450}{10} = 45 \text{ second}$$

Since, we know that

$$v = u - gt + v_r \log_e\left(\frac{m_0}{m}\right)$$

In this problem, we have

$u = 0$, $v_r = 2 \times 10^3 \text{ ms}^{-1}$, $m_0 = 500$ kg and $m = 50$ kg

Substituting these values, we get,

$$v = -(10)(45) + (2 \times 10^3)\log_e\left(\frac{500}{50}\right)$$

$$\Rightarrow \quad v = -450 + 4606 = 4156 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = 4.156 \text{ kms}^{-1}$$

4. The thrust acting on the container due to ejection of water is bv_0 in the upward direction. Let us first write the equations of motion for mass and container by using the Newton's Second Law.

For mass, we have

$$m_1 g - T = m_1 a \qquad \ldots(1)$$

For container, we have

$$T + bv_0 - (m_0 - bt)g = (m_0 - bt)a \qquad \ldots(2)$$

Adding equations (1) and (2), we get

$$m_1 g + bv_0 - m_0 g + btg = (m_0 + m_1 - bt)a$$

$$\Rightarrow \quad a = \frac{(m_1 - m_0 + bt)g + bv_0}{m_0 + m_1 - bt}$$

5. For v to remain constant, net pulling force on the chain should be zero. This is possible only when

$$\begin{pmatrix}\text{Thrust}\\ \text{Force}\end{pmatrix} + \begin{pmatrix}\text{Weight}\\ \text{of Left}\\ \text{Side of}\\ \text{Chain}\end{pmatrix} = \begin{pmatrix}\text{Pull}\\ P\end{pmatrix} + \begin{pmatrix}\text{Weight}\\ \text{of Right}\\ \text{Side of}\\ \text{Chain}\end{pmatrix}$$

$$\Rightarrow \quad (\lambda v^2)+(\lambda h)g = P+(\lambda y)g$$

$$\Rightarrow \quad P = \lambda v^2 + \lambda g(h-y)$$

6. Rocket will start to lift when upthrust on rocket will balance its weight. So, we have

$$kv_0 = (M-kt)g$$

$$\Rightarrow \quad t = \frac{Mg-kv_0}{kg} = \frac{M}{k}-\frac{v_0}{g}$$

7. Since, $a = \dfrac{\text{Upward Force}}{\text{Mass}}$

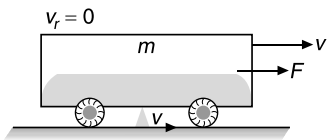
$$\Rightarrow \quad a = \frac{-u\left(\frac{dm}{dt}\right)}{m}$$

$$\Rightarrow \quad \frac{dm}{m} = -\left(\frac{a}{u}\right)dt$$

$$\Rightarrow \quad \int_{m_0}^{m}\frac{dm}{m} = -\left(\frac{a}{u}\right)\int_0^t dt$$

$$\Rightarrow \quad m = m_0 e^{-\left(\frac{a}{u}\right)t}$$

8. In this problem the sand spills through a hole at the bottom of the cart. Hence, the relative velocity of the sand v_r will be zero because it will acquire the same velocity as that of the cart at the moment.



Since the thrust force is given by

$$F_{\text{thrust}} = v_r\frac{dm}{dt}$$

$$\Rightarrow \quad F_{\text{thrust}} = 0$$

So, net force equals the external force F, hence

$$F_{\text{net}} = F$$

$$\Rightarrow \quad m\left(\frac{dv}{dt}\right) = F$$

Since, $m = m_0 - \mu t$

$$\Rightarrow \quad (m_0-\mu t)\frac{dv}{dt} = F$$

$$\Rightarrow \quad \int_0^v dv = \int_0^t \frac{Fdt}{m_0-\mu t}$$

$$\Rightarrow \quad v = -\frac{F}{\mu}\ln(m_0-\mu t)\Big|_0^t$$

$$\Rightarrow \quad v = \frac{F}{\mu}\ln\left(\frac{m_0}{m_0-\mu t}\right)$$

Since the acceleration is given by

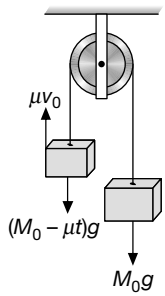
$$a = \frac{dv}{dt} = \frac{F}{m}$$

$$\Rightarrow \quad a = \frac{F}{m_0-\mu t}$$

9. Net pulley force at time t is,

$$F = \left[M_0 g + v_0\left(\frac{dm}{dt}\right)\right]-\left[M_0 - t\frac{dm}{dt}\right]g$$

$$\Rightarrow \quad F = (M_0 g + \mu v_0)-(M_0-\mu t)g = \mu(v_0+tg)$$



Total mass of system at time t is $M = 2M_0 - \mu t$

Since $F = M\dfrac{dv}{dt}$

$$\Rightarrow \quad (2M_0-\mu t)\frac{dv}{dt} = \mu(v_0+tg)$$

$$\Rightarrow \quad \int_0^v dv = \mu\int_0^t\left(\frac{v_0+gt}{2M_0-\mu t}\right)dt$$

Integrating, we get

$$v = \left(\frac{2M_0 g+\mu v_0}{\mu}\right)\log_e\left(\frac{2M_0}{2M_0-\mu t}\right)+gt$$

Test Your Concepts-VI (Based on Head-On Collisions)

1. Since $v_1 = \left(\dfrac{m_1-em_2}{m_1+m_2}\right)u_1+\left(\dfrac{(1+e)m_2}{m_1+m_2}\right)u_2$

and $v_2 = \left(\dfrac{(1+e)m_1}{m_1+m_2}\right)u_1+\left(\dfrac{m_2-em_1}{m_1+m_2}\right)u_2$

So, for $m_1 = m_2 = m$ (say) and $u_2 = 0$, we have

$$v_1 = \left(\frac{1+e}{2}\right)u \text{ and } v_2 = \left(\frac{1-e}{2}\right)u$$

Given that $K_f = \dfrac{3}{4}K_i$

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$$\Rightarrow \quad \frac{1}{2}mv_1^2+\frac{1}{2}mv_2^2=\frac{3}{4}\left(\frac{1}{2}mu^2\right)$$

Substituting the value, we get

$$\left(\frac{1+e}{2}\right)^2+\left(\frac{1-e}{2}\right)^2=\frac{3}{4}$$

$$\Rightarrow \quad (1+e)^2+(1-e)^2=3$$

$$\Rightarrow \quad 2+2e^2=3$$

$$\Rightarrow \quad e^2=\frac{1}{2}$$

$$\Rightarrow \quad e=\frac{1}{\sqrt{2}}$$

2. $u_1=12\ \text{ms}^{-1}$

$$\Rightarrow \quad m_1=4\ \text{kg}$$

$$\Rightarrow \quad u_2=4\ \text{ms}^{-1}$$

$$\Rightarrow \quad m_2=8\ \text{kg}$$

Let v_1 and v_2 be the velocities after impact

Conservation of momentum

$$m_1v_1+m_2v_2=m_1u_1+m_2u_2$$

$$\Rightarrow \quad 4v_1+8v_2=80 \qquad \ldots(1)$$

Newton's experimental law

$$v_1-v_2=-e(u_1-u_2)$$

$$v_1-v_2=-0.5(12-4)=-4 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_1=4\ \text{ms}^{-1},\ v_2=8\ \text{ms}^{-1}$$

Loss in kinetic energy is

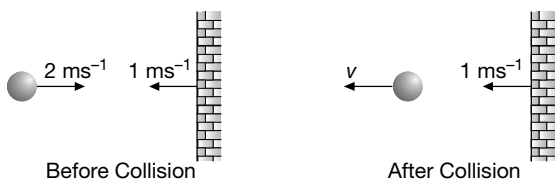
$$-\Delta K=K_i-K_f$$

$$\Rightarrow \quad -\Delta K=\frac{1}{2}\left(m_1u_1^2+m_2u_2^2-m_1v_1^2-m_2v_2^2\right)$$

$$\Rightarrow \quad -\Delta K=\frac{1}{2}[4(144)+8(16)-4(16)-8(64)]$$

$$\Rightarrow \quad -\Delta K=64\ \text{J}$$

3. The speed of wall will not change after the collision. So, let v be the velocity of the ball after collision in the direction shown in figure. Since collision is elastic $(e=1)$,



$$\left(\begin{array}{c}\text{Velocity of Approach of}\\ \text{Bodies just Before Impact}\end{array}\right)=\left(\begin{array}{c}\text{Velocity of Separation of}\\ \text{Bodies just After Impact}\end{array}\right)$$

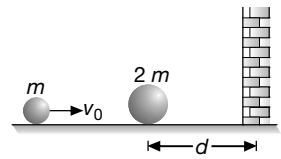
$$\Rightarrow \quad 2+1=v-1$$

$$\Rightarrow \quad v=4\ \text{ms}^{-1}$$

4. $$v_{2m}=\left(\frac{m+\frac{m}{4}}{m+2m}\right)v_0=\frac{5v_0}{12}$$

$$v_m=\left(\frac{m-\frac{2m}{4}}{m+2m}\right)v_0=\frac{v_0}{6}$$

So, $$t_1=\frac{d}{\frac{5v_0}{12}}=\frac{2.4d}{v_0}$$



In this time, the ball m will move a distance.

$$s_1=\left(\frac{v_0}{6}\right)\left(\frac{2.4d}{v_0}\right)=0.4d$$

Next collision will take place after a time

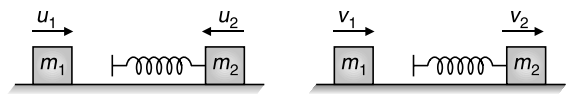
$$t_2=\frac{0.6d}{\left(\frac{v_0}{6}\right)+\left(\frac{5v_0}{48}\right)}=\frac{2.22d}{v_0}$$

In this time, the ball $2m$ will move a distance

$$s_2=\left(\frac{5v_0}{48}\right)\left(\frac{2.22d}{v_0}\right)$$ from the wall

$$\Rightarrow \quad s_2=0.23d$$

5. If we take the 'block + spring' as a system, no external forces acting on the system. Hence linear momentum of the system will remain conserved. Let the velocities of the blocks when the spring is again relaxed be v_1 and v_2 respectively. Now using conservation of linear momentum



$$m_1u_1-m_2u_2=m_1v_1+m_2v_2$$

$$\Rightarrow \quad 2\times4-1\times2=2\times v_1+1\times v_2$$

$$\Rightarrow \quad 2v_1+v_2=6 \qquad \ldots(1)$$

The collision of the spring block system can be treated as perfectly elastic collision. Hence, we can use the coefficient of restitution equation

$$e=\frac{v_2-v_1}{u_1-u_2}=1$$

where, $e=1$, $u_1=4\ \text{ms}^{-1}$, $u_2=-2\ \text{ms}^{-1}$

$$\Rightarrow \quad 1 = \frac{v_2 - v_1}{4 - (-2)}$$

$$\Rightarrow \quad v_2 - v_1 = 6 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_1 = 0 \text{ and } v_2 = 6 \text{ ms}^{-1}$$

6. Since the collision is elastic between the blocks, the two identical blocks will exchange their velocities. First block will come to rest while the second will start moving with velocity v_0. The retardation in this block is μg. Hence, the velocity of this block before colliding with the wall will be given by

$$v^2 = v_0^2 - 2\mu g d$$

$$\Rightarrow \quad v = \sqrt{v_0^2 - 2\mu g d} \qquad \ldots(1)$$

Now, let v' be the velocity of this block in opposite direction after colliding with the wall, then as the collision is elastic $(e = 1)$, hence, relative speed of separation is equal to the relative speed of approach.

$$v' - u = v + u$$

$$\Rightarrow \quad v' = (2u + v) \qquad \ldots(2)$$

Now, to avoid the second collision between the blocks

$$0 = (v')^2 - 2\mu g d$$

$$\Rightarrow \quad (2u + v) = \sqrt{2\mu g d}$$

$$\Rightarrow \quad u = \frac{\sqrt{2\mu g d} - v}{2} = \frac{\sqrt{2\mu g d} - \sqrt{v_0^2 - 2\mu g d}}{2}$$

Therefore, the maximum values of u is

$$\frac{\sqrt{2\mu g d} - \sqrt{v_0^2 - 2\mu g d}}{2}$$

7. Let velocities of sphere 1 and 2 after collision be v_1 and v_2 respectively, then by conservation of linear momentum, we have

$$mv_2 + mv_1 = mu \qquad \ldots(1)$$

Also, $e = \dfrac{v_2 - v_1}{u} \qquad \ldots(2)$

From equations (1) and (2), we get

$$v_2 = \left(\frac{1+e}{2}\right)u$$

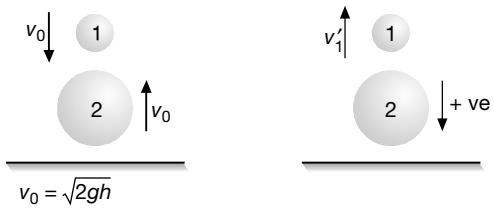
Now, when sphere 2 collides with sphere 3, then after collision velocity of 3 will be

$$v_3 = \left(\frac{1+e}{2}\right)^2 u \text{ and so on.}$$

Similarly, velocity of nth sphere will be

$$v_n = \left(\frac{1+e}{2}\right)^{n-1} u$$

8. Let us assume that the balls are slightly separated when the superball hits the floor.
Let v_1 be the velocity of small ball (1) just after collision.
Let v_1' be the velocity of small ball 1 just after collision with big ball 2. Then,



$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

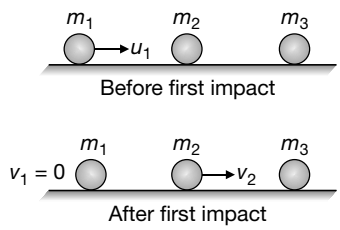
where $v_1 = v_0$, $v_2 = -v_0$ and $\dfrac{m_1}{m_2} = \dfrac{m}{M} \approx 0$, we get

$$v_1' = -3v_0 = -3\sqrt{2gh}$$

So, height gained by small ball after collision, is

$$h_1 = \frac{v_1'^2}{2g} = \frac{9(2gh)}{2g} = 9h$$

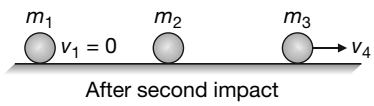
9. After first impact $v_1 = 0$



$$\Rightarrow \quad v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 = 0$$

$$\Rightarrow \quad m_1 = em_2 \qquad \ldots(1)$$

After second impact, $v_3 = 0$



$$\Rightarrow \quad \left(\frac{m_2 - em_3}{m_2 + m_3}\right)v_2 = 0$$

$$\Rightarrow \quad m_2 = em_3 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$m_2 = \sqrt{m_1 m_3}$$

10. By Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}m_1 v^2 = \frac{1}{2}kx^2 + m_1 gx$$

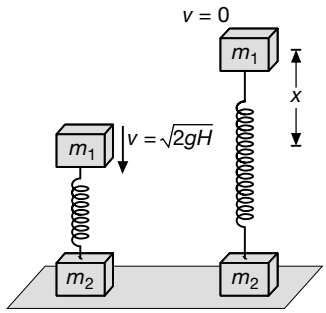
Since, $v = \sqrt{2gH}$

CHAPTER 2

$$\Rightarrow \quad 2m_1gH = kx^2 + 2m_1gx \qquad \ldots(1)$$

The lower block will rebounce only when

$$x > \frac{m_2g}{k} \qquad \{\because\ kx = m_2g\}$$



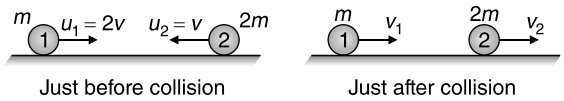
Substituting, $x = \frac{m_2g}{k}$ in Equation (1), we get

$$2m_1gH = k\left(\frac{m_2g}{k}\right)^2 + 2m_1g\left(\frac{m_2g}{k}\right)$$

$$\Rightarrow \quad H = \frac{m_2g}{k}\left(\frac{m_2 + 2m_1}{2m_1}\right)$$

$$\Rightarrow \quad H_{\min} = \frac{m_2g}{k}\left(\frac{m_2 + 2m_1}{2m_1}\right)$$

11. Let us assume the final velocities of the particles m and $2m$ be v_1 and v_2, respectively, as shown in Figure.



Applying conservation of linear momentum, we get

$$m(2v) + 2m(-v) = m(v_1) + 2m(v_2)$$

$$\Rightarrow \quad 0 = mv_1 + 2mv_2$$

$$\Rightarrow \quad v_1 + 2v_2 = 0 \qquad \ldots(1)$$

Since collision is elastic, so relative velocity of approach of bodies equals the relative velocity of separation of bodies. Hence, we get

$$v_2 - v_1 = 2v - (-v)$$

$$\Rightarrow \quad v_2 - v_1 = 3v \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_1 = -2v \text{ and } v_2 = v$$

This simply means that both masses reverse their direction of motion after the collision as shown in Figure.

12. Consider the two blocks plus the spring to be the system. No external force acts on this system in horizontal direction. Hence, the linear momentum will remain constant. Suppose, the block of mass M moves with a speed V and the other block with a speed v after losing contact with the spring. From Law of Conservation of Linear Momentum in horizontal direction, we get

$$MV - mv = 0$$

$$\Rightarrow \quad V = \frac{m}{M}v \qquad \ldots(1)$$

The initial energy of the system is $E_i = \frac{1}{2}kx^2$

The final energy of the system is

$$E_f = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

Since there is no friction so, mechanical energy will remain conserved.

$$\Rightarrow \quad \frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{1}{2}kx^2 \qquad \ldots(2)$$

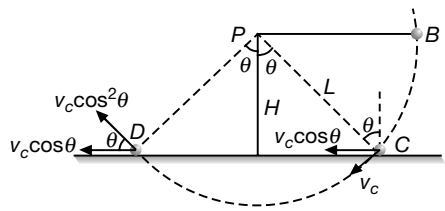
Solving equations (1) and (2), we get

$$v = x\sqrt{\frac{kM}{m(M+m)}} \text{ and}$$

$$V = x\sqrt{\frac{km}{M(M+m)}}$$

Test Your Concepts-VII (Based on Oblique Collisions)

1. In perfectly inelastic collision with the horizontal surface, the component parallel to the surface will remain unchanged.



$$\cos\theta = \frac{H}{L}$$

By Law of Conservation of Energy, we have

$$v_c = \sqrt{2gH}$$

Similarly, when the string becomes taut again, the component perpendicular to its length will remain unchanged. So,

$$v_c\cos^2\theta = \left(\sqrt{2gH}\right)\frac{H^2}{L^2} = v \qquad \{\text{say}\}$$

Again, using Law of Conservation of Energy, we get

$$h = \frac{v^2}{2g} = \frac{(2gH)\frac{H^4}{L^4}}{2g} = \frac{H^5}{L^4}$$

2. Let $\vec{v}$ be the velocity of the sphere after impact. To find $\vec{v}$ we must separate the velocity components parallel and perpendicular to the wall.

Since, after impact the component of velocity parallel to the wall remains unchanged while component perpendicular to the wall becomes e times in opposite direction. So, we have

$$\vec{v} = -\frac{3}{2}\hat{i} + \hat{j}$$

Therefore, the velocity of the sphere after impact is $-\frac{3}{2}\hat{i} + \hat{j}$

The loss in kinetic energy is

$$-\Delta K = K_i - K_f$$

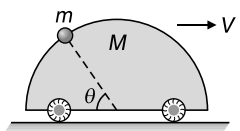
$$\Rightarrow \quad -\Delta K = \frac{1}{2}m\left(3^2 + 1^2 - \left(\frac{3}{2}\right)^2 - 1^2\right) = \frac{27m}{8}\ \text{J}$$

Since impulse $\vec{J} = \Delta\vec{p}$, so we have

$$\vec{J} = m\left(-\frac{3}{2}\hat{i} + \hat{j}\right) - m\left(3\hat{i} + \hat{j}\right) = -\left(\frac{9m}{2}\right)\hat{i}$$

3. Since the collision is perfectly inelastic, so the putty wedge system of mass $(M+m)$ moves as a single body with a velocity V. Applying the conservation of linear momentum, we get

$$mv_0\cos\theta = (M+m)V$$



$$\Rightarrow \quad V = \frac{mv_0\cos\theta}{M+m}$$

The loss in kinetic energy of the system is

$$\text{Loss} = -\Delta K = K_i - K_f$$

$$\Rightarrow \quad -\Delta K = \frac{1}{2}mv_0^2 - \frac{1}{2}(M+m)V^2$$

where, $V = \frac{mv_0\cos\theta}{M+m}$

$$\Rightarrow \quad -\Delta K = \frac{1}{2}mv_0^2 - \frac{1}{2}(M+m)\left(\frac{mv_0\cos\theta}{M+m}\right)^2$$

$$\Rightarrow \quad -\Delta K = \frac{mv_0^2}{2}\left(1 - \frac{m\cos^2\theta}{M+m}\right)$$

$$\Rightarrow \quad -\Delta K = \frac{mv_0^2}{2}\left(\frac{M+m\sin^2\theta}{M+m}\right)$$

4. Since impulse equals change in momentum, so we have

$$\vec{J} = m\left(\hat{i} + 3\hat{j}\right) - m\left(4\hat{i} - \hat{j}\right)$$

$$\Rightarrow \quad \vec{J} = m\left(-3\hat{i} + 4\hat{j}\right)$$

To find the coefficient of restitution we require the velocity components, before and after impact, in the direction of impulse i.e., along the line of impact, i.e., in the direction $-3\hat{i} + 4\hat{j}$. The unit vector in the direction of $\vec{J}$ is $\frac{1}{5}\left(-3\hat{i} + 4\hat{j}\right)$. So, the magnitudes of the velocity components in this direction just before impact is

$$\left(4\hat{i} - \hat{j}\right)\cdot\frac{1}{5}\left(-3\hat{i} + 4\hat{j}\right) = -\frac{16}{5}$$

and just after impact is

$$\left(\hat{i} + 3\hat{j}\right)\cdot\frac{1}{5}\left(-3\hat{i} + 4\hat{j}\right) = \frac{9}{5}$$

The significance of the negative sign in $-\frac{16}{5}$ is simply an indication that this component is in a direction opposite to that of $\vec{J}$.

The speed of approach to the wall is therefore, $\frac{16}{5}$ and the speed of separation is $\frac{9}{5}$. Since

$$e = \frac{\text{Speed of Separation}}{\text{Speed of Approach}}$$

$$\Rightarrow \quad e = \frac{9/5}{16/5} = \frac{9}{16}$$

5. Applying Law of Conservation of Linear Momentum in vector form, we get

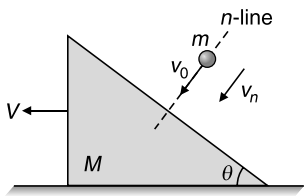
$$mu\hat{i} = 0 + mv\hat{j} + m\vec{v}'$$

$$\Rightarrow \quad \vec{v}' = u\hat{i} - v\hat{j}$$

$$\Rightarrow \quad |\vec{v}'| = \sqrt{u^2 + v^2}$$

6. **Method I: By using conservation of linear momentum**

If we take wedge and ball as a system, then no external force is acting along the horizontal direction, so linear momentum is conserved along the horizontal direction. Since the ball is moving along the normal direction, so it has no component of velocity along the inclined surface, i.e. the common tangent direction or along the t-line.



Let velocity of the ball and wedge after collision be v_n and V respectively, then by conservation of linear momentum applied along the horizontal direction, we have

$$mv_0 \sin\theta = MV + mv_n \sin\theta \quad \ldots(1)$$

Also, $v_2 - v_1 = e(u_1 - u_2)$

$$\Rightarrow \quad V\sin\theta - v_n = e(v_0 - 0) \quad \ldots(2)$$

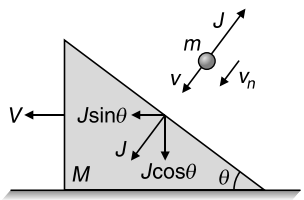
Solving equations (1) and (2), we get

$$V = \left[\frac{(1+e)m\sin\theta}{M + m\sin^2\theta}\right]v_0$$

and $v_n = V\sin\theta - ev_0 = \left(\frac{m\sin^2\theta - Me}{M + m\sin^2\theta}\right)v_0$

Method 2: By using the impulse method

The impulse acting on the ball and wedge is shown in Figure.



Since impulse equals the change in momentum of the body, so

for the wedge, we have

$$J\sin\theta = MV \quad \ldots(1)$$

for the ball, we have

$$-J = mv_n - mv_0 \quad \ldots(2)$$

Also, $v_2 - v_1 = e(u_1 - u_2)$

$$\Rightarrow \quad V\sin\theta - v_n = e(v_0 - 0) \quad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$V = \left[\frac{(1+e)m\sin\theta}{M + m\sin^2\theta}\right]v_0 \text{ and}$$

$$v_n = \left(\frac{m\sin^2\theta - Me}{M + m\sin^2\theta}\right)v_0$$

7. Vertical component of velocity and hence the time of flight becomes e times. Since,

$$\text{Range} = u_H T$$

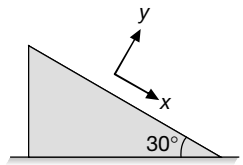
where, u_H is the horizontal component of velocity which remains unchanged due to collision. Hence the new range will become e times i.e.,

$$L_2 = eL_1$$

8. $$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{4 + \left(\frac{R}{\sqrt{3}}\right)}{g}}$$

Since, $R = v_A t$

$$\Rightarrow \quad R = 2\sqrt{\frac{4 + \left(\frac{R}{\sqrt{3}}\right)}{g}}$$



Squaring and solving, we get

$$R \approx 2 \text{ m} \text{ and } t \approx 1 \text{ s}$$

Now, $u_x = 2\cos(30°) = \sqrt{3}$ ms^{-1} and

$u_y = 2\sin(30°) = 1$ ms^{-2}

$a_x = g\sin(30°) = 4.9$ ms^{-2} and

$a_y = -g\cos 30° = -4.9\sqrt{3}$ ms^{-2}

Speed after bouncing is

$$v = \sqrt{v_x^2 + (ev_y)^2}$$

$$\Rightarrow \quad v = \sqrt{\left(\sqrt{3} + 4.9\times 1\right)^2 + \left(0.6\left(1 - 4.9\sqrt{3}\times 1\right)\right)^2}$$

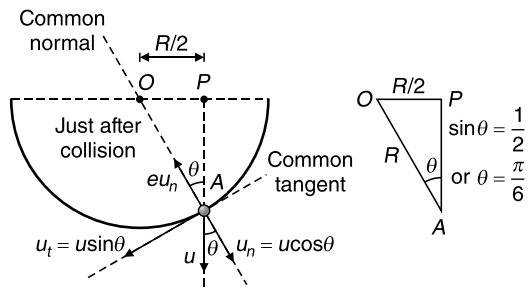
$$\Rightarrow \quad v \cong 8 \text{ ms}^{-1}$$

9. If after collision, the particle moves along the track, then this means that there is no normal component of velocity and hence $e = 0$.

Just before collision, the components of velocity along the normal and along the tangent are

$$u_n = u\cos\theta \text{ and } u_t = u\sin\theta$$

When the velocity of particle becomes horizontal just after collision with the track, then we have



The component of velocity along the common tangent direction will remain unchanged because in this direction there is no impulse. Hence

$$v_t = u_t = u\sin\theta \quad \ldots(1)$$

Also, speed of the particle just after impact along the n-line is e times the speed of particle before collision along the n-line.

$$v_n = eu_n = eu\cos\theta \quad \ldots(2)$$

Since after collision, the particle moves in horizontal direction, so vertical component of particle velocity should be zero. For this condition to be obeyed, we have

$$eu_n\cos\theta = v_t\sin\theta \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$(eu\cos\theta)\cos\theta = (u\sin\theta)\sin\theta$$

$$\Rightarrow \quad e = \tan^2\theta = \tan^2\left(\frac{\pi}{6}\right) = \frac{1}{3}$$

10. (a) $u_r = 0$, $a_r = g$

$$\Rightarrow \quad v_r = \sqrt{2gh_1}$$

After collision relative velocity $v_r' = e\sqrt{2gh_1}$ and relative retardation is still g (downwards). So,

$$h_2 = \frac{(v_r')^2}{2g} = e^2h_1$$

(b) $u_r = 0$, $a_r = g + \frac{g}{4} = \frac{5g}{4}$

So, just before collision, we have

$$v_r = \sqrt{2\left(\frac{5g}{4}\right)h_1}$$

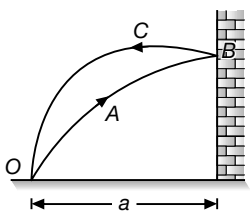
Just after collision, $v_r' = ev_r$

Relative retardation is still $\frac{5g}{4}$

Hence, $h_2 = \dfrac{(v_r')^2}{2\left(\frac{5g}{4}\right)} = e^2h_1$

11. The horizontal component of the velocity of ball during the path OAB is $u\cos\alpha$ while in its return journey BCO it is $eu\cos\alpha$. The time of flight T also remains unchanged. Hence,

$$T = t_{OAB} + t_{BCO}$$



$$\Rightarrow \quad \frac{2u\sin\alpha}{g} = \frac{a}{u\cos\alpha} + \frac{a}{eu\cos\alpha}$$

$$\Rightarrow \quad \frac{a}{eu\cos\alpha} = \frac{2u\sin\alpha}{g} - \frac{a}{u\cos\alpha}$$

$$\Rightarrow \quad \frac{a}{eu\cos\alpha} = \frac{2u^2\sin\alpha\cos\alpha - ag}{gu\cos\alpha}$$

$$\Rightarrow \quad e = \frac{ag}{2u^2\sin\alpha\cos\alpha - ag}$$

$$\Rightarrow \quad e = \frac{1}{\left(\frac{u^2\sin(2\alpha)}{ag} - 1\right)}$$

Single Correct Choice Type Questions

1. Horizontal and vertical components of initial velocity are

$$u_x = 20\sqrt{2}\cos 45° = 20 \text{ ms}^{-1}$$

and $u_y = 20\sqrt{2}\sin 45° = 20 \text{ ms}^{-1}$

After 1 s, horizontal component remains unchanged while the vertical component becomes

$$v_y = u_y - gt$$

$$\Rightarrow \quad v_y = 20 - (10)(1) = 10 \text{ ms}^{-1}$$

Due to explosion one part comes to rest. Hence, from Conservation of Linear Momentum, vertical component of second part will become $v_y' = 20 \text{ ms}^{-1}$. Therefore, maximum height attained by the second part will be

$$H = h_1 + h_2$$

Where, h_1 be the height attained in 1 s.

$$\Rightarrow \quad h_1 = (20)(1) - \frac{1}{2}(10)(1)^2 = 15 \text{ m}$$

and h_2 be the height attained after 1 s.

$$\Rightarrow \quad h_2 = \frac{v_y'^2}{2g} = \frac{(20)^2}{2\times 10} = 20 \text{ m}$$

$$\Rightarrow \quad H = 20 + 15 = 35 \text{ m}$$

Hence, the correct answer is (B).

2. Let speed of block be V. Then by Law of Conservation of Linear Momentum (applied along the horizontal direction), velocity of cylinder will be v in opposite direction, then

$$mv - MV = 0$$

$$\Rightarrow \quad \frac{Mv}{2} = MV$$

$$\Rightarrow \quad v = 2V$$

By Law of Conservation of Mechanical Energy, we have

$$mgh = \frac{1}{2}MV^2 + \frac{1}{2}m(2V)^2$$

where, $h = R - r = 1$ m

Substituting the values, we get

$$(1)(10)(1) = \frac{1}{2}(2)(V^2) + \frac{1}{2}(1)(4V^2)$$

$$\Rightarrow \quad 3v^2 = 10$$

$$\Rightarrow \quad v = \sqrt{\frac{10}{3}} \text{ ms}^{-1}$$

Hence, the correct answer is (A).

3. Since centre of mass of 1 g, 2 g and 3 g is at $(2,\ 2,\ 2)$

$$\Rightarrow$$ a mass of $(1+2+3)$ g $= 6$ g is placed at $(2,\ 2,\ 2)$.

CHAPTER 2

Let the new mass of 4 g be placed at $\vec{r}$, such that the new centre of mass is new at origin $(0, 0, 0)$. Hence

$$(0, 0, 0) = \frac{6(2,\ 2,\ 2) + 4\vec{r}}{10}$$

$$\Rightarrow \quad \vec{r} = (-3,\ -3,\ -3)$$

Hence, the correct answer is (C).

4. Let the displacement of wedge be x (leftwards). Horizontal displacement of A and B with respect to wedge is $10\cos 45°$ or $5\sqrt{2}$ cm (rightwards) or the horizontal displacement of A and B with respect to ground is $(5\sqrt{2} - x)$ cm rightwards. The centre of mass of the whole system will not move in horizontal direction. So,

$$(2m)x = (5\sqrt{2} - x)(m + 2m)$$

$$\Rightarrow \quad 5mx = 15\sqrt{2} \text{ m}$$

$$\Rightarrow \quad x = 3\sqrt{2} \text{ cm}$$

Hence, the correct answer is (B).

5. $m = m_0 e^{-\lambda t}$

$$\Rightarrow \quad \left(-\frac{dm}{dt}\right) = m_0 \lambda e^{-\lambda t}$$

Now thrust force $F = u\left(-\frac{dm}{dt}\right) = um_0\lambda e^{-\lambda t}$

$$\Rightarrow \quad m\left(\frac{dv}{dt}\right) = um_0\lambda e^{-\lambda t}$$

$$\Rightarrow \quad \left(m_0 e^{-\lambda t}\right)\frac{dv}{dt} = um_0\lambda e^{-\lambda t}$$

$$\Rightarrow \quad dv = u\lambda dt$$

$$\Rightarrow \quad \int_0^v dv = u\lambda \int_0^t dt$$

$$\Rightarrow \quad v = u\lambda t$$

Hence, the correct answer is (B).

6. By Work-Energy Theorem

$$\frac{1}{2}m(v^2 - u^2) = Fs \qquad \ldots(1)$$

For the second case

$$\frac{1}{2}m(v'^2 - u'^2) = Fs \qquad \ldots(2)$$

where F is the retarding force offered by the plank to the bullet.

Equating (1) and (2), we get

$$v^2 - u^2 = v'^2 - u'^2$$

$$\Rightarrow \quad (150)^2 - (125)^2 = v'^2 - (90)^2$$

$$\Rightarrow \quad v'^2 = (35)^2$$

$$\Rightarrow \quad v' = 35 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

7. Let J be the impulse and v the common speed. Since,

Impulse = Change in Linear Momentum

So, for mass m, we have

$$J = mv_0 - mv \qquad \ldots(1)$$

where, $v_0 = \sqrt{2gh}$

and for mass 2m, we have

$$J = 2mv \qquad \ldots(2)$$

From equations (1) and (2), we get

$$v = \frac{\sqrt{2gh}}{3}$$

Hence, the correct answer is (D).

8. Before explosion, particle was moving along x-axis, i.e., it has no y-component of velocity. Therefore, the centre of mass will not move in y-direction

$$\Rightarrow \quad y_{cm} = 0$$

$$\Rightarrow \quad y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

$$\Rightarrow \quad 0 = \frac{\left(\frac{m}{4}\right)(+15) + \left(\frac{3m}{4}\right)(y)}{\left(\frac{m}{4} + \frac{3m}{4}\right)}$$

$$\Rightarrow \quad y = -5 \text{ cm}$$

Hence, the correct answer is (A).

9. Since $\vec{F}_{ext} \neq 0$

$\vec{a}_{cm} = a_o$, $\vec{v}_{cm} = v_o$ at any time t.

Since $\vec{a}_{cm} = \frac{\vec{F}_{ext}}{M}$ so $a_0 \neq 0$

$v_0 = 0$ and $a_0 \neq 0$

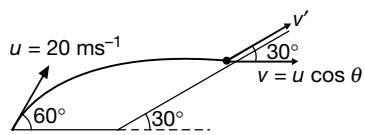
Hence, the correct answer is (B).

10. Speed of ball before collision is

$$v = u\cos 60° = (20)\left(\frac{1}{2}\right) = 10 \text{ ms}^{-1}$$

Since, collision is perfectly inelastic $(e = 0)$, the ball will not bounce. It will move along the plane with velocity

$$v' = v\cos 30° = \frac{10\sqrt{3}}{2} = 5\sqrt{3} \text{ ms}^{-1}$$



Maximum height attained by the ball is

$$H = \frac{u^2\sin^2 60°}{2g} + \frac{v'^2}{2g}$$

$$\Rightarrow \quad H = \frac{(20)^2\left(\frac{\sqrt{3}}{2}\right)^2}{20} + \frac{(5\sqrt{3})^2}{20}$$

$$\Rightarrow \quad H = \frac{300}{20} + \frac{75}{20} = \frac{375}{20} = 18.75 \text{ m}$$

Hence, the correct answer is (B).

11. Retardation due to friction

$$a = \mu g = (0.25)(10) = 2.5 \text{ ms}^{-2}$$

Since collision is elastic, i.e., after collision first block comes to rest and the second block acquires the velocity of first block. Distance travelled by it will be

$$s = \frac{v^2}{2a} = \frac{(5)^2}{(2)(2.5)} = 5 \text{ m}$$

So, final separation x is

$$x = 5 - 2 = 3 \text{ m}$$

Hence, the correct answer is (C).

12. In perfectly inelastic collision between two particles, linear momentum is conserved. If θ be the angle between the velocities of the two particles before collision, then

$$p^2 = p_1^2 + p_2^2 + 2p_1p_2\cos\theta$$

$$\Rightarrow \quad \left(2m\frac{v}{2}\right)^2 = (mv)^2 + (mv)^2 + 2(mv)(mv)\cos\theta$$

$$\Rightarrow \quad 1 = 1 + 1 + 2\cos\theta$$

$$\Rightarrow \quad \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \quad \theta = 120^\circ$$

Hence, the correct answer is (D).

13. In one dimensional collision of two particles, the velocities are interchanged when collision is elastic and masses are equal.

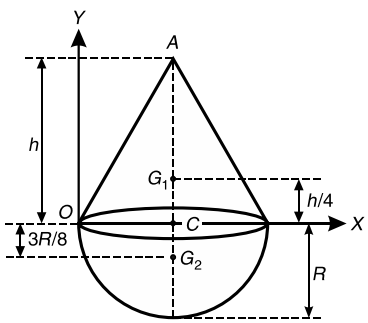
Hence, the correct answer is (A).

14. $M_1 =$ Mass of cone $= \frac{1}{3}\pi R^2 h\rho$

$M_2 =$ Mass of hemisphere $= \frac{2}{3}\pi R^3\rho$

Since $CG_1 = \frac{h}{4}$ therefore $AG_1 = \frac{3h}{4}$

Also $CG_2 = \frac{3R}{8}$



The position vectors of points G_1 and G_2 are

$$\vec{r}_1 = R\hat{i} + \frac{h}{4}\hat{j} \text{ and}$$

$$\vec{r}_2 = R\hat{i} + \frac{3R}{8}(-\hat{j}) = R\hat{i} - \frac{3R}{8}\hat{j}$$

According to the problem the centre of mass lies at C and hence $\vec{r}_{cm} = R\hat{i}$

Since $\vec{r}_{cm} = \frac{M_1\vec{r}_1 + M_2\vec{r}_2}{M_1 + M_2}$

Substituting values to get $\frac{h}{R} = \sqrt{3}$

Hence, the correct answer is (A).

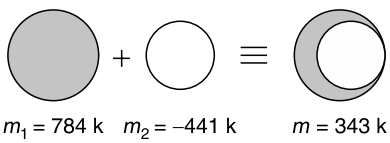
15. $\begin{pmatrix}\text{Kinetic energy}\\ \text{of a system}\\ \text{of particles}\end{pmatrix} = \begin{pmatrix}\text{KE of}\\ \text{centre of}\\ \text{mass}\end{pmatrix} + \begin{pmatrix}\text{KE of different}\\ \text{particles in the}\\ \text{frame of reference}\\ \text{of centre of mass}\end{pmatrix}$

Here, KE of centre of mass is $\frac{1}{2}mv^2$

So, KE of the system of particles $\geq \frac{1}{2}mv^2$

Hence, the correct answer is (C).

16. This problem can be done by using the concept of "Negative Mass being added to a system".



$$r_{cm} = \frac{(784k)(0) + (-441k)7}{784k + (-441k)}$$

$$\Rightarrow \quad r_{cm} = 9 \text{ cm}$$

Hence, the correct answer is (B).

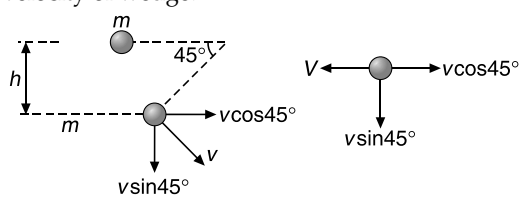
17. Fraction of energy retained $= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$

$$\Rightarrow \quad \text{Fraction} = \left(\frac{m_{\text{nucleus}} - m_n}{m_{\text{nucleus}} + m_n}\right)^2$$

$$\Rightarrow \quad \text{Fraction} = \left(\frac{Am_n - m_n}{Am_n + m_n}\right)^2 = \left(\frac{A-1}{A+1}\right)^2$$

Hence, the correct answer is (B).

18. Let v be the velocity of ball w.r.t. wedge and V be the velocity of wedge.



CHAPTER 2

Applying conservation of linear momentum, we get

$$mV = m(v\cos 45° - V)$$

$$\Rightarrow \quad V = \frac{v}{2\sqrt{2}}$$

$$\Rightarrow \quad v = 2\sqrt{2}V$$

Applying conservation of energy, we get

$$\frac{1}{2}m\left[(v\cos 45° - V)^2 + (v\sin 45°)^2 + V^2\right] = mgh$$

$$\Rightarrow \quad \left(\frac{v}{\sqrt{2}} - V\right)^2 + \left(\frac{v}{\sqrt{2}}\right)^2 + V^2 = 2gh$$

$$\Rightarrow \quad \left(\frac{2\sqrt{2}V}{\sqrt{2}} - V\right)^2 + \left(\frac{2\sqrt{2}V}{\sqrt{2}}\right)^2 + V^2 = 2g\left(\frac{g}{\sqrt{2}}\right)$$

$$\Rightarrow \quad V^2 + 4V^2 + V^2 = gR\sqrt{2}$$

$$\Rightarrow \quad 6V^2 = gR\sqrt{2}$$

$$\Rightarrow \quad V = \sqrt{\frac{gR}{6}\sqrt{2}}$$

$$\Rightarrow \quad V = \sqrt{\frac{gR}{3\sqrt{2}}}$$

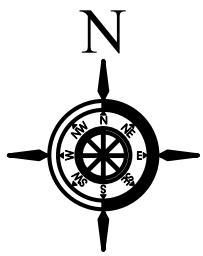
Hence, the correct answer is (A).

19. By Law of Conservation of Momentum

along *x*-axis

$$mv = 2mV\cos\theta$$

$$\Rightarrow \quad V\cos\theta = \frac{v}{2} \qquad \ldots(1)$$



along *y*-axis

$$mv = 2mV\sin\theta$$

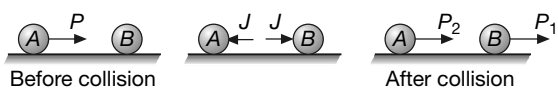
$$\Rightarrow \quad V\sin\theta = \frac{v}{2} \qquad \ldots(2)$$

Squaring (1) and (2) and adding, we get

$$V = \frac{v}{\sqrt{2}}$$

Hence, the correct answer is (C).

20. Let P_1 and P_2 be the momenta of A and B after collision.



Since, Impulse = Change in Linear Momentum, so we get

For *B*: $J = P_1$ …(1)

For *A*: $J = P - P_2$

$$\Rightarrow \quad P_2 = P - J \qquad \ldots(2)$$

Coefficient of restitution $e = \dfrac{P_1 - P_2}{P}$

$$\Rightarrow \quad e = \frac{P_1 - P + J}{P}$$

$$\Rightarrow \quad e = \frac{J - P + J}{P} = \frac{2J}{P} - 1$$

Hence, the correct answer is (B).

21. $e = \dfrac{v_1' - v_2'}{u_2 - u_1} = \dfrac{v_1 - v_2}{u - 3u}$

$$\Rightarrow \quad 2eu = v_2 - v_1$$

Hence, the correct answer is (C).

22. $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

Since $m_1 = m_2 = m$ {say}

$$\Rightarrow \quad \vec{v}_{cm} = \frac{\vec{v}_1 + \vec{v}_2}{2} \qquad \{m_1 = m_2\}$$

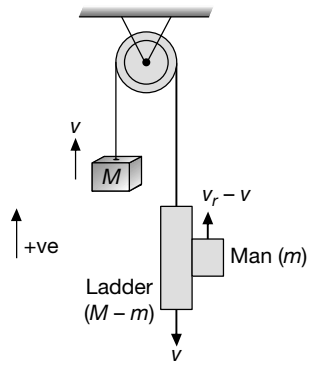
$$\Rightarrow \quad \vec{v}_{cm} = \left(\hat{i} + \hat{j}\right)\ \text{ms}^{-1}$$

Similarly, $\vec{a}_{cm} = \dfrac{\vec{a}_1 + \vec{a}_2}{2} = \dfrac{3}{2}\left(\hat{i} + \hat{j}\right)\ \text{ms}^{-2}$

Since, $\vec{v}_{cm}$ is parallel to $\vec{a}_{cm}$, so the path will be a straight line.

Hence, the correct answer is (C).

23. The rope tension is the same both on the left and right hand side at every instant, and consequently momentum of both sides are equal



$$\Rightarrow \quad Mv = (M - m)(-v) + m(v_r - v)$$

$$\Rightarrow \quad v = \frac{m}{2M}v_r$$

Momentum of the centre of mass is

$$P = P_1 + P_2$$

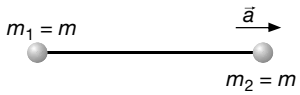
$$\Rightarrow \quad 2Mv_{com} = Mv + Mv$$

$$\Rightarrow \quad v_{com} = v = \frac{m}{2M}v_r$$

Hence, the correct answer is (B).

24. $\vec{a}_{cm} = \dfrac{m(0) + m\vec{a}}{m+m}$

$\vec{a}_{cm} = \dfrac{\vec{a}}{2}$



Hence, the correct answer is (B).

25. At maximum extension, velocity of both the blocks will be same. Let v be the common velocity of the blocks (towards right). By Law of Conservation of Linear Momentum, we get

$6(2) + 3(-1) = (3+6)\ v$

$\Rightarrow \quad v = 1\ \text{ms}^{-1}$

If x be the maximum extension in the spring, then by Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}(3)(1)^2 + \frac{1}{2}(6)(2)^2 = \frac{1}{2}(200)x^2 + \frac{1}{2}(9)(1)^2$$

$\Rightarrow \quad 3 + 24 = 200x^2 + 9$

$\Rightarrow \quad x = \sqrt{\dfrac{18}{200}}$

$\Rightarrow \quad x = 0.3\ \text{m}$

$\Rightarrow \quad x = 30\ \text{cm}$

Hence, the correct answer is (D).

26. Since, both have equal mass and collision is elastic, so pendulum will have a velocity v after collision

By Law of Conservation of Energy

$$\frac{1}{2}mv^2 = mgh$$

$\Rightarrow \quad h = \dfrac{v^2}{2g}$

Hence, the correct answer is (B).

27. For completely inelastic collision, the ball will stick to the pendulum

$\Rightarrow \quad mv = (m+m)V$

$\Rightarrow \quad V = \dfrac{v}{2}$

$\Rightarrow \quad h = \dfrac{V^2}{2g} = \dfrac{v^2}{8g}$

Hence, the correct answer is (D).

28. In an elastic oblique collision if two particles are of equal masses and the second particle is at rest, then after collisions the particles scatter at right angles.

Hence, the correct answer is (A).

29. On breaking off from the wall the speed of m_1 is zero and the spring is unstretched. If m_2 has a speed v_2 at that instant, then by Law of Conservation of Energy.

$$\begin{pmatrix}\text{Loss in Elastic}\\ \text{Potential Energy}\end{pmatrix} = \begin{pmatrix}\text{Gain in Kinetic}\\ \text{Energy of } m_2\end{pmatrix}$$

$\Rightarrow \quad \dfrac{1}{2}kx^2 = \dfrac{1}{2}m_2v_2^2$

$\Rightarrow \quad \dfrac{1}{2}(5)\left(\dfrac{4}{100}\right)^2 = \dfrac{1}{2}(5)v_2^2$

$\Rightarrow \quad v_2 = 0.04\ \text{ms}^{-1}$

So, $v_{CM} = \dfrac{m_1v_1 + m_2v_2}{m_1+m_2} = \dfrac{0+5(0.04)}{10+5} = \dfrac{0.04}{3}\ \text{ms}^{-1}$

$\Rightarrow \quad v_{CM} = \dfrac{4}{3}\ \text{cms}^{-1}$

Hence, the correct answer is (B).

30. $v_1 = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)u_1$

Since, $m_1 = \dfrac{4}{3}\pi r_1^3$ and $m_2 = \dfrac{4}{3}\pi r_2^3$

$\Rightarrow \quad v_1 = \left(\dfrac{r_1^3 - r_2^3}{r_1^3 + r_2^3}\right)u_1$ ...(1)

Similarly, $v_2 = \left(\dfrac{2m_1}{m_1+m_2}\right)u_1$

$\Rightarrow \quad v_2 = \left(\dfrac{2r_1^3}{r_1^3 + r_2^3}\right)u_1$ ...(2)

Put $r_1 = 4\ \text{cm}$ and $r_2 = 2\ \text{cm}$ in (1) and (2), we get

$$v_1 = \left(\frac{64-8}{64+8}\right)81 = 63\ \text{cms}^{-1}$$

$$v_2 = \frac{2(64)}{64+8} \times 81 = 144\ \text{cms}^{-1}$$

Hence, the correct answer is (A).

31. $(1)(12) + (2)(-24) = 1v_1 + 2v_2$

$\Rightarrow \quad v_1 + 2v_2 = -36$ ...(1)

Further $e = -\left(\dfrac{v_2 - v_1}{u_2 - u_1}\right)$

$\Rightarrow \quad \dfrac{2}{3} = -\left(\dfrac{v_2 - v_1}{-24 - 12}\right)$

$\Rightarrow \quad \dfrac{2}{3} = \dfrac{v_2 - v_1}{36}$

$\Rightarrow \quad v_2 - v_1 = 24$ ...(2)

Add (1) and (2)

$3v_2 = -12$

$\Rightarrow \quad v_2 = -4\ \text{ms}^{-1}$

$\Rightarrow \quad v_1 = -28\ \text{ms}^{-1}$

$\Rightarrow \quad$ Loss = Total Initial – Total Final

Since

$$\text{Total Initial} = \frac{1}{2}(1)(144) + \frac{1}{2}(2)(576)$$

CHAPTER 2

$\Rightarrow \quad E_i = 72 + 576 = 648$ J

Similarly, $E_f = \frac{1}{2}(1)(784) + \frac{1}{2}(2)(16)$

$\Rightarrow \quad E_f = 392 + 16$

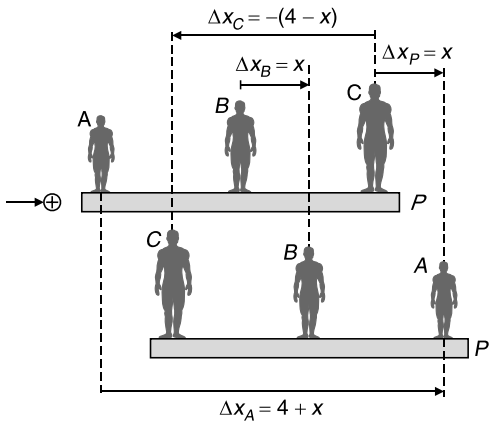
$\Rightarrow \quad E_f = 408$ J

$\Rightarrow \quad$ Loss $= 648 - 408$

$\Rightarrow \quad$ Loss $= 240$ J

Hence, the correct answer is (C).

32. When A and C exchange their position, then displacements of A, B, C and plank (P) are shown in Figure.



Since x_{cm} system remains fixed, so

$\Delta x_{cm} = 0$

$\Rightarrow \quad m_A \Delta x_A + m_B \Delta x_B + m_C \Delta x_C + m_P \Delta x_P = 0$

$\Rightarrow \quad 40(4 + x) + 50x - 60(4 - x) + 90x = 0$

$\Rightarrow \quad 240x = 80$

$\Rightarrow \quad x = +\frac{1}{3}$ m

So, B shifts $\frac{1}{3}$ m, rightwards

Hence, the correct answer is (B).

33. $4Mx = M(10R - x)$

$\Rightarrow \quad 4Mx = 10MR - Mx$

$\Rightarrow \quad x = 2R$

Hence, the correct answer is (B).

34. In an inelastic collision, only the momentum of system (ball and earth) may remain conserved. Some energy can be lost in the form of heat, sound etc.

Hence, the correct answer is (C).

35. Let x be the displacement of bead. Displacement of particle with respect to bead is $L(1-\cos\theta)$, i.e., displacement of particle with respect to ground will be $L(1-\cos\theta) - x$. Since net force in horizontal direction on the system is zero. Therefore, the centre of mass will not move in horizontal direction.

$\Rightarrow \quad 2mx = m[L(1-\cos\theta) - x]$

$\Rightarrow \quad 3mx = mL(1-\cos\theta)$

$\Rightarrow \quad x = \frac{L}{3}(1-\cos\theta)$

Hence, the correct answer is (D).

36. $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1$

$\Rightarrow \quad v_1 = 0$

Hence, the correct answer is (D).

37. $\begin{pmatrix}\text{Relative}\\ \text{velocity of}\\ \text{separation}\end{pmatrix} = \begin{pmatrix}\text{Relative}\\ \text{velocity of}\\ \text{approach}\end{pmatrix} = v \qquad \{\text{as } e = 1\}$

$\Rightarrow \quad$ Time of next collision $= \frac{2\pi r}{v}$

Hence, the correct answer is (B).

38. By Law of Conservation of Momentum

$2.9(0) + 0.1(150) = (2.9 + 0.1)v$

$\Rightarrow \quad v = \frac{(0.1)(150)}{3}$

$\Rightarrow \quad v = 5 \text{ ms}^{-1}$

By Law of Conservation of Energy

$\begin{pmatrix}\text{Loss in K.E. of}\\ \text{combined system}\end{pmatrix} = \begin{pmatrix}\text{Gain in P.E. of}\\ \text{combined system}\end{pmatrix}$

$\Rightarrow \quad \frac{1}{2}(M+m)v^2 = (M+m)gh$

$\Rightarrow \quad v^2 = 2gh$

But $h = \ell(1-\cos\theta)$

$\Rightarrow \quad v^2 = 2g\ell(1-\cos\theta)$

$\Rightarrow \quad 25 = 2(10)(2\cdot5)(1-\cos\theta)$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 60°$

Hence, the correct answer is (C).

39. Fraction Transferred

Fraction Lost $= \frac{4m_1m_2}{(m_1+m_2)^2}$

$\Rightarrow \quad$ Fraction $= \frac{4n}{(n+1)^2}$

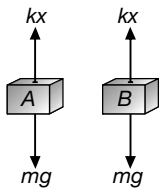
Hence, the correct answer is (D).

40. At maximum extension

$\frac{1}{2}kx^2 = mgx$

$\Rightarrow \quad x = \frac{2mg}{k}$ ...(1)

Free body diagram of both the blocks at this instant is shown in Figure.



So, $a_{cm} = \dfrac{2kx - 2mg}{2m} = \dfrac{kx - mg}{m}$ ...(2)

$\Rightarrow \quad a_{cm} = g$

Hence, the correct answer is (A).

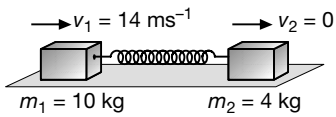
41. $x_{cm} = \dfrac{m\left(\dfrac{R}{2}\right) + 2m(x)}{m + 2m}$

$\Rightarrow \quad R = \dfrac{\dfrac{R}{2} + 2x}{3}$

$\Rightarrow \quad x = \dfrac{5}{4}R$

Hence, the correct answer is (C).

42.



$v_{cm} = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

$\Rightarrow \quad v_{cm} = \dfrac{10 \times 14 + 4 \times 0}{10 + 4} = \dfrac{10 \times 14}{14} = 10 \text{ ms}^{-1}$

Hence, the correct answer is (C).

43. $v_{cm} = \dfrac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \dfrac{10 \times 14 + 4 \times 0}{10 + 4} = 10 \text{ ms}^{-1}$

Hence, the correct answer is (C).

44. The system is stationary in air implies equilibrium, i.e. net force is zero. If the man now climbs up to the balloon using the rope, the centre of mass of the 'man plus balloon' system will remain stationary

Hence, the correct answer is (A).

45. Let $m_A = m$, then $m_B = 2m$. If f be the friction between the two blocks, then net force on block A is

$F_1 = mg\sin\theta - f$ {down the plane}

Similarly, net force on block B is

$F_2 = 2mg\sin\theta + f$ {down the plane}

Where, $f = \mu mg\cos\theta$

$\Rightarrow \quad a_{cm} = \dfrac{F_1 + F_2}{m + 2m} = \dfrac{3mg\sin\theta}{3m}$

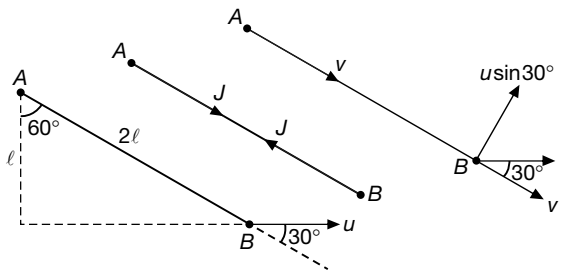
$\Rightarrow \quad a_{cm} = g\sin\theta$

Hence, the correct answer is (D).

46. When the string becomes taut both particles begin to move with velocity components v in the direction AB. Applying Law of Conservation of Linear Momentum in the direction AB, we get

$mu\cos 30° = mv + mv$

$\Rightarrow \quad v = \dfrac{u\sqrt{3}}{4}$



Hence, the velocity of ball A just after the string becomes taut is $\dfrac{u\sqrt{3}}{4}$

Hence, the correct answer is (A).

47. Since this situation is satisfied only for an elastic collision, so both momentum and energy are conserved.

Hence, the correct answer is (D).

48. By Law of Conservation of Momentum

$0 = mu + M(-v)$

$\Rightarrow \quad v = \dfrac{mu}{M}$

is the velocity of the man in the upward direction.

Since, no gravity is existing hence ball will reach the floor in a time $t = \dfrac{h}{u}$

During this time the man will move up by a distance $d = vt$

$\Rightarrow \quad d = \left(\dfrac{mu}{M}\right)\dfrac{h}{u} = \dfrac{mh}{M}$

So, total distance of man from floor when ball reaches the floor is

$s = d + h = h\left(1 + \dfrac{m}{M}\right)$

Hence, the correct answer is (A).

49. $v_1 = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)u_1$

$\Rightarrow \quad \dfrac{2}{3} = \dfrac{m_1 - m_2}{m_1 + m_2}$

$\Rightarrow \quad 2m_1 + 2m_2 = 3m_1 - 3m_2$

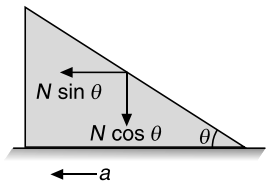
$\Rightarrow \quad 5m_2 = m_1$

CHAPTER 2

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{5}{1}$$

Hence, the correct answer is (C).

50. Let a be the acceleration of wedge leftwards and a_r the relative acceleration of block down the plane. Then absolute acceleration of block in horizontal direction will be $(a_r \cos\theta - a)$ towards right. Net force on the system in horizontal direction is zero. Therefore, acceleration of COM in horizontal direction will be zero or acceleration of wedge towards left is equal to the acceleration of block towards right.



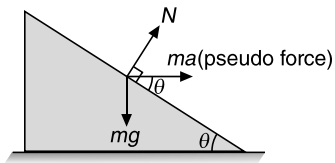
So, $a_r \cos\theta - a = a$

$$\Rightarrow \quad 2a = a_r \cos\theta \qquad \ldots(1)$$

Now let N be the normal reaction between the block and the wedge. Then free body diagram of wedge gives

$$N \sin\theta = ma \qquad \ldots(2)$$

Free body diagram of block with respect to wedge is

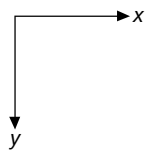


Net force on block perpendicular to plane is zero.

Hence, $N + ma\sin\theta = mg\cos\theta \qquad \ldots(3)$

Solving equations (1), (2) and (3), we get

$$a_r = \frac{2g\sin\theta}{1+\sin^2\theta}$$



Acceleration of block vertically downwards is

$$a_y = a_r \sin\theta$$

$$\Rightarrow \quad a_y = \frac{2g\sin^2\theta}{1+\sin^2\theta}$$

So, acceleration of COM is

$$a_{cm} = \frac{a_y}{2} = \frac{g\sin^2\theta}{(1+\sin^2\theta)}$$

Hence, the correct answer is (C).

51. If the shape of the body is spherical with its centre at origin then

$$R_{cm} = (0,0)$$

and $R_{cm} \frac{1}{M}\int r_i dm_i$

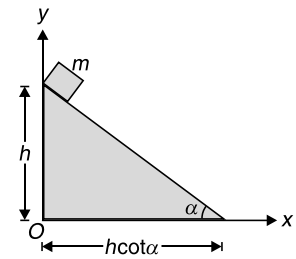
$$R_{cm} = R$$

$$\Rightarrow \quad R_{cm} \le R$$

Hence, the correct answer is (B).

52. Let the origin O be located just below the top position of m. Let the centre of mass of M be located at point (x, y) before m is released.

When m comes down to the floor, let the centre of mass of M move to the left by ℓ. The centre of mass of m has travelled through $(h\cot\alpha - \ell)$ to the right.



Initially, $(x_{cm})_{initial} = \frac{Mx+0}{M+m} = \frac{Mx}{M+m} \qquad \ldots(1)$

Finally, $(x_{cm})_{final} = \frac{M(x-\ell)+m(h\cot\alpha-\ell)}{M+m} \qquad \ldots(2)$

Further no external force is acting on the system and the system (M plus m) is initially at rest, so we must conclude that the position of centre of mass remains fixed.

$$\Rightarrow \quad \frac{Mx}{M+m} = \frac{M(x-\ell)+m(h\cot\alpha-\ell)}{M+m}$$

$$\Rightarrow \quad 0 = -\frac{M\ell}{M+m} + \frac{m}{M+m}(h\cot\alpha - \ell)$$

$$\Rightarrow \quad \ell = \frac{mh\cot\alpha}{M+m}$$

Hence, the correct answer is (C).

53. When the car C accelerates from rest to a velocity v_0 relative to the double-boat system, the two boats accelerate to the left, so we have

$$v_C(\text{to right}) + v_A(\text{to left}) = v_0$$

By conservation of momentum, we get

$$mv_C = 2Mv_A$$

Solving, we get

$$v_A = \frac{mv_0}{m+2M}, \; v_C = \frac{2Mv_0}{m+2M}$$

After the car applies brakes to come to rest, then the tension in the string connecting A, B also becomes zero. Applying conservation of momentum to A and C, we get

$$mv_C - Mv_A = (m+M)v'_A$$

So, velocity of A (to right) is

$$v'_A = \frac{mMv_0}{(m+M)(m+2M)}$$

Hence, the correct answer is (A).

54. $F = \rho a v^2 = \dfrac{\rho(av)^2}{a}$

where, $\rho = 10^3$ kgm^{-3}, $av = 20\times10^{-6}$ m^{3}s^{-1}

and $a = \dfrac{\pi d^2}{4} = \dfrac{\pi}{4}(10^{-2})^2$

Substituting the values, we get

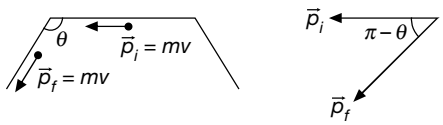
$$F = \frac{(10^3)(20\times10^{-6})^2}{\frac{\pi}{4}\times10^{-4}} = 5.1\times10^{-3}\ \text{N}$$

Hence, the correct answer is (D).

55. Angle $\theta = \left(\pi - \dfrac{\pi}{n}\right)$

$$\Rightarrow \quad \pi - \theta = \frac{\pi}{n}$$

Impulse = change in linear momentum $= \vec{p}_f - \vec{p}_i$



$$\Rightarrow \quad \Delta p = \sqrt{(mv)^2 + (mv)^2 - 2(mv)(mv)\cos(\pi-\theta)}$$

$$\Rightarrow \quad \Delta p = mv\sqrt{2(1+\cos\theta)} = 2mv\cos\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \Delta p = 2mv\cos\left(\frac{\pi}{2} - \frac{\pi}{2n}\right) = 2mv\sin\left(\frac{\pi}{2n}\right)$$

Hence, the correct answer is (D).

56. When two bodies of equal mass collide elastically head-on, then they exchange their velocity.
Hence, the correct answer is (A).

57. Let v' be the velocity of block. Then by Law of Conservation of Linear Momentum, we have

$$mu = mv + mnv'$$

$$\Rightarrow \quad v' = \left(\frac{u-v}{n}\right)$$

Velocity of bullet relative to block is

$$v_r = v - v' = v - \left(\frac{u-v}{n}\right)$$

$$\Rightarrow \quad v_r = \frac{(1+n)v - u}{n}$$

Hence, the correct answer is (C).

58. $T = \dfrac{d}{v/\sqrt{2}} + \dfrac{d}{ev/\sqrt{2}} = \left(1+\dfrac{1}{e}\right)\dfrac{\sqrt{2}d}{v}$

where $T = \dfrac{2v\sin(45°)}{g}$

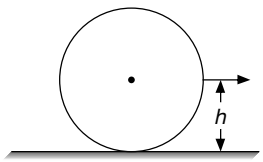
$$\Rightarrow \quad \frac{2v/\sqrt{2}}{g} = \left(1+\frac{1}{e}\right)\frac{\sqrt{2}d}{v}$$

$$\Rightarrow \quad e = \frac{gd}{v^2 - gd}$$

Hence, the correct answer is (C).

59. $\vec{F} = ma_{cm}$

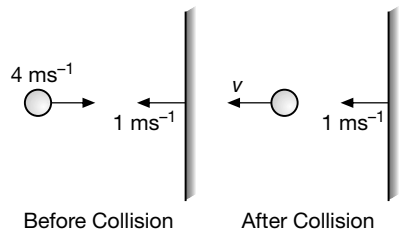
$$a_{cm} = \frac{\vec{F}}{m}$$



which is independent of h.
Hence, the correct answer is (D).

60. Let v be the velocity of ball after an elastic collision with the wall, so

$e = 1$



$$\Rightarrow \quad \begin{pmatrix}\text{Relative velocity}\\ \text{of separation}\end{pmatrix} = \begin{pmatrix}\text{Relative velocity}\\ \text{of approach}\end{pmatrix}$$

$$\Rightarrow \quad v - 1 = 4 + 1$$

$$\Rightarrow \quad v = 6\ \text{ms}^{-1} \qquad \{\text{away from the wall}\}$$

Hence, the correct answer is (D).

61. Let v_r be the speed of bullet with respect to gun and v the velocity of gun. Then from the two figures it is clear that $\phi > 45°$.

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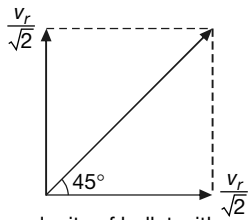
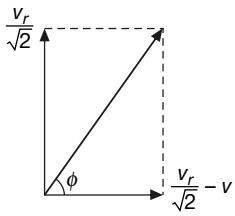


velocity of bullet with respect to gun



velocity of bullet with respect to ground

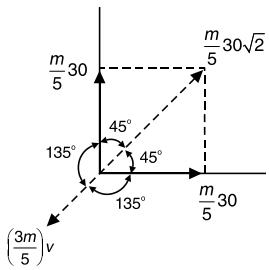
Hence, the correct answer is (C).

62. If mass is non-uniformly distributed, then centre of mass of ring may lie from origin to circumference. Hence, $0 \le b \le a$.

Hence, the correct answer is (D).

63. $0 = \frac{m}{5}(30\sqrt{2}) + \frac{3m}{5}v$

$\Rightarrow \quad v = -10\sqrt{2}\ ms^{-1}$

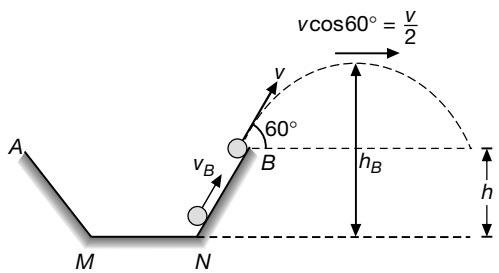


Hence, the correct answer is (B).

64. After collision balls exchange their velocities, so

$$v_A = \sqrt{2gh} \quad \text{and} \quad v_B = \sqrt{2g(4h)} = 2\sqrt{2gh}$$

Height attained by A will be $h_A = \frac{v_A^2}{2g} = h$



But path of B will be first straight line and then parabolic as shown in figure. For N to B, Loss in KE = Gain in PE

$\Rightarrow \quad \frac{1}{2}mv_B^2 + \frac{1}{2}mv^2 = mgh$

$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}m(8gh) - mgh$

$\Rightarrow \quad \frac{1}{2}mv^2 = 3mgh$

$\Rightarrow \quad v = \sqrt{6gh}$

So, $h_B = h + \frac{v^2 \sin^2(60)}{2g}$

$\Rightarrow \quad h_B = h + \frac{9h}{4} = \frac{13h}{4}$

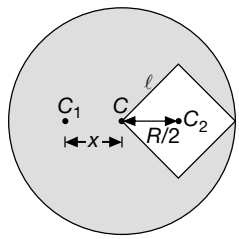
$\Rightarrow \quad \frac{h_A}{h_B} = \frac{4}{13}$

Hence, the correct answer is (C).

65. Let σ be surface mass density. Then

$m_{\text{disc}} = (\pi R^2)\sigma$

$m_{\text{cavity}} = -\ell^2\sigma$



where ℓ is length of square. Now,

$\ell\cos(45°) = \frac{R}{2}$

$\Rightarrow \quad \ell = \frac{R}{\sqrt{2}}$

$\Rightarrow \quad m_{\text{cavity}} = -\frac{R^2\sigma}{2}$

Now assume origin to be at centre of disc, then

$$r_{\text{CM remainder}} = \frac{(\pi R^2\sigma)0 - \left(\frac{R^2\sigma}{2}\right)\left(\frac{R}{2}\right)}{\pi R^2\sigma - \frac{R^2\sigma}{2}}$$

$\Rightarrow \quad r_{\text{CM remainder}} = -\frac{R}{2(2\pi - 1)}$

Negative sign indicates that CM of remained is to the left of centre of disc i.e., origin.

Hence, the correct answer is (C).

66. Thrust $= v\frac{dm}{dt}$

$\Rightarrow \quad \text{Thrust} = (2)\frac{\left(\frac{2}{1000}\right)}{5}$

$\Rightarrow \quad \text{Thrust} = 8\times10^{-4}\ \text{N}$

Hence, the correct answer is (D).

67. By Law of Conservation of Momentum

$$mu - mv = m(0) + mv'$$

$$\Rightarrow \quad v' = u - v \qquad \ldots(1)$$

Further

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right)$$

$$\Rightarrow \quad e = -\left(\frac{v' - 0}{-v - u}\right)$$

$$\Rightarrow \quad v' = e(v + u) \qquad \ldots(2)$$

Equate (1) and (2), we get

$$u - v = ev + eu$$

$$\Rightarrow \quad (1 - e)u = (1 + e)v$$

$$\Rightarrow \quad \frac{u}{v} = \frac{1 + e}{1 - e}$$

Hence, the correct answer is (A).

68. At the highest point velocity before explosion is $v\cos 60$ along $+x$-axis. By Law of Conservation of Momentum

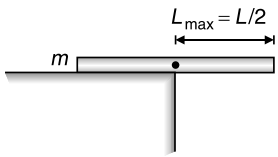
$$(mv\cos 60)\hat{i} = \frac{m}{3}\left(100\hat{j}\right) + \frac{m}{3}\left(-100\hat{j}\right) + \frac{mv'}{3}$$

$$\Rightarrow \quad v' = \frac{3v}{2}\hat{i} = \frac{3(200)}{2}\hat{i}$$

$$\Rightarrow \quad v' = 300\hat{i} \text{ ms}^{-1}$$

Hence, the correct answer is (B).

69. Let us first consider maximum offset/overhang of one brick as shown in Figure.

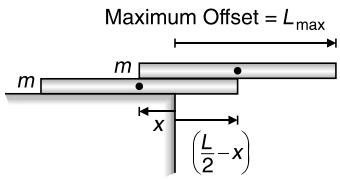


For One Brick

So, for one brick, we have

$$L_{max} = \frac{L}{2}(1) = \frac{L}{2}$$

Now let us consider two bricks and calculate maximum offset (or overhang) for combination.



For Two Bricks

By Law of Conservation of Moments, we have

$$mgx = mg\left(\frac{L}{2} - x\right)$$

$$\Rightarrow \quad 2x = \frac{L}{2}$$

$$\Rightarrow \quad x = \frac{L}{4}$$

So, maximum offset is

$$L_{max} = \left(\frac{L}{2} - x\right) + \frac{L}{2} = \frac{3L}{4}$$

For two bricks this can be generalised as

$$L_{max} = \frac{L}{2}\left(1 + \frac{1}{2}\right)$$

For three bricks, we have

$$L_{max} = \frac{L}{2}\left(1 + \frac{1}{2} + \frac{1}{3}\right) = \frac{11L}{12}$$

For four bricks, we have

$$L_{max} = \frac{L}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \text{ and so on.}$$

Hence, the correct answer is (D).

70. $p^2 = p_x^2 + p_y^2$

$$\Rightarrow \quad p^2 = 12^2 + 5^2$$

$$\Rightarrow \quad p = 13 \text{ kgms}^{-1}$$

Hence, the correct answer is (C).

71. $v = \frac{dx}{dt} = 6t^2$

Impulse $(J) = \Delta p = m\left(v_f - v_i\right)$

$$\Rightarrow \quad J = 2(6 \times 1 - 0) = 12 \text{ Ns}$$

Hence, the correct answer is (B).

72. When the man moves forward with a velocity v with respect to the plank, then the plank moves backwards with a velocity V with respect to ground. So, velocity of the man with respect to ground in the forward direction is $(v - V)$. Applying conservation of linear momentum, we get

$$MV = m(v - V)$$

$$\Rightarrow \quad V = \frac{mv}{m + M} = \left(\frac{m}{m + M}\right)\left(\frac{L}{t}\right)$$

Hence, the correct answer is (B).

73. By Law of Conservation of Mechanical Energy

$$\frac{1}{2}kx^2 = \frac{1}{2}\mu v_r^2 \qquad \ldots(1)$$

Here, μ = reduced mass of the blocks, so

$$\mu = \frac{(m)(2m)}{m + 2m} = \frac{2}{3} \text{ m}$$

and v_r = relative velocity of the two blocks

Substituting in equation (1), we get

$$kx^2 = \frac{2}{3}mv_r^2$$

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$$\Rightarrow \quad v_r = \left(\sqrt{\frac{3k}{2m}}\right)x$$

Hence, the correct answer is (A).

74. $|\vec{v}_{b,w}| = |\vec{v}'_{b,w}| = 2\sqrt{2}\ \text{ms}^{-1}$

So, $|\vec{v}'_b| = |\vec{v}'_{b,w} + \vec{v}_w|$

$$\Rightarrow \quad |\vec{v}'_b| = \sqrt{(2)^2 + (2\sqrt{2})^2 + 2(2)(2\sqrt{2})\cos 45°}$$

$$\Rightarrow \quad |\vec{v}'_b| = \sqrt{4+8+8} = \sqrt{20}$$

$$\Rightarrow \quad |\vec{v}'_b| = 2\sqrt{5}\ \text{ms}^{-1}$$

Hence, the correct answer is (D).

75. $M_0 g = \text{Thrust} = v\dfrac{dM}{dt}$

$$\Rightarrow \quad \frac{dM}{dt} = \frac{M_0 g}{v}$$

$$\Rightarrow \quad \frac{dM}{dt} = \frac{60000}{1000} = 60\ \text{kgs}^{-1}$$

Hence, the correct answer is (B).

76. $F - Mg = Ma$

$$\Rightarrow \quad F = M(g+a)$$

$$\Rightarrow \quad F = (6000)(10+20)$$

$$\Rightarrow \quad F = 18\times 10^4\ N$$

Further

$$F = v\frac{dM}{dt}$$

$$\Rightarrow \quad 18\times 10^4 = (1000)\frac{dM}{dt}$$

$$\Rightarrow \quad \frac{dM}{dt} = 180\ \text{kgs}^{-1}$$

Hence, the correct answer is (D).

77. Since the system is free from external force, hence $a_{cm} = 0$ and since initially they are at rest, so

$$V_{cm} = 0$$

Hence, the correct answer is (A).

78. After collision $v_2 = \left(\dfrac{1+e}{2}\right)u$

and $v_1 = \left(\dfrac{1-e}{2}\right)u$

Here, u = initial speed of ball A

Now, $v_2 = 2v_1$

$$\Rightarrow \quad e = \frac{1}{3}$$

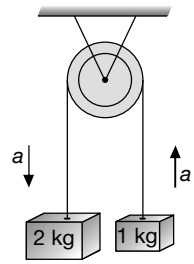
Hence, the correct answer is (B).

79. By Law of Conservation of Linear Momentum

Hence, the correct answer is (C).

80. $a = \dfrac{2g-1g}{2+1} = \dfrac{10}{3}\ \text{ms}^{-2}$

$$\Rightarrow \quad a_{cm} = \frac{(2)(a)-(1)(a)}{2+1}$$



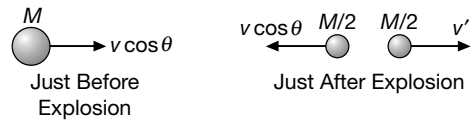
$$\Rightarrow \quad a_{cm} = \frac{a}{3} = \frac{10}{9}\ \text{ms}^{-2} \qquad \{\text{downwards}\}$$

Since, $s_{cm} = \dfrac{1}{2}a_{cm}t^2$

$$\Rightarrow \quad s_{cm} = \frac{1}{2}\left(\frac{10}{9}\right)(2)^2 = \frac{20}{9}\ \text{m} = 2.22\ \text{m}$$

Hence, the correct answer is (B).

81. At the highest point velocity of the shell is $v\cos\theta$. At the highest point, it explodes into two pieces of equal masses out of which one piece retraces the path i.e. has a velocity $-v\cos\theta$.



So, by Law of Conservation of Momentum

$$M(v\cos\theta) = \frac{M}{2}(-v\cos\theta) + \frac{Mv'}{2}$$

$$\Rightarrow \quad v' = 3v\cos\theta$$

Hence, the correct answer is (A).

82. Since Impulse = Change in linear momentum

$$\Rightarrow \quad \vec{F}\Delta t = m(\vec{v}_f - \vec{v}_i)$$

$$\Rightarrow \quad (2\hat{i}+\hat{j}+3\hat{k})(2) = (1)\left(\vec{v}_f - (2\hat{i}+\hat{j})\right)$$

$$\Rightarrow \quad \vec{v}_f = 6\hat{i}+3\hat{j}+6\hat{k}$$

$$\Rightarrow \quad |\vec{v}_f| = \sqrt{(6)^2+(3)^2+(6)^2} = 9\ \text{ms}^{-1}$$

Hence, the correct answer is (C).

83. When the block just reaches the top of the wedge then the velocity of block with respect to wedge at the top of the wedge is zero. Let v be the horizontal velocity of

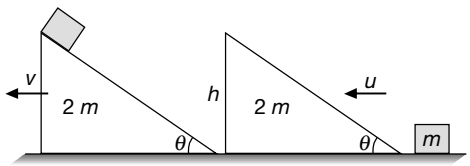
both at this instant. By Law of Conservation of Linear Momentum, we have

$$(2m+m)v = mu$$

$$\Rightarrow \quad v = \frac{u}{3}$$

By Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}mu^2 = \frac{1}{2}(3m)v^2 + mgh$$



$$\Rightarrow \quad u^2 = 3\left(\frac{u^2}{9}\right) + 2gh$$

$$\Rightarrow \quad \frac{2}{3}u^2 = 2gh$$

$$\Rightarrow \quad u = \sqrt{3gh}$$

Hence, the correct answer is (B).

84. Fraction Lost $= \dfrac{4m_1m_2}{(m_1+m_2)^2} = \dfrac{8}{9}$

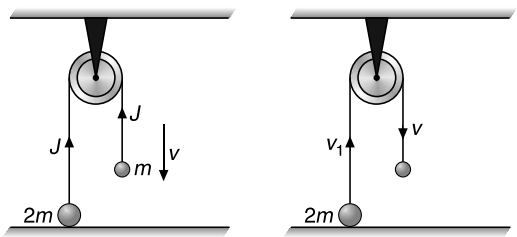
Hence, the correct answer is (D).

85. The centre of mass cannot move towards left. It will always move towards right. Since wedge has a tendency to move towards left so the only external force on the system in horizontal direction is friction which acts towards right.

Hence, the correct answer is (D).

86. Velocity of mass m just before string becomes taut is

$$v = \sqrt{2gh} = \sqrt{4gl} = 2\sqrt{gl}$$



Since, impulse e quals the change in momentum, so we have

for mass $2m$, $J = 2mv_1$

and for mass m, $J = mv - mv_1$

$$\Rightarrow \quad mv - J = mv_1$$

$$\Rightarrow \quad mv - 2mv_1 = mv_1$$

$$\Rightarrow \quad 3mv_1 = mv$$

$$\Rightarrow \quad v_1 = \frac{v}{3} = \frac{2\sqrt{gl}}{3}$$

Hence, the correct answer is (B).

87. $y_{cm} = \dfrac{m(-)\ M(\ )}{m+M} = \dfrac{mL}{(m+M)} = y_{initial}$

Finally, when ice melts y_{final} = zero

$$\text{Shift} = \Delta y = -\frac{mL}{2(m+M)}$$

Hence, the correct answer is (C).

88. If v_{cm} be the velocity of the centre of mass, then

$$v_{cm} = \frac{mu+0}{m+\eta m} = \frac{u}{1+\eta}$$

Further by Law of Conservation of Energy

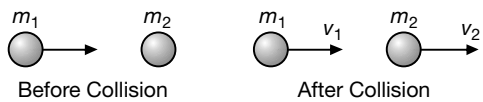
$$\begin{pmatrix}\text{Loss in}\\ \text{K.E. of m}\end{pmatrix} = \begin{pmatrix}\text{Gain in K.E. of}\\ \text{centre of mass}\\ \text{of system}\end{pmatrix} + \begin{pmatrix}\text{Gain in}\\ \text{P.E. of m}\end{pmatrix}$$

$$\frac{1}{2}mu^2 = \frac{1}{2}(m+\eta m)v_{cm}^2 + mgh$$

$$\Rightarrow \quad u = \sqrt{2gh\left(1+\frac{1}{\eta}\right)}$$

Hence, the correct answer is (C).

89. Let v_1 and v_2 be the velocities of the two masses after collision in the same direction. Then



$$m_1v = m_1v_1 + m_2v_2 \qquad \ldots(1)$$

and $v_2 - v_1 = ev$ $\qquad \ldots(2)$

Solving equations (1) and (2) we get,

$$v_1 = \frac{v(m_1 - em_2)}{m_1+m_2}$$

For v_1 to be positive $m_1 > em_2$

$$\Rightarrow \quad \frac{m_1}{m_2} > e$$

Hence, the correct answer is (B).

90. Since, the lower end moves towards positive x-axis the force of friction will be along negative x-direction. Therefore, centre of mass of the rod will finally fall at $x < 0$. Hence, the lower end will be at $x < \dfrac{\ell}{2}$.

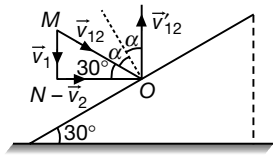
Hence, the correct answer is (C).

91. $\vec{a}_{CM} = \dfrac{M\vec{a}_1 + (2M)\vec{a}_2}{3M} = \dfrac{M\left(\dfrac{\vec{F}_{12}}{M}\right) + 2M\left(\dfrac{\vec{F}_{21}}{2M}\right)}{3M}$

$\Rightarrow \quad \vec{a}_{CM} = \dfrac{\vec{F}_{12} + \vec{F}_{21}}{3M} = 0$

Hence, the correct answer is (A).

92. In the figure let, $\vec{v}_{12}$ = velocity of ball w.r.t. wedge before collision and
$\vec{v}_{12}$ = velocity of ball w.r.t. wedge after collision, which must be in vertically upward direction as shown.



In elastic collision $\vec{v}_{12}$ and $\vec{v}'_{12}$ will make equal angle (say α) with the normal to the plane.

We can show that $\alpha = 30°$

$\Rightarrow \quad \angle MON = 30°$

Now, $\dfrac{v_2}{v_1} = \cot 30° = \sqrt{3}$

Hence, the correct answer is (A).

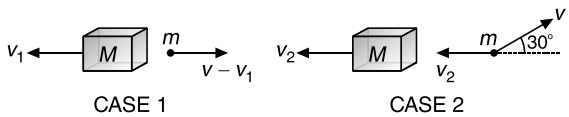
93. $r_{cm} = \dfrac{\int x\,dm}{\int dm}$

$\Rightarrow \quad r_{cm} = \dfrac{\int_0^L x\left(m_0\dfrac{x^2}{L}\right)dx}{\int_0^L \dfrac{m_0x^2}{L}dx} = \dfrac{\dfrac{L^4}{4}}{\dfrac{L^3}{3}}$

$\Rightarrow \quad r_{cm} = \dfrac{3L}{4}$

Hence, the correct answer is (D).

94. Let mass of gun to be M and that of shell to be m. The two cases are shown in figure as below



Here, v_1 and v_2 are the recoil speeds of the gun in two cases. Applying Law of Conservation of Linear Momentum in horizontal direction in two cases, we get

CASE-1: $m(v - v_1) = Mv_1$

$\Rightarrow \quad v_1 = \dfrac{mv}{M+m}$ …(1)

CASE-2: $m(v\cos 30° - v_2) = Mv_2$

$\Rightarrow \quad v_2 = \dfrac{\sqrt{3}mv}{2(M+m)}$ …(2)

From equations (1) and (2), we get

$\dfrac{v_1}{v_2} = \dfrac{2}{\sqrt{3}}$

Hence, the correct answer is (B).

95. $(0)A = 4v + (A-4)V$

$\Rightarrow \quad V = -\dfrac{4v}{A-4}$

Negative sign indicates the recoil speed.

Hence, the correct answer is (A).

96. If m be mass of each block, then acceleration of system is

$a = \dfrac{mg\sin(60°) - mg\sin(30°)}{2m}$

$\Rightarrow \quad a = \left(\dfrac{\sqrt{3}-1}{4}\right)g$

Now $\vec{a}_{cm} = \dfrac{m\vec{a}_1 + m\vec{a}_2}{2m} = \dfrac{\vec{a}_1 + \vec{a}_2}{2}$

Here, $\vec{a}_1$ and $\vec{a}_2$ are $\left(\dfrac{\sqrt{3}-1}{4}\right)g$ directed at right angles to each other.

$\Rightarrow \quad |\vec{a}_{cm}| = \dfrac{\sqrt{a^2+a^2}}{2} = \left(\dfrac{\sqrt{3}-1}{4\sqrt{2}}\right)g$

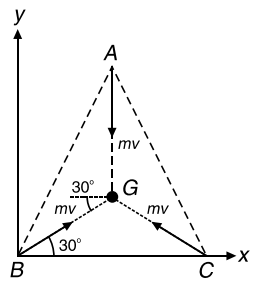
Hence, the correct answer is (A).

97. Let h be the height of each step. Then

$e = \dfrac{v}{u} = \dfrac{\sqrt{2gh}}{\sqrt{2g(2h)}} = \dfrac{1}{\sqrt{2}}$

Hence, the correct answer is (B).

98. Let Origin be placed at B and x-axis along BC. By Law of Conservation of Momentum



along *x*-axis

$0 + mv\cos 30 - mv\cos 30 = 0 - mv\cos 30 + mv_x$

$\Rightarrow \quad v_x = v\cos 30$ …(1)

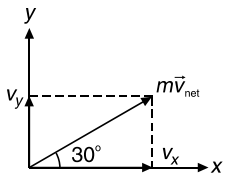
along *y*-axis

$-mv + mv\sin 30 + mv\sin 30 = 0 - mv\sin 30 + mv_y$

$\Rightarrow \quad v_y = v\sin 30$ ...(2)

Squaring (1) and (2) and adding, we get

$v_{net} = \sqrt{v_x^2 + v_y^2} = v$ at



Hence, the particle C will have the same magnitude of velocity but directed along the original direction (direction before collision) of particle B.

Hence, the correct answer is (C).

99. By Law of Conservation of Momentum

$0 = 8(6) + 4v$

$\Rightarrow \quad v = -12 \text{ ms}^{-1}$

Negative sign indicates the recoil of second piece

$\text{KE} = \frac{1}{2}(4)(12)^2 = 288 \text{ J}$

Hence, the correct answer is (D).

100. $\vec{F} = \frac{d\vec{p}}{dt} = 2\vec{B}t$

When $\vec{a}$ and $\vec{u}$ are at 45°, $\vec{F}$ and $\vec{p}$ will also be inclined at 45°. This will happen at $t = \sqrt{\frac{A}{B}}$

$\Rightarrow \quad \vec{F} = 2\vec{B}\sqrt{\frac{A}{B}}$

Hence, the correct answer is (C).

101. Net external force on the two blocks (whether they move with same retardation or not) is

$F_{\text{ext}} = (0.2)(2+1)(10) = 6 \text{ N}$

$\Rightarrow \quad a_{\text{cm}} = \frac{F_{\text{ext}}}{2+1} = 2 \text{ ms}^{-2}$

Hence, the correct answer is (B).

102. K.E. Retained $= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 E$

$\Rightarrow$ K.E. Retained $= \left(\frac{8-2}{8+2}\right)^2 E$

$\Rightarrow$ K.E. Retained $= 0.36E$

Hence, the correct answer is (C).

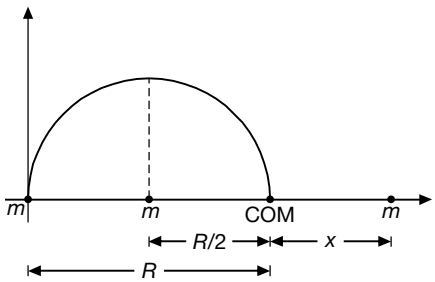
103. Change in linear momentum $\Delta\vec{p} = \vec{F}\Delta t$

Here, $\vec{F}$ is weight, i.e., mg

$\Rightarrow \quad |\Delta\vec{p}| = (mg)(\Delta t) = (1)(10)(1) = 10 \text{ kgms}^{-1}$

Hence, the correct answer is (C).

104.



The centre of mass will follow the same path, so

$mx = m\frac{R}{2} + mR$

$\Rightarrow \quad x = \frac{3R}{2} = \frac{3\times 100}{2} = 150 \text{ m}$

Hence, the desired distance is $(100 + 150)$ m $= 250$ m

Hence, the correct answer is (C).

105. $6(9) = (12+6)v$

$\Rightarrow \quad v = 3 \text{ ms}^{-1}$

Hence, the correct answer is (A).

106. $\hat{j}$ component, i.e., component of velocity parallel to wall remains unchanged while $\hat{i}$ component will become $\left(-\frac{1}{2}\right)(2\hat{i}) = -\hat{i}$. Therefore, velocity vector of the sphere after it hits the wall is $-\hat{i} + 2\hat{j}$.

Hence, the correct answer is (B).

107. Since $\Delta x_{\text{cm}} = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2} = 0$

$\Rightarrow \quad M(4L - x) = 5Mx$

$\Rightarrow \quad x = \frac{2}{3}L$ to left

Hence, the correct answer is (D).

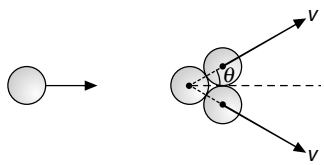
108. $M\frac{d^2\vec{R}}{dt^2} = M\vec{g}$

$\Rightarrow \quad \frac{d^2\vec{R}}{dt^2} = \vec{g}$

Hence, the correct answer is (C).

109. $\sin\theta = \frac{r}{2r} = \frac{1}{2}$

$\Rightarrow \quad \theta = 30°$



CHAPTER 2

By Law of Conservation of Linear Momentum

$$mu = 2mv\cos(30°)$$

$$\Rightarrow \quad v = \frac{u}{\sqrt{3}} \qquad \ldots(1)$$

Now $e = -\left[\dfrac{(v_2)_n - (v_1)_n}{(u_2)_n - (u_1)_n}\right]$

$$\Rightarrow \quad e = \frac{v}{u\cos(30°)} = \frac{\frac{u}{\sqrt{3}}}{\frac{u\sqrt{3}}{2}} = \frac{2}{3}$$

Hence, the correct answer is (C).

110. Unless $m_1 = m_3$ the COM of all the four particles can never be at the centre of the square.
Hence, the correct answer is (D).

111. Let u be the velocity of ball before collision. Speed of ball after collision will become.

$$v = \sqrt{\left(\frac{u}{\sqrt{2}}\right)^2 + \left(\frac{u}{2\sqrt{2}}\right)^2} = \sqrt{\frac{5}{8}}u$$

$$\text{Fraction of KE lost in collision } = \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2}$$

$$\Rightarrow \quad \text{Fraction Lost } = 1 - \left(\frac{v}{u}\right)^2 = 1 - \frac{5}{8} = \frac{3}{8}$$

Hence, the correct answer is (C).

112. $\left|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)\right| = \Delta\vec{p}$

$$\Rightarrow \quad \Delta\vec{p} = |\text{change in momentum of the two particles}|$$

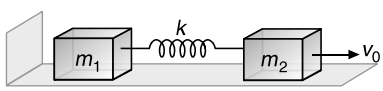
$$\Rightarrow \quad \Delta\vec{p} = \left|\begin{matrix}\text{External force on}\\ \text{the system}\end{matrix}\right| \times \text{time interval}$$

$$\Rightarrow \quad \Delta\vec{p} = (m_1 + m_2)g(2t_0)$$

$$\Rightarrow \quad \Delta\vec{p} = 2(m_1 + m_2)gt_0$$

Hence, the correct answer is (C).

113. Let common velocity of both blocks after spring is completely extended be v, then applying conservation of linear momentum, we get



$$m_2v_0 = (m_1 + m_2)v$$

$$\Rightarrow \quad v = \frac{m_2v_0}{m_1 + m_2}$$

Applying conservation of energy, we get

$$\frac{1}{2}m_2v_0^2 - \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}kx^2 = \frac{1}{2}m_2v_0^2 - \frac{1}{2}(m_1 + m_2)\frac{m_2^2v_0^2}{(m_1 + m_2)^2}$$

$$\Rightarrow \quad kx^2 = m_2v_0^2 - \frac{m_2^2v_0^2}{m_1 + m_2}$$

$$kx^2 = \frac{m_1m_2v_0^2 + m_2^2v_0^2 - m_2^2v_0^2}{m_1 + m_2} = \left(\frac{m_1m_2}{m_1 + m_2}\right)v_0^2$$

$$\Rightarrow \quad x = v_0\sqrt{\frac{m_1m_2}{(m_1 + m_2)k}}$$

Hence, the correct answer is (D).

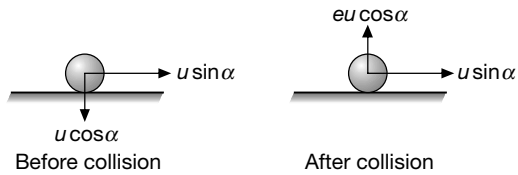
114. Velocity of man with respect to ground is 1 ms^{-1} in opposite direction.

$$\Rightarrow \quad v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$\Rightarrow \quad v_{cm} = \frac{40\times 2 - 80\times 1}{40 + 80} = 0$$

Hence, the correct answer is (A).

115. Impulse = Change in Linear Momentum



$$\Rightarrow \quad Ft = m(eu\cos\alpha + u\cos\alpha)$$

$$\Rightarrow \quad F = \frac{mu\cos\alpha(1 + e)}{t}$$

Hence, the correct answer is (D).

116. $v_1 = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\dfrac{2m_2}{m_1 + m_2}\right)u_2$

For $m_1 << m_2$

$$m_1 - m_2 \quad -m_2$$

$$m_1 + m_2 \simeq m_2$$

$$\Rightarrow \quad v_1 = -u_1 + 2u_2$$

$$\Rightarrow \quad v_1 = -12 + 2(10)$$

$$\Rightarrow \quad v_1 = 8 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

117. Since, $h \ll$ radius of earth, acceleration due to gravity g can be assumed to be constant. Let v be the velocity of block at height $\dfrac{h}{2}$. Then by Law of Conservation of Linear Momentum, velocity of earth will be

$$MV - \frac{Mv}{3} = 0$$

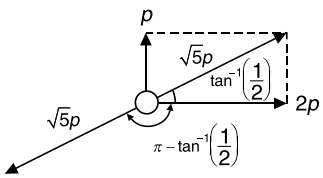
By Law of Conservation of Mechanical Energy, we get

$$\left(\frac{M}{3}\right)g\left(\frac{h}{2}\right)=\frac{1}{2}\left(\frac{M}{3}\right)v^2+\frac{1}{2}(M)\left(\frac{v}{3}\right)^2$$

$$\Rightarrow \quad v=\frac{\sqrt{3gh}}{2}$$

Hence, the correct answer is (C).

118.



Hence, the correct answer is (D).

119. Since the compartment (including passengers) is stationary, so the combined centre of mass of compartment and passengers is fixed i.e. C_2 is fixed. When the passengers move here and there in the compartment then in an attempt to keep C_2 fixed C_1 has to move.
Hence, the correct answer is (C).

120. By Law of Conservation of Momentum
along *x*-axis

$$m(9)+m(0)=mv_1\cos 30+mv_2\cos 30$$

$$\Rightarrow \quad v_1+v_2=\frac{18}{\sqrt{3}}=6\sqrt{3}$$

along *y*-axis

$$0=mv_1\sin 30-mv_2\sin 30$$

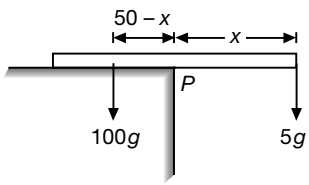
$$\Rightarrow \quad v_1=v_2$$

$$\Rightarrow \quad v_1=v_2=3\sqrt{3}\ \text{ms}^{-1}$$

Hence, the correct answer is (D).

121. Transfer of energy is the maximum when both bodies are of equal masses.
Hence, the correct answer is (A).

122. For the rule not to tilt about P, anticlockwise torque about P > clockwise torque about P. In critical case these two torques are equal



$$\Rightarrow \quad 100g(50-x)=5gx$$

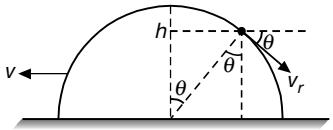
$$\Rightarrow \quad 105x=50\times 100$$

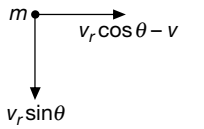
$$\Rightarrow \quad x=47.6\ \text{cm}$$

Hence, the correct answer is (D).

123. Centre of mass will move in a vertical line if $v_1\cos\theta_1=v_2\cos\theta_2$. Else, for any other values, it will follow a parabolic path.
Hence, the correct answer is (B).

124. Let v_r be the velocity of particle relative to hemisphere and v the linear velocity of hemisphere at this moment. Then from Law of Conservation of Linear Momentum, we have





Absolute components of velocity of particle

$$4mv=m(v_r\cos\theta-v)$$

$$\Rightarrow \quad 5v=v_r\cos\theta$$

$$\Rightarrow \quad v_r=\frac{5v}{\cos\theta}$$

$$\Rightarrow \quad \omega=\frac{v_r}{R}=\frac{5v}{R\cos\theta}$$

Hence, the correct answer is (C).

125. $a_{cm}=\dfrac{F}{m+2m}=\dfrac{F}{3m}$

$$\Rightarrow \quad x_{cm}=\frac{1}{2}a_{cm}t^2=\frac{Ft^2}{6m}$$

Since, $x_{cm}=\dfrac{m(x)+2m(x')}{m+2m}$

$$\Rightarrow \quad \frac{Ft^2}{6m}=\frac{x+2x'}{3}$$

$$\Rightarrow \quad x'=\frac{Ft^2}{4m}-\frac{x}{2}$$

Hence, the correct answer is (B).

126. $MV=m(0)+(M-m)v$

$$\Rightarrow \quad v=\frac{MV}{M-m}$$

Hence, the correct answer is (C).

127. Range of stone $=\dfrac{u^2\sin 2\theta}{g}=\dfrac{(10)^2\sin 90^\circ}{10}=10$ m

Net force on system in horizontal direction is zero. Therefore, centre of mass will remain stationary in horizontal direction. Hence,

$$(60+40)x=(1)(10)$$

Where, x is the displacement of boy + platform

CHAPTER 2

$$\Rightarrow \quad x = \frac{10}{100} = 0.1 \text{ m} = 10 \text{ cm}$$

Hence, the correct answer is (B).

128. Since both particles move under the influence of gravity, so relative acceleration of one as seen by other is zero. Therefore, the relative motion between the two is uniform. Relative velocity of the two is $v_r = (10+5)\ \text{ms}^{-1} = 15\ \text{ms}^{-1}$ in horizontal direction. Therefore, the collision will take place after a time

$$t = \frac{60}{15} = 4 \text{ s}$$

Net linear momentum in horizontal direction of the two particles is zero. Therefore, after collision the combined mass will fall vertically downwards. The desired distance from A would be

$$x = v_r t = 10 \times 4 = 40 \text{ m}$$

Hence, the correct answer is (A).

Multiple Correct Choice Type Questions

1. Horizontal velocity of the ball B at the time when it strikes the ground is v_B, then $10 = v_B \times t$

$$\Rightarrow \quad 10 = v_B \times \sqrt{\frac{2\times 5}{g}} = v_B \sqrt{\frac{10}{10}} = v_B$$

$$\Rightarrow \quad v_B = 10\ \text{ms}^{-1} \qquad \ldots(1)$$

Since collision is perfectly elastic and mass of ball A and that of B are same, therefore velocity of ball A, before collision is also $10\ \text{ms}^{-1}$.
So, OPTION **(C)** is correct.

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}m\left[\sqrt{(10)^2 + (10)^2}\right]^2$$

$$\Rightarrow \quad \text{KE} = \frac{1}{2}m \times 2 \times 100 = (m \times 100)\ \text{J}$$

So, OPTION **(B)** is also correct.
Hence, (B) and (C) are correct.

2. By Law of Conservation of Linear Momentum,

$$\left(\frac{m}{2}\right)u = \left(m + \frac{m}{2}\right)v$$

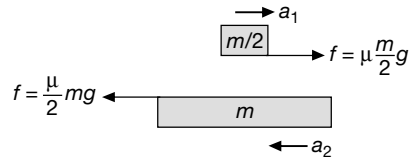
$$\Rightarrow \quad -$$

Work done against friction $= E_i - E_f = W_f$

$$\Rightarrow \quad W_f = \frac{1}{2}\left(\frac{m}{2}\right)u^2 - \frac{1}{2}\left(\frac{3m}{2}\right)\left(\frac{u}{3}\right)^2$$

$$\Rightarrow \quad W_f = \frac{1}{6}mu^2 = \frac{2}{3}\left(\frac{1}{4}mu^2\right)$$

Force of friction on the two blocks before the blocks reach a common velocity is as shown in Figure.



$$a_1 = \mu g \text{ and } a_2 = \frac{\mu}{2}g$$

$$\Rightarrow \quad a_r = \frac{3}{2}\mu g$$

Hence, (A), (B) and (C) are correct.

3. When the string becomes taut, speed of balls is given by

$$V = \frac{mv_0}{2m} = \frac{v_0}{2}$$

If CM having velocity v_0 is raised up by height h, then we have

$$(2m)gh = \frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2$$

$$\Rightarrow \quad h = \frac{v_0^2}{8g}$$

When CM is at highest point, then

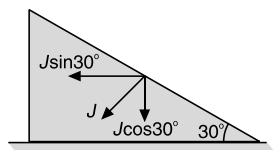
$$K_{\text{system}} = \frac{mv_0^2}{2} - 2mgh = \frac{mv_0^2}{2} - \frac{mv_0^2}{4} = \frac{mv_0^2}{4}$$

Hence, (A) and (D) are correct.

4. Since collision is elastic so, the kinetic energy will be conserved. Let v be the speed of ball after collision. Then,

$$\frac{1}{2}(1)(10)^2 = \frac{1}{2}(4)(4)^2 + \frac{1}{2}(1)v^2$$

$$\Rightarrow \quad v = 6\ \text{ms}^{-1}$$



Let J be the impulse between the ball and wedge during collision.
Since Impulse = Change in Linear Momentum

$$\Rightarrow \quad J\sin 30° = (4)(4)$$

$$\Rightarrow \quad J = 32\ \text{Ns}$$

Hence, (A) and (D) are correct.

5. Since initially the system is at rest so the centre of mass remains fixed.

$$x_{\text{cm}} = \frac{mx_1 + Mx_2}{m+M}$$

$$\Rightarrow \quad \Delta x_{cm} = \frac{m\Delta x_1 + M\Delta x_2}{M+m} = 0$$

$$\Rightarrow \quad m\Delta x_1 + M\Delta x_2 = 0$$

$$\Rightarrow \quad m(L-D) + MD = 0$$

$$\Rightarrow \quad D = \frac{mL}{M+m} < L$$

Hence, (B) and (D) are correct.

6. The impulse imparted to m is equal to change in its momentum, so we have

$$\text{Impulse} = \Delta p = m\left(v_f - v_i\right)$$

Impulse received by m is

$$\Delta\vec{p}_1 = m\left[\left(-2\hat{i} + \hat{j}\right) - \left(3\hat{i} + 2\hat{j}\right)\right]$$

$$\Rightarrow \quad \Delta\vec{p}_1 = m\left(-5\hat{i} - \hat{j}\right)$$

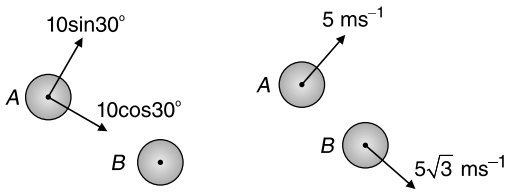
Impulse imparted to M is Δp_2 given by

$$\Delta\vec{p}_2 = -\Delta\vec{p}_1$$

$$\Rightarrow \quad \Delta\vec{p}_2 = m\left(5\hat{i} + \hat{j}\right)$$

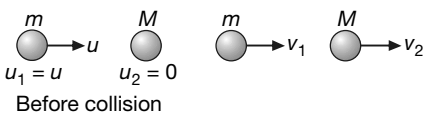
Hence, (B) and (D) are correct.

7.



Velocity components in common tangent direction will remain unchanged while velocity components in common normal direction are interchanged in case of an elastic collision. Hence, both A and B move at right angles after collision with $v_A = 5\ \text{ms}^{-1}$ and $v_B = 5\sqrt{3}\ \text{ms}^{-1}$. Kinetic energy is conserved in an elastic collision, whether it is a head on or an oblique collision.
Hence, (A), (B) and (D) are correct.

8.



$$\text{Since } v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

$$\Rightarrow \quad v_1 = \left(\frac{m-M}{m+M}\right)u$$

$$\Rightarrow \quad v_1 = \left(\frac{1-x}{1+x}\right)u \qquad \ldots(1)$$

Where, $x = \dfrac{M}{m}$

Now speed of ball 1 after impact is one third its initial speed, so

$$v_1 = \pm\frac{u}{3}$$

Substituting this in equation (1), we get

$$x = \frac{M}{m} = \frac{1}{2} \text{ OR } 2$$

Hence, (B) and (C) are correct.

9. Since impulse equals change in momentum, so we have

$$\Delta\vec{p} = 2\left(\vec{v}_2 - \vec{v}_1\right)$$

$$\Rightarrow \quad \Delta\vec{p} = 2\left(3\hat{i} - \hat{j}\right)$$

As impulse is in the direction normal to the colliding surface, so we have

$$\tan\theta = \frac{1}{3}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{3}\right)$$

$$\Rightarrow \quad \alpha = 90° + \tan^{-1}\left(\frac{1}{3}\right)$$

Hence, (B) and (D) are correct.

10. Since, $v_{cm} = \dfrac{mv_o + M(0)}{m+M} = \dfrac{mv_o}{m+M}$

So, OPTION **(A)** is correct.
At the time of maximum compression both the blocks will be moving with same common velocity. Applying Law of Conservation of Linear Momentum and Conservation of Mechanical Energy, we get

$$mv_o = (M+m)v \qquad \ldots(1)$$

$$\frac{1}{2}mv_o^2 = \frac{1}{2}(M+m)v^2 + \frac{1}{2}kx_{max}^2 \qquad \ldots(2)$$

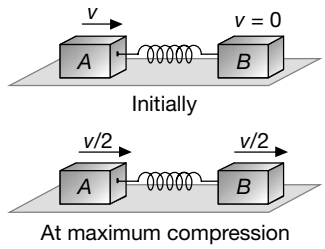
Solving (1) and (2), we get

$$x_{max} = v_o\sqrt{\frac{mM}{(m+M)}\frac{1}{k}}$$

So, OPTION **(C)** is also correct.
In the state of maximum compression, both the blocks are moving with the same velocity. Therefore, velocity of centre of mass in the CM frame is zero.
Hence OPTION **(D)** is also correct.
Hence, (A), (C) and (D) are correct.

11. Here, at maximum compression x_{max}, we introduce the concept of reduced mass μ of the system (details discussed in Rotational Dynamics).

CHAPTER 2



$$\frac{1}{\mu}=\frac{1}{m}+\frac{1}{m}$$

$$\Rightarrow \quad \mu=\frac{m}{2}$$

Further, by Law of Conservation of Energy

$$\frac{1}{2}\mu v^2=\frac{1}{2}kx_{\max}^2$$

$$\Rightarrow \quad x_{\max}=v\sqrt{\frac{m}{2k}} \text{ and}$$

$$\text{KE}=\frac{1}{2}\mu v^2=\frac{1}{2}\left(\frac{m}{2}\right)v^2$$

$$\Rightarrow \quad \text{KE}=\frac{mv^2}{4}$$

Hence, (B) and (D) are correct.

12. Since external force i.e., gravitational force acts on system, therefore momentum of system is not conserved. With the passage of time, mass keeps on decreasing, so acceleration keeps on increasing.
Hence, (B), (C) and (D) are correct.

13. By Law of Conservation of Momentum

$$mv=(M+m)V$$

Momentum of combined system is $(M+m)V=mv$

$$\text{KE}=\frac{1}{2}(M+m)V^2$$

$$\Rightarrow \quad \text{KE}=\frac{m^2v^2}{2(M+m)}$$

Hence, (B) and (C) are correct.

14. B will give impulse $-J$ to A and

$$e=\frac{J-(P-J)}{P}=\frac{2J}{P}-1$$

Hence, (A), (B) and (D) are correct.

15. At minimum separation, the common velocity attained by the bodies equals the velocity of centre of mass. So,

$$v_{\text{cm}}=v_0=\frac{3\times 2}{1+3}=1.5 \text{ ms}^{-1}$$

At time t, for body A, we have

$$v_A=u_A+a_At=0+\left(\frac{F}{m_A}\right)t$$

$$\Rightarrow \quad v_A=6t \qquad \ldots(1)$$

At time t, for body B, we have

$$v_B=u_B+a_Bt=2-\left(\frac{6}{3}\right)t$$

$$\Rightarrow \quad v_B=2-2t \qquad \ldots(2)$$

Since $v_A=v_B$, so we get

$$6t=2-2t$$

$$\Rightarrow \quad t=0.25 \text{ s}$$

Change in kinetic energy is given by

$$\Delta K=\frac{1}{2}(1+3)(1.5)^2-\frac{1}{2}(3)(2)^2=-1.5 \text{ J}$$

During this time let displacement of A be Δs, then from Work-Energy Theorem, work done equals change in kinetic energy, so

$$W=\Delta K=F\Delta s\cos\theta=F\Delta s\cos(180^\circ)$$

$$\Rightarrow \quad -1.5=6\Delta s(-1)$$

$$\Rightarrow \quad \Delta s=\frac{1.5}{6}=0.25 \text{ m}$$

Minimum separation is $1-0.25=0.75$ m
Hence, (B) and (D) are correct.

16. $$a_{\text{cm}}=\frac{\text{Force on centre of mass}}{\text{Mass}}=\frac{M_{\text{cm}}g}{M_{\text{cm}}}=g$$

Hence, (A) and (C) are correct.

17. Before collision: $v_A=\sqrt{2gH}$ and $v_B=0$

If collision is elastic, then velocities after collision will be

$$v_A'=\left(\frac{m_A-m_B}{m_A+m_B}\right)v_A$$

$$\Rightarrow \quad v_A'=\left(\frac{M-3M}{M+3M}\right)\sqrt{2gH}=-\frac{\sqrt{2gH}}{2}$$

and $$v_B'=\left(\frac{2m_A}{m_A+m_B}\right)v_A$$

$$\Rightarrow \quad v_B'=\left(\frac{2M}{M+3M}\right)\sqrt{2gH}=\frac{\sqrt{2gH}}{2}$$

So, $h_A=\frac{v_A'^2}{2g}=\frac{H}{4}$ and $h_B=\frac{v_B'^2}{2g}=\frac{H}{4}$

If collision in perfectly inelastic, velocity of combined mass after collision

$$v=\frac{m_Av_A}{m_A+m_B}=\frac{M\sqrt{2gH}}{M+3M}=\frac{\sqrt{2gH}}{4}$$

The combined mass will rise to a height

$$h = \frac{v^2}{2g} = \frac{H}{16}$$

Hence, (A), (B) and (D) are correct.

18. Since velocity remains same along the common tangent line, so we have

$$u\cos\theta = v\cos\phi \quad \ldots(1)$$

Also, along the line of impact, we have

$$v\sin\phi = eu\sin\theta$$

$$\Rightarrow \quad eu\sin\theta = v\sin\phi \quad \ldots(2)$$

From equations (1) and (2), we get

$$\tan\phi = e\tan\theta$$

Momentum or velocity changes only in vertical direction.

$$\Rightarrow \quad |\text{Impulse}| = |\Delta p|$$

$$\Rightarrow \quad |\text{Impulse}| = m(u\sin\theta + eu\sin\theta)$$

$$\Rightarrow \quad |\text{Impulse}| = m(1+e)u\sin\theta$$

Also, $v = \sqrt{(v\cos\phi)^2 + (v\sin\phi)^2}$

$$\Rightarrow \quad v = \sqrt{(u\cos\theta)^2 + (eu\sin\theta)^2}$$

$$\Rightarrow \quad v = \sqrt{u^2(\cos^2\theta + e^2\sin^2\theta)}$$

$$\Rightarrow \quad v = u\sqrt{1-(1-e^2)\sin^2\theta}$$

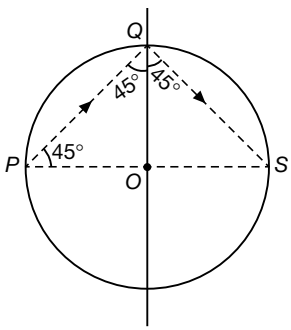
The ratio of final kinetic energy K_f to initial kinetic energy K_i is

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \frac{v^2}{u^2}$$

$$\Rightarrow \quad \frac{K_f}{K_i} = \cos^2\theta + e^2\sin^2\theta$$

Hence, (A), (B), (C) and (D) are correct.

19.



We observe that, $\angle PQS = 90^\circ$

Since, angle made by the body with the normal OQ before collision and after collision are same, therefore, collision is elastic,

Hence, (B) and (C) are correct.

20. In case of perfectly inelastic collision the velocity of combined mass will be $\frac{v}{2}$, so

$$\text{KE} = \frac{1}{2}(2m)\left(\frac{v^2}{4}\right)$$

$$\Rightarrow \quad (0.2) = \frac{1}{2}(0.2)\left(\frac{v^2}{4}\right)$$

$$\Rightarrow \quad v = 2\sqrt{2}\ \text{ms}^{-1}$$

In case of perfectly elastic collision, first particle stops and the second acquires the same velocity v

$$\text{KE} = \frac{1}{2}mv^2$$

$$\Rightarrow \quad 0.2 = \frac{1}{2}(0.1)v^2$$

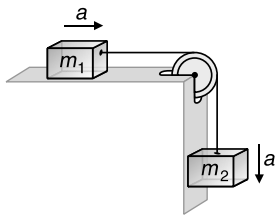
$$\Rightarrow \quad v = 2\ \text{ms}^{-1}$$

Therefore, minimum value of v is $2\ \text{ms}^{-1}$ in case of perfectly elastic collision and maximum value of v is $2\sqrt{2}\ \text{ms}^{-1}$ in case of perfectly inelastic collision.

Hence, (B) and (C) are correct.

21. Acceleration a of the blocks is

$$a = \frac{\text{Net pulling force}}{\text{Total mass}} = \frac{m_2 g}{m_1 + m_2}$$

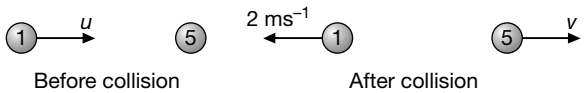


$$\Rightarrow \quad (a_{cm})_x = \frac{m_1 a}{m_1 + m_2} = \frac{m_1 m_2 g}{(m_1 + m_2)^2}$$

and $(a_{cm})_y = \frac{m_2 a}{m_1 + m_2} = \left(\frac{m_2}{m_1 + m_2}\right)^2 g$

Hence, (B) and (C) are correct.

22.



Since collision is elastic, so $e = 1$

$\Rightarrow$ Velocity of Approach = Velocity of Separation

CHAPTER 2

$\Rightarrow \quad u = v + 2$ ...(1)

By Law of Conservation of Linear Momentum, we have

$(1)u = (5)v - (1)(2)$

$\Rightarrow \quad u = 5v - 2$

$\Rightarrow \quad v + 2 = 5v - 2$

$\Rightarrow \quad v = 1 \text{ ms}^{-1}$ and $u = 3 \text{ ms}^{-1}$

Momentum of system $= (1)(3) = 3 \text{ kgms}^{-1}$

Momentum of 5 kg after collision $= (5)(1) = 5 \text{ kgms}^{-1}$

So, kinetic energy of centre of mass is

$$K_{cm} = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 u}{m_1 + m_2}\right)^2$$

$\Rightarrow \quad K_{cm} = \frac{1}{2}(1+5)\left(\frac{1\times 3}{6}\right)^2$

$\Rightarrow \quad K_{cm} = 0.75 \text{ J}$

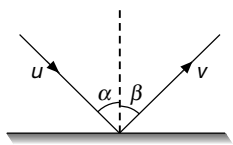
Total kinetic energy $= K = \frac{1}{2}\times 1 \times 3^2$

$\Rightarrow \quad K = 4.5 \text{ J}$

Hence, (A) and (C) are correct.

23. Let velocity of ball before collision and after collision be u and v respectively, then

$$e = \frac{v\cos\beta}{u\cos\alpha} \quad \text{...(1)}$$



and by Law of Conservation of Momentum along horizontal direction, we get

$u\sin\alpha = v\sin\beta$ ...(2)

From (1) and (2), we get

$$e = \frac{\tan\alpha}{\tan\beta}$$

So, OPTION **(A)** is correct.

If $\alpha < \beta$, then $e < 1$, implies that collision is inelastic.

So, OPTION **(B)** is correct.

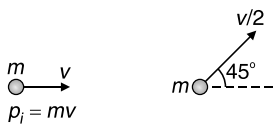
If $\alpha = \beta$, then $e = 1$, collision is elastic.

So, OPTION **(C)** is correct.

Hence, (A), (B) and (C) are correct.

24. By Law of Conservation of Momentum, we have

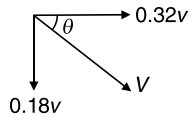
$\vec{p}_i = \vec{p}_f$



$\Rightarrow \quad (mv)\hat{i} = \left(\frac{mv}{2\sqrt{2}}\hat{i} + \frac{mv}{2\sqrt{2}}\hat{j}\right) + (2m)\vec{V}$

$2\vec{V} = v\left(1 - \frac{1}{2\sqrt{2}}\right)\hat{i} - \frac{v}{2\sqrt{2}}\hat{j}$

$\Rightarrow \quad \vec{V} = (0.32v)\hat{i} - (0.18v)\hat{j}$



$\Rightarrow \quad V = 0.37v$

$\Rightarrow \quad \tan\theta = \frac{0.18v}{0.32v} = 0.5625$

$\Rightarrow \quad \theta = 29.35°$

Since $(\theta + 45°) < 90°$. Therefore, the angle of divergence between particles after collision is less than 90°. Further

$$K_i = \frac{1}{2}mv^2 \text{ and } K_f = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}(2m)(0.37v)^2$$

$\Rightarrow \quad K_f < K_i$

So, collision is inelastic.

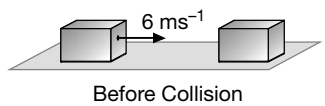
Hence, (B) and (D) are correct.

25. By Law of Conservation of Linear Momentum, we get

$1(6) + 2(0) = 1\,v + 2(4)$

$\Rightarrow \quad v = -2 \text{ ms}^{-1}$

Negative sign indicates, that the 1 kg block has velocity leftwards after impact.



Before Collision

After Collision

$$e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$

$\Rightarrow \quad e = \frac{4-(-2)}{6-0} = \frac{6}{6} = 1$

From $0 < t < 1$ s, we have

$$v_{cm} = \frac{m_1 v_2 + m_2 v_2}{m_1 + m_2} = \frac{1\times 6 + 2\times 0}{1+2} = 2 \text{ ms}^{-1}$$

For $t > 1$ s, we have

$$v_{cm} = \frac{1(-2) + 2(4)}{1+2} = 2 \text{ ms}^{-1}$$

Hence, (B), (C) and (D) are correct.

26. Velocity of ball just before striking the ground is

$$v_0 = \sqrt{2gh} = 10\sqrt{2} \text{ ms}^{-1}$$

After collision, its speed becomes

$$v_1 = ev_0$$

Since the ball rises to a height of 2.5 m after the first collision, so we have

$$h = \frac{v_1^2}{2g} = \frac{e^2 v_0^2}{2g} = \frac{e^2(200)}{20} = 2.5$$

$$\Rightarrow \quad e^2 = 0.25$$

$$\Rightarrow \quad e = 0.5$$

Impulse equals the change in momentum, so we have

$$|J| = |\Delta p| = |m(v_1 - v_0)| = mv_0(1+e)$$

$$\Rightarrow \quad J = \left(\frac{10}{100}\right)(10\sqrt{2})(1+0.5) \approx 2.1 \text{ kgms}^{-1}$$

Hence, (A), (B) and (C) are correct.

27. Since, $v_1 = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_1 + \left[\frac{(1+e)m_2}{m_1 + m_2}\right]u_2$

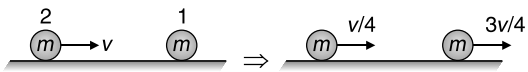
$$\Rightarrow \quad v_1 = \left(\frac{m_1 - m/2}{m+m}\right)v + 0 = \frac{v}{4}$$

Also, $v_2 = \left[\frac{(1+e)m_1}{m_1 + m_2}\right]u_1 + \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_2$

$$\Rightarrow \quad v_2 = \frac{(1+1/2)m}{2m} = \frac{3v}{4}$$

$$|\text{Impulse}| = |\Delta p_1| = |\Delta p_2|$$

$$\Rightarrow \quad |\text{Impulse}| = m\left(\frac{3}{4}v\right) = \frac{3mv}{4}$$



Loss in kinetic energy is $K_i - K_f = -\Delta K$

$$\Rightarrow \quad -\Delta K = \frac{1}{2}mv^2 - \left[\frac{1}{2}m\left(\frac{3v}{4}\right)^2 + \frac{1}{2}m\left(\frac{v}{4}\right)^2\right]$$

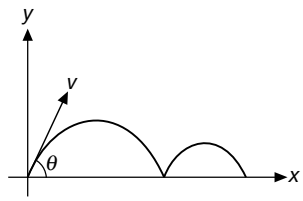
$$\Rightarrow \quad -\Delta K = \frac{3}{16}mv^2$$

Hence, (B) and (C) are correct.

28. If A and B are free to move, no external forces are acting and hence p and E both are conserved. But when B is fixed, (with the help of an external force) then E is conserved but p is not conserved.
Hence, (A) and (C) are correct.

29. As discussed and studied, the horizontal component u_x remains unchanged while the vertical component u_y becomes eu_y. Since

$$T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$$



$$\Rightarrow \quad T \propto u_y$$

$$\Rightarrow \quad a = \frac{T_1}{T_2} = \frac{u_y}{u_y'} = \frac{1}{e}$$

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2g}$$

$$\Rightarrow \quad H \propto u_y^2$$

$$\Rightarrow \quad b = \frac{H_1}{H_2} = \frac{u_y^2}{u_y'^2} = \frac{1}{e^2}$$

$$R = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{2u_x u_y}{g}$$

$$\Rightarrow \quad R \propto u_y$$

$$\Rightarrow \quad c = \frac{R_1}{R_2} = \frac{u_y}{u_y'} = \frac{1}{e}$$

Hence, (A), (B) and (D) are correct.

30. Just after collision, we have

$$v_m = \left(\frac{m-5m}{m+5m}\right)\sqrt{2gl} = -\frac{2}{3}\sqrt{2gl}$$

and $v_{5m} = \left(\frac{2\times m}{m+5m}\right)\sqrt{2gl} = \frac{\sqrt{2gl}}{3}$

Also, $T - mg = \frac{mv_m^2}{l} = \frac{m}{l}\left(\frac{8gl}{9}\right)$

$$\Rightarrow \quad T = \frac{17mg}{9}$$

By Law of Conservation of Energy, we get

$$h_m = \frac{v_m^2}{2g} = \frac{4l}{9}$$

Hence, (B), (C) and (D) are correct.

31. Momentum of the system is always conserved.
Potential energy of the system is maximum when both the particles move with same velocity i.e., mass becomes double while momentum is constant.

$$K = \frac{P^2}{2m} \text{ or } K \propto \frac{1}{m}$$

i.e., at this moment KE becomes half the original i.e., $\frac{K}{2}$. The rest is in the form of elastic potential energy.

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Therefore, minimum kinetic energy of system is $\frac{K}{2}$ and at the same moment, elastic potential energy of the system is maximum i.e., $\frac{K}{2}$.

Hence, (B) and (D) are correct.

32. $a_{cm} = \frac{m_1a_1 + m_2a_2}{m_1 + m_2}$

$a_{cm} = \frac{(m)(g)+(m)(g)}{2m} = g$

i.e., acceleration of centre of mass of particles is g downwards

Horizontal and vertical components of velocity of centre of mass will be 10 ms^{-1} each. So,

$$H = \frac{v_y^2}{2g}$$

Where, v_y is the vertical component of velocity of centre of mass

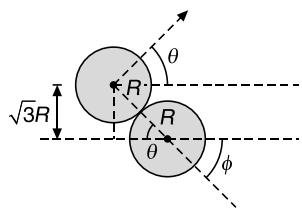
$\Rightarrow \quad H = \frac{(10)^2}{2\times 10} = 5$ m

Therefore, maximum height of centre of mass from the ground will be $(20+5)$ or 25 m.

Hence, (B) and (C) are correct.

33. For elastic collision oblique collision in which bodies are of equal mass and target body is at rest, then

$\theta + \phi = 90°$



From figure, we see that

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$\Rightarrow \quad \theta = 60°$

$\Rightarrow \quad \phi = 30°$

Hence, (B) and (C) are correct.

34. Let after collision, the velocity of $2m$ and m be and v_1 respectively, then

$$e = \frac{v_2 - v_1}{v}$$

$\Rightarrow \quad v_2 - v_1 = ev \qquad \ldots(1)$

Relative velocity $v_r = v_2 - v_1$

Time after which the next collision takes place is

$$t = \frac{2\pi R}{v_r} = \frac{2\pi R}{v_2 - v_1} = \frac{2\pi R}{ev}$$

$\Rightarrow \quad t = \frac{2\pi R}{v} \qquad \{\because$ collision is elastic $e = 1\}$

So, OPTION **(A)** is correct.

Time after which next collision takes place is $\frac{2\pi R}{v}$, which is independent of mass.

So, OPTION **(D)** is correct.

Hence, (A) and (D) are correct.

35. Out of two blocks, one block of mass m is also moving in vertical direction (downwards). Therefore, CM is moving vertically downwards and momentum of the system is not conserved in vertical direction because of application of force. However, since no force is acting on system in horizontal direction, so momentum of system is conserved along the horizontal direction.

Hence, (C) and (D) are correct.

36. Since, $v_{cm} = \frac{mv + mv}{2m} = v$ and

$$a_{cm} = \frac{mg + mg}{2m} = g$$

Electrostatic forces between the particles are equal and opposite. On the lower mass it will be downwards and on upper mass it will be upwards. Therefore, both the particles will always lie on a vertical line and the horizontal displacement of the lower particle will be less and the upper particle will be more than the value which would had been in the absence of charges on them because time of flight of lower mass will decrease while that of upper mass will increase.

Hence, (A), (B), (C) and (D) are correct.

37. If the man walks along the rails with velocity v relative to car, then some velocity say V is also imparted to car. If M be the mass of car, then by Law of Conservation of Linear Momentum, we get

$$MV = m(v - V)$$

$\Rightarrow \quad V = \frac{mv}{m + M}$

So, work done by man is W_{man} given by

$$W_{man} = \frac{1}{2}m(v - V)^2 + \frac{1}{2}mV^2$$

$$W_{man} = \frac{1}{2}\left(\frac{mM}{m+M}\right)v^2 < \frac{1}{2}mv^2$$

Hence, OPTION **(B)** is correct.

If the man moves normal to the rails, then car will not move. Hence, work done by him in this case will just be $\frac{1}{2}mv^2$ and hence OPTION **(C)** is also correct.

Hence, (B) and (C) are correct.

Reasoning Based Questions

1. $E = \frac{p^2}{2m}$

During firing of bullet from gun, momentum is conserved.

So, p = constant

$\Rightarrow \quad E \propto \frac{1}{m}$

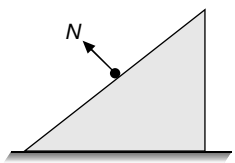
$\Rightarrow \quad \frac{E_{gun}}{E_{bullet}} = \frac{m_{bullet}}{m_{gun}}$

Hence, the correct answer is (A).

2. Centre of mass is the property of mass distribution and it is possible that no mass lies at the centre of mass e.g., a ring, a shell.

Hence, the correct answer is (D).

3.



During collision force exerted by wedge on particle is perpendicular to the inclined face. So linear momentum of wedge is conserved along the face of wedge, because the impact force has got no component along the wedge surface.

Hence, the correct answer is (A).

4. By Law of Conservation of Linear Momentum and Kinetic Energy, we have

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 = \frac{2u_1}{\left(1 + \frac{m_2}{m_1}\right)}$$

Hence, the correct answer is (C).

5. When man moves 4 m w.r.t. boat, let boat move x in opposite direction. So, net displacement of man w.r.t. ground is $(4 - x)$. So,

$$60(40 - x) = 60x$$

$\Rightarrow \quad 240 - 120 = 0$

$\Rightarrow \quad x = 2$ m is distance moved by boat.

Hence, the correct answer is (D).

6. Net external force is zero,

$\Rightarrow \quad \vec{a}_{cm} = \vec{0}$

So, centre of mass may be at rest or may move with a constant velocity.

Hence, the correct answer is (D).

7. Net force acting on the wedge and block system is gravity therefore centre of mass is accelerated in downward direction.

If external force acting on the system is zero, centre of mass may be in rest or moving with constant velocity.

Hence, the correct answer is (C).

8. If m be mass of man, then $M_{\text{ladder}} = M - m$

$$\Delta y_{cm} = \frac{My + (M - m)(-y) + m(h - y)}{2M}$$

$\Rightarrow \quad \Delta y_{cm} = \frac{mh}{2M}$ (upward)

Hence, the correct answer is (D).

9. $(F_{\text{ext}})\Delta t = \text{Impulse} = \Delta p$

Internal forces are always zero, in a system because they always form an action-reaction pair, so no need to say "must be zero".

Hence, the correct answer is (C).

10. The momentum of a system of particles from any frame is given by

$$\vec{p} = m\vec{v}_{cm}$$

From the centre of mass frame, $\vec{v}_{cm} = \vec{0}$

$\Rightarrow \quad \vec{p} = \vec{0}$

Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. Since the collision is elastic velocity of the centre of mass will remain unchanged.

$\Rightarrow \quad v_{\text{cm}} = \frac{Mv_0}{M + m}$

Hence, the correct answer is (B).

2. Since m remains stationary wrt ground, so

$$t_1 = \frac{L}{v_0}$$

Hence, the correct answer is (A).

3. Applying conservation of linear momentum, we get

$$Mv_0 = Mv_1 + mv_2 \qquad \ldots(1)$$

Since the collision is elastic, so we have

$$\frac{v_2 - v_1}{v_0} = 1 \qquad \ldots(2)$$

$\Rightarrow \quad v_2 - v_1 = v_0$

$\Rightarrow \quad v_1 = v_2 - v_0$

CHAPTER 2

$$\Rightarrow \quad Mv_0 = M(v_2 - v_0) + mv_2$$

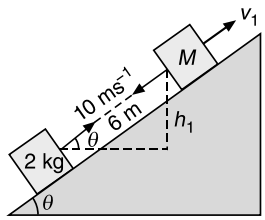
$$\Rightarrow \quad v_2 = \frac{2Mv_0}{M+m}$$

Since $v_{cm} = \frac{Mv_0}{m+M}$, so velocity of bead in the centre of mass frame is

$$v = v_2 - v_{cm} = \frac{Mv_0}{m+M}$$

Hence, the correct answer is (B).

4. Let v_1 be the velocity of block 2 kg just before collision, v_2 be the velocity of block 2 kg just after collision and v_3 be the velocity of block M just after collision. Applying work energy theorem, according to which, change in kinetic energy equals the work done by all forces at different stages as shown in Figure.



$$\Delta KE = W_{friction} + W_{gravity}$$

$$\Rightarrow \quad \left[\frac{1}{2}m\left(v_1^2 - (10)^2\right)\right] = -6\mu mg\cos\theta - mgh_1$$

Since, $m = 2$ kg, so we get

$$v_1^2 - 100 = -2\left[6\mu g\cos\theta + gh_1\right]$$

where, $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-(0.05)^2} \approx 0.99$

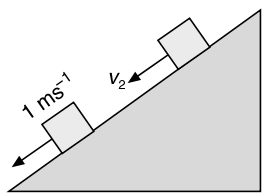
$$\Rightarrow \quad v_1^2 = 100 - 2\left[(6)(0.25)(10)(0.99) + (10)(0.3)\right]$$

$$\Rightarrow \quad v_1 \approx 8 \text{ ms}^{-1}$$

Hence, the correct answer is (B).

5. Since, $\Delta K = W_{friction} + W_{gravity}$

$$\Rightarrow \quad \frac{1}{2}m\left[(1)^2 - \left(v_2^2\right)\right] = -6\mu mg\cos\theta + mgh_1$$



$$\Rightarrow \quad 1 - v_2^2 = 2\left(-6\mu g\cos\theta + gh_1\right)$$

$$\Rightarrow \quad 1 - v_2^2 = 2\left[(-6)(0.25)(10)(0.99) + (10)(0.3)\right]$$

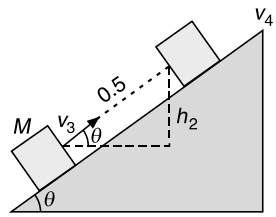
$$\Rightarrow \quad 1 - v_2^2 = -23.7$$

$$\Rightarrow \quad v_2^2 = 24.7$$

$$\Rightarrow \quad v_2 \approx 5 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

6.



Since, $\Delta K = W_{friction} + W_{gravity}$

$$\Rightarrow \quad \frac{1}{2}M\left(0 - v_3^2\right) = -(0.5)(\mu)(M)g\cos\theta - Mgh_2$$

$$\Rightarrow \quad -v_3^2 = -\mu g\cos\theta - 2gh_2$$

$$\Rightarrow \quad v_3^2 = (0.25)(10)(0.99) + 2(10)(0.025)$$

$$\Rightarrow \quad v_3^2 = 2.975$$

$$\Rightarrow \quad v_3 \approx 1.72 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

7. Coefficient of restitution e is given by

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

$$\Rightarrow \quad e = \frac{v_2 + v_3}{v_1} = \frac{5 + 1.72}{8} = \frac{6.72}{8}$$

$$\Rightarrow \quad e \approx 0.84$$

Hence, the correct answer is (D).

8. Applying conservation of linear momentum before and after collision, we get

$$2v_1 = Mv_3 - 2v_2$$

$$\Rightarrow \quad M = \frac{2(v_1 + v_2)}{v_3} = \frac{2(8+5)}{1.72} = \frac{26}{1.72}$$

$$\Rightarrow \quad M \approx 15.12 \text{ kg}$$

Hence, the correct answer is (D).

9. Since, $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$

and $v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$

where

$$m_1 = m$$

$$m_2 = 2m$$

$$u_1 = -\sqrt{2gH}$$

$$u_2 = \sqrt{2gH}$$

$$\Rightarrow \quad v_1 = \left(\frac{m-2m}{3m}\right)\left(-\sqrt{2gH}\right) + \frac{2(2m)}{3m}\sqrt{2gH}$$

$$\Rightarrow \quad v_1 = \frac{5}{3}\sqrt{2gH}$$

Similarly

$$v_2 = \left(\frac{2m}{3m}\right)\left(-\sqrt{2gH}\right) + \left(\frac{2m-m}{3m}\right)\sqrt{2gH}$$

$$\Rightarrow \quad v_2 = -\frac{2}{3}\sqrt{2gH} + \frac{1}{3}\sqrt{2gH}$$

$$\Rightarrow \quad v_2 = -\frac{1}{3}\sqrt{2gH}$$

Hence, the correct answer is (B).

10. $h_2 = \dfrac{v_2^2}{2g} = \dfrac{\frac{1}{9}(2gH)}{2g} = \dfrac{H}{9}$

Hence, the correct answer is (A).

11. $h_1 = \dfrac{v_1^2}{2g} = \dfrac{\frac{25}{9}(2gH)}{2g} = \dfrac{25H}{9}$

Hence, the correct answer is (C).

12. Since the centre of mass of system remains at rest, so we have

$$m(5l - x) = Mx$$

$$\Rightarrow \quad x = \frac{5ml}{M+m}$$

Hence, the correct answer is (D).

13. By Conservation of Linear Momentum, we get

$$Mv_M - mv_m = 0 \qquad \ldots(1)$$

By Conservation of Energy, we get

$$\frac{1}{2}kl^2 = \frac{1}{2}mv_m^2 + \frac{1}{2}Mv_M^2 \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_M = l\sqrt{\frac{km}{M(M+m)}}$$

Hence, the correct answer is (C).

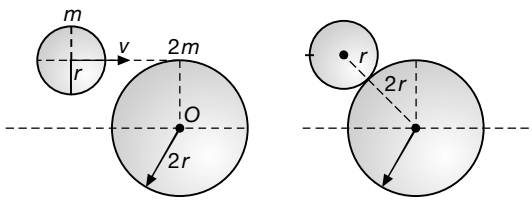
14. Since $v_m = \left(\dfrac{M}{m}\right)v_M$

$$\Rightarrow \quad v_m = \frac{M}{m}\sqrt{\frac{km}{M(M+m)}}\,l = \sqrt{\frac{kM}{m(M+m)}}\,l$$

$$\Rightarrow \quad KE = \frac{1}{2}mv_m^2 = \frac{kMl^2}{2(M+m)}$$

Hence, the correct answer is (B).

15.



$$r_{cm} = \frac{m(3r)}{3m} = r$$

Hence, the correct answer is (A).

16. $v_{cm} = \dfrac{mv}{3m} = \dfrac{v}{3}$

Hence, the correct answer is (D).

17. $I_{cm} = \dfrac{2}{5}mr^2 + m(2r)^2 + \dfrac{2}{5}(2m)(2r)^2 + 2mr^2$

$$\Rightarrow \quad I_{cm} = \frac{18}{5}mr^2 + 6mr^2 = \frac{48}{5}mr^2$$

Hence, the correct answer is (C).

18-20. The correct answer is 18(D), 19(C) and 20(C).

Combined solution to 18, 19, 20

Since $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

So, velocity of 1 and 2 w.r.t. centre of mass frame are given by

$$\vec{v}_{1/cm} = \vec{v}_1 - \vec{v}_{cm} = \frac{m_2(\vec{v}_1 - \vec{v}_2)}{m_1 + m_2}$$

$$\vec{v}_{2/cm} = \vec{v}_2 - \vec{v}_{cm} = \frac{m_1(\vec{v}_2 - \vec{v}_1)}{m_1 + m_2}$$

Total momentum of system $\left(\vec{p}_{system}\right)$ equals

$$\vec{p}_{system} = m_1\vec{v}_{1/cm} + m_2\vec{v}_{2/cm}$$

$$\Rightarrow \quad \vec{p}_{system} = \frac{m_1m_2(\vec{v}_1 - \vec{v}_2)}{m_1 + m_2} + \frac{m_2m_1(\vec{v}_2 - \vec{v}_1)}{m_1 + m_2} = 0$$

Please note that the total momentum of a system in centre of mass reference frame, i.e. C-frame is always zero

$$K_{1/cm} = \frac{1}{2}m_1v_{1/cm}^2 \text{ and } K_{2/cm} = \frac{1}{2}m_2v_{2/cm}^2$$

In C-frame, we have

$$K = \frac{1}{2}\frac{m_1m_2^2}{(m_1+m_2)^2}(\vec{v}_1 - \vec{v}_2)^2 + \frac{1}{2}\frac{m_2m_1^2}{(m_1+m_2)^2}(\vec{v}_2 - \vec{v}_1)^2$$

$$\Rightarrow \quad K = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(\vec{v}_1 - \vec{v}_2)^2$$

CHAPTER 2

Consider two particles of mass m_1 and m_2 having velocity $\vec{v}_1$ and $\vec{v}_2$ respectively in ground frame. Let velocity of centre of mass equals $\vec{v}_{cm}$ in ground frame, then

$$\vec{v}_{1/cm} = \vec{v}_1 - \vec{v}_{cm}$$

$$\vec{v}_{2/cm} = \vec{v}_2 - \vec{v}_{cm}$$

Kinetic energy in ground frame

$$K = \frac{1}{2}m_1\left(v_{1/cm}\right)^2 + \frac{1}{2}m_2\left(v_{2/cm}\right)^2$$

$$\Rightarrow \quad K = \frac{1}{2}m_1\left(v_1^2 + v_{cm}^2 - 2\vec{v}_1\cdot\vec{v}_{cm}\right) + \frac{1}{2}m_2\left(v_2^2 + v_{cm}^2 - 2\vec{v}_2\cdot\vec{v}_{cm}\right)$$

$$\Rightarrow \quad K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}\left(m_1+m_2\right)v_{cm}^2 - \left(m_1\vec{v}_1 + m_2\vec{v}_2\right)\cdot\vec{v}_{cm}$$

$$\Rightarrow \quad K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{1}{2}\left(m_1+m_2\right)v_{cm}^2$$

$$\Rightarrow \quad K_{\text{in C-frame}} = K_{\text{in ground frame}} - \frac{1}{2}\left(m_1+m_2\right)v_{cm}^2$$

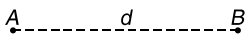
Since $m_1 + m_2 = m$ and $v_{cm} = v$

$$\Rightarrow \quad K_{\text{in ground frame}} = K_{\text{in cm frame}} + \frac{1}{2}mv^2$$

21-23. The correct answer is 21(B), 22(C) and 23(D).

Combined solution to 21, 22, 23

Applying Law of Conservation of Linear Momentum, we get



$$40v + 4\times 5 = 0$$

$$\Rightarrow \quad v = -\frac{1}{2}\ \text{ms}^{-1}$$

$$\Rightarrow \quad |\vec{v}| = 0.5\ \text{ms}^{-1}$$

Applying Law of Conservation of Linear Momentum, we get

$$20 = 44v$$

$$\Rightarrow \quad v = \frac{10}{22} = \frac{5}{11}\ \text{ms}^{-1}$$

24. If v_1 and v_2 are the velocities of object of mass m and block of mass $4m$, just after collision then by conservation of momentum,

$$mv = mv_1 + 4mv_2,\ \textit{i.e.},\ v = v_1 + 4v_2 \qquad \ldots(1)$$

Further, as collision is elastic

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}4mv_2^2,\ \textit{i.e.},\ v^2 = v_1^2 + 4v_2^2 \qquad \ldots(2)$$

Solving, these two equations we get either

$$v_2 = 0 \ \text{ or } \ v_2 = \frac{2}{5}v$$

Therefore, $v_2 = \frac{2}{5}v$

Substituting in equation (1), $v_1 = \frac{3}{5}v$

When $v_2 = 0$, $v_1 = v_2$, but it is physically unacceptable. Now, after collision the block B will start moving with velocity v_2 to the right. Since, there is no friction between blocks A and B the upper block A will stay at its position and will topple if B moves a distance s such that

$$s > 2d \qquad \ldots(3)$$

(Toppling will take place if line of action of weight does not pass through the base area in contact)

However, the motion of B is retarded by frictional force $f = \mu(4m+2m)g$ between table and its lower surface. So, the distance moved by B till it stops

$$0 = v_2^2 - 2\left(\frac{6\mu mg}{4m}\right)s\ \text{, i.e., } s = \frac{v_2^2}{3\mu g}$$

Substituting this value of s in equation (3), we find that for toppling of A

$$v_2^2 > 6\mu gd \ \text{ or } \ \frac{2}{5}v > \sqrt{6\mu gd} \qquad \left\{\text{as } v_2 = \frac{2v}{5}\right\}$$

i.e., $v > \frac{5}{2}\sqrt{6\mu gd}$ or $v_{\min} = v_o = \frac{5}{2}\sqrt{6\mu gd}$

Hence, the correct answer is (C).

25. If $v = 2v_o = 5\sqrt{6\mu gd}$, the object will rebound with speed

$$v_1 = \frac{3}{5}v = 3\sqrt{6\mu gd}$$

and as time taken by it to fall down

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2d}{g}} \qquad \left\{\because \text{ as } h = d\right\}$$

The horizontal distance moved by it to the left of P in this time

$$x = v_1t = 6d\sqrt{3\mu}$$

Hence, the correct answer is (B).

26. $$mu_x = \frac{mu_x}{2} + Mv_x$$

$$\Rightarrow \quad \frac{mu\cos\theta}{2} = Mv_x$$

$$\Rightarrow \quad v_x = \frac{mu\cos\theta}{2M}$$

Hence, the correct answer is (C).

27. Loss in kinetic energy of bullet block system is

$$-\Delta K = \frac{1}{2}mu_x^2 - \left[\frac{1}{2}m\left(\frac{ux}{2}\right)^2 + \frac{1}{2}Mv_x^2\right]$$

$$\Rightarrow \quad -\Delta K = \frac{1}{2}mu_x^2 - \frac{1}{8}mu_x^2 - \frac{1}{2}Mv_x^2$$

$$\Rightarrow \quad -\Delta K = \frac{3}{8}mu_x^2 - \frac{1}{2}Mv_x^2$$

$$\Rightarrow \quad -\Delta K = \frac{3mu^2\cos^2\theta}{8} - \frac{1}{2}M\left(\frac{m^2u^2\cos^2\theta}{4M^2}\right)$$

$$\Rightarrow \quad -\Delta K = \frac{3mu^2\cos^2\theta}{8} - \frac{m^2u^2\cos^2\theta}{8M}$$

$$\Rightarrow \quad -\Delta K = \frac{3mMu^2\cos^2\theta}{8M} - \frac{mu^2\cos^2\theta}{8M}$$

$$\Rightarrow \quad -\Delta K = \frac{m(3M-m)u^2\cos^2\theta}{8M}$$

Hence, the correct answer is (C).

28. $|\Delta v_x| = v_0\cos\theta$

Hence, the correct answer is (C).

29. Since collision is elastic, so for the particle

$|\Delta v_x| = v_0\cos\theta$

Hence, the correct answer is (C).

30. During collision,

$r_\perp = 0$, so $L = mvr_\perp = 0$

Hence, the correct answer is (B).

31. By Newton's Law

$$F - \mu v = (M+\mu t)\frac{dv}{dt} \qquad \ldots(1)$$

$$\Rightarrow \quad \int_0^v \frac{dv}{F-\mu v} = \int_0^t \frac{dt}{M+\mu t}$$

$$\Rightarrow \quad v = \frac{Ft}{M_0+\mu t}$$

Hence, the correct answer is (D).

32. For constant velocity, $\frac{dv}{dt} = 0$

So, from (1), we get $F = \mu v$

$$\Rightarrow \quad P = Fv = \mu v^2$$

Hence, the correct answer is (C).

33. $K = \frac{1}{2}Mv^2$

For constant velocity, $v =$ constant

$$\Rightarrow \quad \frac{dK}{dt} = \frac{v^2}{2}\frac{dM}{dt}$$

Since $M = M_0 + \mu t$

$$\Rightarrow \quad \frac{dM}{dt} = \mu$$

$$\Rightarrow \quad \frac{dK}{dt} = \frac{1}{2}\mu v^2$$

Hence, the correct answer is (D).

34. No additional force is required.

Hence, the correct answer is (C).

35. $v_1 = \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1$

$v_2 = \left(\frac{2m_1}{m_1+m_2}\right)u_1$

For $m_1 \gg m_2$

$v_1 \simeq u_1$

$v_2 \cong 2u_1$

Hence, the correct answer is (C).

36. If the two bodies are of equal mass and second body is at rest, then after impact they exchange their velocities i.e. body one comes to rest and body two moves with velocity of body 1 and hence the maximum transfer of momentum takes place when $m_1 = m_2$ and m_2 is at rest

$$\Rightarrow \quad 3 - x^2 + x = 1$$

$$\Rightarrow \quad x^2 - x - 2 = 0$$

$$\Rightarrow \quad x^2 - 2x + x - 2 = 0$$

$$\Rightarrow \quad x(x-2) + 1(x-2) = 0$$

$$\Rightarrow \quad (x+1)(x-2) = 0$$

$$\Rightarrow \quad x = -1 \text{ or } x = 2$$

Hence, the correct answer is (B, D).

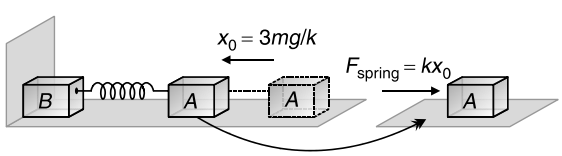
37. Fraction of energy lost by m_1 is $f = \dfrac{\frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2}{\frac{1}{2}m_1u_1^2}$

Put $v_1 = \left(\frac{m_1-m_2}{m_1+m_2}\right)u_1$, we get

$$f = \frac{4m_1m_2}{(m_1+m_2)^2}$$

Hence, the correct answer is (A).

38. The normal reaction by the wall provides external force to the system of blocks A and B. The block B will not move but the block A will move due to initial compression of spring.



CHAPTER 2

Acceleration of block A just after release is

$$a_A = \frac{F_{sp}}{m} = \frac{k(3mg/k)}{m}$$

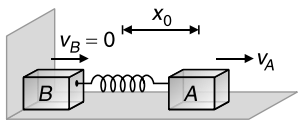
$\Rightarrow \quad a_A = 3g$, towards right

and acceleration of block B is zero. Hence acceleration of the system is

$$a_{cm} = \frac{m_A a_A + m_B a_B}{m_A + m_B} = \frac{m(3g) + m(0)}{(m+m)} = \frac{3}{2}g$$

Hence, the correct answer is (C).

39. The block B will lose contact with wall when block is moving towards right and spring attains its natural length.



For velocity of block A, using $\Delta K + \Delta U = 0$

$$\left(\frac{1}{2}mv_A^2 - 0\right) + \left(0 - \frac{1}{2}kx_0^2\right) = 0$$

$\Rightarrow \quad v_A^2 = \frac{k}{m}x_0^2$

$\Rightarrow \quad v_A = \sqrt{\frac{k}{m}}x_0 = \left(\sqrt{\frac{k}{m}}\right)\left(\frac{3mg}{k}\right)$

$\Rightarrow \quad v_A = 3g\sqrt{\frac{m}{k}}$

The velocity of block B just at the time of losing contact with wall will be zero. So, we have

$$v_{cm} = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{m\left(3g\sqrt{\frac{m}{k}}\right)}{m+m} = \frac{3g}{2}\sqrt{\frac{m}{k}}$$

Hence, the correct answer is (C).

40. When both the blocks move with same velocity, the extension spring will be maximum. The common velocity of the blocks is equal to the velocity of CM. Applying Conservation of Mechanical Energy to the initial and the final state when extension is maximum, we get

$$\Delta K + \Delta U = 0$$

$\Rightarrow \quad \left(\frac{1}{2}(m+m)v_{cm}^2 - 0\right) + \left(\frac{1}{2}kx^2 - \frac{1}{2}kx_0^2\right) = 0$

$\Rightarrow \quad \frac{1}{2}(2m)\left(\frac{9g^2m}{4k}\right) + \frac{1}{2}k\left(x^2 - \frac{9m^2g^2}{k^2}\right) = 0$

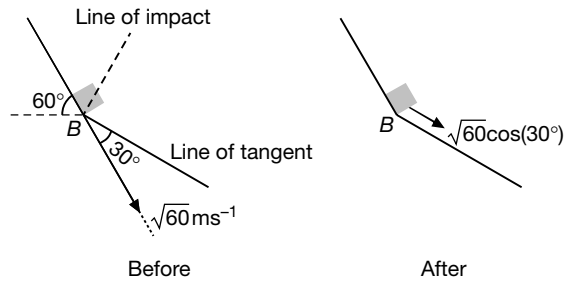
$\Rightarrow \quad \frac{9m^2g^2}{4k} + \left(\frac{1}{2}kx^2 - \frac{9m^2g^2}{2k}\right) = 0$

$\Rightarrow \quad kx^2 = \frac{9m^2g^2}{2k}$

$\Rightarrow \quad x = \frac{3mg}{k\sqrt{2}}$

Hence, the correct answer is (C).

41.



Let speed of block just before it strikes the second inclined plane be v then,

$$\frac{1}{2}mv^2 = mg\left(\sqrt{3}\tan 60°\right)$$

$\Rightarrow \quad v = \sqrt{60}$ ms^{-1}

Speed of block immediately after it strikes the second incline is $\sqrt{45}$ ms^{-1}. because in perfectly inelastic collision the component of velocity along line of impact becomes zero.

Hence, the correct answer is (B).

42. By Conservation of Mechanical Energy

$$\frac{1}{2}mv_C^2 = \frac{1}{2}m\left(\sqrt{45}\right)^2 + mg\left(3\sqrt{3}\tan 30°\right)$$

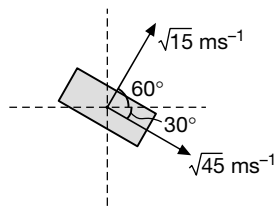
$\Rightarrow \quad v_C^2 = 45 + 60 = 105$

$\Rightarrow \quad v_C = \sqrt{105}$ ms^{-1}

Hence, the correct answer is (B).

43. If collision is completely elastic, then vertical component of velocity becomes,

$$\sqrt{45}\sin 30° - \sqrt{15}\sin 60° = \frac{\sqrt{45}}{2} - \frac{\sqrt{15}\times\sqrt{3}}{2} = 0$$



Hence, the correct answer is (C).

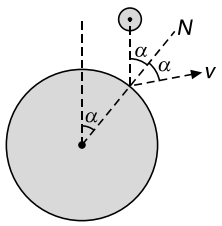
44. Maximum potential energy is stored when the relative velocities of the balls become zero (i.e., both have the same velocities) and the potential energy is stored at the cost of kinetic energy.

Since, in both the cases, when the relative velocity becomes zero, the balls have same velocities.
Hence, the correct answer is (C).

45. Since, there is no external force, center of mass will move with uniform speed all the time.
Hence, the correct answer is (B).

46. Since, the collision is inelastic kinetic energy will not be conserved but the velocity of center of mass will not change because there is no external force.
Hence, the correct answer is (B).

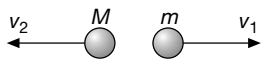
47. Since collision is elastic, so angle of reflection is same as that of incidence angle. In frame of big sphere, small sphere will rebound at an angle 2α with vertical as shown in Figure.



Hence, the correct answer is (B).

48. In frame of larger sphere, the smaller sphere is travelling with $2v_0$ before second collision. Since larger sphere is massive in comparison to smaller sphere, so the smaller sphere will rebound with same velocity $2v_0$ as collision is elastic and surface is frictionless. The small sphere will reflect at same angle.
Hence, the correct answer is (C).

49. At an instant when m is just leaving M, let v_1 and v_2 be the velocities of m and M (both horizontal).



By Law of Conservation of Linear Momentum, we have

$$1\times v_1 = 4v_2 \qquad \ldots(1)$$

By Law of Conservation of Mechanical Energy, we have

$$(1)(10)(4-2) = \frac{1}{2}(1)v_1^2 + \frac{1}{2}(4)v_2^2$$

$$\Rightarrow \quad v_1^2 + 4v_2^2 = 40 \qquad \ldots(2)$$

Solving these two equations, we get

$$v_2 = \sqrt{2}\ \text{ms}^{-1} \text{ and } v_1 = 4\sqrt{2}\ \text{ms}^{-1}$$

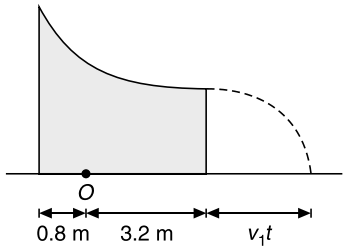
Hence, the correct answer is (D).

50. Let d be the displacement of M, when m just leaves M, then position of centre of mass remains fixed, so

$$\sum m_R x_L = \sum m_L x_L$$

$$\Rightarrow \quad 4d = 1(4-d)$$

$$\Rightarrow \quad d = 0.8\ \text{m}$$



i.e., m is already at $x = 3.2$ m at this instant. Time taken by m to fall to ground from this instant will be

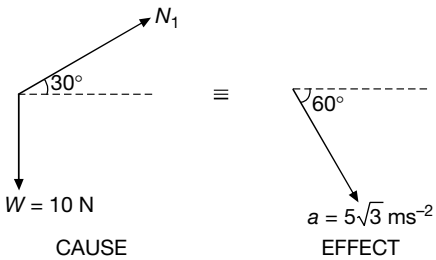
$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 2}{10}} = \sqrt{\frac{2}{5}}\ \text{s}$$

The x-coordinate of m where it strikes the ground will be

$$x = 3.2 + v_1 t = 3.2 + 4\sqrt{2}\times\sqrt{\frac{2}{5}} \approx 6.8\ \text{m}$$

Hence, the correct answer is (C).

51. Let N be normal reaction between M and m. Then free body diagram of m will be as shown below.



$$\sum F_H = ma_H$$

$$\Rightarrow \quad N\cos 30^\circ = (1)5\sqrt{3}\cos(60^\circ)$$

$$\Rightarrow \quad N = 5\ \text{Newton}$$

Hence, the correct answer is (B).

52. Since, $N' = Mg + N\sin(30^\circ)$

$$\Rightarrow N' = 40 + 2.5$$

$$\Rightarrow N' = 42.5\ \text{N}$$

Hence, the correct answer is (C).

53. Relative acceleration of the bullet is zero

Relative velocity of the bullet is $100\ \text{ms}^{-1}$

$$\Rightarrow \quad t = \frac{\text{Initial Separation}}{\text{Relative Velocity of Approach}} = \frac{100}{100} = 1\ \text{s}$$

Hence, the correct answer is (A).

CHAPTER 2

54. $|\Delta\vec{p}| = F\Delta t = (m_1 + m_2)g\Delta t$

$\Rightarrow \quad |\Delta\vec{p}| = \left(\frac{30+20}{1000}\right)(10)(10^{-4})$

$\Rightarrow \quad |\Delta\vec{p}| = 5\times10^{-5}$ Ns

Hence, the correct answer is (B).

55. Since collision is perfectly inelastic, so

$(20)(90) - (30)(10) = (20+30)v$

$\Rightarrow \quad v = 30 \text{ ms}^{-1}$

Hence, the correct answer is (D).

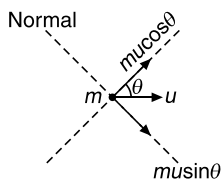
56. The block is not free to move along horizontal or vertical. It can move along the inclined surface when the bullet collides, jerk or impulse will be generated by the inclined surface. As the surface is smooth, only normal impulse will be generated. So, the forces along the inclined will not be impulsive and conservation of linear momentum is applicable along the inclined.

Hence, the correct answer is (D).

57. As discussed above, the impulse by inclined plane will be along normal to surface. The impulsive force between bullet and block will be internal. So net impulse will be along normal to the incline.

Hence, the correct answer is (C).

58. By impulse momentum theorem, $I = \Delta p$



Initial momentum along the normal is

$p_i = -mu\sin\theta$

Also, $p_f = 0$

$\Rightarrow \quad I = \Delta p = mu\sin\theta$

Hence, the correct answer is (A).

59. Applying conservation of linear momentum along the incline, we get

$mu\cos\theta = (m+M)u'$

$\Rightarrow \quad u' = \frac{mu\cos\theta}{m+M}$

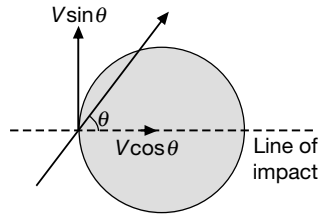
Stopping distance $s = \frac{u^2}{2a}$

$\Rightarrow \quad s = \left(\frac{mu\cos\theta}{m+M}\right)^2 \times \frac{1}{2g\sin\theta}$

$\Rightarrow \quad s = \frac{m^2u^2\cos^2\theta}{2g\sin\theta(m+M)^2}$

Hence, the correct answer is (C).

60. For the following collision line of impact is as shown in the Figure.



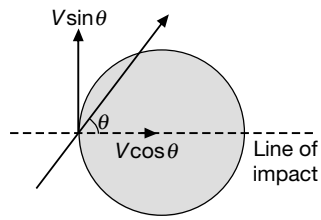
So, velocity of approach along line of impact is

$v_n = V\cos\theta$ …(1)

Hence, the correct answer is (C).

61. Let V_{cm} be the velocity of cm of the disc after collision and V' be velocity of ball after collision, then

$e = \frac{V_{cm} - V}{V\cos\theta}$ …(2)



Conserving Linear Momentum along the line of impact, we get

$MV\cos\theta = MV_{cm} + MV'$

From equation (2), we get

$MV\cos\theta = MV_{cm} + M(-eV\cos\theta + V_{cm})$

$\Rightarrow \quad V_{cm} = \frac{V\cos\theta(1+e)}{2}$

Hence, the correct answer is (D).

62. Speed of ball after collision is

$V' = V\sqrt{\frac{\cos^2\theta(1-e)^2}{4} + \sin^2\theta}$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (p); B → (r); C → (s); D → (r)

(A) $K = \frac{p^2}{2m}$

$K' = \frac{(3p)^2}{2m} = 9\left(\frac{p^2}{2m}\right) = 9K$

% increase in $K = 800\% = 8(100\%)$

(B) $p = \sqrt{2\ mK}$

$p' = \sqrt{2(4K)m} = 2\sqrt{2Km} = 2p$

% increase in $p = 100\% = 1(100\%)$

(C) $K = \dfrac{p^2}{2m}$

For small percentage changes,

% increase in $K = 2$ (% increase in p) $= 2\% = 4(0.5\%)$

(D) $p = \sqrt{2mK}$

% increase in $p = \dfrac{1}{2}$ (% increase in K) $= 0.5\% = 1(0.5\%)$

2. A → (p); B → (p); C → (q); D → (r)

$$\frac{1}{2}mv^2 = \mu mgx$$

$$\Rightarrow \quad v = \sqrt{2Mgx} = \sqrt{2 \times 0.5 \times 10 \times 2.5} = 5\ \text{ms}^{-1}$$

Apply Law of Conservation of Energy, we get

$$\frac{1}{2}mv^2 = mgR(1 - \cos\theta)$$

$$\Rightarrow \quad v = 5\ \text{ms}^{-1}$$

Since, $v^2 = u^2 + 2as$

$$\Rightarrow \quad v^2 = 25 - 2 \times \mu g \times 1.6 = 3\ \text{ms}^{-1}$$

Since, $e = \dfrac{v_2 - v_1}{u_1 - u_2}$

$$\Rightarrow \quad e = 1$$

3. A → (q); B → (q); C → (p); D → (s)

The time of flight of the particle launched from the ground is

$$t_1 = \frac{2u}{g} = \frac{2 \times 20}{10} = 4\ \text{s}$$

The time taken by the second particle dropped from a height of 180 m is

$$t_2 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(180)}{10}} = 6\ \text{s}$$

Taking the downward direction as positive, we have at $t = 0$

$$a_{CM} = \frac{m_1a_1 + m_2a_2}{m_1 + m_2} = \frac{(m)(10) + (m)(10)}{2m} = 10\ \text{ms}^{-2}$$

$$v_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(m)(-20) + (m)(0)}{2m} = -10\ \text{ms}^{-1}$$

At $t = 5$ s, the first particle will come to rest whereas the second particle will still be moving with a downward velocity

$$v_2 = gt = 50\ \text{ms}^{-1}$$

So, at $t = 5$ s, we have

$$a_{CM} = \frac{(m)(0) + (m)(10)}{2m} = 5\ \text{ms}^{-2}$$

$$v_{CM} = \frac{(m)(0) + (m)(50)}{2m} = 25\ \text{ms}^{-1}$$

4. A → (q); B → (s); C → (r); D → (p)

$$\vec{F}_{cm} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 = \left(2\hat{i} + 8\hat{j}\right)$$

$$\Rightarrow \quad \left|\vec{F}_{cm}\right| = \sqrt{4 + 64} = \sqrt{68}\ \text{unit}$$

$$\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

$$\Rightarrow \quad \vec{V}_{cm} = \frac{(1)\left(4\hat{i}\right) + (2)\left(4\hat{j}\right)}{3} = \frac{4\hat{i} + 8\hat{j}}{3}$$

$$\Rightarrow \quad \left|\vec{v}_{cm}\right| = \frac{1}{3}\sqrt{16 + 64} = \frac{\sqrt{80}}{3}\ \text{unit}$$

$$\Rightarrow \quad m\left|\vec{v}_{cm}\right| = \sqrt{80}\ \text{unit}$$

Now, $\vec{r}_1 = \displaystyle\int_0^2 \vec{v}_1 dt = \left(4\hat{i}\right)$

and $\vec{r}_2 = \displaystyle\int_0^2 \vec{v}_2 dt = \left(\frac{8}{3}\hat{j}\right)$

$$\Rightarrow \quad \vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{(1)\left(4\hat{i}\right) + (2)\left(\frac{8}{3}\hat{j}\right)}{3}$$

$$\Rightarrow \quad \vec{r}_{cm} = \left(\frac{4}{3}\hat{i} + \frac{16}{9}\hat{j}\right)$$

$$\Rightarrow \quad \left|\vec{r}_{cm}\right| = \sqrt{\frac{16}{9} + \frac{256}{81}} = \frac{20}{9}$$

5. A → (p, r, s); B → (p, r, s); C → (p); D → (p, q)

Since, $v_{cm} = \dfrac{\text{Total momentum}}{\text{Total mass}} = \dfrac{\Sigma p}{\Sigma m}$

$$\Rightarrow \quad v_{cm} = \frac{2 \times 3}{3 + 6} = \frac{2}{3}\ \text{ms}^{-1} = \text{constant}$$

At maximum deformation in the spring, both the blocks move together with the velocity that equals the velocity of centre of mass, so we have

$$v_{3\ kg} = v_{6\ kg} = v_{cm} = \frac{2}{3}\ \text{ms}^{-1}$$

Up to the maximum extension in spring, the spring force on 6 kg is towards left. So 6 kg block will accelerate and its velocity will be maximum.

CHAPTER 2

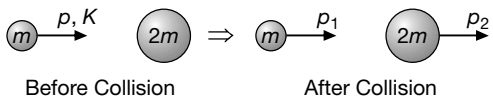
6. A → (r); B → (p); C → (q); D → (s)

$$p_1 + p_2 = p \quad \ldots(1)$$

Further, $K_1 + K_2 = K$

$$\Rightarrow \quad \frac{p_1^2}{2m} + \frac{p_2^2}{4m} = \frac{p^2}{2m}$$

$$\Rightarrow \quad 2p_1^2 + p_2^2 = 2p^2 \quad \ldots(2)$$



Solving these two equations we get,

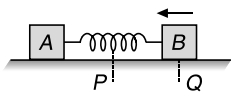
$$p_2 = \frac{4}{3}p$$

and $p_1 = -\frac{p}{3}, K_1 = \frac{K}{9}$

and $K_2 = \frac{8K}{9}$

7. A → (r); B → (q); C → (s); D → (p)

Let the spring be compressed to state P from its natural state at Q as shown in Figure.



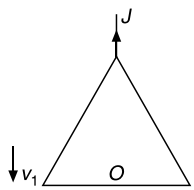
When released, the block B goes from P to Q, so we have

$$a_B = \frac{kx}{m_B}$$

and $a_{CM} = \frac{m_B a_B}{m_A + m_B} = \frac{(m_B)(kx)}{m_A + m_B}$

When the block goes from P to Q, we see that the compression x decreases, so a_{CM} decreases. After Q, we observe that A leaves contact with the wall and spring comes to its natural length, hence net force on system is zero. Therefore, a_{CM} is zero. Also, we see that from P to Q, the velocity of B increases and hence velocity of centre of mass also increases. After this the acceleration of centre of mass, i.e. a_{CM} becomes zero and hence v_{CM} becomes constant.

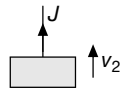
8. A → (q); B → (r); C → (p); D → (p)



According to Impulse - Momentum Theorem, we have

$$J = \vec{P}_2 - \vec{P}_1$$

$$\Rightarrow \quad -J = (0.5 + 1.5)v_2 - 0.5 \times 16 \quad \ldots(1)$$



and $J = 3 \times v_2$ …(2)

Solving (1) and (2), we get

$$v_2 = 1.6 \text{ ms}^{-1}$$

Since, $a = \frac{(m_2 - m_1)g}{(m_1 + m_2)} = \left(\frac{3-2}{3+2}\right)g = \frac{g}{5}$

$$\Rightarrow \quad a = 2 \text{ ms}^{-2}$$

Also, $h_{max} = \frac{v^2}{2a} = \frac{1.6^2}{2 \times 2} = 0.64 \text{ m}$

and $t = \frac{24}{a} = 0.64 \text{ s}$

9. A → (p, r); B → (p, r); C → (q); D → (q)

The velocity of the centre of mass is

$$v_{cm} = \frac{\text{Total momentum}}{\text{Total mass}} = \frac{\Sigma p}{\Sigma m}$$

$$\Rightarrow \quad v_{cm} = \frac{(1)(10) + (2)(-5)}{3} = 0 = \text{constant}$$

$$\Rightarrow \quad p_{cm} = Mv_{cm} = 0 = \text{constant}$$

Finally, both blocks the will stop.

10. A → (q); B → (p, r); C → (p, r); D → (r, s)

Since, $\vec{F} = \frac{Md\vec{v}_{cm}}{dt} = M\vec{a}_{cm}$

$$\Rightarrow \quad M\frac{d\vec{v}_{cm}}{dt} = M\vec{a}_{cm} = \vec{0}$$

$$\Rightarrow \quad \vec{a}_{cm} = \vec{0}$$

So, (A) → (q)

But $\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt} = \vec{0}$

$$\Rightarrow \quad \vec{v}_{cm} = \text{constant}$$

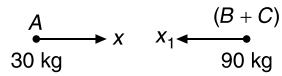
and this constant value may be zero also.

So, (B) → (p, r) and similarly (C) → (p, r)

No comments can be given about velocities of individual particles. So, (D) → (r, s)

11. A → (s); B → (p); C → (p); D → (r)

(a) When A moves x towards right, then B and C will move towards left as shown in Figure.

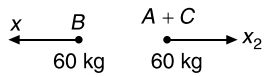


Under these conditions the centre of mass of the system will remain fixed, so we have

$$30x = 90x_1$$

$$\Rightarrow \quad x_1 = \frac{x}{3}$$

(b) When B moves x towards left, then A and C will move towards right as shown in Figure

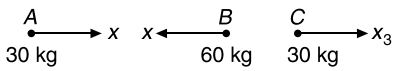


Under these conditions the centre of mass of the system will remain fixed, so we have

$$60x = 60x_2$$

$$\Rightarrow \quad x_2 = x$$

(c) When A moves x towards right and B moves x towards left, then C will move towards right as shown in Figure.

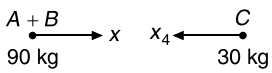


Under these conditions the centre of mass of the system will remain fixed, so we have

$$30x + 30x_3 = 60x$$

$$\Rightarrow \quad x_3 = x$$

(d) When A and B both move x towards right, then C will move towards left as shown in Figure.



Under these conditions the centre of mass of the system will remain fixed, so we have

$$90x = 30x_4$$

$$\Rightarrow \quad x_4 = 3x$$

12. A → (s); B → (s); C → (p, q, r); D → (p, q, r)
Conceptual

13. A → (r); B → (s); C → (p); D → (q)
By conservation of linear momentum, we have

$$mv = mv_A + mv_B$$

$$\Rightarrow \quad v_A + v_B = v \qquad \ldots(1)$$

Also, applying the definition of e, we get

$$e = -\left(\frac{v_A - v_B}{0 - v}\right)$$

$$\Rightarrow \quad v_A - v_B = ev \qquad \ldots(2)$$

From equations (1) and (2), we get

$$v_B = \left(\frac{1-e}{2}\right)v \text{ and } v_A = \left(\frac{1+e}{2}\right)v$$

For elastic collision, $e = 1$, so $v_A = v$

For perfectly inelastic collision, $e = 0$, so $v_A = \frac{v}{2}$

For inelastic collision having $e = \frac{1}{2}$, $v_A = \frac{3v}{4}$

For inelastic collision having $e = \frac{1}{4}$, $v_A = \frac{5v}{8}$

14. A → (q); B → (q); C → (q); D → (p)

$$v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = 1 \text{ ms}^{-1}$$

So, (A) → (q)
At maximum compression by Law of Conservation of Linear Momentum, we get

$$(2)(2) + (2)(0) = (2+2)\, v_{combined}$$

$$\Rightarrow \quad v_{combined} = 1 \text{ ms}^{-1}$$

So, (B) → (q)
Now, $U_{max} = K_i - K_f$

$$\Rightarrow \quad U_{max} = \frac{1}{2} \times 2 \times (2)^2 - \frac{1}{2} \times 4 \times (1)^2 = 2 \text{ J}$$

So, (D) → (p)
Since, $\frac{1}{2}Kx_{max}^2 = 2$ J

$$\Rightarrow \quad x_{max} = 1 \text{ m}$$

So, (C) → (q)

15. A → (r); B → (r, s); C → (p, q); D → (p)

(a) Let T be the tension in string connecting m_1 to m_2, then tension in the string held by the man is $2T$. For equilibrium of m_2, we have

$$T = m_2g = 200 \text{ N}$$

For equilibrium of $(M + m_1)$, we have

$$3T = W_{man} + m_1g$$

$$\Rightarrow \quad 600 = W_{man} + 100$$

$$\Rightarrow \quad W_{man} = 500 \text{ N}$$

(b) The net upward force acting on the system is $4T$ and the net downward force acting on the system is the weight of all. For the centre of mass to accelerate upwards, we must have

$$4T > 200 + 100 + 500$$

$$\Rightarrow \quad T > 200 \text{ N}$$

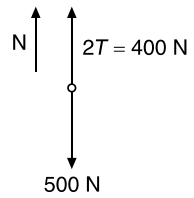
(c) For the centre of mass to accelerate downwards, we must have

$$4T < 200 + 100 + 500$$

$$\Rightarrow \quad T < 200 \text{ N}$$

(d) When in equilibrium, the forces acting on the man are shown in Figure.

CHAPTER 2



Since, $\Sigma F = 0$

$\Rightarrow \quad N + 400 = 500$

$\Rightarrow \quad N = 100$ N

16. A → (q); B → (p); C → (r); D → (r)

Apply conservation of linear momentum in horizontal direction and conservation of mechanical energy.

17. A → (q, r); B → (q, r); C → (p, s); D → (p)

In the absence of external force, the state of centre of mass will remain unchanged and the linear momentum of system remains conserved e.g. a man walking on a platform which lies on a frictionless horizontal surface.

Centre of mass of a body can lie inside or outside it. Centre of mass of a solid cube will lie inside the cube.

18. A → (r); B → (r); C → (q); D → (p)

For (A), $p = \sqrt{2mK} = \sqrt{2 \times 0.5 \times 4} = 2$ kgms^{-1}

For (B), $p_{CM} = p_A + p_B = 0 + 2 = 2$ kgms^{-1}

For (C), i.e. at maximum compression the blocks move with a common velocity and will have momentum equal to half the momentum of centre of mass of the particles.

$\Rightarrow \quad p_A = p_B = \dfrac{p_{CM}}{2} = 1$ kgms^{-1}

For (D), since we know that for an elastic collision between two equal masses, the velocities are interchanged. Hence, B comes to rest and A starts moving.

19. A → (q); B → (s); C → (p); D → (r)

Using $x_{cm} = \dfrac{m_1x_1 + ... + m_nx_n}{m_1 + ... + m_n}$

and $y_{cm} = \dfrac{m_1y_1 + ... + m_ny_n}{m_1 + ... + m_n}$

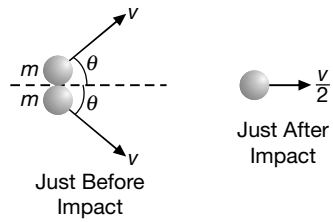
Integer/Numerical Answer Type Questions

1. $mv\cos\theta + mv\cos\theta = (m+m)\dfrac{v}{2}$

$2mv\cos\theta = mv$

$\Rightarrow \quad \cos\theta = \dfrac{1}{2}$

$\Rightarrow \quad \theta = 60°$



So, angle between the two before collision is

$\Rightarrow \quad \phi = 2\theta = 120°$

$\Rightarrow \quad \dfrac{\phi}{20} = 6°$

2. $v'_A = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_A = \left(\dfrac{3-2}{3+2}\right)(2) = 0.4$ ms^{-1}

$v'_B = \left(\dfrac{2m_A}{m_A + m_B}\right)v_B = \left(\dfrac{6}{3+2}\right)(2) = 2.4$ ms^{-1}

Retardation of both the blocks will be,

$a = \mu_K g = 0.4 \times 10 = 4$ ms^{-2}

$s_A = \dfrac{v'^2_A}{2a}$ and $s_B = \dfrac{v'^2_B}{2a}$

Therefore, the desired distance is,

$d = s_B - s_A = \dfrac{v'^2_B - v'^2_A}{2a} = \dfrac{(2.4)^2 - (0.4)^2}{8}$

$d = \dfrac{5.6}{8} = 0.7$ m $= 70$ cm

3. For stable circular motion of centre of mass of rope, we use

$T = m\left(\dfrac{l}{2}\right)\omega^2$

For the rope not to break, we must have $T \le 40$ N

$\Rightarrow \quad m\left(\dfrac{l}{2}\right)\omega^2 \le 40$

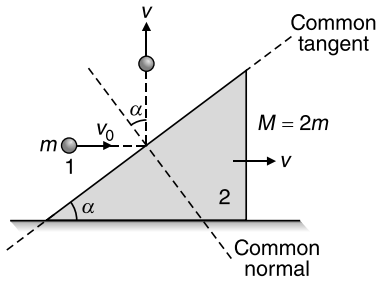
$\Rightarrow \quad (0.4)\left(\dfrac{2}{2}\right)\omega^2 \le 40$

$\Rightarrow \quad \omega^2 \le 100$

$\Rightarrow \quad \omega \le 10$ rads^{-1}

$\Rightarrow \quad \omega_{max} = 10$ rads^{-1}

4. Along common tangent direction, the speed of the particle remains unchanged. So, we have



$$v_0 \cos\alpha = v \sin\alpha$$

$$\Rightarrow \quad v = v_0 \cot\alpha \qquad \ldots(1)$$

Also, we can conserve linear momentum of the 'particle + wedge' system along horizontal direction. So,

$$mv_0 + 0 = 2mV$$

$$\Rightarrow \quad V = \frac{v_0}{2} \qquad \ldots(2)$$

Coefficient of restitution

$$e = \frac{(\text{Relative velocity of separation})_{n\text{-line}}}{(\text{Relative velocity of approach})_{n\text{-line}}}$$

$$\Rightarrow \quad e = \frac{V\sin\alpha + v\cos\alpha}{v_0 \sin\alpha} = \frac{V}{v_0} + \frac{v}{v_0}\cot\alpha$$

Using (1) and (2), we get

$$e = \frac{1}{2} + \cot^2\alpha$$

Given that $\tan\alpha = 2$

$$\Rightarrow \quad \cot\alpha = \frac{1}{2}$$

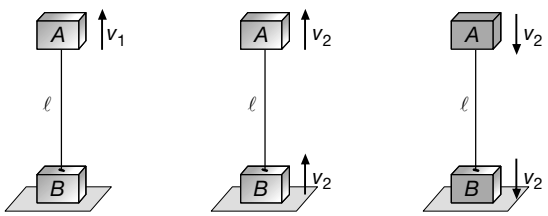
$$\Rightarrow \quad e = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} = 0.75$$

5. String becomes taut when A moves upwards by a distance ℓ under influence of gravity. Let v_1 be the velocity of A at this moment, then

$$v_1^2 = \left(\sqrt{10gl}\right)^2 - 2gl = 8gl$$

$$\Rightarrow \quad v_1 = \sqrt{8gl}$$

Let v_2 be the common velocities of both A and B, just after string becomes taut. Then from Law of Conservation of Linear Momentum, we have



$$m\sqrt{8g\ell} + m(0) = (m+m)v_2$$

$$\Rightarrow \quad v_2 = \frac{v_1}{2} = \frac{\sqrt{8gl}}{2}$$

Both particles return to their original height with the same speed v_2 and the string becomes loose as soon as B strikes the ground and the speed v with which A strikes the ground is,

$$v^2 = v_2^2 + 2g\ell = \frac{8g\ell}{4} + 2g\ell$$

$$\Rightarrow \quad v^2 = 4g\ell$$

$$\Rightarrow \quad v = 2\sqrt{g\ell} = \sqrt{4g\ell}$$

$$\Rightarrow \quad x = 4$$

6. Since no external force acts in z direction, hence z coordinate of the centre of mass of the ball should be zero. To make z coordinate zero other ball should fall symmetrically with respect to z axis.
Hence, z coordinate of other ball is $z_2 = -5$ m.
The balls do not have any external force in x direction. Hence in x direction the centre of mass should move with constant velocity. The x coordinate of centre of mass at $t = 2.0$ s is

$$x_{cm} = 200 \times 2 = 400 \text{ m}$$

Since, $x_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$

$$\Rightarrow \quad 400 = \frac{20 \times 250 + 20x_2}{20 + 20}$$

$$\Rightarrow \quad x_2 = 800 - 250 = 550 \text{ m}$$

Height fallen by centre of mass at $t = 2$ s is

$$h = \frac{1}{2} \times 10 \times (2)^2 = 20 \text{ m}$$

Hence, y-coordinate of centre of mass is

$$y_{cm} = 30 - 20 = 10 \text{ m}$$

Since, $y_{cm} = \dfrac{m_1y_1 + m_2y_2}{m_1 + m_2}$

$$\Rightarrow \quad 10 = \frac{20 \times 0 + 20 \times y_2}{20 + 20}$$

$$\Rightarrow \quad y_2 = 20 \text{ m}$$

$$\Rightarrow \quad x_2 + y_2 + z_2 = -5 + 20 + 20 = 35$$

7. Assuming wedge, block 1 and block 2 as a system.
Since no external force acts on the system in horizontal direction, so both the linear momentum and mechanical energy will be conserved for the system.
Final step: Let velocity of block 1 in vertical direction be v. Hence velocity of block 2 with respect to wedge will be v towards left. Let velocity of wedge towards right be V.

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Hence velocity of block 2 towards left (with respect to ground) will be $(v - V)$.

Applying conservation of linear momentum for the system in horizontal direction, we get

$$(M+m)V - m(v-V) = 0$$

Since $M = 3m$

$$\Rightarrow \quad v = 5V \qquad \ldots(1)$$

Applying conservation of mechanical energy for the system, we get

$$\Delta K + \Delta U = 0$$

$$\Rightarrow \quad \frac{(M+m)}{2}V^2 + \frac{m}{2}(v-V)^2 + \frac{m}{2}v^2 - 0 + (-mgh) = 0$$

$$\Rightarrow \quad \frac{1}{2}(3m+m)V^2 + \frac{1}{2}m(5V-V)^2 + \frac{1}{2}m(5V)^2 = mgh$$

$$\Rightarrow \quad V = \sqrt{\frac{2gh}{45}}$$

$$\Rightarrow \quad \alpha = 45$$

8. (a) $y_{cm} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

$$24 = \frac{(m_1)(0) + \left(\frac{600}{1000}\right)(80)}{m_1 + 0.6} = \frac{48}{m_1 + 0.6}$$

$$\Rightarrow \quad m_1 = 1.4 \text{ kg}$$

So, total mass $= (1.4 + 0.6)$ kg $= 2$ kg

(b) $\vec{v}_{cm} = (6 \text{ ms}^{-3})t^2\hat{j}$

$$\Rightarrow \quad \vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt} = (12 \text{ ms}^{-3})t\hat{j}$$

$$\vec{a}_{cm}|_{t=2\text{ s}} = (12)(2) = 24 \text{ ms}^{-2}$$

(c) $\vec{a}_{cm}$ at $t = 3$ s is $(36 \text{ ms}^{-2})\hat{j}$

$$\Rightarrow \quad \vec{F}_{cm} = (m_1 + m_2)\vec{a}_{cm} = (2)(36)\hat{j} = (72 \text{ N})\hat{j}$$

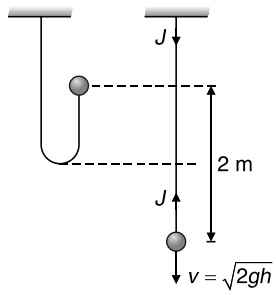
9. The system will be in equilibrium about the centre of mass. Considering origin at the left end of the rod we get

$$x_c = \frac{25\times 0 + 8.5\times 30 + 10\times 50 + 5\times 80 + 5\times 90}{53.5}$$

$$\Rightarrow \quad x_c = \frac{1605}{53.5} = 30 \text{ cm}$$

10. The string will become taut when the particle will fall through a distance 2 m in downward direction. Applying impulse momentum equation, we get

$$mv - J = 0$$



$$\Rightarrow \quad J = mv = m\sqrt{2gh}$$

$$\Rightarrow \quad J = 1\sqrt{2\times 10\times 2} = 2\sqrt{10} \text{ kgms}^{-1} = \sqrt{40} \text{ kgms}^{-1}$$

$$\Rightarrow \quad \alpha = 40$$

11. At the bottommost point, just before collision,

$$v_m = \sqrt{4ga} \text{ and } v_{2m} = 0$$

Also, we know that

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1$$

and $v_2 = \left[\dfrac{(1+e)m_1}{m_1 + m_2}\right]u_1$

$$\Rightarrow \quad v_m' = \left(\frac{m - \frac{1}{2}\times 2m}{m + 2m}\right)\sqrt{4ga} = 0$$

$$\Rightarrow \quad v_{2m}' = \left(\frac{\left(1 + \frac{1}{2}\right)\text{m}}{m + 2m}\right)\sqrt{4ga} = \frac{1}{2}\sqrt{4ga}$$

After second impact,

$$v_m'' = \left(\frac{\left(1+\frac{1}{2}\right)2m}{m+2m}\right)\left(\frac{-\sqrt{4ga}}{2}\right) = -\frac{\sqrt{4ga}}{2} = -\frac{\sqrt{4ga}}{2}$$

$$v_{2m}'' = \left(\frac{2m - \frac{1}{2}\times m}{2m+m}\right)\left(-\frac{\sqrt{4ga}}{2}\right) = -\frac{\sqrt{-4ga}}{4}$$

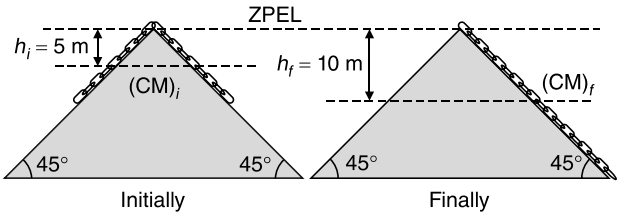
So, $h_m = \dfrac{(v_m'')^2}{2g} = \dfrac{a}{2}$ and $h_{2m} = \dfrac{(v_{2m}'')^2}{2g} = \dfrac{a}{8}$

$$\Rightarrow \quad x = 2 \text{ and } y = 8$$

12. When chain leaves the vertex, then P.E. is decreased by the same factor by which K.E. is increased

$$\sin(45°) = \frac{h_f}{10\sqrt{2}}$$

$$\Rightarrow \quad h_f = 10 \text{ m}$$



Since, $U_f = -mgh_f = -10\times10\times10$

$\Rightarrow \quad U_f = -1000$ J

Also, $U_i = -mgh_i = -10\times10\times5$

$\Rightarrow \quad U_i = -500$ J

Since $\Delta U + \Delta K = 0$

$\Rightarrow \quad -500 + \frac{1}{2}\times10\times v^2 = 0$

$\Rightarrow \quad v^2 = 100$

$\Rightarrow \quad v = 10 \text{ ms}^{-1}$

13. Let us consider ball A and B as a system, then the initial height of centre of mass from ground is

$$(y_{cm})_i = \frac{m(0)+m(40)}{2m} = 20 \text{ m}$$

Initial velocity of centre of mass is

$$u_{cm} = \frac{m(50)+m(30)}{2m} = 40 \text{ ms}^{-1}$$

Acceleration of centre of mass is

$$a_{cm} = -g = -10 \text{ ms}^{-2}$$

Since $v_{cm}^2 - u_{cm}^2 = 2a_{cm}\Delta y_{cm}$

where Δy_{cm} is displacement of centre of mass along y direction (taken vertically upwards).

$\Rightarrow \quad 0^2 - 40^2 = -2\times10\times\Delta y_{cm}$

$\Rightarrow \quad \Delta y_{cm} = \frac{1600}{20} = 80$ m

$\Rightarrow \quad (y_{cm})_f - (y_{cm})_i = 80$

$\Rightarrow \quad (y_{cm})_f = 20 + 80 = 100$ m

Hence, maximum height reach by centre of mass from ground level will be

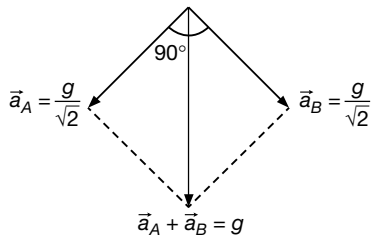
$$h_{max} = 100 \text{ m}$$

14. Acceleration of both the blocks will be

$$a_1 = a_2 = g\sin45° = \frac{g}{\sqrt{2}}$$

at right angles to each other. Since,

$$\vec{a}_{cm} = \frac{m_A\vec{a}_A + m_B\vec{a}_B}{m_A + m_B}$$



Also, $m_A = m_B$

$\Rightarrow \quad \vec{a}_{cm} = \frac{1}{2}(\vec{a}_A + \vec{a}_B) = \frac{g}{2\sqrt{2}}(\hat{i}+\hat{j})$, downwards

$\Rightarrow \quad |\vec{a}_{cm}| = \left(\frac{g}{2\sqrt{2}}\right)(\sqrt{2}) = \frac{g}{2} = 5 \text{ ms}^{-2}$

15. When bock reaches ground and if B is displaced toward right by x, the displacement of centre of mass of system in horizontal direction must be zero hence we use

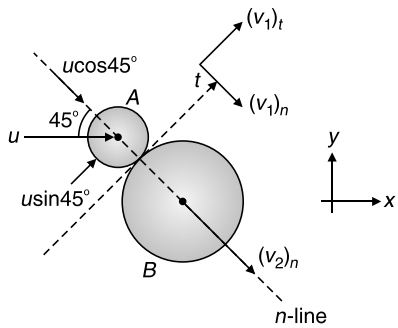
$$4M(l-x) = 20Mx$$

$$x = \frac{l}{6} = \frac{12}{6} = 2 \text{ m}$$

16. Velocity of disc A in tangential direction is

$$(v_1)_t = u\sin45° = \frac{u}{\sqrt{2}} \qquad \ldots(1)$$

The disc B will move along common normal direction only.



Since both the discs move perpendicular to each other, so disc A should move along common tangent direction.

So, $v_A = \frac{u}{\sqrt{2}}$, along common tangent direction.

Applying conservation of linear momentum along common normal direction, we get

$$m\left(\frac{u}{\sqrt{2}}\right) + 0 = m(0) + (2m)(v_2)_n$$

$\Rightarrow \quad (v_2)_n = \frac{u}{2\sqrt{2}}$

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Hence, velocity of disc B along common normal is $\frac{u}{2\sqrt{2}}$.

Since, $e=\frac{(\text{Relative velocity of separation})_{n\text{-line}}}{(\text{Relative velocity of approach})_{n\text{-line}}}$

$\Rightarrow \quad e=\frac{(v_2)_n-0}{u\cos 45^\circ}=\frac{u/2\sqrt{2}}{u/\sqrt{2}}=\frac{1}{2}$

$\Rightarrow \quad \frac{1}{e}=2$

17. Maximum transfer of momentum takes place when both the masses are equal i.e.,

$x^2-9x+21=1$

$\Rightarrow \quad x^2-9x+20=0$

$\Rightarrow \quad (x-4)(x-5)=09$

$\Rightarrow \quad x=4 \text{ and } x=5$

18. Let ρ be the mass density, then mass of the removed cone is

$$m_1=\rho\left(\frac{1}{3}\pi\left(\frac{R}{2}\right)^2 R\right)=\frac{\pi R^3\rho}{12}$$

Also, centre of mass of cone is at height $\frac{R}{4}$

Mass of remaining hemisphere is

$$m_2=\left(\frac{2}{3}\pi R^3-\frac{\pi R^3}{12}\right)\rho=\frac{7\pi R^3\rho}{12}$$

Let its centre of mass be at a height y

Since common centre of mass of m_1+m_2 (i.e. a hemisphere) is at a height $\frac{3R}{8}$ so we get

$$\frac{3R}{8}=\frac{\rho\left(\frac{\pi R^3}{12}\right)\frac{R}{4}+\rho\left(\frac{7}{12}\pi R^3\right)y}{\rho\left(\frac{2}{3}\pi R^3\right)}$$

$\Rightarrow \quad \frac{2}{3}\left(\frac{3R}{8}\right)=\frac{R}{48}+\frac{7y}{12}$

$\Rightarrow \quad \frac{7y}{12}=\frac{R}{4}-\frac{R}{48}=\frac{11R}{48}$

$\Rightarrow \quad y=\frac{11R}{28}=\frac{11}{28}(28)=11 \text{ cm}$

19. Velocity of B after collision is

$$v_2=\frac{(1+e)m_1u_1}{m_1+m_2}=\left[\frac{(1+e)m}{m+m}\right](10)$$

$\Rightarrow \quad v_2=5(1+e)$

Height to which B rises is $h=\frac{v_2^2}{2g}$

Now, $\frac{v_2^2}{2g}=3.2$

$\Rightarrow \quad \frac{[5(1+e)]^2}{2\times 10}=3.2$

$\Rightarrow \quad e=\frac{3}{5}=0.6$

20. Since gases are ejected out during motion we use

$$\int_0^v m\,dv=\int_{m_0}^{m_0/2} u\,dm$$

$\Rightarrow \quad v=u\ln 2=2\ln(2) \text{ ms}^{-1}\approx 1.4 \text{ ms}^{-1}$

21. According to Work-Energy Theorem, we have

$$\frac{1}{2}mV^2=(\mu mg)(0.06)+\frac{1}{2}kx^2$$

$\Rightarrow \quad \frac{1}{2}(0.18)V^2=(0.1)(0.18)(10)(0.06)+\frac{1}{2}(2)(0.06)^2$

$\Rightarrow \quad V=0.4 \text{ ms}^{-1}$

$\Rightarrow \quad 0.4=\frac{N}{10}$

$\Rightarrow \quad N=4$

22. Since no force acts along the horizontal direction and initial velocity of centre of mass along a horizontal direction is also zero, so x coordinate of centre of mass remains zero. However, $(a_{cm})_y=-g$

Initial velocity of centre of mass along y-direction is

$$(u_{cm})_y=\frac{m[10\sin(30^\circ)]+m[10\sin(30^\circ)]}{2m}$$

$\Rightarrow \quad (u_{cm})_y=5 \text{ ms}^{-1}$

At $t=10$ s

$$y_{cm}=(u_{cm})_y t+\frac{1}{2}(a_{cm})_y t^2$$

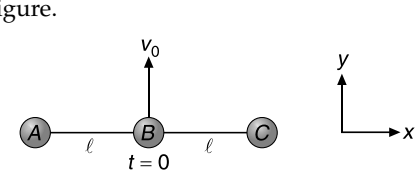
$\Rightarrow \quad y_{cm}=(5)(10)+\frac{1}{2}(-10)(10)^2$

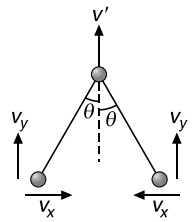
$\Rightarrow \quad y_{cm}=50-500=-450 \text{ m}$

$\Rightarrow \quad x=0 \text{ and } y=-450$

$\Rightarrow \quad x-y=450$

23. Initial state at $t=0$ and final state at time t are shown in Figure.





Applying conservation of momentum at time t, we get

$$mv_0 = mv' + 2mv_y$$

$$\Rightarrow \quad v_0 = v' + 2v_y$$

At the time of collision $\theta = 0$ and $v' = v_y$

$$\Rightarrow \quad v' = v_y = \frac{v_0}{3}$$

Applying conservation of energy, we get

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv'^2 + 2\left[\frac{1}{2}m\left(v_x^2 + v_y^2\right)\right]$$

$$\Rightarrow \quad v_0^2 = \frac{v_0^2}{9} + 2\left(v_x^2 + \frac{v_0^2}{9}\right)$$

$$\Rightarrow \quad v_0^2\left(1 - \frac{1}{9} - \frac{2}{9}\right) = 2v_x^2$$

$$\Rightarrow \quad v_0^2\left(\frac{9-3}{9}\right) = 2v_x^2$$

$$\Rightarrow \quad v_x = \frac{v_0}{\sqrt{3}}$$

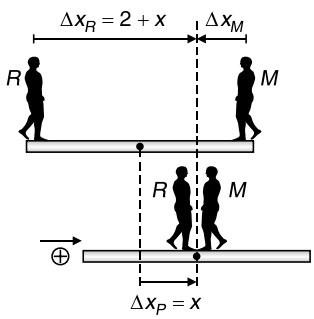
So, net velocity of A at the time of collision is

$$v_A = \sqrt{v_x^2 + v_y^2}$$

$$\Rightarrow \quad v_A = \sqrt{\frac{v_0^2}{3} + \frac{v_0^2}{9}} = v_0\sqrt{\frac{4}{9}} = \frac{2v_0}{3}$$

$$\Rightarrow \quad v_A = \frac{2v_0}{3} = \left(\frac{2}{3}\right)(9) = 6 \text{ ms}^{-1}$$

24. Let displacement of the platform be $\Delta x_p = x$, towards right as shown in Figure.



The displacement of Rahul is $\Delta x_R = (2 + x)$, rightwards

The displacement of Mohit is $\Delta x_M = (2 - x)$, leftwards

Since no external forces are acting on the system, the displacement of the centre of mass of the system should be zero. Hence

$$\Delta x_{cm} = \frac{m_P\Delta x_P + m_R\Delta x_R + m_M\Delta x_M}{m_P + m_R + m_M} = 0$$

$$\Rightarrow \quad \frac{40x + 50(2+x) - 60(2-x)}{150} = 0$$

$$\Rightarrow \quad x = \frac{2}{15} \text{ m} \approx 13 \text{ cm}$$

25. $$H = \frac{u^2\sin^2\theta}{2g} = \frac{(20)^2 \times \frac{3}{4}}{2\times 10} = 15 \text{ m}$$

i.e., the shell strikes the ball at highest point of its trajectory. Velocity of (ball + shell) just after collision, using Law of Conservation of Linear Momentum is

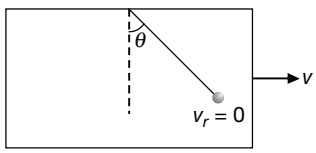
$$v = \frac{u\cos 60°}{2}$$

$$\Rightarrow \quad v = \frac{20}{2\times 2} = 5 \text{ ms}^{-1}$$

At the highest point, combined mass is at rest relative to the trolley. Let v be the velocity of trolley at this instant. Then by Law of Conservation of Momentum, we get

$$2\times 5 = (2+18)v$$

$$\Rightarrow \quad v = \frac{1}{2} \text{ ms}^{-1}$$



Using Law of Conservation of Mechanical Energy, we get

$$\left(\frac{1}{2}\right)(2)(5)^2 - \left(\frac{1}{2}\right)\times(2+18)\left(\frac{1}{2}\right)^2 = (2)(10)(1)(1-\cos\theta)$$

$$\Rightarrow \quad 25 - 2.5 = 20(1-\cos\theta)$$

$$\Rightarrow \quad 1 - \cos\theta = 1.125$$

$$\Rightarrow \quad \cos\theta = 0.125$$

$$\Rightarrow \quad \cos\theta = \frac{1}{8}$$

$$\Rightarrow \quad * = 8$$

26. Let v be velocity of the ball after collision along the normal, v' be velocity of the ball after collision along incline and J be impulse on ball, then

$$J = v - (-2\cos 30°) = v + \sqrt{3}$$

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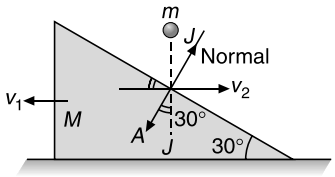


Impulse on wedge is

$$J\sin 30° = Mv_1 = 2v_1$$

$$\Rightarrow \quad \frac{v+\sqrt{3}}{2} = 2v_1$$

$$\Rightarrow \quad v = 4v_1 - \sqrt{3} \qquad \ldots(1)$$

Coefficient of restitution is defined as

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right)$$

$$\Rightarrow \quad \frac{1}{2} = \frac{\left(v + \frac{v_1}{2}\right)}{2\cos 30°}$$

$$\Rightarrow \quad v = \frac{\sqrt{3}}{2} - \frac{v_1}{2} \qquad \ldots(2)$$

Solving (1) and (2), we get

$$v_1 = \frac{1}{\sqrt{3}} \text{ ms}^{-1} \text{ and } v = \frac{1}{\sqrt{3}} \text{ ms}^{-1}$$

For the ball, velocity along incline remains constant, so

$$v' = 2\sin(30°) = 1 \text{ ms}^{-1}$$

Hence final velocity of ball is

$$v_2 = \sqrt{1^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{2}{\sqrt{3}} \text{ ms}^{-1}$$

$$\Rightarrow \quad \alpha = 2$$

27. If all the particles of the system have same acceleration, $\vec{a}$ say, then acceleration of centre of mass will also be equal to $\vec{a}$. Now, in this problem since both particles are projected in the air, so both particles have acceleration g downwards.

$$\Rightarrow \quad \vec{a}_{CM} = -g\hat{j}$$

Now, if $\vec{a}_{CM}$ remains constant, then equation of motion can be applied to centre of mass. In this question, maximum height attained by the centre of mass is asked, thus we need not to think about the vertical motion of centre of mass.

$$(v_{CM})_y = \frac{5(10\sin 30°) + 2(17\sin 30°)}{5+2}$$

$$\Rightarrow \quad (v_{CM})_y = \frac{25+17}{7} = \frac{42}{7} = 6 \text{ ms}^{-1}$$

Also, $(a_{CM})_y = -10 \text{ ms}^{-2}$

$$\Rightarrow \quad (h_{CM})_{max} = \frac{\left[(v_{CM})_y\right]^2}{2(a_{CM})_y} = \frac{36}{2\times 10} = 1.8 \text{ m}$$

$$\Rightarrow \quad (h_{CM})_y = 180 \text{ cm}$$

28. Before we can draw working diagrams, the velocity components, before and after impact, parallel and perpendicular to the line of centres must be found. To do this we need unit vectors in these two directions. Let the unit vector along the line of centres i.e., along n-line be $\hat{n}$, where

$$\hat{n} = \frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$$

and a unit vector perpendicular to the line of centres i.e., along t-line be $\hat{t}$ where

$$\hat{t} = \frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$$

The velocity components along these directions are calculated in table.

Sphere	Magnitude of component	
	Parallel to $\hat{n}$	Parallel to $\hat{t}$
A	$(\hat{i}+2\hat{j})\cdot\frac{1}{\sqrt{2}}(\hat{i}-\hat{j}) = -\frac{1}{\sqrt{2}}$	$(\hat{i}+2\hat{j})\cdot\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}) = \frac{3}{\sqrt{2}}$
B	$(-\hat{i}+3\hat{j})\cdot\frac{1}{\sqrt{2}}(\hat{i}-\hat{j}) = -2\sqrt{2}$	$(-i+3\hat{j})\cdot\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}) = \sqrt{2}$

Using Law of Conservation of Linear Momentum and the Law of Restitution along the line of impact i.e., n-line, we get

$$m\left(\frac{1}{\sqrt{2}}\right) + 2m\left(2\sqrt{2}\right) = mu + 2mv$$

and $\frac{1}{2}\left(2\sqrt{2} - \frac{1}{\sqrt{2}}\right) = u - v$

Solving, we get

$$u = 2\sqrt{2} \text{ and } v = \frac{5}{4}\sqrt{2}$$

The velocity of A after impact is

$$\vec{v}_A = -u\hat{n} + \frac{3}{\sqrt{2}}\hat{t} = (-2\sqrt{2})\frac{1}{\sqrt{2}}(\hat{i}-\hat{j}) + \left(\frac{3}{\sqrt{2}}\right)\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$$

$$\vec{v}_A = \frac{1}{2}(-\hat{i}+7\hat{j})$$

$\Rightarrow \quad \alpha = 2,\ x_1 = 1$ and $y_1 = 7$

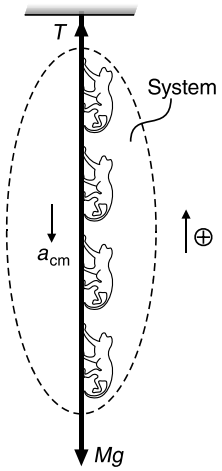
The velocity of B after impact is

$$\vec{v}_B = -v\hat{n} + \sqrt{2}\hat{t} = \left(-\frac{5}{4}\sqrt{2}\right)\frac{1}{\sqrt{2}}\left(\hat{i}-\hat{j}\right) + \left(\sqrt{2}\right)\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$$

$$\Rightarrow \quad \vec{v}_B = \frac{1}{4}\left(-\hat{i}+9\hat{j}\right)$$

$\Rightarrow \quad \beta = 4,\ x_2 = 1$ and $y_2 = 9$

29. Taking four monkeys together as system, the external forces acting on the system is tension and gravitational force.



Acceleration of centre of mass of system is

$$a_{cm} = \frac{m_1a_1 + m_2a_2 + m_3a_3 + m_4a_4}{m_1 + m_2 + m_3 + m_4}$$

Taking upward direction is positive, we get

$$a_{cm} = \frac{15(1) + 25(-3) + 20(0) + 40(0)}{15 + 25 + 20 + 40}$$

$$\Rightarrow \quad a_{cm} = -\frac{60}{100} = \frac{6}{10}\ \text{ms}^{-2}, \text{ downwards}$$

Since, $\Sigma F_{ext} = Ma_{cm}$

$\Rightarrow \quad Mg - T = Ma$

$\Rightarrow \quad (100)(9.8) - T = 100(0.6)$

$\Rightarrow \quad T = 980 - 60 = 920$ N

30. Mass of gun is $80m - 5m = 75m$

After first shot, if speed of car is v_1, then we have

$$75mv = 5m(100 - v)$$

$$\Rightarrow \quad v_1 = \frac{500}{80} = \frac{100}{6}\ \text{ms}^{-1}$$

After second shot, if speed of car is v_2, then we have

$$75m\left(\frac{100}{16}\right) = 70mv_2 - 5m(100 - v_2)$$

$$\Rightarrow \quad 75v_2 = \frac{75\times 100}{16} + 500$$

$$\Rightarrow \quad v_2 = \frac{100}{16} + \frac{100}{15}\ \text{ms}^{-1}$$

$$\Rightarrow \quad v_2 = 100\left(\frac{1}{16} + \frac{1}{15}\right)\ \text{ms}^{-1} \approx 13\ \text{ms}^{-1}$$

31. (a) Let v be velocity of block when the bullet emerges from it. Applying Law of Conservation of Linear Momentum, we get

$$(1)v + (4\times 10^{-3})(100) = (4\times 10^{-3})(600)$$

$$\Rightarrow \quad v = 2\ \text{ms}^{-1}$$

Using $v^2 = u^2 - 2as$, we get

$$0 = (2)^2 - 2(\mu_k g)\ \text{s}$$

$$0 = 4 - 2\mu_k(10)(0.4)$$

$$\Rightarrow \quad \mu_k = 0.5 = \frac{1}{2}$$

$\Rightarrow \quad * = 2$

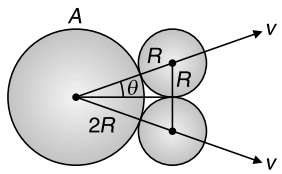
(b) Decrease in kinetic energy of bullet is

$$-\Delta K = \frac{1}{2}(4\times 10^{-3})\left[(600)^2 - (100)^2\right] = 700\ \text{J}$$

(c) Kinetic energy of block

$$K = \frac{1}{2}\times 1\times (2)^2 = 2\ \text{J}$$

32. Just before collision, the arrangement is shown in Figure.



So, $\cos\theta = \frac{2\sqrt{2}R}{2R+R} = \frac{2\sqrt{2}}{3}$

Let initial velocity of A be u and final velocity of each small balls be v

Applying conservation of linear momentum, we get

$$mu = 2mv\cos\theta$$

Since, $e = \frac{(\text{Relative velocity of separation})_{n\text{-line}}}{(\text{Relative velocity of approach})_{n\text{-line}}}$

$$\Rightarrow \quad e = \frac{v}{u\cos\theta}$$

$$\Rightarrow \quad e = \frac{1}{2\cos^2\theta} = \frac{1}{2\left(\frac{8}{9}\right)} = \frac{9}{16} \approx 0.56$$

33. If final velocity of shuttle is v, then by conservation of momentum, we get

$$M(4000) = \left(\frac{4M}{5}\right)v + \frac{M}{5}(3880)$$

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$$\Rightarrow \quad \frac{4v}{5} = 4000 - \frac{3880}{5} = 3224$$

$$\Rightarrow \quad v = 4030 \text{ kph}$$

34. For elastic collision between A and B, we have

$$v_B = \left(\frac{2m}{2m+m}\right)9$$

$$\Rightarrow \quad v_B = 6 \text{ ms}^{-1}$$

Now, for completely inelastic collision between B and C, we have

$$v_C = \frac{2m(v_B) + m(0)}{2m+m}$$

$$\Rightarrow \quad v_C = \frac{(2m)(6)}{3m}$$

$$\Rightarrow \quad v_C = 4 \text{ ms}^{-1}$$

35. Applying conservation of momentum along x and y direction. Along x direction,

$$m_B v = m_A v_A \cos\theta \qquad \ldots(1)$$

If A moves at an angle θ to initial direction of B which is taken as x axis, then along y direction

$$m_B \frac{v}{2} = m_A v_A \sin\theta \qquad \ldots(2)$$

Dividing (2) by (1), we get

$$\tan\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \alpha = 2$$

Since collision is elastic, so we have

$$\frac{1}{2}m_B v^2 = \frac{1}{2}m_B\left(\frac{v^2}{4}\right) + \frac{1}{2}m_A v_A^2$$

Using equation (1), we get

$$(m_A v_A \cos\theta)v = (m_A v_A \cos\theta)\frac{v}{4} + m_A v_A^2$$

$$\Rightarrow \quad v\cos\theta = \frac{1}{4}v\cos\theta + V_A$$

$$\Rightarrow \quad V_A = \frac{3}{4}v\cos\theta$$

Since, $\tan\theta = \frac{1}{2}$, so $\cos\theta = \frac{2}{\sqrt{5}}$

$$\Rightarrow \quad V_A = \left(\frac{3}{4}\right)\left(\frac{2}{\sqrt{5}}\right)v = \frac{3v}{2\sqrt{5}} = \left(\sqrt{\frac{9}{4\times 5}}\right)v = \sqrt{0.45}v$$

$$\Rightarrow \quad \beta = 0.45$$

36. At maximum compression (x_m), velocity of both the blocks is same, say it is v. Applying Law of Conservation of Linear Momentum, we get

$$(m_A + m_B)v = m_B v_0$$

$$\Rightarrow \quad (1+1)v = (1)v_0$$

$$\Rightarrow \quad v = \frac{v_0}{2} = \frac{2}{2} = 1 \text{ ms}^{-1}$$

By Law of Conservation of Mechanical Energy, we get

$$\frac{1}{2}m_B v_0^2 = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}kx_m^2$$

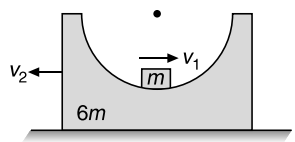
Substituting the values, we get

$$\frac{1}{2}\times(1)\times(2)^2 = \frac{1}{2}\times(1+1)\times(1)^2 + \frac{1}{2}\times(200)\times x_m^2$$

$$\Rightarrow \quad 2 = 1 + 100x_m^2$$

$$\Rightarrow \quad x_m = 0.1 \text{ m} = 10 \text{ cm}$$

37. Let smaller block be moving with speed v_1 relative to bigger block when it reaches the bottommost position and at this instant, bigger block is moving at v_2 (say). Applying conservation of momentum in horizontal direction we get



$$6mv_2 = m(v_1 - v_2) \qquad \ldots(1)$$

Applying conservation of energy, we get

$$mga = \frac{1}{2}m(v_1 - v_2)^2 + \frac{1}{2}(6m)v_2^2 \qquad \ldots(2)$$

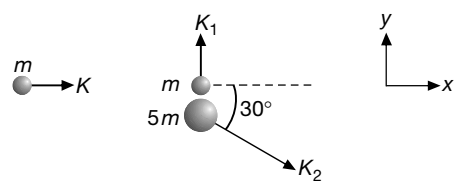
Solving equations (1) and (2), we get

$$2ga = 36v_2^2 + 6v_2^2$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{ga}{21}}$$

$$\Rightarrow \quad * = 21$$

38. Let K be the initial kinetic energy of mass m and K_1 and K_2 the kinetic energies after collision in the direction shown in figure. Initial Momentum of the particle is $\sqrt{2mK}$. So,



Applying conservation of linear momentum in vector form, we get,

$$\sqrt{2mK}\,\hat{i} = \sqrt{2mK_1}\,\hat{j} + \sqrt{2MK_2}\left(\cos 30\hat{i} - \sin 30\hat{j}\right)$$

$$\Rightarrow \quad \sqrt{2mK} = \sqrt{10mK_2}\frac{\sqrt{3}}{2} \text{ and}$$

$$\sqrt{2mK_1} = \sqrt{10mK_2}\frac{1}{2}$$

Solving, we get

$$K_2 = \frac{4}{15}K \text{ and } K_1 = \frac{K}{3}$$

$$\Rightarrow \quad K_1 + K_2 = \frac{9}{15}K = 0.6K$$

So, loss in KE $= K - 0.6K = 0.4K$

Hence, KE will be decreased by 40%.

39. After time t, let us calculate velocity of centre of mass of system by using impulse momentum equation. As spring force is an internal force of system, so we have

$$p_{\text{system}} = (2+3)v_{\text{cm}} = \int_0^t 5t\,dt = \frac{5t^2}{2} \qquad \ldots(1)$$

At $t = 10$ s, from equation (1) we get

$$v_{\text{cm}} = \frac{5(10)^2/2}{5} = 50 \text{ ms}^{-1}$$

If at this instant, let velocity of 2 kg mass be v_2, then we have

$$v_{\text{cm}} = 50 = \frac{(3)(30)+2v_2}{5}$$

$$\Rightarrow \quad v_2 = \frac{(5)(50)-(3)(30)}{2} = \frac{160}{2} = 80 \text{ ms}^{-1}$$

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CHAPTER 2

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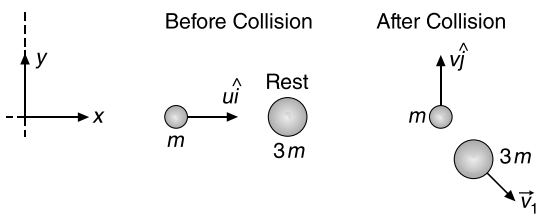
1. Applying Law of Conservation of Momentum, we get

$$\vec{p}_i = \vec{p}_f$$



$$\Rightarrow \quad (m)(u\hat{i}) + (3m)(\vec{0}) = (m)(v\hat{j}) + (3m)\vec{v}_1$$

$$\Rightarrow \quad mui - mvj = 3m\vec{v}_1$$

$$\Rightarrow \quad \vec{v}_1 = \frac{u\hat{i} - v\hat{j}}{3}$$

$$\Rightarrow \quad |\vec{v}_1| = \frac{\sqrt{u^2+v^2}}{3} \qquad \ldots(1)$$

$$\Rightarrow \quad v_1^2 = \frac{u^2+v^2}{9}$$

Since collision is perfectly elastic, so we have

$$\Sigma K_{\text{initial}} = \Sigma K_{\text{final}}$$

$$\Rightarrow \quad \frac{1}{2}mu^2 + \frac{1}{2}(3m)(0)^2 = \frac{1}{2}mv^2 + \frac{1}{2}(3m)v_1^2$$

$$\Rightarrow \quad u^2 = v^2 + 3v_1^2$$

Substituting value of v_1 from equation (1), we get

$$u^2 = v^2 + 3\left(\frac{u^2+v^2}{9}\right)$$

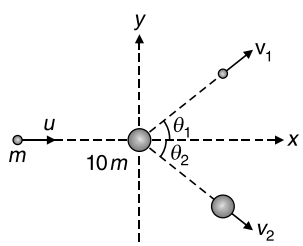
$$\Rightarrow \quad 3u^2 = 3v^2 + u^2 + v^2$$

$$\Rightarrow \quad 2u^2 = 4v^2$$

$$\Rightarrow \quad v = \frac{u}{\sqrt{2}}$$

Hence, the correct answer is (D).

2. Applying momentum conservation along y-axis, we get



$$m_1 v_1 \sin\theta_1 = m_2 v_2 \sin\theta_2$$

$$\Rightarrow \quad m v_1 \sin\theta_1 = 10 m v_2 \sin\theta_2$$

$$\Rightarrow \quad v_1 \sin\theta_1 = 10 v_2 \sin\theta_2 \qquad \ldots(1)$$

Since it is given that

$$(K_f)_m = \frac{1}{2}(K_i)_m$$

$$\Rightarrow \quad \frac{1}{2} m v_1^2 = \frac{1}{2}\left(\frac{1}{2} m u^2\right)$$

$$\Rightarrow \quad v_1 = \frac{u}{\sqrt{2}} \qquad \ldots(2)$$

Also, collision is elastic, so we have $K_f = K_i$

$$\Rightarrow \quad \frac{1}{2} m u^2 = \frac{1}{2} m v_1^2 + \frac{1}{2}(10m) v_2^2$$

$$\Rightarrow \quad \frac{1}{2} m u^2 = \frac{1}{2}\left(\frac{1}{2} m u^2\right) + \frac{1}{2}(10m) v_2^2$$

$$\Rightarrow \quad \frac{1}{4} m u^2 = \frac{1}{2}(10m) v_2^2$$

$$\Rightarrow \quad v_2 = \frac{u}{\sqrt{20}} \qquad \ldots(3)$$

Substituting equations (2) and (3) in equation (1), we get

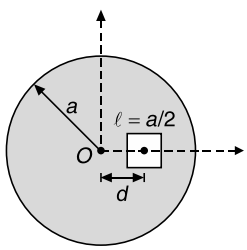
$$\frac{u}{\sqrt{2}} \sin\theta_1 = 10 \frac{u}{\sqrt{20}} \sin\theta_2$$

$$\Rightarrow \quad \sin\theta_1 = \sqrt{10} \sin\theta_2$$

$$\Rightarrow \quad n = 10$$

Hence, the correct answer is (10.00).

3. Since $x_{cm} = \dfrac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$



where, m_1 is mass of complete disc and m_2 is removed mass.

Let σ be the surface mass density of disc material, then

$$x_{cm} = \frac{\sigma\pi a^2 (0) - \sigma\left(a^2/4\right) d}{\sigma\pi a^2 - \sigma\left(a^2/4\right)} = \frac{-a^2 d/4}{\pi a^2 - \left(a^2/4\right)}$$

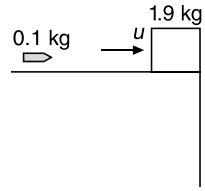
$$\Rightarrow \quad x_{cm} = \frac{-d}{4\pi - 1} = -\frac{a}{2(4\pi - 1)}$$

$$\Rightarrow \quad X = 2(4\pi - 1) = (8\pi - 2) = 23.12$$

So, nearest integer value of $X = 23$

Hence, the correct answer is (23.00).

4. Since, $p_i = p_f$



$$\Rightarrow \quad 0.1 \times 20 = 2v$$

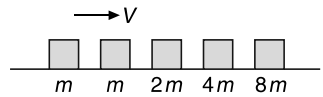
$$\Rightarrow \quad v = 1 \text{ ms}^{-1}$$

So, final kinetic energy is

$$K_f = mgh + \frac{1}{2} m v^2 = 21 \text{ J}$$

Hence, the correct answer is (A).

5. Since all collisions are perfectly inelastic, so after the final collision, all blocks are moving together. So, if the final velocity be V, then on applying momentum conservation, we get



$$mv = 16mV$$

$$\Rightarrow \quad V = \frac{v}{16}$$

Now initial energy is $E_i = \frac{1}{2} m v^2$

Final energy is $E_f = \frac{1}{2}(16m)\left(\frac{v}{16}\right)^2$

$$\Rightarrow \quad E_f = \frac{m v^2}{32}$$

Energy loss is $E_i - E_f$ given by

$$E_i - E_f = \frac{1}{2} m v^2 - \frac{1}{2} m \frac{v^2}{16}$$

$$\Rightarrow \quad E_i - E_f = \frac{1}{2} m v^2 \left(1 - \frac{1}{16}\right)$$

$$\Rightarrow \quad E_i - E_f = \frac{1}{2} m v^2 \left(\frac{15}{16}\right)$$

So, percentage loss in KE is

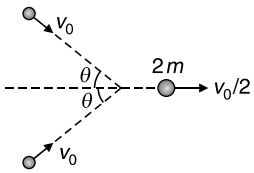
$$\left|\frac{\Delta K}{K}\right| \times 100\% \frac{\text{Energy loss}}{\text{Original energy}} \times 100\%$$

$$\Rightarrow \left|\frac{\Delta K}{K}\right| \times 100\% = \frac{\frac{1}{2}mv^2\left(\frac{15}{16}\right)}{\frac{1}{2}mv^2} \times 100 = 93.75\%$$

$$\Rightarrow \quad p \approx 94$$

Hence, the correct answer is (D).

6. Momentum conservation along x direction, we get



$$2mv_0\cos\theta = (2m)\frac{v_0}{2}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 60^\circ$$

Angle is $2\theta = 120^\circ$

Hence, the correct answer is (120.00).

7. Given that the respective initial velocity of A and B be

$$\vec{u}_1 = \left(\sqrt{3}\hat{i} + \hat{j}\right) \text{ ms}^{-1} \text{-and } \vec{u}_2 = \vec{0}$$

Also, $m_1 = 2m_2$ and after collision, we have

$$\vec{v}_1 = \left(\hat{i} + \sqrt{3}\hat{j}\right) \text{ ms}^{-1} \text{ and } \vec{v}_2 = ?$$

Applying Law of Conservation of Linear Momentum, we get

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\Rightarrow \quad 2m_2\left(\sqrt{3}\hat{i} + \hat{j}\right) + 0 = 2m_2\left(\hat{i} + \sqrt{3}\hat{j}\right) + m_2\vec{v}_2$$

$$\Rightarrow \quad \vec{v}_2 = 2\left(\sqrt{3}\hat{i} + \hat{j}\right) - 2\left(\hat{i} + \sqrt{3}\hat{j}\right)$$

$$\Rightarrow \quad \vec{v}_2 = 2\left(\sqrt{3}\hat{i} - \hat{j}\right) + 2\left(i - \sqrt{3}\hat{j}\right)$$

$$\Rightarrow \quad \vec{v}_2 = 2\left(\sqrt{3} - 1\right)\left(\hat{i} - \hat{j}\right)$$

If θ be the angle between $\vec{v}_1$ and $\vec{v}_2$, then we have

$$\cos\theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1||\vec{v}_2|} = \frac{2\left(\sqrt{3}-1\right)\left(1-\sqrt{3}\right)}{(2)\left[2\sqrt{2}\left(\sqrt{3}-1\right)\right]}$$

$$\Rightarrow \quad \cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}} \qquad \ldots(1)$$

Since we can write, $\cos(105^\circ) = \cos(60^\circ + 45^\circ)$

$$\Rightarrow \quad \cos(105^\circ) = \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$\Rightarrow \quad \cos(105^\circ) = \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$$

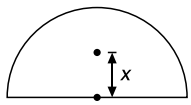
$$\Rightarrow \quad \cos(105^\circ) = \frac{1-\sqrt{3}}{2\sqrt{2}} \qquad \ldots(2)$$

So, from equations (1) and (2), we get

$$\theta = 105^\circ$$

Hence, the correct answer is (D).

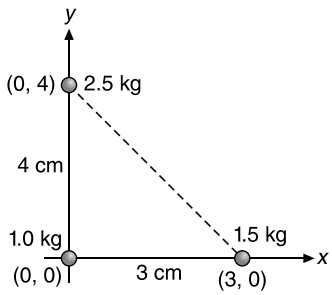
8. Since, $x = \frac{3R}{8} = 3$ cm



$$\Rightarrow \quad x = 3$$

Hence, the correct answer is (3).

9. Let 1 kg be taken as origin and xy axis is shown in Figure.

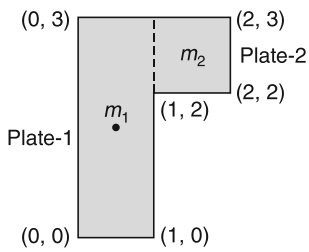


So, $x_{cm} = \frac{1(0)+1.5(3)+2.5(0)}{5} = 0.9$ cm and

$$y_{cm} = \frac{1(0)+1.5(0)+2.5(4)}{5} = 2 \text{ cm}$$

Hence, the correct answer is (B).

10. Let lamina be divided in two parts such that $m_1 = 3$ kg and $m_2 = 1$ kg



Mass of plate-1 is assumed to be concentrated at $(0.5, 1,.5)$ and mass of plate-2 is assumed to be concentrated at $(1.5, 2.5)$, so we have

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{3\times0.5 + 1\times1.5}{4} = 0.75 \text{ and}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{3\times1.5 + 1\times2.5}{4} = 1.75$$

Hence, the correct answer is (D).

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11. By conservation of linear momentum, we have

$$(0.1)(3\hat{i})+(0.1)(5\hat{j})=(0.1)(4)(\hat{i}+\hat{j})+(0.1)\vec{v}$$

$$\Rightarrow \quad \vec{v}=-\hat{i}+\hat{j}$$

Speed of B after collision is $|\vec{v}|=\sqrt{2}$

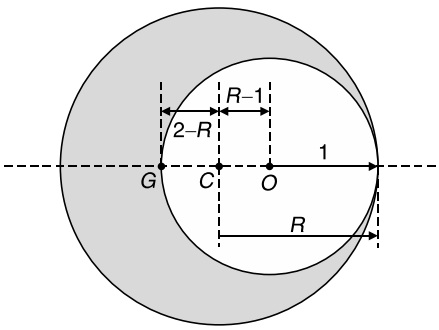
Now, kinetic energy of B is

$$K_B=\frac{1}{2}mv^2=\frac{1}{2}(0.1)(2)=\frac{1}{10}$$

$$\Rightarrow \quad x=1$$

Hence, the correct answer is (1.00).

12. Applying Law of Conservation of Moments about the centre C of the full disc, we have



$m_1r_1=m_2r_2$, where

$m_1=m_{\text{remainder}}=\frac{4}{3}\pi\left(R^3-(1^3)\right)\rho$, $r_1=2-R$ and

$m_2=m_{\text{cavity}}=\frac{4}{3}\pi(1)^3\rho$, $r_1=R-1$

$$\Rightarrow \quad m_{\text{remainder}}(2-R)=m_{\text{cavity}}(R-1)$$

$$\Rightarrow \quad \left[\frac{4}{3}\pi R^3\rho-\frac{4}{3}\pi(1)^3\rho\right](2-R)=\left[\frac{4}{3}\pi(1)^3\rho\right](R-1)$$

$$\Rightarrow \quad (R^3-1)(2-R)=R-1$$

$$\Rightarrow \quad (R^2+R+1)(2-R)=1$$

Alternatively, we can also get the desired result by applying the formula

$$x_{\text{cm}}=\frac{m_{\text{full disc}}x_1-m_{\text{removed disc}}x_2}{m_{\text{full disc}}-m_{\text{removed disc}}}$$

Considering origin to be at the centre of the full disc, we have

$$x_{\text{cm}}=-(2-R),\ x_1=0 \text{ and } x_2=R-1$$

$$\Rightarrow \quad -(2-R)=\frac{0-\frac{4}{3}\pi(1)^3\rho(R-1)}{\frac{4}{3}\pi R^3\rho-\frac{4}{3}\pi(1)^3\rho}=\frac{-(R-1)}{R^3-(1)^3}$$

$$\Rightarrow \quad -(2-R)(R^3-1)=-(R-1)$$

$$\Rightarrow \quad (2-R)(R-1)(R^2+R+1)=(R-1)$$

$$\Rightarrow \quad (R^2+R+1)(2-R)=1$$

Hence, the correct answer is (C).

13. Applying Law of Conservation of Momentum, we get

$$(mu)\hat{i}+mu\left(\frac{\hat{i}+\hat{j}}{2}\right)=(m+m)\vec{v}$$

$$\Rightarrow \quad \vec{v}=\frac{3}{4}u\hat{i}+\frac{u}{4}\hat{j}$$

$$\Rightarrow \quad |\vec{v}|=\frac{u}{4}\sqrt{10}$$

So, final kinetic energy is

$$K_{\text{final}}=\frac{1}{2}2m\left(\frac{u}{4}\sqrt{10}\right)^2=\frac{5}{8}mu^2$$

and initial kinetic energy is

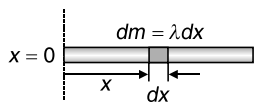
$$K_i=\frac{1}{2}mu^2+\frac{1}{2}m\left(\frac{u}{\sqrt{2}}^2\right)=\frac{6}{8}mu^2$$

So, loss in kinetic energy is

$$-\Delta K=K_i-K_f=\frac{1}{8}mu^2$$

Hence, the correct answer is (B).

14. Since, $x_{\text{cm}}=\dfrac{\int x\,dm}{\int dm}=\dfrac{\int(\lambda dx)x}{\int dm}$

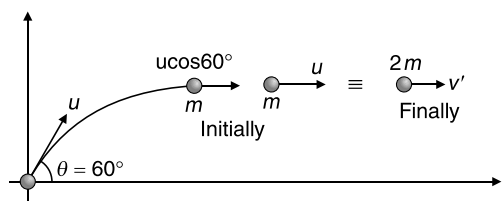


$$\Rightarrow \quad x_{\text{cm}}=\frac{\int_0^L\left(a+\frac{bx^2}{L^2}\right)x\,dx}{\int_0^L\left(a+\frac{bx^2}{L^2}\right)dx}=\frac{\frac{aL^2}{2}+\frac{b}{L^2}\frac{L^4}{4}}{aL+\frac{b}{L^2}\frac{L^3}{3}}$$

$$\Rightarrow \quad x_{\text{cm}}=\frac{\frac{(4a+2b)}{8}}{\frac{(3a+b)}{3}}=\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$$

Hence, the correct answer is (D).

15. Applying Law of Conservation of Momentum along x direction, we get



$$\frac{mu}{2} + mu = 2mv'$$

$$\Rightarrow \quad v' = \frac{3u}{4}$$

Range after collision is

$$R = v't = \left(\frac{3u}{4}\right)t = \frac{3u}{4}\sqrt{\frac{2H}{g}} \qquad \left\{\because\ H = \frac{1}{2}gt^2\right\}$$

where, $H = \dfrac{u^2\sin^2(60°)}{2g}$

$$\Rightarrow \quad R = \frac{3u}{4}\sqrt{\frac{2u^2\sin^2(60°)}{g(2g)}} = \left(\frac{3u}{4}\right)\left(\frac{u\sqrt{3}}{2g}\right)$$

$$\Rightarrow \quad R = \frac{3\sqrt{3}u^2}{8g}$$

Hence, the correct answer is (C).

16. $\vec{a}_{CM} = \dfrac{(2m)a\hat{j} + 3m\times a\hat{i} + ma(-\hat{i}) + 4m\times a(-\hat{j})}{2m+3m+4m+m}$

$$\Rightarrow \quad \vec{a}_{CM} = \frac{2a\hat{i} - 2a\hat{j}}{10} = \frac{a}{5}(\hat{i} - \hat{j})$$

Hence, the correct answer is (D).

17. Applying Conservation of Linear Momentum, we get

$$m_1v_1 + m_2v_2 = m_1v_3 + m_2v_4$$

$$\Rightarrow \quad m_1v_1 + 0.5m_1v_2 = 0.5m_1v_1 + 0.5m_1v_4$$

$$\Rightarrow \quad v_1 = v_4 - v_2$$

Hence, the correct answer is (B).

18. X-coordinate of CM of remaining sheet is

$$x_{cm} = \frac{MX - mx}{M-m}$$

$$\Rightarrow \quad x_{cm} = \frac{(4m)\times\left(\frac{a}{2}\right) - M\left(\frac{3a}{4}\right)}{4m-m} = \frac{5a}{12}$$

Similarly, $y_{cm} = \dfrac{5b}{12}$

So, centre of mass is located at $\left(\dfrac{5a}{12}, \dfrac{5b}{12}\right)$

Hence, the correct answer is (B).

19. Applying Conservation of Linear Momentum, we get

$$2V = \frac{2V}{4} + m_2V_2 \qquad \ldots(1)$$

Since $e = -\left(\dfrac{v_2 - v_1}{u_2 - u_1}\right) = 1$

$$\Rightarrow \quad V = V_2 - \frac{V}{4} \qquad \ldots(2)$$

Substituting (2) in (1), we get

$$\frac{3V}{2} = m_2\left(V + \frac{V}{4}\right)$$

$$\Rightarrow \quad m_2 = \frac{6}{5}\ \text{kg}$$

Hence, the correct answer is (D).

20. Applying momentum conservation, we get

$$mv = (4m+m)v'$$

So, common speed is $v' = \dfrac{v}{5}$

Applying energy conservation, we get

$$mgh + \frac{1}{2}5m\left(\frac{v^2}{25}\right) = \frac{1}{2}mv^2$$

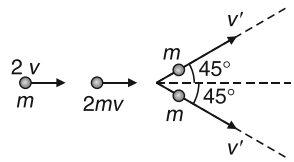
$$\Rightarrow \quad mgh = \frac{1}{2}mv^2\left(1 - \frac{1}{5}\right) = \frac{1}{2}mv^2\frac{4}{5}$$

$$\Rightarrow \quad h = \frac{2mv^2}{5\times mg} = \frac{2v^2}{5g}$$

Hence, the correct answer is (B).

21. Initial momentum is $p_i = 2mv + 2mv = 4mv$

Let v' be the speed of each splitted particle, then by momentum conservation, we get



$$\Rightarrow \quad 2\frac{mv'}{\sqrt{2}} = 4mv$$

$$\Rightarrow \quad v' = 2\sqrt{2}v$$

Hence, the correct answer is (D).

22 Applying conservation of linear momentum along x and y directions.

Along x direction, initial and final momentum are

$$(\Sigma p_x)_{\text{initial}} = M(10\cos 30°) + 2M(5\cos 45°)$$

$$(\Sigma p_x)_{\text{final}} = 2M(v_1\cos 30°) + M(v_2\cos 45°)$$

Since, $(\Sigma p_x)_{\text{initial}} = (\Sigma p_x)_{\text{final}}$

$$\Rightarrow \quad 5\sqrt{3} + \frac{10}{\sqrt{2}} = \sqrt{3}v_1 + \frac{v_2}{\sqrt{2}}$$

$$\Rightarrow \quad 5\sqrt{3} + 5\sqrt{2} = \sqrt{3}v_1 + \frac{v_2}{\sqrt{2}} \qquad \ldots(1)$$

Along y direction, initial and final momentum are

$$(\Sigma p_y)_{\text{initial}} = 2M(5\sin 45°) - M(10\sin 30°)$$

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$$(\Sigma p_y)_{\text{final}} = 2M(v_1 \sin 30°) - M(v_2 \sin 45°)$$

Since, $(\Sigma p_y)_{\text{initial}} = (\Sigma p_y)_{\text{final}}$

$$\Rightarrow \quad 5\sqrt{2} - 5 = v_1 - \frac{v_2}{\sqrt{2}} \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$(\sqrt{3}+1)v_1 = 5\sqrt{3} + 10\sqrt{2} - 5$$

$$\Rightarrow \quad v_1 = 6.5 \text{ ms}^{-1} \text{ and } v_2 = 6.3 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

23. Applying momentum conservation, we get

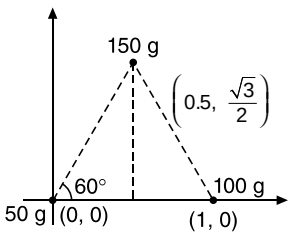
$$50V_1 = 20V_2$$

Also, $V_1 + V_2 = 0.70$

$$\Rightarrow \quad V_1 = 0.20$$

Hence, the correct answer is (A).

24.

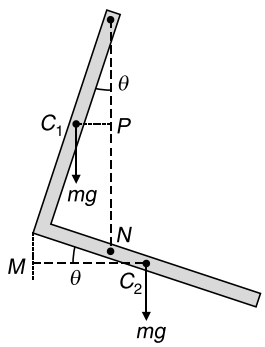


$$x_{\text{CM}} = \frac{(m)(0)+(2m)(1)+(3m)\left(\frac{1}{2}\right)}{6m} = \frac{7}{12} \text{ m}$$

$$y_{\text{CM}} = \frac{(m)(0)+(2m)(0)+(3m)\left(\frac{\sqrt{3}}{2}\right)}{6m} = \frac{\sqrt{3}}{4} \text{ m}$$

Hence, the correct answer is (D).

25. Let mass of one rod be m, then balancing the torque about one end of the rod, we get



$$mg(C_1P) = mg(C_2N)$$

$$\Rightarrow \quad mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta = L\sin\theta\right)$$

$$\Rightarrow \quad \frac{3}{2}mgL\sin\theta = \frac{1}{2}mgL\cos\theta$$

$$\Rightarrow \quad \tan\theta = \frac{1}{3}$$

Hence, the correct answer is (C).

26. Since, $\Delta E_1 = \frac{1}{2}\mu v_{\text{rel}}^2$

$$\Rightarrow \quad \Delta E_1 = \frac{1}{2}\left(\frac{m}{2}\right)v^2 = \frac{1}{4}mv^2$$

Velocity after collision is $V = \frac{v}{2}$

$$\Rightarrow \quad \Delta E_2 = \frac{1}{2}\left(\frac{2mM}{2m+M}\right)\left(\frac{v}{2}\right)^2$$

Since, $\Delta E_1 + \Delta E_2 = \frac{5}{6}\left(\frac{1}{2}mv^2\right)$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{m}{2}\right)v^2 + \left(\frac{1}{2}\right)\left(\frac{m}{2}\right)v^2\left(\frac{M}{2m+M}\right) = \frac{5}{6}\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \quad 1 + \frac{M}{2m+M} = \frac{10}{6} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{M}{m} = 4$$

Hence, the correct answer is (B).

27. Let bullet meet the piece of wood in time t, then we have

$$100 - \frac{1}{2}10t^2 = 100t - 5t^2$$

$$\Rightarrow \quad t = 1 \text{ s}$$

Applying Law of Conservation of Momentum, we get

$$90(0.02) - 10(0.03) = v(0.05)$$

$$\Rightarrow \quad v = \frac{1.8 - 0.3}{0.05} = \frac{1.5}{0.05} = \frac{150}{5} = 30 \text{ ms}^{-1}$$

$$\Rightarrow \quad s_2 = \frac{30^2}{2\times 10} = \frac{30\times 30}{2\times 10} = 45 \text{ m}$$

So, maximum height above the building is

$$h_{\max} = 45 - 5 = 40 \text{ m}$$

Hence, the correct answer is (B).

28. Initial compression is $x_i = \frac{3\times 10}{k}$

Since spring constant is high, so initial compression is low. Let v_1 be velocity after collision, then by Conservation of Momentum, we have

$$4v_1 = v_0 \qquad \ldots(1)$$

where, $v_0 = \sqrt{2gh} = \sqrt{2g \times 100}$

$\Rightarrow \quad \frac{1}{2}(4)v_1^2 = \frac{1}{2}kx^2$

$\Rightarrow \quad x = 2 \text{ cm}$

*No given option is correct.

29. Since, $u = \omega\theta_0$ and $v = \omega\theta_1$

$\Rightarrow \quad \frac{u}{v} = \frac{\theta_0}{\theta_1}$

Also, $v = \left(\frac{M-m}{M+m}\right)u$

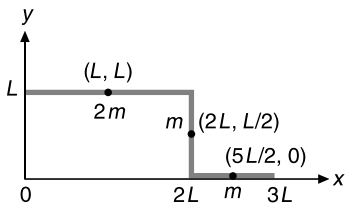
$\Rightarrow \quad \frac{M+m}{M-m} = \frac{u}{v} = \frac{\theta_0}{\theta_1}$

$\Rightarrow \quad \frac{M}{m} = \frac{\theta_0+\theta_1}{\theta_0-\theta_1}$

$\Rightarrow \quad M = m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$

Hence, the correct answer is (A).

30. Let assume linear mass density is λ. The corresponding coordinates of the rod are shown in Figure.



then, $m_1 = 2L\lambda$ and $r_{1\,cm} \equiv (L, L)$

$m_2 = L\lambda$ and $r_{2\,cm} \equiv \left(2L, \frac{L}{2}\right)$ and

$m_3 = L\lambda$ and $r_{3\,cm} \equiv \left(\frac{5L}{2}, 0\right)$

Since, $x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$

$\Rightarrow \quad x_{cm} = \frac{(2m)L + m(2L) + m(5L/2)}{2m + m + m} = \frac{13}{8}L$

and $y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$

$\Rightarrow \quad x_{cm} = \frac{(2m)L + m(L/2) + m(0)}{2m + m + m} = \frac{5}{8}L$

Hence, the correct answer is (C).

31.

Applying Conservation of Linear Momentum, we get

$mv_0 = mv_1 + mv_2$

$\Rightarrow \quad v_1 + v_2 = v_0 \qquad \ldots(1)$

According to the question, we have

$K_f = \frac{3}{2}K_i \qquad \ldots(2)$

$\Rightarrow \quad \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$

$\Rightarrow \quad v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \qquad \ldots(3)$

From equation (1), we have

$(v_1 + v_2)^2 = v_0^2$

$\Rightarrow \quad v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$

Using equation (3), we get

$2v_1v_2 = v_0^2 - \frac{3}{2}v_0^2$

$\Rightarrow \quad 2v_1v_2 = -\frac{v_0^2}{2}$

$\Rightarrow \quad 4v_1v_2 = -v_0^2$

Since, $(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$

$\Rightarrow \quad (v_1 - v_2)^2 = v_0^2 - (-v_0^2) = 2v_0^2$

$\Rightarrow \quad v_1 - v_2 = \sqrt{2}v_0$

Hence, the correct answer is (B).

32. In an elastic oblique 2-D collision, if two bodies are of equal masses and second body is at rest, then after collision, the bodies scatter at right angles. So, the bodies have equal mass i.e.

$m_{unknown} = m$

Hence, the correct answer is (A).

33. The kinetic energy of an object just after it hits the ground is 50% of K.E. of the object. So, we have

$\frac{1}{2}mv'^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$

$\Rightarrow \quad v' = \frac{v}{\sqrt{2}}$

By definition, we have

$e = \frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}}$

$\Rightarrow \quad e = \frac{v'}{v} = \frac{1}{\sqrt{2}}$

CHAPTER 2

When a ball dropped from a height h, total distance covered when

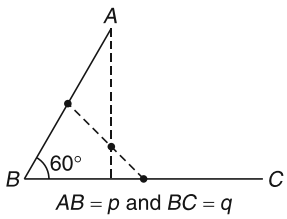
$t \to \infty$ is $\ell = h\left(\dfrac{1+e^2}{1-e^2}\right)$

$\Rightarrow \quad \ell = h\left(\dfrac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) = 3h$

None of the given OPTIONS is correct.

34. Let the length $AB = p$ and $BC = q$

If λ be the linear mass density of the rod, then according to question, centre of mass of the rod lies vertically below point A, so $x_{cm} = p\cos 60°$



$$\Rightarrow \quad x_{CM} = p\cos 60° = \frac{(\lambda q)\left(\frac{q}{2}\right)+(\lambda p)\left(\frac{p}{2}\right)}{\lambda(p+q)}\cos 60°$$

$$\Rightarrow \quad \frac{p}{2} = \frac{\frac{q^2}{2}+\frac{p^2}{4}}{(p+q)}$$

$$\Rightarrow \quad p^2 + pq = q^2 + \frac{p^2}{2}$$

$$\Rightarrow \quad 1+\frac{q}{p} = \frac{q^2}{p^2}+\frac{1}{2}$$

$$\Rightarrow \quad \left(\frac{q}{p}\right)^2 - \frac{q}{p} - \frac{1}{2} = 0$$

$$\Rightarrow \quad \frac{q}{p} = \frac{-(-1)\pm\sqrt{(-1)^2-4(1)\left(-\frac{1}{2}\right)}}{2\times 1} = \frac{1\pm\sqrt{3}}{2}$$

$$\Rightarrow \quad \frac{q}{p} = \frac{1+\sqrt{3}}{2} = 1.366 \approx 1.37$$

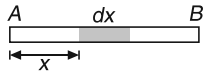
Hence, the correct answer is (C).

35. The centre of mass of a uniform solid cone of height h lies at a distance of $\dfrac{h}{4}$ from base or $\dfrac{3h}{4}$ from vertex.

Hence, the correct answer is (B).

36. Consider a small segment dx of the rod at a distance x from A. Mass of this small segment

$$dm = \mu dx = \left(a+\frac{bx}{L}\right)dx$$



Then CM of the rod AB (from A) is given by

$$x_{CM} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^L (\mu dx)x}{\int_0^L \mu dx}$$

$$\Rightarrow \quad \frac{7}{12}L = \frac{\int_0^L \left(ax+\frac{bx^2}{L}\right)dx}{\int_0^L \left(a+\frac{bx}{L}\right)dx} = \frac{\left(\frac{aL^2}{2}+\frac{bL^2}{3}\right)}{\left(aL+\frac{bL}{2}\right)}$$

$$\Rightarrow \quad \frac{7}{12} = \frac{\frac{a}{2}+\frac{b}{3}}{a+\frac{b}{2}}$$

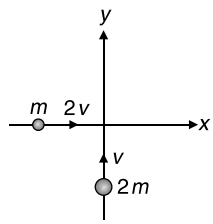
$$\Rightarrow \quad b = 2a$$

Hence, the correct answer is (C).

37. $\vec{v}_{combined} = \dfrac{m(2v)\hat{i}+(2m)\hat{j}}{m+2m}$

$$\Rightarrow \quad \vec{v}_{combined} = \frac{2v}{3}\left(\hat{i}+\hat{j}\right)$$

$$\Rightarrow \quad |\vec{v}_{combined}| = \frac{2\sqrt{2}v}{3}$$



Now loss in KE is

$$-\Delta E = E_i - E_f$$

$$\Rightarrow \quad \text{Loss} = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 - \frac{1}{2}(3m)\left(\frac{2\sqrt{2}v}{3}\right)^2$$

$$\Rightarrow \quad \text{Loss} = 3mv^2 - \frac{4}{3}mv^2 = \frac{5}{3}mv^2$$

$$\Rightarrow \quad \%\ \text{Loss} = \frac{5mv^2/3}{2mv^2+mv^2}\times 100\% = 56\%$$

Hence, the correct answer is (C).

38. **Hence, the correct answer is (B).**

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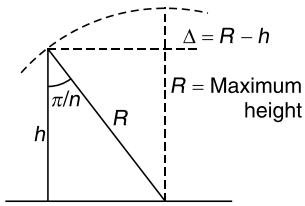
Single Correct Choice Type Problems

1.

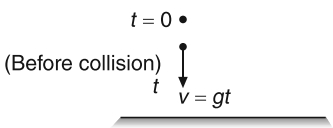


Since, $\cos\left(\frac{\pi}{n}\right)=\frac{h}{R}$

$\Rightarrow \quad \Delta = R-h=\frac{h}{\cos(\pi/n)}-h=h\left[\frac{1}{\cos(\pi/n)-1}\right]$

Hence, the correct answer is (D).

2.



Since, $K=\frac{1}{2}mv^2=\frac{1}{2}mg^2t^2 \qquad \{\because v=gt\}$

$\Rightarrow \quad K \propto t^2$

Therefore, K-t graph is parabola.
However, during collision, retarding force is just like the spring force $(F \propto x)$, therefore kinetic energy first decreases to elastic potential energy and then increases.
Hence, the correct answer is (B).

3. Final momentum of object

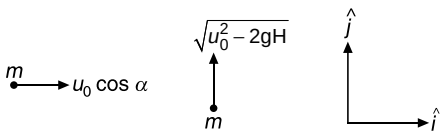
$$p=\frac{E}{c}=\frac{\text{Power}\times\text{Time}}{\text{Speed of light}}=\frac{Pt}{c}$$

$\Rightarrow \quad p=\frac{30\times10^{-3}\times100\times10^{-9}}{3\times10^{8}}$

$\Rightarrow \quad p=1.0\times10^{-17}\ \text{kgms}^{-1}$

Hence, the correct answer is (B).

4. From momentum conservation, we have



$\Sigma\vec{p}_i=\Sigma\vec{p}_f$

$\Rightarrow \quad m(u_0\cos\alpha)\hat{i}+m\left(\sqrt{u_0^2-2gH}\right)\hat{j}=(2m)\vec{v} \qquad \ldots(1)$

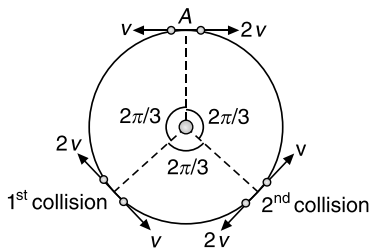
Since, $H=\frac{u_0^2\sin^2\alpha}{2g} \qquad \ldots(2)$

From equations (1) and (2), we get

$$\vec{v}=\left(\frac{u_0\cos\alpha}{2}\right)\hat{i}+\left(\frac{u_0\cos\alpha}{2}\right)\hat{j}$$

Since both x and y components of $\vec{v}$ are equal, so $\vec{v}$ makes an angle of $45°$ with horizontal.
Hence, the correct answer is (A).

5. Before the first collision, the particle having speed $2v$ will rotate $240°$ $\left(\text{or } \frac{4\pi}{3}\right)$ while other particle having speed v will rotate $120°$ $\left(\text{or } \frac{2\pi}{3}\right)$. After first collision, they will exchange their velocities. So, after two collisions they will again reach at point A as shown in Figure.



Hence, the correct answer is (C).

6. Since, $R=u\sqrt{\frac{2h}{g}}$

$\Rightarrow \quad 20=v_1\sqrt{\frac{2\times5}{10}}$ and $100=v_2\sqrt{\frac{2\times5}{10}}$

$\Rightarrow \quad v_1=20\ \text{ms}^{-1},\ v_2=100\ \text{ms}^{-1}$

Applying momentum conservation just before and just after the collision, we get

$$(0.01)(v)=(0.2)(20)+(0.01)(100)$$

$\Rightarrow \quad v=500\ \text{ms}^{-1}$

Hence, the correct answer is (D).

7. Since, $y_{CM}=\frac{m_1y_1+m_2y_2+m_3y_3+m_4y_4+m_5y_5}{m_1+m_2+m_3+m_4+m_5}$

$\Rightarrow \quad y_{CM}=\frac{(6m)(0)+(m)(a)+m(a)+m(0)+m(-a)}{6m+m+m+m+m}$

$\Rightarrow \quad y_{CM}=\frac{a}{10}$

Hence, the correct answer is (A).

CHAPTER 2

8. On a system of particles if, $\sum F_{ext.} = 0$ then,

$$p_{\text{system}} = \text{constant}$$

Hence, the correct answer is (A).

9. Since, $\vec{p} = A\cos(kt)\hat{i} - A\sin(kt)\hat{j}$

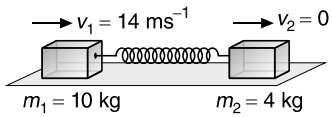
$$\Rightarrow \quad \vec{F} = \frac{d\vec{p}}{dt} = -kA\sin(kt)\hat{i} - kA\cos(kt)\hat{j}$$

Since, $\vec{F}\cdot\vec{p} = 0$

So, angle between $\vec{F}$ and $\vec{p}$ is $90°$

Hence, the correct answer is (D).

10.



Since, $v_{\text{cm}} = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2}$

$$\Rightarrow \quad v_{\text{cm}} = \frac{10\times 14 + 4\times 0}{10+4} = \frac{10\times 14}{14} = 10 \text{ ms}^{-1}$$

Hence, the correct answer is (C).

11. Change in momentum of two particles is

$$\Delta\vec{p} = \left|\left(m_1\vec{v}_1' + m_2\vec{v}_2'\right) - \left(m_1\vec{v}_1 + m_2\vec{v}_2\right)\right|$$

Since, $\Delta\vec{p} = \left|\begin{matrix}\text{External force on}\\ \text{the system}\end{matrix}\right| \times \text{time interval}$

$$\Rightarrow \quad \Delta\vec{p} = F_{\text{ext}}\Delta t = \left(m_1 + m_2\right)g\left(2t_0\right)$$

$$\Rightarrow \quad \Delta\vec{p} = 2\left(m_1 + m_2\right)gt_0$$

Hence, the correct answer is (C).

12. Before explosion, particle was moving along x-axis , i.e., it has no y-component of velocity. Therefore, the centre of mass will not move along the y-direction

$$\Rightarrow \quad y_{\text{cm}} = 0$$

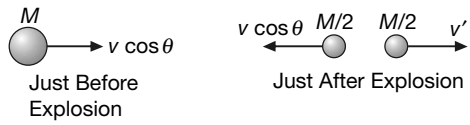
$$\Rightarrow \quad y_{\text{cm}} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

$$\Rightarrow \quad 0 = \frac{\left(\dfrac{m}{4}\right)(+15) + \left(\dfrac{3m}{4}\right)(y)}{\left(\dfrac{m}{4} + \dfrac{3m}{4}\right)}$$

$$\Rightarrow \quad y = -5 \text{ cm}$$

Hence, the correct answer is (A).

13. At the highest point velocity of the shell is $v\cos\theta$. At the highest point, it explodes into two pieces of equal masses out of which one piece retraces the path i.e., has a velocity $-v\cos\theta$.



So, by Law of Conservation of Momentum

$$M(v\cos\theta) = \frac{M}{2}(-v\cos\theta) + \frac{Mv'}{2}$$

$$\Rightarrow \quad v' = 3v\cos\theta$$

Hence, the correct answer is (A).

14. In an inelastic collision, only the momentum of system (ball and earth) may remain conserved. Some energy can be lost in the form of heat, sound etc.

Hence, the correct answer is (C).

15. Since the system is free from external force, hence $a_{cm} = 0$ and since initially they are at rest, so

$$V_{cm} = 0$$

Hence, the correct answer is (A).

Multiple Correct Choice Type Problems

1. The rate of collision of particle with the piston is

$$f = \frac{1}{2L/v} = \frac{v}{2L}$$

The speed of the particle after collision (when it collides with a speed v) with the piston is $v + 2V$. Assuming that the piston moves inwards by dL, then speed of the particle increases by

$$dv = 2V\left(\frac{dL}{V}\right)\left(\frac{v}{2L}\right)$$

$$\Rightarrow \quad \frac{dv}{v} = \frac{dL}{L}$$

Since L is decreasing, so we have

$$\frac{dv}{v} = -\frac{dL}{L}$$

$$\Rightarrow \quad \int_{v_0}^{v}\frac{dv}{v} = -\int_{L_0}^{L_0/2}\frac{dL}{L}$$

$$\Rightarrow \quad \left(\log_e v\right)\Big|_{v_0}^{v} = -\left(\log_e L\right)\Big|_{L_0}^{L_0/2}$$

$$\Rightarrow \quad \log_e\left(\frac{v}{v_0}\right) = -\log_e\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad v = 2v_0$$

So, kinetic energy at L_0 is $K_{L_0} = \dfrac{1}{2}mv_0^2$ and kinetic energy at $L_0/2$ is

$$K_{L_0/2} = \frac{1}{2}m(2v_0)^2 = 4K_{L_0}$$

Hence, (B) and (C) are correct.

2. Since, Δx_{cm} of the block and point mass system is zero i.e., CM coil not move along x-direction .

$\Rightarrow \quad m(x+R)+Mx=0$

where, x is displacement of the block.

$$\Rightarrow \quad x = -\frac{mR}{M+m}$$

Applying conservation of momentum, we get

$$0 = mv - MV$$

$$\Rightarrow \quad mv = MV$$

Applying conservation of energy, we get

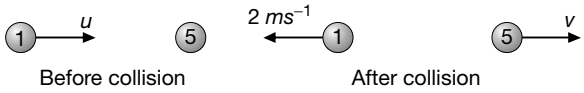
$$mgR = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

Solving these two equations, we get

$$v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$$

Hence, (A) and (B) are correct.

3.



Since collision is elastic, so $e=1$

$\Rightarrow$ Velocity of Approach = Velocity of Separation

$$\Rightarrow \quad u = v+2 \qquad \ldots(1)$$

By Law of Conservation of Linear Momentum, we get

$$(1)u = (5)v - (1)(2)$$

$$\Rightarrow \quad u = 5v - 2 \qquad \ldots(2)$$

Equating (1) and (2), we get

$$v+2 = 5v-2$$

$\Rightarrow \quad v = 1 \text{ ms}^{-1}$ and $u = 3 \text{ ms}^{-1}$

Momentum of system $p_{system} = (1)(3) = 3 \text{ kgms}^{-1}$

Momentum of 5 kg after collision is

$$p_2 = (5)(1) = 5 \text{ kgms}^{-1}$$

So, kinetic energy of centre of mass is

$$K_{cm} = \frac{1}{2}(m_1+m_2)\left(\frac{m_1u}{m_1+m_2}\right)^2 = \frac{1}{2}(1+5)\left(\frac{1\times 3}{6}\right)^2$$

$$\Rightarrow \quad K_{cm} = 0.75 \text{ J}$$

Total kinetic energy is $K = \frac{1}{2}\times 1\times 3^2$

$$\Rightarrow \quad K = 4.5 \text{ J}$$

Hence, (A) and (C) are correct.

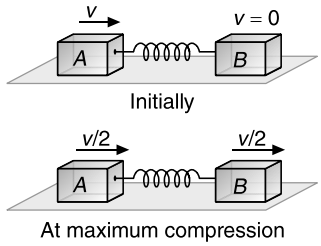
4. Initial momentum of the system is $\Sigma p_{initial} = \vec{p}_1 + \vec{p}_2 = 0$

So, final momentum of system $\Sigma p_{final} = \vec{p}_1' + \vec{p}_2' = 0$

OPTION (B) is allowed because if we put $c_1 = -c_2 \neq 0$, $\vec{p}_1' + \vec{p}_2'$ will be zero. Similarly, we can check other options.

Hence, (A) and (D) are correct.

5. Here, at maximum compression x_{max}, we introduce the concept of reduced mass μ of the system (details discussed in Rotational Dynamics) given by



$$\frac{1}{\mu} = \frac{1}{m} + \frac{1}{m}$$

$$\Rightarrow \quad \mu = \frac{m}{2}$$

Further, by Law of Conservation of Energy we get

$$\frac{1}{2}\mu v^2 = \frac{1}{2}kx_{max}^2$$

$$\Rightarrow \quad x_{max} = v\sqrt{\frac{m}{2k}}$$

So, $\text{K.E.} = \frac{1}{2}\mu v^2 = \frac{1}{2}\left(\frac{m}{2}\right)v^2$

$$\Rightarrow \quad \text{K.E.} = \frac{mv^2}{4}$$

Hence, (B) and (D) are correct.

Reasoning Based Questions

1. If a force is applied at centre of mass of a rigid body, its torque about centre of mass will be zero, but acceleration will be non-zero. Hence, velocity will change.

Hence, the correct answer is (D).

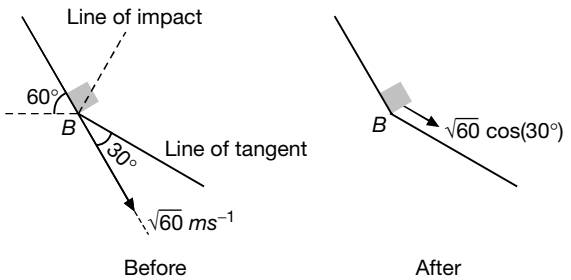
2. In case of elastic collision, coefficient of restitution $e=1$.

Magnitude of relative velocity of approach equals magnitude of relative velocity of separation.

But relative speed of approach is not equal to relative speed of separation.
Hence, the correct answer is (D).

Linked Comprehension Type Questions

1.



Let speed of block just before it strikes the second inclined plane be v then,

$$\frac{1}{2}mv^2 = mg\left(\sqrt{3}\tan 60^\circ\right)$$

$$\Rightarrow \quad v = \sqrt{60}\ \text{ms}^{-1}$$

Speed of block immediately after it strikes the second incline is $\sqrt{45}\ \text{ms}^{-1}$. Because in perfectly inelastic collision the component of velocity along line of impact becomes zero.
Hence, the correct answer is (B).

2. By Conservation of Mechanical Energy

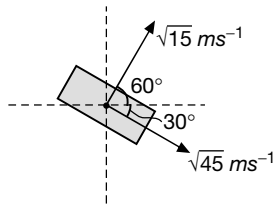
$$\frac{1}{2}mv_C^2 = \frac{1}{2}m\left(\sqrt{45}\right)^2 + mg\left(3\sqrt{3}\tan 30^\circ\right)$$

$$\Rightarrow \quad v_C^2 = 45 + 60 = 105$$

$$\Rightarrow \quad v_C = \sqrt{105}\ \text{ms}^{-1}$$

Hence, the correct answer is (B).

3. If collision is completely elastic, then vertical component of velocity becomes,



$$\sqrt{45}\sin 30^\circ - \sqrt{15}\sin 60^\circ = \frac{\sqrt{45}}{2} - \frac{\sqrt{15}\times\sqrt{3}}{2} = 0$$

Hence, the correct answer is (C).

Integer/Numerical Answer Type Questions

1. For elastic collision between A and B, we have

$$v_B = \left(\frac{2m}{2m+m}\right)9$$

$$\Rightarrow \quad v_B = 6\ \text{ms}^{-1}$$

Now, for completely inelastic collision between B and C, we have

$$v_C = \frac{2m(v_B)+m(0)}{2m+m} = \frac{(2m)(6)}{3m}$$

$$\Rightarrow \quad v_C = 4\ \text{ms}^{-1}$$

Test Your Concepts-I (Based on Moment of Inertia and Applications)

1. Axis A: $I_A = m_1 d_1^2 + m_2 d_2^2 = (3\text{ kg})(1\text{ m})^2 + (5\text{ kg})(2\text{ m})^2 = 23\text{ kgm}^2$

Axis B: $I_B = m_1(0) + m_2(d_1 + d_2)^2 = 45\text{ kgm}^2$

Axis C: $I_C = m_1(d_1 + d_2)^2 + m_2(0) = 27\text{ kgm}^2$

Axis D: $I_D = 0$

I_D is zero because we treated the masses as point particles and perpendicular distances are equal to zero.

2. For each mass we need its perpendicular distance from the axis. For each axis, two masses do not contribute to the moment of inertia. The other two are at the same distance

$3\sin(53°) = 2.4\text{ m}$

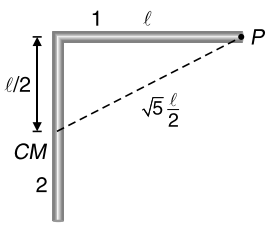
$I_A = (4\text{ kg})(2.4\text{ m})^2 + (2\text{ kg})(2.4\text{ m})^2$

$\Rightarrow \quad I_A = 34.6\text{ kgm}^2$

$I_B = (1\text{ kg})(2.4\text{ m})^2 + (3\text{ kg})(2.4\text{ m}^2) = 23\text{ kgm}^2$

3. Moment of inertia of rod 1 about axis P is

$$I_1 = \frac{ml^2}{3}$$



Moment of inertia of rod 2 about axis P,

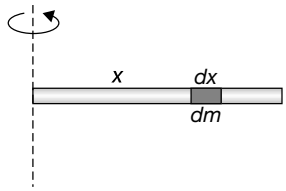
$$I_2 = \frac{ml^2}{12} + m\left(\sqrt{5}\frac{l}{2}\right)^2$$

So, moment of inertia of a system about axis P,

$$I = I_1 + I_2 = \frac{ml^2}{3} + \frac{ml^2}{12} + m\left(\sqrt{5}\frac{l}{2}\right)^2$$

$$\Rightarrow \quad I = \frac{ml^2}{3}$$

4. (a) $dI = x^2(dm) = x^2(\lambda_0 x)dx \qquad \{\because\ dm = \lambda dx\}$



$$\Rightarrow \quad I = \lambda_0 \int_0^{2L} x^3 dx$$

$$\Rightarrow \quad I = \lambda_0 \left(\frac{x^4}{4}\Big|_0^{2L}\right)$$

$$\Rightarrow \quad I = \frac{\lambda_0}{4}(16L^4)$$

$$\Rightarrow \quad I = 4\lambda_0 L^4 \qquad \ldots(1)$$

Now $dm = \lambda dx = \lambda_0 x dx$

$$\Rightarrow \quad M = \int dm = \lambda_0 \int_0^{2L} x dx$$

$$\Rightarrow \quad M = \lambda_0 \left(\frac{x^2}{2}\Big|_0^{2L}\right)$$

$$\Rightarrow \quad M = 2\lambda_0 L^2$$

So, $I = 2ML^2$

(b) Similarly, in this case

$$I = \frac{1}{3}ML^2$$

5. $I_1 = \frac{1}{12}ML^2 \qquad \ldots(1)$

Now, $L = 2\pi r$

$$\Rightarrow \quad r = \frac{L}{2\pi}$$

$$I_2 = 2Mr^2 = \frac{ML^2}{2\pi^2}$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \left(\frac{\cancel{ML^2}}{12}\right)\left(\frac{2\pi^2}{\cancel{ML^2}}\right) = \frac{\pi^2}{6}$$

6. Using parallel axis theorem

$$I_1 = I_C + ma^2 \quad \ldots(1)$$

$$I_2 = I_C + mb^2 \quad \ldots(2)$$

From (1) and (2)

$$I_1 - I_2 = m(a^2 - b^2)$$

7. (a) $I = 0 + m(2d)^2 + m(\sqrt{2}d)^2 + m(\sqrt{2}d)^2$

$$\Rightarrow \quad I = 4md^2 + 2md^2 + 2md^2$$

$$\Rightarrow \quad I = 8md^2$$

(b) $I = 0 + m\ell^2 + m\ell^2 + m(\sqrt{2}\ell)^2$

$$\Rightarrow \quad I = 4m\ell^2$$

8. $I = I_{\text{total}} = I_{\text{can}} + 2I_{\text{lids}}$

$$\Rightarrow \quad I = MR^2 + 2\left(\frac{1}{2}mR^2\right)$$

$$\Rightarrow \quad I = (2\pi Rh\sigma)R^2 + (\pi R^2\sigma)R^2$$

$$\Rightarrow \quad I = \sigma\pi R^3(2h + R)$$

9. $I = I_{\text{total}} = I_{\text{rod}} + I_{\text{spheres}}$

$$\Rightarrow \quad I = \frac{1}{12}[m(3R)^2] + 2\left[\frac{2}{5}mR^2 + m\left(\frac{3R}{2} + R\right)^2\right]$$

$$\Rightarrow \quad I = \frac{3}{4}mR^2 + 2\left(\frac{2}{5}mR^2 + \frac{25}{4}mR^2\right)$$

$$\Rightarrow \quad I = mR^2\left(\frac{3}{4} + \frac{4}{5} + \frac{25}{2}\right)$$

$$\Rightarrow \quad I = \frac{mR^2}{20}(15 + 16 + 250)$$

$$\Rightarrow \quad I = \frac{mR^2}{20}(281)$$

$$\Rightarrow \quad I \cong 14mR^2$$

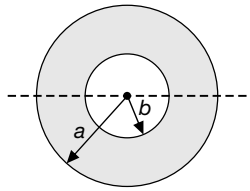
10. Given $M = \rho\left(\frac{4}{3}\pi a^3 - \frac{4}{3}\pi b^3\right)$

$$\Rightarrow \quad M = \frac{4}{3}\pi\rho(a^3 - b^3) \quad \ldots(1)$$

$$I = I_{\substack{\text{sphere of}\\ \text{radius } a}} - I_{\substack{\text{sphere of}\\ \text{radius } b}}$$

$$\Rightarrow \quad I = \frac{2}{5}\left[\left(\frac{4}{3}\pi a^3\right)\rho\right]a^2 - \frac{2}{5}\left[\left(\frac{4}{3}\pi b^3\right)\rho\right]b^2$$

$$\Rightarrow \quad I = \left(\frac{2}{5}\right)\left(\frac{4}{3}\pi\rho\right)(a^5 - b^5)$$



From (1), $\frac{4}{3}\pi\rho = \frac{M}{a^3 - b^3}$

$$\Rightarrow \quad I = \frac{2}{5}M\left(\frac{a^5 - b^5}{a^3 - b^3}\right)$$

11. Let σ be mass per unit area i.e., surface mass density, then

$$\sigma = \frac{M}{\pi(R^2 - a^2)} \quad \ldots(1)$$

Also $I = I_{\text{disc}} - I_{\text{cavity}}$

Now, about the said axis, we have

$$\Rightarrow \quad I_{\text{disc}} = \frac{1}{2}(\pi R^2\sigma)R^2 \text{ and}$$

By Parallel Axis Theorem, we have

$$I_{\text{cavity}} = \frac{1}{2}(\pi a^2\sigma)a^2 + (\pi a^2\sigma)b^2$$

$$\Rightarrow \quad I_{\text{cavity}} = \pi a^2\sigma\left(\frac{a^2}{2} + b^2\right)$$

$$\Rightarrow \quad I = \frac{1}{2}(\pi R^2\sigma)R^2 - \pi a^2\sigma\left(\frac{a^2}{2} + b^2\right)$$

$$\Rightarrow \quad I = \frac{1}{2}\pi\sigma(R^4 - a^4 - 2a^2b^2)$$

Using (1), we get

$$I = \frac{1}{2}M\left(\frac{R^4 - a^4 - 2a^2b^2}{R^2 - a^2}\right)$$

12. Moment of inertia of a semi-circular disc about an axis passing through centre and perpendicular to plane of disc, $I = \frac{MR^2}{2}$

Using parallel axis theorem $I = I_{\text{cm}} + Md^2$, d is the perpendicular distance between two parallel axis passing through centre C and CM.

$$I = \frac{MR^2}{2},\ d = \frac{4R}{3\pi}$$

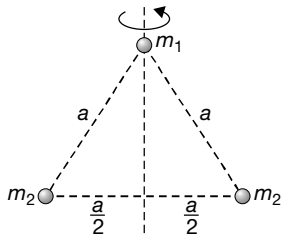
$$\Rightarrow \quad \frac{MR^2}{2} = I_{\text{cm}} + M\left(\frac{4R}{3\pi}\right)^2$$

$$\Rightarrow \quad I_{cm} = \left[\frac{MR^2}{2} - M\left(\frac{4R}{3\pi}\right)^2\right]$$

$$\Rightarrow \quad I_{cm} = MR^2\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)$$

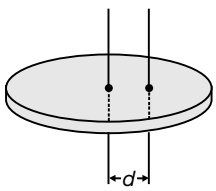
13. $I = m_1(0)^2 + m_2\left(\frac{a}{2}\right)^2 + m_3\left(\frac{a}{2}\right)^2$

$$\Rightarrow \quad I = \frac{a^2}{4}(m_1 + m_2)$$



14. $k = R$

$$\Rightarrow \quad I = Mk^2 = MR^2$$



By Parallel Axis Theorem, we have

$$I = I_G + Md^2$$

$$\Rightarrow \quad MR^2 = \frac{1}{2}MR^2 + Md^2$$

$$\Rightarrow \quad Md^2 = \frac{1}{2}MR^2$$

$$\Rightarrow \quad d = \frac{R}{\sqrt{2}}$$

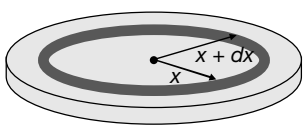
15. $dm = \sigma dA$

$$\Rightarrow \quad dm = (kx^2)(2\pi x dx) \qquad \ldots(1)$$

$$\Rightarrow \quad M = \int dm = 2\pi k\int_0^R x^3 dx$$

$$\Rightarrow \quad M = 2\pi k\left(\frac{R^4}{4}\right)$$

$$\Rightarrow \quad M = \frac{\pi kR^4}{2} \qquad \ldots(2)$$



If dI be the moment of inertia of element, then

$$dI = x^2 dm$$

$$\Rightarrow \quad dI = x^2(2\pi kx^3 dx)$$

$$\Rightarrow \quad I = 2\pi kR\int_0^R x^5 dx$$

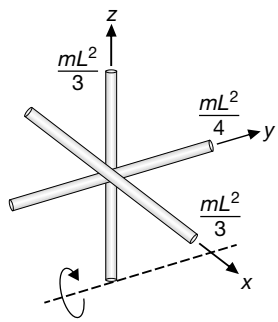
$$\Rightarrow \quad I = 2\pi k\left(\frac{R^6}{6}\right) = \frac{\pi kR^6}{3}$$

But from (2) $\pi kR^4 = 2M$

$$\Rightarrow \quad I = \frac{2MR^2}{3}$$

16. So, $I = \frac{mL^2}{3} + \frac{mL^2}{4} + \frac{mL^2}{3}$

$$\Rightarrow \quad I = \frac{11}{12}mL^2$$



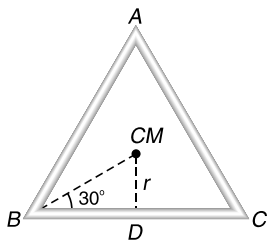
17. Moment of inertia of rod BC about an axis perpendicular to plane of triangle ABC and passing through the mid-point of rod BC (i.e., D) is

$$I_1 = \frac{m\ell^2}{12}$$

According to Parallel Axis Theorem, moment of inertia of this rod about the asked axis is

$$I_2 = I_1 + mr^2$$

$$\Rightarrow \quad I_2 = \frac{m\ell^2}{12} + m\left(\frac{\ell}{2\sqrt{3}}\right)^2 = \frac{m\ell^2}{6}$$



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So, moment of inertia of the system is

$$I = 3I_2 = 3\left(\frac{m\ell^2}{6}\right)$$

$$\Rightarrow \quad I = \frac{m\ell^2}{2}$$

Since, $I = (M_{\text{total}})K^2 = \frac{m\ell^2}{2}$

$$\Rightarrow \quad (3m)K^2 = \frac{m\ell^2}{2}$$

$$\Rightarrow \quad K = \frac{\ell}{\sqrt{6}}$$

Test Your Concepts-II (Based on Rotational Kinematics, Combined Effect of Rotation and Translation Motion)

1. Angular velocity $\omega = \frac{d\theta}{dt} = \frac{d}{dt}(4t - 3t^2 + t^3)$

$$\Rightarrow \quad \omega = 4 - 6t + 3t^2$$

(a) At $t = 2$ s

$$\omega = 4 - (6)(2) + 3(2)^2$$

$$\Rightarrow \quad \omega = 4 \text{ rads}^{-1}$$

At $t = 4$ s

$$\omega = 4 - (6)(4) + 3(4)^2$$

$$\Rightarrow \quad \omega = 28 \text{ rads}^{-1}$$

(b) Average angular acceleration

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{28 - 4}{4 - 2}$$

$$\Rightarrow \quad \alpha_{av} = 12 \text{ rads}^{-2}$$

(c) Instantaneous angular acceleration is,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(4 - 6t + 3t^2)$$

$$\Rightarrow \quad \alpha = -6 + 6t$$

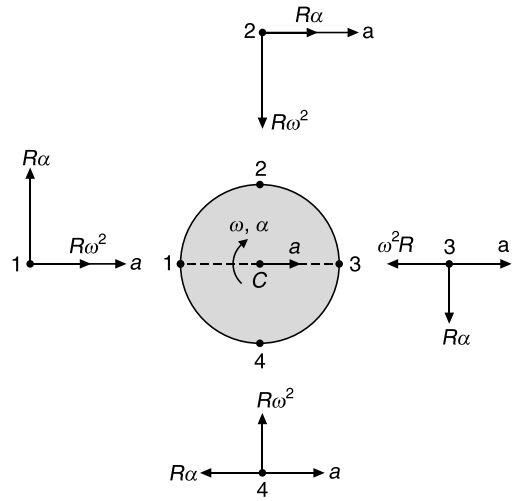
At $t = 2$ s, $\alpha = -6 + (6)(2) = 6 \text{ rads}^{-2}$

At $t = 4$ s, $\alpha = -6 + (6)(4) = 18 \text{ rads}^{-2}$

2. The motion of disc is both translational and rotational, so the acceleration of any point P on the disc can be expressed by

$$\vec{a}_P = \vec{a}_{PC} + \vec{a}_C$$

For all the specified points 1, 2, 3 and 4, the accelerations are shown in Figure.



The acceleration of point 1 is

$$\vec{a}_1 = \vec{a}_{1C} + \vec{a}_C$$

But $\vec{a}_{1C} = (\vec{a}_{1C})_{\text{tangential}} + (\vec{a}_{1C})_{\text{radial}}$

$$\Rightarrow \quad \vec{a}_{1C} = (R\alpha)\hat{j} + (R\omega^2)\hat{i} \text{ and } \vec{a}_C = a\hat{i}$$

$$\Rightarrow \quad \vec{a}_1 = (a + R\omega^2)\hat{i} + (R\alpha)\hat{j}$$

Similarly, the acceleration of point 2 is

$$\vec{a}_2 = \vec{a}_{2C} + \vec{a}_C$$

where, $\vec{a}_{2C} = (R\alpha)\hat{i} - (R\omega^2)\hat{j}$ and $\vec{a}_C = a\hat{i}$

$$\Rightarrow \quad \vec{a}_2 = (a + R\alpha)\hat{i} - (R\omega^2)\hat{j}$$

Acceleration for the point 3 is

$$\vec{a}_3 = \vec{a}_{3C} + \vec{a}_C$$

where, $\vec{a}_{3C} = -(R\omega^2)\hat{i} - (R\alpha)\hat{j}$ and $\vec{a}_C = a\hat{i}$

$$\Rightarrow \quad \vec{a}_3 = (a - R\omega^2)\hat{i} - (R\alpha)\hat{j}$$

Acceleration for the point 4 is

$$\vec{a}_4 = \vec{a}_{4C} + \vec{a}_C$$

where, $\vec{a}_{4C} = -(R\alpha)\hat{i} + (R\omega^2)\hat{j}$ and $\vec{a}_C = a\hat{i}$

$$\Rightarrow \quad \vec{a}_4 = (a - R\alpha)\hat{i} + (R\omega^2)\hat{j}$$

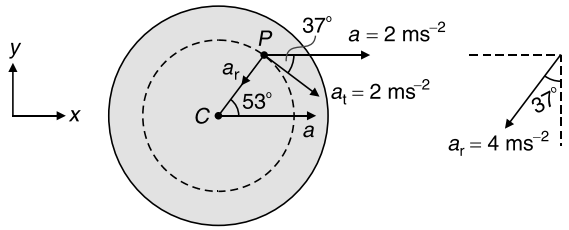
3. For particle at P, $r = CP = 1$ m

$$\Rightarrow \quad \alpha = \frac{d\omega}{dt} = \frac{d}{dt}(2t) = 2 \text{ rads}^{-2}$$

At $t = 1$ s, $\omega = 2 \text{ rads}^{-1}$ and $\alpha = 2 \text{ rads}^{-2}$

So, $a_t = r\alpha = 2 \text{ ms}^{-2}$, $a_r = r\omega^2 = 4 \text{ ms}^{-2}$, $a = 2 \text{ ms}^{-2}$

Net acceleration of P is the vector sum of three terms a, a_r and a_t as shown in Figure below.



$$\Rightarrow \quad \vec{a}_P = 2\hat{i} + \left(2\cos 37°\hat{i} - 2\sin 37°\hat{j}\right) + \left(-4\sin 37°\hat{i} - 4\cos 37°\hat{j}\right)$$

$$\Rightarrow \quad \vec{a}_P = 2\hat{i} + 1.6\hat{i} - 1.2\hat{j} - 2.4\hat{i} - 3.2\hat{j}$$

$$\Rightarrow \quad \vec{a}_P = \left(1.2\hat{i} - 4.4\hat{j}\right) \text{ ms}^{-2}$$

4. Here $\alpha = \pi$ rads^{-2}

$\omega_0 = 0$

$t = 4$ s

(a) $\omega = 0 + (\pi \text{ rads}^{-2}) \times 4 \text{ s} = 4\pi \text{ rads}^{-1}$

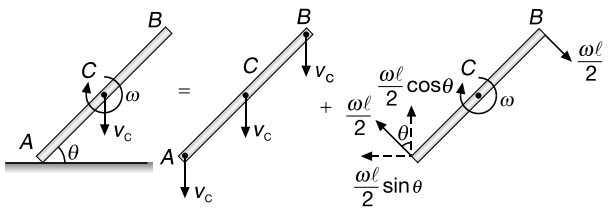
(b) $\theta = \omega_0 t + \frac{1}{2}(\pi \text{ rads}^{-2}) \times (16 \text{ s}^2) = 8\pi$ rad

(c) Let the number of turns be n

$\Rightarrow \quad n(2\pi)\text{rad} = 8\pi$ rad

$\Rightarrow \quad n = 4$

5. The motion of the rod can be considered as super position of pure translation and pure rotation.



As end A, the rod slide along horizontal surface, the component of its velocity normal to ground should be zero. So, we get

$$-v_C + \frac{\omega l}{2}\cos\theta = 0$$

$$\Rightarrow \quad v_C = \frac{\omega l}{2}\cos\theta$$

6. (a) The tangential acceleration is constant and given by

$$a_T = \alpha r = (60 \text{ rads}^{-2})(0.2 \text{ m}) = 12 \text{ ms}^{-2}$$

In order to calculate the radial acceleration, we first need to find the angular velocity at the given time. So,

$$\omega = \omega_0 + \alpha t = 0 + (60 \text{ rads}^{-2})(0.15 \text{ s})$$

$$\Rightarrow \quad \omega = 9 \text{ rads}^{-1}$$

Since $a_C = r\omega^2$, so

$$a_C = \omega^2 r = (81 \text{ rad}^2\text{s}^{-2})(0.2 \text{ m}) = 16.2 \text{ ms}^{-2}$$

The magnitude of the net linear acceleration is

$$a = \sqrt{a_C^2 + a_T^2} = 20.2 \text{ ms}^{-2}$$

(b) Since $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(60 \text{ rads}^{-2})(0.25 \text{ s})^2$

$$\Rightarrow \quad \theta = 1.88 \text{ rad}$$

$$\Rightarrow \quad n = \frac{\theta}{2\pi} = \frac{1.88}{2\pi} = 0.3 \text{ rev}$$

7. Given that $\omega_0 = 0.2$ rads^{-1}, $R = 10$ cm $= 0.1$ m. Since $\vec{r}$ rotates relative to O with constant angular velocity ω_0. So angular velocity relative to C is

$$\omega_c = 2\omega_0 = 0.4 \text{ rads}^{-1}$$

Modulus of velocity $|\vec{v}| = R\omega_c = (0.1)(0.4)$

$$\Rightarrow \quad |\vec{v}| = 0.04 \text{ ms}^{-1} = 4 \text{ cms}^{-1}$$

Modulus of total acceleration is

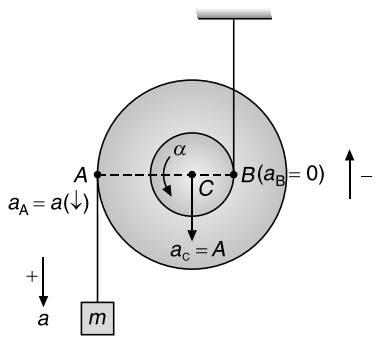
$$|\vec{a}| = R\omega_c^2$$

$$\Rightarrow \quad |\vec{a}| = (0.1)(0.4)^2 = 0.016 \text{ ms}^{-2}$$

$$\Rightarrow \quad |\vec{a}| = 1.6 \text{ cms}^{-2}$$

The direction of its total acceleration (centripetal acceleration) will be towards centre C.

8. Let the angular acceleration of the bobbin is α as shown in Figure.



Point B is directly connected with roof through string. It means net acceleration of point B should be zero.

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$$\vec{a}_B = \vec{a}_{BC} + \vec{a}_C$$

$$\Rightarrow \quad 0 = -\alpha r + A$$

$$\Rightarrow \quad \alpha = \frac{A}{r} \qquad \ldots(1)$$

Similarly, point A on the bobbin is directly connected with block. This means that the acceleration of point A should be same as the acceleration of the block. So, we have

$$\vec{a}_A = \vec{a}_{AC} + \vec{a}_C$$

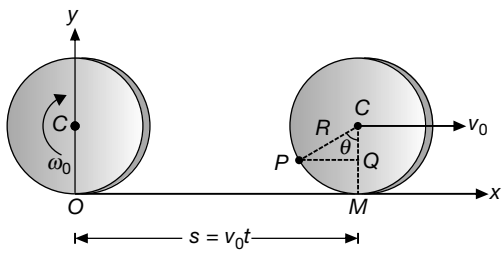
$$\Rightarrow \quad a = \alpha r + A \qquad \ldots(2)$$

From equations (1) and (2), we get

$$a = \left(\frac{A}{r}\right)R + A$$

$$\Rightarrow \quad \frac{A}{a} = \frac{r}{R+r}$$

9. At time t the bottommost point will rotate an angle $\theta = \omega_0 t$ with respect to the centre of the disc C. The centre C will travel a distance $s = v_0 t$.



From the figure, $PQ = R\sin\theta = R\sin(\omega_0 t)$

and $CQ = R\cos\theta = R\cos(\omega_0 t)$

Coordinates of point P at time t are

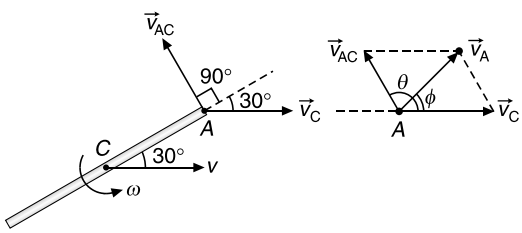
$$x = OM - PQ = v_0 t - R\sin\omega_0 t$$

and $y = CM - CQ = R - R\cos\omega_0 t$

So, $P(x, y) \equiv \left(v_0 t - R\sin(\omega_0 t),\ R - R\cos(\omega_0 t)\right)$

10. The point A is translating as well as rotating, so the velocity of the end A is the combined effect of rotation and translation. The combined velocity of end A is given by

$$\vec{v}_A = \vec{v}_{AC} + \vec{v}_C$$



where, $\omega = \dfrac{2v}{l}$, $v_C = v_{cm} = v$ and

$$v_{AC} = \frac{l\omega}{2} = \left(\frac{l}{2}\right)\left(\frac{2v}{l}\right) = v$$

So, the resultant velocity of A is given by

$$v_A = \sqrt{v_{AC}^2 + v_C^2 + 2v_{AC}v_C\cos\theta}$$

where, $\theta = 90° + 30° = 120°$

$$\Rightarrow \quad v_A = \sqrt{v^2 + v^2 + 2v^2\cos 120°} = v$$

If $\vec{v}_A$ makes angle ϕ with the direction of $\vec{v}_C$, then

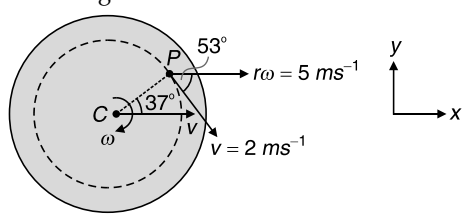
$$\phi = \tan^{-1}\left(\frac{v_{AC}\sin\theta}{v_{AC}\cos\theta + v_C}\right)$$

$$\Rightarrow \quad \phi = \tan^{-1}\left(\frac{v\sin 120°}{v\cos 120° + v}\right) = \frac{\pi}{3}$$

11. For particle P, $r = CP = 1$ m

$$\Rightarrow \quad r\omega = (1)(5) = 5\ \text{ms}^{-1}$$

Net velocity of P is the vector sum of v and $r\omega$ as shown in Figure.



$$\vec{v}_P = 2\hat{i} + \left(5\cos 53°\hat{i} - 5\sin 53°\hat{j}\right)$$

$$\Rightarrow \quad \vec{v}_P = 2\hat{i} + 3\hat{i} - 4\hat{j} = \left(5\hat{i} - 4\hat{j}\right)\ \text{ms}^{-1}$$

Test Your Concepts-III (Based on Instantaneous Axis of Rotation, Pure Rolling and Conservation of Energy)

1. For pure rolling, we have

$$\frac{K_R}{K_T} = \frac{k^2}{r^2} = \frac{2}{5}$$

So, translational kinetic energy at topmost point is

$$K_T = \frac{5}{7}mg(h - 2R)$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{5}{7}mg(h - 2R)$$

$$\Rightarrow \quad v^2 = \frac{10}{7}g(h - 2R) \qquad \ldots(1)$$

For the ball not to leave the track, we have

$$mg = \frac{mv^2}{R} \qquad \ldots(2)$$

Solving, equation (1) and (2), we have

$$h = 2.7R$$

In case of frictionless track, there will be no rotational kinetic energy, so

$$\frac{1}{2}mv^2 = mg(h-2R)$$

$$\Rightarrow \quad v^2 = 2g(h-2R) \qquad \ldots(3)$$

Solving equation (2) and (3), we get

$$h = 2.5R$$

2. If m be the mass of the ball, then total kinetic energy at B is $E_B = mgh$

The ratio of rotational to translational kinetic energy would be,

$$\frac{K_R}{K_T} = \frac{r^2}{k^2} = \frac{2}{5}$$

$$\Rightarrow \quad K_R = \frac{2}{7}mgh \text{ and } K_T = \frac{5}{7}mgh$$

In portion BC, friction is absent, so rotational kinetic energy will remain constant and translational kinetic energy will get converted to potential energy. So, if H be the height to which ball climbs in BC, then

$$mgH = K_T$$

$$\Rightarrow \quad mgH = \frac{5}{7}mgh$$

$$\Rightarrow \quad H = \frac{5}{7}h$$

3. Since speed of rod is v, so speed of centre of mass of the cylinder and the sphere is $v/2$.
For a body rolling without slipping, the kinetic energy is

$$K = K_T + K_R = \frac{1}{2}mv_{cm}^2\left(1+\frac{R^2}{K^2}\right)$$

So, for the rolling sphere and the cylinder, we have

$$K_{cylinder} = \frac{1}{2}m_2\left(\frac{v}{2}\right)^2\left(1+\frac{1}{2}\right) = \frac{3}{16}m_2v^2$$

and $K_{sphere} = \frac{1}{2}m_1\left(\frac{v}{2}\right)^2\left(1+\frac{2}{5}\right) = \frac{7}{40}m_1v^2$

The motion of the rod is purely translational, so kinetic energy of rod is

$$K_{rod} = \frac{1}{2}mv^2$$

The total kinetic energy of the system is

$$K_{system} = K_{rod} + K_{cylinder} + K_{sphere}$$

$$\Rightarrow \quad K_{system} = \frac{1}{2}mv^2 + \frac{3}{16}m_2v^2 + \frac{7}{40}m_1v^2$$

$$\Rightarrow \quad K_{system} = \frac{1}{2}mv^2 + \frac{3}{16}(4m)v^2 + \frac{7}{40}(5m)v^2$$

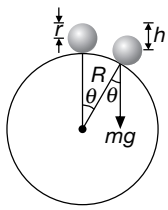
$$\Rightarrow \quad K_{system} = \frac{1}{2}mv^2 + \frac{3}{4}mv^2 + \frac{7}{8}mv^2$$

$$\Rightarrow \quad K_{system} = \frac{1}{2}mv^2\left(1+\frac{3}{2}+\frac{7}{4}\right) = \frac{17}{8}mv^2$$

4. The equation of motion for the centre of the sphere at the moment of breaking off, $N = 0$ is

$$\frac{mv^2}{R+r} = mg\cos\theta \qquad \ldots(1)$$

where v is the speed of the centre of the sphere at that moment and θ is the corresponding angle. The speed v can be found by using the Law of Conservation of Energy, according to which



$$mgh = \frac{mv^2}{2} + \frac{I\omega^2}{2}$$

where $I = \frac{2}{5}mr^2$, $v = r\omega$

and $h = (R+r)(1-\cos\theta)$

From these equations we get

$$\omega = \sqrt{\frac{10g(R+r)}{17r^2}}$$

5. When the bead comes to its lowest position, then change in gravitational potential energy of the bead is

$$\Delta U = -mgh = -mg(2R) = -2mgR \qquad \ldots(1)$$

Similarly, change in kinetic energy will take place both for the bead and the disc, so we have

$$\Delta K = \Delta K_{bead} + \Delta K_{disc}$$

$$\Rightarrow \quad \Delta K = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\omega^2$$

Since $\omega = \frac{v}{R}$

$$\Rightarrow \quad \Delta K = \frac{mv^2}{2} + \frac{mv^2}{4}$$

$$\Rightarrow \quad \Delta K = \frac{3}{4}mv^2 \qquad \ldots(2)$$

By Law of Conservation of Energy, we have

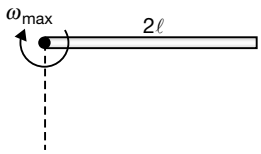
$$\Delta U + \Delta K = 0$$

$$\Rightarrow \quad -2mgR + \frac{3}{4}mv^2 = 0$$

$$\Rightarrow \quad v = \sqrt{\frac{8}{3}gR}$$

6. Angular velocity will be maximum, when the rod is in the vertical position. Then by Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in Gravitational}\\ \text{Potential Energy of}\\ \text{Centre of Mass of}\\ \text{the Rod}\end{pmatrix} = \begin{pmatrix}\text{Gain in Rotational}\\ \text{Kinetic Energy}\\ \text{of the}\\ \text{Rod}\end{pmatrix}$$



$$mgh = \frac{1}{2}\mathrm{I}\omega_{\max}^2$$

where, $h = \dfrac{2\ell}{2} = \ell$

$$\Rightarrow \quad mg(\ell) = \frac{1}{2}\frac{(m)(2\ell)^2}{3}\omega_{\max}^2$$

$$\Rightarrow \quad \omega_{\max} = \sqrt{\frac{3g}{2\ell}}$$

7. By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in Gravitational}\\ \text{Potential Energy}\\ \text{of the point mass}\end{pmatrix} = \begin{pmatrix}\text{Gain in Rotational}\\ \text{Kinetic Energy of}\\ \textit{point mass} + \textit{Disc}\end{pmatrix}$$

$$\Rightarrow \quad mg(R) = \frac{1}{2}\left(\frac{1}{2}MR^2 + mR^2\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{4mg}{(2m+M)R}}$$

8. If v_{cm} be the velocity of the centre of mass of the rod, then for end A we have

$$v_A = v = \left(\frac{l}{2}\right)\omega - v_{cm} \qquad \ldots(1)$$

Similarly, for end B, we have

$$v_B = 2v = v_{cm} + \left(\frac{l}{2}\right)\omega \qquad \ldots(2)$$

Solving Equations (1) and (2), we have

$$\omega = \frac{3v}{l} \text{ and } v_C = \frac{v}{2}$$

Angular velocity of the rod can also be directly calculated as

$$\omega = \frac{v_{\text{rel}}}{r_\perp} = \frac{3v}{l}$$

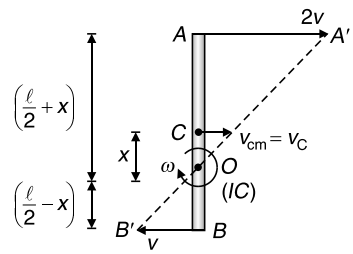
Since, we know that the kinetic energy

$$K = K_T + K_R = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

$$\Rightarrow \quad K = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}\left(\frac{ml^2}{12}\right)\left(\frac{3v}{l}\right)^2$$

$$\Rightarrow \quad K = \frac{1}{2}mv^2$$

Alternatively, we can calculate angular velocity of the rod and velocity of centre of mass by IC method. Let the IC of the rod be at a distance x from the centre of mass of the rod as shown in Figure.



Since the rod will be in pure rotation about the IC so for end A, we have

$$v = \left(\frac{l}{2} - x\right)\omega \qquad \ldots(3)$$

and for end B, we have

$$2v = \left(\frac{l}{2} + x\right)\omega \qquad \ldots(4)$$

From equations (3) and (4), we get

$$x = \frac{l}{6} \text{ and } \omega = \frac{3v}{l}$$

Also, the velocity of centre of mass is given by

$$v_C = x\omega = \left(\frac{l}{6}\right)\left(\frac{3v}{l}\right) = \frac{v}{2}$$

Rotational kinetic energy of the rod is

$$K_R = \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}\left(\frac{ml^2}{12}\right)\left(\frac{3v}{l}\right)^2 = \frac{3}{8}mv^2$$

Translational kinetic energy of the rod is

$$K_T = \frac{1}{2}mv_{cm}^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{8}mv^2$$

Total kinetic energy of the rod is

$$K = K_R + K_T$$

$$\Rightarrow \quad K = \frac{3}{8}mv^2 + \frac{1}{8}mv^2 = \frac{1}{2}mv^2$$

Total kinetic energy of the rod also equals the rotational kinetic energy of the rod about the instantaneous centre of zero velocity (IC) and is given by

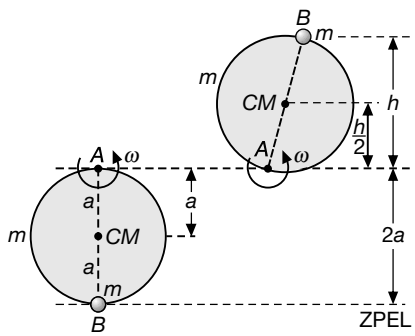
$$K = \frac{1}{2} I_{IC} \omega^2 = \frac{1}{2}\left(I_{cm} + mx^2\right)\omega^2$$

$$\Rightarrow \quad K = \frac{1}{2}\left[\frac{ml^2}{12} + m\left(\frac{l}{6}\right)^2\right]\left(\frac{3v}{l}\right)^2$$

$$\Rightarrow \quad K = \frac{1}{2}mv^2$$

9. The initial angular velocity imparted to the system is

$$\omega = \frac{v}{2a} = \frac{3\sqrt{ga}}{2a} = \frac{3}{2}\sqrt{\frac{g}{a}}$$



By Law of Conservation of Mechanical Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{RKE of}\\ \text{System}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{GPE of}\\ \text{Particle}\end{pmatrix} + \begin{pmatrix}\text{Gain in}\\ \text{GPE of}\\ \text{CM of Ring}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}\left(m(2a)^2 + 2ma^2\right)\left(\frac{3}{2}\sqrt{\frac{g}{a}}\right)^2 = mg(2a+h) + mg\left(a + \frac{h}{2}\right)$$

$$\Rightarrow \quad \frac{27}{4}mga = 3mga + \frac{3mgh}{2}$$

$$\Rightarrow \quad h = 2.5a$$

10. $mgh = K_R + K_T = \frac{3}{4}mv^2$

where $h = s\sin\theta$

$$\Rightarrow \quad gs\sin\theta = \frac{3}{4}v^2$$

$$\Rightarrow \quad s = \frac{3v^2}{4g\sin\theta} = \frac{(3)(4)}{(4)(10)\left(\frac{1}{2}\right)} = 0.6 \text{ m}$$

11. By Law of Conservation of Energy, we get

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega_0^2$$

Since $v = R\omega_0$ and $I = \frac{1}{2}mR^2$, so we get

$$kx^2 = m\left(R^2\omega_0^2\right) + \left(\frac{1}{2}mR^2\right)\omega_0^2$$

$$\Rightarrow \quad kx^2 = \frac{3}{2}mR^2\omega_0^2$$

$$\Rightarrow \quad x = R\omega_0\sqrt{\frac{3m}{2k}}$$

12. In rolling without sliding on a stationary ground, work done by friction is zero. Hence

$$\begin{pmatrix}\text{Work done by}\\ \text{Applied Force}\end{pmatrix} = \begin{pmatrix}\text{Change in KE of}\\ \text{Cradle and Disks}\end{pmatrix}$$

$$\Rightarrow \quad (30)(0.25) = \frac{1}{2}(9)(v^2) + 2\left(\frac{1}{2}(6)(v^2) + \frac{1}{2}\left(\frac{1}{2}\times 6\times r^2\right)\frac{v^2}{r^2}\right)$$

$$\Rightarrow \quad 7.5 = 13.5v^2$$

$$\Rightarrow \quad v = 0.745 \text{ ms}^{-1}$$

Test Your Concepts-IV (Based on Torque and Applications)

1. Decrease in the height of centre of mass of the rod

$$h = \frac{\ell}{2}(\cos 37° - \cos 60°)$$

$$\Rightarrow \quad h = \frac{\ell}{2}(0.8 - 0.5) = 0.15\ell$$

By Law of Conservation of Mechanical Energy, we get

$$mgh = \frac{1}{2}I_O\omega^2 = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

$$\Rightarrow \quad 0.15\ell mg = \left(\frac{m\ell^2}{6}\right)\omega^2$$

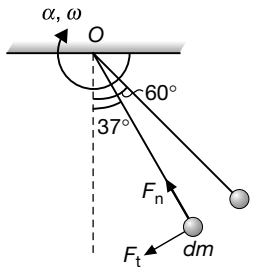
$$\Rightarrow \quad a_n = \ell\omega^2 = 0.9g \qquad \ldots(1)$$

Angular acceleration,

$$\alpha = \frac{\tau_O}{I_O} = \frac{\left(mg\frac{\ell}{2}\right)\sin(37°)}{\frac{1}{3}m\ell^2}$$

$$\Rightarrow \quad a_t = \ell\alpha = 0.9g \qquad \ldots(2)$$

CHAPTER 3



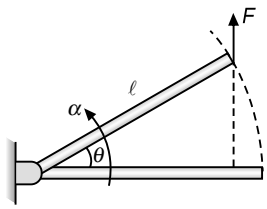
Normal force, $F_n = (dm)\ell\omega^2 = 0.9gdm = 9dm$

Tangential force, $F_t = (dm)\ell\alpha = 0.9gdm = 9dm$

So, net force, $F = \sqrt{F_n^2 + F_t^2} = 9\sqrt{2}dm$

2. $\tau = F\ell\cos\theta$

Since, $\alpha = \dfrac{\tau}{I} = \dfrac{F\ell\cos\theta}{m\ell^2/3} = \dfrac{3F\cos\theta}{m\ell}$



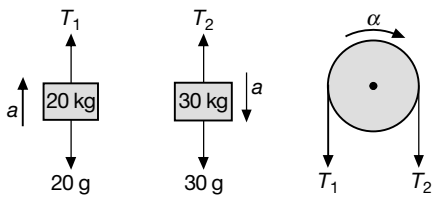
$$\Rightarrow \quad \omega\frac{d\omega}{d\theta} = \frac{3F\cos\theta}{m\ell}$$

$$\Rightarrow \quad \omega d\omega = \frac{3F\cos\theta}{m\ell}d\theta$$

$$\Rightarrow \quad \int_0^{\omega}\omega d\omega = \frac{3F}{m\ell}\int_0^{\phi}\cos\theta d\theta$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6F\sin\phi}{m\ell}}$$

3. The free body diagrams of the bodies and the pulley are shown in Figure, then



for 20 kg mass, we get

$$T_1 - 20g = 20a \qquad \ldots(1)$$

for 30 kg mass, we get

$$30g - T_2 = 30a \qquad \ldots(2)$$

for pulley, we get

$$\alpha = \frac{(T_2 - T_1)R}{\frac{1}{2}mR^2} = \frac{2(T_2 - T_1)}{mR}$$

$$\Rightarrow \quad \alpha = \frac{2(T_2 - T_1)}{5R} = \frac{0.4(T_2 - T_1)}{R} \qquad \ldots(3)$$

For no slipping, $a = R\alpha$ $\qquad \ldots(4)$

Solving the above equations, we have

$a = 1.87\ \text{ms}^{-2}$, $T_1 = 233.5$ N and

$T_2 = 238.2$ N

$$\Rightarrow \quad v = \sqrt{2as} = \sqrt{2\times 1.87\times 2} = 2.73\ \text{ms}^{-1}$$

$$\Rightarrow \quad \omega = \frac{v}{R} = \frac{2.73}{0.1} = 27.3\ \text{rads}^{-1}$$

$$\Rightarrow \quad t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 2}{1.87}} = 1.47\ \text{s}$$

4. At angle θ, we have by Law of Conservation of Energy

$$\frac{1}{2}\text{I}\omega^2 = mg\frac{\ell}{2}(1-\cos\theta)$$

$$\Rightarrow \quad \frac{1}{2}\frac{m\ell^2}{3}\omega^2 = \frac{mg\ell}{2}(1-\cos\theta)$$

$$\Rightarrow \quad \omega^2 = \frac{3g}{\ell}(1-\cos\theta) \qquad \ldots(1)$$

Since $\alpha = \dfrac{\tau}{I}$

$$\Rightarrow \quad \alpha = \frac{mg\left(\frac{\ell}{2}\sin\theta\right)}{\frac{m\ell^2}{3}} = \frac{3g\sin\theta}{2\ell} \qquad \ldots(2)$$

Since, $a_n = r\omega^2$

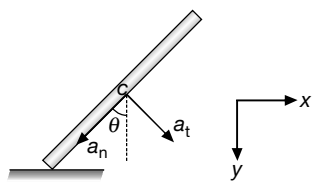
$$\Rightarrow \quad a_n = \frac{\ell}{2}\omega^2 = \frac{3g}{2}(1-\cos\theta)$$

and $a_t = r\alpha = \dfrac{\ell}{2}\alpha = \dfrac{3}{4}g\sin\theta$

$$f = ma_x = m(a_t\cos\theta - a_n\sin\theta)$$

$$\Rightarrow \quad f = m\left(\frac{3}{4}g\sin\theta\cos\theta - \frac{3g\sin\theta}{2}(1-\cos\theta)\right)$$

$$\Rightarrow \quad f = \frac{3}{2}mg\sin\theta\left(\frac{3}{2}\cos\theta - 1\right)$$



Further, $mg - N = ma_y$

$\Rightarrow \quad N = m(g - a_y)$

$\Rightarrow \quad N = m\left[g - (a_t \sin\theta + a_n \cos\theta)\right]$

$\Rightarrow \quad N = m\left[g - \frac{3}{4}g\sin^2\theta - \frac{3g\cos\theta}{2}(1-\cos\theta)\right]$

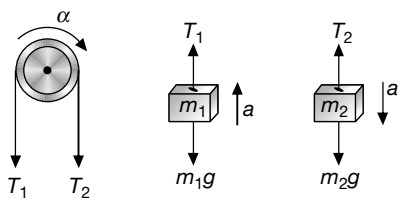
$\Rightarrow \quad N = \frac{mg}{4}(4 - 3\sin^2\theta - 6\cos\theta + 6\cos^2\theta)$

$\Rightarrow \quad N = \frac{mg}{4}(1 - 3\cos\theta)^2$

So, $N = 0$ at $\theta = \cos^{-1}\left(\frac{1}{3}\right)$

Hence, it will certainly slip beyond that.

5. Let α be the angular acceleration of the cylinder and a be the linear acceleration of two bodies



For mass m_1

$$T_1 - m_1 g = m_1 a \qquad \ldots(1)$$

For mass m_2

$$m_2 g - T_2 = m_2 a \qquad \ldots(2)$$

For cylinder

$$\alpha = \frac{(T_2 - T_1)R}{\frac{1}{2}mR^2} \qquad \ldots(3)$$

For no slipping condition, we have

$$a = R\alpha \qquad \ldots(4)$$

Solving these equations, we get

$$\alpha = \frac{2(m_2 - m_1)g}{(2m_1 + 2m_2 + m)R}$$

and $\frac{T_1}{T_2} = \frac{m_1(m + 4m_2)}{m_2(m + 4m_1)}$

6. (a) $F\cos\theta = Mg\sin\theta$

$\Rightarrow \quad F = Mg\tan\theta$

(b) $f = 0$

7. $\alpha = \frac{2mga}{\frac{1}{2}ma^2} = \frac{4g}{a}$

Since $\ell = r\theta$

$\Rightarrow \quad \theta = \frac{4a}{a} = \frac{1}{2}\alpha t^2 = \frac{2g}{a}t^2 \qquad \{\because \ \ell = 4a\}$

$\Rightarrow \quad t = \sqrt{\frac{2a}{g}}$

Since, $\omega = \alpha t = \frac{4g}{a}\sqrt{\frac{2a}{g}}$

$\Rightarrow \quad \omega = 4\sqrt{\frac{2g}{a}}$

8. Equations of motion are,

$$m_2 g\sin\theta - T = m_2 a \qquad \ldots(1)$$

$$\alpha = \frac{TR}{\frac{1}{2}m_1 R^2} = \frac{2T}{m_1 R} \qquad \ldots(2)$$

and $a = R\alpha$ $\qquad \ldots(3)$

Solving these equations, we get

$$a = \left(\frac{2m_2 g\sin\theta}{2m_2 + m_1}\right) \text{ and } T = \left(\frac{m_1 m_2 g\sin\theta}{2m_2 + m_1}\right)$$

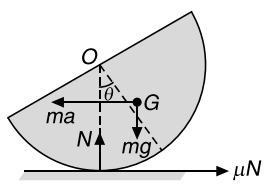
Total energy of system when m_2 is at a height h is mgh. When m_2 reaches the bottom total energy, due to energy conservation will still be mgh.

However, this mgh gets distributed to pulley as rotational kinetic energy $\left(=\frac{1}{2}I\omega^2\right)$ and to mass m_2 as translation kinetic energy $\left(=\frac{1}{2}m_2 v^2\right)$, so speed at bottom is

$$v = \sqrt{2as} = \sqrt{\left(\frac{4m_2 g\sin\theta}{2m_2 + m_1}\right)\left(\frac{h}{\sin\theta}\right)}$$

$\Rightarrow \quad v = \sqrt{\frac{2gh}{1 + (m_1/2m_2)}}$

9. Let a be the acceleration of truck. The free body diagram of hemisphere in the frame of reference of the truck is shown in Figure.



The centre of gravity is at G such that

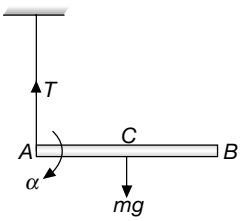
$$OG = \frac{3r}{8}$$

The hemisphere is in equilibrium in this frame of reference. Hence, net moment of all the forces about bottommost point must be zero.

$$\Rightarrow \quad mg\left(\frac{3r}{8}\sin\theta\right) = ma\left(r - \frac{3r}{8}\cos\theta\right)$$

$$\Rightarrow \quad a = \frac{3g\sin\theta}{8 - 3\cos\theta}$$

10. $\alpha = \dfrac{\tau}{I} = \dfrac{mg\left(\frac{\ell}{2}\right)}{\frac{m\ell^2}{3}} = \dfrac{3}{2}\dfrac{g}{\ell}$



(a) $a_B = \ell\alpha = \dfrac{3g}{2}$

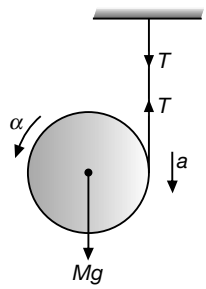
(b) $a_C = \dfrac{\ell}{2}\alpha = \dfrac{3g}{4}$

(c) Since $mg - T = ma_C$

$$\Rightarrow \quad mg - T = m\left(\frac{3g}{4}\right)$$

$$\Rightarrow \quad T = \frac{mg}{4}$$

11. From the free body diagram, we get



$$a = \frac{Mg - T}{M} \qquad \ldots(1)$$

$$\alpha = \frac{TR}{\frac{1}{2}MR^2} = \frac{2T}{MR} \qquad \ldots(2)$$

$$a = R\alpha \qquad \ldots(3)$$

Solving these three equations, we get

$$a = \frac{2g}{3} \text{ and } T = \frac{Mg}{3}$$

12. $\alpha = \dfrac{\tau}{I} = \dfrac{4mga}{\frac{1}{2}ma^2} = \dfrac{8g}{a}$

Since $\omega = \omega_0 + \alpha t$

$$\Rightarrow \quad \omega = \alpha t = \frac{32g}{a} \qquad \{\because\ \omega_0 = 0\}$$

13. (a) $W = \Delta K$

$$\Rightarrow \quad \tau\theta = \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad (40 \times 0.25)\left(\frac{5}{0.25}\right) = \frac{1}{2} \times 4 \times \omega^2$$

$$\Rightarrow \quad 3\omega^2 = 200$$

$$\Rightarrow \quad \omega = 8.16 \text{ rads}^{-1}$$

(b) Mass of the block is $m = \dfrac{40}{10} = 4$ kg

By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{GPE of}\\ \text{4 kg mass}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{KE of}\\ \text{4 kg mass}\end{pmatrix} + \begin{pmatrix}\text{Gain in}\\ \text{RKE}\\ \text{of pulley}\end{pmatrix}$$

$$\Rightarrow \quad mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad mgh = \frac{1}{2}m(R\omega)^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad (4)(10)(5) = \frac{1}{2}(4)\left(\frac{1}{16}\right)\omega^2 + \frac{1}{2}(6)\omega^2$$

$$\Rightarrow \quad 200 = \frac{\omega^2}{8} + 3\omega^2$$

$$\Rightarrow \quad \omega^2 = 64$$

$$\Rightarrow \quad \omega = 8 \text{ rads}^{-1}$$

The difference in answers is due to the fact that in the first case $T = 40$ N and in the second case $T < 40$ N.

14. Equations of motion for the discs are,

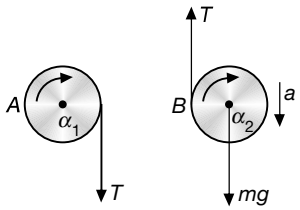
$$a = \frac{mg - T}{m} \qquad \ldots(1)$$

$$\alpha_2 = \frac{TR}{mR^2/2} = \frac{2T}{mR} \qquad \ldots(2)$$

Similarly, we get

$$\alpha_1 = \frac{TR}{mR^2/2} = \frac{2T}{mR} \qquad \ldots(3)$$

$$\Rightarrow \quad \alpha_1 = \alpha_2$$



For no slipping, $a = R\alpha_2$ ...(4)

Further, $a - R\alpha_2 = R\alpha_1$ ...(5)

Solving these equations, we get

$$T = \frac{mg}{5},\ a = \frac{4g}{5} \text{ and } \alpha_2 = \frac{2g}{5R}$$

15. By Law of Conservation of Mechanical Energy,

$$\begin{pmatrix}\text{Loss in}\\ \text{GPE of}\\ \text{CM of}\\ \text{Ring}\end{pmatrix} + \begin{pmatrix}\text{Loss in}\\ \text{GPE of}\\ \text{Particle}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{RKE of}\\ \text{Ring and}\\ \text{Particle}\end{pmatrix}$$

$$\Rightarrow \quad mgr\sin 60° + mg(2r\sin 60°) = \frac{1}{2}\left(\frac{3mr^2}{2} + m(2r)^2\right)\omega^2$$

$$\Rightarrow \quad \frac{3\sqrt{3}mgr}{2} = \frac{11}{4}mr^2\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{6g\sqrt{3}}{11r}}$$

Since, $\tau = mgr\cos(60°) + mg2r\cos(60°)$

$$\Rightarrow \quad \tau = \frac{3}{2}mgr$$

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{\frac{3}{2}mgr}{\left(\frac{3mr^2}{2} + m(2r)^2\right)}$$

$$\Rightarrow \quad \alpha = \frac{3mgr/2}{11mr^2/2} = \frac{3g}{11r}$$

16. Let us calculated the torque about O, so

$$\tau_0 = I_0\alpha$$

$$\Rightarrow \quad Mgx = \left(\frac{M\ell^2}{12} + Mx^2\right)\alpha$$

$$\Rightarrow \quad \alpha = \frac{gx}{k^2 + x^2}, \text{ where } k^2 = \frac{\ell^2}{12}$$

For α to be maximum, we have

$$\frac{d\alpha}{dx} = 0$$

$$\Rightarrow \quad x(-1)(k^2 + x^2)^{-2}(2x) + (k^2 + x^2)^{-1}(1) = 0$$

$$\Rightarrow \quad \frac{2x^2}{k^2 + x^2} = 1$$

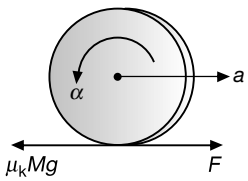
$$\Rightarrow \quad 2x^2 = \frac{\ell^2}{12} + x^2 \qquad \left\{\because k^2 = \frac{\ell^2}{12}\right\}$$

$$\Rightarrow \quad x = \frac{\ell}{2\sqrt{3}}$$

$$\Rightarrow \quad \alpha_{max} = \alpha\big|_{x=\frac{\ell}{2\sqrt{3}}} = \frac{g\sqrt{3}}{\ell}$$

Test Your Concepts-V (Based on Uniform and Accelerated Pure Rolling)

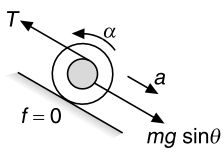
1.



$$a = \frac{F - \mu_k Mg}{M} = \frac{F}{M} - \mu_k g$$

and $\alpha = \frac{\tau}{I} = \frac{(F - \mu_k Mg)R}{\frac{1}{2}MR^2} = \frac{2}{R}\left(\frac{F}{M} - \mu_k g\right)$

2.



$$a = \frac{mg\sin\theta - T}{m} \quad ...(1)$$

$$\alpha = \frac{Tr}{I} \quad ...(2)$$

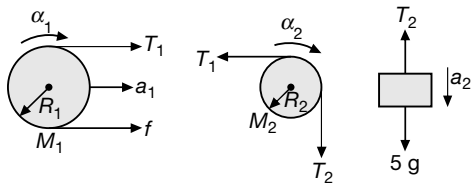
$$a = r\alpha \quad ...(3)$$

Solving these three equations, we get

$$a = \frac{g\sin\theta}{1 + \frac{I}{mr^2}}$$

NOTE: Compare the above result with rolling on a rough ground and think what are the similarities between the two.

3. Let us first draw the free body diagrams for cylinder, pulley and mass. Then



CHAPTER 3

for 5 kg mass, $5g - T_2 = 5a_2$ ...(1)

for pulley, $\dfrac{(T_2 - T_1)R_2}{\frac{1}{2}M_2R_2^2} = \alpha_2$ ...(2)

for cylinder, $T_1 + f = M_1a_1$ ...(3)

$$\frac{(T_1 - f)R_1}{\frac{1}{2}M_1R_1^2} = \alpha_1 \quad \text{...(4)}$$

Also, $a_1 = R_1\alpha_1$ ...(5)

$a_2 = R_2\alpha_2 = a_1 + R_1\alpha_1$ ...(6)

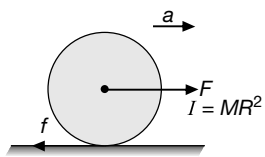
Solving above equations, we get

$$a_2 = \frac{4}{11}g$$

$$\Rightarrow \quad v = a_2t = \frac{4gt}{11}$$

4. For thin walled lawn roller, we have

$$I = MR^2$$



For pure rolling to take place, we have

$$a = R\alpha$$

Also $F - f = Ma$

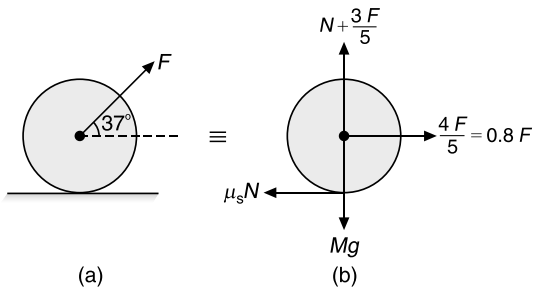
where $a = R\alpha = R\left(\dfrac{\tau}{I}\right)$

$$\Rightarrow \quad a = R\left(\frac{fR}{MR^2}\right) = \frac{f}{M}$$

$$\Rightarrow \quad f = \frac{F}{2}$$

and $a = \dfrac{F - f}{M} = \dfrac{F}{2M}$

5. The free body diagram for cylindrical wheel is shown in Figure (b).



$$N = Mg - \frac{3F}{5} = Mg - 0.6F$$

For pure rolling $a = R\alpha$

$$\Rightarrow \quad \frac{0.8F - \mu_sN}{M} = \frac{(R)(\mu_sN)(R)}{\frac{1}{2}MR^2} = \frac{2\mu_sN}{M}$$

$$\Rightarrow \quad 0.8F = 3(0.4)(Mg - 0.6F)$$

$$\Rightarrow \quad F = 0.79Mg$$

Therefore, maximum value of F is $0.79Mg$

6. $\dfrac{I}{mR^2} = \dfrac{2}{5} = 0.4$ for sphere

$\dfrac{I}{mR^2} = \dfrac{1}{2} = 0.5$ for disc and $= 1$ for hoop

$$s = \frac{2}{\sin(30°)} = 4 \text{ m}$$

For sphere, we have

$$a = \frac{g\sin\theta}{1 + \dfrac{I}{mR^2}} = \frac{(9.8)\left(\frac{1}{2}\right)}{1 + 0.4} = 3.5 \text{ ms}^{-1}$$

$$\Rightarrow \quad v = \sqrt{2as} = \sqrt{2 \times 3.5 \times 4} = 5.29 \text{ ms}^{-1}$$

$$f = \frac{mg\sin\theta}{1 + \dfrac{mR^2}{I}} = \frac{3 \times 9.8 \times \frac{1}{2}}{1 + \left(\frac{1}{0.4}\right)} = 4.2 \text{ N}$$

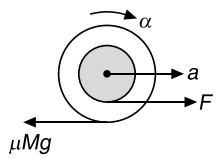
$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 4}{3.5}} = 1.51 \text{ s}$$

Similarly, the values for disc and hoop can be obtained.

7. Force of friction in this case will be backwards. For pure rolling,

$$a = R\alpha$$

$$\Rightarrow \quad \frac{1}{M}(F - \mu Mg) = R\left(\frac{\mu MgR - Fb}{\frac{1}{2}MR^2}\right)$$



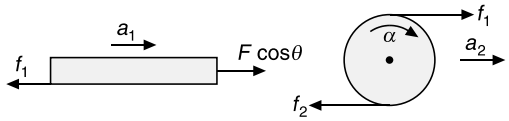
Solving this equation, we get

$$F = \frac{3\mu MgR}{R + 2b}$$

Thus, maximum value of F can be

$$F_{max} = \frac{3\mu MgR}{R + 2b}$$

8. The free body diagrams for plank and cylinder are shown in Figure. Also



for plank, we have

$$F\cos\theta - f_1 = ma_1 \qquad \ldots(1)$$

for cylinder, we have

$$f_1 - f_2 = Ma_2 \qquad \ldots(2)$$

$$\alpha = \frac{(f_1 + f_2)R}{\frac{1}{2}MR^2} \qquad \ldots(3)$$

$$a_2 = R\alpha \qquad \ldots(4)$$

and $a_1 = a_2 + R\alpha \qquad \ldots(5)$

Solving these equations, we get

$$a_2 = \frac{4F\cos\theta}{3M+8m},\ f_1 = \frac{3MF\cos\theta}{3M+8m}$$

and $f_2 = \frac{MF\cos\theta}{3M+8m}$

9. Equation of motion for the block is

$$mg\sin\theta - T = ma$$

$$\Rightarrow \quad (1)(9.8)\sin 30° - T = (1)a$$

$$\Rightarrow \quad T + a = 4.9 \qquad \ldots(1)$$

Equation of motion for disk is

$$\frac{TR}{\frac{1}{2}MR^2} = \alpha$$

$$\Rightarrow \quad \alpha = \frac{2T}{MR}$$

For no slipping, we have

$$R\alpha = a = \frac{2T}{M}$$

$$\Rightarrow \quad a = \frac{2T}{5}$$

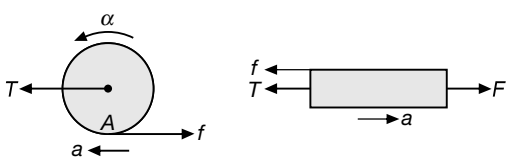
$$\Rightarrow \quad T = 2.5a$$

Substituting in equation (1), we get

$$3.5a = 4.9$$

$$\Rightarrow \quad a = 1.4\ \text{ms}^{-2}$$

10. Free body diagram for cylinder and plank are shown below, then



for plank, we have

$$F - T - f = Ma \qquad \ldots(1)$$

for cylinder, we have

$$T - f = ma \qquad \ldots(2)$$

$$\alpha = \frac{fR}{\frac{1}{2}mR^2} = \frac{2f}{mR} \qquad \ldots(3)$$

for no slipping at A, we have

$$R\alpha - a = a$$

$$\Rightarrow \quad R\alpha = 2a \qquad \ldots(4)$$

Solving these equations, we get

$$a = \frac{F}{3m+M}$$

11. At P, we have

$$h = (R-r)(1-\cos\theta) \qquad \ldots(1)$$

By Law of Conservation of Energy, we have

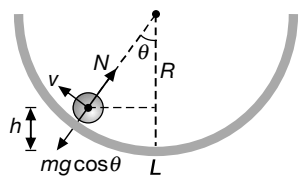
$$\begin{pmatrix}\text{Loss in KE of}\\ \text{Rolling}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{GPE}\end{pmatrix}$$

$$\Rightarrow \quad \left(\frac{1}{2}mv_0^2 - \frac{1}{2}mv^2\right)\left(\frac{k^2}{R^2}\right) = mgh$$

$$\Rightarrow \quad \left(\frac{1}{2}mv_0^2 - \frac{1}{2}mv^2\right)\left(1+\frac{2}{5}\right) = mgh$$

$$\Rightarrow \quad \frac{1}{2}mv_0^2 - \frac{1}{2}mv^2 = \frac{5}{7}mgh$$

$$\Rightarrow \quad v^2 = v_0^2 - \frac{10}{7}mgh$$



Equation of motion at angle θ is,

$$N - mg\cos\theta = \frac{mv^2}{(R-r)}$$

$$\Rightarrow \quad N = \frac{m}{(R-r)}\left(v_0^2 - \frac{10}{7}gh\right) + mg\cos\theta$$

Substitute value of h from (1), we get

$$N = \left(\frac{m}{R-r}\right)\left(v_0^2 - \frac{10g}{7}(R-r)(1-\cos\theta)\right) + mg\cos\theta$$

$$\Rightarrow \quad F_n = N = \frac{mg}{7}(17\cos\theta - 10) + \frac{mv_0^2}{(R-r)}$$

Tangential force is the force of friction given by

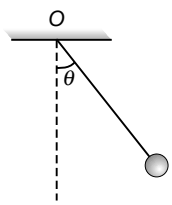
$$f=\frac{mg\sin\theta}{1+\left(k^2/r^2\right)}=\frac{mg\sin\theta}{1+(5/2)}$$

$$\Rightarrow \quad F_t=f=\frac{2}{7}mg\sin\theta$$

Test Your Concepts-VI (Based on Angular Momentum and its Conservation)

1. Let v_0 be the initial velocity and v its velocity in the position when the thread forms an angle $\frac{\pi}{2}$ with the vertical. The ball experiences two forces, the gravitational force and the tension of the thread. Moment of these forces is zero relative to the vertical axis passing through the point O. Consequently, the angular momentum of the ball relative to the given axis is constant. So, we have

$$mv_0\ell\sin\theta=mv\ell \qquad \ldots(1)$$



The ball moves in the Earth's gravitational field under the influence of an external force, the tension of the thread. That force is always perpendicular to the velocity vector of the sphere and, therefore, does not perform any work. It follows that mechanical energy of the ball remains constant

$$\frac{mv_0^2}{2}=\frac{mv^2}{2}+mg\ell\cos\theta \qquad \ldots(2)$$

Solving these two equations, we get

$$v_0=\sqrt{2g\ell\sec\theta}$$

2. (a) Applying the Law of Conservation of Angular Momentum about the pivot, we get

$$\sum mvr_\perp=\left(I_{\text{system}}\right)\omega$$

$$\Rightarrow \quad m_{\text{bullet}}v\left(\frac{L}{2}\right)=\left[\frac{m_{\text{rod}}L^2}{3}+m_{\text{bullet}}\left(\frac{L}{2}\right)^2\right]\omega$$

If m be the mass of the rod, then

$$m_{\text{bullet}}=\frac{m_{\text{rod}}}{6}=\frac{m}{6}$$

$$\Rightarrow \quad \left(\frac{m}{6}\right)v\left(\frac{L}{2}\right)=\left(\frac{mL^2}{3}+\frac{mL^2}{24}\right)\omega$$

$$\Rightarrow \quad \frac{mvL}{12}=\frac{9mL^2}{24}\omega$$

$$\Rightarrow \quad \omega=\frac{2v}{9L}$$

(b) The desired ratio is,

$$\frac{K_{\text{system}}}{K_{\text{bullet}}}=\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}\left(\frac{m}{6}\right)v^2}=\frac{\left(\frac{9mL^2}{24}\right)\left(\frac{2v}{9L}\right)^2}{\left(\frac{mv^2}{6}\right)}$$

$$\Rightarrow \quad \frac{K_{\text{system}}}{K_{\text{bullet}}}=\frac{1}{9}$$

3. Since, net external torque on the system is zero. Therefore, angular momentum will remain conserved. So,

$$I_1\omega_1=I_2\omega_2$$

$$\Rightarrow \quad \omega_2=\frac{I_1\omega_1}{I_2}$$

where, $I_1=I$, $\omega_1=\omega_0$, $I_2=I+mR^2$

$$\Rightarrow \quad \omega_2=\frac{I\omega_0}{I+mR^2}$$

4. By Law of Conservation of Angular Momentum about centre of disc, we have

$$I_1\omega_1=I_2\omega_2$$

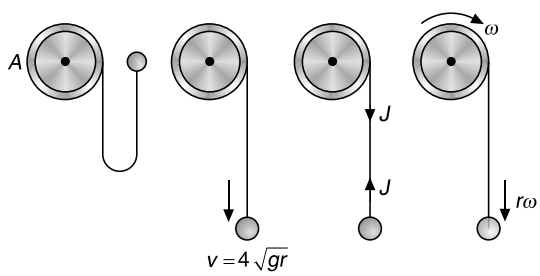
$$\Rightarrow \quad \omega_2=\left(\frac{I_1}{I_2}\right)\omega_1=\left[\frac{\frac{1}{2}ma^2}{\frac{1}{2}ma^2+2m\left(\frac{a}{2}\right)^2}\right]\omega$$

$$\Rightarrow \quad \omega_2=\frac{\omega}{2}$$

5. Applying Law of Conservation of Angular Momentum about axis of rotation, we get

$$mvr=(mr\omega)r+I\omega$$

where, $I=\frac{1}{2}(2m)r^2$



$$\Rightarrow \quad \left(4m\sqrt{gr}\right)r=(mr\omega)r+\frac{1}{2}(2m)r^2\omega=2mr^2\omega$$

$$\Rightarrow \quad \omega = 2\sqrt{\frac{g}{r}}$$

Considering the sudden change in the linear momentum of the particle caused by the impulsive tension J, we have, by

Impulse – Momentum Theorem,

Impulse = Change in Momentum

$$\Rightarrow \quad J = -mr\omega - \left(-4m\sqrt{gr}\right)$$

$$\Rightarrow \quad J = 2m\sqrt{gr}$$

6. (a) By Law of Conservation of Angular Momentum, we get

$$I_1\omega_1 = I_2\omega_2$$

$$\Rightarrow \quad \omega_2 = \left(\frac{I_1}{I_2}\right)\omega_0 = \frac{\left(\frac{1}{2}MR^2 + mR^2\right)}{\left(\frac{1}{2}MR^2\right)}\omega_0$$

$$\Rightarrow \quad \omega_2 = \left(1 + \frac{2m}{M}\right)\omega_0$$

(b) $W = E_f - E_i = \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}I_1\omega_1^2$

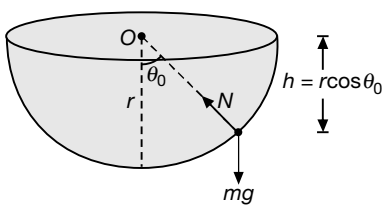
$$\Rightarrow \quad W = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(1 + \frac{2m}{M}\right)^2\omega_0^2 - \frac{1}{2}\left(\frac{1}{2}MR^2 + mR^2\right)\omega_0^2$$

$$\Rightarrow \quad W = \frac{1}{2}m\omega_0^2R^2\left(1 + \frac{2m}{M}\right)$$

7. The following two forces are acting on the particle.

(i) Normal Reaction (N)

(ii) Weight (mg)



The torque due to the normal reaction N about centre O is zero. The torque due to the weight is not zero, but its component along a vertical axis passing through O is zero. Hence, angular momentum of the particle about the same axis will remain conserved. So, we have

$$mv_0r\sin\theta_0 = mvr \qquad \ldots(1)$$

where, v is the speed of the particle at the topmost position.

Since friction is absent, so, mechanical energy will remain conserved

$$\Rightarrow \quad v^2 = v_0^2 - 2gh \qquad \ldots(2)$$

Solving equation (1) and (2), we get

$$v_0 = \sqrt{2gr\sec\theta_0}$$

$$\Rightarrow \quad (v_0)_{\min} = \sqrt{2gr\sec\theta_0}$$

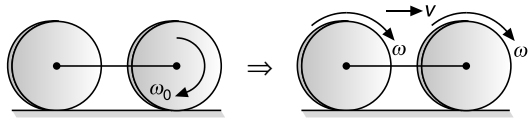
8. According to Angular Impulse – Angular Momentum Theorem, we have

$$J\ell = \left(I_{\text{system}}\right)\omega = \frac{m(2\ell)^2}{3}\omega$$

$$\Rightarrow \quad J = \frac{4m\ell\omega}{3}$$

9. By Law of Conservation of Angular Momentum, we have

$L_i = L_f$ {about bottom most point}



$$\Rightarrow \quad I\omega_0 = 2(I\omega + mRv)$$

$$\Rightarrow \quad \left(\frac{1}{2}mR^2\right)\omega_0 = 2\left(\frac{1}{2}mR^2\omega + mR(\omega R)\right)$$

$$\Rightarrow \quad \omega = \frac{\omega_0}{6}$$

$$\Rightarrow \quad v = \omega R = \frac{\omega_0 R}{6}$$

10. Applying Law of Conservation of Angular Momentum, Angular Velocity of disc is

$$\sum mvr_\perp = \left(I_{\text{system}}\right)\omega$$

$$\Rightarrow \quad m_1vR = \left(\frac{1}{2}m_2R^2 + m_1R^2\right)\omega$$

$$\Rightarrow \quad \omega = \frac{m_1R}{\frac{1}{2}m_2R^2 + m_1R^2}v = \left(\frac{2m_1}{m_2 + 2m_1}\right)\left(\frac{v}{R}\right)$$

where v is the relative speed of man with respect to disc.

So, $\frac{v}{R} = \omega_r$ (the relative angular speed of man)

$$\Rightarrow \quad \omega = \frac{2m_1}{(m_2 + 2m_1)} \cdot \omega_r$$

$$\Rightarrow \quad \frac{d\phi}{dt} = \left(\frac{2m_1}{m_2 + 2m_1}\right)\frac{d\theta}{dt}$$

CHAPTER 3

Integrating we get,

$$\phi = \left(\frac{2m_1}{m_2 + 2m_1}\right)\theta$$

11. Applying conservation of angular momentum, we get

$$Mv_1R = I\left(\frac{v_2}{R}\right) + Mv_2R$$

$$\Rightarrow \quad v_2 = \frac{v_1}{1 + \frac{I}{MR^2}}$$

$$\Rightarrow \quad \omega = \frac{v_2}{R} = \frac{v_1}{R\left(1 + \frac{I}{MR^2}\right)}$$

12. Let J be the linear impulse applied at the end B and ω the angular speed of rod, then by Impulse – Momentum Theorem, we have

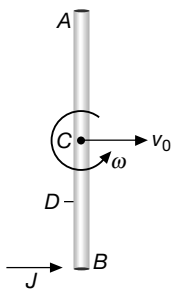
$$J = mv_0 \qquad \ldots(1)$$

Also, on applying Angular Impulse – Angular Momentum theorem, we get

$$J\left(\frac{\ell}{2}\right) = I\omega = \frac{m\ell^2}{12}\omega \qquad \ldots(2)$$

Solving these two equations, we get

$$\omega = \frac{6v_0}{\ell}$$



Linear speed of D (mid-point of CB i.e., centre of mass of lower half) relative to C is

$$v = \omega\left(\frac{\ell}{4}\right) = \frac{3}{2}v_0$$

Force exerted by upper half on the lower half,

$$F = \frac{mv^2/2}{l/4}$$

Substituting $v = \frac{3}{2}v_0$, we get

$$F = \frac{9mv_0^2}{2\ell}$$

13. Linear momentum, angular momentum and kinetic energy are conserved in the process.
Applying Law of Conservation of Linear Momentum, we get

$$MV = mv$$

$$\Rightarrow \quad V = \frac{m}{M}v \qquad \ldots(1)$$

Applying Law of Conservation of Angular Momentum about the centre of the rod, we get

$$\sum mvr = \left(I_{\text{system}}\right)\omega$$

$$\Rightarrow \quad mvd = \left(\frac{M\ell^2}{12}\right)\omega$$

$$\Rightarrow \quad \omega = \left(\frac{12mvd}{M\ell^2}\right) \qquad \ldots(2)$$

Since the collision is elastic, so $e = 1$

$$\Rightarrow \quad \left(\begin{matrix}\text{Relative Speed}\\ \text{of Approach}\end{matrix}\right) = \left(\begin{matrix}\text{Relative Speed}\\ \text{of Separation}\end{matrix}\right)$$

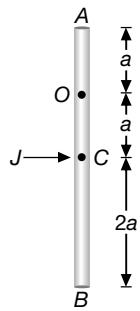
$$\Rightarrow \quad v = V + \omega d$$

Substituting the values, we get

$$v = \frac{m}{M}v + \frac{12mvd^2}{M\ell^2}$$

$$\Rightarrow \quad m = \frac{M\ell^2}{12d^2 + \ell^2}$$

14. Let ω be the angular velocity of rod just after the impulse is applied. Applying Angular Impulse Angular Momentum Theorem, we get



$$Ja = I\omega = \left(\frac{m(4a)^2}{12} + m(a^2)\right)\omega$$

$$\Rightarrow \quad \omega = \frac{3J}{7ma}$$

For describing the complete revolution

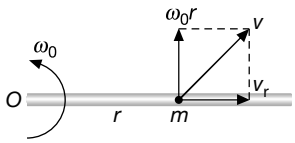
$\frac{1}{2}I\omega^2 > mgh$, where $h = 2a$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{7}{3}ma^2\right)\left(\frac{3J}{7ma}\right)^2 > mg(2a)$$

$$\Rightarrow \quad J > 2m\sqrt{\frac{7ga}{3}}$$

15. In the process of motion of the given system the kinetic energy and the angular momentum relative to the axis of rotation do not vary. So, by Law of Conservation of Energy, we have

$$\frac{1}{2}I\omega_0^2 = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$



$$\Rightarrow \quad I\omega_0^2 = I\omega^2 + mv^2 \qquad \ldots(1)$$

By Law of Conservation of Angular Momentum, we have

$$I\omega_0 = (I + mr^2)\omega \qquad \ldots(2)$$

where $v^2 = v_r^2 + \omega_0^2 r^2$ $\qquad \ldots(3)$

Solving these equations, we get

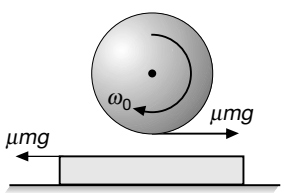
$$v_r = \frac{\omega_0 r}{\sqrt{1 + \frac{mr^2}{I}}}$$

Test Your Concepts-VII (Based on Rolling with Slipping)

1. Let, a_1 be the linear acceleration of sphere (towards right), a_2 be the linear acceleration of plank (towards left) and α be the angular retardation of sphere, then

$$a_1 = a_2 = \frac{\mu mg}{m} = \mu g$$

$$\alpha = \frac{\mu mg}{2mr^2/5} = \frac{5\mu g}{2r}$$



Let pure rolling start after time t, then for pure rolling to occur, we have

$$\omega r - v = v$$

$$\Rightarrow \quad \omega r = 2v$$

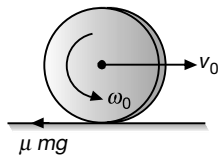
$$\Rightarrow \quad (\omega_0 - \alpha t)r = 2(a_1 t)$$

Substituting the values, we get

$$t = \frac{2r\omega_0}{9\mu g}$$

$$\Rightarrow \quad s = \frac{1}{2}a_2 t^2 = \frac{2\omega_0^2 r^2}{81\mu g}$$

2. Initially there is forward slipping so, the friction is backwards and maximum.



Let velocity becomes zero in time t_1 and angular velocity becomes zero in time t_2. Then, $0 = v_0 - at_1$

$$\Rightarrow \quad t_1 = \frac{v_0}{a} = \frac{v_0}{\mu g} \qquad \ldots(1)$$

and $0 = \omega_0 - \alpha t_2$

$$\Rightarrow \quad t_2 = \frac{\omega_0}{\alpha}$$

where, $\alpha = \frac{\mu mgR}{mR^2/2} = \frac{2\mu g}{R}$

$$\Rightarrow \quad t_2 = \frac{\omega_0 R}{2\mu g} \qquad \ldots(2)$$

Disk will return when

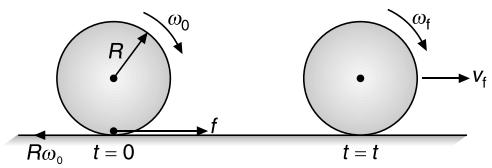
$$t_2 > t_1$$

$$\Rightarrow \quad \frac{\omega_0 R}{2\mu g} > \frac{v_0}{\mu g}$$

$$\Rightarrow \quad \omega_0 > \frac{2v_0}{R}$$

$$\Rightarrow \quad (\omega_0)_{\min} = \frac{2v_0}{R}$$

3. The situation given in the problem is shown in Figure.



In this case sliding friction acts on cylinder in forward direction which increases its linear speed and decreases its angular speed. Let pure rolling start after time t, when final velocity be v_f and final angular velocity be ω_f, then we must have

CHAPTER 3

$v_f = R\omega_f$ (as pure rolling starts)

For translational motion, applying the impulse momentum equation, we get

$ft = mv_f - 0$

Initial momentum of the cylinder is taken as zero as it does not have any translational speed.

$\Rightarrow \quad \mu mgt = mv_f$ …(1)

For rotational motion, applying the angular impulse angular momentum equation, we get

$I\omega_0 - fRt = I\omega_f$

$\Rightarrow \quad \left(\frac{1}{2}mR^2\right)\omega_0 - \mu mgRt = \left(\frac{1}{2}mR^2\right)\frac{v_f}{R}$ …(2)

Dividing equations (1) and (2), we get

$$\frac{\mu gt}{R\omega_0 - 2\mu gt} = 1$$

$\Rightarrow \quad \mu gt = R\omega_0 - 2\mu gt$

$\Rightarrow \quad t = \frac{\omega_0 R}{3\mu g}$

During sliding, friction is the only force acting on the cylinder and we know that the work done by the friction is always negative or loss in kinetic energy of system.

At $t = 0$, the kinetic energy of the cylinder is

$E_i = \frac{1}{2}I\omega_0^2$

$\Rightarrow \quad E_i = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_0^2 = \frac{1}{4}mR^2\omega_0^2$ …(3)

Finally, when pure rolling starts, kinetic energy of cylinder is

$E_f = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$ …(4)

During sliding the acceleration of cylinder due to friction $(f = \mu mg)$ is $a = \frac{f}{m} = \mu g$, thus after time t it gains a velocity

$v_f = \mu gt = \frac{1}{3}\omega_0 R$

At this moment pure rolling starts, thus its angular velocity at this instant is

$\omega_f = \frac{v_f}{R} = \frac{\mu gt}{R} = \frac{1}{3}\omega_0$

From equation (4), final kinetic energy is

$E_f = \frac{1}{2}m\left(\frac{R\omega_0}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{\omega_0}{3}\right)^2$

$\Rightarrow \quad E_f = \frac{1}{12}m\omega_0^2R^2$ …(5)

According to Work Energy theorem, work done by friction is

$W_{\text{friction}} = E_f - E_i$

Substituting the values of E_i and E_f from equations (3) and (5), we get

$W_{\text{friction}} = \frac{1}{12}m\omega_0^2R^2 - \frac{1}{4}m\omega_0^2R^2$

$\Rightarrow \quad W_{\text{friction}} = -\frac{1}{6}m\omega_0^2R^2$

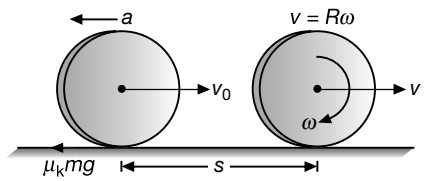
4. Applying Law of Conservation of Angular Momentum about bottommost point, we get

$L_i = L_f$

$\Rightarrow \quad mv_0R = mvR + \left(\frac{2}{5}mR^2\right)\omega$

$\Rightarrow \quad mv_0R = mvR + \frac{2}{5}mvR \qquad \left\{\because\ \omega = \frac{v}{R}\right\}$

$\Rightarrow \quad v = \frac{5}{7}v_0$



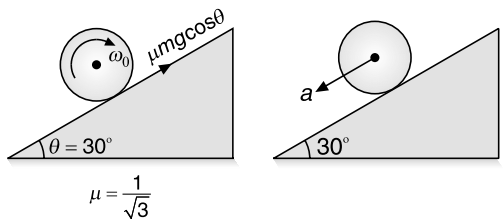
Since, $a = \frac{\mu_k mg}{m} = \mu_k g$ and $v^2 = v_0^2 - 2as$

$\Rightarrow \quad s = \frac{v_0^2 - v^2}{2a} = \frac{v_0^2 - \frac{25}{49}v_0^2}{2\mu_k g}$

$\Rightarrow \quad s = \frac{12v_0^2}{49\mu_k g}$

5. Till the pure rolling starts, friction (f_1) is upwards and it is maximum. Linear acceleration for $\mu = 1/\sqrt{3}$ is

$$a = \frac{\mu mg\cos 30° - mg\sin 30°}{m} = 0$$



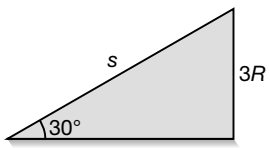
i.e., the cylinder will keep on rotating at the same place till its angular velocity becomes zero. Angular retardation, for $\mu = \frac{1}{\sqrt{3}}$ and $\theta = 30°$ is

$$\alpha = \frac{(\mu mg\cos\theta)R}{mR^2/2} = \frac{g}{R}$$

If t_1 be the time when the angular velocity of cylinder becomes zero, then

$$0 = \omega_0 - \alpha t_1$$

$$\Rightarrow \quad t_1 = \frac{\omega_0}{\alpha} = \frac{\omega_0 R}{g} \qquad \ldots(1)$$



Once, the angular velocity becomes zero, cylinder starts rolling downwards with an acceleration

$$a = \frac{g\sin\theta}{1+(I/mR^2)} = \frac{g\sin 30°}{1+(1/2)} = \frac{g}{3}$$

Also, $s = 3R\,\text{cosec}(30°) = 6R$

Let t_2 be the time taken by the cylinder to reach the bottom, then

$$t_2 = \sqrt{\frac{2s}{a}} = \sqrt{\frac{12R}{g/3}} = 6\sqrt{\frac{R}{g}}$$

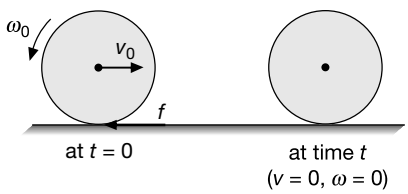
So, total time $t = t_1 + t_2$

$$\Rightarrow \quad t = \frac{\omega_0 R}{g} + 6\sqrt{\frac{R}{g}}$$

6. $a = \frac{f}{m} = \mu g$

$$\alpha = \frac{\tau}{I} = \frac{fR}{\frac{2}{5}MR^2} = \frac{\mu(Mg)R}{\frac{2}{5}MR^2}$$

$$\Rightarrow \quad \alpha = \frac{5\mu g}{2R}$$



Since $v = v_0 + (-\mu g)t$

$$\Rightarrow \quad 0 = v_0 - (\mu g)t$$

$$\Rightarrow \quad t = \frac{v_0}{\mu g} \qquad \ldots(1)$$

Also, $\omega = \omega_0 + \alpha t$

$$\Rightarrow \quad 0 = \omega_0 + \left(-\frac{5\mu g}{2R}\right)t$$

$$\Rightarrow \quad t = \frac{2R\omega_0}{5\mu g} \qquad \ldots(2)$$

Distance travelled is

$$s = v_0 t + \frac{1}{2}(-\mu g)t^2$$

$$\Rightarrow \quad s = v_0\left(\frac{v_0}{\mu g}\right) - \frac{1}{2}(\mu g)\frac{v_0^2}{\mu^2 g^2}$$

$$\Rightarrow \quad s = \frac{v_0^2}{2\mu g}$$

Equating (1) and (2), we get

$$\frac{v_0}{\mu g} = \frac{2R\omega_0}{5\mu g}$$

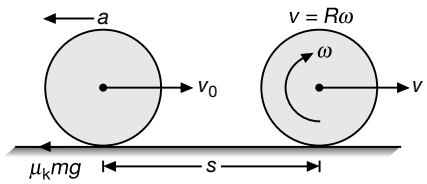
$$\Rightarrow \quad \omega_0 = \frac{5v_0}{2R}$$

7. From conservation of angular momentum about bottommost point.

$$L_i = L_f$$

$$\Rightarrow \quad mv_0 R = mvR + \frac{2}{5}mR^2\cdot\omega$$

$$\Rightarrow \quad mv_0 R = mvR + \frac{2}{5}mvR \qquad \left\{\because\ \omega = \frac{v}{R}\right\}$$



$$\Rightarrow \quad v = \frac{5}{7}v_0$$

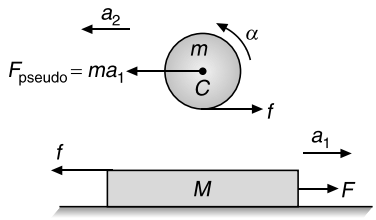
Since, $a = \frac{\mu_k mg}{m} = \mu_k g$

So, from $v^2 = v_0^2 - 2as$

We get $s = \frac{v_0^2 - v^2}{2a} = \frac{v_0^2 - \frac{25}{49}v_0^2}{2\mu_k g}$

$$\Rightarrow \quad s = \frac{12v_0^2}{49\mu_k g}$$

8. Since the cylinder does not slip over the plank, so it is the case of pure rolling and hence we can take friction on cylinder in any direction, say towards right as shown in Figure.



Since friction on cylinder is acting towards right, so it must act on the plank towards left. Let plank moves toward right with an acceleration a_1, due to which the cylinder will experience a pseudo force ma_1 towards left, and hence it will roll towards left with respect to plank with an acceleration a_2.

Since we have used the concept of pseudo force, so we must say that the acceleration a_2 must be with respect to the plank. Let the angular acceleration of the cylinder during rolling be α, so we have

$$a_2 = R\alpha$$

For translational motion of plank, we have

$$F - f = Ma_1 \qquad \ldots(1)$$

For translational motion of cylinder with respect to plank we have

$$ma_1 - f = ma_2 \qquad \ldots(2)$$

For rotational motion of cylinder with respect to plank, we have

$$fR = I\alpha$$

$$\Rightarrow \quad fR = \left(\frac{1}{2}mR^2\right)\left(\frac{a_2}{R}\right)$$

$$\Rightarrow \quad f = \frac{1}{2}ma_2 \qquad \ldots(3)$$

From equation (2) and (3), we get

$$ma_1 - \frac{1}{2}ma_2 = ma_2$$

$$\Rightarrow \quad a_1 = \frac{3}{2}a_2 \qquad \ldots(4)$$

Using equations (1), (2) and (3), we get

$$F - \frac{1}{2}ma_2 = \frac{3}{2}Ma_2$$

$$\Rightarrow \quad a_2 = \frac{2F}{3M+m}$$

From equation (4), we get

$$a_1 = \frac{3F}{3M+m}$$

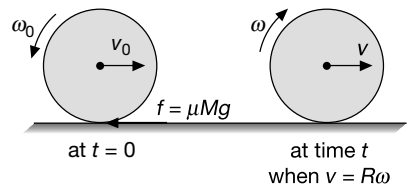
Since, a_2 is acceleration of cylinder relative to the plank, so net acceleration of the cylinder is

$$a_{net} = a_1 - a_2 = \frac{F}{3M+m}$$

Please note that in the equation $a_2 = R\alpha$, a_2 is the acceleration of cylinder with respect to plank. So,

$$\alpha = \frac{a_1 - a_2}{R} = \frac{F}{R(3M+m)}$$

9. Let us draw the situations at $t = 0$ and at time t.



Taking forward direction as positive and CW sense as positive, we get

$$a = \mu g$$

$$\alpha = \frac{\tau}{I} = \frac{\mu MgR}{\frac{2}{5}MR^2} = \frac{5\mu g}{2R}$$

Now, at time t, we have

$$v = v_0 - (\mu g)t \qquad \ldots(1)$$

and $$\omega = -\omega_0 + \left(\frac{5\mu g}{2R}\right)t \qquad \ldots(2)$$

Since at time t, we have

$$v = R\omega$$

$$\Rightarrow \quad v_0 - (\mu g)t = R\left[-\omega_0 + \left(\frac{5\mu g}{2R}\right)t\right]$$

$$\Rightarrow \quad t = \frac{2}{7}\left(\frac{v_0 + R\omega_0}{\mu g}\right)$$

So, speed of ball at time t is

$$v = v_0 - (\mu g)t$$

$$\Rightarrow \quad v = v_0 - \frac{2}{7}(v_0 + R\omega_0)$$

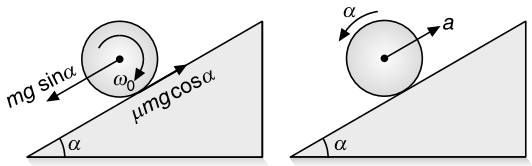
$$\Rightarrow \quad v = \frac{1}{7}(5v_0 + 2R\omega_0)$$

10. Given $\mu > \tan\alpha$

$$\Rightarrow \quad \mu mg\cos\alpha > mg\sin\alpha$$

Since, $a = (\mu g\cos\alpha - g\sin\alpha)$

Also, $\alpha = \dfrac{\tau}{I} = \dfrac{(\mu mg\cos\alpha)r}{\frac{1}{2}mr^2} = \dfrac{2\mu g\cos\alpha}{r}$



Slipping will stop say at time t, then

$v = r\omega$

$\Rightarrow \quad at = r(\omega_0 - \alpha t)$

$\Rightarrow \quad t = \dfrac{r\omega_0}{a + r\alpha} = \left(\dfrac{r\omega_0}{3\mu g\cos\alpha - g\sin\alpha}\right)$

If d_1 is the distance travelled during this time t, then

$$d_1 = \frac{1}{2}at^2 = \frac{r^2\omega_0^2(\mu\cos\alpha - \sin\alpha)}{2g(3\mu\cos\alpha - \sin\alpha)^2}$$

Also, $v = at = \dfrac{r\omega_0(\mu\cos\alpha - \sin\alpha)}{(3\mu\cos\alpha - \sin\alpha)}$

Once slipping has stopped, pure rolling continues if

$$\mu > \frac{\tan\alpha}{1 + (mR^2/I)}$$

$\Rightarrow \quad \mu > \dfrac{\tan\alpha}{1+2}$

$\Rightarrow \quad \mu > \dfrac{\tan\alpha}{3}$

Since, in the question it is already given that $\mu > \tan\alpha$, so the retardation of cylinder is given by

$$a' = \frac{g\sin\alpha}{1 + (I/mr^2)} = \frac{g\sin\alpha}{1 + 1/2} = \frac{2}{3}g\sin\alpha$$

If it travels a further distance d_2 under this new retardation a', then

$$d_2 = \frac{v^2}{2a'} = \frac{3r^2\omega_0^2(\mu\cos\alpha - \sin\alpha)^2}{(3\mu\cos\alpha - \sin\alpha)^2(4g\sin\alpha)}$$

So, $d_{max} = d_1 + d_2$

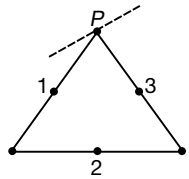
$$\Rightarrow \quad d_{max} = \left(\frac{r^2\omega_0^2(\mu\cos\alpha - \sin\alpha)}{2g(3\mu\cos\alpha - \sin\alpha)^2}\right) \times \left(1 + \frac{3(\mu\cos\alpha - \sin\alpha)}{2\sin\alpha}\right)$$

$$\Rightarrow \quad d_{max} = \frac{r^2\omega_0^2(\mu\cos\alpha - \sin\alpha)}{4g\sin\alpha(3\mu\cos\alpha - \sin\alpha)}$$

Single Correct Choice Type Questions

1. Since axis passes through ends of two rods and is perpendicular to their length, so

$$I_{1P} = I_{3P} = \frac{1}{3}m\ell^2$$



Moment of inertia of rod 2 about it CG is $\dfrac{1}{12}m\ell^2$ and point P is at a distance $\dfrac{\sqrt{3}}{2}\ell$, so by Parallel Axis Theorem

$$I_{2P} = \frac{1}{12}m\ell^2 + m\left(\frac{\sqrt{3}}{2}\ell\right)^2$$

$\Rightarrow \quad I_{2P} = \dfrac{1}{12}m\ell^2 + \dfrac{3}{4}m\ell^2$

$\Rightarrow \quad I_{2P} = \dfrac{10}{12}m\ell^2$

$\Rightarrow \quad I_{total} = \left(\dfrac{10}{12} + \dfrac{1}{3} + \dfrac{1}{3}\right)m\ell^2$

$\Rightarrow \quad I_{total} = \left(\dfrac{10+4+4}{12}\right)m\ell^2$

$\Rightarrow \quad I_{total} = \dfrac{3}{2}m\ell^2$

Further $I_{total} = (3m)k^2$

$\Rightarrow \quad (3m)k^2 = \dfrac{3}{2}m\ell^2$

$\Rightarrow \quad k = \dfrac{\ell}{\sqrt{2}}$

Hence, the correct answer is (C).

2. If we assume complete disc to be present, then it would have a mass 4 times the mass of the sector. Then, moment of inertia of the complete disc is

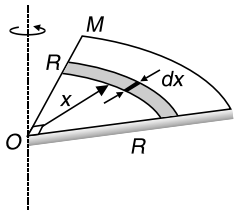
$$I_{disc} = \frac{1}{2}M_{disc}R^2 = \frac{1}{2}(4M_{sector})R^2$$

Hence, $I_{sector} = \dfrac{I_{disc}}{4}$

$\Rightarrow \quad I_{sector} = \dfrac{1}{2}MR^2$

CHAPTER 3

Calculus Method:



Consider an element of mass dm, thickness dx and radius x of the sector.

$$dm = \frac{M}{\left(\frac{\pi R^2}{4}\right)} \frac{2\pi x dx}{4} = \frac{2M}{R^2} x dx$$

If dI be the moment of inertia of the element, then $dI = x^2 dm$

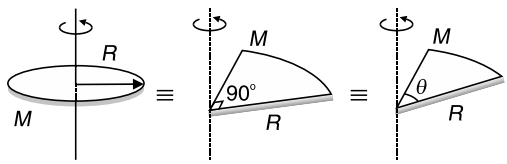
$$\Rightarrow \quad dI = x^2\left(\frac{2M}{R^2} x\, dx\right)$$

$$\Rightarrow \quad dI = \frac{2M}{R^2}\left(x^3\, dx\right)$$

$$\Rightarrow \quad I = \frac{2M}{R^2}\int_0^R x^3\, dx$$

$$\Rightarrow \quad I = \frac{1}{2} MR^2$$

General Funda:



$$\left(\begin{array}{c}\text{Moment of Inertia}\\ \text{of a DISC of mass}\\ \text{M with radius R}\end{array}\right) \equiv \left(\begin{array}{c}\text{Moment of Inertia}\\ \text{of a SECTOR of}\\ \text{mass M with radius R}\end{array}\right)$$

Hence, the correct answer is (A).

3. Since, Impulse = Change in Momentum, so

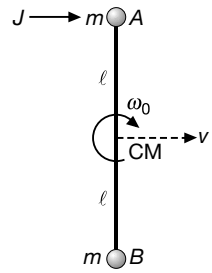
$$J = (2m)v$$

$$\Rightarrow \quad v = \frac{J}{2m}$$

$$\text{Also,} \left(\begin{array}{c}\text{Angular}\\ \text{Impulse}\end{array}\right) = \left(\begin{array}{c}\text{Change in Angular}\\ \text{Momentum}\end{array}\right)$$

$$\Rightarrow \quad J\ell = \left(I_{\text{system}}\right)\omega$$

$$\Rightarrow \quad J\ell = \left(2m\ell^2\right)\omega$$



$$\Rightarrow \quad \omega = \frac{J\ell}{2m\ell^2} = \frac{J}{2m\ell}$$

So, $v_A = v + \ell\omega$

$$\Rightarrow \quad v_A = \frac{J}{2m} + \frac{J}{2m} = \frac{J}{m}$$

Hence, the correct answer is (C).

4. Work done $W = \frac{1}{2} I\omega^2$

If x is the distance of mass 0.3 kg from the centre of mass, we will have,

$$I = (0.3)x^2 + (0.7)(1.4 - x)^2$$

For work to be minimum, the moment of inertia (I) should be minimum

$$\Rightarrow \quad \frac{dI}{dx} = 0$$

$$\Rightarrow \quad 2(0.3x) - 2(0.7)(1.4 - x) = 0$$

$$\Rightarrow \quad (0.3)x = (0.7)(1.4 - x)$$

$$\Rightarrow \quad x = \frac{(0.7)(1.4)}{0.3 + 0.7} = 0.98 \text{ m}$$

Hence, the correct answer is (C).

5. Assuming mass of body to be m, the mechanical energy is conserved in both the cases, so

$$\text{Hence,} \quad \frac{1}{2} m\left(\frac{5}{4} v_0\right)^2 = \frac{1}{2} m v_0^2 + \frac{1}{2} I\omega^2$$

$$\Rightarrow \quad \frac{25}{16} v_0^2 = v_0^2 + \frac{I}{m}\left(\frac{v_0}{R}\right)^2$$

If K be the radius of gyration, then

$$I = mK^2, \text{ so}$$

$$\Rightarrow \quad \frac{25}{16} = 1 + \frac{mK^2}{mR^2}$$

$$\Rightarrow \quad K^2 = \frac{9}{16} R^2$$

$$\Rightarrow \quad K = \frac{3}{4} R$$

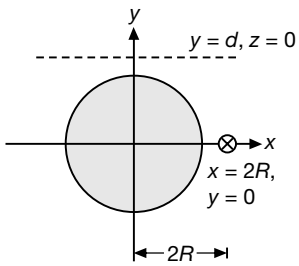
Hence, the correct answer is (B).

6. Moment of inertia about the axis passing through $x = 2R$, $y = 0$ shown by $\otimes$ direction in figure is

$$I_1 = \frac{1}{2}mR^2 + m(2R)^2$$

$$\Rightarrow \quad I_1 = \frac{9}{2}mR^2 \qquad \ldots(1)$$

Moment of inertia about the axis passing through $y = d$, $z = 0$, shown as dotted line in figure is



$$I_2 = \frac{1}{4}mR^2 + md^2 \qquad \ldots(2)$$

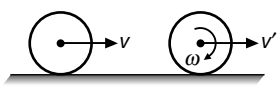
Equating (1) and (2), we get

$$\frac{1}{4}mR^2 + md^2 = \frac{9}{2}mR^2$$

$$\Rightarrow \quad d = \frac{\sqrt{17}}{2}R$$

Hence, the correct answer is (B).

7. Let v be the velocity of centre of mass of ring just after the impulse is applied and v' be its velocity when pure rolling starts. Angular velocity ω of the ring at this instant will be $\omega = \frac{v'}{r}$



Since, Impulse = Change in Linear Momentum, so we have

$$J = mv$$

$$\Rightarrow \quad v = \frac{J}{m} \qquad \ldots(1)$$

Between the two positions shown in figure, force of friction on the ring acts backwards. Angular momentum of the ring about bottommost point will be conserved, so

$$L_i = L_f$$

$$\Rightarrow \quad mvr = mv'r + I\omega$$

$$\Rightarrow \quad mvr = mv'r + (mr^2)\frac{v'}{r} = 2mv'r$$

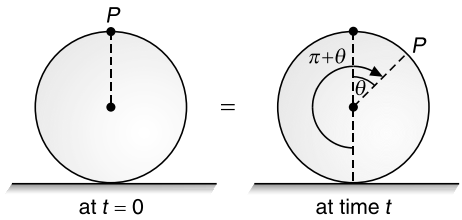
So, from (1), we get

$$\Rightarrow \quad v' = \frac{v}{2} = \frac{J}{2m}$$

Hence, the correct answer is (B).

8. Since at $t = 0$, P is at the top of rim, so

$$v_P = 2v_0 \sin\left(\frac{\pi + \theta}{2}\right)$$



$$\Rightarrow \quad v_p = 2v_0 \cos\left(\frac{\theta}{2}\right) = 2v_0 \cos\left(\frac{v_0 t}{2R}\right)$$

$$\Rightarrow \quad v_P^2 = 4v_0^2 \cos^2\left(\frac{v_0 t}{2R}\right)$$

At $t = 0$, $v_P^2 = 4v_0^2$

Also, $v_P = 0$ at point of contact i.e., when P reaches the bottom i.e., in a time $t = \frac{T}{2} = \frac{\pi R}{v_0}$

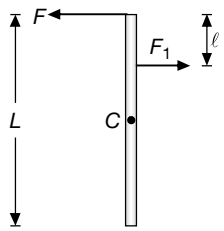
Hence, the correct answer is (B).

9. Let L be the length of the rod and F_1 the magnitude of other force. If C is the centre of the rod, then

$$F_1 - F = ma$$

$$\Rightarrow \quad F_1 = F + ma \qquad \ldots(1)$$

Since the rod moves translationally, so the net torque about C is zero.



$$\Rightarrow \quad F\left(\frac{L}{2}\right) = F_1\left(\frac{L}{2} - \ell\right)$$

$$\Rightarrow \quad F\left(\frac{L}{2}\right) = (F + ma)\left(\frac{L}{2} - \ell\right)$$

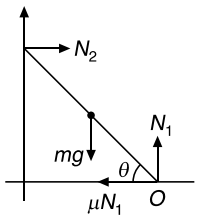
$$\Rightarrow \quad F\left(\frac{L}{2}\right) = (F + ma)\ell$$

$$\Rightarrow \quad L = \frac{2(F+ma)\ell}{ma}$$

$$\Rightarrow \quad L = 2\ell\left(1+\frac{F}{ma}\right)$$

Hence, the correct answer is (B).

10. The free body diagram of arrangement is shown in Figure.



$$mg = N_1$$

Also, $\mu N_1 = N_2$

$$\Rightarrow \quad N_2 = \mu mg$$

Taking moment of forces about O, we get

$$\mu mgl\sin\theta = mg\frac{l}{2}\cos\theta$$

$$\Rightarrow \quad \tan\theta = \frac{1}{2\mu}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{2\mu}\right)$$

Hence, the correct answer is (D).

11. Given, that I_0 is the moment of inertia of table and gun and m the mass of bullet.

Initial angular momentum of system about centre

$$L_i = \left(I_0 + mr^2\right)\omega_0 \qquad \ldots(1)$$

Let ω be the angular velocity of table after the bullet is fired. Final angular momentum

$$L_f = I_0\omega - m(v - r\omega)r \qquad \ldots(2)$$

where $(v - r\omega)$ is absolute velocity of bullet to the right, then

From (1) and (2), we get

$$(\omega - \omega_0) = \frac{mvr}{I_0 + mr^2}$$

This is also the increase in angular velocity

Hence, the correct answer is (B).

12. The rod will rotate about point A with angular acceleration given by

$$\alpha = \frac{\tau}{I} = \frac{Fx}{\left(\frac{m\ell^2}{3}\right)} = \frac{3Fx}{m\ell^2}$$

$$\Rightarrow \quad a = \frac{\ell}{2}\alpha = \frac{3}{2}\frac{Fx}{m\ell}$$

$$\Rightarrow \quad a \propto x$$

So, a-x graph is a straight line passing through origin with slope $\frac{3F}{2m\ell}$.

Hence, the correct answer is (B).

13. The angular momentum of a body $\vec{L}$ may be expressed as the sum of two parts,

(a) one arising from the motion of the centre of mass of the body and

(b) the other from the motion of the body with respect to its centre of mass.

i.e. $\vec{L}_{\text{total}} = \vec{L}_{C.M.} + \vec{r}_{C.M.} \times \vec{p}$

$$\Rightarrow \quad \vec{L}_{\text{total}} = \vec{L}_{C.M.} + M\left(\vec{r}_{C.M.} \times \vec{v}_{C.M.}\right)$$

For this Problem

$$L_{C.M.} = I\omega = \frac{1}{2}MR^2\omega \text{ and}$$

$$M\left(\vec{r}_{C.M.} \times \vec{v}_{C.M.}\right) = MRv_{CM} = MR(R\omega)$$

$$\Rightarrow \quad M\left(\vec{r}_{C.M.} \times \vec{v}_{C.M.}\right) = MR^2\omega$$

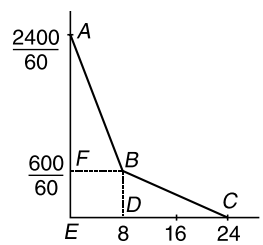
$$\Rightarrow \quad L_{\text{total}} = \frac{1}{2}MR^2\omega + MR^2\omega = \frac{3}{2}MR^2\omega$$

Hence, the correct answer is (B).

14. Total Revolutions is equal to area under the curve.

Area= $Ar\Delta ABF + Ar\Delta BCD + Ar$

Rectangle $BDEF$



$$\Rightarrow \quad \text{Area} = \frac{1}{2}\frac{(1800)}{60}(8) + \frac{1}{2}(16)\frac{(600)}{60} + 8\times\frac{600}{60}$$

$$\Rightarrow \quad \text{Area} = \frac{7200+4800+4800}{60} = 280$$

Hence, the correct answer is (B).

15. Applying conservation of energy, we get

$$\frac{1}{2}mv^2 = \frac{1}{2}M\left(\frac{mv}{M}\right)^2 + \frac{1}{2}\left(\frac{ML^2}{12}\right)(\omega)^2 \qquad \ldots(1)$$

Applying conservation of angular momentum, we get

$$mv\left(\frac{L}{2}\right)=\frac{ML^2}{12}\omega$$

$$\Rightarrow \quad \omega=\frac{6mv}{ML} \qquad \ldots(2)$$

From (1) and (2), we get

$$\frac{1}{2}mv^2=\frac{1}{2}\frac{m^2v^2}{M}+\frac{1}{24}ML^2\left(\frac{36m^2v^2}{M^2L^2}\right)$$

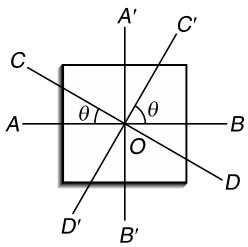
$$\Rightarrow \quad 1=\frac{m}{M}+\frac{3m}{M}$$

$$\Rightarrow \quad 1=\frac{4m}{M}$$

$$\Rightarrow \quad m=\frac{M}{4}$$

Hence, the correct answer is (B).

16.



$I_{AB}=I_{A'B'}=I$ and $I_{CD}=I_{C'D'}$

If I_0 be the moment of inertia of the square plate about an axis passing through O and perpendicular to the plate, then by perpendicular axis theorem

$$I_0=I_{AB}+I_{A'B'}=2I_{AB} \qquad \ldots(1)$$

OR

$$I_0=I_{CD}+I_{C'D'}=2I_{CD} \qquad \ldots(2)$$

From (1) and (2)

$$I_{CD}=I_{AB}=I \qquad \textbf{\{OPTION (A)\}}$$

Hence, the correct answer is (A).

17. Net torque about O due to weights acting at midpoints of the respective rods should be zero. Hence,

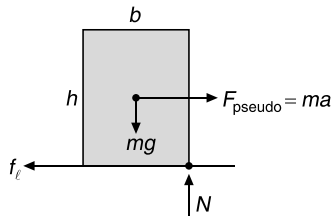
$$mg\left(\frac{\ell}{2}\sin 60^\circ\right)=Mg\left(\frac{\ell}{2}\sin 30^\circ\right)$$

$$\Rightarrow \quad \frac{M}{m}=\frac{\sin 60^\circ}{\sin 30^\circ}=\sqrt{3}$$

Hence, the correct answer is (C).

18. Let a be the acceleration of the truck, then from FBD of the block, taking torque about right edge from where normal reaction is passing, we get

$$ma\left(\frac{h}{2}\right)\le mg\left(\frac{b}{2}\right)$$



$$\Rightarrow \quad a\le\frac{b}{h}g$$

Final velocity of truck is zero, so we have

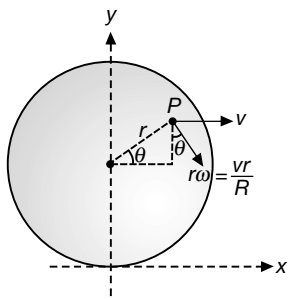
$$0=v^2-2\left(\frac{b}{h}g\right)l$$

$$\Rightarrow \quad l=\frac{hv^2}{2bg}$$

Hence, the correct answer is (B).

19. Since, for pure rolling, we have

$$\omega=\frac{v}{R}$$



Velocity of point P is resultant of two velocity vectors as shown in figure. So,

$$\vec{v}_P=v_x\hat{i}+v_y\hat{j}$$

$$\Rightarrow \quad \vec{v}_P=\left(v+\frac{vr\sin\theta}{R}\right)\hat{i}-\left(\frac{vr\cos\theta}{R}\right)\hat{j}$$

$$\Rightarrow \quad \vec{v}_P=\omega\left[(R+r\sin\theta)\hat{i}-(r\cos\theta)\hat{j}\right]$$

Hence, the correct answer is (C).

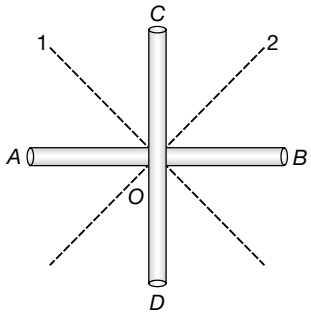
20. Moment of inertia of both the rods about an axis passing through their point of intersection O and perpendicular to the plane of the rods is

$$I_0=\frac{m\ell^2}{12}+\frac{m\ell^2}{12}=\frac{m\ell^2}{6}$$

Now axis 1 and axis 2 are two mutually perpendicular axes passing through O lying in the plane of two rods. Then by symmetry, we have

$$I_1=I_2=I \qquad \{\text{say}\}$$

CHAPTER 3



Now, according to Perpendicular Axis Theorem, we have

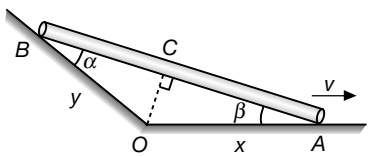
$$I_1 + I_2 = I_0$$

$$\Rightarrow \quad 2I_1 = I_0 = \frac{m\ell^2}{6}$$

$$\Rightarrow \quad I_1 = \frac{m\ell^2}{12}$$

Hence, the correct answer is (C).

21. Let $OA = x$ and $OB = y$ as shown in figure



Then $BC + CA = \ell$

$$\Rightarrow \quad y\cos\alpha + x\cos\beta = \ell \qquad \ldots(1)$$

Differentiating both sides of (1) w.r.t. time t, we get

$$\left(\frac{dy}{dt}\right)\cos\alpha + \left(\frac{dx}{dt}\right)\cos\beta = 0$$

Since, $\frac{dx}{dt} = v_A = v$

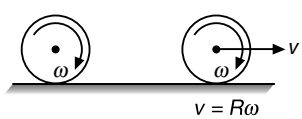
So, $\left(\frac{dy}{dt}\right) = v_B = -\frac{v\cos\beta}{\cos\alpha}$

Here, negative sign implies that y decreases as t increases.

Hence, the correct answer is (D).

22. Applying Conservation of Angular Momentum about point of contact, we get

$$I\omega_0 = I\omega + mvR$$



$$\Rightarrow \quad I\omega_0 = I\left(\frac{v}{R}\right) + mvR$$

$$\Rightarrow \quad v = \frac{I\omega_0}{\frac{I}{R} + mR}$$

$$\Rightarrow \quad v = \frac{\omega_0}{\frac{1}{R} + \frac{mR}{I}}$$

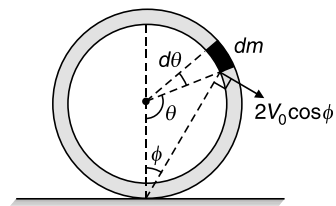
Now $I_{\text{solid sphere}} < I_{\text{hollow}}$

$$\Rightarrow \quad v_{\text{solid}} < v_{\text{hollow}}$$

$$\Rightarrow \quad v_1 < v_2$$

Hence, the correct answer is (C).

23. Considering an elemental mass dm on ring as shown in Figure. Then



$$dm = \left(\frac{M}{2\pi R}\right)(Rd\theta) = \frac{Md\theta}{2\pi}$$

Velocity of this elemental mass is

$$v = 2v_0\cos\phi$$

$$\Rightarrow \quad v = 2v_0\cos\phi = 2v_0\sin\left(\frac{\theta}{2}\right)$$

Kinetic energy of element is

$$dK = \frac{1}{2}(dm)v^2 = \frac{1}{2}(dm)\left(4v_0^2\sin^2\frac{\theta}{2}\right)$$

$$\Rightarrow \quad dK = \frac{Mv_0^2}{2\pi}(1-\cos\theta)d\theta$$

Total kinetic energy of segment ACB is

$$K = K_{ACB} = \frac{Mv_0^2}{2\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}(1-\cos\theta)d\theta$$

$$\Rightarrow \quad K = \frac{Mv_0^2}{2} - \frac{Mv_0^2}{\pi}$$

Hence, the correct answer is (A).

24. $\tau = \frac{dL}{dt} = \frac{d}{dt}(I\omega)$

$$\Rightarrow \quad \tau = \frac{d}{dt}\left(\frac{ML^2}{3} + mx^2\right)\omega$$

$$\Rightarrow \quad \tau = 2mx\frac{dx}{dt}\omega$$

Now, $x = vt$

$$\Rightarrow \quad \tau \propto t$$

Finally, torque becomes zero.

Hence, the correct answer is (B).

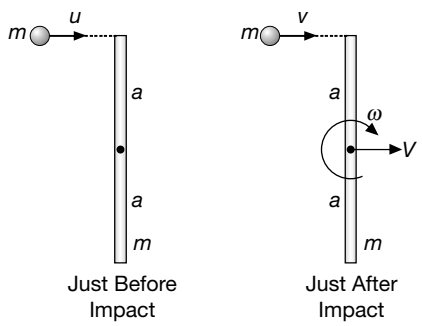
25. No external torque is acting on the system, so angular momentum is conserved. Further there exists no non-conservative forces in the system, so total energy is also conserved.

Hence, the correct answer is (B).

26. By Law of Conservation of Linear Momentum, we have

$$mu = mv + mV$$

$$\Rightarrow \quad u = v + V \qquad \ldots(1)$$



By Law of Conservation of Angular Momentum about centre of rod, we have

$$mua = mva + \left(\frac{ma^2}{3}\right)\omega$$

$$\Rightarrow \quad u = v + \frac{a\omega}{3} \qquad \ldots(2)$$

Further applying the definition of coefficient of restitution $(e = 1)$ at the point of impact, we get

Relative speed of approach = Relative speed of separation

$$\Rightarrow \quad u = (V + a\omega) - v \qquad \ldots(3)$$

Solving these three equations (1), (2) and (3) we get,

$$V = \frac{2}{5}u \text{ and } \omega = \frac{6u}{5a}$$

So, kinetic energy of rod is

$$K = \frac{1}{2}\times m\times\left(\frac{2}{5}u\right)^2 + \frac{1}{2}\times\frac{ma^2}{3}\times\left(\frac{6u}{5a}\right)^2$$

$$\Rightarrow \quad K = \frac{8}{25}mu^2$$

Hence, the correct answer is (D).

27. $\left(\frac{1}{2}MR^2 + M_bR^2\right)\omega = \frac{1}{2}MR^2\omega'$

(Since the boy reaches the centre, so final angular momentum of boy is zero).

$$\Rightarrow \quad \left(\frac{1}{2}M + M_b\right)\omega = \frac{1}{2}M\omega'$$

$$\Rightarrow \quad (100 + 50)\omega = 100\omega'$$

$$\Rightarrow \quad \omega' = \frac{3\omega}{2} = 15 \text{ rpm}$$

Hence, the correct answer is (C).

28. For the plank P

$$(\text{KE})_{\text{Plank}} = \frac{1}{2}m(2v)^2$$

$$(\text{KE})_{\text{Plank}} = 2mv^2$$

For the cylinder

$$(\text{KE})_{\text{Cylinder}} = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

$$\Rightarrow \quad (\text{KE})_{\text{Cylinder}} = \frac{1}{2}mv^2\left(1 + \frac{1}{1}\right) = mv^2$$

$$\Rightarrow \quad \frac{(\text{KE})_{\text{Plank}}}{(\text{KE})_{\text{Cylinder}}} = \frac{2}{1}$$

Hence, the correct answer is (B).

29. Let M be the mass of the square plate before the holes are cut. If m be the mass of one hole, then

$$m = \left(\frac{M}{16R^2}\right)\pi R^2 = \frac{\pi M}{16}$$

Moment of inertia of remaining portion is

$$I = I_{\text{square}} - 4I_{hole}$$

$$\Rightarrow \quad I = \frac{M}{12}(16R^2 + 16R^2) - 4\left[\frac{mR^2}{2} + m(2R^2)\right]$$

$$\Rightarrow \quad I = \frac{8}{3}MR^2 - 10mR^2$$

$$\Rightarrow \quad I = \left(\frac{8}{3} - \frac{10\pi}{16}\right)MR^2$$

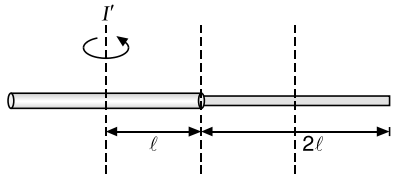
Hence, the correct answer is (C).

30. **Conceptual the correct answer is (B).**

31. Applying angular momentum conservation, we get

$$2\left[\frac{m(2l)^2}{12}\right]\omega_0 = I'\omega' \qquad \ldots(1)$$

CHAPTER 3



where, $I' = \frac{m(2l)^2}{12} + \left[\frac{m(2l)^2}{12} + m(2l)^2\right]$

$\Rightarrow \quad I' = \frac{ml^2}{3} + \frac{13ml^2}{3} = \frac{14ml^2}{3}$

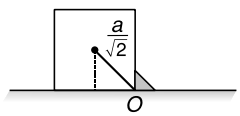
So, equation (1) becomes

$$\frac{2}{3}ml^2\omega_0 = \frac{14}{3}ml^2\omega'$$

$\Rightarrow \quad \omega' = \frac{\omega_0}{7}$

Hence, the correct answer is (B).

32. By Law of Conservation of Angular Momentum



$$mv\left(\frac{a}{2}\right) = \left(I_{\substack{\text{system}\\ \text{about O}}}\right)\omega$$

$$mv\left(\frac{a}{2}\right) = \left[\frac{1}{6}Ma^2 + M\left(\frac{a}{\sqrt{2}}\right)^2\right]\omega$$

$\Rightarrow \quad \omega = \frac{3v}{4a}$

Hence, the correct answer is (A).

33. By Law of Conservation of Energy

Loss in Potential energy = Gain in Kinetic energy

$$mg\ell(1-\cos\theta) = \frac{1}{2}mv^2$$

$\Rightarrow \quad \frac{mv^2}{\ell} = 2mg(1-\cos\theta)$

At point of breaking

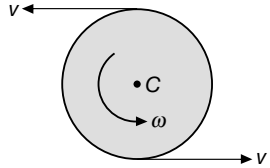
$$mg\cos\theta = \frac{mv^2}{\ell}$$

$\Rightarrow \quad mg\cos\theta = 2mg(1-\cos\theta)$

$\Rightarrow \quad \cos\theta = \frac{2}{3}$

Hence, the correct answer is (B).

34.



$$2(mR^2)(2\pi - \theta) = \frac{1}{2}MR^2\theta$$

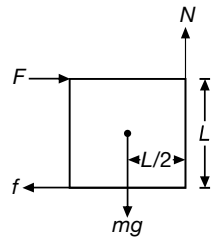
$\Rightarrow \quad 4\pi m - 2m\theta = \frac{M}{2}\theta$

$\Rightarrow \quad \theta = \frac{8\pi m}{4m + M}$

Hence, the correct answer is (D).

35. At the critical condition, normal reaction N will pass through point P. In this condition

$\tau_N = 0 = \tau_f$ (about P)



the block will topple when

$\tau_F > \tau_{mg}$

$\Rightarrow \quad FL > (mg)\frac{L}{2}$

$\Rightarrow \quad F > \frac{mg}{2}$

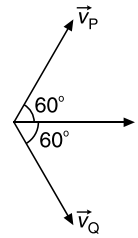
Therefore, the minimum force required to topple the bock is

$$F_{\min} = \frac{mg}{2}$$

Hence, the correct answer is (B).

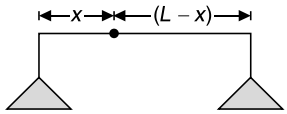
36. Relative velocity is

$$\left|\vec{v}_P - \vec{v}_Q\right| = 2R\omega\cos 30° = \sqrt{3}R\omega$$



Hence, the correct answer is (D).

37. When m is placed in left pan, m_1 is placed in right pan. So,



$$mgx = m_1 g(L-x) \qquad \ldots(1)$$

When m is placed in right pan, m_2 is placed in left pan.
So, $m_2 gx = mg(L-x)$ $\ldots(2)$

Divide (1) by (2), we get

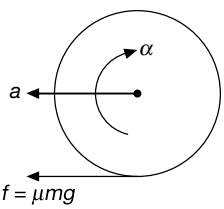
$$m^2 = m_1 m_2$$

$$\Rightarrow \quad m = \sqrt{m_1 m_2}$$

Hence, the correct answer is (B).

38. $f = \mu mg$, $a = \mu g$

$$\alpha = \frac{\tau}{I} = \frac{\mu mgR}{\frac{2}{5}mR^2} = \frac{5}{2}\frac{g\mu}{R}$$



Now, $t = \frac{v_0}{a} = \frac{\omega_0}{\alpha}$

$$\Rightarrow \quad \frac{a}{\alpha} = \frac{v_0}{\omega_0}$$

$$\Rightarrow \quad \frac{2R}{5} = \frac{v_0}{\omega_0}$$

$$\Rightarrow \quad 5v_0 = 2R\omega_0$$

Hence, the correct answer is (B).

39. $T = \frac{mv^2}{r}$

$$\Rightarrow \quad 16 = \frac{(16)v^2}{144}$$

$$\Rightarrow \quad v = 12 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

40. Axis of pure rotation will be the point of intersection of the lines perpendicular to both the straight lines.
A line perpendicular to $2x - 3y = 2$ and passing through $(4, 2)$ is $2y + 3x = 16$ and a line perpendicular to $3x + 4y = 7$ and passing through $(1, 1)$ is $3y - 4x = -1$

The intersection of $2y + 3x = 16$ and $3y - 4x = -1$ is

$$\left(\frac{50}{17}, \frac{61}{17}\right)$$

Hence, the correct answer is (D).

41. Since, retarding torque is constant, so, angular retardation say α will also be constant. Applying $\omega^2 = \omega_0^2 - 2\alpha\theta$ we get,

$$\left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 - 2\alpha\theta_1 \qquad \ldots(1)$$

and $0 = \left(\frac{\omega_0}{2}\right)^2 - 2\alpha\theta_2$ $\ldots(2)$

Solving equations (1) and (2), we get

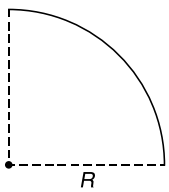
$$\theta_2 = \frac{\theta_1}{3}$$

So, the disc will make $\frac{n}{3}$ more rotations before it comes to rest.

Hence, the correct answer is (B).

42. $\ell = \frac{2\pi R}{4}$

$$\Rightarrow \quad R = \frac{2\ell}{\pi}$$



Since, $I = mR^2 = m\left(\frac{2\ell}{\pi}\right)^2$

$$\Rightarrow \quad I = \frac{2m\ell^2}{5} \qquad \{\because \pi^2 \approx 10\}$$

Hence, the correct answer is (D).

43. $\begin{pmatrix}\text{MI of}\\ \text{Given}\\ \text{Shape}\end{pmatrix} = \begin{pmatrix}\text{MI of Complete}\\ \text{Disc of}\\ \text{Radius } R\end{pmatrix} - \begin{pmatrix}\text{MI of Removed}\\ \text{Portion of}\\ \text{Radius } R/3\end{pmatrix}$

$$\Rightarrow \quad I_{\text{remaining}} = I_{\text{whole}} - I_{\text{removed}}$$

$$\Rightarrow \quad I = \frac{1}{2}(9M)(R)^2 - \left[\frac{1}{2}m\left(\frac{R}{3}\right)^2 + \frac{1}{2}m\left(\frac{2R}{3}\right)^2\right] \qquad \ldots(1)$$

where, $m = \sigma a = \frac{9M}{\pi R^2} \times \pi\left(\frac{R}{3}\right)^2 = M$

Substituting in equation (1), we get

$$I = 4MR^2$$

Hence, the correct answer is (A).

CHAPTER 3

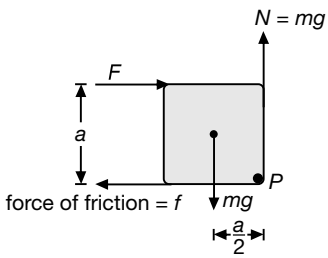
44. If m be the mass of cube and a be the side of cube, then the cube will slide when

$$F > \mu mg \qquad \ldots(1)$$

and it will topple if torque due to F about P, is greater than torque due to mg about P i.e.,

$$Fa > \left(\frac{a}{2}\right)mg$$

$$\Rightarrow \quad F > \frac{1}{2}mg \qquad \ldots(2)$$



From equations (1) and (2), we observe that cube will topple before sliding if

$$\mu > \frac{1}{2}$$

Hence, the correct answer is (D).

45. Mass of cotton pad after time t is

$$m = \mu t$$

Applying conservation of angular momentum, we get

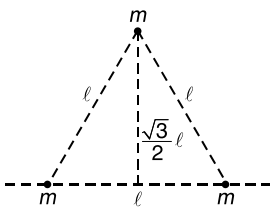
$$\left(\frac{m_0 r^2}{2}\right)\omega = \left(\frac{m_0 r^2}{2} + \mu t r^2\right)\frac{\omega}{2}$$

$$\Rightarrow \quad m_0 r^2 = \frac{m_0 r^2}{2} + \mu t r^2$$

$$\Rightarrow \quad t = \frac{m_0}{2\mu}$$

Hence, the correct answer is (B).

46.



$$I = m(0)^2 + m(0)^2 + m\left(\frac{\sqrt{3}}{2}\ell\right)^2$$

$$\Rightarrow \quad I = \frac{3}{4}m\ell^2$$

Hence, the correct answer is (B).

47. When the string is cut the weight Mg of rod acting at centre produces the torque about P.

$$\Rightarrow \quad \tau = Mg\left(\frac{L}{2}\right)$$

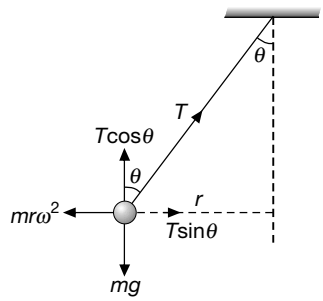
Also, $\tau = I\alpha$

$$\Rightarrow \quad Mg\left(\frac{L}{2}\right) = \left(\frac{ML^2}{3}\right)\alpha$$

$$\Rightarrow \quad \alpha = \frac{3g}{2L}$$

Hence, the correct answer is (D).

48. $T\sin\theta = m(L\sin\theta)\omega^2$



$$\Rightarrow \quad 324 = (0.5)(0.5)\omega^2$$

$$\Rightarrow \quad \omega^2 = \frac{324}{0.5 \times 0.5}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{324}{0.5 \times 0.5}}$$

$$\Rightarrow \quad \omega = \frac{18}{0.5} = 36 \text{ rads}^{-1}$$

Hence, the correct answer is (B).

49. For equilibrium of rod (considering limiting friction), we have

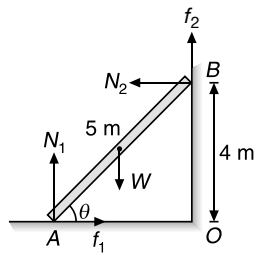
$$f_1 = N_2$$

$$\Rightarrow \quad \mu N_1 = N_2 \qquad \ldots(1)$$

Also, $-W + N_1 + f_2 = 0$

$$\Rightarrow \quad N_1 + \mu N_2 = W$$

$$\Rightarrow \quad N_1 + \mu^2 N_1 = W \qquad \ldots(2)$$



Balancing all torques about point A, we get

$\Rightarrow \quad -W\left(\frac{3}{2}\right)+f_2(3)+N_2(4)=0$

$\Rightarrow \quad 3f_2+4N_2=\frac{3W}{2}$

$\Rightarrow \quad 3\mu N_2+4N_2=\frac{3W}{2}$

$\Rightarrow \quad \mu N_1(3\mu+4)=\frac{3}{2}N_1(1+\mu^2)$

$\Rightarrow \quad 2(3\mu^2+4\mu)=3+3\mu^2$

$\Rightarrow \quad 6\mu^2+8\mu=3+3\mu^2$

$\Rightarrow \quad 3\mu^2+8\mu-3=0$

$\Rightarrow \quad 3\mu^2+9\mu-\mu-3=0$

$\Rightarrow \quad 3\mu(\mu+3)-1(\mu+3)=0$

$\Rightarrow \quad (3\mu-1)(\mu+3)=0$

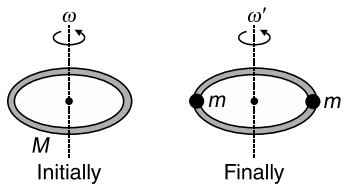
Since $\mu \neq -3$

$\Rightarrow \quad \mu=\frac{1}{3}$

Hence, the correct answer is (B).

50. By Law of Conservation of Angular Momentum

$$(MR^2)\omega=MR^2\omega'+2mR^2\omega'$$



$\Rightarrow \quad \omega'=\left(\frac{M}{M+2m}\right)\omega$

Hence, the correct answer is (B).

51. By Law of Conservation of Angular Momentum

$$\sum mvr=(I_{\text{system}})\omega$$

$\Rightarrow \quad mv\frac{\ell}{2}=\frac{(2m)(2\ell)^2}{12}\omega=\frac{2m(4\ell^2)}{12}\omega$

$\Rightarrow \quad \omega=\frac{3v}{4\ell}(\text{anticlockwise})$

Hence, the correct answer is (A).

52. $t=\frac{v_0}{a}=\frac{\omega_0}{\alpha}$

$\Rightarrow \quad v_0=\left(\frac{a}{\alpha}\right)\omega_0$

where $\alpha=\frac{\tau}{I}=\frac{fR}{I}=\frac{(\mu mg)R}{\frac{1}{2}mR^2}=\frac{2\mu g}{R}$

$\Rightarrow \quad v_0=\frac{(\mu g)}{\frac{2\mu g}{R}}\omega_0=\frac{r\omega_0}{2}$

Since $v_0=\frac{r\omega_0}{2}$ i.e., $v_0<r\omega_0$, so f acts such that it decreases v_0 and hence acts opposite to v_0 i.e., backwards. Also, the point of contact of disc with the ground has non-zero velocity as $v_0<r\omega_0$.

Hence, the correct answer is (B).

53. At the topmost point of the loop minimum value of linear speed of centre of sphere is

$$v=\sqrt{gR}$$

So, translational kinetic energy at topmost point is

$$K_T=\frac{1}{2}mv^2=\frac{1}{2}mgR$$

In case of pure rolling of a solid sphere the ratio of rotational to translational kinetic energy is

$$\frac{K_R}{K_T}=\frac{k^2}{r^2}=\frac{2}{5}$$

So, total kinetic energy at topmost point is

$$K=\left(\frac{5+2}{5}\right)K_T=\frac{7}{5}\left(\frac{1}{2}mgR\right)=\frac{7}{10}mgR$$

Now by Law of Conservation of Mechanical Energy, we have

$$\frac{7}{10}mgR=mg(h-2R)$$

$\Rightarrow \quad h=2.7\text{R}$

Hence, the correct answer is (D).

54. For pure rolling, $a_{\text{cm}}=R\alpha$, where $\alpha=\frac{\tau}{I}=\frac{F(h-R)}{\frac{2}{5}mR^2}$

$\Rightarrow \quad \frac{F}{m}=R\left[\frac{F(h-R)}{\frac{2}{5}mR^2}\right]$

$\Rightarrow \quad h-R=0.4R$

$\Rightarrow \quad h=1.4R$

Hence, the correct answer is (C).

55. For a given perimeter area of a circle is maximum, i.e., distribution of mass is at a maximum distance from the axis in case of a circle.

Hence, the correct answer is (A).

CHAPTER 3

56. Mass of the loop = $M = L\rho$

Further if r is the radius of the loop, then

$$2\pi r = L$$

$$\Rightarrow \quad r = \frac{L}{2\pi}$$

Moment of Inertia about XX' is $I = \frac{3}{2}Mr^2$

$$\Rightarrow \quad I = \frac{3}{2}(L\rho)\frac{L^2}{(2\pi)^2} = \frac{3\rho L^3}{8\pi^2}$$

Hence, the correct answer is (D).

57. By Law of Conservation of Angular Momentum

$$mvr = \left(I_{\text{system}}\right)\omega$$

$$\Rightarrow \quad mvr = \left(I + mR^2\right)\omega$$

$$\Rightarrow \quad \omega = \frac{mvr}{I + mR^2}$$

Hence, the correct answer is (A).

58. Mass of the element dx is $m = \frac{M}{L}dx$

This element needs centripetal force for rotation, so

$$dF = mx\omega^2 = \left(\frac{M}{L}x\omega^2 dx\right)$$

$$\Rightarrow \quad F = \int_0^L dF = \frac{m}{L}\omega^2\int_0^L x dx = \frac{M\omega^2 L}{2}$$

This is the force exerted by the liquid at the other end.
Hence, the correct answer is (A).

59. Friction is not sufficient for pure rolling. Therefore, maximum friction will act upwards in all the three bodies. So, linear acceleration a of all the three bodies will be same and equal to $(g\sin\theta - \mu g\cos\theta)$. Therefore, time taken by all the three will be same.
Hence, the correct answer is (D).

60. Linear velocity of all three will be same. However, ring having maximum moment of inertia will have least angular acceleration and hence least rotational kinetic energy.
Hence, the correct answer is (A).

61. At any angle θ,

$$T = \frac{mv^2}{r} + mg\cos\theta$$

At mean position $\theta = 0°$

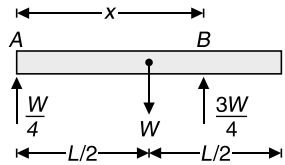
And $v = \sqrt{2gr}$

$$\Rightarrow \quad T = \frac{m(2gr)}{r} + mg$$

$$\Rightarrow \quad T = 3mg$$

Hence, the correct answer is (B).

62.



By Law of Conservation of Moments,

$$\begin{pmatrix}\text{Total Clockwise}\\ \text{Moments}\end{pmatrix} = \begin{pmatrix}\text{Total Anticlockwise}\\ \text{Moments}\end{pmatrix}$$

$$\Rightarrow \quad W\frac{L}{2} = \frac{W}{4}(0) + \left(\frac{3W}{4}\right)x$$

$$\Rightarrow \quad x = \frac{2L}{3}$$

Hence, the correct answer is (C).

63. $\frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2$

$$\Rightarrow \quad \frac{1}{2}(3)(9) = \frac{1}{2}(27)v^2$$

$$\Rightarrow \quad v = 1\ \text{ms}^{-1}$$

Hence, the correct answer is (A).

64. Since $L = I\omega$

$$\Rightarrow \quad L = \left(mr^2\right)\omega$$

when r becomes $\frac{r}{2}$, L becomes $\frac{L}{4}$.

Hence, the correct answer is (A).

65. If v be the linear velocity of centre of mass of the spherical body and ω be its angular velocity about centre of mass, then

$$\omega = \frac{v}{2R}$$

KE of spherical rolling body is

$$K_1 = \frac{1}{2}mv^2 + \frac{1}{2}\text{I}\omega^2$$

$$\Rightarrow \quad K_1 = \frac{1}{2}mv^2 + \frac{1}{2}(2mR^2)\left(\frac{v^2}{4R^2}\right)$$

$$\Rightarrow \quad K_1 = \frac{3}{4}mv^2 \qquad \ldots(1)$$

Speed of the block will be

$$v' = v_{cm} + R\omega = 2R\omega + R\omega = 3R\omega$$

$$\Rightarrow \quad v' = 3R\omega = (3R)\left(\frac{v}{2R}\right) = \frac{3}{2}v$$

So, KE of block is $K_2 = \frac{1}{2}mv'^2$

$$\Rightarrow \quad K_2 = \frac{1}{2}m\left(\frac{3}{2}v\right)^2 = \frac{9}{8}mv^2 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{K_1}{K_2}=\frac{2}{3}$$

Hence, the correct answer is (B).

66. $T = Ar^n$

$$\Rightarrow \quad Ar^n = \frac{mv^2}{r}$$

Further, by Law of Conservation of Angular Momentum

$$L = mvr = \text{constant}$$

$$\Rightarrow \quad v = \frac{L}{mr}$$

$$\Rightarrow \quad Ar^n = \frac{m}{r}\left(\frac{L}{mr}\right)^2$$

$$\Rightarrow \quad Ar^n = \frac{L^2}{m}r^{-3}$$

$$\Rightarrow \quad n = -3 \qquad \left\{\because \frac{L^2}{m} = \text{constant}\right\}$$

Hence, the correct answer is (D).

67. According to Angular Impulse – Angular Momentum Theorem, we have

$$\vec{\tau}_{av}\Delta t = \Delta\vec{L} \qquad \ldots(1)$$

where, Δt = time of flight $= \dfrac{2u\sin\theta}{g}$

$|\Delta\vec{L}| = |\vec{L}_f - \vec{L}_i|$ about point of projection

$$\Rightarrow \quad \Delta\vec{L} = (mu\sin\theta)(\text{range}) - 0$$

$$\Rightarrow \quad \Delta\vec{L} = \frac{(mu\sin\theta)(u^2\sin 2\theta)}{g}$$

$$\Rightarrow \quad \Delta\vec{L} = \frac{mu^3\sin\theta\sin 2\theta}{g}$$

$$\Rightarrow \quad |\vec{\tau}_{av}| = \frac{\Delta\vec{L}}{\Delta t} = \left(\frac{mu^3\sin\theta\sin 2\theta}{g}\right)\left(\frac{g}{2u\sin\theta}\right)$$

$$\Rightarrow \quad |\vec{\tau}_{av}| = \frac{mu^2\sin(2\theta)}{2}$$

Hence, the correct answer is (C).

68. Equilibrium of m gives

$$T = mg \qquad \{T = \text{Tension in string}\}$$

Net torque about point of contact of spool should be zero. Hence,

$$(2R)(Mg\sin\alpha) = TR$$

$$\Rightarrow \quad 2Mg\sin\alpha = mg$$

$$\Rightarrow \quad m = 2M\sin\alpha$$

Hence, the correct answer is (A).

69. $M_1 = M_2$

$$\Rightarrow \quad \left[(\pi R_1^2)t\right]d_1 = \left[(\pi R_2^2)t\right]d_2$$

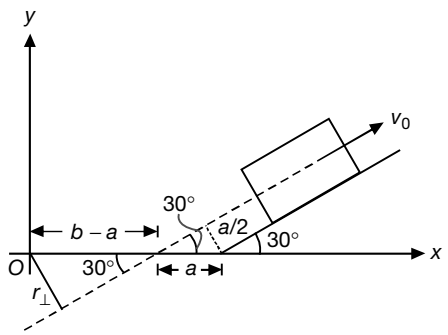
where t is thickness of both discs.

$$\Rightarrow \quad \frac{R_1^2}{R_2^2} = \frac{d_2}{d_1}$$

$$\Rightarrow \quad \frac{\frac{1}{2}MR_1^2}{\frac{1}{2}MR_2^2} = \frac{d_2}{d_1}$$

Hence, the correct answer is (B).

70. $r_\perp = (b-a)\sin 30° = \dfrac{b-a}{2}$



$$\Rightarrow \quad L = mv_0 r_\perp = \frac{mv_0(b-a)}{2}$$

Hence, the correct answer is (D).

71. $Mg = m\ell\omega^2$

$$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{Mg}{m\ell}}$$

Hence, the correct answer is (A).

72. $a = \dfrac{g}{\left(1+\dfrac{I}{mr^2}\right)}$

Since $v^2 - u^2 = 2as$

$$\Rightarrow \quad v^2 = 2ah$$

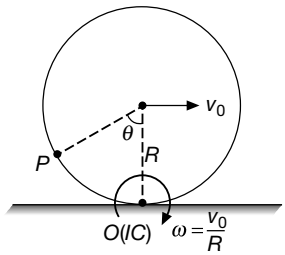
$$\Rightarrow \quad v^2 = \frac{2(mgr^2)h}{I+mr^2}$$

$$\Rightarrow \quad v = r\sqrt{\frac{2\,mgh}{I+mr^2}}$$

$$\Rightarrow \quad \omega = \frac{v}{r} = \sqrt{\frac{2\,mgh}{I+mr^2}}$$

Hence, the correct answer is (B).

73. The combined effect of translation and rotation is equivalent to a single effect of rotation about the point having zero velocity i.e., IC just like the hoop being pinned at IC and rotating about IC.



So, this can be assumed as a pure rotation about point of contact say O (i.e., IC) with angular velocity $\omega = \frac{v}{R}$, where R is the radius of hoop. Speed of P will be

$$v_P = (OP)\omega = \left(2R\sin\frac{\theta}{2}\right)\omega$$

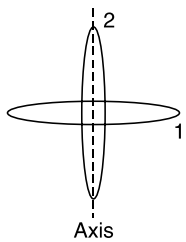
$$\Rightarrow \quad v_P = (2R\omega)\sin\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad v_P = 2v_0\sin\left(\frac{\theta}{2}\right)$$

Hence, the correct answer is (B).

74. $I_1 = mr^2$

I_2 =moment of inertia about any one of the diameters



$$= \frac{1}{2}mr^2$$

$$\Rightarrow \quad I = I_1 + I_2 = \frac{3}{2}mr^2$$

Hence, the correct answer is (D).

75. $\vec{L} = m(\vec{r} \times \vec{v})$

$$\Rightarrow \quad \vec{L} = 2\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 2 & -2 & 2 \end{vmatrix}$$

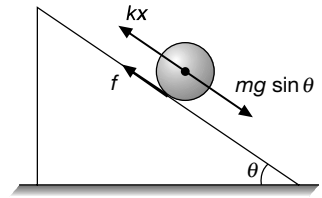
$$\vec{L} = 2\left(2\hat{i} - 2\hat{j} - 4\hat{k}\right)$$

$$\Rightarrow \quad \vec{L} = 4\hat{i} - 4\hat{j} - 8\hat{k}$$

$$\Rightarrow \quad L_z = -8 \text{ kgm}^2\text{s}^{-1}$$

Hence, the correct answer is (D).

76. Initially the spring force kx is less than $mg\sin\theta$, i.e., the cylinder is accelerated downward or force of friction f is upwards. It will reverse its direction when $kx > mg\sin\theta$.

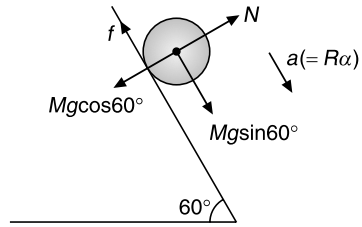


Hence, the correct answer is (D).

77. $\omega = \frac{v}{R}$ and $v_c = 0$

Hence, the correct answer is (C).

78. Let friction act on cylinder in backward direction



Since, $a = R\alpha$ …(1)

$$\Rightarrow \quad mg\sin 60° - f = Ma \qquad \ldots(2)$$

Since $fR = I\alpha$

$$\Rightarrow \quad fR = \frac{MR^2}{2}\left(\frac{a}{R}\right)$$

$$\Rightarrow \quad f = \frac{Ma}{2} \qquad \ldots(3)$$

$$\Rightarrow \quad Mg\sin 60° = \frac{3}{2}Ma$$

$$\Rightarrow \quad a = \frac{2}{3}g\sin 60° = \frac{g}{\sqrt{3}}$$

$$\Rightarrow \quad f = \frac{Mg}{2\sqrt{3}}$$

For sliding, we have

$$\Rightarrow \quad f = \mu Mg\cos 60°$$

$$\Rightarrow \quad \frac{Mg}{2\sqrt{3}} = \left(\frac{2-3x}{\sqrt{3}}\right)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad 2 - 3x = 1$$

$$\Rightarrow \quad 3x = 1$$

$$\Rightarrow \quad x = \frac{1}{3}m$$

Hence, the correct answer is (A).

79. Since spheres are smooth, so no transfer of angular momentum takes place from A to B. However, sphere A only transfers its linear velocity v to sphere B and stops. Hence, we conclude that A stops but continues to rotate with same angular speed ω and B moves with speed of A but with zero angular speed.
Hence, the correct answer is (B).

80. Consider the origin at $x = 0$. Let the equilibrium be established at a distance r from origin.
Then by Law of Conservation of Moments

$$\left[1^2 + 2^2 + 3^2 + \ldots\ldots + (100)^2\right] = (1 + 2 + 3 + \ldots\ldots + 100)r$$

$$\Rightarrow \quad \frac{(100)(100+1)(200+1)}{6} = \frac{(100)(100+1)}{2}r$$

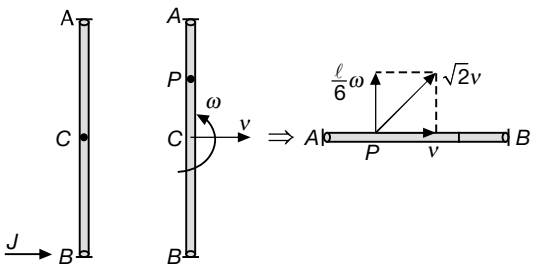
$$\Rightarrow \quad r = \frac{201}{3}$$

$$\Rightarrow \quad r = 67 \text{ cm}$$

Hence, the correct answer is (B).

81. Angular momentum of system cannot remain conserved as some external unbalanced torque is present due to forces at axles. Kinetic energy is not conserved, because slipping is there and work is done against friction.
Hence, the correct answer is (C).

82. Let v and ω be the linear and angular speeds of the rod after applying an impulse J at B. Then
Impulse = Change in Momentum



$$\Rightarrow \quad mv = J$$

$$\Rightarrow \quad v = \frac{J}{m} \qquad \ldots(1)$$

$$\text{Also, } \begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix} = \begin{pmatrix}\text{Change in Angular}\\ \text{Momentum}\end{pmatrix}$$

$$\Rightarrow \quad I\omega = J\left(\frac{\ell}{2}\right)$$

$$\Rightarrow \quad \frac{m\ell^2}{12}\omega = J\left(\frac{\ell}{2}\right)$$

$$\Rightarrow \quad \omega = \frac{6J}{m\ell} \qquad \ldots(2)$$

After the given time $t = \dfrac{\pi m\ell}{12J}$, the rod will rotate an angle

$$\theta = \omega t = \left(\frac{6J}{m\ell}\right)\left(\frac{\pi m\ell}{12J}\right) = \frac{\pi}{2}$$

$$\Rightarrow \quad v = \left(\frac{\ell}{6}\right)\omega = \left(\frac{\ell}{6}\right)\left(\frac{6J}{m\ell}\right) = \frac{J}{m}$$

$$\Rightarrow \quad |\vec{v}_P| = \sqrt{2}v = \sqrt{2}\frac{J}{m}$$

Hence, the correct answer is (C).

83. Net external torque is zero. Therefore, angular momentum of system will remain conserved, i.e.,

$$L_i = L_f$$

Initial angular momentum $L_i = 0$, so final angular momentum should also be zero

$$\begin{pmatrix}\text{Angular}\\ \text{Momentum of}\\ \text{Man in CW sense}\end{pmatrix} + \begin{pmatrix}\text{Angular Momentum}\\ \text{of Platform in}\\ \text{CCW sense}\end{pmatrix} = 0$$

$$\Rightarrow \quad mvr - \left(I_{\text{platform}}\right)\omega = 0$$

$$\Rightarrow \quad \omega = \frac{mv_0 r}{I} = \frac{(70)(1)(2)}{200}$$

$$\Rightarrow \quad \omega = 0.7 \text{ rads}^{-1}$$

Hence, the correct answer is (B).

84. Angular velocity of man relative to platform is

$$\omega_r = \omega + \frac{v_0}{r} = 0.7 + \frac{1}{2} = 1.2 \text{ rads}^{-1}$$

If t be the time taken by man to complete one round around the platform, then

$$t = \frac{2\pi}{\omega_r}$$

$$\Rightarrow \quad t = \frac{2\pi}{1.2} \text{ s}$$

Angle rotated with respect ground in this time is

$$\theta = \left(\frac{v_0}{r}\right)t = \left(\frac{1}{2}\right)\left(\frac{2\pi}{1.2}\right) = \frac{5\pi}{6}$$

Hence, the correct answer is (B).

85. Angular retardation $\alpha = \dfrac{\tau}{I} = \dfrac{\mu mgR}{mR^2} = \dfrac{\mu g}{R}$
Since, $\omega = \omega_0 - \alpha t$

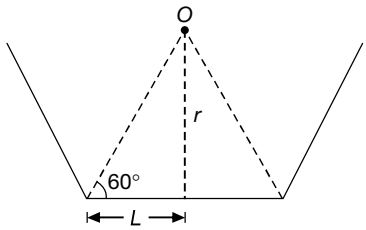
$$\Rightarrow \quad t = \frac{\omega_0 - \omega}{\alpha}$$

$$\Rightarrow \quad t = \frac{\omega_0 - \dfrac{\omega_0}{2}}{\dfrac{\mu g}{R}} = \frac{\omega_0 R}{2\mu g}$$

Hence, the correct answer is (D).

86. Length of each side is $2L$, so if r is the perpendicular distance of wire from the centre of mass (in this case, the geometrical centre), then

$$r = \sqrt{3}\,L$$



The moment of inertia about O is

$$I = 6\left(I_{\text{one side}}\right)$$

$$\Rightarrow \quad I = 6\left(\frac{m(2L)^2}{12} + mr^2\right)$$

$$\Rightarrow \quad I = 6\left(\frac{mL^2}{3} + 3mL^2\right) = 20\ \text{mL}^2$$

Hence, the correct answer is (B).

87. Moment of inertia about z-axis is $\frac{1}{2}\,\text{MR}^2$

Perpendicular distance on $y = x + c$ from origin is

$$r_\perp = d = \left|\frac{0 - 0 + c}{\sqrt{2}}\right| = \frac{c}{\sqrt{2}}$$

According to Parallel Axis Theorem, we have

$$I = I_G + Md^2$$

$$\Rightarrow \quad \frac{1}{2}MR^2 = \frac{1}{4}MR^2 + M\left(\frac{c}{\sqrt{2}}\right)^2$$

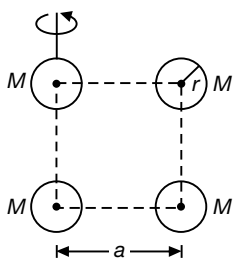
$$\Rightarrow \quad c = \frac{R}{\sqrt{2}}$$

Hence, the correct answer is (A).

88. $I_{\text{system}} = 4I_{\text{sphere}} + 2Ma^2$

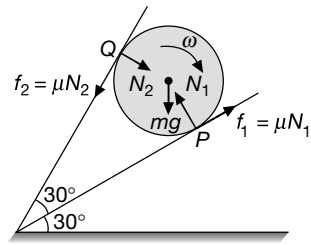
$$\Rightarrow \quad I_{\text{system}} = 4\left(\frac{2}{5}Mr^2\right) + 2Ma^2$$

$$\Rightarrow \quad I_{\text{system}} = \frac{2}{5}M\left(4r^2 + 5a^2\right)$$



Hence, the correct answer is (D).

89. Let μ be the friction coefficient between sphere and each wall. Free body diagram of sphere is



Net force on the sphere in horizontal direction is zero, so

$$N_1\cos 60° + \mu N_2\cos 60° = N_2\cos 30° + \mu N_1\cos 30°$$

$$\Rightarrow \quad N_1 + \mu N_2 = \sqrt{3}\left(N_2 + \mu N_1\right)$$

$$\Rightarrow \quad N_1\left(1 - \sqrt{3}\mu\right) = N_2\left(\sqrt{3} - \mu\right)$$

$$\Rightarrow \quad \frac{N_1}{N_2} = \frac{\sqrt{3} - \mu}{1 - \sqrt{3}\mu}$$

Substituting $\mu = \frac{1}{3}$ we get,

$$\frac{N_1}{N_2} = \frac{\sqrt{3} - \frac{1}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{3\sqrt{3} - 1}{3 - \sqrt{3}} = 1 + \frac{4}{\sqrt{3}}$$

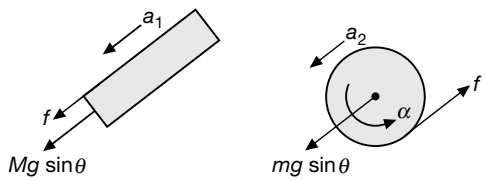
Since, $\frac{f_1}{f_2} = \frac{\mu N_1}{\mu N_2} = 1 + \frac{4}{\sqrt{3}}$

Hence, the correct answer is (B).

90. For Plank, $Mg\sin\theta + f = Ma_1$ …(1)

For Sphere, $mg\sin\theta - f = ma_2$ …(2)

where $a_2 - R\alpha = a_1$ …(3)



Since, $fR = \left(\frac{2}{5}mR^2\right)\alpha$ …(4)

Solving equations (1), (2), (3) and (4), we get

$$f = 0$$

Hence, the correct answer is (A).

91. Since $\tau = I\alpha$

$$\Rightarrow \quad \tau = \left[\frac{m(2\ell)^2}{12}\right]\left(\frac{\omega}{t}\right)$$

$$\Rightarrow \quad \tau = \frac{m\ell^2\omega}{3t}$$

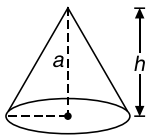
Hence, the correct answer is (B).

92. $KE = \frac{1}{2}m\left(v_0^2\right) + \frac{1}{2}\left(mR^2\right)\left(\frac{v_0}{R}\right)^2$

$+\frac{1}{2}(3m)\left(\sqrt{2}v_0\right)^2 + \frac{1}{2}(m)(2v_0)^2 = 6mv_0^2$

Hence, the correct answer is (B).

93. $I = \frac{3}{10}Ma^2$



Hence, the correct answer is (D).

94. From conservation of angular momentum $(I\omega = \text{constant})$, angular velocity will remain half. As,

$$K = \frac{1}{2}I\omega^2$$

The rotational kinetic energy will become half.
Hence, the correct answer is (B).

95. Moment of inertia of semi-circular portions about x and y axes are same. But moment of inertia of straight portions about x-axis is zero. So, we have

$$I_x < I_y$$

$$\Rightarrow \quad \frac{I_x}{I_y} < 1$$

Hence, the correct answer is (B).

96. By Law of Conservation of Angular Momentum

$$\left(\sum mvr\right)_{\text{about }0} = \left(\mathrm{I}_{\substack{\text{system}\\ \text{about }0}}\right)\omega$$

$$\Rightarrow \quad MV\frac{L}{4} = \left(\mathrm{I}_{\text{rod about }0} + \mathrm{I}_{\text{insect about }0}\right)\omega$$

$$\Rightarrow \quad MV\frac{L}{4} = \left[\frac{ML^2}{12} + M\left(\frac{L}{4}\right)^2\right]\omega$$

$$\Rightarrow \quad MV\frac{L}{4} = ML^2\left(\frac{1}{12} + \frac{1}{16}\right)\omega$$

$$\Rightarrow \quad MV\frac{L}{4} = ML^2\left(\frac{4+3}{48}\right)\omega$$

$$\Rightarrow \quad \omega = \frac{12}{7}\frac{V}{L}$$

Hence, the correct answer is (B).

97. $R.K.E. = \frac{1}{2}\mathrm{I}\omega^2$

$$\Rightarrow \quad R.K.E. = \frac{1}{2}(2mr^2)(2\pi n)^2$$

$$\Rightarrow \quad R.K.E. = 4\pi^2 n^2\ \mathrm{mr}^2$$

Hence, the correct answer is (B).

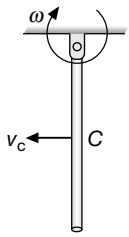
98. The rod will rotate about A. Therefore, by Law of Conservation of Mechanical Energy, we get

$$\begin{pmatrix}\text{Decrease in}\\ \text{Gravitational}\\ \text{Potential Energy}\end{pmatrix} = \begin{pmatrix}\text{Increase in Rotational}\\ \text{Kinetic Energy}\\ \text{about } A\end{pmatrix}$$

$$\Rightarrow \quad mg\left(\frac{\ell}{2}\right) = \frac{1}{2}I_A\omega^2$$

$$\Rightarrow \quad mg\left(\frac{\ell}{2}\right) = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

$$\Rightarrow \quad \omega^2 = \frac{3g}{\ell} \qquad \ldots(1)$$



Centripetal force on CM of rod in this position is

$$F_C = m\left(\frac{\ell}{2}\right)\omega^2 = \frac{3mg}{2} \qquad \{\text{towards } A\}$$

Let F be the force exerted by the hinge on the rod upwards, then

$$F - mg = ma_C = F_C$$

$$\Rightarrow \quad F - mg = \frac{3mg}{2}$$

$$\Rightarrow \quad F = \frac{5}{2}mg$$

So, the force exerted by the rod on the hinge is $\frac{5}{2}mg$, downwards
Hence, the correct answer is (B).

99. Assume mass of cylinder to be m, radius R, then linear acceleration of cylinder is zero when

$mg\sin\theta =$ frictional force (f) upwards

Now, angular acceleration about C is $\alpha = \frac{\tau}{I}$

$$\Rightarrow \quad \alpha = \frac{fR}{\frac{1}{2}mR^2} = \frac{2f}{mR}$$

$$\Rightarrow \quad \alpha = \frac{2mg\sin\theta}{mR} = \frac{2g\sin\theta}{R}$$

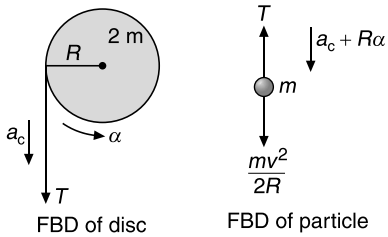
For no slipping between cylinder and plank, we have

$$a = R\alpha = 2g\sin\theta$$

Hence, the correct answer is (C).

CHAPTER 3

100. If a_C is initial acceleration of centre of mass of disc and α is its initial angular acceleration, then we have



$$T = (2m)a_C$$

Also, $TR = \frac{1}{2}(2m)R^2\alpha$

$$T = mR\alpha$$

$$\Rightarrow \quad \frac{m\left(\frac{J}{m}\right)^2}{2R} - T = m(a_c + R\alpha)$$

$$\Rightarrow \quad T + \frac{T}{2} + T = \frac{J^2}{2mR}$$

$$\Rightarrow \quad \frac{5T}{2} = \frac{J^2}{2mR}$$

$$\Rightarrow \quad T = \frac{J^2}{5mR}$$

$$\Rightarrow \quad a_C = \frac{T}{2m} = \frac{J^2}{5mR(2m)} = \frac{J^2}{10m^2R}$$

Substituting all the values, we get

$$a_c = 4 \text{ ms}^{-2}$$

Hence, the correct answer is (B).

101. $$mgh = \frac{1}{2}mv_{CM}^2\left(1 + \frac{K^2}{R^2}\right)$$

$$\Rightarrow \quad \frac{7}{10}mv_{CM}^2 = mgh$$

$$\Rightarrow \quad h = \frac{7}{10}\left(\frac{v_{CM}^2}{g}\right)$$

Hence, the correct answer is (B).

102. KE of ball in position $B = mg(R - r)$

Since, it rolls without slipping the ratio of rotational to translational kinetic energy will be $\frac{k^2}{r^2} = \frac{2}{5}$

$$\Rightarrow \quad \frac{K_R}{K_T} = \frac{k^2}{r^2} = \frac{2}{5}$$

$$\Rightarrow \quad K_T = \frac{5}{7}mg(R - r)$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{5}{7}mg(R - r)$$

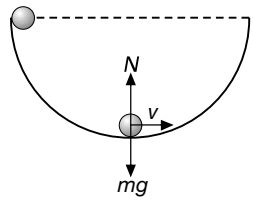
$$\Rightarrow \quad v = \sqrt{\frac{10g(R - r)}{7}}$$

$$\Rightarrow \quad \omega = \frac{v}{R - r} = \sqrt{\frac{10g}{7(R - r)}}$$

Hence, the correct answer is (B).

103. By Law of Conservation of Mechanical Energy, we get

$$mg(R - r) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{7}{10}mv^2 \qquad \ldots(1)$$



Also, $N - mg = \frac{mv^2}{R - r}$ …(2)

Solving equations (1) and (2), we get

$$N = \frac{17}{7} \text{ mg}$$

Hence, the correct answer is (D).

104. Since, $a = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}}$

$$\Rightarrow \quad a = \frac{g\sin(60°)}{1 + \frac{1}{2}}$$

$$\Rightarrow \quad a = \frac{g}{\sqrt{3}}$$

Hence, the correct answer is (C).

105. According to Parallel Axis Theorem, we have

$$I = I_G + md^2$$

In case of a circle, the distance d is equal from centre of mass of the rigid body for all points lying on it.
Hence, the correct answer is (B).

106. Since, $\tau_{ext} = 0$

$$\Rightarrow \quad I\omega = \text{constant}$$

$$\Rightarrow \quad \Delta(I\omega) = 0$$

$$\Rightarrow \quad \omega\Delta I + I\Delta\omega = 0$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega} = -\frac{\Delta I}{I}$$

Now, volume $V = \frac{4}{3}\pi R^3$

$$\Rightarrow \quad \frac{\Delta V}{V}\times 100 = 3\frac{\Delta R}{R}\times 100$$

Percentage increase in volume is 3%, so percentage increase in radius will be 1%

Further, $I = \frac{2}{5}mR^2$

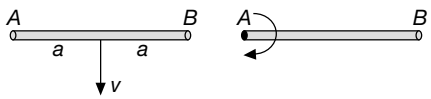
$$\Rightarrow \quad \frac{\Delta I}{I} = \frac{2\Delta R}{R}$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega} = -\frac{\Delta I}{I} = -\frac{2\Delta R}{R} = -2\%$$

Hence, the correct answer is (A).

107. Applying Law of Conservation of Angular momentum about A, we get

$$L_i = L_f$$



$$\Rightarrow \quad mva = I\omega$$

$$\Rightarrow \quad mva = \frac{m(2a)^2}{3}\omega$$

$$\Rightarrow \quad \omega = \frac{3v}{4a}$$

Hence, the correct answer is (D).

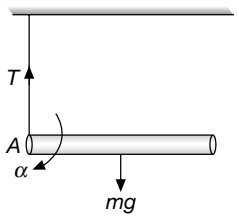
108. $K = K_T + K_R$

$$\Rightarrow \quad K = \frac{1}{2}(2m)(v^2) + \frac{1}{2}\left(mR^2 + \frac{(2R)^2}{12}\right)\left(\frac{v}{R}\right)^2$$

$$\Rightarrow \quad K = \frac{5}{3}mv^2$$

Hence, the correct answer is (D).

109. The rod will rotate about point A. Let a be the linear acceleration of centre of mass of the rod and α the angular acceleration of the rod about A. Then



$$mg - T = ma \qquad \ldots(1)$$

Also, $\alpha = \frac{\tau}{I} = \frac{mg\frac{\ell}{2}}{\frac{m\ell^2}{3}} = \frac{3g}{2\ell}$ $\qquad \ldots(2)$

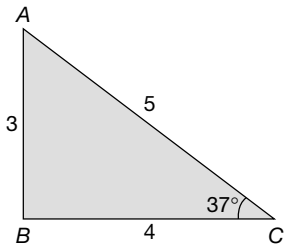
$$\Rightarrow \quad a = \frac{\ell}{2}\alpha \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$T = \frac{mg}{4}$$

Hence, the correct answer is (A).

110. $AC > BC > AB$



Moment of inertia of a body depends on distribution of mass from the axis of rotation. The mass is farthest from the axis AB. So, I_1 is maximum and nearest from the axis AC. So, I_3 is minimum or $I_3 < I_2 < I_1$.

Hence, the correct answer is (C).

111. In case of pure rolling, we have

$$\frac{K_R}{K_T} = \frac{k^2}{R^2}$$

for solid sphere $\frac{K_R}{K_T} = \frac{2}{5}$ and

for solid cylinder $\frac{K_R}{K_T} = \frac{1}{2}$

At the bottom, both have the same linear velocities, i.e., they have the same translational kinetic energies, so

$$\frac{5}{7}mgh_1 = \frac{2}{3}mgh_2$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{14}{15}$$

Hence, the correct answer is (C).

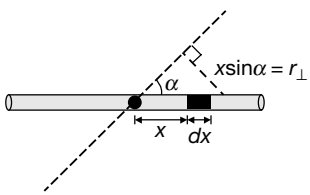
112. $\int \tau dt = \Delta L$

Since, $\tau = Fr_\perp = (2t)(R + r)$

$$\Rightarrow \quad L = \int_0^t 2t(R+r)dt = (R+r)t^2$$

Hence, the correct answer is (C).

113. The desired moment of inertia is



CHAPTER 3

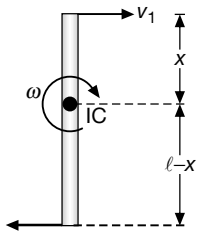
$$I = \int_{x=-\ell}^{x=+\ell} dI = \int_{-\ell}^{\ell} dm(r_\perp)^2$$

$$I = \int_{-\ell}^{\ell} \left(\frac{m}{2\ell}dx\right)(x\sin\alpha)^2$$

$$I = \frac{m\ell^2}{3}\sin^2\alpha$$

Hence, the correct answer is (B).

114.



$$\omega = \frac{v_1}{x} = \frac{v_2}{\ell - x}$$

$$\Rightarrow \quad x = \frac{v_1}{v_1 + v_2}\ell$$

Hence, the correct answer is (C).

115. By Law of Conservation of Angular Momentum

$$I\omega = \text{constant}$$

Since R goes to $\frac{R}{n}$, so I goes to $\frac{I}{n^2}$, hence ω goes to $n^2\omega$

$\{\because I\omega = \text{constant}\}$

$$\Rightarrow \quad \omega' = n^2\omega$$

$$\Rightarrow \quad \frac{2\pi}{T'} = n^2\frac{2\pi}{T}$$

$$\Rightarrow \quad T' = \frac{T}{n^2} = \frac{24}{n^2}$$

So, here we formulate a rule to calculate the new time period. If Radius of earth shrinks by $\frac{1}{n}(n>1)$ then new time period is $\frac{T}{n^2}$ i.e. $\frac{24}{n^2}$. So new time period of rotation also decreases.

Hence, the correct answer is (B).

116. Angular retardation $\alpha = \frac{\tau}{I} = \frac{(\mu mgR)}{\frac{2}{5}mR^2}$

$$\Rightarrow \quad \alpha = \frac{5\mu g}{2R}$$

$$\Rightarrow \quad \alpha = \frac{5\times 0.1\times 10}{2\times 1}$$

$$\Rightarrow \quad \alpha = 2.5 \text{ rads}^{-2}$$

Now, $0 = \omega_0 - \alpha t$

$$\Rightarrow \quad t = \frac{\omega_0}{\alpha} = \frac{40}{2.5} = 16 \text{ s}$$

Hence, the correct answer is (C).

117. Since $\theta = \frac{1}{2}\alpha t^2$

$$\Rightarrow \quad \theta = \frac{1}{2}(2)(10)^2$$

$$\Rightarrow \quad \theta = 100 \text{ rads}^{-1}$$

$$\Rightarrow \quad n = \frac{\theta}{2\pi} = \frac{100}{2\pi}$$

$$\Rightarrow \quad n \simeq 16$$

Hence, the correct answer is (B).

118. $\frac{I_2}{I_1} = 4$

$$\Rightarrow \quad \frac{M_2R_2^2}{M_1R_1^2} = 4$$

Let λ be the linear mass density. Then

$$M_1 = (2\pi R_1)\lambda \text{ and}$$

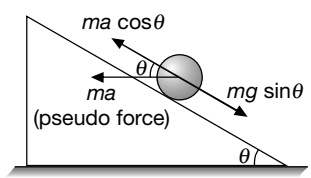
$$M_2 = (2\pi R_2)\lambda$$

$$\Rightarrow \quad \frac{R_2^3}{R_1^3} = 4$$

$$\Rightarrow \quad \frac{R_2}{R_1} = 4^{\frac{1}{3}}$$

Hence, the correct answer is (B).

119.



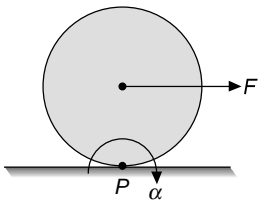
The sphere will continue pure rolling when

$$ma\cos\theta = mg\sin\theta$$

$$\Rightarrow \quad a = g\tan\theta$$

Hence, the correct answer is (D).

120. In case of rolling without slipping, the point of contact P on the ground is at rest. So, the cylinder will rotate about P with angular acceleration



$$\alpha = \frac{\tau_P}{I_P} = \frac{FR}{\frac{3}{2}MR^2}$$

$$\Rightarrow \quad \alpha = \frac{2F}{3MR}$$

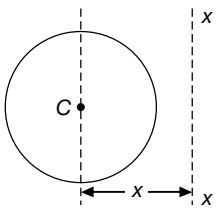
Hence, the correct answer is (B).

121. If m be the mass of sphere, then

$$I(x) = I = I_c + mx^2$$

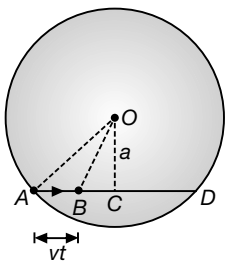
$I = I_c$ at $x = 0$

Therefore, I versus x graph is a parabola with minimum value of $I = I_c$ at $x = 0$. Therefore, the correct graph is (B).



Hence, the correct answer is (B).

122. Since, there is no external torque, angular momentum will remain conserved. The moment of inertia will first decrease till the tortoise moves from A to C and then increase as it moves from C and D. Therefore, ω will initially increase and then decrease.



Let R be the radius of platform, m the mass of disc and M is the mass of platform.

Moment of inertia when the tortoise is at A

$$I_1 = mR^2 + \frac{MR^2}{2}$$

and moment of inertia when the tortoise is at B

$$I_2 = mr^2 + \frac{MR^2}{2}$$

Here, $r^2 = a^2 + \left[\sqrt{R^2 - a^2} - vt\right]^2$

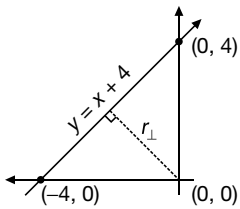
From conservation of angular momentum

$$\omega_0 I_1 = \omega(t) I_2$$

Substituting the values, we can see that variation of $\omega(t)$is non-linear.

Hence, the correct answer is (B).

123.



$$L = mvr_\perp$$

where $r_\perp = \frac{|0 - 0 - 4|}{\sqrt{2}} = \frac{4}{\sqrt{2}}$

$$\Rightarrow \quad L = (5)(3\sqrt{2})\left(\frac{4}{\sqrt{2}}\right)$$

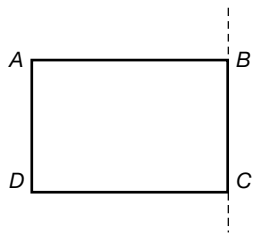
$$\Rightarrow \quad L = 60 \text{ cgs unit}$$

Hence, the correct answer is (D).

124. **Conceptual the correct answer is (C).**

125. $\frac{AB}{BC} = 2$

$$\Rightarrow \quad AB = DC = \frac{\ell}{3} \text{ and } BC = AD = \frac{\ell}{6}$$



Similarly, $m_{AB} = m_{DC} = \frac{m}{3}$

and $m_{BC} = m_{AD} = \frac{m}{6}$

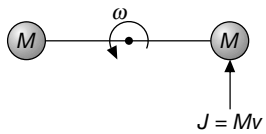
Now $I = 2I_{AB} + I_{AD} + I_{BC}$

$$\Rightarrow \quad I = 2\left\{\left(\frac{1}{3}\right)\left(\frac{m}{3}\right)\left(\frac{\ell}{3}\right)^2\right\} + \left(\frac{m}{6}\right)\left(\frac{\ell}{3}\right)^2 + 0$$

$$\Rightarrow \quad I = \frac{189}{4374}m\ell^2 = \frac{7}{162}\mathrm{m}\ell^2$$

Hence, the correct answer is (D).

126. Let ω be the angular velocity of the rod. Applying, angular impulse equals change in angular momentum about centre of mass of the system, we get



$$J\left(\frac{L}{2}\right) = \mathrm{I}_c\omega$$

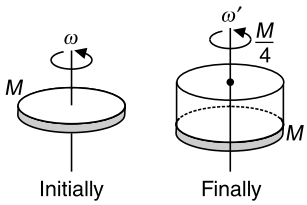
$$\Rightarrow \quad (Mv)\left(\frac{L}{2}\right) = (2)\left(\frac{ML^2}{4}\right)\omega$$

$$\Rightarrow \quad \omega = \frac{v}{L}$$

Hence, the correct answer is (A).

127. $mg\sin\theta$ component is always down the plane whether it is rolling up or rolling down. Therefore, for no slipping, sense of angular acceleration should also be same in both the cases. Therefore, force of friction f always act upwards.
Hence, the correct answer is (B).

128.



By Law of Conservation of Angular Momentum

$$\left(\frac{1}{2}MR^2\right)\omega = \left(\frac{1}{2}MR^2 + \frac{1}{2}\left(\frac{M}{4}\right)R^2\right)\omega'$$

$$\Rightarrow \quad \omega = \frac{5}{4}\omega'$$

$$\Rightarrow \quad \omega' = \frac{4\omega}{5}$$

Hence, the correct answer is (B).

129. Force of friction passes through point P. Hence, its torque about P will be zero. Hence, only $mg\sin\theta$ will provide torque to ring about P.

Since, $\tau t = \Delta L$

$$\Rightarrow \quad (mgR\sin\theta)t = L$$

Hence, the correct answer is (B).

130. Let m be the mass of the disc. Then translational kinetic energy of the disc is

$$K_T = \frac{1}{2}mv^2 \qquad \ldots(1)$$

As it ascends on a smooth track its rotational kinetic energy will remain same while translational kinetic energy will go on decreasing. At highest point.

$$K_T = mgh$$

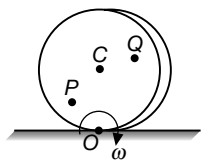
$$\Rightarrow \quad \frac{1}{2}mv^2 = mgh$$

$$\Rightarrow \quad h = \frac{v^2}{2g} = \frac{(6)^2}{2\times 10} = 1.8 \text{ m}$$

Hence, the correct answer is (B).

131. Since, net force on centre of mass is zero so, the centre of mass will not move at all. Hence, the body can only rotate about centre of mass.
Hence, the correct answer is (B).

132. In case of pure rolling bottommost point is the instantaneous centre of zero velocity.



Velocity of any point on the disc, $v = r\omega$, where r is the distance of point from O.

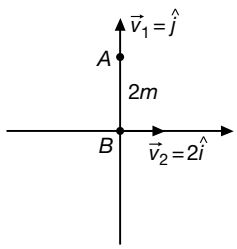
$$r_Q > r_C > r_P$$

$$\Rightarrow \quad v_Q > v_C > v_P$$

Hence, the correct answer is (A).

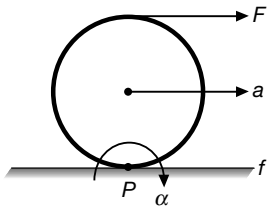
133. At $t = 2$ s, A and B are at $(0,\ 2\text{ m})$ and $(0,\ 0)$ as shown. Component of relative velocity perpendicular to AB is 2 ms^{-1}. So,

$$\omega = \frac{2}{2} = 1 \text{ rads}^{-1}$$



Hence, the correct answer is (B).

134. Let f be the friction on the ring towards right, a be the linear acceleration and α be the angular acceleration of the ring about centre of mass.



Point of contact P is momentarily at rest, about Instantaneous Centre of Zero Velocity IC (located at P), i.e., ring will rotate about P, so

$$\alpha = \frac{\tau_P}{I_P} = \frac{F(2R)}{2MR^2} = \frac{F}{MR}$$

For translational motion of ring, we have

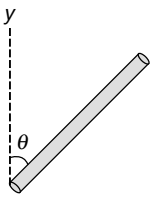
$$F + f = Ma = MR\alpha = F$$

$$\Rightarrow \quad f = 0$$

Hence, the correct answer is (B).

135. Since moment of inertia of a rod about an axis passing through one end, making angle θ with it is

$$I = \frac{1}{3}m\ell^2 \sin^2\theta$$



So, $I_x = I_y = 2\left\{\frac{m\ell^2}{3}\sin^2 45°\right\} = \frac{m\ell^2}{3}$

$$I_z = 2\left(\frac{m\ell^2}{3}\right) = \frac{2}{3}m\ell^2$$

Hence, $I_x = I_y < I_z$

Hence, the correct answer is (B).

136. $\left(\begin{array}{c}\text{Loss in}\\ \text{Gravitational}\\ \text{P.E. of}\\ \text{C.M.}\end{array}\right) = \left(\begin{array}{c}\text{Gain in}\\ \text{Rotational}\\ \text{K.E. of}\\ \text{C.M.}\end{array}\right) + \left(\begin{array}{c}\text{Gain in}\\ \text{Translational}\\ \text{K.E. of}\\ \text{C.M.}\end{array}\right)$

$$\Rightarrow \quad MgR - M'gR' = \left(\frac{1}{2}I'\omega^2\right) + \frac{1}{2}M'v_{cm}^2$$

$$\left\{\text{where } R' = \frac{R}{2} \;\&\; M' = \pi R'^2\ell\rho = \frac{\pi R^2\ell\rho}{4} = \frac{M}{4}\right\}$$

$$\Rightarrow \quad \frac{7}{8}MgR = \frac{1}{2}\left(\frac{1}{2}M'R'^2\right)\left(\frac{v_{cm}}{R'}\right)^2 + \left(\frac{1}{2}\left(\frac{M}{4}\right)v_{cm}^2\right)$$

$$\Rightarrow \quad \frac{7}{8}MgR = \frac{1}{2}\left(\frac{M}{8}\right)v_{cm}^2 + \left(\frac{M}{8}\right)v_{cm}^2$$

$$\Rightarrow \quad \frac{7}{8}MgR = \left(\frac{M}{8}\right)\left(\frac{3}{2}\right)v_{cm}^2$$

$$\Rightarrow \quad v_{cm} = \sqrt{\frac{14}{3}gR}$$

Hence, the correct answer is (B).

137. Tangential Force $= F_T = ma = m(\alpha L) = N$

$\therefore$ Limiting value of friction $(f_s)_{\max} = \mu N = \mu F_T$

$$\Rightarrow \quad (f_s)_{\max} = \mu N = \mu m\alpha L \qquad \ldots(1)$$

Further if ω is the angular velocity at time t, then

$$\omega = \alpha t \qquad \ldots(2)$$

Also, the centripetal force is

$$F_C = mL\omega^2 = mL(\alpha^2 t^2) \qquad \ldots(3)$$

For bead to slide $F_C > (f_s)_{\max}$

$$\Rightarrow \quad mL(\alpha^2 t^2) > \mu m\alpha L$$

$$\Rightarrow \quad t > \sqrt{\frac{\mu}{\alpha}}$$

So, the minimum time after which the bead begins to start is $\sqrt{\frac{\mu}{\alpha}}$

Hence, the correct answer is (A).

138. In pure rolling mechanical energy remains conserved, therefore speed will be same in both the cases. Acceleration of the sphere, a, down the plane is proportional to $\sin\theta$. So,

$$a \propto \sin\theta$$

i.e., acceleration and hence, time of descend will be different.

Hence, the correct answer is (B).

139. $\vec{v} = \vec{\omega} \times \vec{r}$

$$\Rightarrow \quad \vec{v} = (2\hat{k}) \times (2\hat{i} + 2\hat{j}) = 4(\hat{j} - \hat{i})$$

Hence, the correct answer is (B).

140. In uniform circular motion the only force acting on the particle is centripetal (towards centre). Torque of this force about the centre is zero. Hence, angular momentum about centre remain conserved.

Hence, the correct answer is (A).

CHAPTER 3

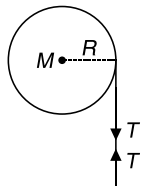
141. For Angular Momentum to be conserved $\vec{\tau}_{ext} = \vec{0}$

$$\Rightarrow \quad \frac{a}{2} = \frac{3}{-6} = \frac{6}{-12}$$

$$\Rightarrow \quad a = -1$$

Hence, the correct answer is (B).

142. Torque $\tau = TR$



$$\Rightarrow \quad I\alpha = TR$$

$$\Rightarrow \quad \left(\frac{1}{2}MR^2\right)\alpha = TR$$

$$\Rightarrow \quad T = \frac{1}{2}MR\alpha$$

$$\Rightarrow \quad T = \frac{1}{2}(50)(0.5)(4\pi)$$

$$\Rightarrow \quad T = 157 \text{ N}$$

Hence, the correct answer is (B).

143. $\tau = FR = I\alpha$

$$\Rightarrow \quad FR = \left(\frac{1}{2}MR^2\right)\alpha$$

$$\Rightarrow \quad \alpha = \frac{2F}{MR}$$

$$\Rightarrow \quad a = \frac{2F}{M}$$

$$s = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{2F}{M}\right)t^2$$

$$\Rightarrow \quad s = \left(\frac{F}{M}\right)t^2 \text{ and } \theta = \frac{1}{2}\alpha t^2$$

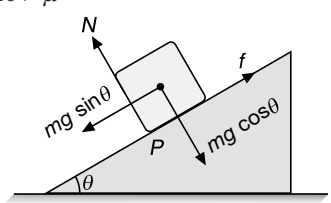
$$\Rightarrow \quad \theta = \left(\frac{F}{MR}\right)t^2$$

Hence, the correct answer is (C).

144. If m be the mass of the cube and a be the side of the cube, then the cube will slide when

$$mg\sin\theta > \mu mg\cos\theta$$

$$\Rightarrow \quad \tan\theta > \mu \qquad \ldots(1)$$



The cube will topple if torque due to $mg\sin\theta$ about P is greater than torque due to $mg\cos\theta$ about P i.e.,

$$(mg\sin\theta)\left(\frac{a}{2}\right) > (mg\cos\theta)\left(\frac{a}{2}\right)$$

Since f, the force of friction passes through P, so torque due to f about P is zero.

$$\Rightarrow \quad \tan\theta > 1 \qquad \ldots(2)$$

From equations (1) and (2), we observe that cube slides before toppling if

$$\mu < 1$$

Hence, the correct answer is (B).

145. MI of the sphere about the diameter parallel to PQ is

$$I = \frac{2}{5}MR^2$$

By parallel axis theorem, MI of the sphere about the centre of mass is

$$I = I_{cm} + Md^2$$

$$\Rightarrow \quad I_{cm} = \frac{2}{5}MR^2 - M\left(\frac{3R}{8}\right)^2 \qquad \ldots(1)$$

The MI about the axis PQ at a distance $d' = \frac{3R}{4} - \frac{3R}{8} = \frac{3R}{8}$ from the CM is

$$I_{PQ} = I_{cm} + M(d')^2$$

$$\Rightarrow \quad I_{PQ} = \left[\frac{2}{5}MR^2 - M\left(\frac{3R}{8}\right)^2\right] + M\left(\frac{3R}{8}\right)^2$$

$$\Rightarrow \quad I_{PQ} = \frac{2}{5}MR^2 = MK^2$$

$$\Rightarrow \quad K = R\sqrt{\frac{2}{5}}$$

Hence, the correct answer is (D).

146. $\vec{L} = m(\vec{r} \times \vec{v})$

Direction of $(\vec{r} \times \vec{v})$, hence the direction of angular momentum remains the same.

Hence, the correct answer is (B).

147. By Law of Conservation of Energy

$$Mgh = \frac{1}{2}Mv_{CM}^2\left(1 + \frac{K^2}{R^2}\right)$$

Hence, the correct answer is (C).

148. $R.K.E. = \frac{1}{2}I\omega^2 = 1500$

$$\Rightarrow \quad \frac{1}{2}I(\alpha t)^2 = 1500$$

$\Rightarrow \quad (1.2)(25)^2 t^2 = 3000$

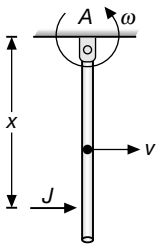
$\Rightarrow \quad t^2 = 4$

$\Rightarrow \quad t = 2$

Hence, the correct answer is (B).

149. Since, Impulse = Change in Momentum, so we get

$$J = mv \qquad \ldots(1)$$



where v is the linear speed of centre of mass of rod. Also, Angular Impulse = Change in Angular Momentum

$$\Rightarrow \quad Jx = I\omega = \left(\frac{m\ell^2}{3}\right)\omega \qquad \ldots(2)$$

$$\Rightarrow \quad v = \frac{\ell}{2}\omega \qquad \ldots(3)$$

where, ω is the angular speed of rod about point A. Solving these three equations, we get

$$x = \frac{2}{3}\ell$$

Hence, the correct answer is (B).

150. Let the extension produced in the spring due to rotation be x. Then, for the body to be in equilibrium

$$F_{\text{spring}} = F_{\text{centrifugal}}$$

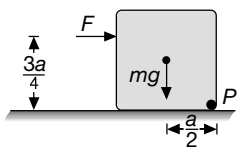
$$\Rightarrow \quad kx = m(\ell + x)\omega^2$$

$$\Rightarrow \quad x = \frac{m\ell\omega^2}{k - m\omega^2}$$

Hence, the correct answer is (B).

151. For the cube to topple about P, we must have

$$\tau_F > \tau_{mg}$$



$$\Rightarrow \quad F\left(\frac{3a}{4}\right) > (mg)\frac{a}{2}$$

$$\Rightarrow \quad F > \frac{2}{3}mg$$

So, the minimum value of F is $\frac{2}{3}mg$

Hence, the correct answer is (C).

152. $I_{\text{RIM}} = 2MR^2$

$$I_{\text{DIAMETER}} = \frac{1}{2}MR^2$$

$\Rightarrow$ Required Ratio= 4 : 1

Hence, the correct answer is (B).

153. **Conceptual the correct answer is (D).**

154. $Pd = I_A\omega$ where $I_A = \frac{1}{3}m\ell^2$

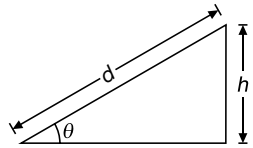
Since the distance from A to C is $\frac{\ell}{2}$, we get $v_C = \frac{\omega\ell}{2}$.

$$\Rightarrow \quad m\left(\frac{\omega\ell}{2}\right)d = \left[\frac{1}{3}m\ell^2\right]\omega$$

$$\Rightarrow \quad d = \frac{2}{3}\ell$$

Hence, the correct answer is (B).

155. $\sin\theta = \frac{h}{d}$



Also, condition for banking is

$$\tan\theta = \frac{v^2}{rg}$$

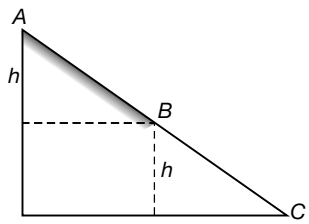
$$\Rightarrow \quad \tan\left[\sin^{-1}\left(\frac{h}{d}\right)\right] = \frac{v^2}{rg}$$

Hence, the correct answer is (B).

156. In case of pure rolling upto B, we have for a cylinder

$$\frac{K_T}{K_R} = 2$$

where $K_T = \frac{2}{3}mgh$ and $K_R = \frac{1}{3}mgh$



After B: Rotational kinetic energy is constant (because no torque is provided due to absence of friction) however translational kinetic energy increases.

CHAPTER 3

At C: $K_T = \frac{2}{3}mgh + mgh = \frac{5}{3}mgh$

while $K_R = \frac{1}{3}mgh$

So, $\frac{K_T}{K_R} = 5$

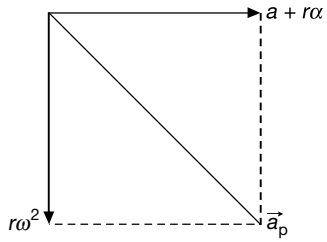
Hence, the correct answer is (B).

157. $\vec{a}_P = \vec{a}_{P_0} + \vec{a}_0$

Here, $\vec{a}_{P_0}$ =acceleration of P with respect to O, so

$$\vec{a}_{P_0} = \vec{a}_{P_{0t}} + \vec{a}_{P_{0n}}$$

$$\Rightarrow \quad \vec{a}_P = \left(\vec{a}_{P_{0t}} + \vec{a}_{P_{0n}}\right) + \vec{a}_0$$



where, $\vec{a}_{P_{0t}}$ =tangential component of $\vec{a}_{P_0}$

and $\vec{a}_{P_{0n}}$ =normal component of $\vec{a}_{P_0}$

So, $\left|\vec{a}_0 + \vec{a}_{P_{0t}}\right| = a + r\alpha$ and

$$\left|\vec{a}_{P_{0n}}\right| = r\omega^2$$

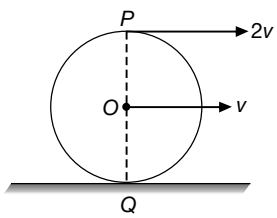
$$\Rightarrow \quad \left|\vec{a}_P\right| = \sqrt{(a + r\alpha)^2 + (r\omega^2)^2}$$

Hence, the correct answer is (A).

158. In case (i) work done by friction is zero, while in case (ii) it is non-zero.

Hence, the correct answer is (B).

159.

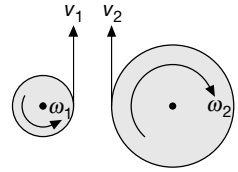


$$v_r = v_{PQ} = 2v$$

$$\Rightarrow \quad a_r = \frac{v_r^2}{2R} = \frac{4v^2}{2R} = \frac{2v^2}{R}$$

Hence, the correct answer is (B).

160. Let ω_1 and ω_2 be the final angular velocities when the slipping has ceased. Then



$$v_1 = v_2$$

$$\Rightarrow \quad \omega_1 R = \omega_2 (2R)$$

$$\Rightarrow \quad \omega_2 = \frac{\omega_1}{2} \qquad \ldots(1)$$

Since, $\begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix} = \begin{pmatrix}\text{Change in Angular}\\ \text{Momentum}\end{pmatrix}$, so

$$JR = I\omega_1 \qquad \ldots(2)$$

and $J2R = 4I(\omega_0 - \omega_2)$ …(3)

where, J is the linear impulse due to friction which acts tangentially and is equal for both the cylinders.

Solving equations (1), (2) and (3), we get

$$\omega_1 = \omega_0 \text{ and } \omega_2 = \frac{\omega_0}{2}$$

Hence, the correct answer is (B).

161. The frictional force μmg is the only horizontal force acting on the two bodies. So, each body has an acceleration $\frac{\mu mg}{m} = \mu g$ in opposite direction. So, relative acceleration is $2\mu g$.

Hence, the correct answer is (B).

162. $a = \frac{g}{1 + \frac{M}{2m}}$

So, $v^2 = 2ah$

$\Rightarrow$ v is independent of R.

Hence, the correct answer is (D).

163. $I = Mk^2 = (40)(0.5)^2$

$$\Rightarrow \quad I = 40(0.25) = 10 \text{ kgm}^2$$

$$\omega_0 = 1800 \times \frac{2\pi}{60} = 60\pi \text{ rads}^{-1}$$

$$\Rightarrow \quad \alpha = \frac{\omega_0}{t} = \frac{60\pi}{30} = 2\pi \text{ rads}^{-2}$$

Also, $\tau = I\alpha$

$$\Rightarrow \quad \tau = 20\pi$$

Hence, the correct answer is (B).

164. By Law of Conservation of Energy

$$Mgh = \frac{1}{2}Mv_{CM}^2\left(1+\frac{K^2}{R^2}\right)$$

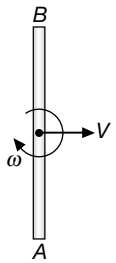
For a cylinder $\frac{K^2}{R^2} = \frac{1}{2}$

$$\Rightarrow \quad v_{CM} = \sqrt{\frac{4}{3}gh}$$

Hence, the correct answer is (B).

165. Let V be the linear velocity of centre of mass of rod just after collision and ω be its angular velocity, then by Law of Conservation of Linear Momentum, we have

$$mv_0 = Mv \qquad \ldots(1)$$



By Law of Conservation of Angular Momentum about centre of rod, we get

$$mv_0 x = \left(\frac{ML^2}{12}\right)\omega \qquad \ldots(2)$$

For A to be at rest just after collision, we have

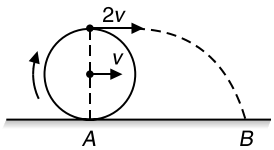
$$\frac{L}{2}\omega = V \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$x = \frac{L}{6}$$

Hence, the correct answer is (B).

166. From the concepts of Projectile Motion.



$$2r = \frac{1}{2}gt^2$$

$$\Rightarrow \quad t = \sqrt{\frac{4r}{g}}$$

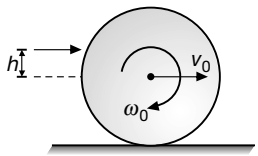
Since $x = AB = (2v)t$

$$\Rightarrow \quad AB = 4v\sqrt{\frac{r}{g}}$$

Hence, the correct answer is (C).

167. Since $L = mvr_\perp$ and as $r_\perp$ is constant, so L is constant.
Hence, the correct answer is (B).

168. Let J be the linear impulse imparted to the ball, then



Impulse =Change in Momentum

$$\Rightarrow \quad J = mv_0 \qquad \ldots(1)$$

Also, $\begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix} = \begin{pmatrix}\text{Change in Angular}\\ \text{Momentum}\end{pmatrix}$

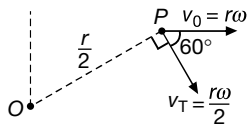
$$\Rightarrow \quad Jh = I\omega_0 = \frac{2}{5}mr^2\omega_0 \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\omega_0 = \frac{5}{2}\frac{v_0 h}{r^2}$$

Hence, the correct answer is (B).

169. The point P will have a velocity v_0 and a tangential velocity $v_T = \left(\frac{r}{2}\right)\omega = \frac{v_0}{2}$ inclined to each other at 60°.

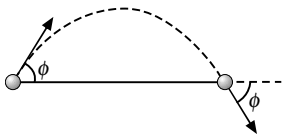


So, $v_P^2 = v_0^2 + \frac{v_0^2}{4} + 2(v_0)\left(\frac{v_0}{2}\right)\cos(60°)$

$$\Rightarrow \quad v_P = \frac{v_0\sqrt{7}}{2}$$

Hence, the correct answer is (C).

170.



$$\langle\omega\rangle = \frac{\Delta\phi}{\Delta t} = \frac{2\phi}{T} = \frac{2\phi}{\left(\frac{2v\sin\phi}{g}\right)} = \frac{g\phi}{v\sin\phi}$$

Hence, the correct answer is (D).

171. Angular momentum, $L = I\omega$

For the said axis, $I = \frac{1}{3}m\ell^2$

$$\Rightarrow \quad L = \frac{m\ell^2}{3}\omega$$

Hence, the correct answer is (B).

CHAPTER 3

Multiple Correct Choice Type Questions

1. $I_1 = \frac{1}{2}MR^2 + \frac{1}{12}M\ell^2$

$\Rightarrow \quad I_1 = \frac{1}{2}MR^2 + \frac{1}{12}M(4R^2)$

$\Rightarrow \quad I_1 = \frac{1}{2}MR^2 + \frac{1}{3}MR^2 = \frac{5}{6}MR^2$

Also, $I_2 = \frac{1}{2}MR^2 + \frac{1}{3}M(4R^2)$

$\Rightarrow \quad I_2 = \frac{1}{2}MR^2 + \frac{4}{3}MR^2 = \frac{11}{6}MR^2$

$\Rightarrow \quad I_2 > I_1$ and $I_2 - I_1 = MR^2$

Hence, (A) and (B) are correct.

2. Due to Law of Conservation of Angular Momentum $\vec{L}$ =constant

$\Rightarrow \quad \vec{L}\cdot\vec{L} = \text{constant}$

$\Rightarrow \quad \frac{d}{dt}(\vec{L}\cdot\vec{L}) = 0$

$\Rightarrow \quad 2\vec{L}\cdot\frac{d\vec{L}}{dt} = 0$

$\Rightarrow \quad \vec{L} \perp \frac{d\vec{L}}{dt}$

Since $\vec{\tau} = \vec{A}\times\vec{L}$

$\Rightarrow \quad \frac{d\vec{L}}{dt} = \vec{A}\times\vec{L}$

$\Rightarrow \quad \frac{d\vec{L}}{dt}$ must be perpendicular to $\vec{A}$ as well as $\vec{L}$

Further component of $\vec{L}$ along $\vec{A}$ is $\frac{\vec{A}\cdot\vec{L}}{A} = x$(say). Since $\vec{A} \perp \frac{d\vec{L}}{dt}$ and $\frac{d\vec{A}}{dt} = \vec{0}$

$\Rightarrow \quad \frac{d}{dt}(\vec{A}\cdot\vec{L}) = \vec{A}\cdot\frac{d\vec{L}}{dt} + \vec{L}\cdot\frac{d\vec{A}}{dt} = 0$

$\Rightarrow \quad \vec{A}\cdot\vec{L} = \text{constant}$

$\Rightarrow \quad \frac{\vec{A}\cdot\vec{L}}{A} = x = \text{constant}$

Since $\frac{d\vec{L}}{dt}$ (or $\vec{\tau}$) is perpendicular to $\vec{L}$, hence it cannot change magnitude of $\vec{L}$ but can surely change direction of $\vec{L}$.

Hence, (A), (B) and (C) are correct.

3. By Law of Conservation of Linear Momentum, we have

$$mv\hat{i} + mv\hat{j} + mv(-\hat{j}) + \vec{0} = \vec{p}_{\substack{\text{Triangular}\\ \text{Wedge } ABC}} + \vec{0}$$

$\Rightarrow \quad \vec{p}_{\substack{\text{Triangular}\\ \text{Wedge } ABC}} = (mv)\hat{i}$

Since the net linear momentum imparted to the triangular wedge is along x-axis and is non-zero, so the centre of mass of wedge ABC will move along x-axis.

Hence, the correct answer is (B).

4. In the frame of rod, the small vertical rods will experience centrifugal forces which forms a couple in clockwise direction in the state given in problem. To balance this couple force by hinge at A on the rod must be downward and the force by hinge at B must be upward.

The angular momenta of the vertical rod particles about point O will be inclined to rod hence option (D) is also correct.

Hence, (A), (B) and (D) are correct.

5. $V_c = \frac{2m(-v) + m(2v) + 8m(0)}{2m + m + 8m} = 0$ {OPTION **(A)**}

Further, by Law of Conservation of Angular Momentum

$$\left(I_{\substack{\text{system}\\ \text{about } O}}\right)\omega = \sum_{\substack{\text{system}\\ \text{about O}}} mvr_\perp$$

$$I_{\substack{\text{system}\\ \text{about } O}} = \left[\frac{1}{12}(8m)(6a)^2 + m(2a)^2 + 2ma^2\right] = 30ma^2$$

$$\sum_{\substack{\text{system}\\ \text{about } O}} mvr_\perp = (8m)(0)(0) + 2m(-v)(-a) + m(2v)(2a)$$

$\Rightarrow \quad \sum_{\substack{\text{system}\\ \text{about } O}} mvr_\perp = 6mva$

$\Rightarrow \quad (30ma^2)\omega = 6mva$

$\Rightarrow \quad \omega = \frac{v}{5a}$ {OPTION **(C)**}

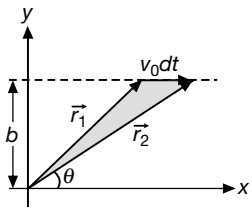
Further total energy of the system E is

$E = \frac{1}{2}I\omega^2 = \frac{3}{5}mv^2$ {OPTION **(D)**}

Hence, (A), (C) and (D) are correct.

6. $L = mv_0 r\sin\theta$

$L = mv_0 b = \text{constant}$



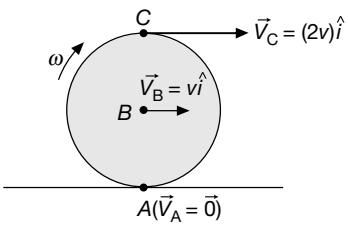
Also, $|\vec{r}_1| \approx |\vec{r}_2| = r$

Since, $dA = \frac{1}{2}(v_0 dt)r\sin\theta$

$\Rightarrow \quad \frac{dA}{dt} = \frac{mv_0 r\sin\theta}{2m} = \frac{L}{2m}$

Hence, (B) and (D) are correct.

7.



Hence, (B) and (C) are correct.

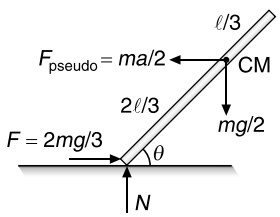
8. The mass M of the rod is

$$M = \int_0^l \lambda dr = \int_0^l \left(\frac{\lambda_0}{l^2} r\right) dr = \frac{\lambda_0}{2}$$

Position of centre of mass of rod from end lying on the surface is

$$r_{cm} = \frac{\int_0^l \left(\frac{\lambda_0}{l^2} rdr\right) r}{\int_0^l \frac{\lambda_0}{l^2} rdr} = \frac{2l}{3}$$

Free body diagram of rod is shown in Figure.



Given that $F = \frac{4W}{3} = \frac{4Mg}{3} = \frac{2\lambda_0 g}{3}$

If $N = \frac{\lambda_0 g}{2}$, then for equilibrium of the rod, we have $\Sigma\tau = 0$ and $\Sigma F = 0$.

$\Sigma F = 0$ gives, $F_{applied} = F_{pseudo} = Ma$

$\Rightarrow \quad \frac{2\lambda_0 g}{3} = \frac{\lambda_0 a}{2}$

$\Rightarrow \quad a = \frac{4}{3}g$

Similarly, $\Sigma\tau = 0$ gives $F\sin\theta = N\cos\theta$

$\Rightarrow \quad \tan\theta = \frac{g}{a} = \frac{3}{4}$

$\Rightarrow \quad \theta = 37°$

Hence, (A), (B) and (C) are correct.

9. $F\hat{i} = 3m\vec{a} \quad \{\because \text{Net force} = (\text{Total mass})(\text{acceleration})\}$

$\Rightarrow \quad \vec{a} = \vec{a}_0 = \frac{1}{3}\frac{F}{m}\hat{i}$

Since $\tau = I_0\alpha$

$\Rightarrow \quad Fb = (3mr^2)\alpha$

$\Rightarrow \quad \alpha = \frac{Fb}{3mr^2}$

Hence, (A) and (C) are correct.

10. $L_1 = I\omega = MK^2\omega$ ...(1)

$L_2 = I\omega + MvR$

$\Rightarrow \quad L_2 = MK^2\omega + MR(\omega R) \quad \{\because v = R\omega\}$

$\Rightarrow \quad L_2 = M\omega(K^2 + R^2)$ ...(2)

From equations (1) and (2), we can see that

$L_2 = 2L_1$, when $K = R$ and

$L_2 > 2L_1$, when $K > R$

Hence, (B) and (D) are correct.

11. **Conceptual (C) and (D) are correct.**

12. $\omega = \omega_0 - a\phi$

$\Rightarrow \quad \frac{d\phi}{dt} = \omega_0 - a\phi$

$\Rightarrow \quad \int_0^\phi \frac{d\phi}{\omega_0 - a\phi} = \int_0^t dt$

$\Rightarrow \quad -\frac{1}{a}\log_e(\omega_0 - a\phi)\Big|_0^\phi = t$

$\Rightarrow \quad \log_e(\omega_0 - a\phi) - \log_e\omega_0 = -at$

$\Rightarrow \quad \frac{\omega_0 - a\phi}{\omega_0} = e^{-at}$

$\Rightarrow \quad \phi = \frac{\omega_0}{a}(1 - e^{-at})$ {OPTION **(A)**}

Further, $\omega = \frac{d\phi}{dt} = \frac{\omega_0}{a}\frac{d}{dt}[1 - e^{-at}]$

$\Rightarrow \quad \omega = \frac{\omega_0}{a}[-e^{-at}(-a)]$

$\Rightarrow \quad \omega = \omega_0 e^{-at}$ {OPTION **(C)**}

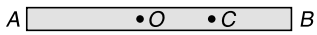
Hence, (A) and (C) are correct.

13. When cylinder comes down, at the point where string leaves contact with the cylinder is point of instantaneous rest, thus string does zero work.

$$\frac{K_R}{K_T} = \frac{\frac{1}{2}I_C\omega^2}{\frac{1}{2}mv^2} = \frac{\frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v}{R}\right)^2}{\frac{1}{2}mv^2} = \frac{1}{2}$$

Hence, (A) and (C) are correct.

14. The farther the mass is distributed from the Axis of Rotation (AOR), the more the Moment of Inertia.



Here C must be towards heavier side and hence lies between O and B.

$\Rightarrow \quad I_A > I_O > I_C > I_B$

Hence, (A) and (C) are correct.

15. Since $L = J\ell$

Where $J = mv$ is the impulse.

Also, $L = I\omega$

$\Rightarrow \quad \omega = \frac{L}{I} = \frac{J\ell}{\frac{1}{3}m\ell^2}$

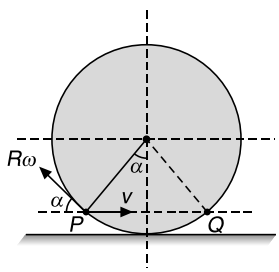
$\Rightarrow \quad \omega = \frac{3J}{m\ell}$

Kinetic energy $= \frac{L^2}{2I} = \frac{3J^2}{2m}$

and $v_C = \omega\frac{\ell}{2} = \frac{3J}{2m}$

Hence, (A), (B), (C) and (D) are correct.

16. Particle P will have a velocity in vertical direction, when



$R\omega\cos\alpha = v$

$\Rightarrow \quad \alpha = \cos^{-1}\left(\frac{v}{R\omega}\right)$

So, the required angle θ is

$$\theta = \pi - \cos^{-1}\left(\frac{v}{R\omega}\right) \text{ and } \theta = \pi + \cos^{-1}\left(\frac{v}{R\omega}\right)$$

Point P corresponds to $\theta = \pi - \cos^{-1}\left(\frac{v}{R\omega}\right)$ and it has velocity in vertically upward direction, while Q corresponds to $\theta = \pi + \cos^{-1}\left(\frac{v}{R\omega}\right)$ and it has velocity in vertically downward direction.

Hence, (C) and (D) are correct.

17. If v_{cm} be the velocity of centre of mass of the cylinder, ω be the angular velocity of the cylinder, then for no sliding between plank A and cylinder C, we have

$$R\omega - v_{cm} = v \qquad \ldots(1)$$

Similarly, for no sliding between plank B and cylinder C, we have

$$v_{cm} + R\omega = 2v \qquad \ldots(2)$$

Solving equations (1) and (2), we get

$$v_{cm} = \frac{v}{2} \text{ and } \omega = \frac{3v}{2R}, \text{ clockwise}$$

Below the centre of the cylinder at a distance x (say), we observe a point which happens to be at instantaneous rest and this point is called the instantaneous centre (IC) of zero velocity and the axis passing through it is called the Instantaneous Axis of Rotation. At this point, we have

$v_{cm} - x\omega = 0$

$\Rightarrow \quad x = \frac{v_{cm}}{\omega} = \frac{v/2}{3v/2R} = \frac{R}{3}$

The translational kinetic energy of the system is

$K_T = K_A + K_B + (K_C)_T$

$\Rightarrow \quad K_T = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 + \frac{1}{2}(8m)\left(\frac{v}{2}\right)^2$

$\Rightarrow \quad K_T = 4mv^2$

Since the cylinder is in pure rolling, so the kinetic energy of the system is

$K_T = K_A + K_B + (K_C)_{\text{rolling}}$

The kinetic energy in rolling is the sum of translational kinetic energy and rotational kinetic energy and is given by

$$K_{\text{rolling}} = \frac{1}{2}Mv_{cm}^2\left(1 + \frac{K^2}{R^2}\right)$$

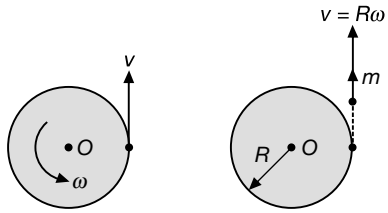
So, $(K_C)_{\text{rolling}} = \frac{1}{2}(8m)\left(\frac{v}{2}\right)^2\left(1+\frac{1}{2}\right) = \frac{3}{2}mv^2$

$\Rightarrow \quad K = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 + \frac{3}{2}mv^2$

$\Rightarrow \quad K_T = \frac{9}{2}mv^2$

Hence, (A), (B), (C) and (D) are correct.

18. Since, the broken away piece of mass m (say) has the same linear speed as it had before detaching from the disc, the angular speed of the remaining disc will remain stay unchanged. Further, the linear speed of the piece will first decrease and then increase due to gravity. So, its angular speed about an axis passing through O will first decrease and then increase.



Hence, (C) and (D) are correct.

19. Since $\begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix} = \begin{pmatrix}\text{Change in Angular}\\ \text{Momentum}\end{pmatrix}$, so

$L = \tau t$

$\Rightarrow \quad L = F(2R)t$

i.e., L varies linearly with time.

Hence, (B) and (C) are correct.

20. Velocity of particle is $\vec{v} = \vec{\omega} \times \vec{r}$ and centripetal acceleration is

$\vec{a}_c = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{r})$

Hence, (B) and (D) are correct.

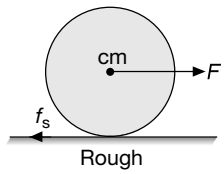
21. On a smooth horizontal surface, it can roll without slipping if $v = R\omega$ and no external force is acting on it.

Hence, (A), (C) and (D) are correct.

22. On melting, moment of inertia increases and hence ω decreases.

Hence, (B) and (D) are correct.

23. Since cylinder will roll towards right and required torque about centre of mass can be provided by friction if friction acts in the backward direction.



Now $F - f_s = ma$ …(1)

Also, $\tau = f_s R = I_{\text{cm}}\alpha = \left(\frac{1}{2}mR^2\right)\frac{a}{R}$

$\Rightarrow \quad f_s = \frac{1}{2}ma$ …(2)

Solving (1) and (2), we get

$a = \frac{2F}{3M}$ and $f_s = \frac{F}{3}$

Hence, (B), (C) and (D) are correct.

24. Point of contact P is at rest. Taking torque due to all the forces about point P we observe that

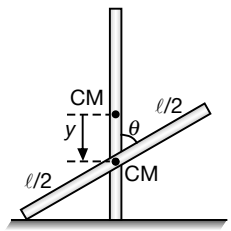
Torque is clockwise if F_1 is applied i.e., spool will rotate clockwise.

Torque due to F_2 is zero i.e., spool will not rotate if F_2 is applied, because F_2 passes through P.

Torque due to F_3 and F_4 is anticlockwise i.e., spool will rotate anticlockwise if only F_3 or F_4 is applied.

Hence, (B) and (D) are correct.

25. Since surface is frictionless, the centre of mass of the rod will fall along the straight line



$y_{\text{cm}} = \frac{\ell}{2}(1-\cos\theta)$

$\Rightarrow \quad \frac{dy}{dt} = \left(\frac{\ell}{2}\sin\theta\right)\frac{d\theta}{dt} = \frac{\ell\omega}{2}\sin\theta$

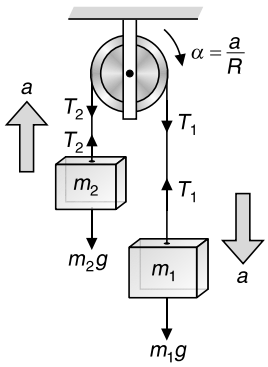
$\Rightarrow \quad a = \frac{d^2y}{dt^2} = \frac{\ell}{2}\omega^2\cos\theta + \left(\frac{\ell}{2}\sin\theta\right)\alpha$

$\Rightarrow \quad a = \frac{\omega^2\ell}{2}\cos\theta + \frac{\ell\alpha}{2}\sin\theta$

Hence, (A) and (D) are correct.

CHAPTER 3

26. Since the pulley has mass, so tension in both branches of string must be different.



Assume T_1 to be absent then T_2 provides a torque T_2R (in anticlockwise sense).
Now assume T_2 to be absent then T_1 provides a torque T_1R (in clockwise sense). So, Net Torque

$$\tau = T_1R - T_2R$$

$$\Rightarrow \quad I\alpha = (T_1 - T_2)R \qquad \{\because\ \tau = I\alpha\}$$

$$\Rightarrow \quad I\left(\frac{a}{R}\right) = (T_1 - T_2)R \qquad \left\{\because\ \alpha = \frac{a}{R}\right\}$$

$$\Rightarrow \quad (T_1 - T_2) = \frac{Ia}{R^2} \qquad \ldots(1)$$

Further for m_1

$$m_1g - T_1 = m_1a \qquad \ldots(2)$$

and for m_2

$$T_2 - m_2g = m_2a \qquad \ldots(3)$$

Adding (2) and (3) and using (1), we get

$$a = \left[\frac{m_1 - m_2}{m_1 + m_2 + (I/R^2)}\right]g$$

Hence, (A), (B), (C) and (D) are correct.

27. When the centre of mass moves a distance S, then the distance covered by the point of application of force F is $2S$, therefore work done is $W = F(2S)$
From Work Energy Theorem, we have

$$W = 2FS = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow \quad v = \sqrt{\frac{20FS}{7M}}$$

Hence, (A) and (B) are correct.

28. Moment of inertia will decrease but radius of gyration will increase because the distance of remaining mass from the axis has been increased. Radius of gyration depends on the distribution of mass from the axis.

Hence, (A), (B) and (D) are correct.

29. From symmetry we have

$$I_1 = I_2 \text{ and } I_4 = I_3 \qquad \ldots(1)$$

Also, by perpendicular Axis Theorem

$$I_0 = I_1 + I_2 = I_4 + I_3 \qquad \ldots(2)$$

From (2), we get

$$2I_1 = 2I_3 \qquad \{\because \text{ of } (1)\}$$

$$\Rightarrow \quad I_1 = I_3$$

$$\Rightarrow \quad I_0 = I_1 + I_2 = I_4 + I_3 = I_1 + I_3$$

Hence, (A), (B) and (C) are correct.

30. $\vec{L}_{\text{total}} = \vec{L}_{\text{about cm}} + \vec{L}_{\text{cm about } O}$

$$\Rightarrow \quad |\vec{L}_O| = \underbrace{(I_{\text{cm}})\omega}_{\text{CCW}} + \underbrace{Mv_{\text{cm}}(r_{\perp \text{ cm from } O})}_{CW}$$

Taking CW as positive, we get

$$|\vec{L}_O| = -\left(\frac{1}{2}MR^2\right)\omega + M(R\omega)(3R)$$

$$\Rightarrow \quad |\vec{L}_O| = \left(\frac{5}{2}MR^2\right)\omega$$

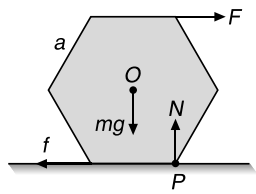
Similarly, angular momentum about A is

$$\vec{L}_A = \vec{L}_{\text{about cm}} + \vec{L}_{\text{cm about } A}$$

$$\Rightarrow \quad |\vec{L}_A| = \left(\frac{1}{2}mR^2\right)\omega + 0 = \left(\frac{1}{2}mR^2\right)\omega$$

Hence, (A) and (C) are correct.

31. When the body is on the verge of toppling, then we can apply the equilibrium condition to get the desired results. Also, if the body topples about the point P, then the normal reaction will shift to this point as shown in Figure.



So, we have $\Sigma F = 0$ and $\Sigma\tau = 0$
For translational equilibrium of the body

$$N = mg \text{ and } F = f_l = \mu N = \mu mg$$

For rotational equilibrium of the body, taking torque about P, we get

$$F\sqrt{3} = mg\left(\frac{a}{2}\right)$$

$$\Rightarrow \quad F = \frac{mg}{2\sqrt{3}}$$

For rotational equilibrium of the body, taking torque about O, we get

$$\frac{mg}{2\sqrt{3}}\left(\frac{\sqrt{3}a}{2}\right)+\mu mg\left(\frac{\sqrt{3}a}{2}\right)=mg\left(\frac{a}{2}\right)$$

$$\Rightarrow \quad \mu=\frac{1}{2\sqrt{3}}$$

When $\mu=2\mu_{\min}=\frac{1}{\sqrt{3}}$ and $F=\frac{mg}{\sqrt{3}}$, taking torque about P, we get

$$\frac{mg}{\sqrt{3}}(\sqrt{3}a)-mg\left(\frac{a}{2}\right)=\left(\frac{17}{12}ma^2\right)\alpha$$

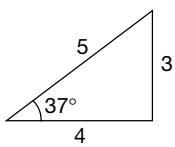
$$\Rightarrow \quad \alpha=\frac{6g}{17a}$$

Hence, (A) and (C) are correct.

32. Relative velocity between A and B along AB should be zero. Hence

$$v_A\sin\theta=v_B\cos\theta$$

$$\Rightarrow \quad v_A=v_B\cot\theta=\frac{4v_0}{3}$$



$$\text{Since, } \omega=\frac{\left(\begin{array}{c}\text{component of relative velocity}\\ \text{perpendicular to } AB\end{array}\right)}{AB}$$

$$\Rightarrow \quad \omega=\frac{v_A\cos\theta+v_B\sin\theta}{\ell}$$

$$\Rightarrow \quad \omega=\frac{\left(\frac{4v_0}{3}\right)\left(\frac{4}{5}\right)+v_0\left(\frac{3}{5}\right)}{\ell}=\frac{5v_0}{3\ell}$$

Hence, (B) and (D) are correct.

33. **Conceptual (A), (B), (C) and (D) are correct.**

34. In case of pure rolling, $\frac{K_R}{K_T}=\frac{k^2}{k^2+r^2}$, so

$\frac{K_R}{K_T}=1$ for a ring and $\frac{K_R}{K_T}=\frac{2}{5}$ for a solid sphere.

where, K_R =rotational kinetic energy and

K_T =translational kinetic energy

Therefore, fraction of its total energy associated with rotation is

$\alpha=\frac{1}{1+1}=\frac{1}{2}$ for ring and

$\beta=\frac{2}{2+5}=\frac{2}{7}$ for solid sphere

Hence, (A) and (D) are correct.

35. If F_1 or F_2 is applied, the centre of mass of spool will move towards right and it will rotate anticlockwise about centre of mass. Hence, friction will be towards left.

If F_3 is applied, centre of mass of spool will move towards left and it will rotate anticlockwise about centre of mass. Hence, friction will be towards left.

If F_4 is applied, then the centre of mass of spool will move towards left and it will rotate counter clockwise and F_4 in itself has a tendency of both. So, friction may be zero, leftwards or rightwards depending upon the moment of inertia, mass and radius.

Hence, (A), (B) and (D) are correct.

36. Since no external torque is acting on the system hence $\vec{L}$ is conserved.

$$(\vec{L}_{\text{student+stool}})_{\text{initial}}+(\vec{L}_{\text{wheel}})_{\text{initial}}=(\vec{L}_{\text{student+stool}})_{\text{final}}+(\vec{L}_{\text{wheel}})_{\text{final}}$$

$$\Rightarrow \quad 0+\vec{L}_0=(\vec{L}_{\text{student+stool}})_{\text{final}}+(-\vec{L}_0)$$

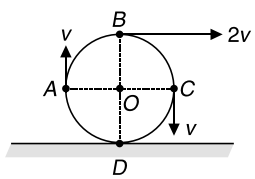
$$\Rightarrow \quad (\vec{L}_{\text{student+stool}})_{\text{final}}=2\vec{L}_0$$

Hence, (A) and (C) are correct.

37. Speed of the bottommost point is zero but acceleration is not zero. Friction force may be there if it is an accelerated motion but work done by friction is always zero.

Hence, (A) and (D) are correct.

38. This figure makes us conclude that section AB and BC have equal kinetic energy. Also, section ABC has greater kinetic energy than section ADC and BC has greater kinetic energy than CD.

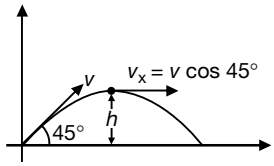


Hence, (A) and (B) are correct.

CHAPTER 3

39. $L = mv_x h$

$\Rightarrow \quad L = m(v\cos 45)\left(\dfrac{v^2 \sin^2 45}{2g}\right)$



$\Rightarrow \quad L = \dfrac{mv^3}{4\sqrt{2}\,g} \qquad \left\{\because h\dfrac{v^2 \sin^2 45}{2g}\right\}$

Also, from $h = \dfrac{v^2}{4g}$

$\Rightarrow \quad v = 2\sqrt{gh}$

Also, $L = \dfrac{mv}{\sqrt{2}}\, h$

$$L = \frac{m\left(2\sqrt{gh}\right)h}{\sqrt{2}}$$

$\Rightarrow \quad L = m\sqrt{2gh^3}$

Hence, (B) and (D) are correct.

40. When the net force on a rigid body is zero, then the summation of moments about any point is constant.

Hence, (A) and (C) are correct.

41. By Law of Conservation of Angular Momentum

$$\left(\frac{1}{2}MR^2\right)\omega = \left(\frac{1}{2}MR^2 + MR^2\right)\omega'$$

$\Rightarrow \quad \omega' = \dfrac{\omega}{3}$

Loss in Kinetic energy= $K_i - K_f$

Where $K_i = \dfrac{1}{2}I\omega^2 = \dfrac{1}{4}MR^2\omega^2$

$$K_f = \frac{1}{2}(I_{\text{total}})\omega'^2$$

$\Rightarrow \quad K_f = \dfrac{1}{2}\left(\dfrac{3}{2}MR^2\right)\left(\dfrac{1}{9}\omega^2\right)$

$\Rightarrow \quad K_f = \dfrac{1}{12}MR^2\omega^2$

$\Rightarrow \quad \text{Loss} = \left(\dfrac{1}{4} - \dfrac{1}{12}\right)MR^2\omega^2$

$\Rightarrow \quad \text{Loss} = \dfrac{1}{6}MR^2\omega^2$

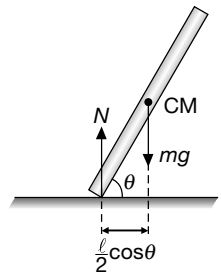
$\Rightarrow \quad \text{Loss} = \dfrac{2}{3}\left(\dfrac{1}{4}MR^2\omega^2\right)$

$\Rightarrow \quad \text{Loss} = \dfrac{2}{3}K_i$

Obviously, the lost energy must change to heat and actually this energy is lost due to the presence of friction which helps the two bodies attain a common angular velocity.

Hence, (A), (B) and (D) are correct.

42. Initially, when rod is vertical, magnitude of normal reaction is mg and only gravitational force is acting on the centre of mass of the rod in the vertical downward direction.



Now consider the situation in the figure, then

$$Mg - N = Ma \qquad \ldots(1)$$

Also, $N\left(\dfrac{\ell}{2}\cos\theta\right) = \mathrm{I}\alpha = \left(\dfrac{M\ell^2}{12}\right)\alpha \qquad \ldots(2)$

$\Rightarrow \quad \alpha\dfrac{\ell}{2} = a\cos\theta \qquad \ldots(3)$

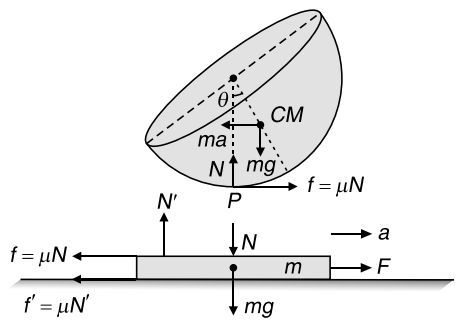
After solving (1), (2) and (3), we get

$$N = \frac{Mg}{4}$$

Hence, (A), (B) and (D) are correct.

43. When seen from the frame attached to the plank, net torque about P is zero, so we have

$$ma\left(R - \frac{R}{2}\cos\theta\right) = mg\frac{R}{2}\sin\theta$$



$\Rightarrow \quad a = \dfrac{g\sin\theta}{2 - \cos\theta}$

When $\theta = 37°$, we get $a = 5\ \text{ms}^{-2}$

Also, $f = \mu N = ma$

$\Rightarrow \quad \mu mg = ma$ …(1)

$\Rightarrow \quad \mu_{\min} = \dfrac{a}{g} = \dfrac{1}{2}$

For the plank, we have

$F - f - f' = ma$

$\Rightarrow \quad F - \mu mg - 2\mu mg = ma$ …(2)

Adding equations (1) and (2), we get

$F - 2\mu mg = 2ma$

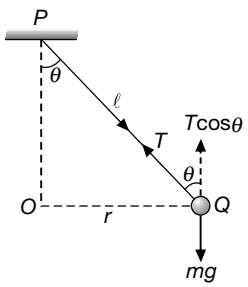
$\Rightarrow \quad F = 100\ \text{N}$

Hence, (B) and (D) are correct.

44. $T\cos\theta = mg$ and $T\sin\theta = \dfrac{mv^2}{r}$

$\Rightarrow \quad \dfrac{v^2}{rg} = \tan\theta = \tan(30°) = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad v = \sqrt{\dfrac{rg}{\sqrt{3}}} = \sqrt{\dfrac{0.5\times 10}{1.731}} \approx 1.7\ \text{ms}^{-1}$



Now, $PO = \ell\cos\theta$

$\Rightarrow \quad PO = (1)(0.86) = 0.86\ \text{m}$

Also, $r = \ell\sin\theta = (1)\left(\dfrac{1}{2}\right) = 0.5\ \text{m}$

Horizontal component of angular momentum is

$L_H = (m)(v)(PO)$

$\Rightarrow \quad L_H = (2)(1.7)(0.86) = 2.9\ \text{kgm}^2\text{s}^{-1}$

Vertical component of angular momentum is

$L_V = (m)(v)(OQ)$

$L_V = (2)(1.7)(0.5) = 1.7\ \text{kgm}^2\text{s}^{-1}$

Also, $\left|\dfrac{d\vec{L}}{dt}\right| = |\vec{\tau}| = mgr = (2)(10)(0.5)$

$\Rightarrow \quad \dfrac{d\vec{L}}{dt} = 10\ \text{kgm}^2\text{s}^{-2}$

Hence, (A), (B) and (C) are correct.

Reasoning Based Questions

1. When earth shrinks its angular momentum remains constant i.e.,

$$L = \text{I}\omega = \frac{2}{5}mR^2 \times \frac{2\pi}{T} = \text{constant}$$

$T \propto I \propto R^2$ it means if size of earth changes then it's moment of inertia changes. If radius is half so time period will become $\dfrac{T}{4} = \dfrac{24}{4} = 6$ hr.

Hence, the correct answer is (A).

2. In the case of smooth surface, $f_r = 0$, whereas in pure rolling motion friction can be present or absent.

Hence, the correct answer is (B).

3. Perpendicular axis theorem is not valid for a sphere.

Hence, the correct answer is (D).

4. If angular velocity is constant then frictional force acting on sphere is zero. In case of pure rolling velocity of contact point is zero.

Hence, the correct answer is (B).

5. If system has only rotational kinetic energy than momentum may be zero.

Hence, the correct answer is (D).

6. As the person climbs up, normal reaction and friction between the ladder and the wall both increases. This decreases normal reaction from the floor, decreasing limiting value of friction there. This increases the possibility of the ladder slipping.

Hence, the correct answer is (C).

7. Since net torque is zero angular velocity remains constant.

Hence, the correct answer is (A).

8. Moment of inertia is then sum of mr^2 terms. We cannot change all the r's, keep m's the same, and expect $\Sigma m_i r_i^2$ to remain unchanged.

Hence, the correct answer is (A).

9. Velocity of point of contact

$v = v_{cm} - R\omega$

When pure rolling occurs $v_{cm} = R\omega$

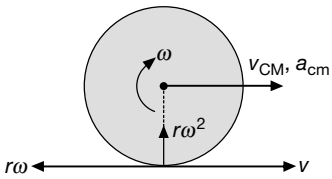
$\Rightarrow \quad V = 0$

Also, frictional force provides torque which further helps in achieving the pure rolling condition.

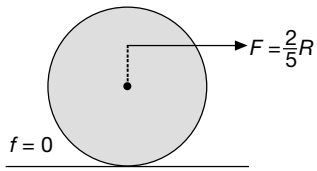
Hence, the correct answer is (B).

CHAPTER 3

10.

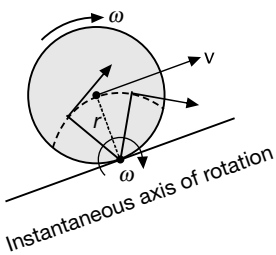


During the motion of rolling there is radial acceleration toward the centre. Hence contact point moves vertically upward.



Hence, the correct answer is (D).

11.



Hence, the correct answer is (A).

12. In sliding down, the entire potential energy is converted into kinetic energy. While in rolling down same part of potential energy is converted into kinetic energy of rotation, therefore linear velocity acquired is less.

Hence, the correct answer is (A).

13. Frictional force on an inclined plane, for a disc is

$$f = \frac{1}{3}g\sin\alpha$$

Hence, the correct answer is (B).

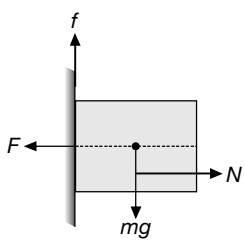
14. As the block remains stationary

$\Sigma fx = 0$ i.e., $F = N$

$\Sigma fy = 0$ i.e., $f = mg$

and $\Sigma\tau = 0$

$\Rightarrow \quad \vec{\tau}_f + \vec{\tau}_N = 0$



Since $\vec{\tau}_f \neq 0$

$\Rightarrow \quad \vec{\tau}_N \neq 0$ and torque by friction and normal reaction will be in opposite.

Hence, the correct answer is (A).

15. $I_1\omega_1 = I_2\omega_2$

If $I_2 < I_1$

$\Rightarrow \quad \omega_2 > \omega_1$

The diver does work by pulling his limps and thus, ω increases or rotational kinetic energy increases.

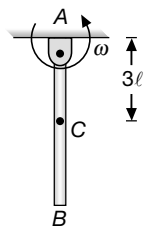
Hence, the correct answer is (A).

Linked Comprehension Type Questions

1. By Law of Conservation of Mechanical Energy, we have

$$(3m)(g)(2\ell) = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[\frac{(3m)(4\ell)^2}{3}\right]\omega^2 = 8m\ell^2\omega^2$$

$$\Rightarrow \quad \omega = \frac{1}{2}\sqrt{\frac{3g}{\ell}}$$



Applying, $\begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix} = \begin{pmatrix}\text{Change in}\\ \text{Angular}\\ \text{Momentum}\end{pmatrix}$, we get

$$J_1(3\ell) = I\omega$$

$$\Rightarrow \quad 3J_1\ell = (16m\ell^2)\left(\frac{1}{2}\sqrt{\frac{3g}{\ell}}\right)$$

$$\Rightarrow \quad J_1 = \frac{8}{3}m\ell\sqrt{\frac{3g}{\ell}}$$

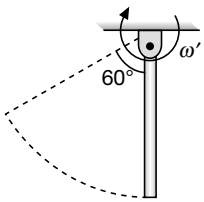
$$\Rightarrow \quad J_1 = \frac{8}{3}m\sqrt{3g\ell} = 8m\sqrt{\frac{g\ell}{3}}$$

Hence, the correct answer is (C).

2. Let ω' be the angular speed in opposite direction. Again, applying Law of Conservation of Mechanical Energy, we get

$$(3m)(g)(\ell) = \frac{1}{2}I(\omega')^2 = 8m\ell^2(\omega')^2$$

$$\Rightarrow \quad \omega' = \frac{1}{2\sqrt{2}}\sqrt{\frac{3g}{\ell}}$$



Since, $\begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix}=\begin{pmatrix}\text{Change in Angular}\\ \text{Momentum}\end{pmatrix}$, so

$$J_2(3\ell)=I(\omega+\omega')=(16m\ell^2)\frac{1}{2}\sqrt{\frac{3g}{\ell}}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad J_2=\frac{4}{3}m\sqrt{6g\ell}\left(\sqrt{2}+1\right)=4m\left(\sqrt{2}+1\right)\sqrt{\frac{2g\ell}{3}}$$

Hence, the correct answer is (B).

3. Torque about point A is

$$\tau=F(2R)=I\alpha$$

$$\Rightarrow \quad F(2R)=\left(\frac{3mR^2}{2}\right)\alpha$$

$$\Rightarrow \quad \alpha=\frac{4F}{3mR}$$

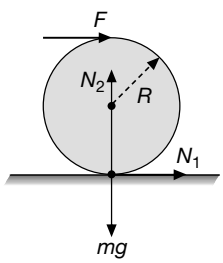
So tangential acceleration of center of mass is

$$a_t=R\alpha$$

$$\Rightarrow \quad a_t=\frac{4F}{3m}$$

Hence, the correct answer is (D).

4.



$$N_2=mg$$

Hence, the correct answer is (B).

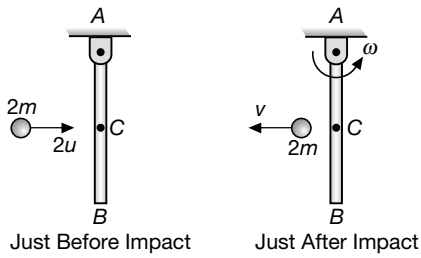
5. $F+N_1=ma_t$

$$\Rightarrow \quad N_1=\frac{F}{3} \qquad \left\{\because\ a_t=\frac{4F}{3m}\right\}$$

Hence, the correct answer is (C).

6. By Law of Conservation of Angular Momentum, we have

$$(2m)(2u)(\ell)=\frac{(3m)(2\ell)^2}{3}\omega-(2m)(v_1)(\ell)$$



$$\Rightarrow \quad 4u=4\ell\omega-2v_1$$

$$\Rightarrow \quad 2u=2\ell\omega-v_1 \qquad \ldots(1)$$

Since, for an elastic collision we have, $e=1$, so

$$\begin{pmatrix}\text{Relative}\\ \text{speed of}\\ \text{approach}\end{pmatrix}=\begin{pmatrix}\text{Relative}\\ \text{speed of}\\ \text{separation}\end{pmatrix}, \text{ at point } C$$

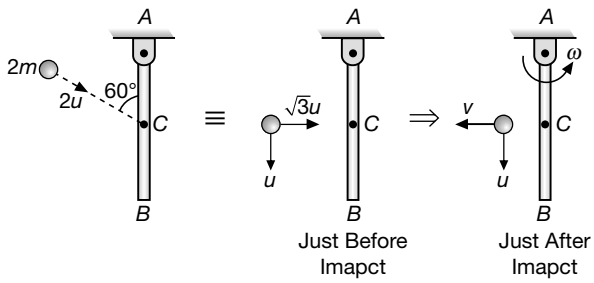
$$\Rightarrow \quad 2u=v_1+\ell\omega \qquad \ldots(2)$$

From equation (1) and (2), we get

$$v_1=\frac{2u}{3}$$

Hence, the correct answer is (C).

7. In this case, the component of velocity along AC remains unchanged. Proceeding in the same manner, we can just replace $2u$ by $2u\cos(30°)=\sqrt{3}u$ and find v_2.



Hence, $v=\left(\frac{2}{3}\right)\frac{\sqrt{3}}{2}u=\frac{u}{\sqrt{3}}$

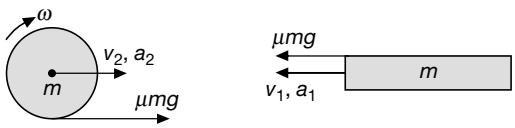
Speed of particle after impact is

$$v_2=\sqrt{v^2+u^2}=\sqrt{\frac{u^2}{3}+u^2}=\frac{2}{\sqrt{3}}u$$

Hence, the correct answer is (B).

CHAPTER 3

8.



$$a_1 = \frac{\mu mg}{m} = \mu g$$

$$a_2 = \frac{\mu mg}{m} = \mu g$$

If v_1 and v_2 be the velocities of plank and cylinder at time t, then relative velocity of centre of mass of sphere w.r.t. plank is $(v_1 + v_2)$. So, when pure rolling begins, then

$$\omega = \frac{v_1 + v_2}{r}$$

where $v_1 = (\mu g)t$ and $v_2 = (\mu g)t$

Now, $\alpha = \frac{\tau}{I} = \frac{(\mu mg)r}{\frac{2}{5}mr^2}$

$$\Rightarrow \quad \alpha = \frac{5}{2}\left(\frac{\mu g}{r}\right)$$

when pure rolling begins at time t, then

$$v_1 + v_2 = r\omega$$

$$\Rightarrow \quad (2\mu g)t = r(\omega_0 - \alpha t)$$

$$\Rightarrow \quad (2\mu g)t = r\omega_0 - \left(\frac{5}{2}\mu g\right)t$$

$$\Rightarrow \quad t = \frac{2}{9}\left(\frac{r\omega_0}{\mu g}\right)$$

Hence, the correct answer is (B).

9. Since $v_2 = (\mu g)t$

$$\Rightarrow \quad v_2 = \frac{2}{9}r\omega_0$$

Hence, the correct answer is (A).

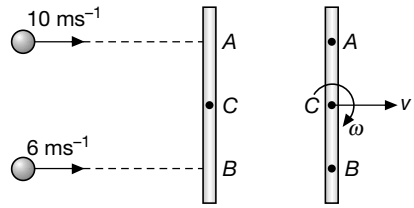
10. $x_P = \frac{1}{2}a_1 t^2$

$$\Rightarrow \quad x_P = \frac{1}{2}(\mu g)\left(\frac{2}{9}\frac{r\omega_0}{\mu g}\right)^2$$

$$\Rightarrow \quad x_P = \frac{2}{81}\left(\frac{r^2\omega_0^2}{\mu g}\right)$$

Hence, the correct answer is (D).

11. Let v be the velocity of centre of mass (also at C) of rod and two particles and ω the angular velocity of the system.



By Law of Conservation of Linear Momentum, we have

$$(0.08)(10+6) = (0.08+0.08+0.16)v$$

$$\Rightarrow \quad v = 4 \text{ ms}^{-1}$$

Hence, the correct answer is (D).

12. Also, $AB = CB = 0.5$ m

Similarly applying Conservation of Angular Momentum about the point C, we get

$$(0.08)(10)(0.5) - (0.08)(6)(0.5) = I_{\text{system}}\omega \quad \ldots(1)$$

where, $I_{\text{system}} = I_{\text{rod}} + I_{\text{two particles}} = \frac{(1.6)\left(\sqrt{3}\right)^2}{12} + 2(0.08)(0.5)^2$

$$\Rightarrow \quad I_{\text{system}} = 0.08 \text{ kgm}^2$$

Substituting in equation (1), we get

$$\omega = 2 \text{ rads}^{-1}$$

Hence, the correct answer is (C).

13. Loss of kinetic energy $= -\Delta K = K_i - K_f$

$$\Rightarrow \quad -\Delta K = \frac{1}{2}(0.08)(10)^2 + \frac{1}{2}(0.08)(6)^2 - \frac{1}{2}(0.08+0.08+0.16)(4)^2 - \frac{1}{2}(0.08)(2)^2$$

$$\Rightarrow \quad -\Delta K = 4 + 1.44 - 2.56 - 0.16$$

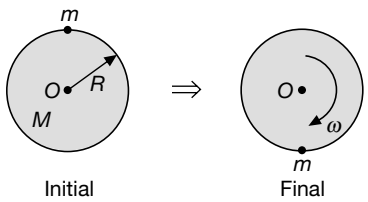
$$\Rightarrow \quad -\Delta K = 2.72 \text{ J}$$

Hence, the correct answer is (B).

14. By Law of Conservation of Mechanical Energy, we have

$$mg(2R) = \frac{1}{2}\left(\frac{1}{2}MR^2 + mR^2\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{8mg}{(2m+M)R}}$$



Since, $m = \frac{M}{2}$, so we have

$$\omega = \sqrt{\frac{2g}{R}}$$

Hence, the correct answer is (C).

15. Let F be the force exerted by disk on the particle (upwards). Then,

$$F - mg = mR\omega^2$$

$$\Rightarrow \quad F = mg + \frac{8m^2 g}{(2m+M)}$$

$$\Rightarrow \quad F = \frac{mg(10m+M)}{(2m+M)}$$

Since, $m = \frac{M}{2}$, so we have

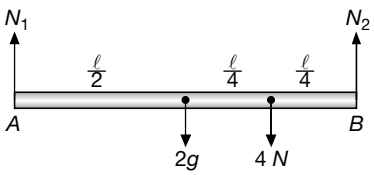
$$F = \frac{3}{2}Mg$$

Hence, the correct answer is (D).

16. Since the rod is in equilibrium, so

$$N_1 + N_2 = (2)(10) + 4$$

$$\Rightarrow \quad N_1 + N_2 = 24 \text{ N} \qquad \ldots(1)$$



Taking torque due to all forces about A, we get

$$\underbrace{N_1(0)}_{CW} + \underbrace{(20)\left(\frac{\ell}{2}\right)}_{CW} + \underbrace{4\left(\frac{3\ell}{4}\right)}_{CW} + \underbrace{N_2\ell}_{CCW} = 0$$

$$\Rightarrow \quad 10\ell + 3\ell - N_2\ell = 0$$

$$\Rightarrow \quad N_2 = 13 \text{ N} \qquad \ldots(2)$$

From (1), we get

$$N_1 = 11 \text{ N}$$

Hence, the correct answer is (B).

17. Just after hinge B breaks (and F is removed too) the rod will not be in equilibrium. Let the new reaction at A be N, then

$$2g - N = 2a \qquad \ldots(1)$$

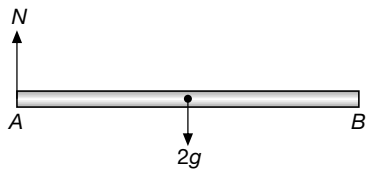
$$\tau = I\alpha$$

$$\Rightarrow \quad (2g)\frac{\ell}{2} = \left(\frac{(2)\ell^2}{3}\right)\alpha$$

$$\Rightarrow \quad g\ell = \left(\frac{2\ell^2}{3}\right)\alpha$$

$$\Rightarrow \quad \alpha = \frac{3g}{2\ell}$$

Since $a = \left(\frac{\ell}{2}\right)\alpha$



$$\Rightarrow \quad a = \frac{\ell}{2}\left(\frac{3g}{2\ell}\right)$$

$$\Rightarrow \quad a = \frac{3g}{4}$$

So, from (1), we get

$$20 - N = 2\left(\frac{3g}{4}\right)$$

$$\Rightarrow \quad 20 - N = \frac{60}{4}$$

$$\Rightarrow \quad N = 5 \text{ N}$$

Hence, the correct answer is (D).

18. By Law of Conservation of Energy

$$\begin{pmatrix}\text{Loss in GPE}\\ \text{of CM of Rod}\end{pmatrix} = \begin{pmatrix}\text{Gain in RKE}\\ \text{of Rod}\end{pmatrix}$$

$$\Rightarrow \quad (2)g\left(\frac{\ell}{2}\right) = \frac{1}{2}\left(\frac{(2)\ell^2}{3}\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3g}{\ell}}$$

So, acceleration is

$$a_n = \ell\omega^2 = 3g = 30 \text{ ms}^{-2}$$

Hence, the correct answer is (C).

19. Applying Conservation of Angular Momentum, we get

$$\sum mvr_\perp = \left(I_{\text{system}}\right)\omega$$

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$$\Rightarrow \quad (mu)L = \left(\frac{1}{3}ML^2 + mL^2\right)\omega$$

$$\Rightarrow \quad \omega = \frac{3mu}{(M+3m)L}$$

Since $m \ll M$, so, we get

$$\omega \cong \frac{3mu}{ML} \qquad \ldots(1)$$

Hence, the correct answer is (C).

20. By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in RKE}\\ \text{of Ball-Rod}\\ \text{System}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{GPE of}\\ \text{CM of Rod}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{GPE of}\\ \text{Ball}\end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{3}ML^2 + mL^2\right)\omega^2 = Mg\left(\frac{L}{2}\right)(1-\cos\theta) + mgL(1-\cos\theta)$$

$$(M+3m)L\omega^2 = 3g(1-\cos\theta)(M+2m)$$

Since $M \gg m$, so

$$M+3m \cong M \text{ and } M+2m \cong M$$

$$\Rightarrow \quad \omega \cong \sqrt{\frac{3g}{L}(1-\cos\theta)}$$

$$\Rightarrow \quad \frac{3mu}{ML} \cong \sqrt{\frac{3g}{L}\left[2\sin^2\left(\frac{\theta}{2}\right)\right]}$$

$$\Rightarrow \quad u \cong \frac{M}{m}\sqrt{\frac{2}{3}gL}\sin\left(\frac{\theta}{2}\right)$$

Hence, the correct answer is (C).

21. Since angular momentum is conserved about the hinge so there will be no change in angular momentum of system, however the rod is hinged at support and due to this the linear momentum changes. So,

$$\Delta p = p_f - p_i$$

$$\Rightarrow \quad \Delta p = \left[m(L\omega) + M\left(\frac{L\omega}{2}\right)\right] - mu$$

$$\Rightarrow \quad \Delta p = \left(m + \frac{M}{2}\right)L\omega - mu$$

$$\Rightarrow \quad \Delta p = \left(m + \frac{M}{2}\right)\left(\frac{3mu}{M}\right) - mu$$

$$\Rightarrow \quad \Delta p = \left(\frac{m}{M} + \frac{1}{2}\right)3mu - mu$$

$$\Rightarrow \quad \Delta p \simeq \frac{3}{2}mu - mu$$

$$\Rightarrow \quad \Delta p \cong \frac{1}{2}mu$$

But $u = \frac{M}{m}\sqrt{\frac{2}{3}gL}\sin\left(\frac{\theta}{2}\right)$

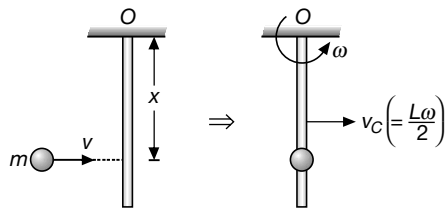
$$\Rightarrow \quad \Delta p \cong \frac{1}{2}(m)\left(\frac{M}{m}\right)\sqrt{\frac{2}{3}gL}\sin\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \quad \Delta p \cong M\sqrt{\frac{gL}{6}}\sin\left(\frac{\theta}{2}\right)$$

Hence, the correct answer is (D).

22. Applying Law of Conservation of Angular Momentum about the hinge we get

$$mvx = I\omega = \left(\frac{ML^2}{3}\right)\omega \qquad \ldots(1)$$



Also, by Law of Conservation of Linear Momentum, we get

$$P_i = P_f$$

$$\Rightarrow \quad mv = Mv_c = M\left(\frac{L}{2}\omega\right) \qquad \ldots(2)$$

From equations (1) and (2), we get

$$x = \frac{2L}{3}$$

Hence, the correct answer is (C).

23. $I = \frac{m\ell^2}{3}$

$$\Rightarrow \quad m\ell^2 = 3I \qquad \ldots(1)$$

Further, let R be the radius of ring, then

$$\ell = 2\pi R$$

$$\Rightarrow \quad R = \frac{\ell}{2\pi}$$

$$I_1 = mR^2 = m\left(\frac{\ell^2}{4\pi^2}\right) = \left(\frac{3I}{4\pi^2}\right)$$

Hence, the correct answer is (B).

24. $\ell = 2(2\pi R)$

$$\Rightarrow \quad R' = \frac{\ell}{4\pi}$$

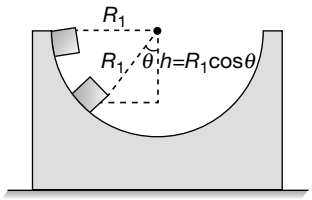
$$I_2 = 2m(R')^2 = \frac{2m\ell^2}{16\pi^2} = \left(\frac{3}{8\pi^2}\right)I$$

Hence, the correct answer is (D).

25. For block

$$v(\theta)=\sqrt{2gR_1\cos\theta}$$

$$\Rightarrow \quad R_1\left(-\frac{d\theta}{dt}\right)=\sqrt{2gR_1\cos\theta}$$



$$\Rightarrow \quad \frac{-d\theta}{\sqrt{\cos\theta}}=\sqrt{\frac{2g}{R_1}}dt$$

$$\Rightarrow \quad \sqrt{\frac{2g}{R_1}}\int_0^{t_1}dt=\int_{\pi/2}^{0^\circ}\frac{-d\theta}{\sqrt{\cos\theta}}$$

$$\Rightarrow \quad t_1=\sqrt{\frac{R_1}{2g}}\int_{\pi/2}^{0^\circ}\frac{-d\theta}{\sqrt{\cos\theta}} \qquad \ldots(1)$$

For the ball, we have, $\frac{K_R}{K_T}=\frac{2}{5}$

$$\Rightarrow \quad \frac{1}{2}mv^2=\frac{5}{7}E$$

where E is the total mechanical energy, so

$E=mgh$, where $h=gR_2\cos\theta$

$$\Rightarrow \quad v(\theta)=\sqrt{\frac{10}{7}gR_2\cos\theta}$$

Now again proceeding in the similar manner, we get

$$t_2=\sqrt{\frac{7R_2}{10g}}\int_{\pi/2}^{0^\circ}\frac{-d\theta}{\sqrt{\cos\theta}} \qquad \ldots(2)$$

From equations (1) and (2), on dividing and using $R_1=2R_2$, we get

$$\frac{t_1}{t_2}=\sqrt{\frac{10}{7}}$$

$$\Rightarrow \quad t_1>t_2$$

So, the ball will reach the bottom of the track first.

Hence, the correct answer is (B).

26. From equations (1) and (2), already calculated, when

$$t_1=t_2$$

then $\frac{R_1}{2}=\frac{7R_2}{10}$

$$\Rightarrow \quad \frac{R_1}{R_2}=\frac{7}{5}$$

Hence, the correct answer is (C).

27. $\tau_{av}=\left|\frac{\Delta L}{\Delta t}\right|=\frac{1}{2}=0.5$ Nm

Hence, the correct answer is (C).

28. $\theta=\left(\frac{\omega+\omega_0}{2}\right)t=\left(\frac{L+L_0}{2I}\right)t$

$$\Rightarrow \quad \theta=\left[\frac{3+2}{2(0.4)}\right](2)=12.5 \text{ rad}$$

So, number of revolutions is

$$N=\frac{\theta}{2\pi}=\frac{12.5}{2(3.14)}=1.99\approx 2$$

Hence, the correct answer is (A).

29. $W=\Delta K=\frac{L^2}{2I}-\frac{L_0^2}{2I}$

$$\Rightarrow \quad W=\frac{1}{2(0.4)}(4-9)=-6.25 \text{ J}$$

$$\Rightarrow \quad |W|=6.25 \text{ J}$$

Hence, the correct answer is (D).

30. C is the centre of mass of rod and O the combined centre of mass of system (ball + rod) after collision. If we consider the origin at the centre of the rod, C, then the combined centre of mass of ball + rod is at a distance $\frac{x}{2}$ from C, as shown. Conserving angular momentum about point O.

$$mv_0\left(\frac{x}{2}\right)=I_0\omega=\left[m\left(\frac{x}{2}\right)^2+\frac{m\ell^2}{12}+m\left(\frac{x}{2}\right)^2\right]\omega$$

$$\Rightarrow \quad \omega=\frac{v_0\left(\frac{x}{2}\right)}{\left(\frac{\ell^2}{12}\right)+\left(\frac{x^2}{2}\right)}=\frac{6v_0x}{(\ell^2+6x^2)}$$

$$\Rightarrow \quad \omega=\frac{6v_0}{\left(6x+\frac{\ell^2}{x}\right)}$$

For ω to be maximum or minimum quantity $\left(6x+\frac{\ell^2}{x}\right)$ should be minimum or maximum

$$\frac{d}{dx}\left(6x+\frac{\ell^2}{x}\right)=0$$

$$\Rightarrow \quad 6-\frac{\ell^2}{x^2}=0$$

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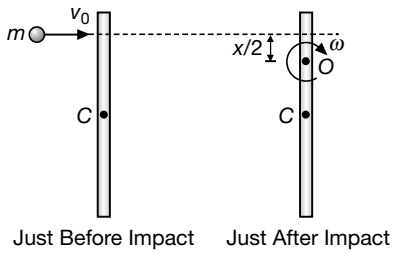


$$\Rightarrow \quad x = \frac{\ell}{\sqrt{6}}$$

Further $\dfrac{d^2\left(6x + \dfrac{\ell^2}{x}\right)}{dx^2}$ at $x = \dfrac{\ell}{\sqrt{6}}$ is positive, hence at $x = \dfrac{\ell}{\sqrt{6}}$, the denominator is minimum and hence ω is maximum

Further at $x = 0$, $\omega = 0$, therefore ω will first increase and then it will decrease.

Hence, the correct answer is (B).

31. Impulse $|J| = \Delta p = |p_f - p_i|$

$$\Rightarrow \quad |J| = p_i - p_f \left(\text{as } p_i > p_f\right)$$

Now, $|J|$ will be maximum when p_f or v_f for ball is least. v_f will be least when the ball strikes at centre of rod, because in that case $v_f = \dfrac{v_0}{2}$ (by Law of Conservation of Linear Momentum). In all other cases, one more term $r\omega$, will be added to this.

$$\text{So, } |J|_{\max} = m\left(v_0 - \frac{v_0}{2}\right) = \frac{mv_0}{2}$$

Hence, the correct answer is (A).

32. Translational kinetic energy in both cases will be equal but rotational kinetic energy in CASE-1 will be less. This is because perpendicular distance of linear impulse from the centre is less. So, angular impulse imparted to the sphere will be less or ω will be less.

Hence, the correct answer is (C).

33. Friction will be in forwards direction in the case of backward slipping just after being hit by the cue, i.e.,

$$R\omega > v$$

By Angular Impulse – Angular Momentum Theorem, we have

$$Jh = \mathrm{I}\omega$$

where J is the linear impulse

$$\Rightarrow \quad R\left(\frac{Jh}{\mathrm{I}}\right) > \frac{J}{m}$$

$$\Rightarrow \quad h > \frac{\mathrm{I}}{mR}$$

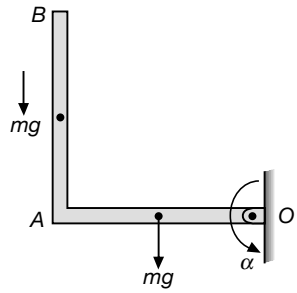
$$\Rightarrow \quad h > \frac{\frac{2}{5}mR^2}{mR}$$

$$\Rightarrow \quad h > 0.4R$$

i.e., in CASE-2

Hence, the correct answer is (D).

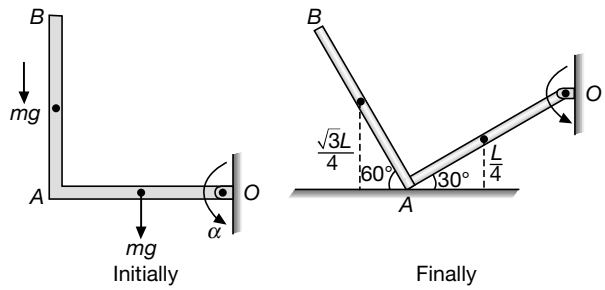
34.



$$\alpha = \frac{\tau_0}{I_0} = \frac{mg\dfrac{L}{2} + mgL}{\dfrac{mL^2}{3} + mL^2} = \frac{9g}{8L}$$

Hence, the correct answer is (B).

35. Let ω be the angular speed just before striking with ground.



$$\begin{pmatrix}\text{Decrease in}\\ \text{Gravitational}\\ \text{Potential Energy}\end{pmatrix} = \begin{pmatrix}\text{Increase in}\\ \text{Rotational}\\ \text{Kinetic Energy}\end{pmatrix}$$

$$\Rightarrow \quad mg\left(\frac{L}{2} - \frac{L}{4}\right) + mg\left(L - \frac{\sqrt{3}L}{4}\right) = \frac{1}{2}\left(\frac{mL^2}{3} + mL^2\right)\omega^2$$

Solving this equation, we get,

$$\omega = 1 \cdot 1\sqrt{\frac{g}{L}}$$

$$\Rightarrow \quad v_A = (OA)\omega = L\omega = 1 \cdot 1\sqrt{gL}$$

Hence, the correct answer is (D).

36. The distance of centre of mass of the system from point A is

$$r = \frac{\ell}{\sqrt{3}}$$

Therefore, the magnitude of horizontal force exerted by the hinge on the body is

F =centripetal force

$$\Rightarrow \quad F = (3m)r\omega^2$$

$$\Rightarrow \quad F = (3m)\left(\frac{\ell}{\sqrt{3}}\right)\omega^2$$

$$\Rightarrow \quad F = \sqrt{3}\, m\ell\omega^2$$

Hence, the correct answer is (C).

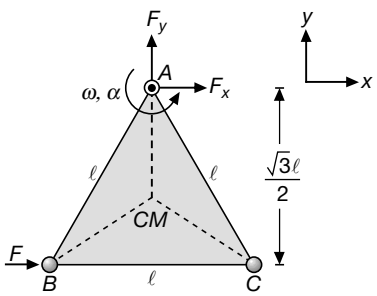
37. Angular acceleration of system about point A is

$$\alpha = \frac{\tau_A}{I_A} = \frac{(F)\left(\frac{\sqrt{3}}{2}\ell\right)}{2m\ell^2} = \frac{\sqrt{3}F}{4m\ell}$$

Now, acceleration of centre of mass along x-axis is

$$a_x = r\alpha = \left(\frac{\ell}{\sqrt{3}}\right)\left(\frac{\sqrt{3}F}{4m\ell}\right)$$

$$\Rightarrow \quad a_x = \frac{F}{4m}$$



Now, let F_x be the force applied by the hinge along x-axis. Then,

$$F_x + F = (3m)a_x$$

$$\Rightarrow \quad F_x + F = (3m)\left(\frac{F}{4m}\right)$$

$$\Rightarrow \quad F_x + F = \frac{3}{4}F$$

$$\Rightarrow \quad F_x = -\frac{F}{4}$$

$$\Rightarrow \quad F_x = \frac{F}{4}, \text{ along negative } x\text{-axis}$$

Hence, the correct answer is (D).

38. If F_y be the force applied by the hinge along y-axis. Then,

F_y = Centripetal Force

$$\Rightarrow \quad F_y = \sqrt{3}\, m\ell\omega^2$$

Hence, the correct answer is (C).

39. If ground is smooth the body will rotate about O, so

$$K = \frac{1}{2}I_0\omega^2$$

$$\Rightarrow \quad K = \frac{1}{2}I_0(\alpha t)^2$$

$$\Rightarrow \quad K = \frac{1}{2}I_0\left(\frac{\tau}{I_0}\right)^2 t^2$$

$$\Rightarrow \quad K = \frac{\tau^2 t^2}{2I_0}$$

where $\tau = F(2R) - F(R) = FR$

$$\Rightarrow \quad K = \frac{F^2R^2t^2}{2I_0}$$

Substituting the values, we get

$$K = \frac{(10)^2(1)^2(2)^2}{2\times 4} = 50 \text{ J}$$

Hence, the correct answer is (B).

40. In this case the body will rotate about bottommost point, so

$$K = \frac{\tau^2 t^2}{2I}$$

$$\Rightarrow \quad K = \frac{[F(4R) - F(3R)]^2 t^2}{2\left[I_0 + m(2R)^2\right]}$$

$$\Rightarrow \quad K = \frac{F^2R^2t^2}{2\left[I_0 + 4mR^2\right]}$$

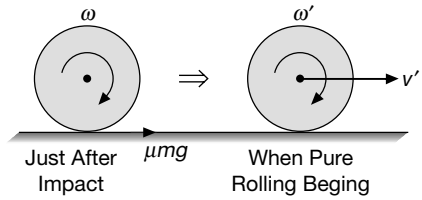
$$\Rightarrow \quad K = \frac{(10)^2(1)^2(6)^2}{2[4+8]}$$

$$\Rightarrow \quad K = 150 \text{ J}$$

Hence, the correct answer is (D).

41. Since both spheres are of equal masses and the second sphere is at rest, so just after collision the first sphere comes to rest and it will have only ω i.e., it will slip backwards. So, friction will be maximum and in forward direction. Let v' be its linear speed and ω' its angular speed when it again starts pure rolling. Friction is passing through its bottommost point, so we can conserve angular momentum about an axis passing through its bottommost point and perpendicular to plane of motion. So,

CHAPTER 3



$$L_i = L_f$$

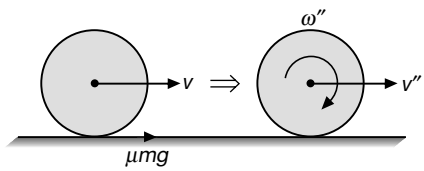
$$\Rightarrow \quad \frac{2}{5}(mR^2)\omega = \frac{2}{5}(mR^2)\omega' + mv'R$$

$$\Rightarrow \quad \frac{2}{5}(mR^2)\omega = \frac{2}{5}\left(mR^2\left(\frac{v'}{R}\right) + mv'R\right)$$

$$\Rightarrow \quad v' = \frac{2}{7}R\omega$$

Hence, the correct answer is (B).

42. Thinking on the same concept as discussed above, we again have



$$L_i = L_f$$

$$\Rightarrow \quad mvR = \frac{2}{5}(mR^2)\omega'' + m(\omega''R)R$$

$$\Rightarrow \quad \omega'' = \frac{5}{7}\frac{v}{R}$$

Now, angular impulse imparted to the sphere is

$$\Delta L = I\omega'' = \left(\frac{2}{5}MR^2\right)\left(\frac{5}{7}\frac{v}{R}\right)$$

$$\Rightarrow \quad \Delta L = \frac{2}{7}mvR$$

Hence, the correct answer is (A).

43. In case of pure rolling

$$a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}$$

a is minimum when I is maximum which is for ring equal to mR^2. So,

$$a_{\min} = \frac{g\sin\theta}{2} = \frac{10}{4}\ \text{ms}^{-2}$$

$$\Rightarrow \quad t_{\max} = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 5}{\left(\frac{10}{4}\right)}} = 2\ \text{s}$$

Hence, the correct answer is (D).

44. Minimum value of μ required in this case is

$$\mu_{\min} = \frac{\tan\theta}{1 + \frac{mR^2}{I}}$$

I is maximum for ring, hence minimum value of μ required for pure rolling will have maximum value for ring. So, it will be the first to start slipping.

Hence, the correct answer is (A).

45. $$M = \int_0^a \frac{\rho_0(a+x)}{a}dx = \frac{\rho_0}{a}\left(ax + \frac{x^2}{2}\right)\Big|_0^a = \frac{3a\rho_0}{2}$$

Hence, the correct answer is (D).

46. $$x_{\text{cm}} = \frac{\int_0^a (dm)x}{M} = \frac{\int_0^a \frac{\rho_0(a+x)}{a}xdx}{\frac{3a\rho_0}{2}}$$

$$\Rightarrow \quad x_{\text{cm}} = \frac{2}{3a}\left(\frac{ax^2}{2} + \frac{x^3}{2}\right)\Big|_0^a = \frac{5a}{9}$$

Hence, the correct answer is (C).

47. $$I_A = \int_0^a (dm)x^2 = \int_0^a \frac{\rho_0(a+x)}{a}x^2dx = \frac{\rho_0}{a}\left(\frac{ax^3}{3} + \frac{x^4}{4}\right)\Big|_0^a$$

$$\Rightarrow \quad I_A = \frac{7\rho_0 a^3}{12}$$

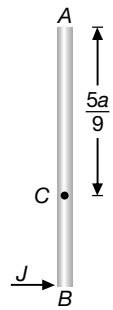
$$\Rightarrow \quad I_A = \frac{7Ma^2}{18} \qquad \left\{\because\ M = \frac{3a\rho_0}{2}\right\}$$

Hence, the correct answer is (A).

48. Since, $\begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix} = \begin{pmatrix}\text{Change in Angular}\\ \text{Momentum}\end{pmatrix}$, so we get

$$Ja = I_A\omega = \frac{7Ma^2}{18}\omega$$

$$\Rightarrow \quad \omega = \frac{18J}{7Ma}$$



B will pass through a point vertically above A when

$$\frac{1}{2}I_A\omega^2 > Mg(2AC)$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{7Ma^2}{18}\right)\left(\frac{18J}{7Ma}\right)^2 > 2Mg\left(\frac{5a}{9}\right)$$

$$\Rightarrow \quad J > \frac{M}{9}\sqrt{70ag}$$

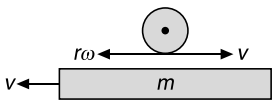
$$\Rightarrow \quad J_{\min} = \frac{M}{9}\sqrt{70ag}$$

Hence, the correct answer is (B).

49. At the time of pure rolling, we have

$$-v = v - r\omega$$

$$\Rightarrow \quad v = \frac{r\omega}{2}$$



Also, we have

$$\frac{1}{2}\left(\frac{1}{2}I_{cm}\omega_0^2\right) = \frac{1}{2}mv^2 + \left(\frac{1}{2}mv^2 + \frac{1}{2}I_{cm}\omega^2\right)$$

$$\Rightarrow \quad \frac{r^2\omega_0^2}{8} = 2v^2$$

$$\Rightarrow \quad v = \frac{r\omega_0}{4}$$

Hence, the correct answer is (A).

50. When slipping ceases, we have

$$0 = v - (\mu g)t$$

$$\Rightarrow \quad v = (\mu g)t$$

$$\Rightarrow \quad t = \frac{r\omega_0}{4\mu g}$$

Hence, the correct answer is (D).

51. $L_i = \left(\frac{1}{2}mr^2\right)\omega_0$

$$L_f = \left(\frac{1}{2}mr^2\right)\omega$$

Since $\omega = \frac{\omega_0}{4}$, so

$$|\Delta L| = |L_f - L_i| = \frac{3}{8}mr^2\omega_0$$

Hence, the correct answer is (D).

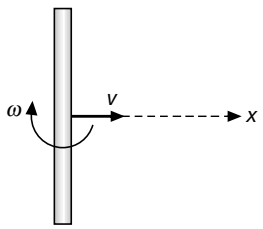
52. Since $0^2 - v^2 = 2(-\mu g)s$

$$\Rightarrow \quad s = \frac{v^2}{2\mu g}$$

$$\Rightarrow \quad s = \frac{r^2\omega_0^2}{32\mu g}$$

Hence, the correct answer is (D).

53. $v = \frac{J}{m} = \frac{10}{2} = 5 \text{ ms}^{-1}$



$$\omega = \frac{\tau}{I} = \frac{J\left(\frac{\ell}{2}\right)}{I} = \frac{J\ell}{2\left(\frac{m\ell^2}{12}\right)} = \frac{6J}{m\ell}$$

$$\Rightarrow \quad \omega = \frac{6\times 10}{2\times 2} = 15 \text{ rads}^{-1}$$

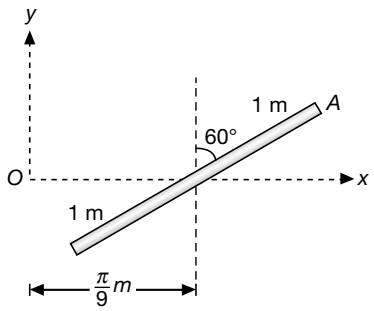
At point P, which is say at distance r from centre where if, v and $r\omega$ cancel each other then that point will be at rest just after impact.

Therefore, $r = \frac{v}{\omega} = \frac{5}{15} = \frac{1}{3}$ m

Hence, the correct answer is (C).

54. In the given time angle through which the rod rotates is

$$\theta = \omega t = 15\left(\frac{\pi}{45}\right) = \frac{\pi}{3} = 60°$$



The displacement of centre of mass from origin O is

$$s = vt = (5)\left(\frac{\pi}{45}\right) = \frac{\pi}{9} \text{ m}$$

CHAPTER 3

$$\Rightarrow \quad x_A = \frac{\pi}{9} + (1)\sin(60°) = \left(\frac{\pi}{9} + \frac{\sqrt{3}}{2}\right) \text{ m}$$

$$\Rightarrow \quad y_A = 1\cos(60°) = \frac{1}{2} \text{ m}$$

Hence, the correct answer is (D).

55. Acceleration of block $a_B = g\sin(45°) - \mu g\cos(45°)$

$$\Rightarrow \quad a_B = \frac{g}{\sqrt{2}} - \frac{g}{2\sqrt{2}} = \frac{g}{2\sqrt{2}}$$

and acceleration of cylinder $a_C = \dfrac{g\sin 45°}{1 + \dfrac{I}{2mR^2}}$ …(1)

where, $I = \frac{1}{2}(2m)R^2$

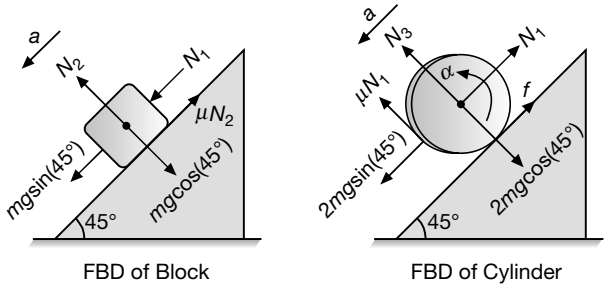
$$\Rightarrow \quad \frac{I}{2mR^2} = \frac{1}{2}$$

Substituting in equation (1), we get

Acceleration of cylinder $a_C = \dfrac{\sqrt{2}g}{3}$

Hence, the correct answer is (D).

56. The free body diagrams of the block and the cylinder are as shown.



FBD of Block FBD of Cylinder

Equations of motion are,

$$N_2 = mg\cos(45°) + \mu N_1 \quad \ldots(1)$$

$$mg\sin(45°) + N_1 - \mu N_2 = ma \quad \ldots(2)$$

$$\mu N_1 + N_3 = 2mg\cos(45°) \quad \ldots(3)$$

$$2mg\sin(45°) - N_1 - f = 2ma \quad \ldots(4)$$

Also, $\alpha = \dfrac{(f - \mu N_1)R}{\frac{1}{2}(2m)R^2}$ …(5)

For no slipping, we have

$$a = R\alpha \quad \ldots(6)$$

Solving these equations, we get

$$a = \frac{3g}{5\sqrt{2}}$$

Hence, the correct answer is (B).

57. In process A, $T = 5$ N

In process B, $mg - T' = ma$

$$\Rightarrow \quad T' = (5 - ma) < T$$

So, tangential acceleration will be greater in process A than that in B.

Hence, the correct answer is (B).

58. The mechanical energy is conserved in both A and B.

Hence, the correct answer is (A).

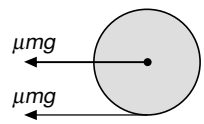
59. Let the a be the acceleration of C.M. w.r.t. plank, then

$$2\mu mg = ma$$

$$\Rightarrow \quad a = 2\mu g \quad \ldots(1)$$

Further, $\tau = \mu(mg)r = \dfrac{mr^2}{2}\alpha$

$$\Rightarrow \quad \alpha = \frac{2\mu g}{r}$$



Let a_0 be the acceleration of point of contact due to rotation.

$$a_0 = r\alpha$$

$$\Rightarrow \quad a_0 = 2\mu g \quad \ldots(2)$$

Acceleration a' of point of contact w.r.t. the plank is

$$a' = a + a_0$$

$$\Rightarrow \quad a' = 4\mu g$$

$$\Rightarrow \quad a' = 4 \text{ ms}^{-2}$$

Hence, the correct answer is (C).

60. $t = \dfrac{v}{a'} = \dfrac{v}{4\mu g}$

$$\Rightarrow \quad t = 2.5 \text{ s}$$

Hence, the correct answer is (B).

61. Distance travelled by the cylinder w.r.t. plank in 2.5 s is

$$s = -vt + \frac{1}{2}(2\mu g)t^2$$

$$\Rightarrow \quad s = -18.75 \text{ m}$$

$$\Rightarrow \quad |s| = 18.75 \text{ m}$$

After $t = 2.5$ s the velocity of cylinder is

$$v' = -v + (2\mu g)(2.5)$$

$$\Rightarrow \quad v' = -5 \text{ ms}^{-1}$$

Remaining distance = 40 – 18.75 = 21.25 m

Time taken to cover 21.25 m is

$$t' = \frac{21.25}{5} = 4.25 \text{ s}$$

So, total time $T = 2.5 + 4.25 = 6.75$ s

Hence, the correct answer is (D).

62. Let v_{cm} be the velocity of centre of mass, then

$$v_P = v_{cm} - \frac{R\omega}{2}$$

$$\Rightarrow \quad v_P = R\omega - \frac{R\omega}{2} = \frac{R\omega}{2} = \frac{v_{cm}}{2}$$

Since $v_P = 2 \text{ ms}^{-1}$

$$\Rightarrow \quad v_{cm} = R\omega = 4 \text{ ms}^{-1}$$

$$\Rightarrow \quad \omega = 4 \text{ rads}^{-1} \qquad \{\because R = 1 \text{ m}\}$$

Now, acceleration of point P is

$$a_P = 10 = \sqrt{a_T^2 + a_N^2}$$

$$\Rightarrow \quad 10 = \sqrt{a_T^2 + \left(\omega^2 \frac{R}{2}\right)^2}$$

$$\Rightarrow \quad a_T = \sqrt{36} = 6 \text{ ms}^{-2}$$

If a_{cm} be the acceleration of the centre of mass then

$$a_T = a_{\text{cm}} - \frac{R\alpha}{2} = 6$$

$$\Rightarrow \quad a_{\text{cm}} - \frac{a_{\text{cm}}}{2} = 6$$

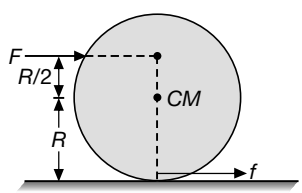
$$\Rightarrow \quad a_{\text{cm}} = 12 \text{ ms}^{-2}$$

Hence, the correct answer is (C).

63. If f be the force of friction, then

$$F\left(\frac{R}{2}\right) - fR = I\alpha = \left(\frac{1}{2}mR^2\right)\alpha$$

$$\Rightarrow \quad \frac{F}{2} - f = (5)(12) = 60 \qquad \ldots(1)$$



Also, $F + f = ma_{\text{cm}} = 10 \times 12 = 120$ …(2)

From (1) and (2), we get

$$\frac{3F}{2} = 180$$

$$\Rightarrow \quad F = (180)\left(\frac{2}{3}\right) = 120 \text{ N}$$

Hence, the correct answer is (C).

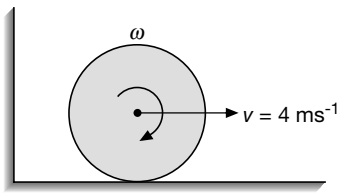
64. Let N be normal force between the wall and sphere at the time of impact and Δt the time of impact, then

$$N\Delta t = \Delta p = 8 \text{ m} \qquad \ldots(1)$$

$$\{\because \Delta v = 8 \text{ ms}^{-1}\}$$

Further $R(\mu N \Delta t) = \Delta(\text{I}\omega) = \left(\frac{2}{5}mR^2\right)(5)$ …(2)

$$\{\because \Delta\omega = 5 \text{ rads}^{-1}\}$$



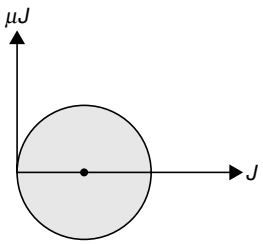
Dividing equation (2) by (1), we get

$$\mu = \frac{1}{4}$$

Hence, the correct answer is (A).

65. Net linear impulse is $J_{\text{net}} = \sqrt{J^2 + \mu^2 J^2}$

$$\Rightarrow \quad J_{\text{net}} = \left(\sqrt{1 + \mu^2}\right) \text{J}$$



where, $J = \Delta P = 16$ Ns

So, net impulse is $J_{\text{net}} = \left(\sqrt{1 + \frac{1}{16}}\right)(16) = 4\sqrt{17}$ Ns

Hence, the correct answer is (C).

66. $$\frac{\text{Rotational KE}}{\text{Translational KE}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2} = \frac{K^2}{R^2} \qquad \{\because I = MK^2, v = R\omega\}$$

Hence, the correct answer is (D).

67. $$\frac{\text{RKE}}{\text{TE}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2\left(1 + \frac{R^2}{K^2}\right)} = \frac{K^2}{R^2 + K^2}$$

Hence, the correct answer is (D).

68. $$\frac{\text{TKE}}{\text{TE}}=\frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)}=\frac{R^2}{R^2+K^2}$$

Hence, the correct answer is (B).

69. At maximum height, both the sphere and wedge have same horizontal velocity say v, then by Impulse – Momentum Theorem, we have

$$(\text{Im}\,pulse)=\left(\begin{array}{c}\text{Total Change in}\\ \text{Momentum of Wedge}\end{array}\right)+(\text{sphere})$$

$$\Rightarrow \quad P=(4m)v+(m)v$$

$$\Rightarrow \quad P=5mv$$

Just after sharp impulse is given to the sphere, let ω_0 be the angular velocity and v_0 be the velocity of centre of mass of sphere, then by Law of Conservation of Energy, we have

$$\frac{1}{2}mv_0^2+\frac{1}{2}I_{cm}\omega_0^2=\left(\frac{1}{2}mv^2+\frac{1}{2}I_{cm}\omega_0^2\right)+\frac{1}{2}(4m)v^2$$

$$\Rightarrow \quad \frac{1}{2}mv_0^2=\frac{1}{2}(5m)v^2+mgh$$

$$\Rightarrow \quad h=\frac{4P^2}{10m^2g}=\frac{2P^2}{5m^2g}$$

Hence, the correct answer is (A).

70. $$KE=\left(\frac{1}{2}mv^2+\frac{1}{2}I_{cm}\omega_0^2\right)+\frac{1}{2}(4m)v^2$$

$$\Rightarrow \quad KE=\frac{1}{2}(5m)v^2+\frac{1}{2}I_{cm}\omega_0^2 \qquad \ldots(1)$$

Now, by Angular Impulse – Angular Momentum Theorem, we have

$$Ph=I_{cm}\omega_0, \text{ where } h=0.4r=\frac{2}{5}r$$

$$\Rightarrow \quad \omega_0=\frac{Ph}{I_{cm}}=\frac{P\left(\frac{2}{5}r\right)}{\frac{2}{5}mr^2}$$

$$\Rightarrow \quad \omega_0=\frac{P}{mr}$$

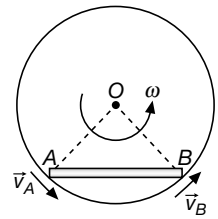
So, from (1), we get

$$KE=\frac{1}{2}(5m)\left(\frac{P}{5m}\right)^2+\frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{P}{mr}\right)^2$$

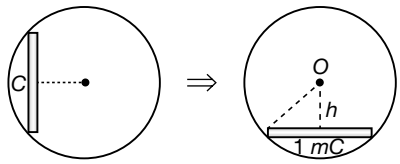
$$\Rightarrow \quad KE=\frac{3P^2}{10m}$$

Hence, the correct answer is (C).

71. Direction of velocities of two points of the rod A and B are tangential as shown.



When we draw perpendicular on them, they meet at O, the centre of sphere. So, the rod can be assumed in pure rotation about an axis passing through O and perpendicular to plane of paper (also called $IAOR$). Since, the surface is smooth mechanical energy will remain conserved. So, decrease in gravitational potential energy of CM of rod = Increase in rotational kinetic energy about $IAOR$



$$\Rightarrow \quad mgh=\frac{1}{2}I_{IAOR}\omega^2=\frac{1}{2}\left(\frac{m\ell^2}{12}+mh^2\right)\omega^2$$

where, $h=\sqrt{4-1}=\sqrt{3}$ m

$$\Rightarrow \quad 4\times10\times\sqrt{3}=\left(\frac{1}{2}\frac{(4)(4)}{12}+(4)(3)\right)\omega^2$$

$$\Rightarrow \quad \omega=3.2 \text{ rads}^{-1}$$

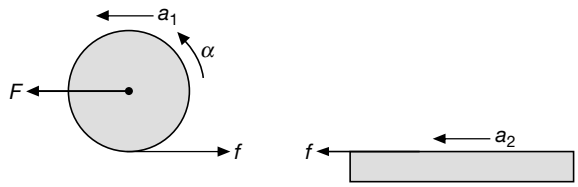
Hence, the correct answer is (A).

72. $v_c=h\omega=(\sqrt{3})(3.2)=5.5 \text{ ms}^{-1}$

Hence, the correct answer is (B).

73-76. The correct answer is 73(A), 74(C), 75(B) and 76(A).

Combined solution to 73, 74, 75, 76



$$F-f=ma_1 \qquad \ldots(1)$$

$$f=ma_2 \qquad \ldots(2)$$

$$\text{Also, } \alpha=\frac{\tau}{I}=\frac{fR}{\frac{1}{2}mR^2}=\frac{2f}{mR} \qquad \ldots(3)$$

$$\text{and } a_1-R\alpha=a_2 \qquad \ldots(4)$$

Solving equations, (1), (2), (3) and (4), we get

$$a_1 = \frac{3F}{4m},\ a_2 = \frac{F}{4m},\ f = \frac{F}{4},\ \alpha = \frac{F}{2mR}$$

77. Moment of inertia of the system about the given axis,

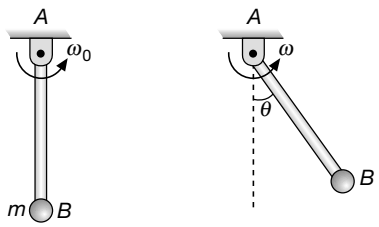
$$I = \frac{(3m)(2\ell)^2}{3} + m(2\ell)^2 = 8m\ell^2 \qquad \ldots(1)$$

By Law of Conservation of Mechanical Energy in the two positions shown in figure, we get

$$\frac{1}{2}I\left(\sqrt{\frac{kg}{\ell}}\right)^2 = \frac{1}{2}I\omega^2 + mg2\ell(1-\cos\theta) + 3mg\ell(1-\cos\theta)$$

Substituting the value of I from equation (1), we get

$$4\ell\omega^2 = 4kg - 5g(1-\cos\theta) \qquad \ldots(2)$$

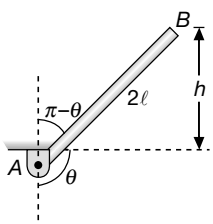


Initial Position General Position

When the rod comes to instantaneous rest, $\omega = 0$, so, we get

$$4kg - 5g(1-\cos\theta) = 0$$

Substituting, $k = 2$, we get $\cos\theta = -\frac{3}{5}$



In this position, θ is an obtuse angle, so the height h of B above A is given by,

$$h = 2\ell\cos(\pi-\theta) = 2\ell\left(\frac{3}{5}\right) = \frac{6\ell}{5}$$

Hence, B rises to $\frac{6\ell}{5}$ above the level of A.

Hence, the correct answer is (C).

78. Since the particle is attached to a rigid rod, its motion is restricted to a circular path. It will therefore, describe complete circles if its speed at the highest point is greater than zero. Since, $4\ell\omega^2 = 4kg - 5g(1-\cos\theta)$ has a real ω at $\theta = \pi$, when

$$4\text{kg} - 5g(1-\cos\pi) > 0$$

The particle will therefore, describe complete circle, only when

$$k > \frac{5}{2}$$

$$\Rightarrow \quad k_{\min} = \frac{5}{2}$$

Hence, the correct answer is (B).

79. $1g - T = 1a \qquad \ldots(1)$

Also, $T(1) = \frac{1}{2}(2)a$

$$\Rightarrow \quad T = a \qquad \ldots(2)$$

$$\Rightarrow \quad 1g - 1a = 1a$$

$$\Rightarrow \quad a = \frac{g}{2} = 5\ \text{ms}^{-2}$$

$$\Rightarrow \quad T = a = 5\ \text{N} \qquad \{\because R = 1\ \text{m}\}$$

Hence, the correct answer is (B).

80. $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(5)(16) = 40\ \text{rad}$

Hence, the correct answer is (A).

81. $W = \tau\theta = (5)(40) = 200\ \text{J}$

Hence, the correct answer is (D).

82. $\Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}(2)(1)^2\right)(5\times 4)^2$

$$\Rightarrow \quad \Delta K = 200\ \text{J}$$

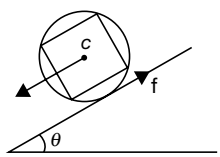
Hence, the correct answer is (C).

83. $I = 4\left\{\frac{M(R\sqrt{2})^2}{12} + M\left(\frac{R}{\sqrt{2}}\right)^2\right\} + mR^2$

$$\Rightarrow \quad I = \frac{8}{3}MR^2 + mR^2 = 20\ \text{kgm}^2$$

Hence, the correct answer is (B).

84.



$$(4M+m)g\sin\theta - f = (4M+m)a \qquad \ldots(1)$$

CHAPTER 3

Also, $\tau = fR = I\left(\frac{a}{R}\right)$ …(2)

Solving (1) and (2)

$$a = \frac{7g}{24}$$

Hence, the correct answer is (D).

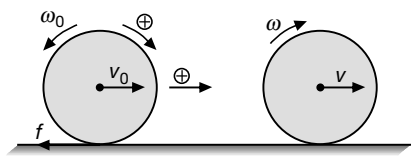
85. From above we get

$$f = 20\ a = \mu \times 28g\cos(30°)$$

$$\Rightarrow \quad \mu = \frac{5}{12\sqrt{3}}$$

Hence, the correct answer is (D).

86.



Let pure rolling start at time t, then

$$I\omega - I(-\omega_0) = (fR)t$$

$$\Rightarrow \quad I\omega + I\omega_0 = (fR)t$$

$$\Rightarrow \quad \frac{1}{2}MR^2\left(\frac{v}{R} + \frac{v_0}{R}\right) = fRt$$

$$\Rightarrow \quad Mv + Mv_0 = 2ft \qquad \ldots(1)$$

where $f = \mu Mg$

Also, we have

$$Mv - Mv_0 = -ft \qquad \ldots(2)$$

So, from (1) and (2), we get

$$t = \frac{2v_0}{3\mu g}$$

Hence, the correct answer is (C).

87. Applying Law of Conservation of Angular Momentum about the point of contact, because net torque on the disc about the point of contact is zero, so

$$L_{\text{initial}} = L_{\text{final}}$$

Since, for a body in combined effect of rotation and translation, we have

$$\vec{L} = \vec{L}_{\text{cm}} + m(\vec{r} \times \vec{v})$$

So, $L_{\text{initial}} = Mv_0R - I_{\text{cm}}\omega_0$ and

$$L_{\text{final}} = MvR + I_{\text{cm}}(0)$$

$$\Rightarrow \quad Mv_0R - \left(\frac{1}{2}MR^2\right)\frac{v_0}{R} = MvR$$

$$\Rightarrow \quad \frac{Mv_0R}{2} = MvR$$

$$\Rightarrow \quad v = \frac{v_0}{2}$$

Hence, the correct answer is (B).

88. Since $Mv - Mv_0 = -ft$

$$\Rightarrow \quad Mv - Mv_0 = -\mu(Mg)t$$

$$\Rightarrow \quad v - v_0 = -\mu gt \qquad \ldots(1)$$

Now, since pure rolling begins at $t = \frac{2v_0}{3\mu g}$

$$\Rightarrow \quad v - v_0 = -\mu g\left(\frac{2v_0}{3\mu g}\right)$$

$$\Rightarrow \quad v = v_0 - \frac{2v_0}{3}$$

$$\Rightarrow \quad v = \frac{v_0}{3}$$

This result can also be obtained by using Law of Conservation of Angular Momentum, according to which

$$Mv_0R - I_{\text{cm}}\omega_0 = MvR + I_{\text{cm}}\omega$$

$$\Rightarrow \quad Mv_0R - \left(\frac{1}{2}MR^2\right)\frac{v_0}{R} = MvR + \left(\frac{1}{2}MR^2\right)\frac{v}{R}$$

$$\Rightarrow \quad Mv_0R - \frac{Mv_0R}{2} = MvR + \frac{MvR}{2}$$

$$\Rightarrow \quad \frac{Mv_0R}{2} = \frac{3MvR}{2}$$

$$\Rightarrow \quad v = \frac{v_0}{3}$$

Hence, the correct answer is (C).

89. $L = mvr\sin\theta$

$$\Rightarrow \quad L = (2)(4)(3)\sin 30$$

$$\Rightarrow \quad L = 12\ \text{kgm}^2\text{s}^{-1}$$

Direction is found by Right Hand Thumb Rule

Hence, the correct answer is (A).

90. $\tau = Fr\sin\theta$

$$\Rightarrow \quad \tau = (2)(3)\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \tau = 3\ \text{Nm}$$

Direction is found by Right Hand Thumb Rule.

Hence, the correct answer is (D).

Matrix Match/Column Match Type Questions

1. A → (r)
B → (p)
C → (p)
D → (s)

$$mv - mv_0 = -\int F dt = -J \quad \ldots(1)$$

$$mV = J \quad \ldots(2)$$

$$mv_0 = mv + mV$$

$$\Rightarrow \quad v_0 = v + V \quad \ldots(3)$$

$$e = 1 = -\left[\frac{\left(V + \frac{\ell\omega}{2}\right) - v}{0 - v_0}\right]$$

$$\Rightarrow \quad V + \frac{\ell\omega}{2} - v = v_0$$

$$\Rightarrow \quad v + v_0 = V + \frac{\ell w}{2} \quad \ldots(4)$$

Also, $mv_0\left(\frac{\ell}{2}\right) = mv\left(\frac{\ell}{2}\right) + \left(\frac{m\ell^2}{12}\right)\omega$

$$\Rightarrow \quad v_0 = v + \frac{\ell\omega}{6} \quad \ldots(5)$$

From (3) and (5), we get

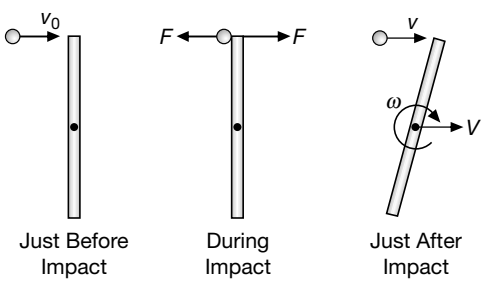
$$V = \frac{\ell\omega}{6}$$

So, from (4), we get

$$v + v_0 = \frac{2}{3}\ell\omega \quad \ldots(6)$$

Solving (5) and (6), we get

$$\omega = \frac{12v_0}{5\ell}$$



So, we get

$$v = \frac{3v_0}{5},\ V = \frac{2v_0}{5}$$

$$\Rightarrow \quad \frac{\text{Final KE of Ball}}{\text{Initial KE of Ball}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv_0^2} = \frac{9}{25}$$

$$\Rightarrow \quad \frac{\text{Impulse Delivered to Rod } (J)}{\text{Initial Momentum of Ball } (mv_0)} = \frac{2}{5}$$

$$\frac{(L_f)_{\text{rod}}}{(L_i)_{\text{ball about CM}}} = \frac{I\omega}{mv_0\left(\frac{\ell}{2}\right)}$$

$$\Rightarrow \quad \frac{(L_f)_{\text{rod}}}{(L_i)_{\text{ball}}} = \frac{\left(\frac{m\ell^2}{12}\right)\omega}{mv_0\left(\frac{\ell}{2}\right)} = \frac{2}{5}$$

$$\Rightarrow \quad \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}mV^2} = \frac{\left(\frac{m\ell^2}{12}\right)\omega^2}{mV^2}$$

$$\Rightarrow \quad \frac{(K_{\text{rod}})_{\text{rotation}}}{(K_{\text{rod}})_{\text{translation}}} = \frac{\ell^2\omega^2}{12V^2} = 3$$

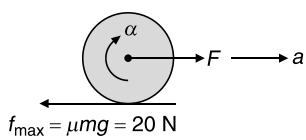
2. A → (q, r)
B → (p)
C → (s)
D → (q)

From graph in question, we see that

$$F = 10t$$

Since $a = R\alpha = R\left(\frac{\tau}{I}\right)$, where $a = \frac{F - f_\ell}{m}$

$$\Rightarrow \quad \frac{F - 20}{10} = (1)\left(\frac{20 \times 1}{\frac{1}{2} \times 10 \times 1^2}\right)$$



$$\Rightarrow \quad F = 60 \text{ N}$$

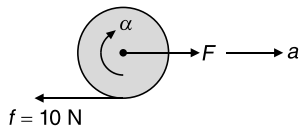
So, when $F = 60$ N, friction reaches its maximum value, i.e. slipping will start and F becomes 60 N at 6 s.

When $f = 10$ N, then

$$\frac{F - 10}{10} = (1)\left(\frac{10 \times 1}{\frac{1}{2} \times 10 \times 1^2}\right)$$

$$\Rightarrow \quad F = 30 \text{ N}$$

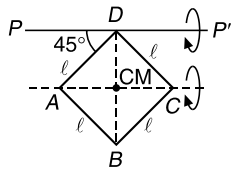
Since $F = 10t$, so F becomes 30 N at 3 s



3. A → (r, s)
B → (p, q)
C → (s)
D → (p, q, r, s)
Conceptual

4. A → (s)
B → (q)
C → (p)
D → (r)

(A) Moment of inertia of the frame about an axis passing through AC



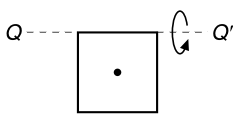
$$I_{AC} = 4\left(\frac{1}{3}ml^2\sin^2 45°\right) = \frac{2}{3}ml^2$$

Using parallel axis theorem, we get

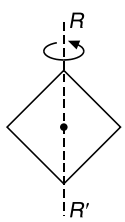
$$I_{PP'} = I_{AC} + 4m\left(\frac{l}{\sqrt{2}}\right)^2 = \frac{2}{3}ml^2 + 2ml^2$$

$$\Rightarrow \quad I_{PP'} = \frac{8}{3}ml^2$$

(B) $I_{QQ'} = \dfrac{2ml^2}{3} + ml^2 = \dfrac{5ml^2}{3}$

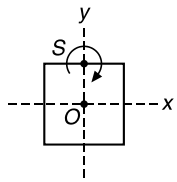


(C) $I_{RR'} = 4 \times \dfrac{1}{3}ml^2\sin^2 45° = \dfrac{2}{3}ml^2$



(D) $I_x = I_y = 2\left[\dfrac{1}{12}ml^2 + m\left(\dfrac{l}{2}\right)^2\right] = \dfrac{2ml^2}{3}$

Since, $I_z = I_x + I_y$



$$\Rightarrow \quad I_z = \frac{4ml^2}{3}$$

Moment of inertia about an axis passing through D

$$I_D = I_z + 4m\left(\frac{l}{2}\right)^2 = \frac{4}{3}ml^2 + ml^2 = \frac{7ml^2}{3}$$

5. A → (q)
B → (p)
C → (s)
D → (r)

In general, as $v_P = 2v_0\sin\left(\dfrac{\theta}{2}\right)$

6. A → (r)
B → (q)
C → (t)
D → (p)

For (A), $I = \dfrac{5}{4}MR^2$

For (B), $I = \dfrac{7}{5}MR^2$

For (C), $I = \dfrac{3}{2}MR^2$

For (D), $I = \dfrac{1}{2}MR^2$

7. A → (s)
B → (r)
C → (q)
D → (p)

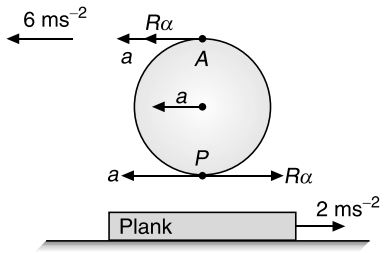
For rolling without slipping, acceleration of point P at bottom of cylinder equals the acceleration of plank, i.e. 2 ms^{-2}. So, we get

$$\Rightarrow \quad R\alpha - a = 2 \qquad \ldots(1)$$

For rolling without slipping, acceleration of point A at the top of cylinder equals acceleration of the block B, i.e. 6 ms^{-2}. So, we get

$$\Rightarrow \quad R\alpha + a = 6 \qquad \ldots(2)$$

where a is the acceleration of centre of mass of cylinder towards left.



Solving (1) and (2), we get

$a = 2 \text{ ms}^{-2}$ and $\alpha = 1 \text{ rads}^{-2}$

$\Rightarrow \quad 2a = 4 \text{ ms}^{-2}$ and $\alpha = 1 \text{ rads}^{-2}$

Further we have, for B,

$m_B g - T = m_B a_B$

$\Rightarrow \quad m_B g - T = m_B (6)$

For cylinder, we have

$T = (m_{cyl})a = 2m_{cyl}$

$\Rightarrow \quad m_B g - 2m_{cyl} = 6m_B$

$\Rightarrow \quad 2m_{cyl} = 4m_B$

$\Rightarrow \quad \dfrac{m_{cyl}}{m_B} = 2$

Also, $\ell = \ell_0 + \dfrac{1}{2}a't^2$

where a' is the acceleration at which the thread unwraps from the cylinder, so

$a' = R\alpha$

$\Rightarrow \quad \ell = 20 + \dfrac{1}{2}(R\alpha)(2)^2$

$\Rightarrow \quad \ell = 20 + 8$

$\Rightarrow \quad \ell = 28 \text{ m}$

$\Rightarrow \quad \Delta\ell = \ell - \ell_0 = 28 - 20 = 8 \text{ m}$

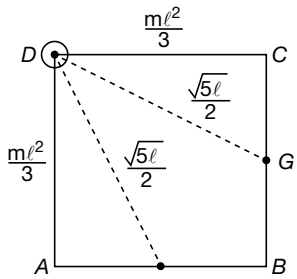
8. A → (p, s)
B → (p, r)
C → (q, r)
D → (p, r)
For the case of pure rolling (upwards or downwards), the required value of friction acts in upward direction and in case of slip (forward or backward) maximum friction will act in backward or forward direction.

9. A → (q)
B → (s)
C → (q)
D → (r)

$$I_1 = 2\left(\frac{m\ell^2}{12}\right) + 2(m)\left(\frac{\ell}{2}\right)^2 = \frac{2}{3}m\ell^2$$

$$I_2 = 0 + 2\left(\frac{m\ell^2}{3}\right) + m\ell^2 = \left(\frac{5}{3}\right)m\ell^2$$

$$I_3 = 4\left[\frac{m\ell^2}{3}\sin^2 45°\right] = \frac{2}{3}m\ell^2 = I_1$$



Note that $I_1 = I_3$ {$\because$ of symmetry}

MI of rods AB and BC about D i.e., 4 is

$$(I_{AB})_4 = (I_{BC})_4 = \frac{m\ell^2}{12} + m\left(\frac{\sqrt{5}\ell}{2}\right)^2 = \frac{4}{3}m\ell^2$$

$$\Rightarrow \quad I_4 = 2\left(\frac{m\ell^2}{3}\right) + 2\left(\frac{4}{3}m\ell^2\right) = \frac{10}{3}m\ell^2$$

10. A → (p, q, r)
B → (p, q, r)
C → (p, q, r)
D → (s)
Conceptual

11. A → (r)
B → (p)
C → (p)
D → (q)

If total mass of disc be M, then mass of cut out portion is $m_1 = \dfrac{M}{4}$

So, $I_1 = \dfrac{1}{2}MR^2 - \dfrac{1}{2}m_1\left(\dfrac{R}{2}\right)^2 = \dfrac{15}{32}MR^2$

Also, by parallel axis theorem, we have

$$I_2 = I_1 + \frac{3M}{4}\left(\frac{R}{2}\right)^2 = \frac{21}{32}MR^2$$

By perpendicular axis theorem, we have

$I_3 + I_4 = I_2$

Since, $I_2 = I_1 + \dfrac{3M}{4}\left(\dfrac{R}{2}\right)^2$

and $I_3 = \dfrac{I_1}{2} + \dfrac{3M}{4}\left(\dfrac{R}{2}\right)^2$ $\left\{\because I_{\text{diameter}} = \dfrac{I_1}{2}\right\}$

$\Rightarrow \quad I_2 - I_3 = \dfrac{I_1}{2}$

CHAPTER 3

12. A → (p)
B → (s)
C → (p)
D → (r)
For all cases if J is the linear impulse, then

$$v = \frac{J}{m}$$

Also, $\omega = \frac{Jr_{\perp}}{I_E}$

where, $r_{\perp}$ is the perpendicular distance from centre E and v is rightwards.
For pure rolling ω should be clockwise and hence J should be applied at A.
If it is below A, angular velocity is anticlockwise and it will cause forward slip.

13. A → (q)
B → (r)
C → (p)
D → (r)

Since, $\tau = 0$, so $L = \text{constant}$, $K = \frac{L^2}{2I}$ and $I\omega = \text{constant}$.
Insect first moves away from the axis, then towards it.
Hence, I will first increase and then decrease.

14. A → (t)
B → (s)
C → (p)
D → (p)

$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$ but $\frac{I}{mR^2} = 1$ for a thin walled cylindrical shell

$$\Rightarrow \quad a = \frac{g\sin\theta}{2} = \frac{g}{4}$$

$$\Rightarrow \quad \alpha = \frac{a}{R} = \frac{g\sin\theta}{2R} = \frac{g}{4R}$$

$$K_T = \frac{1}{2}mv^2 = \frac{1}{2}m(at)^2 = \frac{1}{2}m\left(\frac{g\sin\theta}{2}\right)^2(t^2)$$

$$\Rightarrow \quad K_T = \frac{mg^2t^2\sin^2\theta}{2} = \frac{mg^2t^2}{8}$$

$$K_R = \frac{1}{2}\mathrm{I}\omega^2 = \frac{1}{2}\mathrm{I}(\alpha t)^2 = \frac{1}{2}(mR^2)\left(\frac{g\sin\theta}{2R}\right)^2(t^2)$$

$$\Rightarrow \quad K_R = \frac{mg^2t^2\sin^2\theta}{2} = \frac{mg^2t^2}{8}$$

15. A → (q)
B → (s)
C → (q)
D → (p)

Since $\ell = \frac{1}{2}at^2$

$$\Rightarrow \quad t = \sqrt{\frac{2S}{a}}, \text{ where } a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$$

Moment of inertia I of hollow sphere is maximum, hence a is minimum and t is maximum
Total kinetic energy equals mgh for all

$\frac{K_R}{K_T} = \frac{2}{5}$, for solid sphere

$\frac{K_R}{K_T} = \frac{2}{3}$, for hollow sphere

$\frac{K_R}{K_T} = \frac{1}{2}$, for cylinder

So, rotational kinetic energy is maximum for hollow sphere.

Since $\frac{K_T}{K_R}$ is maximum for solid sphere, hence it has maximum translational kinetic energy.

16. A → (p, r, s)
B → (q, r, s)
C → (q, r, s)
D → (p, r, s)
Conceptual

17. A → (q)
B → (s)
C → (p)
D → (s)
For A: At 2, $K_{\text{Total}} = mgh = E$ (say)

Since, $\frac{K_R}{K_T} = \frac{2}{5}$

$$\Rightarrow \quad K_R = \frac{2}{7}E = \frac{2}{7}mgh$$

For B: At 3, $K_{\text{Total}} = mg\left(\frac{h}{2}\right)$

Since, $\frac{K_R}{K_T} = \frac{2}{5}$

$$\Rightarrow \quad K_T = \frac{5}{7}E = \frac{5}{7}\left(mg\frac{h}{2}\right) = \frac{5}{14}mgh$$

For C: From 3 to 4, K_R will be constant, so

$$\text{At 4, } K_R = \frac{2}{7}\left(mg\frac{h}{2}\right) = \frac{1}{7}mgh$$

For D: At 4, $K_T = K_{\text{Total}} - K_R$

$$\Rightarrow \quad K_T = mgh - \frac{1}{7}mgh = \frac{6}{7}mgh$$

18. A → (p, r)
B → (p, r)
C → (q, s)
D → (p, s)
Conceptual

19. A → (s)
B → (p)
C → (p)
D → (q, r)
Conceptual

20. A → (q)
B → (r)
C → (p)
D → (s)
Acceleration of 1 w.r.t. centre of mass is $\vec{a}_{1/\text{cm}} = r\alpha\hat{i} - \omega^2 r\hat{j}$

Since $\vec{a}_{1/\text{cm}^-} = \vec{a}_1 - \vec{a}_{\text{cm}}$

$\Rightarrow \quad \vec{a}_1 = \vec{a}_{1/\text{cm}} + \vec{a}_{\text{cm}}$ where $\vec{a}_{\text{cm}} = (R\alpha)\hat{i}$

$\Rightarrow \quad \vec{a}_1 = r\alpha\hat{i} - \omega^2 r\hat{j} + R\alpha\hat{i} = (R+r)\alpha\hat{i} - \omega^2 r\hat{j}$

Similarly, we have

$$\vec{a}_2 = -r\alpha\hat{j} - \omega^2 r\hat{i} + R\alpha\hat{i} = (R\alpha - \omega^2 r)\hat{i} - r\alpha\hat{j}$$

$$\vec{a}_3 = -r\alpha\hat{i} + \omega^2 r\hat{j} + R\alpha\hat{i} = (R\alpha - r\alpha)\hat{i} + \omega^2 r\hat{j}$$

$$\vec{a}_4 = r\alpha\hat{j} + \omega^2 r\hat{i} + R\alpha\hat{i} = (R\alpha + \omega^2 r)\hat{i} + r\alpha\hat{j}$$

21. A → (q)
B → (s)
C → (s)
D → (p)

Since $K = \sqrt{\frac{I}{m}}$

For axis 1, $K = \sqrt{\frac{m(2a)^2}{3}} = \frac{2}{\sqrt{3}}a$

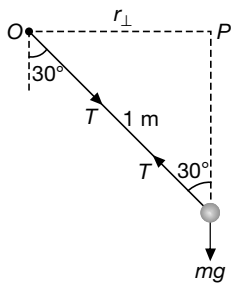
For axis 2, $K = \sqrt{\frac{m(2a)^2}{12}} = \frac{a}{\sqrt{3}}$

For axis 3, $K = \sqrt{\frac{ma^2}{3}} = \frac{a}{\sqrt{3}}$

For axis 4, $K = \sqrt{\frac{ma^2}{12}} = \frac{a}{\sqrt{12}}$

Integer/Numerical Answer Type Questions

1. Two forces are acting on the ball.
(i) tension (T)
(ii) weight (mg)



Since the tension passes through O, so torque due to tension about point O is zero.

So, $\tau_{mg} = F \times r_\perp$

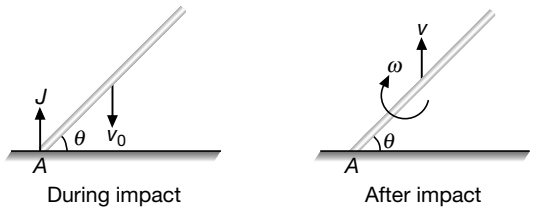
where, $r_\perp = OP = 1\sin(30°) = 0.5$ m

$\Rightarrow \quad \tau_{mg} = (mg)(0.5)$

$\Rightarrow \quad \tau_{mg} = (1)(10)(0.5)$

$\Rightarrow \quad \tau_{mg} = 5$ Nm

2. If v be the linear velocity of rod after impact (upwards), ω be the angular velocity of rod and J be the linear impulse at A during impact, then by



Impulse – Momentum Theorem, we have

$$J = \Delta p = p_f - p_i$$

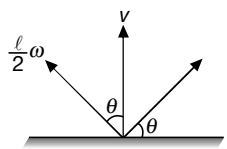
$\Rightarrow \quad J = mv - (-mv_0)$

$\Rightarrow \quad J = m(v + v_0)$ …(1)

Further by $\begin{pmatrix}\text{Angular}\\ \text{Impulse}\end{pmatrix} - \begin{pmatrix}\text{Angular}\\ \text{Momentum}\\ \text{Theorem}\end{pmatrix}$, we have

$$J\left(\frac{\ell}{2}\cos\theta\right) = I\omega = \frac{m\ell^2}{12}\omega \quad \ldots(2)$$

Since the collision is elastic, so at the point of impact, $e = 1$.



$\Rightarrow \quad \begin{pmatrix}\text{Relative speed}\\ \text{of approach}\end{pmatrix} = \begin{pmatrix}\text{Relative speed}\\ \text{of separation}\end{pmatrix}$

CHAPTER 3

$$\Rightarrow \quad v_0 = v + \frac{\ell}{2}\omega\cos\theta \qquad \ldots(3)$$

Solving equations (1), (2) and (3), we get

$$\omega = \frac{6v_0\cos\theta}{\ell(1+3\cos^2\theta)}$$

For ω to be maximum, we have

$$\frac{d\omega}{d\theta} = 0$$

$$\Rightarrow \quad \frac{6v_0}{\ell}\frac{d}{d\theta}\left(\frac{\cos\theta}{1+3\cos^2\theta}\right) = 0$$

$$\Rightarrow \quad \frac{(1+3\cos^2\theta)(-\sin\theta) - \cos\theta(-6\sin\theta\cos\theta)}{(1+3\cos^2\theta)^2} = 0$$

$$\Rightarrow \quad 1+3\cos^2\theta = 6\cos^2\theta$$

$$\Rightarrow \quad 3\cos^2\theta = 1$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{3}}$$

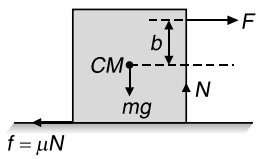
$$\Rightarrow \quad * = 3$$

3. For the block to be in equilibrium

$$F = f = \mu mg \qquad \ldots(1)$$

For block not to topple, we have

$$F\left(b + \frac{a}{2}\right) < mg\frac{a}{2} \qquad \ldots(2)$$



$$\Rightarrow \quad \mu mg\left(b + \frac{a}{2}\right) < mg\left(\frac{a}{2}\right)$$

$$\Rightarrow \quad \left(b + \frac{a}{2}\right)\mu < \frac{a}{2}$$

$$\Rightarrow \quad 0.4 = \mu < \frac{a}{2b+a}$$

$$\Rightarrow \quad 0.8b + 0.4a < a$$

$$\Rightarrow \quad 0.8b < 0.6a$$

$$\Rightarrow \quad \frac{b}{a} < \frac{3}{4}$$

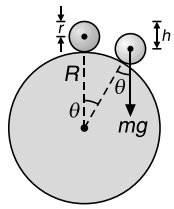
So, $\left(100\frac{b}{a}\right)_{\max} = 75$

4. The equation of motion for the centre of the sphere at the moment of breaking off, $N = 0$ is

$$\frac{mv^2}{R+r} = mg\cos\theta \qquad \ldots(1)$$

where v is the speed of the centre of the sphere at that moment and θ is the corresponding angle. The speed v can be found by using the Law of Conservation of Energy, according to which

$$mgh = \frac{mv^2}{2} + \frac{I\omega^2}{2}$$



where $I = \frac{2}{5}mr^2$, $v = r\omega$ and $h = (R+r)(1-\cos\theta)$

From these equations we get

$$\omega = \sqrt{\frac{10g(R+r)}{17r^2}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{10g\left(R + \frac{R}{16}\right)}{17\left(\frac{R}{16}\right)^2}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{10\times10\times17\times16}{17\times1}}$$

$$\Rightarrow \quad \omega = 40 \text{ rads}^{-1}$$

5. $I = 2\left(\frac{2}{5}MR^2\right) + 2\left(\frac{2}{5}MR^2 + Md^2\right)$

$$\Rightarrow \quad I = \frac{8}{5}MR^2 + 2md^2$$

$$\Rightarrow \quad I = \left[\frac{8}{5}(0.5)\left(\frac{\sqrt{5}}{2}\right)^2 + 2(0.5)(4)(2)\right]10^{-4}$$

$$\Rightarrow \quad I = \left(\frac{5}{5} + 8\right)\times10^{-4}$$

$$\Rightarrow \quad I = 9\times10^{-4}$$

$$\Rightarrow \quad N = 9$$

6. $I = \frac{1}{2}\times40\times(0.5)^2 = 5 \text{ kgm}^2$

Given, $\omega = \frac{240}{\pi}\times\frac{2\pi}{60} = 8 \text{ rads}^{-1}$

Since, $\omega = \alpha t_0$

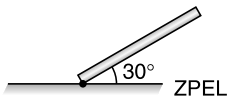
$$\Rightarrow \quad \alpha = \frac{\omega}{t_0} = \frac{8}{2} = 4 \text{ rads}^{-2}$$

$$\Rightarrow \quad \tau = I\alpha = (5)(4) = 20 \text{ Nm}$$

Further, $\alpha' = \frac{FR}{I} = \frac{10 \times 0.5}{5} = 1 \text{ rads}^{-2}$

$\Rightarrow \quad t = \frac{\omega}{\alpha'} = \frac{8}{1} = 8 \text{ s}$

7. From mechanical energy conservation, we get



$U_i + K_i = U_f + K_f$

$\Rightarrow \quad mg\left(\frac{l}{2}\sin 30°\right) + 0 = 0 + \frac{1}{2}I\omega^2$

$\Rightarrow \quad mg\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + 0 = 0 + \frac{1}{2}\left[\frac{m(1)^2}{3}\right]\omega^2$

$\Rightarrow \quad \omega^2 = \frac{3g}{2} = 15$

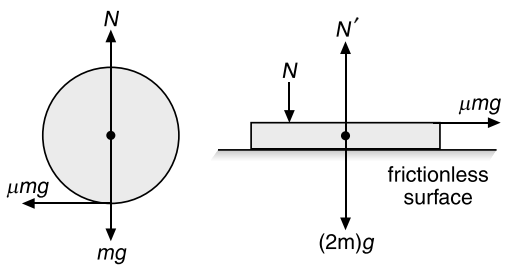
$\Rightarrow \quad \omega = \sqrt{15}$

$\Rightarrow \quad n = 15$

8. Initially the cylinder will slip on the plank, therefore kinetic friction will act between the cylinder and the plank. If a_p be the acceleration of plank, a_c and α_c be the linear and angular acceleration of the cylinder, then

$a_c = -\frac{\mu mg}{m} = -\mu g$

$a_p = +\frac{\mu g}{2}$



$\alpha_c = +\frac{(\mu mg)(R)}{\left(\frac{mR^2}{2}\right)} = +\frac{2\mu g}{R}$

For pure rolling, we have

$(v_{\text{net}})_{\text{cylinder}} = v_{\text{plank}}$

$\Rightarrow \quad v_c - R\omega_c = v_p$

$\Rightarrow \quad \frac{\mu g}{2}t = v_0 - \mu g t - (R)\left(\frac{2\mu g}{R}\right)(t)$

$\Rightarrow \quad t = \frac{v_0}{3.5\mu g} = \frac{7}{3.5 \times 0.1 \times 10} = 2 \text{ s}$

Now, $s_c - s_p = \left(v_0 t - \frac{1}{2}\mu g t^2\right) - \frac{1}{2}\left(\frac{\mu g}{2}\right)t^2$

$\Rightarrow \quad s_c - s_p = (7 \times 2) - \frac{1}{2}(0.1)(10)(4) - \frac{1}{2}\left(\frac{0.1 \times 10}{2}\right)(4)$

$\Rightarrow \quad s_c - s_p = 11 \text{ m}$

Also, $v_c - v_p = (v_0 - \mu g t) - \left(\frac{\mu g}{2}\right)(t)$

$\Rightarrow \quad v_c - v_p = 7 - 0.1 \times 10 \times 2 - \frac{0.1 \times 10 \times 2}{2}$

$\Rightarrow \quad v_c - v_p = 4 \text{ ms}^{-1}$

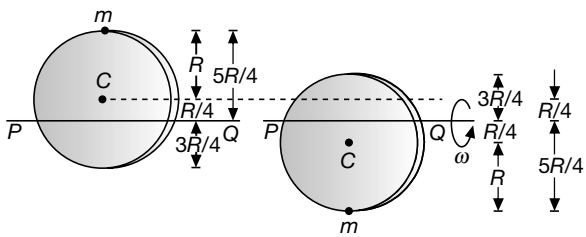
Hence, the remaining distance $(12 - 11 = 1 \text{ m})$ is travelled in a time,

$t' = \frac{1}{4} = 0.25 \text{ s}$

So, total time equals $T = 2 + 0.25 = 2.25 \text{ s}$

9. Initial and final positions are shown in the figure. By Law of Conservation of Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{GPE}\\ \text{of m}\end{pmatrix} + \begin{pmatrix}\text{Loss in}\\ \text{GPE of}\\ \text{CM of disc}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{RKE of}\\ \text{mass + disc}\end{pmatrix} \quad \ldots(1)$$



Loss in gravitational potential energy of mass m is

$mg\left(2 \times \frac{5R}{4}\right) = \frac{5mgR}{2}$

Loss in gravitational potential energy of CM of disc is

$mg\left(2 \times \frac{R}{4}\right) = \frac{mgR}{2}$

Therefore, total decrease in gravitational potential energy of system is

$-\Delta U = \frac{5mgR}{2} + \frac{mgR}{2} = 3mgR \quad \ldots(2)$

Gain in rotational kinetic energy of system (mass + disc) is $\Delta K = \frac{1}{2}I\omega^2$

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where I is the moment of inertia of system (disc + mass) about axis PQ

$\Rightarrow$ I = moment of inertia of disc + moment of inertia of mass

$\Rightarrow$ $I = \left(\frac{mR^2}{4} + m\left(\frac{R}{4}\right)^2\right) + m\left(\frac{5R}{4}\right)^2$

$\Rightarrow$ $I = \frac{15mR^2}{8}$

$\Rightarrow$ $\Delta K = \frac{1}{2}\left(\frac{15mR^2}{8}\right)\omega^2$ ...(3)

Substituting (2) and (3) in (1), we get

$$3mgR = \frac{1}{2}\left(\frac{15mR^2}{8}\right)\omega^2$$

$\Rightarrow$ $\omega = \sqrt{\frac{16g}{5R}}$

Therefore, linear speed of particle at its lowest point is

$$v = \left(\frac{5R}{4}\right)\omega = \frac{5R}{4}\sqrt{\frac{16g}{5R}}$$

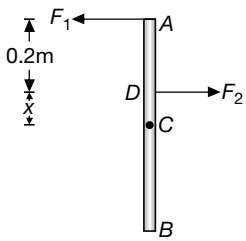
$\Rightarrow$ $v = \sqrt{5gR}$

$\Rightarrow$ $x = 5$

10. Let x be the distance of centre point C of rod from D. Then,

$$F_2 - F_1 = ma$$

$\Rightarrow$ $F_1 = 3$ N



Further, $\tau_c = 0$

$\Rightarrow$ $F_2 x = F_1(0.2 + x)$

$\Rightarrow$ $5x = F_1(0.2 + x)$

$\Rightarrow$ $5x = 3(0.2 + x)$

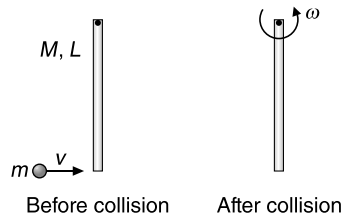
$\Rightarrow$ $x = 0.3$ m

So, length of rod is $L = 2(x + 0.2) = 1$ m

11. Applying conservation of angular momentum, we get

$$\vec{L}_i = \vec{L}_f$$

$\Rightarrow$ $mvL = I\omega$



$\Rightarrow$ $mvL = \left(\frac{mL^2}{3} + mL^3\right)\omega$

$\Rightarrow$ $0.1 \times 80 \times 1 = \left(\frac{0.9 \times 1^2}{3} + 0.1 \times 1^2\right)\omega$

$\Rightarrow$ $8 = \left(\frac{3}{10} + \frac{1}{10}\right)\omega$

$\Rightarrow$ $8 = \frac{4}{10}\omega$

$\Rightarrow$ $\omega = 20$ rads^{-1}

12. (a) Angular retardation $\alpha = \frac{\tau}{I} = \frac{4}{16} = 0.25$ rads^{-2}

Since α is constant, so

$$0 = \omega_0 - \alpha t_0$$

$\Rightarrow$ $t_0 = \frac{\omega_0}{\alpha} = \frac{9}{0.25} = 36$ second

(b) $\alpha = -\frac{d\omega}{dt} = \frac{8t}{16}$

$\Rightarrow$ $\int_0^{t_0} 8t\,dt = -16\int_9^0 d\omega$

$\Rightarrow$ $4t_0^2 = -16(0 - 9)$

$\Rightarrow$ $t_0^2 = 4 \times 9$

$\Rightarrow$ $t_0 = \sqrt{36}$

$\Rightarrow$ $t_0 = 6$ s

13. (a) Maximum frictional force on the block can be

$$f_{max} = \mu mg$$

Therefore, maximum acceleration of block can be

$$a_{max} = \frac{f_{max}}{m} = \frac{\mu mg}{m} = \mu g$$

$\Rightarrow$ $a_{max} = (0.4)(10) = 4$ ms^{-2}

If acceleration of truck, exceeds this value, then slipping will start between the block and the truck.

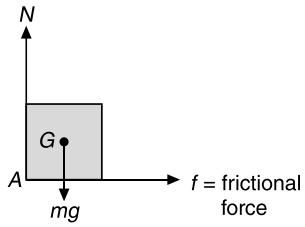
(b) In the critical case when the block is about to topple about A, normal reaction will pass through A. So, the block is on the verge of motion and hence

$$f = ma$$ ...(1)

$N = mg$ ...(2)

The block will topple when,

Torque due to f about G > torque due to N about G



Taking the limiting value, we get

$\tau_f = \tau_N$

$\Rightarrow \quad (f)(h) = (N)(b)$

$\Rightarrow \quad (ma)(h) = (mg)(b)$

$\Rightarrow \quad a = \left(\frac{b}{h}\right)g = \left(\frac{0.6}{1.2}\right)(10) = 5 \text{ ms}^{-2}$

So, if the acceleration of truck exceeds this value, the block will topple about A.

(c) The maximum acceleration/retardation at which the block is not disturbed, is the smaller of the two values, obtained above, i.e., 4 ms^{-2}

Hence, the maximum retardation can be 4 ms^{-2}

Since, $u = 72 \text{ kmhr}^{-1} = 20 \text{ ms}^{-1}$

So, applying $v^2 = u^2 - 2as$ $\{\because \ v = 0\}$

$\Rightarrow \quad s = \frac{u^2}{2a}$ $\{\because \ a = 4 \text{ ms}^{-2}\}$

$\Rightarrow \quad s = \frac{(20)^2}{(2)(4)} = 50 \text{ m}$

14. $L_i = L_f$

$\Rightarrow \quad \left(80R^2 + \frac{200R^2}{2}\right)\omega = \left(0 + \frac{200R^2}{2}\right)\omega_1$

$\Rightarrow \quad \omega_1 = 1.8\omega_0 = (1.8)(5)$

$\Rightarrow \quad \omega_1 = 9 \text{ rpm}$

15. If α be the angular acceleration of the hoop and a be the acceleration of its centre, acceleration of m would be $a + r\alpha$.

If I be the moment of inertia of the hoop, then $I = Mr^2$

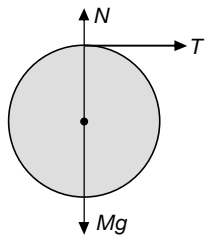
Since, $\tau = Tr = I\alpha$

$\Rightarrow \quad \alpha = \frac{Tr}{I} = \frac{T}{Mr}$

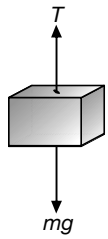
and $T = Ma$

For m, we have

$mg - T = m(a + r\alpha)$



F.B.D. of hoop



F.B.D. of block of mass m

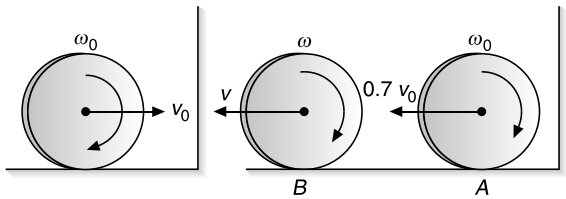
Solving, we get

$a = \frac{mg}{(M + 2m)} = \frac{2}{2} = 1 \text{ ms}^{-2}$

$\Rightarrow \quad T = 1 \text{ N}$

and $\alpha = \frac{Tr}{I} = \frac{T}{Mr} = 5 \text{ rads}^{-2}$

16. Between A and B, there is forward slipping. Therefore, friction will be maximum and backwards (rightwards). At point B where $v = R\omega$, ball starts rolling without slipping and force of friction becomes zero.



By Law of Conservation of Angular Momentum between points A and B about bottommost point (because torque of friction about this point is zero), we get

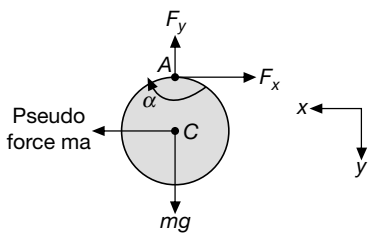
$L_A = L_B$

$\Rightarrow \quad m(0.7v_0)R - I\omega_0 = mvR + I\omega$

Substituting $\omega_0 = \frac{v_0}{R}$, $\omega = \frac{v}{R}$ and $I = \frac{2}{5}mR^2$, we get

$v = \frac{3}{14}v_0 = \left(\frac{3}{14}\right)(7) \text{ ms}^{-1} = 1.5 \text{ ms}^{-1}$

17. FBD of plate w.r.t. truck is shown in figure.



Equations of motion for centre of mass of plate is

$a_x = \frac{ma - F_x}{m}$

CHAPTER 3

$a_y = 0$

$\Rightarrow \quad F_y = mg = 100$ N

Angular acceleration about A is

$$\alpha = \frac{maR}{\frac{3}{2}mR^2} = \frac{2a}{3R}$$

Since $a_x = R\alpha$

$$\Rightarrow \quad \frac{ma - F_x}{m} = \frac{2}{3}a$$

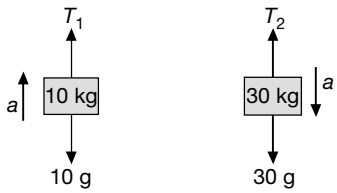
$$\Rightarrow \quad F_x = \frac{1}{3}ma = \frac{1}{3}\left(\frac{100}{10}\right)(0.9) = 3 \text{ N}$$

Absolute acceleration of centre of mass is

$$a_C = \frac{F_x}{m} = \frac{3}{10} = 0.3 \text{ ms}^{-2} = 30 \text{ cms}^{-2}$$

Since, $\alpha = \frac{2}{3}\frac{a}{R} = \frac{2}{3}\left(\frac{0.9}{0.6}\right) = 1 \text{ rads}^{-2}$

18. The free body diagrams of the bodies and the pulley are shown in figure, then

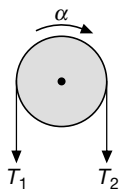


for 10 kg mass, we have

$$T_1 - 10g = 10a \qquad \ldots(1)$$

for 30 kg mass, we have

$$30g - T_2 = 30a \qquad \ldots(2)$$



for the pulley, we have

$$\alpha = \frac{(T_2 - T_1)R}{\frac{1}{2}mR^2} = \frac{2(T_2 - T_1)}{mR}$$

$$\Rightarrow \quad \alpha = \frac{2(T_2 - T_1)}{20R} = \frac{0.1(T_2 - T_1)}{R} \qquad \ldots(3)$$

For no slipping, $a = R\alpha$ $\qquad \ldots(4)$

Solving the above equations, we get

$a = 4 \text{ ms}^{-2}$, $T_1 = 500$ N and $T_2 = 180$ N

$$v = \sqrt{2as} = \sqrt{(2)(4)(2)} = 4 \text{ ms}^{-1}$$

$$\omega = \frac{v}{R} = \frac{4}{0.1} = 40 \text{ rads}^{-1}$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\times 2}{4}} = 1 \text{ second}$$

19. If moment of inertia of bigger disc is $I = \frac{MR^2}{2}$

So, MI of small disc is $I_2 = \frac{M(R/2)^2}{2} = \frac{I}{4}$

By conservation of angular momentum, we have

$$I\omega_1 + \frac{I}{4}(0) = I\omega_2 + \frac{I}{4}\omega_2$$

$$\Rightarrow \quad \omega_2 = \frac{4\omega_1}{5}$$

Initial kinetic energy is $K_i = \frac{1}{2}I\omega_1^2$

Final kinetic energy K_f is

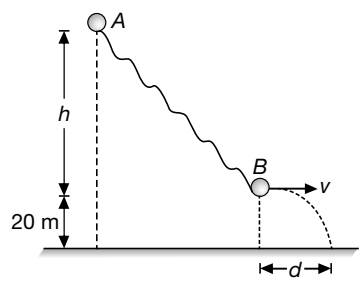
$$K_f = \frac{1}{2}\left(I + \frac{I}{4}\right)\left(\frac{4\omega_1}{5}\right)^2 = \frac{1}{2}I\omega_1^2\left(\frac{4}{5}\right)$$

$$\Rightarrow \quad p\% = \left(\frac{K_i - K_f}{K_i}\right)100\% = \frac{1 - \frac{4}{5}}{1} \times 100 = 20\%$$

20. $h = (48 - 20)$ m $= 28$ m

Total kinetic energy of the sphere at B is

$K = mgh$



In case of pure rolling, we have $\frac{K_T}{K_R} = \frac{r^2}{k^2} = \frac{5}{2}$, for a sphere

$$\Rightarrow \quad K_T = \frac{1}{2}mv^2 = \frac{5}{7}mgh$$

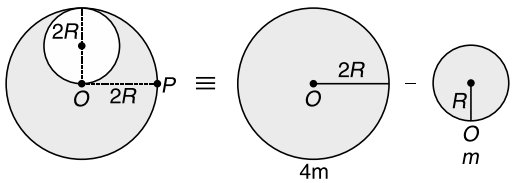
$$\Rightarrow \quad v = \sqrt{\frac{10}{7}gh} = \sqrt{\left(\frac{10}{7}\right)(10)(28)} = 20 \text{ ms}^{-1}$$

Now, $t = \sqrt{\frac{2h}{g}}$

$$\Rightarrow \quad t = \sqrt{\frac{2(20)}{10}} = 2 \text{ s}$$

$$\Rightarrow \quad d = vt = (20)(2) = 40 \text{ m}$$

21.



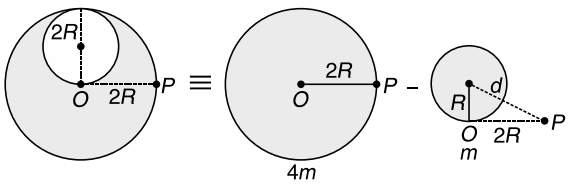
$$I_O = \frac{(4m)(2R)^2}{2} - \frac{3}{2}mR^2$$

$$\Rightarrow \quad I_O = mR^2\left(8 - \frac{3}{2}\right)$$

$$\Rightarrow \quad I_O = \frac{13}{2}mR^2$$

$$I_P = \frac{3}{2}(4m)(2R)^2 - \left(\frac{mR^2}{2} + md^2\right)$$

where $d^2 = R^2 + (2R)^2$

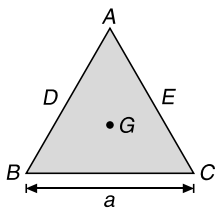


$$\Rightarrow \quad I_P = 24mR^2 - \frac{11}{2}mR^2$$

$$\Rightarrow \quad I_P = \frac{37}{2}mR^2$$

$$\Rightarrow \quad \frac{I_P}{I_O} = \frac{\frac{37}{2}}{\frac{13}{2}} = \frac{37}{13} \approx 3$$

22. Let the side of triangle be a and its mass be m



MI of plate ABC about centroid G is

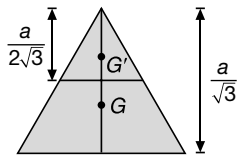
$$I = \frac{ma^2}{12}$$

Triangle ADE is also an equilateral triangular of side $\frac{a}{2}$.

Let moment of inertia of triangular plate ADE about its centroid (G') be I_1 and its mass be m_1

$$m_1 = \frac{m}{(\sqrt{3}a^2/4)} \times \frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 = \frac{m}{4}$$

$$\Rightarrow \quad I_1 = \frac{m_1}{12}\left(\frac{a}{2}\right)^2 = \frac{m}{4 \times 12}\frac{a^2}{4} = \frac{ma^2}{192}$$



Distance $GG' = \frac{a}{\sqrt{3}} - \frac{2}{2\sqrt{3}} = \frac{a}{2\sqrt{3}}$

So, MI of part ADE about centroid G is

$$I_2 = I_1 + m_1\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{ma^2}{192} + \frac{m}{4}\left(\frac{a^2}{12}\right)$$

$$\Rightarrow \quad I_2 = \frac{5ma^2}{192}$$

So, MI of remaining part is

$$= \frac{ma^2}{12} - \frac{5ma^2}{192} = \frac{11ma^2}{12 \times 16} = \frac{11I_0}{16}$$

$$\Rightarrow \quad N = 11$$

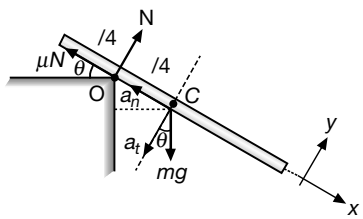
23. $\frac{1}{2}I_0\omega^2 = mg\left(\frac{\ell}{4}\sin\theta\right)$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{m(4a)^2}{12} + m\left(\frac{\ell}{4}\right)^2\right)\omega^2 = mg\left(\frac{\ell}{4}\sin\theta\right)$$

$$\Rightarrow \quad \frac{7}{6}\left(\frac{\ell}{4}\right)\omega^2 = g\sin\theta$$

$$\Rightarrow \quad \omega = \sqrt{\frac{24g\sin\theta}{7\ell}}$$

Since, $\alpha = \frac{\tau}{I_0} = \frac{mg\left(\frac{\ell}{4}\cos\theta\right)}{\frac{7}{3}m\left(\frac{\ell}{4}\right)^2} = \frac{12g\cos\theta}{7\ell}$



CHAPTER 3

Now $\sum F_y = ma_y$

$\Rightarrow \quad mg\cos\theta - N = ma_t$

$\Rightarrow \quad N = mg\cos\theta - ma_t$

$\Rightarrow \quad N = mg\cos\theta - m\left(\frac{\ell}{4}\alpha\right) \qquad \left\{\because\ a_t = \frac{\ell}{4}\alpha\right\}$

$\Rightarrow \quad N = mg\cos\theta - m\left(\frac{3g\cos\theta}{7}\right) = \frac{4}{7}mg\cos\theta$

Rod begins to slip, when

$\mu N - mg\sin\theta = ma_n$

$\Rightarrow \quad \frac{4}{7}\mu mg\cos\theta - mg\sin\theta = m\left(\frac{\ell}{4}\omega^2\right)$

$\Rightarrow \quad \frac{4\mu mg\cos\theta}{7} - mg\sin\theta = \frac{6mg\sin\theta}{7}$

$\Rightarrow \quad \frac{4}{7}\mu mg\cos\theta = \frac{13}{7}mg\sin\theta$

$\Rightarrow \quad \tan\theta = \frac{4\mu}{13}$

So, $* = 4$

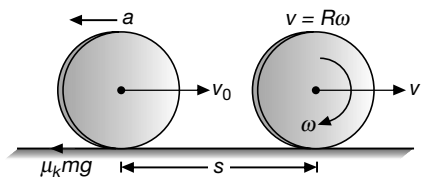
24. Applying Law of Conservation of Angular Momentum about bottommost point, we get

$L_i = L_f$

$\Rightarrow \quad mv_0R = mvR + \left(\frac{2}{5}mR^2\right)\omega$

$\Rightarrow \quad mv_0R = mvR + \frac{2}{5}mvR \qquad \left\{\because\ \omega = \frac{v}{R}\right\}$

$\Rightarrow \quad v = \frac{5}{7}v_0$



Since, $a = \frac{\mu_k mg}{m} = \mu_k g$ and $v^2 = v_0^2 - 2as$

$\Rightarrow \quad s = \frac{v_0^2 - v^2}{2a} = \frac{v_0^2 - \frac{25}{49}v_0^2}{2\mu_k g}$

$\Rightarrow \quad s = \frac{12v_0^2}{49\mu_k g}$ Substituting the values, we get

$s = \frac{(12)(12)^2}{(49)(0.75)(9.8)} = 4.8$ m

25. Equation of motion for the block is

$mg\sin\theta - T = ma$

$\Rightarrow \quad (1)(10)\sin 30° - T = (1)a$

$\Rightarrow \quad T + a = 5 \qquad \ldots(1)$

Equation of motion for disk is

$\frac{TR}{\frac{1}{2}MR^2} = \alpha$

$\Rightarrow \quad \alpha = \frac{2T}{MR}$

For no slipping, we get

$R\alpha = a = \frac{2T}{M}$

$\Rightarrow \quad a = \frac{2T}{3}$

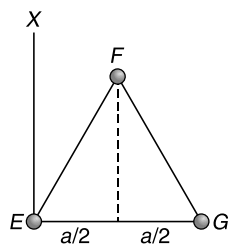
$\Rightarrow \quad T = 1.5a$

Substituting in equation (1), we have

$2.5a = 5$

$\Rightarrow \quad a = 2\ \text{ms}^{-2}$

26. Since, $I = 0 + m\left(\frac{a}{2}\right)^2 + ma^2$



$\Rightarrow \quad I = \frac{5}{4}ma^2 = \frac{25}{20}ma^2$

$\Rightarrow \quad N = 25$

27. (a) By Law of Conservation of Mechanical Energy, we have

$$\begin{pmatrix}\text{Loss in}\\ \text{Gravitational}\\ \text{Potential Energy}\\ \text{of Particle}\end{pmatrix} = \begin{pmatrix}\text{Gain in}\\ \text{Rotational}\\ \text{Kinetic Energy of}\\ \text{disc + Particle}\end{pmatrix}$$

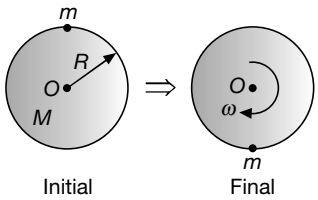
$\Rightarrow \quad mg(2R) = \frac{1}{2}\left(\frac{1}{2}MR^2 + mR^2\right)\omega^2$

$\Rightarrow \quad \omega = \sqrt{\frac{8mg}{(2m+M)R}}$

$\Rightarrow \quad \omega = \sqrt{\frac{(8)(5)(10)}{(10+40)0.5}} = 4\ \text{rads}^{-1}$

(b) If F be the force exerted by disk on the particle (upwards), then

$$F - mg = mR\omega^2$$



$$\Rightarrow \quad F = mg + \frac{8m^2g}{(2m+M)}$$

$$\Rightarrow \quad F = \frac{mg(10m+M)}{(2m+M)}$$

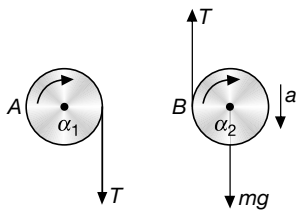
$$\Rightarrow \quad F = \frac{(5)(10)(50+40)}{(10+40)}$$

$$\Rightarrow \quad F = 90 \text{ N}$$

28. Equations of motion for the discs are,

$$a = \frac{mg - T}{m} \qquad \ldots(1)$$

$$\alpha_2 = \frac{TR}{\frac{1}{2}mR^2} = \frac{2T}{mR} \qquad \ldots(2)$$



Similarly, $\alpha_1 = \frac{TR}{\frac{1}{2}mR^2} = \frac{2T}{mR}$ …(3)

$$\Rightarrow \quad \alpha_1 = \alpha_2$$

For no slipping $a = R\alpha_2$ …(4)

Further, $a - R\alpha_2 = R\alpha_1$ …(5)

Solving these equations, we get

$$T = \frac{mg}{5},\ a = \frac{4}{5}g \text{ and } \alpha_2 = \frac{2g}{5R}$$

$$\Rightarrow \quad T = 4 \text{ N},\ a = 8 \text{ ms}^{-2},\ \alpha_2 = 8 \text{ rads}^{-2}$$

29. (a) $\omega = \frac{250}{60} \times 2\pi = 26.18 \text{ rads}^{-1}$

$$\text{I} = \frac{1}{2} \times 30 \times (0.2)^2 = 0.6 \text{ kgm}^2$$

Now, $\omega = \alpha t$

$$\Rightarrow \quad \alpha = \frac{\omega}{t} = 2.618 \text{ rads}^{-2}$$

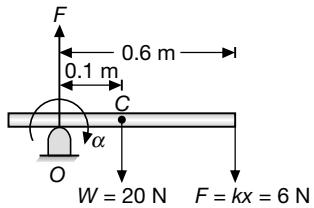
and $\tau = \text{I}\alpha = 1.57$ Nm

(b) $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times (2.618)(100) = 131$ rad

(c) $W = \frac{1}{2} \times \text{I} \times \omega^2 = \frac{1}{2} \times (0.6) \times (26.18)^2$

$$\Rightarrow \quad W \approx 206 \text{ J}$$

30. Just after the thread is burnt, the forces acting on the rod are as shown in Figure.



Torque due to forces about O is

$$\tau = (20)(0.1) + (6)(0.6) = 5.6 \text{ N}$$

Angular acceleration about O is given by

$$\alpha = \frac{\tau}{\text{I}} = \frac{5.6}{\left(\frac{(2)(1)^2}{12} + (2)(0.1)^2\right)}$$

$$\Rightarrow \quad \alpha \cong 30 \text{ rads}^{-2}$$

Now, $a_c = r\alpha = (0.1)\alpha$

$$\Rightarrow \quad a_c = 3 \text{ ms}^{-2} \qquad \{\text{downwards}\}$$

Since $W + kx - F = ma_C$

$$\Rightarrow \quad F = W + kx - ma_C$$

$$\Rightarrow \quad F = 20 + 6 - (2)(3)$$

$$\Rightarrow \quad F = 20 \text{ N}$$

31. $\omega_0 = 600 \text{ rpm} = 600 \times \frac{2\pi}{60} \text{ rads}^{-1}$

{revolutions per minute}

$$\Rightarrow \quad \omega_0 = 20\pi \text{ rads}^{-1}$$

Let α be the constant angular retardation, then

$$\omega = \omega_0 - \alpha t$$

$$\Rightarrow \quad 0 = (20\pi) - 3(\alpha)$$

$$\Rightarrow \quad \alpha = \frac{20}{3}\pi \text{ rads}^{-2}$$

Further, $\alpha = \frac{\tau}{I}$

If R be the radius of fly wheel and F be the tangential force exerted by the brake lining on fly wheel, then

CHAPTER 3

$$\tau = \mu FR$$

where $I = \frac{1}{2}mR^2$

From the above equations, we get

$$\alpha = \frac{20}{3}\pi = \frac{\mu FR}{\frac{1}{2}mR^2} = \frac{2\mu F}{mR}$$

$$\Rightarrow \quad F = \frac{10\pi mR}{3\mu}$$

Substituting the values, we get

$$F = \frac{10 \times 22 \times 20 \times 0.12}{3 \times 7 \times 0.1}$$

$$\Rightarrow \quad F = 251.43 \text{ N}$$

32. By Law of Conservation of Angular Momentum, we get

$$I_1\omega_1 = I_2\omega_2$$

$$\Rightarrow \quad I_1\left(\frac{2\pi}{T_1}\right) = (I_1 + \Delta I)\left(\frac{2\pi}{T_2}\right)$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{I_1 + \Delta I}{I_1} = 1 + \frac{\Delta I}{I_1}$$

$$\Rightarrow \quad \Delta T = (T_2 - T_1) = T_1\left(\frac{\Delta I}{I_1}\right) \qquad \ldots(1)$$

where, $I_1 = \frac{2}{5}MR^2$ and $\Delta I = \frac{2}{3}mR^2$

where, M is the mass of earth, m is the mass of ice. Substituting the values in equation (1), we get

$$\Delta T = \frac{(24 \times 3600)\left(\frac{2}{3} \times 3 \times 10^{19}\right)}{\left(\frac{2}{5} \times 6 \times 10^{24}\right)} = 0.7 \text{ s}$$

33. Since, $\vec{\tau} = (\vec{r}_2 - \vec{r}_1) \times \vec{F}$

$$\Rightarrow \quad \vec{\tau} = \left[\left(4\hat{i} + 3\hat{j} - \hat{k}\right) - \left(\hat{i} + 2\hat{j} + \hat{k}\right)\right] \times \vec{F}$$

$$\Rightarrow \quad \vec{\tau} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) \times \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

$$\Rightarrow \quad \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

$$\Rightarrow \quad |\vec{\tau}| = \sqrt{195} \text{ Nm}$$

$$\Rightarrow \quad x = 195$$

34. (a) By Law of Conservation of Angular Momentum, we get

$$I_1\omega_2 = I_2\omega_2$$

where $\omega_1 = (0.5)(2\pi)$ rads^{-1}

$$I_1 = 1.6 + 2 \times 4 \times (0.9)^2 = 8.08 \text{ kgm}^2$$

$$I_2 = 1.6 + 2 \times 4 \times (0.15)^2 = 1.78 \text{ kgm}^2$$

and $\omega_1 = 0.5$ rads^{-1}

$$\Rightarrow \quad \omega_2 = \left(\frac{8.08}{1.78}\right)(0.5)(2\pi) = 14.3 \text{ rads}^{-1}$$

(b) $E_i = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}(8.08)(2\pi \times 0.5)^2 \approx 40$ J

$$E_f = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}(1.78)(2\pi \times 2.27)^2 = 181 \text{ J}$$

(c) $W = E_f - E_i \approx 141$ J

35. For equilibrium of cylinder in horizontal direction, we have

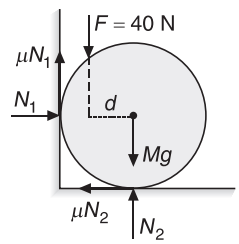
$$N_1 = \mu N_2 \qquad \ldots(1)$$

For equilibrium of cylinder in vertical direction, we have

$$N_2 + \mu N_1 = F + Mg \qquad \ldots(2)$$

Solving these two equations with $F = 40$ N, $M = 2$ kg and $\mu = \frac{1}{3}$, we get

$N_1 = 18$ N and $N_2 = 54$ N



Since, $Fd = \mu(N_1 + N_2)r$

$$\Rightarrow \quad d = \frac{\mu(N_1 + N_2)r}{F} = \frac{\left(\frac{1}{3}\right)(18 + 54)(0.1)}{40}$$

$$\Rightarrow \quad d = 0.06 \text{ m} = 6 \text{ cm}$$

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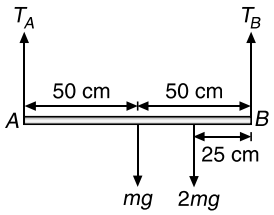
1. For equilibrium, $\tau_B = 0$ (torque about point B is zero)



$$T_A(100) - (mg)(50) - (2mg)(25) = 0$$

$$\Rightarrow \quad 100T_A = 100mg$$

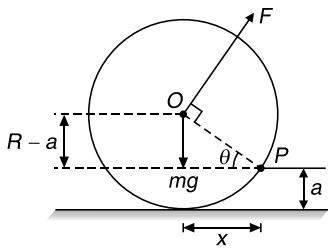
$$\Rightarrow \quad T_A = 1mg$$

Hence, the correct answer is (D).

2. For equilibrium, $(\tau)_P = 0$

$$\Rightarrow \quad FR - mgx = 0$$

$$\Rightarrow \quad F = mg\left(\frac{x}{R}\right)$$



Since, $x = \sqrt{R^2 - (R-a)^2}$

$$\Rightarrow \quad F = mg\sqrt{1 - \left(\frac{R-a}{R}\right)^2}$$

Hence, the correct answer is (D).

3. Since, both discs are rotating in same sense, so applying conservation of an angular momentum to the system, we get

$$L_{\text{initial}} = L_{\text{final}}$$

$$\Rightarrow \quad I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$$

$$\Rightarrow \quad (0.1)(10) + (0.2)(5) = (0.1 + 0.2)(\omega_f)$$

$$\Rightarrow \quad \omega_f = \frac{20}{3}$$

Kinetic energy of combined disc system is

$$\text{RKE} = \frac{1}{2}(I_1 + I_2)\omega_f^2 = \frac{1}{2}(0.1 + 0.2)\left(\frac{20}{3}\right)^2$$

$$\Rightarrow \quad \text{RKE} = \left(\frac{0.3}{2}\right)\left(\frac{400}{9}\right) = \frac{120}{18} = \frac{20}{3}\ \text{J}$$

Hence, the correct answer is (D).

4. Angular momentum conservation gives

$$mv\ell = \left(\frac{M\ell^2}{3}\right)\omega + m\ell^2\omega$$

$$\Rightarrow \quad \omega = \frac{(1)(6)(1)}{\frac{2}{3} + 1} = \frac{18}{5}\ \text{rads}^{-1}$$

Energy conservation gives

$$\frac{1}{2}\left(\frac{M\ell^2}{3}\right)\omega^2 + \frac{1}{2}(m\ell^2)\omega^2 = (m+M)r_{\text{cm}}(1 - \cos\theta)$$

where, $r_{\text{cm}} = \dfrac{m\ell + (M\ell/2)}{m+M}$

$$\Rightarrow \quad \frac{\ell^2\omega^2}{2}\left(\frac{M}{3} + m\right) = (m+M)\left(\frac{m\ell + \frac{M\ell}{2}}{m+M}\right)g(1 - \cos\theta)$$

$$\Rightarrow \quad \left(\frac{5}{6}\right)\left(\frac{18}{5}\right)^2 = 20(1 - \cos\theta)$$

$$\Rightarrow \quad 1 - \cos\theta = \left(\frac{18}{5}\right)\left(\frac{3}{20}\right)$$

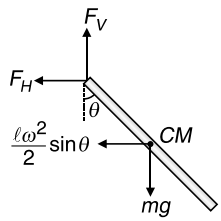
$$\Rightarrow \quad \cos\theta = 1 - \frac{27}{50} = \frac{23}{50}$$

$$\Rightarrow \quad \theta \approx 63°$$

Hence, the correct answer is (B).

5. From free body diagram, we see that $F_V = mg$ and

$$F_H = \frac{m\ell}{2}\omega^2 \sin\theta$$



Net torque about CM due to F_V and F_H is

$$\tau_{\text{net}} = F_V\left(\frac{\ell}{2}\sin\theta\right) - F_H\left(\frac{\ell}{2}\cos\theta\right)$$

According to problem, $\tau_{\text{net}} = \dfrac{m\ell^2\omega^2}{12}\sin\theta\cos\theta$

$$\Rightarrow \quad mg\left(\frac{\ell}{2}\sin\theta\right) - \left(m\omega^2\frac{\ell}{2}\sin\theta\right)\left(\frac{\ell}{2}\cos\theta\right) = \frac{m\ell^2\omega^2}{12}\sin\theta\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{3g}{2\omega^2\ell} \qquad \ldots(2)$$

Hence, the correct answer is (B).

6. $I_1 = \dfrac{MR^2}{2} = \dfrac{\rho(\pi R^2)tR^2}{2}$

$\Rightarrow \quad I \propto R^4$

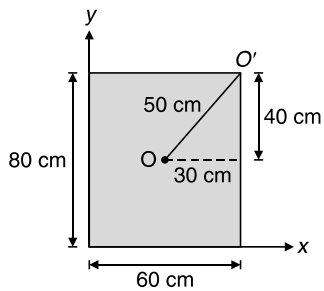
$\Rightarrow \quad \dfrac{I_1}{I_2} = \dfrac{R_1^4}{R_2^4} = \dfrac{1}{16}$

$\Rightarrow \quad \dfrac{R_1}{R_2} = \dfrac{1}{2}$

Hence, the correct answer is (B).

7. Moment of inertia about O is

$$I_O = \frac{M}{12}(L^2 + B^2) = \frac{M}{12}(80^2 + 60^2)$$



Moment of inertia about O' is obtained by applying parallel axis theorem, so we have

$$I_{O'} = I_0 + Md^2$$

$\Rightarrow \quad I_{O'} = \dfrac{M}{12}(80^2 + 60^2) + M(50)^2$

$\Rightarrow \quad \dfrac{I_O}{I_{O'}} = \dfrac{\frac{M}{12}(80^2 + 60^2)}{\frac{M}{12}(80^2 + 60^2) + M(50)^2} = \dfrac{1}{4}$

Hence, the correct answer is (B).

8. Loss in KE is given by

$$\text{Loss} = -\Delta K = \frac{1}{2}\left(\frac{I_1 I_2}{I_1 + I_2}\right)(\omega_1 - \omega_2)^2$$

$\Rightarrow \quad \text{Loss} = \dfrac{1}{2}\left[\dfrac{(I)(3I)}{4I}\right]\omega^2$

$\Rightarrow \quad \text{Loss} = \left(\dfrac{1}{2}I\omega^2\right)\dfrac{3}{4}$

$\Rightarrow \quad \dfrac{\text{Loss}}{K_i} = \dfrac{3}{4}$

Hence, the correct answer is (C).

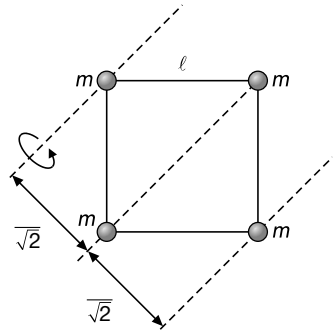
9. Moment of inertia of the cone about the specified axis is

$$I = \frac{MR^2}{2}$$

Hence, the correct answer is (A).

10. The moment of inertia of system about the specified axis is

$$I = m(0)^2 + (2)m\left(\frac{\ell}{\sqrt{2}}\right)^2 + m(\sqrt{2}\ell)^2$$



$\Rightarrow \quad I = \dfrac{2m\ell^2}{2} + 2m\ell^2 = 3m\ell^2$

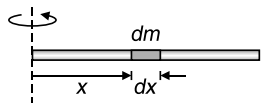
Angular momentum of system about the specified axis is

$$L = I\omega = (3m\ell^2)\omega$$

Hence, the correct answer is (B).

11. Consider an infinitesimal element of mass dm, length dx at a distance x from the specified axis of rotation, then

$$I = \int r^2 dm = \int x^2 \lambda dx$$



$\Rightarrow \quad I = \displaystyle\int_0^L x^2\lambda_0\left(1 + \frac{x}{L}\right)dx$

$\Rightarrow \quad I = \lambda_0\displaystyle\int_0^L\left(x^2 + \frac{x^3}{L}\right)dx = \lambda\left(\frac{L^3}{3} + \frac{L^3}{4}\right)$

$\Rightarrow \quad I = \dfrac{7L^3\lambda_0}{12}$ …(1)

Since $M = \displaystyle\int_0^L \lambda dx = \int_0^L \lambda_0\left(1 + \frac{x}{L}\right)dx$

$\Rightarrow \quad M = \lambda_0\left(L + \dfrac{L}{2}\right) = \dfrac{3\lambda_0 L}{2}$

$\Rightarrow \quad \lambda_0 L = \dfrac{2}{3}M$ …(2)

From (1) and (2), we get

$$I = \frac{7}{12}\left(\frac{2}{3}M\right)L^2 = \frac{7ML^2}{18}$$

Hence, the correct answer is (D).

12. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right) = \frac{3}{4}mv^2$

$\Rightarrow \quad u = \sqrt{\frac{4}{3}gh}$

$\Rightarrow \quad \omega = \frac{u}{r} = \frac{1}{r}\sqrt{\frac{4gh}{3}}$

Hence, the correct answer is (C).

13. $I = I_{cm} + md^2$

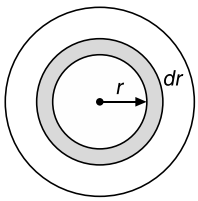
$\Rightarrow \quad m\frac{l^2}{12} + m\frac{l^2}{16} = mk^2$

$\Rightarrow \quad \frac{7l^2}{48} = k^2$

$\Rightarrow \quad k = l\sqrt{\frac{7}{48}}$

Hence, the correct answer is (B).

14. Since $dI = r^2 dm$



$\Rightarrow \quad dI = \sigma(2\pi r dr)r^2$

$\Rightarrow \quad dI = 2\pi(A + Br)r^3 dr$

$\Rightarrow \quad \int dI = 2\pi\int_0^a (Ar^3 + Br^4)dr$

$\Rightarrow \quad I = 2\pi a^4\left(\frac{A}{4} + \frac{aB}{5}\right)$

Hence, the correct answer is (A).

15. Rotational KE is $\text{RKE} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\Rightarrow \quad \text{RKE} = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right) = 8.75\times10^{-4}$ J

Hence, the correct answer is (A).

16. From parallel axis theorem, we get

$$I_0 = 3\times\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10}Md^2$$

Now, $I_A = I_0 + 3M\left(\frac{d}{\sqrt{3}}\right)^2 = \frac{13}{10}Md^2 + Md^2$

$\Rightarrow \quad I_A = \frac{23}{10}Md^2$

$\Rightarrow \quad \frac{I_0}{I_A} = \frac{13}{23}$

Hence, the correct answer is (A).

17. Since $a = \frac{(m_1 - m_2)g}{m_1 + m_2 + \frac{I}{R^2}} = \frac{(m_1 - m_2)gR^2}{(m_1 + m_2)R^2 + I}$

$\Rightarrow \quad v = \sqrt{2ah}\sqrt{2\left[\frac{(m_1 - m_2)gR^2}{(m_1 + m_2)R^2 + J}\right]h}$

$\Rightarrow \quad \omega = \frac{v}{R} = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}$

Hence, the correct answer is (B).

18. Mass of plate is

$$M = \int_0^R \rho_0 r(2\pi r dr) = \frac{2\pi\rho_0 R^3}{3}$$

$\Rightarrow \quad I_C = \int (dm)r^2 = \int_0^R \rho_0 r(2\pi r dr)r^2 = \frac{2\pi\rho_0 R^5}{5}$

$\Rightarrow \quad I = I_C + MR^2 = 2\pi\rho_0 R^5\left(\frac{1}{3} + \frac{1}{5}\right) = \frac{16\pi\rho_0 R^5}{15}$

$\Rightarrow \quad I = \frac{8}{5}\left(\frac{2}{3}\pi\rho_0 R^3\right)R^2 = \frac{8}{5}MR^2$

Hence, the correct answer is (D).

19. If α be initial angular acceleration (before it slips off), then

$$mg\left(\frac{\ell}{2}\right) = I\alpha = \left(\frac{m\ell^2}{3}\right)\alpha$$

$\Rightarrow \quad \alpha = \frac{3g}{2\ell}$

Angular speed acquire by the box in time $\tau = 0.01$ s is

$$\omega = \alpha t = \left(\frac{3g}{2\ell}\right)(0.01) = \frac{3\times10\times0.01}{2\times0.3} = \frac{1}{2}\ \text{rads}^{-1}$$

The angle by which it would rotate when it hits the ground is $\theta = \omega t'$, where, $\omega = \text{constant}$ and $t' = \text{time of fall} = \sqrt{\frac{2H}{g}} = 1$ sec

$\Rightarrow \quad \theta = \frac{1}{2}$ radian

Hence, the correct answer is (D).

20. For sphere, we have $mgh_{sph} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$

$\Rightarrow \quad mgh_{sph} = \frac{7}{10}mv^2$ ...(1)

CHAPTER 3

For cylinder, we have

$$mgh_{\text{cylinder}} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v}{R}\right)^2 = \frac{3}{4}mv^2$$

$$\Rightarrow \quad \frac{h_{\text{sph}}}{h_{\text{cylinder}}} = \frac{7\times 4}{10\times 3} = \frac{14}{15}$$

Hence, the correct answer is (D).

21. Since, $\tau = \frac{dE}{d\theta}$

$$\Rightarrow \quad 2k\theta = I\alpha$$

$$\Rightarrow \quad \alpha = \frac{2k\theta}{I}$$

Hence, the correct answer is (A).

22. Since, $mgh = \frac{1}{2}I_p\omega^2$

For ring, $h_1 = \frac{1}{2}\frac{(2mr'^2)}{mg}\frac{v^2}{(r')^2} = \frac{v^2}{g}$

For cylinder, $h_2 = \frac{1}{2}\left(\frac{3}{2}mr'^2\right)\frac{v^2}{(r')^2} = \frac{3}{4}\left(\frac{v^2}{g}\right)$

For sphere, $h_3 = \frac{1}{2}\times\frac{\frac{7}{5}m(r')^2}{g}\frac{v^2}{(r')^2} = \frac{7}{10}\left(\frac{v^2}{g}\right)$

$$\Rightarrow \quad h_1 : h_2 : h_3 = 2 : \frac{3}{2} : \frac{14}{10} = 20 : 15 : 14$$

No given option is correct.

23. Applying conservation of angular momentum, we get

$$\frac{ML^2}{12}\omega_0 = \left(\frac{ML^2}{12} + 2\frac{mL^2}{4}\right)\omega$$

$$\Rightarrow \quad \omega = \frac{M\omega_0}{M+6m}$$

Hence, the correct answer is (C).

24. Since, $\omega = \alpha t$

Also, $E = \frac{1}{2}I\omega^2 = 1200$

$$\Rightarrow \quad \frac{1}{2}\times 1.5(20t)^2 = 1200 \text{ J}$$

$$\Rightarrow \quad t = 2 \text{ s}$$

Hence, the correct answer is (B).

25. Applying conservation of angular momentum, we get

$$(I_1 + I_2)\omega_{\text{common}} = I_1\omega_1 + I_2\omega_2$$

$$\Rightarrow \quad \omega_{\text{common}} = \omega_c = \frac{I_1\omega_1 + \frac{I_1\omega_1}{4}}{I_1 + \frac{I_1}{2}} = \left(\frac{5}{4}\times\frac{2}{3}\right)\omega_1$$

$$\Rightarrow \quad \omega_c = \frac{5\omega_1}{6}$$

$$\text{Loss in } KE = \left(\frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2\right) - \frac{1}{2}(I_1 + I_2)\omega_c^2$$

$$\Rightarrow \quad \Delta KE = -\frac{I_1\omega_1^2}{24}$$

Also, loss in kinetic energy is

$$\text{Loss} = -\Delta K = \frac{1}{2}\left(\frac{I_1I_2}{I_1+I_2}\right)(\omega_1 - \omega_2)^2$$

$$\Rightarrow \quad -\Delta K = \frac{1}{2}\left(\frac{I_1}{3}\right)\left(\frac{\omega_1^2}{4}\right) = \frac{I_1\omega_1^2}{24}$$

$$\Rightarrow \quad \Delta E = \Delta K = -\frac{I_1\omega_1^2}{24}$$

Hence, the correct answer is (D).

26. Since, $x = x_0 + a\cos\omega_1 t$ and $y = y_0 + b\sin\omega_2 t$

$$\Rightarrow \quad v_x = -a\omega_1\sin(\omega_1 t),\ v_y = b\omega_2\cos(\omega_2 t)$$

$$\Rightarrow \quad a_x = -a\omega_1^2\cos(\omega_1 t),\ a_y = -b\omega_2^2\sin(\omega_2 t)$$

At $t = 0$, $x = x_0 + a$, $y = y_0$

$$a_x = -a\omega_1^2 \text{ and } a_y = 0$$

$$\Rightarrow \quad \vec{\tau} = m\left(-a\omega_1^2\right)\times y_0(-\hat{k})$$

$$\Rightarrow \quad \vec{\tau} = \left(my_0a\omega_1^2\right)\hat{k}$$

Hence, the correct answer is (C).

27. Surface mass density is $\sigma = kr^2$

$$\text{Mass of disc, } M = \int_0^R (kr^2)2\pi r dr$$

$$\Rightarrow \quad M = 2\pi k\frac{R^4}{4} = \frac{\pi kR^4}{2}$$

Moment of inertia of the disc about the axis is

$$I = \int dI = \int (dm)r^2 = \int \sigma dA r^2$$

$$\Rightarrow \quad I = \int (Kr^2)(2\pi r dr)r^2$$

$$\Rightarrow \quad I = \int_0^R 2\pi k r^5 dr = \frac{\pi kR^6}{3} = \frac{2}{3}MR^2$$

Hence, the correct answer is (D).

28. Since, $\vec{r} = 2t\hat{i} - 3t^2\hat{j}$

$$\Rightarrow \quad \vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} - 6t\hat{j}$$

At $t = 2$ s, $\vec{r} = 4\hat{i} - 12\hat{j}$ and $\vec{v} = 2\hat{i} - 12\hat{j}$

$\Rightarrow \quad \vec{L} = m(\vec{r} \times \vec{v}) = 2\left(4\hat{i} - 12\hat{j}\right) \times \left(2\hat{i} - 12\hat{j}\right) = -48\hat{k}$

Hence, the correct answer is (D).

29. Since $\tau = I\alpha$ and $\omega = \omega_0 + \alpha t$

$\Rightarrow \quad 25 \times 2\pi = (\alpha)5$

$\Rightarrow \quad \alpha = 10\pi$

$\Rightarrow \quad \tau = \left(\frac{5}{4}mR^2\right)\alpha \approx \left(\frac{5}{4}\right)(5 \times 10^{-3})(10^{-4})10\pi$

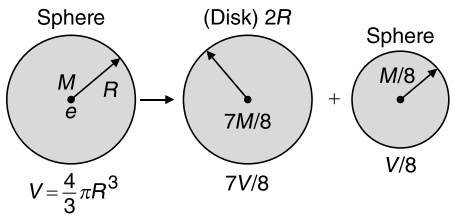
$\Rightarrow \quad \tau = 2.0 \times 10^{-5}$ Nm

Hence, the correct answer is (C).

30. For small sphere, $\frac{4}{3}\pi r^3 \rho = \frac{M}{8} \qquad \left\{\because \ r = \left(\frac{R}{2}\right)\right\}$

For sphere, $I_1 = \left(\frac{7M}{8}\right)(2R)^2 \frac{1}{2} = \frac{14}{8}MR^2$

For disc, $I_2 = \frac{2}{5}\left(\frac{M}{8}\right)r^2$



$\Rightarrow \quad I_2 = \frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R^2}{4}\right) = \frac{MR^2}{80}$

$\Rightarrow \quad \frac{I_1}{I_2} = \frac{14 \times 80}{(8)} = 140$

Hence, the correct answer is (A).

31. Since, $\sigma = \frac{\sigma_0}{r}$

$\Rightarrow \quad dm = \frac{\sigma_0}{r}2\pi r dr$

$\Rightarrow \quad m = \int dm = \sigma_0 2\pi(b-a)$

$\Rightarrow \quad I = 2\pi\sigma_0 \int_a^b r^2 dr = \frac{2\pi\sigma_0}{3}(b^3 - a^3)$

Since, $I = mk^2$

$\Rightarrow \quad 2\pi\sigma_0(b-a)k^2 = \frac{2\pi\sigma_0}{3}(b^3 - a^3)$

$\Rightarrow \quad k^2 = \frac{1}{3}(b^2 + ab + a^2)$

$\Rightarrow \quad k = \sqrt{\frac{1}{3}\frac{(b^3 - a^3)}{b-a}}$

Hence, the correct answer is (D).

32. Applying conservation of mechanical energy, we get

$mg\left(\frac{\ell}{2}\sin 30^\circ\right) = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$

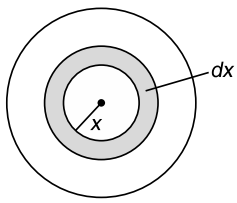
$\Rightarrow \quad \omega^2 = \frac{3g}{2\ell} = \frac{30}{1}$

$\Rightarrow \quad \omega = \sqrt{30}$ rads^{-1}

Hence, the correct answer is (B).

33. The force per unit area P is

$P = \frac{F}{\pi R^2}$



Torque due to frictional force is

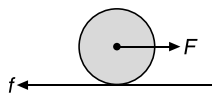
$\tau = \mu \int (dN)x = \frac{\mu F}{\pi R^2} \int_0^R 2\pi x^2 dx$

$\Rightarrow \quad \tau = \frac{\mu F \times 2\pi R^3}{3\pi R^2} = \frac{2}{3}\mu F R$

Hence, the correct answer is (D).

34. Since, $F - f = Ma$

Also, $fR = \frac{MR^2}{2}\frac{a}{R}$



$\Rightarrow \quad f = \frac{Ma}{2}$

$\Rightarrow \quad F = \frac{3Ma}{2}$

$\Rightarrow \quad a = \frac{2F}{3M}$

$\Rightarrow \quad \alpha = \frac{a}{R} = \frac{2F}{3MR}$

Hence, the correct answer is (B).

35. Since, $\tau = I_p\alpha$

$\Rightarrow \quad 5M_0 g\ell - 4M_0 g\ell = \left[2M_0(2\ell)^2 + 5M_0\ell^2\right]\alpha$

$\Rightarrow \quad M_0 g\ell = 13M_0\ell^2\alpha$

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$\Rightarrow \quad \alpha = \dfrac{g}{13\ell}$

Hence, the correct answer is (C).

36. Since, $I = I_{\text{spheres}} + I_{\text{rod}}$

where, $I_{\text{rod}} = \dfrac{M \times 4R^2}{12} = \dfrac{MR^2}{3}$

$$I_{\text{spheres}} = 2\left(\frac{2}{5}MR^2 + 4MR^2\right) = \frac{44}{5}MR^2$$

$\Rightarrow \quad I = \left(\dfrac{44}{5} + \dfrac{1}{3}\right)MR^2$

$\Rightarrow \quad I = \dfrac{137}{15}MR^2$

Hence, the correct answer is (D).

37. Let $I_0 = KML^2$

$\Rightarrow \quad I_1 = K\dfrac{M}{4}\left(\dfrac{L}{2}\right)^2 = K\dfrac{ML^2}{16}$

So, $I = I_0 - I_1$

$\Rightarrow \quad I = \dfrac{15}{16}KML^2 = \dfrac{15}{16}I_0$

Hence, the correct answer is (B).

38. Since, $\vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2$

where, $\vec{\tau}_1 = \left(2\hat{i} + 3\hat{j}\right) \times F\hat{k} = F\left(3\hat{i} - 2\hat{j}\right)$

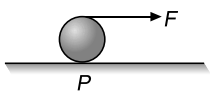
$$\vec{\tau}_2 = 6\hat{j} \times F\left(-\sin 30^\circ \hat{i} - \cos 30^\circ \hat{j}\right) = 3F\hat{k}$$

$\Rightarrow \quad \vec{\tau} = F\left(3\hat{i} - 2\hat{j} + 3\hat{k}\right)$

Hence, the correct answer is (A).

39. Since $\tau_p = I_p\alpha$

$\Rightarrow \quad F(2R) = 2MR^2\alpha$



$\Rightarrow \quad \alpha = \dfrac{F}{MR} = \dfrac{40}{0.5 \times 5}$

$\Rightarrow \quad \alpha = 16 \text{ rads}^{-2}$

Hence, the correct answer is (A).

40. Since $\tau = Fr\sin\theta$

$\Rightarrow \quad 2.5 = 1 \times 5\sin\theta$

$\Rightarrow \quad \sin\theta = 0.5$

$\Rightarrow \quad \theta = \dfrac{\pi}{6}$

Hence, the correct answer is (B).

41. Since $I = I_1 + I_2 + I_3$

$\Rightarrow \quad I = \dfrac{MR^2}{2} + 2\left(\dfrac{MR^2}{4} + MR^2\right) = 3MR^2$

Hence, the correct answer is (A).

42. Since, $\dfrac{M}{2}\left(R_1^2 + R_2^2\right) = MR^2$

$\Rightarrow \quad R = \sqrt{\dfrac{R_1^2 + R_2^2}{2}} = \sqrt{\dfrac{100 + 400}{2}} = \sqrt{250}$

$\Rightarrow \quad R \approx 16 \text{ cm}$

Hence, the correct answer is (C).

43. According to parallel axis theorem, we have

$$I(x) = I_0 + mx^2$$

Hence, the correct answer is (C).

44. Since, $L = mv_0r$

Also, $\dfrac{1}{2}mv_0^2 = \dfrac{1}{2}mv_1^2 + mgh$

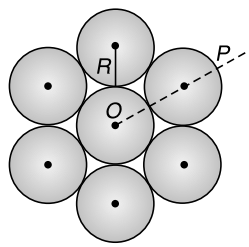
$\Rightarrow \quad v_0^2 = 25 + 2 \times 10 \times 10 = 225$

$\Rightarrow \quad v_0 = 15 \text{ ms}^{-1}$

$\Rightarrow \quad L = 20 \times 10^{-3} \times 15 \times 20 = 6 \text{ kgm}^2\text{s}^{-1}$

Hence, the correct answer is (D).

45. Moment of inertia of one of the outer disc about an axis passing through point O and perpendicular to the plane



$$I_1 = \frac{1}{2}MR^2 + M(2R)^2 = \frac{9}{2}MR^2$$

Moment of inertia of the system about point O,

$$I_O = \frac{1}{2}MR^2 + 6I_1 = \frac{1}{2}MR^2 + 6\left(\frac{9}{2}MR^2\right) = \frac{55}{2}MR^2$$

Required moment of inertia of the system about point P is

$$I_P = I_O + 7M(3R)^2 = \frac{55}{2}MR^2 + 63MR^2 = \frac{181}{2}MR^2$$

Hence, the correct answer is (D).

46. Mass per unit area of disc is $\sigma = \dfrac{9M}{\pi R^2}$

Mass of removed portion is $M' = \dfrac{9M}{\pi R^2} \times \pi \left(\dfrac{R}{3}\right)^2 = M$

Let moment of inertia of removed portion be I_1

$$\Rightarrow \quad I_1 = \frac{M}{2}\left(\frac{R}{3}\right)^2 + M\left(\frac{2R}{3}\right)^3 = \frac{MR^2}{2}$$

Let I_2 be the moment of inertia of the whole disc, so

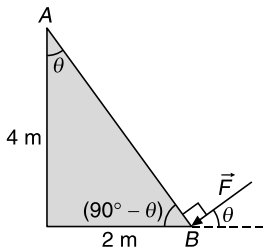
$$I_2 = \frac{9MR^2}{2}$$

Moment of inertia of remaining disc is $I = I_2 - I_1$

$$\Rightarrow \quad I = \frac{9MR^2}{2} - \frac{MR^2}{2} = \frac{8MR^2}{2} = 4MR^2$$

Hence, the correct answer is (D).

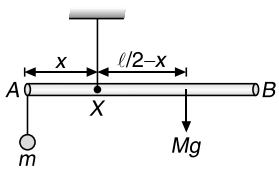
47. Moment of force will be maximum when line of action of force is perpendicular to line AB.



$$\tan\theta = \frac{2}{4} = \frac{1}{2}$$

Hence, the correct answer is (A).

48.



Balancing torque about point of suspension X, we get

$$mgx = Mg\left(\frac{\ell}{2} - x\right)$$

$$\Rightarrow \quad mx = M\frac{\ell}{2} - Mx$$

$$\Rightarrow \quad m = \left(M\frac{\ell}{2}\right)\frac{1}{x} - M$$

This is equation of straight line with variables m and $\dfrac{1}{x}$.

Hence, the correct answer is (D).

49. As coin is at rest on rotating disc, centripetal force is provided by the friction force between the coin and disc.

$$f = m\omega^2 R$$

$$\Rightarrow \quad \mu mg = m\omega^2 r$$

$$\Rightarrow \quad \mu = \frac{\omega^2 r}{g} = \frac{(2\pi\nu)^2 r}{g}$$

$$\Rightarrow \quad \mu = \frac{4\pi^2 (3.5)^2 \times 1.25 \times 10^{-2}}{10} = 604 \times 10^{-3} \approx 0.6$$

Hence, the correct answer is (D).

50. Applying conservation of energy, we get

$$mgh = mg\ell\sin\alpha = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$$

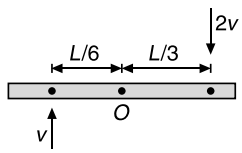
$$\Rightarrow \quad 6g\ell\sin\alpha = v^2$$

$$\Rightarrow \quad v = \sqrt{6g\ell\sin\alpha}$$

$$\Rightarrow \quad v \propto \sqrt{\sin\alpha}$$

Hence, the correct answer is (D).

51. Applying law of conservation of angular momentum, i.e. $L_i = L_f$, we get



$$m(2v)\left(\frac{L}{3}\right) + 2m(v)\left(\frac{L}{6}\right) = I\omega$$

$$\Rightarrow \quad mvL = \left[\frac{1}{12}(8m)L^2 + m\left(\frac{L}{3}\right)^2 + 2m\left(\frac{L}{6}\right)^2\right]\omega$$

$$\Rightarrow \quad v = L\left(\frac{2}{3} + \frac{1}{9} + \frac{1}{18}\right)\omega = \frac{5}{6}\omega L$$

$$\Rightarrow \quad \omega = \frac{6v}{5L}$$

Hence, the correct answer is (A).

52. Moment of inertia about z-axis is $I_z = \dfrac{mR^2}{2}$

Moment of inertia about z' axis is $I_{z'} = I_z + mR^2 = \dfrac{3}{2}mR^2$

$$\Rightarrow \quad I_z : I_{z'} = 1:3$$

Hence, the correct answer is (B).

53. Since, $I = \dfrac{mR^2}{4} + \dfrac{m\ell^2}{12}$

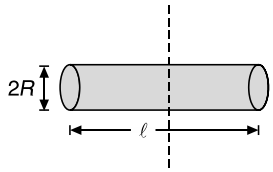
$$\Rightarrow \quad I = \frac{m}{4}\left(R^2 + \frac{\ell^2}{3}\right)$$

If V be volume of cylinder, then $V = (\pi R^2)\ell$

CHAPTER 3

$$\Rightarrow \quad I = \frac{m}{4}\left(\frac{V}{\pi\ell} + \frac{\ell^2}{3}\right)$$

For I to be minimum, we have



$$\frac{dI}{d\ell} = \frac{m}{4}\left(\frac{-V}{\pi\ell^2} + \frac{2\ell}{3}\right) = 0$$

$$\Rightarrow \quad \frac{V}{\pi\ell^2} = \frac{2\ell}{3}$$

$$\Rightarrow \quad V = \frac{2\pi\ell^3}{3}$$

$$\Rightarrow \quad \pi R^2\ell = \frac{2\pi\ell^3}{3}$$

$$\Rightarrow \quad \frac{\ell^2}{R^2} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{\ell}{R} = \sqrt{\frac{3}{2}}$$

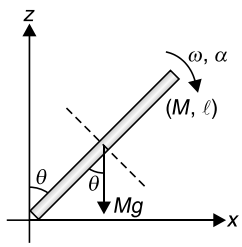
Hence, the correct answer is (D).

54. Torque at angle θ is

$$\tau = Fr_{\perp} = Mg\left(\frac{\ell}{2}\sin\theta\right)$$

Since, $\tau = I\alpha$

$$\Rightarrow \quad I\alpha = \frac{Mg\ell}{2}\sin\theta, \text{ where } I = \frac{M\ell^2}{3}$$



$$\Rightarrow \quad \left(\frac{M\ell^2}{3}\right)\alpha = \frac{Mg\ell}{2}\sin\theta$$

$$\Rightarrow \quad \frac{\ell\alpha}{3} = g\left(\frac{\sin\theta}{2}\right)$$

$$\Rightarrow \quad \alpha = \frac{3g\sin\theta}{2\ell}$$

Hence, the correct answer is (A).

55. From figure, we conclude

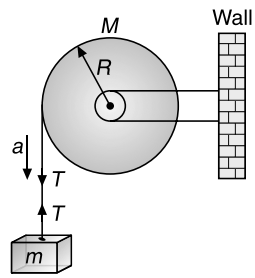
$$mg - T = ma \qquad \ldots(1)$$

Moment of inertia of a uniform disc is $I = \frac{MR^2}{2}$ and an acceleration is $a = \alpha R$

Since, $\tau = TR = I\alpha$

$$\Rightarrow \quad TR = \left(\frac{MR^2}{2}\right)\left(\frac{a}{R}\right)$$

$$\Rightarrow \quad T = \frac{Ma}{2}$$



Substituting this value in equation (1)

$$mg - \frac{Ma}{2} = ma$$

$$\Rightarrow \quad mg = a\left(m + \frac{M}{2}\right)$$

$$\Rightarrow \quad a = \frac{2mg}{M + 2m}$$

Hence, the correct answer is (D).

56. $W_1\ell_1 = W_2\ell_2$, i.e. $W \propto \frac{1}{\ell}$

Hence, the correct answer is (A).

57. Moment of inertia of the disc about the given axis

$$I_D = \frac{MR^2}{2}$$

Mass of removed portion is $M' = \frac{M}{\pi R^2} \times \pi\left(\frac{R}{4}\right)^2 = \frac{M}{16}$

Moment of inertia of removed portion about the given axis, using parallel axes theorem is

$$I_R = \frac{1}{2}\left(\frac{M}{16}\right)\left(\frac{R^2}{16}\right) + \left(\frac{M}{16}\right)\left(\frac{9R^2}{16}\right) = \frac{19MR^2}{512}$$

Required moment of inertia

$$I = I_D - I_R = \frac{1}{2}MR^2 - \frac{19MR^2}{512} = \frac{237}{512}MR^2$$

Hence, the correct answer is (D).

58. Since, $\vec{L} = \vec{r} \times \vec{p} = rp \sin\theta \hat{n}$

$\Rightarrow \quad \vec{L} = pr_{\perp}\hat{n}$

For D to A

$$\vec{L} = \frac{R}{\sqrt{2}} mv(-\hat{k})$$

For A to B

$$\vec{L} = \frac{R}{\sqrt{2}} mv(-\hat{k})$$

For C to D

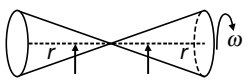
$$\vec{L} = \left(\frac{R}{\sqrt{2}} + a\right) mv(\hat{k})$$

For B to C

$$\vec{L} = \left(\frac{R}{\sqrt{2}} + a\right) mv(\hat{k})$$

Hence, (A) and (C) are correct.

59. Let the distance of central line from instantaneous axis of rotation be r, then r from the point on left becomes lesser than that for right.



So, v_{left} point $= \omega r' < \omega r = v_{\text{right}}$ point

So, the roller will turn to left.

Hence, the correct answer is (D).

60. Since no external torque acts on the system, therefore total angular momentum of the system about point O remains constant. So, $L_i = L_f$

$$\Rightarrow \quad mv\left(\frac{a}{2}\right) = I\omega$$

$$\Rightarrow \quad \omega = \frac{mva}{2I}$$

where, I is the moment of inertia of cube about its edge

$$\Rightarrow \quad I = m\frac{a^2}{6} + m\left(\frac{\sqrt{2}a}{2}\right)^2 = \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2ma^2}{3}$$

$$\Rightarrow \quad \omega = \frac{mva \times 3}{2 \times 2ma^2} = \frac{3v}{4a} = \frac{3 \times 2}{4 \times 0.3} = 5 \text{ rads}^{-1}$$

Hence, the correct answer is (D).

61. For just one complete rotation, speed of the drum at top position,

$v = \sqrt{Rg}$, where $R = 1.25$ m

Angular velocity of the drum is

$$\omega = \frac{v}{R} = \sqrt{\frac{g}{R}}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{10}{1.25}} \text{ rads}^{-1} = \frac{60}{2\pi}\sqrt{\frac{10}{1.25}} \text{ rpm} = 27 \text{ rpm}$$

Hence, the correct answer is (A).

62. (Diameter of Sphere) = (Diagonal of the Cube)

$$\Rightarrow \quad 2R = \sqrt{3}a$$

$$\Rightarrow \quad a = \frac{2R}{\sqrt{3}}$$

Mass of Cube $= m = \rho a^3 = \left(\frac{M}{\frac{4}{3}\pi R^3}\right) a^3$

$$\Rightarrow \quad m = \frac{3M}{4\pi R^3}\left(\frac{2R}{\sqrt{3}}\right)^3$$

$$\Rightarrow \quad m = \frac{2M}{\sqrt{3}\pi}$$

Since MI of cube is $I = \frac{1}{6}ma^2$

$$\Rightarrow \quad I = \frac{4MR^2}{9\sqrt{3}\pi}$$

Hence, the correct answer is (C).

63. Equation of motion for solid cylinder is

$$F - f = ma \qquad \ldots(1)$$

Also, $\tau = fR = I\alpha$

For pure rolling, we have $a = \alpha R$

$$\Rightarrow \quad fR = \frac{mR^2}{2} \cdot \frac{a}{R}$$

$$\Rightarrow \quad f = \frac{ma}{2} \qquad \ldots(2)$$

From equations (1) and (2), we get

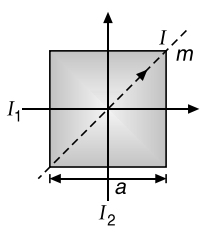
$$F - \frac{ma}{2} = ma$$

$$\Rightarrow \quad F = \frac{3}{2}ma$$

Hence, the correct answer is (C).

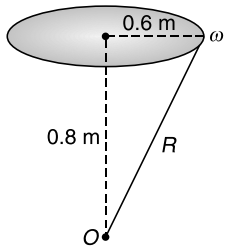
64. For a thin uniform square sheet

$$I_1 = I_2 = I = \frac{ma^2}{12}$$



Hence, the correct answer is (C).

65. $R = \sqrt{(0.8)^2 + (0.6)^2} = 1$ m, $v = r\omega = (0.6)\omega$



Angular momentum of the particle about point O is

$$L = mvR\sin 90° = m(r\omega)R$$

$$\Rightarrow \quad L = (2)(0.6)(12)(1) = 14.4 \text{ kgm}^2\text{s}^{-1}$$

Hence, the correct answer is (D).

66. In this case L changes in direction but not in magnitude.

Hence, the correct answer is (C).

67. Since, $mg - T = ma$ …(1)

Also, $\tau = TR = \text{I}\alpha$

$$\Rightarrow \quad TR = (mR^2)\left(\frac{a}{R}\right)$$

$$\Rightarrow \quad T = ma \qquad \text{…(2)}$$

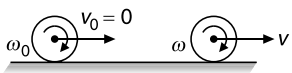
Substituting (2) in (1), we get

$$mg - ma = ma$$

$$\Rightarrow \quad a = \frac{g}{2}$$

Hence, the correct answer is (B).

68. Applying conservation of angular momentum about a point on ground



$$mr^2\omega_r = mr^2\omega + mvr$$

$$\Rightarrow \quad mr^2\omega_0 = 2mvr$$

$$\Rightarrow \quad v = \frac{r\omega_0}{2}$$

Hence, the correct answer is (C).

69. Torque exerted on pulley is $\tau = FR$

$$\Rightarrow \quad \alpha = \frac{FR}{I} \qquad \left\{\because \ \alpha = \frac{\tau}{I}\right\}$$

Given that, $F = (20t - 5t^2)$, $R = 2$ m, $I = 10$ kgm^{-2}

$$\Rightarrow \quad \alpha = \frac{(20t - 5t^2)\times 2}{10}$$

$$\Rightarrow \quad \alpha = (4t - t^2)$$

$$\Rightarrow \quad \frac{d\omega}{dt} = (4t - t^2)dt$$

$$\Rightarrow \quad d\omega = (4t - t^2)dt$$

On integrating, we get

$$\omega = 2t^2 - \frac{t^3}{3}$$

At direction of reversal of motion, $\omega = 0$

$$\Rightarrow \quad t = 6 \text{ s}$$

Since, $\omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$

$$\Rightarrow \quad d\theta = \left(2t^2 - \frac{t^3}{3}\right)dt$$

On integrating, we get

$$\theta = \frac{2t^3}{3} - \frac{t^4}{12}$$

At, $t = 6$ s, $\theta = 36$ rad

$$\Rightarrow \quad 2\pi n = 36$$

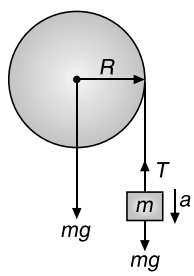
$$\Rightarrow \quad n = \frac{36}{2\pi} < 6$$

Hence, the correct answer is (B).

70. The free body diagram of pulley and mass

$$mg - T = ma$$

$$\Rightarrow \quad a = \frac{mg - T}{m} \qquad \text{…(1)}$$



As per question, pulley to be consider as a circular disc. So, angular acceleration of disc is

$$\alpha = \frac{\tau}{I} = \frac{TR}{I} \qquad \text{…(2)}$$

where, $I = \frac{1}{2}mR^2$ {for circular disc}

$$\Rightarrow \quad T = \frac{mR\alpha}{2} \qquad \{\text{using (2)}\}$$

$$\Rightarrow \quad a = \frac{mg - \frac{mR\alpha}{2}}{m} \qquad \{\text{using (1)}\}$$

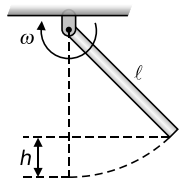
$$\Rightarrow \quad ma = mg - \frac{ma}{2} \qquad \left\{\because\ \alpha = \frac{a}{R}\right\}$$

$$\Rightarrow \quad a = \frac{2g}{3}$$

Hence, the correct answer is (C).

71. By conservation of angular momentum, $I\omega = \text{constant}$
Since I firstly decreases and then increases
So, L first increases and then decreases
Hence, the correct answer is (D).

72. The uniform rod of length ℓ and mass m is swinging about an axis passing through the end. When the centre of mass is raised through h, the increase in potential energy is mgh. This is equal to the kinetic energy $\frac{1}{2}I\omega^2$.



$$\Rightarrow \quad mgh = \frac{1}{2}\left(m\frac{\ell^2}{3}\right)\omega^2$$

$$\Rightarrow \quad h = \frac{\ell^2\omega^2}{6g}$$

Hence, the correct answer is (D).

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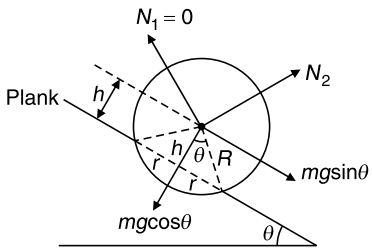
Single Correct Choice Type Problems

1. At the verge of toppling, $N_1 = 0$

$$(mg\sin\theta)h = (mg\cos\theta)r$$



Also, $\cos\theta = \frac{h}{R}$

$$\Rightarrow \quad (mg\sin\theta)h = mg\left(\frac{h}{R}\right)r$$

$$\Rightarrow \quad \sin\theta = \frac{r}{R}$$

Hence, the correct answer is (A).

2. Let the other mass at this instant is at a distance of x from the centre O. Applying Law of Conservation of Angular Momentum, we get

$$I_1\omega_1 = I_2\omega_2$$

$$\Rightarrow \quad (MR^2)(\omega) = \left[MR^2 + \frac{M}{8}\left(\frac{3}{5}R\right)^2 + \frac{M}{8}x^2\right]\left(\frac{8}{9}\omega\right)$$

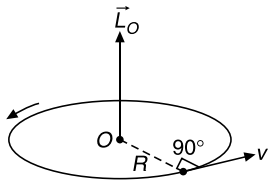
$$x = \frac{4}{5}R$$

Hence, the correct answer is (D).

3. Angular momentum of a particle about a point is

$$\vec{L} = \vec{r}\times\vec{p} = m(\vec{r}\times\vec{v})$$

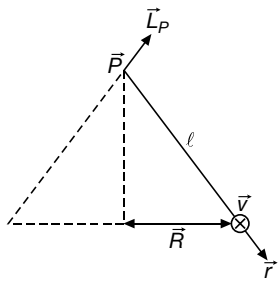
For L_O



$$|\vec{L}| = (mvr\sin\theta) = m(R\omega)(R)\sin 90° = \text{constant}$$

Direction of $\vec{L}_O$ is always upwards. Therefore, complete $\vec{L}_O$ is constant, both in magnitude as well as direction.

For L_P



$$|\vec{L}_P| = (mvr\sin\theta) = (m)(R\omega)(\ell)\sin 90° = (mR\ell\omega)$$

Magnitude of $\vec{L}_P$ will remain constant but direction of $\vec{L}_P$ keeps on changing.

Hence, the correct answer is (C).

CHAPTER 3

4. Since P has most of its mass concentrated at surface, so

$$I_P > I_Q$$

In case of pure rolling, $a = \dfrac{g\sin\theta}{1+\dfrac{I}{mR^2}}$

$\Rightarrow$ $a_Q > a_P$ (as its moment of inertia is less)

Therefore, Q reaches first with more linear speed and more translational kinetic energy.

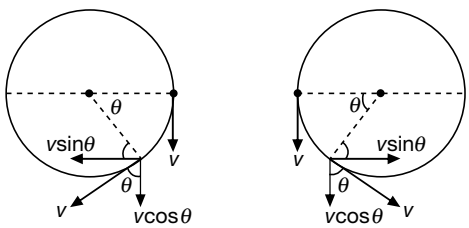
Further, $\omega = \dfrac{v}{R}$

$\Rightarrow$ $\omega \propto v$

$\Rightarrow$ $\omega_Q > \omega_P$ as $v_P > v_Q$

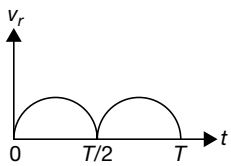
Hence, the correct answer is (D).

5.



$$v_r = |2v\sin\theta|$$

$\Rightarrow$ $v_r = |2v\sin(\omega t)|$



Hence, the correct answer is (D).

6. $\tau = \dfrac{dL}{dt} = \dfrac{d}{dt}(I\omega)$

$\Rightarrow$ $\tau = \dfrac{d}{dt}\left(\dfrac{ML^2}{3} + mx^2\right)\omega$

$\Rightarrow$ $\tau = 2mx\dfrac{dx}{dt}\omega$

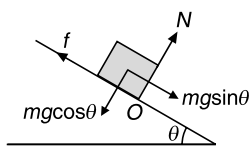
Now, $x = vt$

$\Rightarrow$ $\tau \propto t$

Finally, torque becomes zero.

Hence, the correct answer is (B).

7. Condition of sliding is



$$mg\sin\theta > \mu mg\cos\theta$$

$\Rightarrow$ $\tan\theta > \mu$

$\Rightarrow$ $\tan\theta > \sqrt{3}$ ...(1)

Condition of toppling is

$$\left(\tau_{mg\sin\theta}\right)_{\text{about } O} > \left(\tau_{mg\cos\theta}\right)_{\text{about } O}$$

$\Rightarrow$ $(mg\sin\theta)\left(\dfrac{15}{2}\right) > (mg\cos\theta)\left(\dfrac{10}{2}\right)$

$\Rightarrow$ $\tan\theta > \dfrac{2}{3}$ ...(2)

With increase in value of θ, condition of sliding is satisfied first.

Hence, the correct answer is (D).

8. $\dfrac{1}{2}mv^2 + \dfrac{1}{2}I\left(\dfrac{v}{R}\right)^2 = mg\left(\dfrac{3v^2}{4g}\right)$

$\Rightarrow$ $I = \dfrac{1}{2}mR^2$

$\Rightarrow$ Body is disc.

Hence, the correct answer is (D).

9. $\dfrac{2}{5}MR^2 = \dfrac{1}{2}Mr^2 + Mr^2$

$\Rightarrow$ $\dfrac{2}{5}MR^2 = \dfrac{3}{2}Mr^2$

$\Rightarrow$ $r = \dfrac{2}{\sqrt{15}}R$

Hence, the correct answer is (A).

10. On smooth part BC, due to zero torque, angular velocity and hence the rotational kinetic energy remains constant. While moving from B to C translational kinetic energy converts into gravitational potential energy.

Hence, the correct answer is (D).

11. $\begin{pmatrix}\text{MI of}\\ \text{Given}\\ \text{Shape}\end{pmatrix} = \begin{pmatrix}\text{MI of}\\ \text{Complete Disc}\\ \text{of Radius R}\end{pmatrix} - \begin{pmatrix}\text{MI of}\\ \text{Removed Portion}\\ \text{of Radius } R/3\end{pmatrix}$

$\Rightarrow$ $I_{\text{remaining}} = I_{\text{whole}} - I_{\text{removed}}$

$\Rightarrow$ $I = \dfrac{1}{2}(9M)(R)^2 - \left[\dfrac{1}{2}m\left(\dfrac{R}{3}\right)^2 + \dfrac{1}{2}m\left(\dfrac{2R}{3}\right)^2\right]$ ...(1)

where, $m = \sigma a = \dfrac{9M}{\pi R^2} \times \pi\left(\dfrac{R}{3}\right)^2 = M$

Substituting in equation (1), we get

$$I = 4MR^2$$

Hence, the correct answer is (D).

12. $\vec{L} = m(\vec{r} \times \vec{v})$

Direction of $(\vec{r} \times \vec{v})$, hence the direction of angular momentum remains the same.

Hence, the correct answer is (D).

13. From conservation of angular momentum $(I\omega = \text{constant})$, angular velocity will remain half. As,

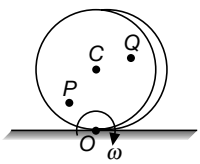
$$K = \frac{1}{2} I\omega^2$$

The rotational kinetic energy will become half.

Hence, the correct answer is (B).

14. In case of pure rolling bottommost point is the instantaneous centre of zero velocity.

Velocity of any point on the disc, $v = r\omega$, where r is the distance of point from O.



$$r_Q > r_C > r_P$$

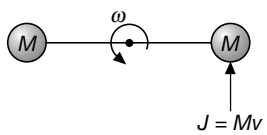
$$\Rightarrow \quad v_Q > v_C > v_P$$

Hence, the correct answer is (A).

15. In uniform circular motion the only force acting on the particle is centripetal (towards centre). Torque of this force about the centre is zero. Hence, angular momentum about centre remain conserved.

Hence, the correct answer is (A).

16. Let ω be the angular velocity of the rod. Applying, angular impulse equals change in angular momentum about centre of mass of the system, we get



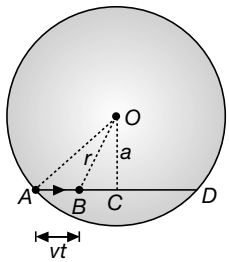
$$J\left(\frac{L}{2}\right) = I_c\omega$$

$$\Rightarrow \quad (Mv)\left(\frac{L}{2}\right) = (2)\left(\frac{ML^2}{4}\right)\omega$$

$$\Rightarrow \quad \omega = \frac{v}{L}$$

Hence, the correct answer is (A).

17. Since, there is no external torque, angular momentum will remain conserved. The moment of inertia will first decrease till the tortoise moves from A to C and then increase as it moves from C and D. Therefore, ω will initially increase and then decrease.



Let R be the radius of platform, m the mass of disc and M is the mass of platform.

Moment of inertia when the tortoise is at A

$$I_1 = mR^2 + \frac{MR^2}{2}$$

and moment of inertia when the tortoise is at B

$$I_2 = mr^2 + \frac{MR^2}{2}$$

Here, $r^2 = a^2 + \left[\sqrt{R^2 - a^2} - vt\right]^2$

From conservation of angular momentum

$$\omega_0 I_1 = \omega(t) I_2$$

Substituting the values, we can see that variation of $\omega(t)$is non-linear.

Hence, the correct answer is (D).

18. $mg\sin\theta$ component is always down the plane whether it is rolling up or rolling down. Therefore, for no slipping, sense of angular acceleration should also be same in both the cases. Therefore, force of friction f always act upwards.

Hence, the correct answer is (B).

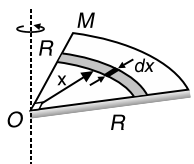
19. If we assume complete disc to be present, then it would have a mass 4 times the mass of the sector. Then, moment of inertia of the complete disc is

$$I_{\text{disc}} = \frac{1}{2} M_{\text{disc}} R^2 = \frac{1}{2}(4M_{\text{sector}})R^2$$

Hence, $I_{\text{sector}} = \frac{I_{\text{disc}}}{4}$

$$\Rightarrow \quad I_{\text{sector}} = \frac{1}{2} MR^2$$

Calculus Method:



Consider an element of mass dm, thickness dx and radius x of the sector.

$$dm = \frac{M}{\left(\frac{\pi R^2}{4}\right)}\frac{2\pi x dx}{4} = \frac{2M}{R^2}x dx$$

If dI be the moment of inertia of the element, then $dI = x^2 dm$

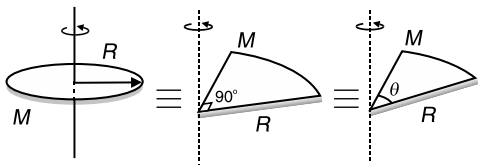
$$\Rightarrow \quad dI = x^2\left(\frac{2M}{R^2}x\,dx\right)$$

$$\Rightarrow \quad dI = \frac{2M}{R^2}\left(x^3\,dx\right)$$

$$\Rightarrow \quad I = \frac{2M}{R^2}\int_0^R x^3\,dx$$

$$\Rightarrow \quad I = \frac{1}{2}MR^2$$

General Funda:



$I_1 = \frac{1}{2}MR^2$ $\quad I_2 = \frac{1}{2}MR^2$ $\quad I_3 = \frac{1}{2}MR^2$

$$\left(\begin{array}{c}\text{Moment of Inertia}\\ \text{of a DISC of mass}\\ M \text{ with radius } R\end{array}\right) \equiv \left(\begin{array}{c}\text{Moment of Inertia}\\ \text{of a SECTOR of}\\ \text{mass } M \text{ with radius } R\end{array}\right)$$

Hence, the correct answer is (A).

20. Mass of the loop = $M = L\rho$

Further if r is the radius of the loop, then

$$2\pi r = L$$

$$\Rightarrow \quad r = \frac{L}{2\pi}$$

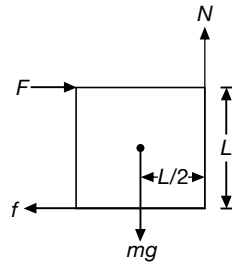
Moment of Inertia about XX' is $I = \frac{3}{2}Mr^2$

$$\Rightarrow \quad I = \frac{3}{2}(L\rho)\frac{L^2}{(2\pi)^2} = \frac{3\rho L^3}{8\pi^2}$$

Hence, the correct answer is (D).

21. At the critical condition, normal reaction N will pass through point P. In this condition

$$\tau_N = 0 = \tau_f \qquad \text{(about } P\text{)}$$



the block will topple when

$$\tau_F > \tau_{mg}$$

$$\Rightarrow \quad FL > (mg)\frac{L}{2}$$

$$\Rightarrow \quad F > \frac{mg}{2}$$

Therefore, the minimum force required to topple the bock is

$$F_{\min} = \frac{mg}{2}$$

Hence, the correct answer is (D).

22. No external torque is acting on the system, so angular momentum is conserved. Further there exists no non-conservative forces in the system, so total energy is also conserved.

Hence, the correct answer is (B).

23. The angular momentum of a body $\vec{L}$ may be expressed as the sum of two parts,

(a) one arising from the motion of the centre of mass of the body and

(b) the other from the motion of the body with respect to its centre of mass.

i.e. $\vec{L}_{\text{total}} = \vec{L}_{C.M.} + \vec{r}_{C.M.} \times \vec{p}$

$$\Rightarrow \quad \vec{L}_{\text{total}} = \vec{L}_{C.M.} + M\left(\vec{r}_{C.M.} \times \vec{v}_{C.M.}\right)$$

For this Problem

$$L_{C.M.} = \text{I}\omega = \frac{1}{2}MR^2\omega \text{ and}$$

$$M\left(\vec{r}_{C.M.} \times \vec{v}_{C.M.}\right) = MRv_{CM} = MR(R\omega)$$

$$\Rightarrow \quad M\left(\vec{r}_{C.M.} \times \vec{v}_{C.M.}\right) = MR^2\omega$$

$$\Rightarrow \quad L_{\text{total}} = \frac{1}{2}MR^2\omega + MR^2\omega = \frac{3}{2}MR^2\omega.$$

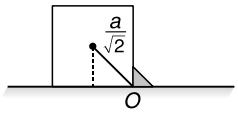
Hence, the correct answer is (C).

24. Since spheres are smooth, so no transfer of angular momentum takes place from A to B. However, sphere A only transfers its linear velocity v to sphere B and stops. Hence, we conclude that A stops but continues to rotate with same angular speed ω and B moves with speed of A but with zero angular speed.

Hence, the correct answer is (C).

25. By Law of Conservation of Angular Momentum

$$mv\left(\frac{a}{2}\right)=\left(I_{\substack{\text{system}\\ \text{about O}}}\right)\omega$$

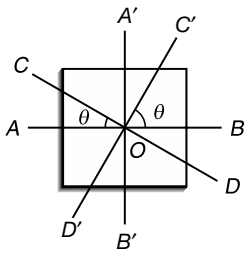


$$\Rightarrow \quad mv\left(\frac{a}{2}\right)=\left[\frac{1}{6}Ma^2+M\left(\frac{a}{\sqrt{2}}\right)^2\right]\omega$$

$$\Rightarrow \quad \omega=\frac{3v}{4a}$$

Hence, the correct answer is (A).

26.



$I_{AB}=I_{A'B'}=I$ and $I_{CD}=I_{C'D'}$

If I_0 be the moment of inertia of the square plate about an axis passing through O and perpendicular to the plate, then by perpendicular axis theorem

$$I_0=I_{AB}+I_{A'B'}=2I_{AB} \qquad \ldots(1)$$

OR

$$I_0=I_{CD}+I_{C'D'}=2I_{CD} \qquad \ldots(2)$$

From (1) and (2)

$$I_{CD}=I_{AB}=I \qquad \textbf{\{OPTION (A)\}}$$

Hence, the correct answer is (A).

27. Since $L=mvr_\perp$ and as $r_\perp$ is constant, so L is constant.

Hence, the correct answer is (B).

28. Work done $W=\frac{1}{2}I\omega^2$

If x is the distance of mass 0.3 kg from the centre of mass, we will have,

$$I=(0.3)x^2+(0.7)(1.4-x)^2$$

For work to be minimum, the moment of inertia (I) should be minimum

$$\Rightarrow \quad \frac{dI}{dx}=0$$

$$\Rightarrow \quad 2(0.3x)-2(0.7)(1.4-x)=0$$

$$\Rightarrow \quad (0.3)x=(0.7)(1.4-x)$$

$$\Rightarrow \quad x=\frac{(0.7)(1.4)}{0.3+0.7}=0.98 \text{ m}$$

Hence, the correct answer is (D).

29. Mass of the element dx is $m=\frac{M}{L}dx$

This element needs centripetal force for rotation, so

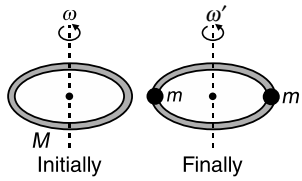
$$dF=mx\omega^2=\left(\frac{M}{L}x\omega^2dx\right)$$

$$\Rightarrow \quad F=\int_0^L dF=\frac{m}{L}\omega^2\int_0^L xdx=\frac{M\omega^2L}{2}$$

This is the force exerted by the liquid at the other end.
Hence, the correct answer is (D).

30. By Law of Conservation of Angular Momentum

$$(MR^2)\omega=MR^2\omega'+2mR^2\omega'$$



$$\Rightarrow \quad \omega'=\left(\frac{M}{M+2m}\right)\omega$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

1. Applying conservation of angular momentum about hinge, we get

$$mvx=\left(mx^2+\frac{mL^2}{3}\right)\omega$$

$$\Rightarrow \quad \omega=\frac{3vx}{L^2+3x^2}$$

For ω to be maximum, we have

$$\frac{d\omega}{dx}=0$$

$$\Rightarrow \quad x_M=\frac{L}{\sqrt{3}}$$

$$\Rightarrow \quad \omega_M=\frac{\sqrt{3}v}{2L}$$

Hence, (A), (C) and (D) are correct.

2. Applying conservation of energy (as the friction is acting at the point of no slipping), we get

$$\frac{MgL}{2}(1-\cos 60^\circ)=\frac{1}{2}I_0\omega^2$$

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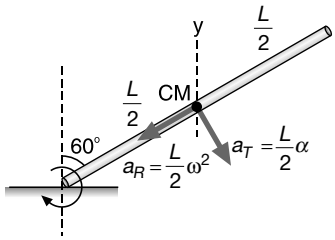


Since $I_0 = \frac{ML^2}{3}$

$\Rightarrow \quad \omega = \sqrt{\frac{3g}{2L}}$

$\Rightarrow \quad a_R = \frac{L}{2}\omega^2 = \frac{3g}{4}$

Also $\tau_0 = Mg\frac{L}{2}\sin(60°) = I_0\alpha$

$\Rightarrow \quad \alpha = \frac{3\sqrt{3}g}{4L}$

For CM of the rod, we have

$$Mg - N = Ma_y$$

where, $a_y = a_T\sin(60°) + a_R\cos(60°)$

$\Rightarrow \quad a_y = \frac{L}{2}\alpha\sin(60°) + \frac{\omega^2 L}{2}\cos(60°)$

$\Rightarrow \quad a_y = \frac{\sqrt{3}}{4}\left(\frac{3\sqrt{3}g}{4}\right) + \left(\frac{3g}{2L}\right)\left(\frac{L}{2}\right)\left(\frac{1}{2}\right)$

$\Rightarrow \quad a_y = \left(\frac{9}{16} + \frac{3}{8}\right)g = \frac{15g}{16}$

$\Rightarrow \quad Mg - N = M\left(\frac{15g}{16}\right)$

$\Rightarrow \quad N = \frac{Mg}{16}$

Hence, (B), (C) and (D) are correct.

3. When the bar makes an angle θ, the height of its CM (mid-point) is $\frac{L}{2}\cos\theta$

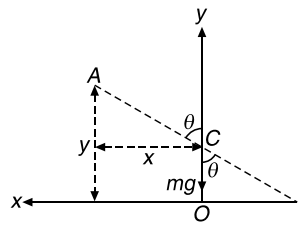
Displacement of CM is

$$y = \frac{L}{2}(1 - \cos\theta)$$

Since, force on CM is only along the vertical direction, hence CM is falling vertically downward. Instantaneous torque about point of contact is

$$\tau = mg\left(\frac{L}{2}\sin\theta\right)$$

$\Rightarrow \quad \tau \propto \sin\theta$



Now, $x = \frac{L}{2}\sin\theta$ and $y = L\cos\theta$

$\Rightarrow \quad \frac{x^2}{\left(\frac{L}{2}\right)^2} + \frac{y^2}{L^2} = 1$

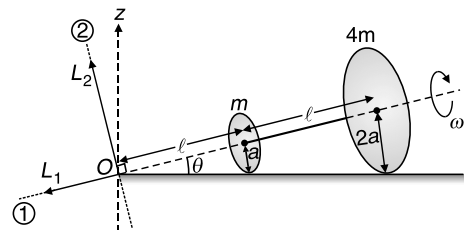
So, path of A is an ellipse.

Hence, (A), (C) and (D) are correct.

4. Let Ω be the angular velocity of assembly perpendicular to axis of the rod, then for pure rolling, we have

$$a\omega = \ell\Omega$$

$\Rightarrow \quad \Omega = \frac{a\omega}{\ell}$



So, component of Ω along z-axis is $\Omega\cos\theta$

$\Rightarrow \quad \Omega\cos\theta = \frac{a\omega}{\ell}\frac{\sqrt{24}}{5} = \frac{\ell\omega}{5\ell} = \frac{\omega}{5} \qquad \{\because \ \ell = \sqrt{24}\,a\}$

So, (A) is correct.

Let L_1 be the angular momentum of assembly about its centre of mass, then

$$L_1 = \left[\frac{ma^2}{2} + \frac{(4m)(2a)^2}{2}\right]\omega = \left(\frac{17}{2}ma^2\right)\omega$$

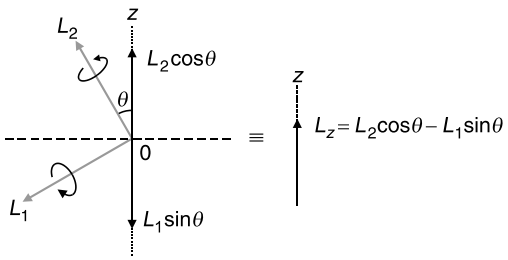
So, (C) is correct

If I_2 be the moment of inertia of assembly about and axis perpendicular to axis of rod (marked as 2), then

$$I_2 = \left(\frac{ma^2}{4} + m\ell^2\right) + \left[\frac{(4m)(2a)^2}{4} + (4m)(2\ell)^2\right]$$

$\Rightarrow \quad I_2 = \frac{17}{4}ma^2 + 17\,m\ell^2 = \frac{17}{4}ma^2 + 17m\left(24a^2\right)$

So, angular momentum of assembly about axis 2 is $L_2 = I_2\Omega$



$$L_2 = (17ma^2)\left(\frac{1}{4}+24\right)\frac{a\omega}{\sqrt{24}a}$$

$$\Rightarrow \quad L_2 = (17ma^2)\left(\frac{97}{4}\right)\frac{\omega}{\sqrt{24}} \cong 17\sqrt{24}\,ma^2\omega$$

$$\Rightarrow \quad L_2\cos\theta = (17\sqrt{24}ma^2\omega)\left(\frac{\sqrt{24}}{5}\right) = \frac{(17)(24)}{5}ma^2\omega$$

Also, $L_1\sin\theta = \left(\frac{17}{2}ma^2\omega\right)\left(\frac{1}{5}\right)$

$$\Rightarrow \quad L_Z = L_2\cos\theta - L_1\sin\theta = \frac{17}{5}ma^2\omega\left(24-\frac{1}{2}\right)$$

$$\Rightarrow \quad L_Z \cong 80ma^2\omega$$

So, (D) is not correct

Angular momentum of CM of assembly about O is

$$L_{CM \text{ about } O} = (5m)(v_{cm})\left(\frac{9\ell}{5}\right)$$

where $v_{cm} = \frac{m(a\omega)+4m((2a)\omega)}{5m} = \frac{9a\omega}{5}$

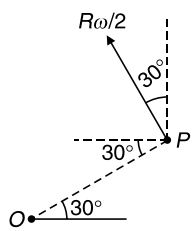
$$L_{CM \text{ about } O} = (5m)\left(\frac{9a\omega}{5}\right)\left(\frac{9\ell}{5}\right) = 81ma^2\omega\left(\frac{\sqrt{24}}{5}\right)$$

So, (B) is not correct

Hence, (A) and (C) are correct.

5. Velocity of point O is $\vec{v}_O = (3R\omega)\hat{i}$

Also, $|\vec{v}_{PO}| = \frac{R\omega}{2}$ in the direction shown in Figure.



In vector form,

$\vec{v}_{PO} = -\frac{R\omega}{2}\sin 30°\hat{i} + \frac{R\omega}{2}\cos 3($

$$\Rightarrow \quad \vec{v}_{PO} = -\frac{R\omega}{4}\hat{i} + \frac{\sqrt{3}R\omega}{4}\hat{k}$$

Since, $\vec{v}_{PO} = \vec{v}_P - \vec{v}_O$

$$\Rightarrow \quad \vec{v}_P = \vec{v}_{PO} + \vec{v}_O$$

$$\Rightarrow \quad \vec{v}_P = \left(-\frac{R\omega}{4}\hat{i} + \frac{\sqrt{3}R\omega}{4}\hat{k}\right) + 3R\omega\hat{i}$$

$$\Rightarrow \quad \vec{v}_P = \frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$$

Hence, (A) and (B) are correct.

6. The data seems to be incomplete.

Let us assume that friction from ground on ring is not impulsive during impact. From linear momentum conservation along horizontal direction, we get

$$(-2\times 1)+(0.1\times 20) = (0.1\times 0)+(2\times v)$$

Here, v is the velocity of CM of ring after impact.

Solving the above equation, we get $v = 0$

So, CM becomes stationary.

Correct OPTION is (A)

Linear impulse during impact

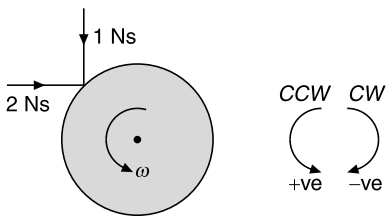
1. Along horizontal direction is

   $J_1 = \Delta p = 0.1\times 20 = 2$ Ns

2. Along vertical direction is

   $J_2 = \Delta p = 0.1\times 10 = 1$ Ns

   Writing the equation (about CM)



Since, angular impulse equals change in angular momentum, so we get

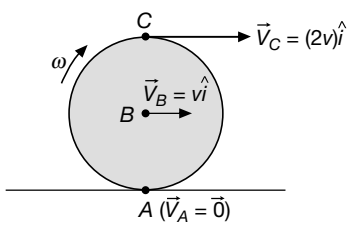
$$1\times\left(\frac{\sqrt{3}}{2}\times\frac{1}{2}\right) - 2\times 0.5\times\frac{1}{2} = 2\times(0.5)^2\left(\omega - \frac{1}{0.5}\right)$$

Solving this equation, ω comes out to be positive i.e. ω is counter clockwise. So just after collision, rightwards slipping is taking place.

Hence, friction is leftwards.

Hence, (A) and (C) are correct.

7.



Hence, (B) and (C) are correct.

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8. In case of pure rolling,

$$f = \frac{mg\sin\theta}{1+\frac{mR^2}{I}} \qquad \text{(upwards)}$$

$\Rightarrow \quad f \propto \sin\theta$

Therefore, as θ decreases force of friction will also decrease.

Hence, (C) and (D) are correct.

9. Due to Law of Conservation of Angular Momentum $\vec{L}$ =constant

$\Rightarrow \quad \vec{L}\cdot\vec{L} = \text{constant}$

$\Rightarrow \quad \frac{d}{dt}(\vec{L}\cdot\vec{L}) = 0$

$\Rightarrow \quad 2\vec{L}\cdot\frac{d\vec{L}}{dt} = 0$

$\Rightarrow \quad \vec{L} \perp \frac{d\vec{L}}{dt}$

Since $\vec{\tau} = \vec{A}\times\vec{L}$

$\Rightarrow \quad \frac{d\vec{L}}{dt} = \vec{A}\times\vec{L}$

$\Rightarrow \quad \frac{d\vec{L}}{dt}$ must be perpendicular to $\vec{A}$ as well as $\vec{L}$

Further component of $\vec{L}$ along $\vec{A}$ is $\frac{\vec{A}\cdot\vec{L}}{A} = x$(say). Also

$$\frac{d}{dt}(\vec{A}\cdot\vec{L}) = \vec{A}\cdot\frac{d\vec{L}}{dt} + \vec{L}\cdot\frac{d\vec{A}}{dt} = 0$$

$$\left\{\because \vec{A} \perp \frac{d\vec{L}}{dt} \text{ and } \frac{d\vec{A}}{dt} = \vec{0}\right\}$$

$\Rightarrow \quad \vec{A}\cdot\vec{L} = \text{constant}$

$\Rightarrow \quad \frac{\vec{A}\cdot\vec{L}}{A} = x = \text{constant}$

Since $\frac{d\vec{L}}{dt}$ (or $\vec{\tau}$) is perpendicular to $\vec{L}$, hence it cannot change magnitude of $\vec{L}$ but can surely change direction of $\vec{L}$.

Hence, (A), (B) and (C) are correct.

10. From symmetry we have

$$I_1 = I_2 \text{ and } I_4 = I_3 \qquad \ldots(1)$$

Also, by perpendicular Axis Theorem

$$I_0 = I_1 + I_2 = I_4 + I_3 \qquad \ldots(2)$$

From (2), we get

$$2I_1 = 2I_3 \qquad \{\because \text{of}(1)\}$$

$\Rightarrow \quad I_1 = I_3$

$\Rightarrow \quad I_0 = I_1 + I_2 = I_4 + I_3 = I_1 + I_3$

Hence, (A), (B) and (C) are correct.

11. $V_c = \frac{2m(-v)+m(2v)+8m(0)}{2m+m+8m} = 0$ {OPTION **(A)**}

Further, by Law of Conservation of Angular Momentum

$$\left(I_{\substack{\text{system}\\\text{about O}}}\right)\omega = \sum_{\substack{\text{system}\\\text{about O}}} mvr_\perp$$

$$I_{\substack{\text{system}\\\text{about O}}} = \left[\frac{1}{12}(8m)(6a)^2 + m(2a)^2 + 2ma^2\right] = 30ma^2$$

$$\sum_{\substack{\text{system}\\\text{about O}}} mvr_\perp = (8m)(0)(0) + 2m(-v)(-a) + m(2v)(2a)$$

$\Rightarrow \quad \sum_{\substack{\text{system}\\\text{about O}}} mvr_\perp = 6mva$

$\Rightarrow \quad (30ma^2)\omega = 6mva$

$\Rightarrow \quad \omega = \frac{v}{5a}$ {OPTION **(C)**}

Further total energy of the system E is

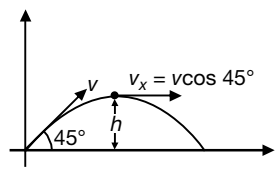
$$E = \frac{1}{2}I\omega^2 = \frac{3}{5}mv^2 \qquad \text{{OPTION \textbf{(D)}}}$$

Hence, (A), (C) and (D) are correct.

12. $L = mv_x h$

$\Rightarrow \quad L = m(v\cos 45)\left(\frac{v^2\sin^2 45}{2g}\right)$

$\Rightarrow \quad L = \frac{mv^3}{4\sqrt{2}\,g} \qquad \left\{\because h\frac{v^2\sin^2 45}{2g}\right\}$



Also, from $h = \frac{v^2}{4g}$

$\Rightarrow \quad v = 2\sqrt{gh}$

$\Rightarrow \quad L = \frac{mv}{\sqrt{2}}h$ gives

$$L = \frac{m\left(2\sqrt{gh}\right)h}{\sqrt{2}}$$

$\Rightarrow \quad L = m\sqrt{2gh^3}$

Hence, (B) and (D) are correct.

Reasoning Based Questions

1. In case of pure rolling on inclined plane,

$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$$

$I_{solid} < I_{hollow}$

$\Rightarrow \quad a_{solid} > a_{hollow}$

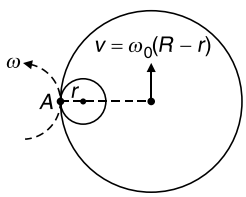
Solid cylinder will reach the bottom first. Further, in case of pure rolling on stationary ground, work done by friction is zero. Therefore, mechanical energy of both the cylinders will remain constant.

$\Rightarrow \quad (KE)_{Hollow} = (KE)_{Solid} = \text{decrease in PE} = mgh$

Hence, the correct answer is (D).

Linked Comprehension Type Questions

1. The point A will be $IAOR$ (i.e. instantaneous axis of rotation). For no slipping, we have



$R\omega = (R-r)\omega_0$

$\Rightarrow \quad \omega = \omega_0\left(\frac{R-r}{R}\right)$

$\Rightarrow \quad KE = \frac{1}{2}(2mR^2)\omega^2 = m\omega_0^2(R-r)^2$

Hence, the correct answer is (A).

2. If height of the cone is h $\quad r$

Then, $\mu N = mg$

$\Rightarrow \quad \mu m(R-r)\omega_0^2 = mg$

$\Rightarrow \quad \omega_0 = \sqrt{\frac{g}{\mu(R-r)}}$

Hence, the correct answer is (C).

3. Force on block along slot is the centripetal force, so

$$m\omega^2 r = ma = m\left(\frac{vdv}{dr}\right)$$

$$\Rightarrow \quad \int_0^v vdv = \int_{\frac{R}{2}}^r \omega^2 r dr$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{\omega^2}{2}\left(r^2 - \frac{R^2}{4}\right)$$

$$\Rightarrow \quad v = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt}$$

$$\Rightarrow \quad \int_{\frac{R}{4}}^r \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_0^t \omega dt$$

$$\Rightarrow \quad \ln\left(\frac{r+\sqrt{r^2-\frac{R^2}{4}}}{\frac{R}{2}}\right) - \ln\left(\frac{\frac{R}{2}+\sqrt{\frac{R^2}{4}-\frac{R^2}{4}}}{\frac{R}{4}}\right) = \omega t$$

$$\Rightarrow \quad r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2}e^{\omega t}$$

$$\Rightarrow \quad r^2 - \frac{R^2}{4} = \frac{R^2}{4}e^{2\omega t} + r^2 - 2r\frac{R}{2}e^{\omega t}$$

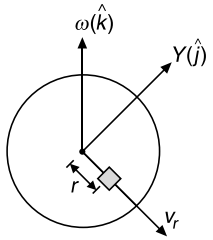
$$\Rightarrow \quad r = \frac{\frac{R^2}{4}e^{2\omega t} + \frac{R^2}{4}}{Re^{\omega t}} = \frac{R}{4}(e^{\omega t} + e^{-\omega t})$$

Hence, the correct answer is (C).

4. Since, $\vec{F}_{rot} = \vec{F}_{in} + 2m(v_{rot}\hat{i}) \times \omega\hat{k} + m(\omega\hat{k} \times r\hat{i}) \times \omega\hat{k}$

$\Rightarrow \quad mr\omega^2\hat{i} = \vec{F}_{in} + 2mv_{rot}\omega(-\hat{j}) + m\omega^2 r\hat{i}$

$\Rightarrow \quad F_{in} = 2mv_r\omega\hat{j}$



Since, $r = \frac{R}{4}(e^{\omega t} + e^{-\omega t})$

$$\Rightarrow \quad \frac{dr}{dt} = v_r = \frac{R}{4}(\omega e^{\omega t} - \omega e^{-\omega t})$$

$$\Rightarrow \quad \vec{F}_{in} = 2m\frac{R\omega}{4}(e^{\omega t} - e^{-\omega t})\omega\hat{j}$$

$$\Rightarrow \quad \vec{F}_{in} = \frac{mR\omega^2}{2}(e^{\omega t} - e^{-\omega t})\hat{j}$$

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Also, reaction is due to the disc surface, so

$$\vec{F}_{\text{reaction}} = \frac{mR\omega^2}{2}\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$$

Hence, the correct answer is (B).

5-6. The correct answer is 5(D), 6(A).

Combined solution to 5, 6

Following points must be kept in mind.

(i) Every particle of the disc is rotating in a horizontal circle.

(ii) Actual velocity of any particle is horizontal.

(iii) Magnitude of velocity of any particle is

$$v = r\omega$$

where, r is the perpendicular distance of that particle from actual axis of rotation (z-axis)

(iv) When it is broken into two parts then actual velocity of any particle is resultant of two velocities

$$v_1 = r_1\omega_1 \text{ and } v_2 = r_2\omega_2$$

Here, r_1 is the perpendicular distance of centre of mass from z-axis, ω_1 is the angular speed of rotation of centre of mass from z-axis, r_2 is the distance of particle from centre of mass and ω_2 is the angular speed of rotation of the disc about the axis passing through centre of mass.

(v) Net v will be horizontal, if v_1 and v_2 both are horizontal. Since, v_1 is already horizontal, because centre of mass is rotating about a vertical z-axis. To make v_2 also horizontal, the second axis should also be vertical.

Hence, the correct answer is (A).

7. $$\frac{1}{2}I(2\omega)^2 = \frac{1}{2}kx_1^2 \qquad \ldots(1)$$

$$\frac{1}{2}(2I)(\omega)^2 = \frac{1}{2}kx_2^2 \qquad \ldots(2)$$

From equations (1) and (2), we have

$$\frac{x_1}{x_2} = \sqrt{2}$$

Hence, the correct answer is (C).

8. Let ω' be the common velocity. Then applying conservation of angular momentum, we get

$$(I + 2I)\omega' = I(2\omega) + 2I(\omega)$$

$$\Rightarrow \quad \omega' = \frac{4}{3}\omega$$

Applying the concept that angular impulse equals change in angular momentum. For any of the disc, we have

$$\tau t = I(2\omega) - I\left(\frac{4}{3}\omega\right)$$

$$\Rightarrow \quad \tau t = \frac{2I\omega}{3}$$

$$\Rightarrow \quad \tau = \frac{2I\omega}{3t}$$

9. Loss of kinetic energy is $-\Delta K = K_i - K_f$

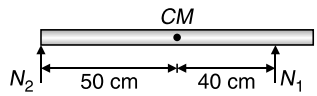
$$\Rightarrow \quad -\Delta K = \left\{\frac{1}{2}I(2\omega)^2 + \frac{1}{2}(2I)(\omega)^2\right\} - \frac{1}{2}(3I)\left(\frac{4}{3}\omega\right)^2$$

$$\Rightarrow \quad -\Delta K = \frac{1}{3}I\omega^2$$

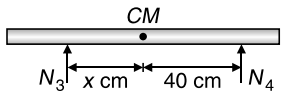
Hence, the correct answer is (B).

Integer/Numerical Answer Type Questions

1. Initially, $40N_1 = 50N_2$



For First Move:



For translational equilibrium, we have

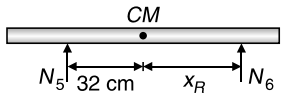
$$\mu_k N_3 = \mu_s N_4 \qquad \ldots(1)$$

For rotational equilibrium, we have

$$xN_3 = 40N_4 \qquad \ldots(2)$$

$$\Rightarrow \quad x = 32 \text{ cm}$$

For Second Move:



For translational equilibrium, we have

$$\mu_s N_5 = \mu_k N_6 \qquad \ldots(1)$$

For rotational equilibrium, we have

$$32(N_5) = X_R N_6 \qquad \ldots(2)$$

$$\Rightarrow \quad X_R = 25.6 \text{ cm}$$

2. In case of pure rolling, mechanical energy remains constant since work done by friction is zero. Further in case of a disc,

$$\frac{\text{Translational Kinetic Energy}}{\text{Rotational Kinetic Energy}} = \frac{K_T}{K_R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}I\omega^2}$$

$$\Rightarrow \quad \frac{K_T}{K_R} = \frac{mv^2}{\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2} = \frac{2}{1}$$

$$\Rightarrow \quad K_T = \frac{2}{3}(\text{Total KE}) = \frac{2}{3}K$$

So, total kinetic energy is

$$K = \frac{3}{2}K_T = \frac{3}{2}\left(\frac{1}{2}mv^2\right) = \frac{3}{4}mv^2$$

Since, decrease in potential energy equals increase in kinetic energy, so

$$mgh = \frac{3}{4}m\left(v_f^2 - v_i^2\right)$$

$$\Rightarrow \quad v_f = \sqrt{\frac{4}{3}gh + v_i^2}$$

Given that the final velocity in both cases is same, so, value of $v_f = \sqrt{\frac{4}{3}gh + v_i^2}$ should be same in both cases.

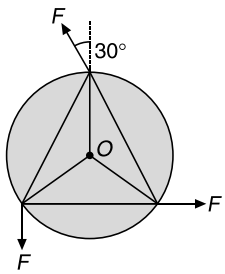
$$\Rightarrow \quad \sqrt{\frac{4}{3}\times 10\times 30 + (3)^2} = \sqrt{\frac{4}{3}\times 10\times 27 + (v_2)^2}$$

$$\Rightarrow \quad v_2 = 7 \text{ ms}^{-1}$$

3. Since angular impulse equals change in angular momentum, so

$$\int \tau dt = I\omega$$

$$\Rightarrow \quad \omega = \frac{\int \tau dt}{I} = \frac{\int_0^t 3F\sin 30° R dt}{I}$$

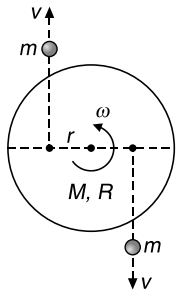


$$\Rightarrow \quad \omega = \frac{3(0.5)(0.5)(0.5)(1)}{1.5(0.5)^2/2} = 2 \text{ rads}^{-1}$$

4. Applying conservation of angular momentum, we get

$$2mvr - \frac{MR^2}{2}\omega = 0$$

$$\Rightarrow \quad \omega = \frac{4mvr}{MR^2}$$



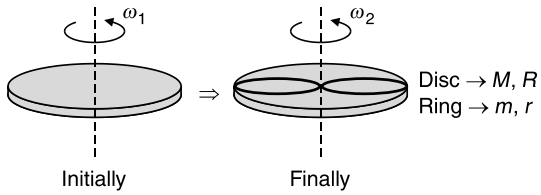
Substituting the values, we get

$$\omega = \frac{(4)(5\times 10^{-2})(9)(0.25)}{45\times 10^{-2}(0.5)^2}$$

$$\Rightarrow \quad \omega = 4 \text{ rads}^{-1}$$

5. By conservation of angular momentum, we have

$$I_1\omega_1 = I_2\omega_2$$

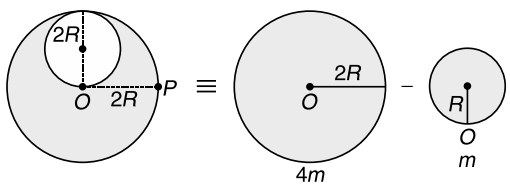


$$\Rightarrow \quad \omega_2 = \left(\frac{I_1}{I_2}\right)\omega_1 = \left(\frac{\frac{1}{2}MR^2}{\frac{1}{2}MR^2 + 2(mr^2)}\right)\omega_1$$

$$\Rightarrow \quad \omega_2 = \left(\frac{50(0.4)^2}{50(0.4)^2 + 8\times(6.25)\times(0.2)^2}\right)(10)$$

$$\Rightarrow \quad \omega_2 = 8 \text{ rads}^{-1}$$

6. Moment of inertia of this lamina about the point O is



$$I_O = \frac{(4m)(2R)^2}{2} - \frac{3}{2}mR^2$$

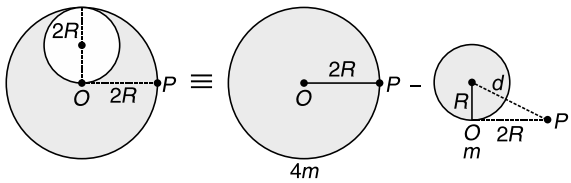
$$\Rightarrow \quad I_O = mR^2\left(8 - \frac{3}{2}\right)$$

$$\Rightarrow \quad I_O = \frac{13}{2}mR^2$$

Similarly, MI about point P is

$$I_P = \frac{3}{2}(4m)(2R)^2 - \left(\frac{mR^2}{2} + md^2\right)$$

where $d^2 = R^2 + (2R)^2$



$$\Rightarrow \quad I_P = 24mR^2 - \frac{11}{2}mR^2$$

$$\Rightarrow \quad I_P = \frac{37}{2}mR^2$$

$$\Rightarrow \quad \frac{I_P}{I_O} = \frac{\frac{37}{2}}{\frac{13}{2}} = \frac{37}{13} \approx 3$$

7. Moment of inertia of system about diagonal of square is

$$I = 2\left(\frac{2}{5}MR^2\right) + 2\left(\frac{2}{5}MR^2 + Md^2\right)$$

$$\Rightarrow \quad I = \frac{8}{5}MR^2 + 2md^2$$

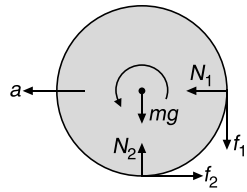
$$\Rightarrow \quad I = \left[\frac{8}{5}(0.5)\left(\frac{\sqrt{5}}{2}\right)^2 + 2(0.5)(4)(2)\right]10^{-4}$$

$$\Rightarrow \quad I = \left(\frac{5}{5} + 8\right) \times 10^{-4}$$

$$\Rightarrow \quad I = 9 \times 10^{-4}$$

$$\Rightarrow \quad N = 9$$

8. There is no slipping between ring and ground, so f_2 is not maximum. However, there is slipping between ring and stick, so, f_1 is maximum.



Moment of inertia of the ring about its centre of mass is

$$I = mR^2 = (2)(0.5)^2 = 0.5 \text{ kgm}^2$$

Since, $N_1 - f_2 = ma$

$$\Rightarrow \quad N_1 - f_2 = (2)(0.3) = 0.6 \text{ N} \qquad \ldots(1)$$

Also, $a = R\alpha = \frac{R\tau}{I} = \frac{R(f_2 - f_1)R}{I} = \frac{R^2(f_2 - f_1)}{I}$

$$\Rightarrow \quad 0.3 = \frac{(0.5)^2(f_2 - f_1)}{(0.5)}$$

$$\Rightarrow \quad f_2 - f_1 = 0.6 \text{ N} \qquad \ldots(2)$$

Since stick applies a force of 2 N on the ring, so we have

$$N_1^2 + f_1^2 = (2)^2 = 4 \qquad \ldots(3)$$

Further, $f_1 = \mu N_1 = \left(\frac{P}{10}\right)N_1 \qquad \ldots(4)$

From equations (1), (2), (3) and (4), we get

$$P \approx 3.6$$

CHAPTER 4: GRAVITATION AND SATELLITES

Test Your Concepts-I
(Based on Acceleration due to Gravity, Gravitational Field and Applications)

1. Escape velocity from the surface of moon is

$$v_e = \sqrt{\frac{2GM_m}{R_m}}$$

Substituting the values, we have

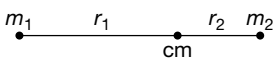
$$v_e = \sqrt{\frac{2\times 6.67\times 10^{-11}\times 7.4\times 10^{22}}{1.74\times 10^{6}}}$$

$$\Rightarrow \quad v_e = 2.4\times 10^{3}\ \text{ms}^{-1} \text{ or } 2.4\ \text{kms}^{-1}$$

2. Both the planet and the sun revolve around their centre of mass with same angular velocity (say ω)

$$r = r_1 + r_2 \qquad \ldots(1)$$

$$m_1 r_1 \omega^2 = m_2 r_2 \omega^2 = \frac{Gm_1 m_2}{r^2} \qquad \ldots(2)$$



Solving these two equations, we get

$$r_1 = r\left(\frac{m_2}{m_1 + m_2}\right)$$

$$r_2 = r\left(\frac{m_1}{m_1 + m_2}\right)$$

and $\omega^2 = \dfrac{G(m_1 + m_2)}{r^3}$

Now, total energy of the system is

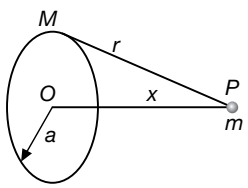
$$E = \text{P.E.} + \text{K.E.}$$

$$\Rightarrow \quad E = -\frac{Gm_1 m_2}{r} + \frac{1}{2}m_1 r_1^2 \omega^2 + \frac{1}{2}m_2 r_2^2 \omega^2$$

Substituting the values of r_1, r_2 and ω^2, we get

$$E = -\frac{Gm_1 m_2}{2r}$$

3. (a) Gravitational potential at point P due to the ring.



$$V = -\frac{GM}{r} = -\frac{GM}{\sqrt{a^2 + x^2}}$$

$$\Rightarrow \quad U = mV = -\frac{GMm}{\sqrt{a^2 + x^2}}$$

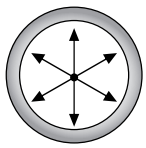
(b) When $x \quad a$, $a^2 + x^2 \approx x^2$

$$\Rightarrow \quad U = -\frac{GMm}{x}$$

{the potential energy of two point masses}

(c) $$F_x = -\frac{dU}{dx} = -GMm \cdot \frac{d}{dx}\left(a^2 + x^2\right)^{-1/2}$$

$$\Rightarrow \quad F_x = -\frac{GMm \cdot x}{\left(x^2 + a^2\right)^{3/2}}$$



(d) When $x \quad a$, $a^2 + x^2 \approx x^2$

and $F_x = -\dfrac{GMm}{x^2}$

{force between two point masses}

(e) At $x = 0$ {centre of the ring}

$F_x = 0$

As the particle is attracted equally from all the four sides.

4. Net force on M due to the pairs $2M$ and $2M$, $4M$ and $4M$, $5M$ and $5M$, $7M$ and $7M$, M and M is zero. So, the only left out mass is $3M$ and net force on M is due to $3M$, given by

$$F = \frac{G(3M)M}{d^2}$$

$$\Rightarrow \quad F = \frac{3GM^2}{d^2}, \text{ along } +x \text{ axis}$$

5. Given that $F = \dfrac{Gm_A m_B}{r^2}$

Since acceleration of A is $a_A = \dfrac{F}{m_A}$

$$\Rightarrow \quad a = a_A = \frac{F}{m_A} = \frac{Gm_B}{r^2} \qquad \ldots(1)$$

Now if, $F = \dfrac{Gm_A m_B}{r^4}$, then

$$a'_A = \frac{F}{m_A} = \frac{Gm_B}{r^4}$$

$$\Rightarrow \quad a'_A = \frac{a}{r^2} \qquad \{\because \text{ of } (1)\}$$

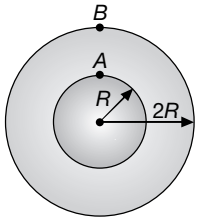
6. Since, $E = \dfrac{F}{m_0}$

$\Rightarrow \quad E = \dfrac{4N}{20 \times 10^{-3}\ \text{kg}}$

$\Rightarrow \quad E = 200\ \text{Nkg}^{-1}$, along $+x$ direction.

7. Let m_1 be the mass of the core and m_2 the mass of outer shell, then

$g_A = g_B$



$\Rightarrow \quad \dfrac{Gm_1}{R^2} = \dfrac{G(m_1 + m_2)}{(2R)^2}$

$\Rightarrow \quad 4m_1 = (m_1 + m_2)$

$\Rightarrow \quad 4\left(\dfrac{4}{3}\pi R^3 \rho_1\right) = \left(\dfrac{4}{3}\pi R^3\right)\rho_1 + \left(\dfrac{4}{3}\pi(2R)^3 - \dfrac{4}{3}\pi R^3\right)\rho_2$

$\Rightarrow \quad 4\rho_1 = \rho_1 + 7\rho_2$

$\Rightarrow \quad \dfrac{\rho_1}{\rho_2} = \dfrac{7}{3}$

8. $L = \pi R$

$\Rightarrow \quad R = \dfrac{L}{\pi}$

Gravitational field at the centre of a semicircular wire of radius R is

$$E = \frac{2G\lambda}{R}$$

where $\lambda = \dfrac{M}{L}$

$\Rightarrow \quad E = \dfrac{2G\left(\dfrac{M}{L}\right)}{\left(\dfrac{L}{\pi}\right)}$

$\Rightarrow \quad E = \dfrac{2\pi GM}{L^2}$

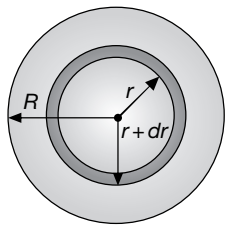
Since $F = mE$ $\qquad \left\{\because E = \dfrac{F}{m}\right\}$

$\Rightarrow \quad F = \dfrac{2\pi GMm}{L^2}$

9. Since $r(=2R)$ lies outside the sphere, so

$$E = \frac{GM_{\text{total}}}{r^2} \text{ for } r \geq R$$

where M_{total} is the total mass of the sphere.



To calculate M_{total}, we consider an infinitesimal shell element of inner radius r and outer radius $r + dr$. If dm be the mass of this infinitesimal element, then

$$dm = (4\pi r^2 dr)\rho$$

But, $\rho = \dfrac{\rho_0 R}{r}$

$\Rightarrow \quad dm = \left(\dfrac{\rho_0 R}{r}\right)(4\pi r^2 dr)$

$\Rightarrow \quad dm = (4\pi\rho_0 R)(r dr)$

$\Rightarrow \quad M_{\text{total}} = \displaystyle\int dm = 4\pi\rho_0 R \int_0^R r dr$

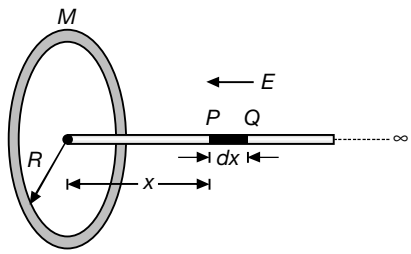
$\Rightarrow \quad M_{\text{total}} = \dfrac{4\pi\rho_0 R^3}{2} = 2\pi\rho_0 R^3$

$\Rightarrow \quad E(r) = \dfrac{GM}{r^2}$

$\Rightarrow \quad E = \dfrac{G(2\pi\rho_0 R^3)}{4R^2} = \dfrac{\pi RG\rho_0}{2}$

10. Field strength at the axis at distance x from the centre of the ring is,

$$E = \frac{GMx}{(R^2 + x^2)^{3/2}}$$



Consider an infinitesimal mass element PQ of length dx, mass dm at a distance x from centre of ring. Then

$$dm = \lambda dx$$

Force on this element will be,

$$dF = Edm = \frac{GM\lambda x}{(R^2 + x^2)^{3/2}} dx$$

$\Rightarrow \quad F = \displaystyle\int_0^\infty dF = \int_0^\infty \frac{GM\lambda x dx}{(R^2 + x^2)^{3/2}}$

Since $\int \frac{dx}{\sqrt{R^2+x^2}} = -\frac{1}{\sqrt{R^2+x^2}}$

$\Rightarrow \quad F = -GM\lambda\left(\frac{1}{\sqrt{R^2+x^2}}\Big|_0^\infty\right)$

$\Rightarrow \quad F = -GM\lambda\left(\frac{1}{\infty}-\frac{1}{R}\right)$

$\Rightarrow \quad F = \frac{GM\lambda}{R}$

11. At the equator, we have

$g_e = g - R\omega^2$

60% of his weight at the pole means

$g_e = \frac{60}{100}g = \frac{3}{5}g$

$\Rightarrow \quad \frac{3}{5}g = g - R\omega^2$

$\Rightarrow \quad \frac{2}{5}g = R\omega^2$

$\Rightarrow \quad \omega = \sqrt{\frac{2}{5}\frac{g}{R}}$

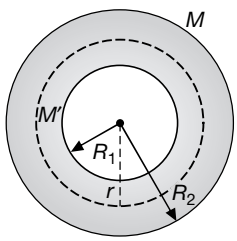
$\Rightarrow \quad \omega = \sqrt{\frac{2\times 9.8}{5\times 6.4\times 10^6}}$

$\Rightarrow \quad \omega = \sqrt{\frac{19.6}{32\times 10^6}}$

$\Rightarrow \quad \omega = 7.8\times 10^{-4}$ rads^{-1}

12. For $r < R_1$, we have $E_r = 0$

For $r > R_2$, we have, $E_r = \frac{GM}{r^2}$



Inside i.e., for $R_1 < r < R_2$, we have

$E_r = \frac{GM'}{r^2}$

where M' is mass of shell from R_1 to r. So,

$M' = \frac{4}{3}\pi\left(r^3 - R_1^3\right)\rho$

where $\rho = \frac{M}{\frac{4}{3}\pi\left(R_2^3 - R_1^3\right)}$ ...(1)

$\Rightarrow \quad E_r = \frac{G\left[\frac{4}{3}\pi\left(r^3 - R_1^3\right)\rho\right]}{r^2}$

From (1), we get

$E_r = \frac{GM}{r^2}\left(\frac{r^3 - R_1^3}{R_2^3 - R_1^3}\right)$

So, $E_r = \begin{cases} 0 & \text{for} \quad r < R_1 \\ \frac{GM}{r^2}\left(\frac{r^3 - R_1^3}{R_2^3 - R_1^3}\right) & \text{for} \quad R_1 < r < R_2 \\ \frac{GM}{r^2} & \text{for} \quad r > R_2 \end{cases}$

13. Reduced by 36% means the value is 64% the original

$\Rightarrow \quad g_h = \frac{64}{100}g$

Since $g_h = \frac{gR^2}{(R+h)^2}$

$\Rightarrow \quad \frac{64}{100} = \left(\frac{R}{R+h}\right)^2$

$\Rightarrow \quad \frac{R}{R+h} = \frac{8}{10}$

$\Rightarrow \quad 10R = 8R + 8h$

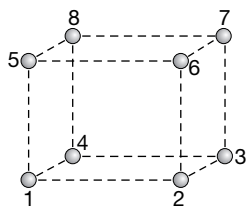
$\Rightarrow \quad h = \frac{R}{4} = 1600$ km

Test Your Concepts-II (Based on Gravitational Potential, Potential Energy and Applications)

1. First of all let us calculate the total number of interactions between these eight particles. Since, total number of interactions (N) between n particles is

$N = {}^nC_2 = \frac{n(n-1)}{2}$

$\Rightarrow \quad N = \frac{8(8-1)}{2} = 28$



Out of these 28 interactions, 12 interactions are due to masses at separation a, 12 interactions are due to masses at separation $\sqrt{2}a$ (the face diagonal) and 4 interactions are due to masses at separation $\sqrt{3}a$ (the body diagonal). So,

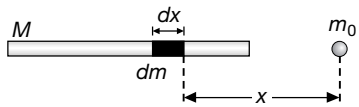
CHAPTER 4

$$U = -12\left(\frac{Gm^2}{a}\right) - 12\left(\frac{Gm^2}{\sqrt{2}a}\right) - 4\left(\frac{Gm^2}{\sqrt{3}a}\right)$$

$$\Rightarrow \quad U = -\frac{Gm^2}{a}\left(12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}}\right)$$

2. Consider an infinitesimal element of the rod of length dx at a distance x from m_0. If dm be the mass of the infinitesimal element, then

$$dm = \frac{M}{L}dx$$



The gravitational potential energy of the infinitesimal mass dm and the point mass m_0 is

$$dU = -\frac{Gm_0 dm}{x} = -\frac{Gm_0\left(\frac{M}{L}dx\right)}{x}$$

$$\Rightarrow \quad U = \int dU = -\frac{GMm_0}{L}\int_a^{a+L}\frac{dx}{x}$$

$$\Rightarrow \quad U = -\frac{GMm_0}{L}\log_e\left(\frac{a+L}{a}\right)$$

3. $U = -\dfrac{GMm}{R}$

So, the binding energy is $|U| = \dfrac{GMm}{R}$

i.e., this much energy is required to displace the particle from the centre of the ring to infinity.

4. Outside the sphere, we have

$$U = -\frac{GMm}{r}$$

as if the sphere were a point mass concentrated at its centre.

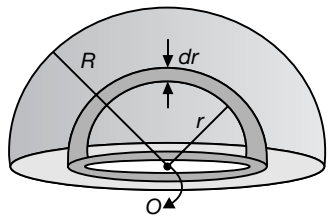
Even if the sphere is replaced by a thin shell, the gravitational potential energy of the particle-shell system will remain the same, because for the shell to the entire mass is concentrated at the centre, so

$$U = -\frac{GMm}{r}$$

5. Consider an elementary hemispherical shell of radius r and thickness dr. If dm is mass of this shell, then

$$dm = \frac{M}{\frac{1}{2}\left(\frac{4}{3}\pi R^3\right)}\frac{1}{2}4\pi r^2 dr$$

$$\Rightarrow \quad dm = \frac{3M}{R^3}r^2 dr$$



Initial potential energy at O due to this element and m is

$$U_i = \int dU_i = -\int\frac{G\,m\,dm}{r} = -\frac{3GMm}{R^3}\int_0^R r\,dr$$

$$\Rightarrow \quad U_i = -\frac{3}{2}\frac{GMm}{R}$$

So, $W_{\text{external}} = U_f - U_i = 0 - \left(-\dfrac{3}{2}\dfrac{GMm}{R}\right) = \dfrac{3}{2}\dfrac{GMm}{R}$

So, the work performed in the process by gravitational force is $\left(-\dfrac{3}{2}\dfrac{GMm}{R}\right)$

6. Let gravitational field be zero at a point lying at distance x from M. Then,

$$\frac{GM}{x^2} = \frac{Gm}{(d-x)^2}$$

$$\Rightarrow \quad \frac{d-x}{x} = \sqrt{\frac{m}{M}}$$

$$\Rightarrow \quad \frac{d}{x} - 1 = \sqrt{\frac{m}{M}}$$

$$\Rightarrow \quad x = \left(\frac{\sqrt{M}}{\sqrt{M}+\sqrt{m}}\right)d \qquad \ldots(1)$$

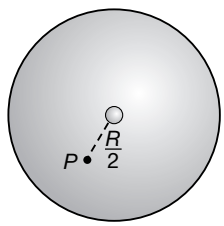
$$\Rightarrow \quad (d-x) = \left(\frac{\sqrt{m}}{\sqrt{M}+\sqrt{m}}\right)d \qquad \ldots(2)$$

Since, $V_P = -\dfrac{Gm}{d-x} - \dfrac{GM}{x}$ $\qquad \ldots(3)$

Substituting (1) and (2) in (3), we get

$$V_P = -\frac{G}{d}\left(\sqrt{m}+\sqrt{M}\right)^2$$

7. $V_P = \left(\begin{matrix}\text{Potential due to}\\ \text{shell at P}\end{matrix}\right) + \left(\begin{matrix}\text{Potential due to}\\ \text{particle at P}\end{matrix}\right)$



Since point P lies inside the shell, so the potential inside the shell is constant and equals the value at the surface i.e., $-\frac{GM}{R}$.

$$\Rightarrow \quad V_P = -\frac{GM}{R} - \frac{GM}{(R/2)}$$

$$\Rightarrow \quad V_P = -\frac{3M}{R}$$

8. (a) Since, $V(m) = -\frac{Gm}{R}$

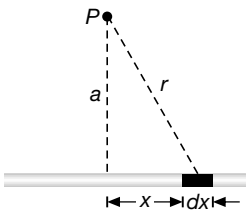
$$\Rightarrow \quad dW = V(dm) = -\left(\frac{Gm}{R}\right)dm$$

$$\Rightarrow \quad W = \int_0^M dW = -\frac{GM^2}{2R} = \text{self energy}$$

(b) See theory.

9. (a) Consider a mass element of length dx, mass dm at a distance x from the centre of rod. Then

$dm = \lambda dx$



The potential at the point P due to this infinitesimal element is

$$dV = -\frac{Gdm}{r} = -\frac{Gdm}{\sqrt{a^2 + x^2}}$$

$$\Rightarrow \quad dV = -\frac{G\lambda dx}{\sqrt{a^2 + x^2}}$$

$$\Rightarrow \quad V = \int dV = -G\lambda \int_{-\ell}^{\ell} \frac{dx}{\sqrt{a^2 + x^2}}$$

Since $\int_{-\ell}^{\ell} f(x)dx = 2\int_0^{\ell} f(x)dx$, iff $f(x)$ is even

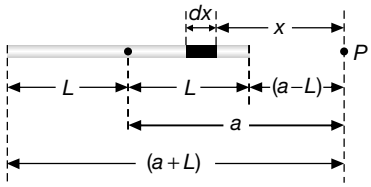
$$\Rightarrow \quad V = -2G\lambda \int_0^{\ell} \frac{dx}{\sqrt{a^2 + x^2}}$$

Also, $\int \frac{dx}{\sqrt{a^2 + x^2}} = \log_e\left(x + \sqrt{a^2 + x^2}\right)$

$$V = -2G\lambda\left[\log_e\left(x + \sqrt{x^2 + a^2}\right)\Big|_0^{\ell}\right]$$

$$\Rightarrow \quad V = -2G\lambda \log_e\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right)$$

(b) Again consider an infinitesimal element of length dx, mass dm at a distance x from P. Then $dm = \lambda dx$



If dV be the gravitational potential due to this element at the point P, then

$$dV = -\frac{Gdm}{x}$$

$$\Rightarrow \quad dV = -\frac{G(\lambda dx)}{x}$$

$$\Rightarrow \quad V = -G\lambda \int_{a-L}^{a+L} \frac{dx}{x}$$

$$\Rightarrow \quad V = -G\lambda\left(\log_e x\Big|_{a-L}^{a+L}\right)$$

$$\Rightarrow \quad V = -G\lambda \log_e\left(\frac{a+L}{a-L}\right)$$

Test Your Concepts-III (Based on Relation between Gravitational Field and Potential)

1. Since, $dV = -\vec{E}\cdot d\vec{\ell}$

$$\Rightarrow \quad \int_B^A dV = -\int_{(0,2,4)}^{(2,1,0)} \left(x\hat{i} - 2y\hat{j} + z\hat{k}\right)\cdot\left(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right)$$

$$\Rightarrow \quad V_A - V_B = -\int_{(0,2,4)}^{(2,1,0)} \left(xdx - 2ydy + zdz\right)$$

$$\Rightarrow \quad V_{AB} = -\left[\left(\frac{x^2}{2} - y^2 + \frac{z^2}{2}\right)\Big|_{(0,2,4)}^{(2,1,0)}\right]$$

$$\Rightarrow \quad V_{AB} = 3 \text{ Jkg}^{-1}$$

2. Since, $dV = -\vec{E}\cdot d\vec{\ell}$

$$\Rightarrow \quad dV = -a\left(ydx + axdy\right) + b\left(zdy + ydz\right)$$

$$\Rightarrow \quad dV = -\left[ad\left(xy\right) + bd\left(yz\right)\right]$$

Integrating we get,

$$V = -\left(axy + byz\right) + C$$

CHAPTER 4

3. (a) $dV = -\vec{E}\cdot d\vec{\ell}$

$$\int_B^A dV = -\int_{(1,1,1)}^{(0,0,0)} \left(y\hat{i}+x\hat{j}\right)\cdot\left(dx\,\hat{i}+dy\,\hat{j}+dz\,\hat{k}\right)$$

$$\Rightarrow\quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} \left(ydx+xdy\right)$$

$$\Rightarrow\quad V_{AB} = -\int_{(1,1,1)}^{(0,0,0)} d\left(xy\right)\quad \left\{\text{as } y\,dx+xdy = d\left(xy\right)\right\}$$

$$\Rightarrow\quad V_{AB} = -\left[\left(xy\right)\Big|_{(1,1,1)}^{(0,0,0)}\right] = 1\ \text{Jkg}^{-1}$$

(b) $dV = -\vec{E}\cdot d\vec{\ell}$

$$\Rightarrow\quad \int_B^A dV = -\int_{(1,1,1)}^{(0,0,0)} \vec{E}\cdot d\vec{\ell}$$

$$\Rightarrow\quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} \left(3x^2\,ydx+x^3\,dy\right)$$

$$\Rightarrow\quad V_A - V_B = -\int_{(1,1,1)}^{(0,0,0)} d\left(x^3y\right)$$

$$\Rightarrow\quad V_{AB} = -\left[\left(x^3y\right)\Big|_{(1,1,1)}^{(0,0,0)}\right] = 1\ \text{Jkg}^{-1}$$

Since in both the cases, the line integral of the field is an exact differential and hence both the fields are conservative in nature.

4. Since $\vec{E} = -\left(\hat{i}\dfrac{\partial V}{\partial x}+\hat{j}\dfrac{\partial V}{\partial y}+\hat{k}\dfrac{\partial V}{\partial z}\right)$

where $\dfrac{\partial V}{\partial x} = 6xy$

$$\frac{\partial V}{\partial y} = 3x^2+3y^2z$$

$$\frac{\partial V}{\partial z} = y^3$$

$$\Rightarrow\quad \vec{E} = -\left(6xy\right)\hat{i}-3\left(x^2+y^2z\right)\hat{j}-y^3\hat{k}$$

5. (a) Given, $V = a\left(x^2-y^2\right)$

So, $\vec{E} = -\vec{\nabla}V = -2a\left(x\hat{i}-y\hat{j}\right)$

(b) Since $V = axy$

So, $\vec{E} = -\vec{\nabla}V = -ay\hat{i}-ax\hat{j}$

6. $V = 5x-3x^2y+2yz^2$

Since $\vec{E} = -\vec{\nabla}V = -\left(\dfrac{\partial V}{\partial x}\hat{i}+\dfrac{\partial V}{\partial y}\hat{j}+\dfrac{\partial V}{\partial z}\hat{k}\right)$

$$\Rightarrow\quad \vec{E} = \left(-5+6xy\right)\hat{i}+\left(3x^2-2z^2\right)\hat{j}-4yz\,\hat{k}$$

$$\Rightarrow\quad \vec{E}\Big|_{(1,\,0,\,-2)} = -5\hat{i}-5\hat{j}$$

$$\Rightarrow\quad E = \sqrt{E_x^2+E_y^2+E_z^2}$$

$$E = \sqrt{(-5)^2+(-5)^2+0^2} = 5\sqrt{2}\ \text{Nkg}^{-1}$$

7. Since, $\vec{E}_g = -\left(\hat{i}\dfrac{\partial V}{\partial x}+\hat{j}\dfrac{\partial V}{\partial y}\right)$

where $\dfrac{\partial V}{\partial x} = 20$

and $\dfrac{\partial V}{\partial y} = 20$

$$\Rightarrow\quad \vec{E}_g = -20\left(\hat{i}+\hat{j}\right)$$

Since, $\vec{F} = m\vec{E}_g = -(0.5)(20)\left(\hat{i}+\hat{j}\right)$

$$\Rightarrow\quad \vec{F} = -10\left(\hat{i}+\hat{j}\right)$$

$$\Rightarrow\quad |\vec{F}| = 10\sqrt{2}\ \text{N}$$

8. Given, $V = a\left(x^2+y^2\right)+bz^2$

Since, $\vec{E} = -\vec{\nabla}V$

$$\Rightarrow\quad \vec{E} = -\left(2ax\hat{i}+2ay\hat{j}+2bz\hat{k}\right)$$

Hence $|\vec{E}| = 2\sqrt{a^2\left(x^2+y^2\right)+b^2z^2}$

9. $\vec{E} = -\left(\dfrac{\partial V}{\partial x}\hat{i}+\dfrac{\partial V}{\partial y}\hat{j}+\dfrac{\partial V}{\partial z}\hat{k}\right)$

where, $\dfrac{\partial V}{\partial x} = \dfrac{\partial}{\partial x}\left(2x+3y-z\right) = 2$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}\left(2x+3y-z\right) = 3$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}\left(2x+3y-z\right) = -1$$

$$\Rightarrow\quad \vec{E} = -2\hat{i}-3\hat{j}+\hat{k}$$

Test Your Concepts-IV (Based on Conservation Laws, Escape Velocity and Applications)

1. (a) Increase in potential energy is $\Delta U = U_f - U_i$

$$\Rightarrow \Delta U = -\frac{GmM}{R+nR} - \left(-\frac{GmM}{R}\right)$$

$$\Rightarrow \Delta U = \frac{GmM}{R}\left(1 - \frac{1}{1+n}\right)$$

$$\Rightarrow \Delta U = \left(\frac{n}{n+1}\right)\left(\frac{GM}{R^2}\right)mR$$

Since $g = \frac{GM}{R^2}$

$$\Rightarrow \Delta U = \left(\frac{n}{n+1}\right)mgR$$

(b) By Law of Conservation of Energy,

$$\begin{pmatrix}\text{Gain in GPE}\\ \text{of } m\end{pmatrix} = \begin{pmatrix}\text{Loss in KE}\\ \text{of } m\end{pmatrix}$$

$$\Rightarrow \left(\frac{n}{n+1}\right)mgR = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{2ngR}{n+1}}$$

2. $(U+K)_{\text{at }\infty} = (U+K)_{\text{surface}}$

$$0 + 0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow v = \sqrt{2\left(\frac{GM}{R^2}\right)R}$$

$$\Rightarrow v = \sqrt{2gR} = 11.2 \text{ kms}^{-1}$$

3. By Law of Conservation of Energy, we have

$$(U+K)_{\text{at 1 m}} = (U+K)_{\text{at 0.5 m}}$$

$$-\frac{Gm_1m_2}{r_1} = -\frac{Gm_1m_2}{r_2} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad \ldots(1)$$

Further by Law of Conservation of Linear Momentum, we have

$$m_1(0) + m_2(0) = m_1v_1 + m_2(-v_2)$$

$$\Rightarrow m_1v_1 = m_2v_2 \qquad \ldots(2)$$

$$\Rightarrow Gm_1m_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2\left(\frac{m_1v_1}{m_2}\right)^2$$

$$\Rightarrow Gm_1m_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = \frac{1}{2}m_1v_1^2\left(1 + \frac{m_1}{m_2}\right)$$

$$\Rightarrow v_1 = \sqrt{\frac{2Gm_2^2}{m_1+m_2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}$$

$$\Rightarrow v_1 = \sqrt{\frac{2G(100)}{30}\left(\frac{1}{0.5} - \frac{1}{1}\right)}$$

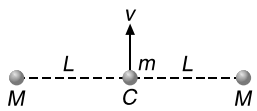
$$\Rightarrow v_1 = \sqrt{\frac{20G}{3}}$$

$$\Rightarrow v_1 = 2.1\times10^{-5} \text{ ms}^{-1}$$

From (2), we get

$$v_2 = 4.2\times10^{-5} \text{ ms}^{-1}$$

4. Let v is the minimum velocity, then by Law of Conservation of Energy, we have



$$(U+K)_C = (U+K)_\infty$$

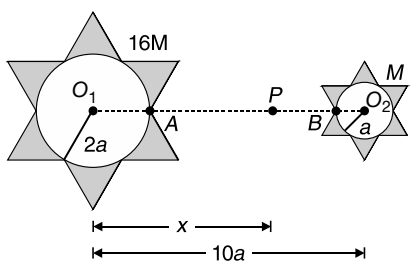
$$\Rightarrow \left(-\frac{GMm}{L}\right)2 + \frac{1}{2}mv^2 = 0 + 0$$

$$\Rightarrow v = 2\sqrt{\frac{GM}{L}}$$

5. Our first job is to find a point where the resultant field due to both is zero. Let the point P be at a distance x from centre of bigger star.

$$\Rightarrow \frac{G(16M)}{x^2} = \frac{GM}{(10a-x)^2}$$

$$\Rightarrow x = 8a \qquad \{\text{from } O_1\}$$



i.e. once the body reaches P the gravitational pull of attraction due to $16M$ vanishes and the gravitation pull due to M takes the lead to make m move towards it automatically.

i.e. a minimum K.E. or velocity has to be imparted to m from surface of $16M$ such that it is just able to overcome the gravitational pull of $16M$.

By Law of Conservation of Energy.

$$\begin{pmatrix}\text{Total Mechanical}\\ \text{Energy at } A\end{pmatrix} = \begin{pmatrix}\text{Total Mechanical}\\ \text{Energy at } P\end{pmatrix}$$

Total mechanical energy at A is

$$E_A = \frac{1}{2}mv_{\min}^2 + \left[-\frac{G(16M)m}{2a} - \frac{GMm}{8a}\right]$$

$$\Rightarrow \quad E_A = \frac{1}{2}mv_{\min}^2 - \frac{GMm}{2a}\left(16 + \frac{1}{4}\right)$$

$$\Rightarrow \quad E_A = \frac{1}{2}mv_{\min}^2 - \frac{65GMm}{8a}$$

Total mechanical energy at B is

$$E_B = \left(-\frac{GMm}{2a} - \frac{G(16M)m}{8a}\right)$$

$$\Rightarrow \quad E_B = -\frac{GMm}{2a}(1+4) = -\frac{5GMm}{2a}$$

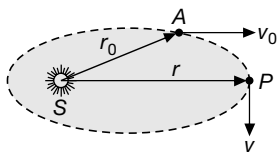
$$\Rightarrow \quad \frac{1}{2}mv_{\min}^2 = \frac{GMm}{8a}(45)$$

$$\Rightarrow \quad v_{\min} = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

6. At minimum and maximum distances, the velocity vector $(\vec{v})$ makes an angle of $90°$ with radius vector. Then, by Law of Conservation of Angular Momentum, we have

$$mv_0 r_0 \sin\phi = mvr \qquad \ldots(1)$$

where, m is the mass of the planet.



Further by Law of Conservation of Energy, we have

$$(U+K)_{\text{at } A} = (U+K)_P$$

$$\Rightarrow \quad \frac{mv_0^2}{2} - \frac{GMm}{r_0} = \frac{mv^2}{2} - \frac{GMm}{r} \qquad \ldots(2)$$

where, M is the mass of the sun.

Solving equations (1) and (2) for r using the concept of quadratic equations we get two values of r, one is $r_{\max}$ and another is $r_{\min}$. So,

$$r_{\max} = \frac{r_0}{2-K}\left(1 + \sqrt{1-K(2-K)\sin^2\phi}\right)$$

and $r_{\min} = \frac{r_0}{2-K}\left(1 - \sqrt{1-K(2-K)\sin^2\phi}\right)$

where, $K = \frac{v_0^2 r_0^2}{GM}$

7. $E = U + K$

$$\Rightarrow \quad E = -\frac{GMm}{R} + \frac{1}{2}mv_e^2$$

But $v_e^2 = \frac{2GM}{R}$

$$\Rightarrow \quad E = -\frac{GMm}{R} + \frac{1}{2}m\left(\frac{2GM}{R}\right)$$

$$\Rightarrow \quad E = 0$$

8. Given that $v_e = 11.2 \text{ kms}^{-1} = \sqrt{\frac{2GM_e}{R_e}}$

By Law of Conservation of Energy, we have

$$K_i + U_i = K_f + U_f$$

$$\Rightarrow \quad \frac{1}{2}mv_s^2 - \frac{GM_s m}{r} - \frac{GM_e m}{R_e} = 0 + 0$$

where, r is the distance of rocket from sun

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2GM_s}{r}}$$

Since, $M_s = 3\times10^5 M_e$ and $r = 2.5\times10^4 R_e$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2G\left(3\times10^5 M_e\right)}{2.5\times10^4 R_e}}$$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e}\left(1 + \frac{3\times10^5}{2.5\times10^4}\right)}$$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e}\times 13}$$

$$\Rightarrow \quad v_s \approx 42 \text{ kms}^{-1}$$

9. Since the gravitational potential energy between the disc of radius a, mass m_1 and a particle of mass m_2 placed at axis of disc at a distance ℓ from centre is

$$U_{\text{axis}} = -\frac{2Gm_1m_2}{a^2}\left(\sqrt{a^2+\ell^2} - \ell\right) \qquad \ldots(1)$$

$$\Rightarrow \quad U_{\text{axis}} = -\frac{2Gm_1m_2\ell}{a^2}\left[\left(1+\frac{a^2}{\ell^2}\right)^{\frac{1}{2}} - 1\right]$$

Since $\frac{a}{\ell} \quad 1$, $\left(1+\frac{a^2}{\ell^2}\right)^{1/2} \quad 1+\frac{a^2}{2\ell^2}$

$$\Rightarrow \quad U_{\text{axis}} = -\left(\frac{2Gm_1m_2\ell}{a^2}\right)\left(\frac{a^2}{2\ell^2}\right)$$

$$\Rightarrow \quad U_{\text{axis}} = -\frac{Gm_1m_2}{\ell}$$

Further, when the particle collides with the disc (at its centre i.e., $\ell = 0$), then

$$U_{\text{centre}} = -\frac{2Gm_1m_2}{a} \qquad \{\because \text{ of } (1)\}$$

Applying Law of Conservation of Energy and using the concept of reduced mass we get

$$(U+K)_{\text{axis}} = (U+K)_{\text{centre}}$$

$$\Rightarrow \quad -\frac{Gm_1m_2}{\ell} + 0 = -2\left(\frac{Gm_1m_2}{a}\right) + \frac{1}{2}\mu v_r^2$$

where $\mu = \dfrac{m_1m_2}{m_1+m_2}$

$$\Rightarrow \quad Gm_1m_2\left(\frac{2}{a}-\frac{1}{\ell}\right) = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)v_r^2$$

$$\Rightarrow \quad v_r = \sqrt{2G(m_1+m_2)\left(\frac{2}{a}-\frac{1}{\ell}\right)}$$

10. $v_{es} = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2G\left(\frac{4}{3}\pi R^3\right)\rho}{R}} = \sqrt{\dfrac{4G\rho}{3}}R$

$v_{es} \propto R$

Surface area of 1 is $A = 4\pi R_1^2$

Surface area of 2 is $4A = 4\pi R_2^2$

$\Rightarrow \quad R_2 = 2R_1$

Given that mass of planet 3 is $M_3 = M_1 + M_2$

$$\Rightarrow \quad \left(\frac{4}{3}\pi R_3^3\right)\rho = \left(\frac{4}{3}\pi R_1^3\right)\rho + \left(\frac{4}{3}\pi R_2^3\right)\rho$$

$$\Rightarrow \quad R_3^3 = R_1^3 + R_2^3$$

$$\Rightarrow \quad R_3^3 = 9R_1^3$$

$$\Rightarrow \quad R_3 = 9^{1/3}R_1$$

$$\Rightarrow \quad R_3 > R_2 > R_1$$

$$\Rightarrow \quad v_3 > v_2 > v_1$$

$$\Rightarrow \quad \frac{v_3}{v_1} = 9^{1/3} \text{ and } \frac{v_2}{v_1} = 2$$

$$\Rightarrow \quad \frac{v_3^3}{v_1^3} = 9 \text{ and } \frac{v_2}{v_1} = 2$$

Test Your Concepts-V (Based on Satellites, Kepler's Laws and Applications)

1. $r_A = R + R = 2R$

and $r_B = R + 3R = 4R$

Since, kinetic energy $K = \dfrac{GMm}{2r}$

$$\Rightarrow \quad K \propto \frac{1}{r}$$

$$\Rightarrow \quad \frac{K_A}{K_B} = \frac{r_B}{r_A} = \frac{4R}{2R} = 2$$

Also, $U = -\dfrac{GMm}{r}$

$$\Rightarrow \quad |U| \propto \frac{1}{r}$$

$$\Rightarrow \quad \frac{U_A}{U_B} = \frac{r_B}{r_A} = 2$$

2. (a) Just before the explosion, the orbital velocity is $v_0 = \sqrt{\dfrac{GM}{R}}$ corresponding to which the total energy associated with the satellite in the orbit is NEGATIVE.

When a mass Δm is expelled very rapidly with a speed v, then for the satellite to still remain within the gravitational pull of the planet, the new total energy must still be negative. So

$$\frac{1}{2}(m-\Delta m)(v_0+v_r)^2 - \frac{GM(m-\Delta m)}{R} \le 0 \qquad \ldots(1)$$

$$\Rightarrow \quad (v_0+v_r) \le \frac{2GM}{R}$$

$$\Rightarrow \quad v_0 + v_r \le \sqrt{\frac{2GM}{R}}$$

$$\Rightarrow \quad \sqrt{\frac{GM}{R}} + v_r \le \sqrt{\frac{2GM}{R}} \qquad \left\{\because v_0 = \sqrt{\frac{GM}{R}}\right\}$$

$$\Rightarrow \quad v_r \le (\sqrt{2}-1)\sqrt{\frac{GM}{R}}$$

(b) Since the term $(m-\Delta m)$ cancels (see equation (1)), so, no modification is needed.

3. Since $T^2 = 4\pi^2\left(\dfrac{r^3}{GM}\right)$

When the satellite is revolving close to the planet, then $r \cong R$

$$\Rightarrow \quad T^2 = 4\pi^2\left(\frac{R^3}{GM}\right)$$

Since $M = \left(\frac{4}{3}\pi R^3\right)\rho$

$$\Rightarrow \quad T^2 = 4\pi^2\frac{R^3}{G\left(\frac{4}{3}\pi R^3\right)\rho}$$

$$\Rightarrow \quad \rho T^2 = \text{constant}$$

4. For the satellite to revolve in a circular orbit of radius r_0 the orbital velocity is given by

$$v_0 = \sqrt{\frac{GM}{r_0}} \qquad \ldots(1)$$

At maximum or minimum distances, velocity is perpendicular to the radius vector. So, on applying Conservation of Angular Momentum and Mechanical Energy, we get

CHAPTER 4

$$mv_0r_0\cos\alpha = mvr \qquad \ldots(2)$$

and $$-\frac{GMm}{2r_0} = -\frac{GMm}{r} + \frac{1}{2}mv^2 \qquad \ldots(3)$$

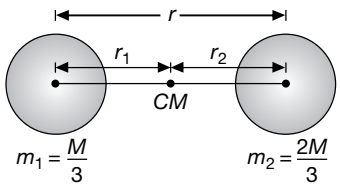
Solving these three equations, we get two values of r i.e., $(1+\sin\alpha)r_0$ and $(1-\sin\alpha)r_0$. Therefore,

$$r_{max} = (1+\sin\alpha)r_0 \text{ and } r_{min} = (1-\sin\alpha)r_0$$

5. Let M be the mass of the sun and r be the distance between the two stars, then

$$r_1 = \frac{m_2}{m_1+m_2}r = \frac{2}{3}r$$

and $$r_2 = \frac{m_1}{m_1+m_2}r = \frac{r}{3}$$



Centripetal force on m_2 is $\frac{Gm_1m_2}{r^2} = \frac{G\left(\frac{2}{9}\right)M^2}{r^2}$

$$\Rightarrow \frac{2}{9}\frac{GM^2}{r^2} = m_2r_2\omega^2 = \left(\frac{2}{3}M\right)\left(\frac{r}{3}\right)\omega^2$$

$$\Rightarrow \omega^2 = \frac{GM}{r^3}$$

$$\Rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r^3}$$

$$\Rightarrow T^2 = \frac{4\pi^2r^3}{GM}$$

Since time period of earth around sun is

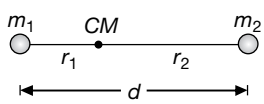
$$T^2 = \frac{4\pi^2R^3}{GM}$$

$$\Rightarrow r = R$$

6. (a) Let the origin be at m_1 and the centre of mass be at a distance r_1 from it. Then

$$r_1 = \frac{m_1(0)+m_2r}{m_1+m_2}$$

$$\Rightarrow r_1 = \left(\frac{m_2}{m_1+m_2}\right)r \text{ and}$$



The centripetal force is provided by gravitational force, so

$$m_1r_1\omega^2 = m_2r_2\omega^2 = \frac{Gm_1m_2}{d^2} \qquad \ldots(1)$$

Solving these equations, we get

$$\omega = \sqrt{\frac{G(m_1+m_2)}{d^3}}$$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{d^3}{G(m_1+m_2)}}$$

(b) $$\frac{K_1}{K_2} = \frac{\frac{1}{2}I_1\omega^2}{\frac{1}{2}I_2\omega^2} = \frac{I_1}{I_2} = \frac{m_1r_1^2}{m_2r_2^2}$$

$$\Rightarrow \frac{K_1}{K_2} = \left(\frac{m_1}{m_2}\right)\left(\frac{r_1}{r_2}\right)^2$$

Since, $m_1r_1\omega^2 = m_2r_2\omega^2$

$$\Rightarrow m_1r_1 = m_2r_2$$

$$\Rightarrow \frac{K_1}{K_2} = \left(\frac{m_1}{m_2}\right)\left(\frac{m_2}{m_1}\right)^2 = \frac{m_2}{m_1}$$

(c) $$\frac{L_1}{L_2} = \frac{I_1\omega}{I_2\omega} = \frac{I_1}{I_2} = \frac{m_2}{m_1}$$

(d) $$L = L_1 + L_2 = (I_1+I_2)\omega$$

$$\Rightarrow L = (m_1r_1^2 + m_2r_2^2)\omega$$

$$\Rightarrow L = \left[\frac{m_1m_2^2d^2}{(m_1+m_2)^2} + \frac{m_2m_1^2d^2}{(m_1+m_2)^2}\right]\omega$$

$$\Rightarrow L = \mu\omega d^2$$

where $\mu = \frac{m_1m_2}{m_1+m_2}$ is the reduced mass

(e) $$K = \frac{1}{2}(I_1+I_2)\omega^2 = \frac{1}{2}\mu\omega^2d^2$$

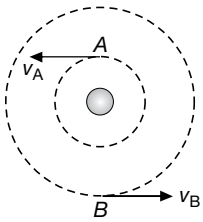
7. For two satellites revolving around same planet, we have

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\Rightarrow T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = 28\left(\frac{2\times10^4}{10^4}\right)^{3/2} = 56\sqrt{2} \text{ hour}$$

Since, $\omega = \frac{2\pi}{T}$

$$\Rightarrow \omega_1 = \frac{2\pi}{28} \text{ radhr}^{-1} \text{ and } \omega_2 = \frac{2\pi}{56\sqrt{2}} \text{ radhr}^{-1}$$



Positions corresponding to maximum separation are shown in figure, so we have

$$v_{BA} = \vec{v}_B - \vec{v}_A = |v_B| + |v_A| \quad \{\text{for shown position}\}$$

$$\Rightarrow \quad v_{BA} = \omega_2 r_2 + \omega_1 r_1$$

$$\Rightarrow \quad v_{BA} = \left[\frac{2\pi}{56\sqrt{2}} \times 2 \times 10^4 + \frac{2\pi}{28} \times 10^4\right] \text{ kmhr}^{-1}$$

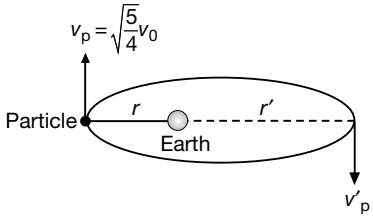
$$\Rightarrow \quad v_{BA} = (0.1587 + 0.2242) \times 10^4 \text{ kmhr}^{-1}$$

$$\Rightarrow \quad v_{BA} = 3829 \text{ kmhr}^{-1}$$

8. Orbital speed of the satellite is $v_0 = \sqrt{\frac{GM}{r}}$, where M is the mass of earth ...(1)

Absolute velocity of particle would be:

$$v_p = v + v_0 = \sqrt{\frac{5}{4}} v_0 = \sqrt{1.25} v_0 \qquad ...(2)$$



Since, v_p lies between orbital velocity and escape velocity, path of the particle would be an ellipse with r being the minimum distance.

Let r' be the maximum distance and v'_p be the velocity at that moment.

Then applying the Law of Conservation of Angular Momentum and Conservation of Mechanical Energy, we get

$$mv_p r = mv'_p r' \qquad ...(3)$$

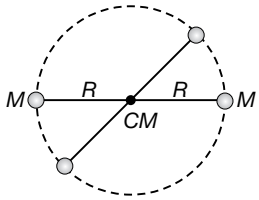
and $$\frac{1}{2}mv_p^2 - \frac{GMm}{r} = \frac{1}{2}mv_p'^2 - \frac{GMm}{r'} \qquad ...(4)$$

Solving the above equations (1), (2), (3) and (4), we get

$$r' = \frac{5r}{3} \text{ and } r' = r$$

Hence, the maximum and minimum distances are $\frac{5r}{3}$ and r respectively.

9. (a) $F = \frac{G(M)(M)}{(2R)^2} = \frac{GM^2}{4R^2}$



(b) $F = MR\omega^2$

$$\Rightarrow \quad \frac{GM^2}{4R^2} = MR\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{F}{MR}} = \sqrt{\frac{GM}{4R^3}}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{4R^3}{GM}} = 4\pi\sqrt{\frac{R^3}{GM}}$$

(c) $$E = -\frac{G(M)(M)}{2R} + 2\left(\frac{1}{2}I\omega^2\right) = -\frac{GM^2}{2R} + I\omega^2$$

$$\Rightarrow \quad E = -\frac{GM^2}{2R} + (MR^2)\left(\frac{GM}{4R^3}\right) = -\frac{GM^2}{4R}$$

So, binding energy is $|E| = \frac{GM^2}{4R}$

Hence, this much amount of energy will be required to separate the two stars to infinity.

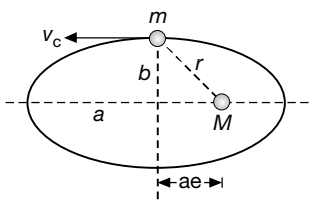
10. Since, we observe that

$$r^2 = a^2e^2 + b^2$$

But $b^2 = a^2(1 - e^2)$

$$\Rightarrow \quad r^2 = a^2e^2 + a^2 - a^2e^2$$

$$\Rightarrow \quad r = a$$



Total energy of a satellite in an elliptical orbit is,

$$E = -\frac{GMm}{2a}$$

$$\Rightarrow \quad E_C = \frac{1}{2}mv_c^2 - \frac{GMm}{r} = -\frac{GMm}{2a}$$

$$\Rightarrow \quad \frac{1}{2}mv_c^2 - \frac{GMm}{a} = -\frac{GMm}{2a} \qquad \{r = a\}$$

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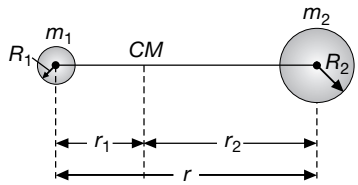
$$\Rightarrow \quad v_c = \sqrt{\frac{GM}{a}}$$

$$\Rightarrow \quad v_c = R\sqrt{\frac{g}{a}} \qquad \{\text{as } GM = gR^2\}$$

11. Both the stars rotate about their centre of mass.



For the position of *CM* let Origin be at m_1 and *CM* be at distance r_1 from origin. Then

$$r_1 = \frac{m_2 r}{m_1 + m_2} \text{ and } r_2 = \frac{m_1 r}{m_1 + m_2}$$

Also, $m_1 r_1 \omega^2 = \frac{G m_1 m_2}{r^2}$ $\qquad \left\{\because \omega = \frac{2\pi}{T}\right\}$

Since, $r_1 = \frac{m_2 r}{m_1 + m_2}$

$$\Rightarrow \quad \omega^2 = \frac{G(m_1 + m_2)}{r^3}$$

$$\Rightarrow \quad r = \left[\frac{G(m_1 + m_2)}{\omega^2}\right]^{1/3} \qquad \ldots(1)$$

Applying Law of Conservation of Mechanical Energy, we get

$$-\frac{G m_1 m_2}{r} = -\frac{G m_1 m_2}{(R_1 + R_2)} + \frac{1}{2}\mu v_r^2 \qquad \ldots(2)$$

where, μ is the reduced mass given by $\mu = \frac{m_1 m_2}{m_1 + m_2}$

and v_r is the relative velocity between the two stars From equation (2), we get

$$v_r^2 = \frac{2G m_1 m_2}{\mu}\left(\frac{1}{R_1 + R_2} - \frac{1}{r}\right)$$

$$\Rightarrow \quad v_r^2 = \frac{2G m_1 m_2}{\left(\frac{m_1 m_2}{m_1 + m_2}\right)}\left(\frac{1}{R_1 + R_2} - \frac{1}{r}\right)$$

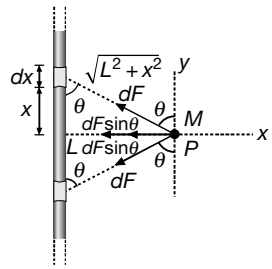
$$\Rightarrow \quad v_r^2 = 2G(m_1 + m_2)\left(\frac{1}{R_1 + R_2} - \frac{1}{r}\right)$$

Substituting the value of r from equation (1), we get

$$v_r = \sqrt{2G(m_1 + m_2)\left[\frac{1}{R_1 + R_2} - \left\{\frac{4\pi^2}{G(m_1 + m_2)T^2}\right\}^{1/3}\right]}$$

Single Correct Choice Type Questions

1.



Let the mass *M* be placed symmetrically.

$$\Rightarrow \quad F_{\text{net}} = \int_{-\infty}^{\infty} dF \sin\theta = \int_{-\infty}^{\infty} \frac{GM(\lambda\, dx)}{x^2 + L^2}\frac{L}{\sqrt{x^2 + L^2}}$$

$$\Rightarrow \quad F_{\text{net}} = GM\lambda L \int_{-\infty}^{\infty} \frac{dx}{(x^2 + L^2)^{3/2}}$$

$$\Rightarrow \quad F_{\text{net}} = \frac{GM\lambda L}{L^2}(2)$$

$$\Rightarrow \quad F_{\text{net}} = \frac{2GM\lambda}{L}$$

Hence, the correct answer is (C).

2. Let M be the mass of the planet and m the mass of satellite. Then

$$mr\omega^2 = \frac{GMm}{r^2}$$

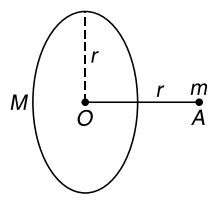
$$\Rightarrow \quad GM = r^2\omega^2$$

Now, $g = \frac{GM}{R^2}$

$$\Rightarrow \quad g = \frac{r^3\omega^2}{R^2}$$

Hence, the correct answer is (C).

3.



$$(U + K)_A = (U + K)_O$$

$$\Rightarrow \quad -\frac{GMm}{\sqrt{r^2 + r^2}} + 0 = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{GM}{r}\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{2GM}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$$

Hence, the correct answer is (D).

4. $$F = \int_{h}^{h+L} \frac{G\left(\frac{M}{L}dx\right)m}{x^2} = \frac{GMm}{L}\int_{h}^{h+L} x^{-2}dx$$

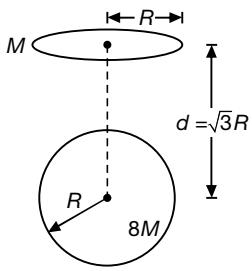
$$\Rightarrow \quad F = \frac{GMm}{L}\left(\frac{x^{-2+1}}{-2+1}\Big|_{h}^{h+L}\right)$$

$$\Rightarrow \quad F = -\frac{GMm}{L}\left(\frac{1}{h+L}-\frac{1}{h}\right)$$

$$\Rightarrow \quad F = \frac{GMm}{h(h+L)}$$

Hence, the correct answer is (C).

5. Gravitational field due to the ring at a distance $d = \sqrt{3}R$ on its axis is



$$E = \frac{GMd}{(R^2+d^2)^{3/2}} = \frac{\sqrt{3}GM}{8R^2}$$

Force on sphere is $F = (8M)E = \frac{\sqrt{3}GM^2}{R^2}$

Hence, the correct answer is (D).

6. By Law of Conservation of Energy, we get

$$(U+K)_{\text{surface}} = (U+K)_{\text{centre}}$$

Now, for a solid sphere, we have

$$U_{surface} = -\frac{GMm}{R} \text{ and } U_{\text{centre}} = -\frac{3}{2}\frac{GMm}{R}$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}m(0)^2 = -\frac{3}{2}\frac{GMm}{R}+\frac{1}{2}mv^2$$

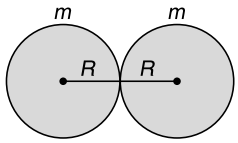
$$\Rightarrow \quad \frac{1}{2}mv^2 = -\frac{GMm}{R}-\left(-\frac{3}{2}\frac{GMm}{R}\right)$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \frac{v_e}{\sqrt{2}}$$

Hence, the correct answer is (D).

7.



Since $F = \frac{Gm^2}{(2R)^2}$

If ρ be the density of each sphere, then

$$m = \left(\frac{4}{3}\pi R^3\right)\rho$$

$$\Rightarrow \quad F = \frac{G\left(\frac{4}{3}\pi R^3\rho\right)^2}{4R^2}$$

$$\Rightarrow \quad F = \frac{4}{9}G\pi^2\rho^2R^4$$

$$\Rightarrow \quad F \propto R^4$$

Hence, the correct answer is (C).

8. Since, $g = \frac{GM}{R^2}$ ...(1)

Also, $M = \left(\frac{4}{3}\pi R^3\right)\rho$

$$\Rightarrow \quad R = \left(\frac{3M}{4\pi\rho}\right)^{1/3}$$

Substituting in (1) we get,

$$g = GM\left(\frac{4\pi\rho}{3M}\right)^{2/3}$$

$$\Rightarrow \quad g \propto M^{1/3}\rho^{2/3}$$

$$\Rightarrow \quad g_{\text{planet}} = g(8)^{1/3}(8)^{2/3} = 8g$$

Hence, the correct answer is (C).

9. $F_g \propto \frac{1}{r^2}$ and does not depend on the medium. Hence,

$$F_1' = \frac{F_1}{4}$$

$$F_c \propto \frac{1}{Kr^2}$$

and when air is sucked, force will slightly increase in vacuum, so

$$F_2' > \frac{F_2}{4}$$

Hence, the correct answer is (C).

10. $g\left(1-\frac{2h}{R}\right) \quad g\left(1-\frac{d}{R}\right)$

$$\Rightarrow \quad d = 2h$$

Hence, the correct answer is (B).

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11. Imagine an inverted hemispherical shell to be placed on this shell so as to complete the spherical shell inside which net gravitational field is zero. Net field can be zero only when field at P is directed along c.
Hence, the correct answer is (C).

12. $(U+K)_{\text{surface}} = (U+K)_{R+7R}$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mv^2 = -\frac{GMm}{R+7R}+0$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mv^2 = -\frac{GMm}{8R}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{GM}{R}-\frac{GM}{8R} \quad \Rightarrow \quad \frac{v^2}{2} = \frac{7}{8}\left(\frac{GM}{R}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{7GM}{4R}}$$

Hence, the correct answer is (D).

13. Since, $g_h = g\left(1-\frac{2h}{R}\right) \qquad \{\because h \quad R\}$

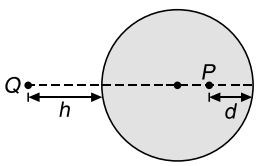
$$\Rightarrow \quad W_h = W\left(1-\frac{2h}{R}\right)$$

Rate of change of weight with height

$$\left|\frac{dW_h}{dh}\right| = \left|0-\frac{2W}{R}\right| = \frac{2mg}{R} \qquad \{\because W = mg\}$$

Hence, the correct answer is (B).

14.



Given that points P and Q located inside and outside the planet have same acceleration due to gravity equal to $\frac{g}{4}$. Thus

$$g\left(1-\frac{d}{R}\right) = \frac{gR^2}{(R+h)^2} = \frac{g}{4}$$

$$\Rightarrow \quad d = \frac{3R}{4} \text{ and } \frac{R}{R+h} = \frac{1}{2}$$

$$\Rightarrow \quad h = R$$

Maximum separation between P and Q will be when these are diametrically opposite. So, maximum separation is

$$r_{\max} = h+R+(R-d)$$

$$\Rightarrow \quad r_{\max} = R+R+\left(R-\frac{3R}{4}\right) = \frac{9R}{4}$$

Hence, the correct answer is (B).

15. $g' = g - R\omega^2\cos^2\phi$

At equator $\phi = 0°$

$$\Rightarrow \quad g' = g - R\omega^2$$

For g' to be zero

$$\omega = \sqrt{\frac{g}{R}} = \frac{1}{800} \text{ rads}^{-1}$$

Hence, the correct answer is (C).

16. $g = \frac{GM}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\rho\right)}{R^2} = \frac{4}{3}\pi G\rho R$

$$\Rightarrow \quad g \propto \rho R$$

R is increased by a factor of 2 i.e., to keep the value of g to be the same, the value of ρ has to be changed by a factor of $\frac{1}{2}$.

Hence, the correct answer is (C).

17. $E = \frac{GM}{R^3}r$

i.e., Slope, $\frac{dE}{dr} = \frac{GM}{R^3}$

but $M = \frac{4}{3}\pi R^3\rho$

$$\Rightarrow \quad \frac{M}{R^3} = \frac{4\pi\rho}{3} \quad (\rho = \text{density})$$

$$\Rightarrow \quad \text{Slope} = \frac{4\pi G\rho}{3} = \text{constant}$$

Hence, the correct answer is (A).

18. $\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{i}+\frac{\partial U}{\partial y}\hat{j}\right)$

$$\Rightarrow \quad \vec{F} = -a\hat{i}-b\hat{j}$$

$$\Rightarrow \quad |\vec{F}| = \sqrt{a^2+b^2}$$

$$\Rightarrow \quad |\text{Acc}| = \frac{|\vec{F}|}{m} = \frac{\sqrt{a^2+b^2}}{m}$$

Hence, the correct answer is (C).

19. Let R be the radius of earth and g the acceleration due to gravity on earth's surface. Then the desired ratio (say x) is

$$x = \frac{g\left(1-\frac{h}{R}\right)}{\frac{g}{\left(1+\frac{h}{R}\right)^2}} = \left(1-\frac{h}{R}\right)\left(1+\frac{h}{R}\right)^2$$

Since $h \quad R$, so $\left(1+\frac{h}{R}\right)^2 \cong 1+\frac{2h}{R}$

$\Rightarrow \quad x=\left(1-\frac{h}{R}\right)\left(1+\frac{2h}{R}\right)$

$\Rightarrow \quad x \approx 1+\frac{h}{R}$

From this expression we see that x increases linearly with h.

Hence, the correct answer is (C).

20. $E(r)=\left(\frac{GM}{R^3}\right)r$ where $M=\left(\frac{4}{3}\pi R^3\right)\rho$

$\Rightarrow \quad E(r)=\frac{Gr}{R^3}\left(\frac{4}{3}\pi R^3\rho\right)=\left(\frac{4}{3}\pi\rho G\right)r$

Hence, the correct answer is (A).

21. Energy required to raise a satellite to a height h is

$$\Delta E = U_f - U_i = -\frac{GMm}{R+h}-\left(-\frac{GMm}{R}\right)$$

$\Rightarrow \quad \Delta E = GMm\left(\frac{1}{R}-\frac{1}{R+h}\right)$

$\Rightarrow \quad \Delta E = \frac{GMmh}{R(R+h)}$

Further, if v_0 be the orbital velocity in the orbit, then

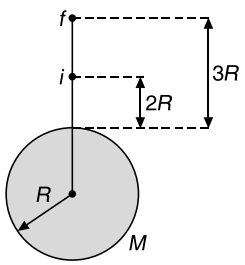
$$v_0 = \sqrt{\frac{GM}{R+h}}$$

$\Rightarrow \quad KE = \frac{1}{2}mv_0^2 = \frac{GMm}{2(R+h)}$

So, $\frac{\Delta E}{KE} = \frac{2h}{R}$

Hence, the correct answer is (C).

22.



Since $U = -\frac{GMm}{R+h}$

So, $U_i = -\frac{GMm}{2R+R} = -\frac{GMm}{3R}$

and $U_f = -\frac{GMm}{R+3R} = -\frac{GMm}{4R}$

Hence, external work done W is given by

$$W = \Delta U = U_f - U_i = \frac{GMm}{12R} = \frac{GM}{R^2}\times\frac{mR}{12}$$

$\Rightarrow \quad W = \frac{mgR}{12}$

Hence, the correct answer is (C).

23. Since, $F_1 = \frac{GMm}{9R^2}$

Also, $F_2 = \frac{GMm}{9R^2} - \frac{G(M/8)m}{(5R/2)^2}$

$\Rightarrow \quad F_2 = \frac{GMm}{9R^2} - \frac{GMm}{50R^2} = \frac{41}{450}\frac{GMm}{R^2}$

$\Rightarrow \quad \frac{F_2}{F_1} = \frac{41}{50}$

Hence, the correct answer is (B).

24. By the principle of superposition of fields

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

where, $\vec{E}$ is net field at the centre of hole due to entire mass, $\vec{E}_1$ is field due to remaining mass and $\vec{E}_2$ is field due to mass in hole = 0.

Since, $\vec{E}_1 = \vec{E} = \left(\frac{GM}{R^3}\right)r$ where $r = \frac{R}{2}$

$\Rightarrow \quad \vec{E} = \frac{GM}{2R^2}$

Hence, the correct answer is (C).

25. Initially field due to both is along positive x-axis. Due to the ring, field will first increase and then decrease to zero at centre. While field due to the solid sphere, will continuously increase in positive x-direction. On the other side of the ring field is now towards negative x-axis.

Hence, the correct answer is (B).

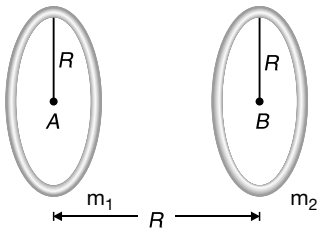
26. Potential due to particle at the surface is $V_1 = -\frac{GM}{R}$ and potential due to shell at its own surface is $V_2 = -\frac{G(3M)}{R}$

So, total potential is $V = V_1 + V_2 = -\frac{4GM}{R}$

Hence, the correct answer is (D).

CHAPTER 4

27.



$$V_A = \begin{pmatrix}\text{Potential at}\\ A \text{ due to } A\end{pmatrix} + \begin{pmatrix}\text{Potential at}\\ A \text{ due to } B\end{pmatrix}$$

$$\Rightarrow \quad V_A = -\frac{Gm_1}{R} - \frac{Gm_2}{\sqrt{2}R} \text{ and}$$

Similarly,

$$V_B = \begin{pmatrix}\text{Potential at}\\ B \text{ due to } A\end{pmatrix} + \begin{pmatrix}\text{Potential at}\\ B \text{ due to } B\end{pmatrix}$$

$$\Rightarrow \quad V_B = -\frac{Gm_2}{R} - \frac{Gm_1}{\sqrt{2}R}$$

Since $W_{A\to B} = m(V_B - V_A)$

$$\Rightarrow \quad W_{A\to B} = \frac{Gm(m_1 - m_2)(\sqrt{2}-1)}{\sqrt{2}R}$$

Hence, the correct answer is (B).

28. $E_g = G\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + \ldots\right]$

$$\Rightarrow \quad E_g = G\left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\right]$$

$$\Rightarrow \quad E_g = G\left(\frac{1}{1-\frac{1}{4}}\right) \qquad \left\{S_\infty = \frac{\text{1st term}}{\text{1-Common Ratio}}\right\}$$

$$\Rightarrow \quad E_g = \frac{4G}{3}$$

Hence, the correct answer is (B).

29. For a satellite revolving near the surface of planet, we have

$$T \cong 2\pi\sqrt{\frac{R^3}{GM}} \qquad \{\because r \cong R\}$$

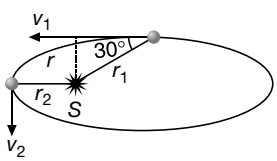
Since, $M = \left(\frac{4}{3}\pi R^3\right)\rho$

$$\Rightarrow \quad T = \sqrt{\frac{3\pi}{\rho G}}$$

i.e., T is independent of R.

Hence, the correct answer is (D).

30.



By Law of Conservation of Angular Momentum, we get

$$mv_1 r = mv_2 r_2$$

$$\Rightarrow \quad mv_1 r_1 \sin 30° = mv_2 r_2$$

$$\Rightarrow \quad v_1 r_1 = 2v_2 r_2$$

Hence, the correct answer is (C).

31. Kinetic energy of satellite is $K = \frac{GMm}{2r}$ and the magnitude of gravitational potential energy is

$$U = \left|-\frac{GMm}{r}\right| = \frac{GMm}{r}$$

So, $K = \frac{U}{2}$

Hence, the correct answer is (B).

32. Total mechanical energy, at earth's surface is

$$E = U + K = -\frac{GMm}{R} + \frac{1}{2}mv_e^2$$

Since $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad E = -\frac{GMm}{R} + \frac{1}{2}m\left(\frac{2GM}{R}\right) = 0$$

Hence, the correct answer is (D).

33. By Law of Conservation of Mechanical Energy, we get

$$-\frac{GmM}{2R} = -\frac{GmM}{\frac{R}{2}} + \frac{1}{2}k\left(\frac{R}{2}\right)^2$$

$$\Rightarrow \quad \frac{GMm}{R}\left(2 - \frac{1}{2}\right) = \frac{kR^2}{8}$$

$$\Rightarrow \quad k = \frac{12GMm}{R^3}$$

Hence, the correct answer is (D).

34. By Law of Conservation of Energy

$$(U+K)_{\text{surface}} = (U+K)_r$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{r} + 0$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mk^2v_e^2 = -\frac{GMm}{r}$$

But $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad -\frac{GM}{R} + \frac{1}{2}k^2\left(\frac{2GM}{R}\right) = -\frac{GM}{r}$$

$$\Rightarrow \quad -\frac{1}{R} + \frac{k^2}{R} = -\frac{1}{r}$$

$$\Rightarrow \quad \frac{1}{r}=\frac{1-k^2}{R}$$

$$\Rightarrow \quad r=\frac{R}{1-k^2}$$

Hence, the correct answer is (B).

35. When kinetic energy $< E$, total energy will be negative, when kinetic energy equals E, total energy is zero and when kinetic energy $> E$, total energy is positive.
Hence, the correct answer is (C).

36. For $r \le r_1$

$$V=\frac{Gm_1}{r_1}-\frac{Gm_2}{r_2}=\text{constant}$$

For $r_1 \le r \le r_2$

$$V=-\frac{Gm_2}{r_2}-\frac{Gm_1}{r}$$

Slope of V-r graph $\frac{dV}{dr}=\frac{Gm_1}{r^2}$

For $V=-\frac{G(m_1+m_2)}{r}$, slope of V-r graph is $\frac{dV}{dr}=\frac{G(m_1+m_2)}{r^2}$

At the boundary of outer shell, slope of V-r graph changes from $\frac{Gm_1}{r_2^2}$ to $\frac{G(m_1+m_2)}{r_2^2}$ i.e., slope increases.
Hence, the correct answer is (D).

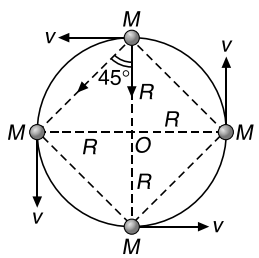
37. $c=\sqrt{\frac{2GM_{star}}{R}}$

$$\Rightarrow \quad 3\times10^8=\sqrt{\frac{2(6.67\times10^{-11})(3\times2\times10^{30})}{R}}$$

$$\Rightarrow \quad R \quad 9 \text{ km}$$

Hence, the correct answer is (D).

38. Gravitational force on each particle due to the other three particles will provide the necessary centripetal force.



$$\Rightarrow \quad \sqrt{2}\frac{GM^2}{\left(\sqrt{2}R\right)^2}\cos45^\circ+\frac{GM^2}{(2R)^2}=\frac{Mv^2}{R}$$

$$\Rightarrow \quad v=\sqrt{\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)}$$

Hence, the correct answer is (D).

39. $dW=\vec{F}\cdot d\vec{\ell}=m_0\left(\vec{E}\cdot d\vec{\ell}\right)$

$$\Rightarrow \quad dW=m_0\left(4\hat{i}+\hat{j}\right)\cdot\left(\hat{i}dx+\hat{j}dy\right)$$

$$\Rightarrow \quad dW=m_0\left(4dx+dy\right)$$

Given that, $W=0$

$$\Rightarrow \quad \int dW=0$$

$$\Rightarrow \quad \int\left(4dx+dy\right)=0$$

$$\Rightarrow \quad \int d\left(4x+y\right)=0$$

$$\Rightarrow \quad 4x+y=\text{constant}$$

$\Rightarrow \quad y+4x=2$, satisfies the condition.

Hence, the correct answer is (A).

40. Total energy of a planet of mass m, in an elliptical orbit is

$$E=-\frac{GMm}{2a} \qquad \{m=\text{mass of planet}\}$$

By Law of Conservation of Energy, we have

$$KE+PE=E$$

$$\Rightarrow \quad \frac{1}{2}mv^2-\frac{GMm}{r}=-\frac{GMm}{2a}$$

$$\Rightarrow \quad v=\sqrt{GM\left(\frac{2}{r}-\frac{1}{a}\right)}$$

Hence, the correct answer is (A).

41. Since $u=\frac{3}{4}v_e$

By Law of Conservation of Energy,

$$(U+K)_{\text{surface}}=(U+K)_r$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mu^2=\frac{-GMm}{r}+0$$

$$\Rightarrow \quad \frac{-GMm}{R}+\frac{1}{2}m\left(\frac{9}{16}v_e^2\right)=-\frac{GMm}{r}$$

Since $v_e^2=\frac{2GM}{R}$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}m\frac{9}{16}\left(\frac{2GM}{R}\right)=-\frac{GMm}{r}$$

$$\Rightarrow \quad -\frac{7}{16}\frac{GMm}{R}=-\frac{GMm}{r}$$

$$\Rightarrow \quad r=\frac{16}{7}R$$

Hence, the correct answer is (B).

CHAPTER 4

42. $\begin{pmatrix}\text{Work}\\ \text{done}\end{pmatrix}=\begin{pmatrix}\text{Increase in Gravitational}\\ \text{Potential Energy}\end{pmatrix}$

Since, $\Delta U=\dfrac{mgh}{1+\dfrac{h}{R}}$

$\Rightarrow \quad W_1=\dfrac{mgR}{1+\dfrac{R}{R}}=\dfrac{mgR}{2}$

and similarly, $W_2=\dfrac{mgh}{1+\dfrac{h}{R}}$

Since, $W_1=2W_2$

$\Rightarrow \quad \dfrac{mgR}{2}=\dfrac{2mgh}{1+\dfrac{h}{R}}$

$\Rightarrow \quad h=\dfrac{R}{3}$

Hence, the correct answer is (C).

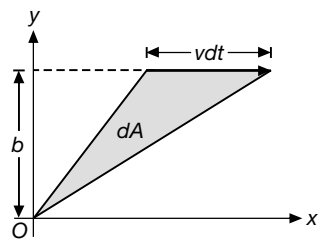
43. $\dfrac{dA}{dt}=\dfrac{1}{2}\times\dfrac{\text{base}\times\text{height}}{dt}$

$\Rightarrow \quad \dfrac{dA}{dt}=\dfrac{1}{2}\dfrac{(vdt)(b)}{dt}$

$\Rightarrow \quad \dfrac{dA}{dt}=\dfrac{bv}{2}$

Since, $v=at$, so

$\Rightarrow \quad \dfrac{dA}{dt}=\dfrac{abt}{2}$



Hence, the correct answer is (B).

44. $\vec{E}_g=-\vec{\nabla}V=-\left(\hat{i}\dfrac{\partial V}{\partial x}+\hat{j}\dfrac{\partial V}{\partial y}\right)=k\left(y\hat{i}+x\hat{j}\right)$

The work done by this field is independent of the path followed between any two states i.e.

$$W_{(0,0)\to(a,a)}=W_{(0,0)\to(0,a)\to(a,a)}=W_{(0,0)\to(a,0)\to(a,a)}$$

Since, $dW=m\vec{E}_g\cdot d\vec{r}$

where $d\vec{r}=\hat{i}\,dx+\hat{j}\,dy$

$\Rightarrow \quad dW=mk\left(y\,dx+x\,dy\right)$

$\Rightarrow \quad W=\int dW=mk\int_{(0,0)}^{(a,a)}d(xy)=mka^2$

$\Rightarrow \quad W=mka^2$ {where m is mass of particle}

$W_{(0,0)\to(a,a)}=W_{(0,0)\to(0,a)\to(a,a)}=W_{(0,0)\to(a,0)\to(a,a)}=mka^2$

Hence the field is conservative.

Hence, the correct answer is (B).

45. Let σ be the surface density, then

$$M_A=\sigma 4\pi R_A^2,\ m_B=\sigma 4\pi R_B^2$$

Since $V_A=\dfrac{-GM_A}{R_A}$ and $V_B=\dfrac{-GM_B}{R_B}$

$\Rightarrow \quad \dfrac{V_A}{V_B}=\dfrac{M_A}{M_B}\dfrac{R_B}{R_A}=\dfrac{\sigma\left(4\pi R_A^2\right)}{\sigma\left(4\pi R_B^2\right)}\dfrac{R_B}{R_A}=\dfrac{R_A}{R_B}$

Given that, $\dfrac{V_A}{V_B}=\dfrac{R_A}{R_B}=\dfrac{3}{4}$

$\Rightarrow \quad R_B=\dfrac{4}{3}R_A$

Since the new shell formed also has same surface mass density. So, we have

$$\sigma=\frac{M_A}{4\pi R_A^2}=\frac{M_B}{4\pi R_B^2}=\frac{M_A+M_B}{4\pi R^2} \qquad \ldots(1)$$

From equation (1), we get

$$\frac{M_A}{M_B}=\frac{R_A^2}{R_B^2} \qquad \ldots(2)$$

and $\dfrac{M_A+M_B}{M_A}=\dfrac{R^2}{R_A^2}$ …(3)

From (2) and (3), we get

$$R^2=R_A^2+R_B^2$$

Now $\dfrac{V}{V_A}=\dfrac{M}{R}\dfrac{R_A}{M_A}=\left(\dfrac{M_A+M_B}{M_A}\right)\dfrac{R_A}{\sqrt{R_A^2+R_B^2}}$ …(4)

From (3), we have $\dfrac{M_A+M_B}{M_A}=\dfrac{R^2}{R_A^2}$

Substituting in equation (4), we get

$$\frac{V}{V_A}=\frac{\sqrt{R_A^2+R_B^2}}{R_A}=\frac{5}{3}$$

Hence, the correct answer is (C).

46. Since, $T^2=kr^3$

$\Rightarrow \quad 2\dfrac{\Delta T}{T}=3\dfrac{\Delta r}{r}$

$\Rightarrow \quad \dfrac{\Delta T}{T}=\dfrac{3}{2}\dfrac{\Delta r}{r}$

Hence, the correct answer is (A).

47. Net mass, $M=\left[\dfrac{4}{3}\pi(2R)^3-\dfrac{4}{3}\pi R^3\right]\rho$

where ρ is the density of the material of sphere. So,

$$\rho=\frac{M}{\frac{4}{3}\pi\left[(2R)^3-(R)^3\right]}=\left(\frac{3M}{28\pi R^3}\right)$$

Now $V=V_{\substack{2R\text{ at}\\ \text{centre}}}-V_{\substack{R\text{ at}\\ \text{centre}}}$ …(1)

$$V_{2R} = -\frac{GM_1}{2R} \text{ and } V_R = -\frac{GM_2}{2R}$$

where, $M_1 = \rho\left(\frac{4}{3}\pi\right)(2R)^3 = \frac{8}{7}M$

and $M_2 = \rho\left(\frac{4}{3}\pi\right)(R)^3 = \frac{M}{7}$

Substituting in equation (1), we get

$$V = -\frac{9GM}{14R}$$

Hence, the correct answer is (B).

48. By Law of Conservation of Energy

$$(U+K)_{\text{surface}} = (U+K)_h$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0$$

$\{\because$ At maximum height velocity is zero$\}$

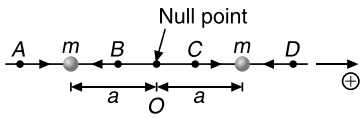
$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mgR = -\frac{GMm}{R+h}$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}\frac{GMm}{R} = -\frac{GMm}{R+h} \qquad \left\{\because g = \frac{GM}{R^2}\right\}$$

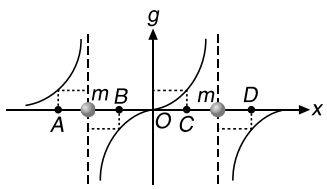
$$\Rightarrow \quad h = R$$

Hence, the correct answer is (B).

49.



The direction of net gravitational intensity at various points is shown above. Taking gravitational intensity towards right as positive, the graph will be



Hence, the correct answer is (A).

50. $\Delta U_1 = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$

when taken from surface to h, we have

$$\Delta U_1 = -\frac{GMm}{R+h} + \frac{GMm}{R}$$

Now, when taken from h to infinity, we have

$$\Delta U_2 = 0 - \left(-\frac{GMm}{R+h}\right)$$

Since $\Delta U_1 = \Delta U_2$

$$\Rightarrow \quad -\frac{GMm}{R+h} + \frac{GMm}{R} = \frac{GMm}{R+h}$$

$$\Rightarrow \quad 2\left(\frac{GMm}{R+h}\right) = \frac{GMm}{R}$$

$$\Rightarrow \quad 2R = R + h$$

$$\Rightarrow \quad h = R$$

Hence, the correct answer is (A).

51. Since $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$

$$\Rightarrow \quad \frac{T_1^2}{T_2^2} = \left(\frac{R}{4R}\right)^3 = \frac{1}{64}$$

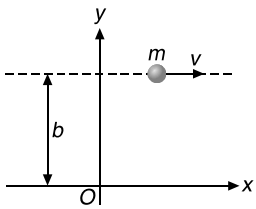
(Three earth radii from the surface means four radii from the centre).

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{1}{8}$$

$$\Rightarrow \quad T_2 = 8T_1 = 8(83) = 664 \text{ minute}$$

Hence, the correct answer is (C).

52.



Since angular momentum of the particle about origin O, $L = mvb = \text{constant}$, therefore, areal velocity of the particle about origin O is constant. So, it sweeps equal areas in equal intervals of time. Thus, area swept in smaller time interval will also be smaller. Hence

$$A_1 > A_2 \text{ if } (t_4 - t_3) < (t_2 - t_1)$$

Hence, the correct answer is (B).

53. $v_0 = \sqrt{\frac{Gm}{5R}}$

After perfectly inelastic collision, velocity of combined mass will become zero. Therefore, if v be the desired speed, then by Law of Conservation of Mechanical Energy, we have

$$(U+K)_{r=5R} = (U+K)_{\text{surface}}$$

$$-\left(\frac{GmM}{5R}\right) - \left(\frac{GmM}{5R}\right) + 0 = -\frac{GM(2m)}{R} + \frac{1}{2}(2m)v^2$$

$$\Rightarrow \quad \frac{1}{2}(2m)v^2 = -\frac{GM(2m)}{5R} + \frac{GM(2m)}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{8GM}{5R}} = 2\sqrt{2}v_0$$

Hence, the correct answer is (A).

CHAPTER 4

54. Since Areal velocity $= \dfrac{\text{Area Swept}}{\text{Time for one Revolution of Earth about the Sun}}$

Further Areal velocity $= \dfrac{L}{2M}$

$\Rightarrow$ Area Swept $= \left(\dfrac{L}{2M}\right)\left(\begin{array}{c}\text{Time for one}\\ \text{Revolution of Earth}\\ \text{about the Sun}\end{array}\right)$

Area Swept $= \dfrac{1}{2}(4.4\times10^{15})(365\times24\times60\times60)$

$\Rightarrow$ Area Swept $\quad 7\times10^{22}\ \text{m}^2$

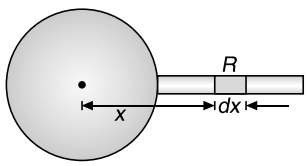
Hence, the correct answer is (D).

55. Force on element dx on rod is

$$dF = \frac{GM(dm)}{x^2}$$

where $dm = \dfrac{M}{2R}dx$

$$\Rightarrow \quad dF = \left(\frac{GM^2}{2R}\right)\frac{dx}{x^2}$$



Total force on the rod is

$$F = \int dF = \frac{GM^2}{2R}\int_R^{2R}\frac{1}{x^2}dx$$

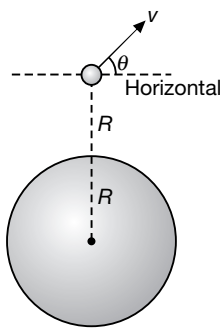
$$\Rightarrow \quad F = \frac{GM^2}{2R}\left(-\frac{1}{x}\right)\Big|_R^{2R} = -\frac{GM^2}{2R}\left(\frac{1}{2R}-\frac{1}{R}\right)$$

$$\Rightarrow \quad F = \frac{GM^2}{4R^2}$$

Hence, the correct answer is (A).

56. At height $h = R$, the distance of the particle from centre of earth is $r = 2R$.

Let v be the velocity at that point. Then by Law of Conservation of Mechanical Energy, we have



$$(U+K)_{\text{surface}} = (U+K)_{\text{at } h}$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}mv_e^2 = -\frac{GMm}{R+R}+\frac{1}{2}mv^2$$

Since, $v_e = \sqrt{\dfrac{2GM}{R}}$

$$\Rightarrow \quad \frac{1}{2}mv^2 = -\frac{GMm}{R}+\frac{GMm}{R}+\frac{GMm}{2R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$$

Let θ be the angle of its velocity with horizontal, then by Law of Conservation of Angular Momentum about centre of earth, we have

$$mv_eR\cos(45°) = mvr\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{v_eR\cos(45°)}{vr}$$

$$\Rightarrow \quad \cos\theta = \frac{R\sqrt{\dfrac{2GM}{R}}\dfrac{1}{\sqrt{2}}}{(2R)\sqrt{\dfrac{GM}{R}}} = \frac{1}{2}$$

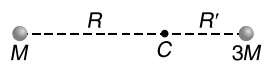
$$\Rightarrow \quad \theta = 60°$$

Hence, the correct answer is (C).

57. $$U(r) = \begin{cases} -\dfrac{GMm}{r} & , \quad r \geq R \\ -\dfrac{GMm}{R} & , \quad r < R \end{cases}$$

Hence, the correct answer is (C).

58. In a double star system, both stars revolve around centre of mass of system (C).



From the definition of centre of mass

$$MR = 3MR'$$

$$\Rightarrow \quad R' = \frac{R}{3}$$

Hence, distance between two stars is

$$r = R+\frac{R}{3} = \frac{4R}{3}$$

The gravitational force between them will provide the centripetal force to revolve in the circle. Therefore, for the smaller star, we have

$$\frac{GM\times3M}{\left(\dfrac{4R}{3}\right)^2} = MR\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{27GM}{16R^3}}$$

$$\Rightarrow \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{16R^3}{27GM}}$$

Hence, the correct answer is (C).

59. By Law of Conservation of Energy, we have

$$(U+K)_{\text{surface}} = (U+K)_h$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}m\left(\frac{v_e}{2}\right)^2 = -\frac{GMm}{R+h} = U_h$$

$$\Rightarrow \quad U_h = -\frac{GMm}{R} + \frac{1}{8}mv_e^2 = -\frac{GMm}{R} + \frac{1}{8}m\left(\frac{2GM}{R}\right)$$

$$\Rightarrow \quad U_h = -\frac{3}{4}\frac{GMm}{R}$$

Hence, the correct answer is (C).

60. $E_i = -\dfrac{GMm}{2r}$, $E_f = -\dfrac{GMm}{2(3r)} = -\dfrac{GMm}{6r}$

Energy Required is

$$\Delta E = E_f - E_i = \frac{GMm}{3r}$$

Hence, the correct answer is (C).

61. Since, $W = U_f - U_i$

Where, $U_f = -\dfrac{3Gm^2}{r_f}$ and $U_i = -\dfrac{3Gm^2}{r_i}$

$$\Rightarrow \quad W = U_f - U_i = 3Gm^2\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

where, $G = 6.67\times10^{-11}$ $\text{Nm}^2\text{kg}^{-2}$, $m = 0.1$ kg, $r_f = 0.4$ m and $r_i = 0.2$ m

Substituting the values, we get

$$W = 5\times10^{-12} \text{ J}$$

Hence, the correct answer is (C).

62. The whole space can be divided into three regions

(i) $0 < r < R_1$, we have $F(r) = 0$

(ii) $R_1 < r < R_2$, we have $F(r) = \dfrac{4}{3}\pi G\rho m\left(r - \dfrac{R_1^3}{r^2}\right)$

(iii) $R_2 < r < \infty$, we have $F(r) = \dfrac{4}{3}\pi G\rho m\left(\dfrac{R_2^3 - R_1^3}{r^2}\right)$

where, ρ is the density of material of the sphere.

Hence, the correct answer is (C).

63. The weight of the traveler first decreases, then becomes zero and after that again increases.

Hence, the correct answer is (C).

64. Since $E = -\dfrac{dV}{dx}$

$$\Rightarrow \quad -dV = Edx$$

$$\Rightarrow \quad -dV = Kx^{-3}dx$$

$$\Rightarrow \quad -\int dV = K\int x^{-3}\,dx$$

$$\Rightarrow \quad -V = K\left(\frac{x^{-3+1}}{-3+1}\right)$$

$$\Rightarrow \quad V = \frac{K}{2x^2}$$

Hence, the correct answer is (D).

65. Total number of interactions $= {}^nC_2 = \dfrac{n(n-1)}{2}$

Hence, the correct answer is (D).

66. Gravitational field due to element dx at origin is

$$dE_g = \frac{G(a+bx^2)dx}{x^2}$$

Net field at O is

$$E_g = \int dE_g = G\int_{\alpha}^{l+\alpha}\left(\frac{a}{x^2} + b\right)dx$$

$$\Rightarrow \quad E_g = \left(-\frac{Ga}{x}\right)\Big|_{\alpha}^{l+\alpha} + Gbl$$

$$\Rightarrow \quad E_g = Ga\left(\frac{1}{\alpha} - \frac{1}{l+\alpha}\right) + Gbl$$

Hence force on mass m is given by

$$F = mE_g = Gm\left[a\left(\frac{1}{\alpha} - \frac{1}{l+\alpha}\right) + bl\right]$$

Hence, the correct answer is (A).

67. Net gravitational field inside a shell is zero. Hence, net force on the particle will be zero i.e., the particle stays at rest in its original position.

Hence, the correct answer is (D).

68. Angular momentum of earth remains constant even after the hit because the meteor is coming radially inward. However, the MOI increases by mR^2 as meteorite strikes the earth at equator.

$$I\omega = \text{constant}$$

$$\Rightarrow \quad Id\omega + \omega dI = 0$$

$$\Rightarrow \quad \frac{d\omega}{\omega} = -\frac{dI}{I}$$

Since $\omega = \dfrac{2\pi}{T}$, we get $\dfrac{dT}{T} = \dfrac{dI}{I}$

Hence $dT = \dfrac{dI}{I}T = \dfrac{mR^2}{\frac{2}{5}MR^2}T = \dfrac{5mT}{2M}$

Hence, the correct answer is (A).

CHAPTER 4

69. The gravitational potential at the centre of the ring is

$$V = -\frac{GM}{R}$$

irrespective of the distribution of mass

Since, $W_{C\to\infty} = m(V_\infty - V_C)$

$$\Rightarrow \quad W_{C\to\infty} = m\left[0 - \left(-\frac{GM}{R}\right)\right]$$

$$\Rightarrow \quad W_{C\to\infty} = \frac{GMm}{R}$$

Hence, the correct answer is (B).

70. When mass of the shell is m, the potential is $V = -\frac{Gm}{R}$

To add an elementary mass dm, the work done is $dW = Vdm = -\frac{Gm}{R}dm$

This, net work done equals the gravitational self energy, so

$$U = -\int_0^M \frac{Gm\,dm}{R} = -\frac{GM^2}{2R}$$

Hence, the correct answer is (D).

71. Given that $v_0 = \frac{v_e}{2}$

$$\Rightarrow \quad \sqrt{\frac{gR^2}{R+h}} = \frac{1}{2}\sqrt{2gR}$$

$$\Rightarrow \quad \frac{R}{R+h} = \frac{1}{2}$$

$$\Rightarrow \quad h = R$$

Since the satellite now stops and falls from this height, using conservation of mechanical energy $U_i + K_i = U_f + K_f$

$$\Rightarrow \quad -\frac{GMm}{R+R} + 0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

Hence, the correct answer is (A).

72. Potential energy at a height of $2R$ from surface of earth is

$$U = -\frac{GMm}{R+2R} = -\frac{GMm}{3R}$$

and total energy of a satellite at a height h is $E = -\frac{GMm}{2(R+h)}$

Given that $U = E$

$$\Rightarrow \quad -\frac{GMm}{3R} = -\frac{GMm}{2(R+h)}$$

$$\Rightarrow \quad h = \frac{R}{2}$$

Hence, the correct answer is (B).

73. **CONCEPT OF REDUCED MASS (μ)**

Let m_1 be at rest and think that m_2 has been replaced by μ and is moving with velocity v. Then by Law of Conservation of Energy

$$\frac{1}{2}m_1(0)^2 + \frac{1}{2}\mu v^2 = -\frac{Gm_1m_2}{r} + 0$$

where, μ = reduced mass of system = $\frac{m_1m_2}{m_1+m_2}$

$$\Rightarrow \quad v = \sqrt{\frac{2G(m_1+m_2)}{r}}$$

Hence, the correct answer is (B).

74. Since $U = v_O = \sqrt{\frac{GM}{R}}$

By Law of Conservation of Energy, we have

$$(U+K)_{\text{surface}} = (U+K)_{at\ h}$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}mu^2 = -\frac{GMm}{R+h} + \frac{1}{2}m(0)^2$$

$$\Rightarrow \quad -\frac{GMm}{R} + \frac{GMm}{2R} = -\frac{GMm}{R+h}$$

$$\Rightarrow \quad h = R$$

Hence, the correct answer is (C).

75. Gravitational force between balls is

$$F = \frac{Gm^2}{(a-x)^2}$$

For equilibrium of each ball, we have

$$T\sin\theta = \frac{Gm^2}{(a-x)^2} \qquad \ldots(1)$$

$$T\cos\theta = mg \qquad \ldots(2)$$

$$\Rightarrow \quad \tan\theta = \frac{Gm}{(a-x)^2 g}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left[\frac{Gm}{(a-x)^2 g}\right]$$

Hence, the correct answer is (B).

76. Since the torque on the planet due to gravitational pull about the sun is zero, so angular momentum of the planet about the sun is constant.

$$\Rightarrow \quad mvr_\perp = \text{constant}$$

Hence, the correct answer is (B).

77. By Law of Conservation of Energy

$$(U+K)_{\text{surface}} = (U+K)_{\infty}$$

$$\Rightarrow \quad \frac{-GMm}{R} + \frac{1}{2}m(3v_e)^2 = 0 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad -\frac{GM}{R} + \frac{9v_e^2}{2} = \frac{1}{2}v^2$$

Since $v_e^2 = \dfrac{2GM}{R}$

$$\Rightarrow \quad -\frac{v_e^2}{2} + \frac{9v_e^2}{2} = \frac{1}{2}v^2$$

$$\Rightarrow \quad v^2 = 8v_e^2$$

$$\Rightarrow \quad v = 2\sqrt{2}v_e$$

Hence, the correct answer is (D).

78. A lighter body inside a denser medium behaves like a negative mass so far the gravitational force is concerned even the air bubbles with negative masses will attract each other.

Hence, the correct answer is (B).

79. According to Kepler's Laws the planets sweep equal areas in equal intervals of time. So,

for

$$\Delta A_1 = \Delta A_2$$

$$t_1 = t_2$$

Hence, the correct answer is (C).

80. By Kepler's Second Law, we have

$$\frac{\Delta A}{\Delta t} = \frac{L}{2M} = \frac{\pi ab}{T}$$

$$\Rightarrow \quad \frac{L}{2M} = \frac{\pi ab}{T}$$

where a is semi major and b is semi minor axis of elliptical orbit.

$$\Rightarrow \quad L = \frac{m\pi(r_a + r_p)\sqrt{r_a r_p}}{T}$$

Hence, the correct answer is (A).

81. Areal velocity $\dfrac{dA}{dt} = \dfrac{L}{2m} = \text{constant}$

Since, $L = mvr\sin\theta$

$$\Rightarrow \quad \frac{dA}{dt} = \frac{vr\sin\theta}{2}$$

$$\Rightarrow \quad \frac{dA}{dt} \propto m^0$$

Hence, the correct answer is (D).

82. Since, $T = \dfrac{2\pi r}{v}$

$$\Rightarrow \quad \frac{2\pi R}{v_0} = 2 \text{ and } \frac{2\pi R'}{v_0'} = 16$$

This is possible when $R' = 4R$ and $v_0' = \dfrac{v_0}{2}$

Hence, the correct answer is (D).

83. Distance of second satellite from the centre of earth becomes four times.

Since, according to Kepler's Laws we have

$$T^2 \propto r^3$$

$$\Rightarrow \quad T \propto r^{3/2}$$

So, time period of second satellite must become $(4)^{3/2} = 8$ times

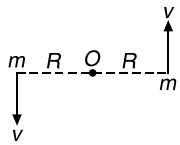
Hence, $T' = 8T = (8)(90) = 720$ min

Hence, the correct answer is (D).

84. The gravitational force provides the necessary centripetal force for circular motion

$$\Rightarrow \quad \frac{Gm^2}{(2R)^2} = \frac{mv^2}{R}$$

$$\Rightarrow \quad v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$$



Hence, the correct answer is (C).

85. Energy of the satellite at the surface of earth is $E_i = -\dfrac{GMm}{R}$ and energy of satellite at a distance $r = (2R + R) = 3R$ from the centre of earth is

$$E_f = -\frac{GMm}{2(3R)} = -\frac{GMm}{6R}$$

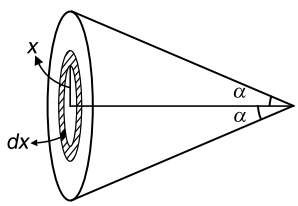
So, the energy required to launch the satellite is

$$E = E_f - E_i = \frac{5GMm}{6R}$$

Hence, the correct answer is (C).

86. $E = -2\pi\sigma G\int_0^{\alpha} \sin\theta\, d\theta$

$$\Rightarrow \quad E = -2\pi\sigma G(1 - \cos\alpha)$$



$$\Rightarrow \quad E = -2\pi\sigma G\left(1 - \frac{r}{\sqrt{r^2+R^2}}\right)$$

$$V = -2\pi\sigma r G\left(1 - \frac{r}{\sqrt{r^2+R^2}}\right)$$

Hence, the correct answer is (C).

87. For all points lying in the plane of base of a hemispherical shell, gravitational field is normal to the base and hence this surface is equipotential. So, $V_A = V_B = V_C$
Hence, the correct answer is (D).

88. Since, Total energy $=$ – Kinetic energy $= -E$
So, energy E will have to be supplied to escape the electron to infinity.
Hence, the correct answer is (C).

89. $U_h = -\dfrac{GMm}{R+h}$

$$\Rightarrow \quad U_h = -\frac{GMm}{R}\left(1+\frac{h}{R}\right)^{-1} = -\frac{GMm}{R}\left(1-\frac{h}{R}\right) \qquad \{\because\ h \quad R\}$$

$$\Rightarrow \quad U_h = -\frac{GMm}{R} + mgh \qquad \left\{\because\ g = \frac{GM}{R^2}\right\}$$

Hence, the correct answer is (B).

90. Since $W = \int \vec{F}\cdot d\vec{l}$

where $\vec{F} = m_0\vec{I}$ and $d\vec{l} = dx\hat{i} + dy\hat{j}$

$$\Rightarrow \quad W = m_0 \int_{(1,\,1)}^{\left(2,\,\frac{1}{3}\right)} 2dx + 3dy$$

Since $3y + 2x = 5$

$\Rightarrow \quad 3dy + 2dx = 0$

$\Rightarrow \quad W = 0$

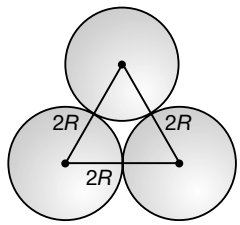
Hence, the correct answer is (A).

91. By Law of Conservation of Energy, we have

$$(U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

$$\Rightarrow \quad -\frac{3Gm^2}{d} + 3\left(\frac{1}{2}m(0)^2\right) = -\frac{3Gm^2}{2R} + 3\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow \quad 3\left(\frac{1}{2}mv^2\right) = 3\left(\frac{Gm^2}{2R} - \frac{Gm^2}{d}\right)$$



$$\Rightarrow \quad v^2 = Gm\left(\frac{1}{R} - \frac{2}{d}\right)$$

$$\Rightarrow \quad v = \sqrt{Gm\left(\frac{1}{R} - \frac{2}{d}\right)}$$

Hence, the correct answer is (D).

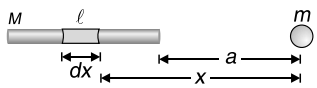
92. $(U+K)_{\text{initial}} = (U+K)_{\infty}$

$$-\frac{GM_e m}{\frac{r}{2}} - \frac{GM_m m}{\frac{r}{2}} + \frac{1}{2}mv^2 = 0$$

$$\Rightarrow \quad v = \sqrt{\frac{4G}{r}(M_e + M_m)}$$

Hence, the correct answer is (A).

93. $dU = -\dfrac{Gm\left(\frac{M}{\ell}dx\right)}{x}$



$$\Rightarrow \quad U = \int dU = -\frac{GmM}{\ell}\int_{a}^{a+\ell}\frac{dx}{x}$$

$$\Rightarrow \quad U = -\frac{GMm}{\ell}\log_e\left(\frac{a+\ell}{a}\right)$$

Hence, the correct answer is (C).

94. By Law of Conservation of Mechanical Energy

$$U_i + K_i = U_f + K_f$$

$$-\frac{GMm}{R+h_i} + 0 = -\frac{GMm}{R+h_f} + \frac{1}{2}mv^2$$

$$-\frac{GM}{R+R} + 0 = -\frac{GM}{R+0.5R} + \frac{1}{2}v^2$$

$$\Rightarrow \quad -\frac{GM}{2R} + \frac{2GM}{3R} = \frac{1}{2}v^2$$

$$\Rightarrow \quad \frac{GM}{6R} = \frac{1}{2}v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{3R}}$$

Hence, the correct answer is (D).

95. Given that $\frac{U}{m} = E$. By Law of Conservation of Energy, we have

$$(U+K)_{\text{surface}} = (U+K)_{\infty}$$

$$-mE + \frac{1}{2}mv_e^2 = 0 \qquad \left\{\because U = -\frac{GmM}{R}\right\}$$

$$\Rightarrow \quad v_e = \sqrt{2E}$$

Hence, the correct answer is (A).

96. Total initial energy is $E_i = -\frac{GMm}{R}$

Total final energy is $E_f = -\frac{GMm}{2(2R+R)}$

Energy required $= E_f - E_i$

$$\Rightarrow \quad \Delta E = -\frac{GMm}{6R} + \frac{GMm}{R} = \frac{5}{6}\frac{GMm}{R}$$

$$\Rightarrow \quad \Delta E = \frac{5}{6}mgR$$

Hence, the correct answer is (D).

97. Time period of a satellite above the earth's surface is

$$T^2 = \frac{4\pi^2R^3}{GM} = \frac{3\pi}{G\left(\frac{M}{\frac{4}{3}\pi R^3}\right)} = \frac{3\pi}{G\rho}$$

$$\Rightarrow \quad \rho T^2 = \frac{3\pi}{G} = \text{a universal constant}$$

Hence, the correct answer is (A).

98. $\int_S \vec{E}_g \cdot d\vec{A} = -4\pi m_0 G$ (Gauss Theorem for Gravitation)

Hence, the correct answer is (B).

99. $M_e = 81\,M_m$ — m — M_m, with R and r

$$U = -Gm\left(\frac{M_e}{R} + \frac{M_m}{r}\right)$$

$$\Rightarrow \quad U = -GmM_m\left(\frac{81}{R} + \frac{1}{r}\right)$$

Hence, the correct answer is (C).

100. Since, we know that from Work – Energy Theorem,

$$W_{\text{ext}} = \Delta U + \Delta K$$

$$\Rightarrow \quad W_{A\to B} = (U_B - U_A) + (K_B - K_A)$$

$$\Rightarrow \quad W_{A\to B} = m(V_B - V_A) + (K_B - K_A)$$

Substituting the values, we get

$$-10 = 2(V_B - V_A) + 4$$

$$\Rightarrow \quad V_B - V_A = -7 \text{ Jkg}^{-1}$$

Hence, the correct answer is (D).

101. Gravitational pull on satellite revolving in an orbit of radius $\frac{3R}{2}$ is

$$F = \frac{GMm}{\left(\frac{3R}{2}\right)^2} = \frac{4}{9}\frac{GMm}{R^2}$$

As weight of 1 kg body on the surface of earth is

$$\frac{GM \times 1}{R^2} = 10 \text{ N} \qquad \text{(given)}$$

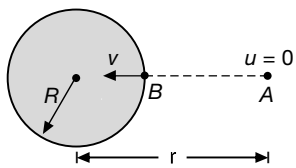
$$\Rightarrow \quad F = \frac{4}{9}\frac{GMm}{R^2} = \frac{4}{9} \times 10 \times 200 = \frac{8000}{9}$$

$$\Rightarrow \quad F = 889 \text{ N}$$

Hence, the correct answer is (B).

102. Let r be the radius of the satellite. Then

$$v_0^2 = \frac{GM}{r}$$



By Law of Conservation of Mechanical Energy,

$$(U+K)_A = (U+K)_B$$

$$\Rightarrow \quad -\frac{GMm}{r} + \frac{1}{2}m(0)^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v^2 = \frac{2GM}{R} - \frac{2GM}{r} = v_e^2 - 2v_0^2$$

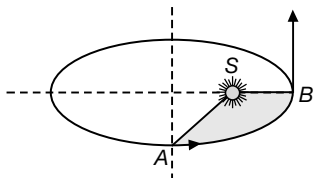
$$\Rightarrow \quad v = \sqrt{v_e^2 - 2v_0^2}$$

Hence, the correct answer is (D).

103. L is conserved both in magnitude and direction.

Hence, the correct answer is (C).

104. Since, we know that areal velocity of planet is constant. So, we have



$$\frac{\text{Area of Ellipse}}{\text{Period of Revolution}} = \frac{\text{Area } SAB}{t_{AB}}$$

$$\Rightarrow \quad t_{AB} = \left(\frac{\text{Area } SAB}{\text{Area of ellipse}}\right)T$$

$$\Rightarrow \quad t_{AB} = \frac{T\left(\frac{\pi ab}{4} - \frac{1}{2}(b)(ea)\right)}{\pi ab}$$

$$\Rightarrow \quad t_{AB} = T\left(\frac{1}{4} - \frac{e}{2\pi}\right)$$

Hence, the correct answer is (B).

105. Gravitational field inside the cavity is

$$\vec{E} = \frac{4}{3}\pi\rho G\vec{r}$$

where ρ is mass density and $\vec{r}$ is separation between centre of sphere and centre of cavity. Escape velocity from point A is calculated by applying Law of Conservation of Energy according to which

$$(U+K)_A = (U+K)_\infty$$

$$\frac{1}{2}mv_{esc}^2 - \frac{GMm}{R} + \frac{G\left(\frac{M}{9}\right)m}{\left(\frac{5R}{3}\right)} = 0$$

$$\Rightarrow \quad v_{esc} = \sqrt{\frac{28GM}{15R}}$$

Hence, the correct answer is (A).

106. $L = mvr$ ...(1)

Also, $\frac{mv^2}{r} = \frac{GMm}{r^2}$ ...(2)

From equations (1) and (2), we get

$$L = m\sqrt{GMr}$$

$$\Rightarrow \quad L \propto r^{1/2}$$

Hence, the correct answer is (D).

107. Let us consider the shell when a mass m is already piled on it by the agency. If V is the potential on the shell, then

$$V = -\frac{Gm}{R}$$

To add a mass dm further, we have

$$dW = Vdm$$

$$\Rightarrow \quad dW = -\frac{Gm}{R}dm$$

$$\Rightarrow \quad W = -\frac{G}{R}\int_0^M m\,dm$$

$$\Rightarrow \quad W = -\frac{1}{2}\frac{GM^2}{R}$$

$$\Rightarrow \quad W = \text{Potential Energy of Interaction}$$

Hence, the correct answer is (B).

108. Total energy of satellite in the first case is $E_i = -\frac{GMm}{2r}$

and in the second case is $E_f = \frac{GMm}{2\left(\frac{3r}{2}\right)} = -\frac{GMm}{3r}$

Energy increased $\Delta E = \frac{GMm}{r}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{GMm}{6r}$

So, percentage increase $= \frac{\left(\frac{GMm}{6r}\right)}{\left(\frac{GMm}{2r}\right)} \times 100\% = 33.33\%$

Hence, the correct answer is (C).

109. $\Delta E = U_f - U_i = -\frac{GM^2}{2(2R)} + \frac{GM^2}{2R} = \frac{GM^2}{4R}$

Hence, the correct answer is (C).

110. With respect to earth, satellite appears after every 6 hours over same place. Since it is revolving in opposite direction to that of earth, angular speed of satellite relative to earth is

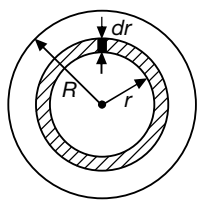
$$\omega_{relative} = \omega_{satellite} + \omega_{earth} = \frac{2\pi}{T} = \frac{2\pi}{6}$$

$$\Rightarrow \quad \frac{2\pi}{24} + \omega_{satellite} = \frac{2\pi}{6}$$

$$\Rightarrow \quad \omega_{satellite} = \frac{2\pi}{6} - \frac{2\pi}{24} = \frac{\pi}{4} \text{ radhr}^{-1}$$

Hence, the correct answer is (C).

111. $dU = -\frac{G\,m\,dm}{r}$



$$\Rightarrow \quad dU = -\frac{G\left(\frac{4}{3}\pi r^3\rho\right)\left(4\pi r^2 dr\rho\right)}{r}$$

$$\Rightarrow \quad dU = -\frac{16\pi^2 G\rho^2}{3}r^4 dr$$

$$\Rightarrow \quad U = -\frac{16}{3}\pi^2 G\left(\frac{M}{\frac{4}{3}\pi R^3}\right)^2\int_0^R r^4 dr$$

$$\Rightarrow \quad U=\left(-\frac{16}{3}\pi^2 G\right)\left(\frac{M^2}{\frac{16}{9}\pi^2 R^6}\right)\left(\frac{R^5}{5}\right)$$

$$\Rightarrow \quad U=-\frac{3}{5}\frac{GM^2}{R}$$

Hence, the correct answer is (C).

112. $\frac{dA}{dt}=\frac{A}{T}=\frac{L}{2m}$

$$\Rightarrow \quad L=\frac{2mA}{T}$$

Hence, the correct answer is (C).

113. $\text{K.E.}=\frac{GMm}{2r}=-E_0$, and $\text{P.E.}=-\frac{GMm}{r}=2E_0$

$$\Rightarrow \quad \text{T.E.}=\text{K.E.}+\text{P.E.}=-\frac{GMm}{2r}=E_0$$

Hence, the correct answer is (C).

114. Since the satellite appears after every 8 hours above the same place on earth, therefore its angular speed relative to earth is $\frac{2\pi}{8}$. Hence

$$\frac{2\pi}{8}=\omega_{\text{satellite}}-\omega_{\text{earth}}$$

$$\Rightarrow \quad \frac{2\pi}{8}=\frac{2\pi}{T_{\text{satellite}}}-\frac{2\pi}{24}$$

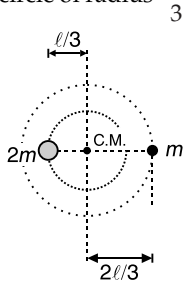
$$\Rightarrow \quad \frac{2\pi}{T_{\text{satellite}}}=\frac{2\pi}{8}+\frac{2\pi}{24}=\frac{2\pi}{6}$$

$$\Rightarrow \quad T_{\text{satellite}}=6 \text{ hr}$$

Hence, the correct answer is (C).

115. The system will revolve/rotate about an axis passing through the centre of mass of the combined system. Considering origin at the particle of mass $2m$, we have the centre of mass at a distance $\frac{\ell}{3}$ from $2m$ and $\frac{2\ell}{3}$ from m.

The gravitational force of attraction between $2m$ and m provides the necessary centripetal force to the mass to revolve in a circle of radius $\frac{2\ell}{3}$ for m or $\frac{\ell}{3}$ for $2m$.



$$\Rightarrow \quad m\left(\frac{2\ell}{3}\right)\omega^2=\frac{Gm(2m)}{\ell^2}$$

$$\Rightarrow \quad \omega=\sqrt{\frac{3Gm}{\ell^3}}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{\ell^3}{3Gm}}$$

Hence, the correct answer is (C).

116. $W'=\frac{3}{4}$

$$\Rightarrow \quad g_{\text{eff}}=\frac{3}{4}g$$

At equator, we have

$$g_{\text{eff}}=g-R\omega^2=\frac{3}{4}g$$

$$\Rightarrow \quad \omega=\sqrt{\frac{g}{4R}}=\frac{1}{2}\left(\sqrt{\frac{g}{R}}\right)=\frac{1}{2}\times\frac{1}{800}$$

$$\Rightarrow \quad \omega=\frac{1}{1600} \text{ rads}^{-1}$$

Hence, the correct answer is (C).

117. Let gravitational field be zero at a point lying at distance x from M. Then,

$$\frac{GM}{x^2}=\frac{Gm}{(d-x)^2}$$

$$\Rightarrow \quad \frac{d-x}{x}=\sqrt{\frac{m}{M}}$$

$$\Rightarrow \quad \frac{d}{x}-1=\sqrt{\frac{m}{M}}$$

$$\Rightarrow \quad x=\left(\frac{\sqrt{M}}{\sqrt{M}+\sqrt{m}}\right)d \qquad \ldots(1)$$

$$\Rightarrow \quad (d-x)=\left(\frac{\sqrt{m}}{\sqrt{M}+\sqrt{m}}\right)d \qquad \ldots(2)$$

Since, $V_P=-\frac{Gm}{d-x}-\frac{GM}{x}$ $\qquad \ldots(3)$

Substituting (1) and (2) in (3), we get

$$V_P=-\frac{G}{d}\left(\sqrt{m}+\sqrt{M}\right)^2$$

Hence, the correct answer is (D).

118. $\left(\begin{matrix}\text{Total}\\ \text{Mechanical Energy}\end{matrix}\right)_P=\left(\begin{matrix}\text{Total}\\ \text{Mechanical Energy}\end{matrix}\right)_O$

$$\Rightarrow \quad \frac{1}{2}m(0)^2-\frac{GMm}{\sqrt{(\sqrt{3}R)^2+R^2}}=\frac{1}{2}mv^2-\frac{GMm}{R}$$

$$\Rightarrow \quad -\frac{GMm}{2R}=\frac{1}{2}mv^2-\frac{GMm}{R}$$

CHAPTER 4

$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$

Hence, the correct answer is (A).

119. By Law of Conservation of Energy, we have

$(U+K)_{axis} = (U+K)_C$

If m_0 be the mass of the particle, then

$$-\frac{Gmm_0}{\sqrt{r^2+r^2}} + \frac{1}{2}m_0(0)^2 = -\frac{Gmm_0}{r} + \frac{1}{2}m_0v^2$$

$$\Rightarrow \quad \frac{1}{2}m_0v^2 = \frac{Gmm_0}{r}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad v = \sqrt{\frac{2Gm}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$$

Hence, the correct answer is (C).

120. $U_i = -\frac{GMm}{R}$ and $U_f = 0$

$\Rightarrow \quad W = \Delta U = \frac{GMm}{R}$

Hence, the correct answer is (A).

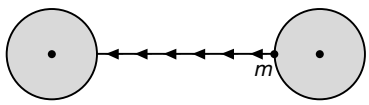
121. Use Gauss Theorem for Gravitation.

Hence, the correct answer is (C).

122. $E = -\frac{dV}{dr}$

Hence, the correct answer is (C).

123. Centre point is the unstable equilibrium position where potential energy is maximum.



Hence, the correct answer is (C).

124. Since, $g = \frac{GM}{R^2} = GMR^{-2}$

Fractional change in g is

$$\frac{\Delta g}{g} = \frac{\Delta M}{M} - \frac{2\Delta R}{R}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = 0.5\% - 2(0.5\%) = -0.5\%$$

So, g will decrease by 0.5%

$$v_e = \sqrt{\frac{2GM}{R}}$$

Fractional change in escape velocity is

$$\frac{\Delta v_e}{v_e} = \frac{1}{2}\frac{\Delta M}{M} - \frac{1}{2}\frac{\Delta R}{R}$$

Percentage change in escape velocity is

$$\frac{\Delta v_e}{v_e} = \frac{1}{2}(0.5-0.5) = 0\%$$

So, escape velocity will remain same.

Potential energy of an object of mass m on the surface of earth is

$$U = -\frac{GMm}{R}$$

Fractional change in potential energy is

$$\frac{\Delta U}{U} = -\frac{\Delta M}{M} + \frac{\Delta R}{R}$$

Percentage change in PE is

$$\frac{\Delta U}{U} - 0.5 + 0.5 = 0\%$$

So, potential energy will remain same.

Hence, the correct answer is (C).

Multiple Correct Choice Type Questions

1. $T^2 = \frac{4\pi^2}{GM}(R_e+h)^3$ and $v_0 = \sqrt{\frac{GM}{R_e+h}}$

If $h \quad R_e$, then

$$T = 2\pi\sqrt{\frac{R_e}{g}}$$

$\Rightarrow \quad T = 84.4$ minute

Hence, (B) and (C) are correct.

2. $U_i = \frac{-GMm}{R} = U$ (given)

$$\Delta U = U_f - U_i = \frac{-GMm}{(R+R)} + \frac{GMm}{R}$$

$$\Rightarrow \quad \Delta U = \frac{GMm}{2R} = -\frac{U}{2}$$

Same is the case with potential.

Hence, (A) and (D) are correct.

3. At a point P lying outside both the shells, the field will be due to both the shells as if they were point masses with total mass (m_1+m_2) concentrated at the centre. So, $E = G\left(\frac{m_1+m_2}{r^2}\right)$ for $r > r_1$ and $r > r_2$.

If P lies inside one shell and outside the other. Then field at point P due to the shell enclosing it is zero and is $\frac{Gm_2}{r^2}$ due to the other shell.

If P lies inside both the shells, then $E_p = 0$.

Hence, (A), (B) and (C) are correct.

4. Similarly, as in SOLUTION to PROBLEM 3,

$$V_p = -\frac{G(m_1+m_2)}{r} \text{ for } r > R_1,\ r > R_2$$

Which satisfies OPTION **(A)**

For the point P lying inside shell 1 and outside shell 2, the total potential at P equals the potential at P due to shell 1 plus potential at P due to shell 2.

$$V_p = V_{p1} + V_{p2}$$

$$\Rightarrow \quad V_p = \frac{-Gm_1}{r_1} + \frac{-Gm_2}{r}$$

$$\Rightarrow \quad V_p = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r}\right)$$

Which satisfies OPTION **(C)**

(Since field inside shell 1 is zero, so potential at a point lying inside shell 1 is a constant and that equals the value of the potential at the surface of shell 1 i.e. $-\frac{Gm_1}{r_1}$).

Similarly, for point P lying inside both the shells, we have

$$V = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2}$$

$$V = -G\left(\frac{m_1}{r_1} + \frac{m_2}{r_2}\right) \text{ for } r < r_2$$

Which satisfies OPTION **(D)**

Hence, (A), (C) and (D) are correct.

5. **Conceptual (A), (B), (C) and (D) are correct.**

6. Inside the inner sphere field is zero but potential is constant.

Between two, field is due to inner sphere and potential is due to both but it is constant due to outer shell.

Outside the outer shell, field and potential are due to both and it decreases in both the cases.

Hence, (A) and (C) are correct.

7.

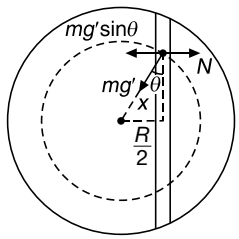
Velocity of Satellite	Nature of Path
$v = v_0$	Circular path around the earth.
$v < v_0$	Elliptical path and body returns to earth.
$v > v_0$ but $< v_e$	Elliptical path around the earth and will not escape.
$v = v_e$	Parabolic path and it escapes from the earth.
$v > v_e$	Hyperbolic path and escapes from earth.

Hence, (A), (B), (C) and (D) are correct.

8. Net force towards centre of earth is mg', where $g' = g\left(\frac{x}{R}\right)$

$$\Rightarrow \quad mg' = \frac{mgx}{R}$$

Normal force $N = mg'\sin\theta$



Since $\sin\theta = \frac{\frac{R}{2}}{x}$

$$\Rightarrow \quad \sin\theta = \frac{R}{2x}$$

$$\Rightarrow \quad N = \left(\frac{mgx}{R}\right)\left(\frac{R}{2x}\right)$$

$$\Rightarrow \quad N = \frac{mg}{2}$$

Constant and independent of x

Tangential force is given by

$$F = ma = mg'\cos\theta$$

$$\Rightarrow \quad a = g'\cos\theta$$

Since $\cos\theta = \frac{\sqrt{\frac{R^2}{4} - x^2}}{x}$

$$\Rightarrow \quad a = \frac{gx}{R}\sqrt{R^2 - 4x^2}$$

So, curve is parabolic and at $x = \frac{R}{2}$ we have $a = 0$

Hence, (B) and (C) are correct.

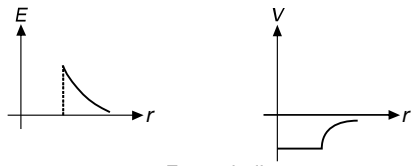
9. $V = -\frac{GM}{R}$

$$\Rightarrow \quad V = -\frac{GM}{R^2}R$$

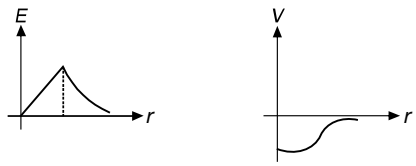
$$\Rightarrow \quad V = -gR$$

Hence, (C) and (D) are correct.

10. E-r and V-r graphs for a spherical shell and a solid sphere are shown here.



For a shell



For a solid sphere

Hence, (A), (B) and (C) are correct.

11. $E = -\dfrac{GMm}{2a}$

So, $E \propto m$

$E \propto m_s$

$E \propto \dfrac{1}{a}$

Hence, (A), (B) and (C) are correct.

12. $T^2 = \dfrac{4\pi^2}{GM}\left(\dfrac{r_A + r_P}{2}\right)^3 \qquad \left\{\because r = \dfrac{r_A + r_P}{2}\right\}$

$\Rightarrow \quad T^2 = \dfrac{\pi^2}{2GM}(r_A + r_P)^3$

By Law of Conservation of Angular Momentum

$mv_A r_A = mv_P r_P$

$\Rightarrow \quad v_A r_A = v_P r_P$

Hence, (B), (C) and (D) are correct.

13. Time period in both the cases is

$$T_1 = T_2 = 2\pi\sqrt{\frac{R^3}{GM}} \approx 84.6 \text{ min}$$

However, $v_1 > v_2$, because the difference in potential energy between the extreme position and mean position will be more in the first case.
Hence, (B) and (D) are correct.

14. $E_g = -\dfrac{dV}{dx}$

If $E_g = 0$, then $V = \text{constant}$ and this constant may also be zero.
Hence, (A) and (C) are correct.

15. At two positions, when the planet is closest to the sun (perigee) and when it is farthest from the sun (apogee), velocity vector is perpendicular to force vector i.e., work done is zero. In one complete revolution work done is zero.
Hence, (A) and (D) are correct.

16. The field inside the shell is zero and so potential inside the shell is constant equal to the value that exists at the surface i.e. $-\dfrac{GM}{a}$.
Hence, (A) and (D) are correct.

17. **Hence, (B) and (D) are correct.**

18. By Law of Conservation of Mechanical Energy, we get

$(U + K)_\infty = (U + K)_r$

$\Rightarrow \quad 0 + 0 = -\dfrac{Gm(4m)}{r} + \dfrac{1}{2}\mu v_r^2$

$\Rightarrow \quad \dfrac{G(m)(4m)}{r} = \dfrac{1}{2}\mu v_r^2 \qquad \ldots(1)$

where, μ = reduced mass $= \dfrac{(m)(4m)}{m + 4m} = \dfrac{4m}{5}$ and

v_r = relative velocity of approach

Substituting in equation (1), we get

$\Rightarrow \quad v_r = \sqrt{\dfrac{10Gm}{r}}$

From equation (1), the total kinetic energy is

$K = \dfrac{G(m)(4m)}{r}$

$\Rightarrow \quad K = \dfrac{4Gm^2}{r}$

Net torque of two equal and opposite forces acting on two objects is zero. Therefore, angular momentum will remain conserved. Initially both the objects were stationary i.e., angular momentum about any point was zero. Hence, angular momentum of both the particles about any point will be zero at all instants.
Hence, (A), (B), (C) and (D) are correct.

19. When total force is zero, $g_{\text{eff}} = 0$

$V = -\dfrac{Gm}{r}$

For a shell of radius R,

$V_{\substack{\text{inside}\\\text{shell}}} = V_{\text{surface}} = -\dfrac{GM}{R}$

$I_{\text{inside}} = 0$ whereas

$I_{\text{outside}} = \dfrac{GM}{r^2}$

So, plot of V vs r is continuous whereas plot of I vs r is discontinuous
Hence, (A), (B) and (C) are correct.

20. Orbital radius of first satellite is $2R$ and that of second satellite is $8R$.
For a satellite

$K.E. = \dfrac{GMm}{2r}$

$P.E. = -\dfrac{GMm}{r}$

$T.E. = -\dfrac{GMm}{2r}$

Hence, (A), (B) and (C) are correct.

21. **Conceptual (A), (B) and (C) are correct.**

22. According to Kepler's Laws Areal Speed is constant. Also, by Law of Conservation of Angular Momentum L must be constant.
Hence, (B) and (C) are correct.

23. According to Right Hand Thumb Rule curl the fingers of right hand in the direction of rotation then thumb gives the direction of the areal velocity/angular momentum.
Hence, (A) and (C) are correct.

24. At centre, $V = -\frac{3}{2}\frac{GM}{R}$

and $E = 0$

Hence, (A) and (D) are correct.

25. Force on P is $\frac{Gm_1m_3}{r^2}$, attractive i.e. towards O.

Hence, (A), (B) and (D) are correct.

26. $v_0 = \sqrt{\frac{GM}{r}}$

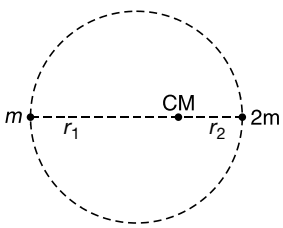
$$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

$$U = \frac{-GMm}{2r}$$

$$K = \frac{GMm}{2r}$$

Hence, (A) and (D) are correct.

27.



Let CM of system be at a distance r_1 from m and r_2 from $2m$. Then, $mr_1 = 2m(r - r_1)$

$\Rightarrow \quad r_1 = \frac{r}{3}$ and $r_2 = r - r_1 = \frac{2r}{3}$

Since, $T_2^2 = \frac{4\pi^2 r_2^3}{Gm}$

$\Rightarrow \quad T_2^2 = \frac{32\pi^2 r^3}{27Gm}$

$\Rightarrow \quad T_2 \propto r^{\frac{3}{2}}$ and $T_2 \propto m^{-\frac{1}{2}}$

Hence, (A) and (D) are correct.

28. **Conceptual (A), (B) and (D) are correct.**

29. Distance from centre of sun and hence the kinetic energy and potential energy keep changing.
Hence, (C) and (D) are correct.

30. $U_i = -\frac{GMm}{R}$

$\Rightarrow \quad U_f = -\frac{GMm}{R+h}$

$(\text{Work done}) = (\text{Change in G.P.E.})$

$\Rightarrow \quad W = U_f - U_i = -GMm\left(\frac{1}{R+h} - \frac{1}{R}\right)$

$\Rightarrow \quad W = -\frac{GMm}{R}\left[\left(1+\frac{h}{R}\right)^{-1} - 1\right]$

$\Rightarrow \quad W = -\frac{GMm}{R}\left[1 - \frac{h}{R} - 1\right]$

$\Rightarrow \quad W \approx \frac{GMmh}{R^2}$

$\Rightarrow \quad W \approx mgh$ for $h \quad R$

Also, $U_i = -\frac{GMm}{R}$, and

$$U_f(h = R) = -\frac{GMm}{R+R} = -\frac{GMm}{2R}$$

$\Rightarrow \quad W = U_f - U_i = \frac{GMm}{2R}$

$\Rightarrow \quad W = \frac{1}{2}mgR \qquad \left\{\because g = \frac{GM}{R^2}\right\}$

Hence, (A) and (D) are correct.

31. $T^2 = \frac{4\pi^2}{GM}(R+h)^3$

Since $h \quad R$

$\Rightarrow \quad T^2 = \text{MIN} = \frac{4\pi^2 R^3}{GM}$

Also, $v = \frac{2\pi R}{T}$

$\Rightarrow \quad v$ is MAXIMUM

and total energy is $E = -\frac{GMm}{2R} = \text{MINIMUM}$

Hence, (A), (B) and (C) are correct.

32. Since, $t = nT$

$\Rightarrow \quad T = \frac{t}{n} = \frac{40}{20}$

$\Rightarrow \quad T = 2$ s

Now, $\Delta T = n\Delta T$

$$\Rightarrow \quad \frac{\Delta t}{t}=\frac{\Delta T}{T}$$

So, $\frac{1}{40}=\frac{\Delta T}{2}$

$$\Rightarrow \quad \Delta T=0.05$$

Since, time period, $T=2\pi\sqrt{\frac{\ell}{g}}$

$$\Rightarrow \quad \frac{\Delta T}{T}=-\frac{1}{2}\frac{\Delta g}{g}$$

$$\Rightarrow \quad -\frac{\Delta g}{g}=2\frac{\Delta T}{T}$$

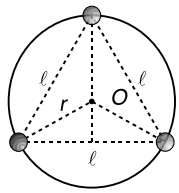
So, percentage error in g is $\frac{\Delta g}{g}\times 100\%$

$$\%\ \text{Error}\ =-2\frac{\Delta T}{T}\times 100\%=-2\times\frac{0.05}{2}\times 100\%$$

$$\Rightarrow \quad \%\ \text{Error}\ =5\%$$

Hence, (A) and (C) are correct.

33. The force of attraction between any two point masses is responsible for providing the necessary centripetal force to a mass to revolve in a circle of radius r. Using trigonometry, we get



$$\cos 30=\frac{\ell/2}{r}=\frac{\ell}{2r}$$

$$\Rightarrow \quad r=\frac{\ell}{2\cos 30}$$

$$\Rightarrow \quad r=\frac{\ell}{\sqrt{3}}$$

$$\Rightarrow \quad mr\omega^2=\frac{Gmm}{\ell^2}$$

$$\Rightarrow \quad m\frac{\ell}{\sqrt{3}}\omega^2=\frac{Gm^2}{\ell^2}$$

$$\Rightarrow \quad \omega=\sqrt{\frac{\sqrt{3}Gm}{\ell^3}}$$

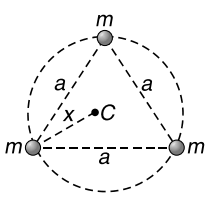
$$\Rightarrow \quad T=2\pi\sqrt{\frac{\ell^3}{\sqrt{3}Gm}}$$

$$\Rightarrow \quad T\propto \ell^{\frac{3}{2}} \text{ and} \qquad \{\text{OPTION } \textbf{(A)}\}$$

$$T\propto m^{-\frac{1}{2}} \qquad \{\text{OPTION } \textbf{(D)}\}$$

Hence, (A) and (D) are correct.

34.



Distance of any mass from centre is

$$r=\frac{a}{\sqrt{3}}$$

So, radius of circular path followed is

$$r=\frac{a}{\sqrt{3}}$$

Mass is moving in circular path of radius $r=\frac{a}{\sqrt{3}}$

Such that gravitational force on the particle due to other two provides the necessary centripetal force.

$$\Rightarrow \quad \frac{mv^2}{\left(\frac{a}{\sqrt{3}}\right)}=\frac{\sqrt{3}Gm^2}{a}$$

$$\Rightarrow \quad v=\sqrt{\frac{Gm}{a}}$$

$$\Rightarrow \quad T=\frac{2\pi\left(\frac{a}{\sqrt{3}}\right)}{v}=\sqrt{\frac{2\pi a^3}{3Gm}}$$

Total kinetic energy of the particles is

$$K=3\left(\frac{1}{2}mv^2\right)=\frac{3}{2}\frac{Gm^2}{a}$$

Potential energy U of the system is

$$U=-3\left(\frac{Gm^2}{a}\right)$$

$$\Rightarrow \quad \text{Total energy } E \text{ is}$$

$$E=U+K=-\frac{3}{2}\left(\frac{Gm^2}{a}\right)$$

$$\Rightarrow \quad \text{Binding Energy}\ =\frac{3}{2}\left(\frac{Gm^2}{a}\right)$$

Hence, (B) and (C) are correct.

35. Kinetic energy, $KE=\frac{GMm}{2r}$

Potential energy, $PE=-\frac{GMm}{r}$

and the total energy, $E=-\frac{GMm}{2r}$

Kinetic energy is always positive and $KE\propto\frac{1}{r}$

Potential energy is negative and $|PE|\propto\frac{1}{r}$

Similarly, total energy is also negative and $|E|\propto\frac{1}{r}$

Also $|E|<|PE|$, so from the graph we observe that A is kinetic energy, B is potential energy and C is total energy of the satellite.

Hence, (A), (B) and (D) are correct.

36. At all the places potential will increase by $\frac{GM}{R}$, so $V_\infty = \frac{GM}{R}$ and $V_{\text{centre}} = -\frac{3GM}{2R} + \frac{GM}{R} = -\frac{GM}{2R}$

Hence, (C) and (D) are correct.

37. **Conceptual (A), (B), (C) and (D) are correct.**

38. Both the stars will revolve about their centre of mass. So, if the centre of mass be at a distance x from $2m$, then

$$x = \frac{2m(0) + mr}{3m} = \frac{r}{3}$$

So, $r_1 = \frac{2r}{3}$ and $r_2 = \frac{r}{3}$

ω and T will be same for both the stars, so

$$K_1 = \frac{1}{2} I_1 \omega^2 \text{ and } K_2 = \frac{1}{2} I_2 \omega^2$$

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{I_1}{I_2} = \frac{m\left(\frac{2r}{3}\right)^2}{2m\left(\frac{r}{3}\right)^2} = 2$$

$$L_1 = I_1 \omega \text{ and } L_2 = I_2 \omega$$

$$\Rightarrow \quad \frac{L_1}{L_2} = \frac{I_1}{I_2} = 2$$

Hence, (A), (B) and (C) are correct.

Reasoning Based Questions

1. $\frac{d\vec{A}}{dt} = \frac{\vec{L}}{2m}$

Since $\vec{L} = \text{constant}$

$$\Rightarrow \quad \frac{d\vec{A}}{dt} = \text{constant}$$

Also, $\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega$

$$\Rightarrow \quad \frac{dA}{dt} = \frac{mr^2\omega}{2m} = \frac{L}{2m} = \text{constant}$$

Hence, the correct answer is (A).

2. The torque on a body is given by $\vec{\tau} = \frac{d\vec{L}}{dt}$. In case of planet orbiting around Sun no torque is acting on it. So,

$$\frac{d\vec{L}}{dt} = 0$$

$$\Rightarrow \quad \vec{L} = \text{constant}$$

Hence, the correct answer is (A).

3. Using energy conservation $\frac{1}{2} m v_e^2 - \frac{GMm}{R} = 0$

$$\Rightarrow \quad v_e = \sqrt{\frac{2Gm}{R}} = \sqrt{2gR}$$

Hence, the correct answer is (A).

4. Although no gravitational field is produced inside a symmetric shell, it produces a field at points outside of shell.

Hence, the correct answer is (D).

5. Work will be done only in bringing the unit mass from infinity upto the surface of shell because inside shell there is no gravitational field and in moving inside the shell no work will be done.

Hence, the correct answer is (D).

6. $g' = g\left(1 - \frac{2h}{R}\right)$

$$\Rightarrow \quad \Delta g = g - g' = g\left(\frac{2h}{R}\right)$$

$$\Rightarrow \quad \frac{dW}{dh} = \frac{d}{dh}(m\Delta g) = m\frac{d}{dh}(\Delta g) = m\frac{d}{dh}\left(\frac{2gh}{R}\right)$$

$$\Rightarrow \quad \frac{dW}{dh} = \frac{2mg}{R} = \text{constant}$$

Hence, the correct answer is (B).

7. $V_{in} = \frac{GM}{2R^3}(3R^2 - r^2)$

At surface, $r = R$, so $V_s = \frac{GM}{R}$

At centre, $r = 0$, so $V_{\text{centre}} = \frac{3}{2}\frac{GM}{R}$

$$\Rightarrow \quad V_{\text{centre}} = \frac{3}{2} V_S$$

$$\Rightarrow \quad V_{in} > V_S$$

V is not same everywhere as indicated by V_{in}.

Hence, the correct answer is (C).

8. The time period of satellite which is very near to earth is given by $T = 2\pi\sqrt{\frac{R}{g}} = 84 \text{ min} = 1 \text{ hr } 24 \text{ min}$

Hence, the correct answer is (A).

9. Work done in conservative field in cyclic process is zero.

Hence, the correct answer is (A).

10. Acceleration due to gravity is given by $g = \frac{GM}{r^2}$, so it does not depend on mass of body on which it is acting. Also, it is not a constant quantity because it changes with the change in value of both M and r (distance between two bodies).

Hence, the correct answer is (C).

CHAPTER 4

Linked Comprehension Type Questions

1. According to Kepler's Law, we have

$$T^2 \propto r^3$$

$$\Rightarrow \quad \frac{T_m^2}{T_e^2} = \frac{r_m^3}{r_e^3}$$

$$\Rightarrow \quad T_m = \left(\frac{r_m}{r_e}\right)^{3/2} T_e$$

Since $T_e = 1$ yr

$$\Rightarrow \quad T_m = \left(\frac{6\times10^{10}}{1.5\times10^{11}}\right)^{3/2} (1\text{ yr})$$

$$\Rightarrow \quad T_m = \left(\frac{6\times10^{10}}{15\times10^{10}}\right)^{3/2} \text{yr}$$

$$\Rightarrow \quad T_m = \left(\frac{2}{5}\right)^{3/2} \text{yr}$$

Hence, the correct answer is (D).

2. Since orbital velocity v_o is given by

$$v_o = \sqrt{\frac{GM}{r}}$$

$$\Rightarrow \quad (v_o)_m = \sqrt{\frac{GM_{\text{sun}}}{r_m}}$$

$$\Rightarrow \quad (v_o)_e = \sqrt{\frac{GM_{\text{sun}}}{r_e}}$$

$$\Rightarrow \quad \frac{(v_o)_m}{(v_o)_e} = \sqrt{\frac{r_e}{r_m}}$$

$$\Rightarrow \quad \frac{(v_o)_m}{(v_o)_e} = \sqrt{\frac{1.5\times10^{11}}{6\times10^{10}}} = \sqrt{\frac{15}{6}}$$

$$\Rightarrow \quad \frac{(v_o)_m}{(v_o)_e} = \sqrt{\frac{5}{2}}$$

Hence, the correct answer is (A).

3. $U_i = -\dfrac{GMm}{R} = -\dfrac{GMm}{R+\frac{R}{4}}$

$$\Rightarrow \quad \Delta U = U_f - U_i = \frac{1}{5}\frac{GMm}{R}$$

$$\Rightarrow \quad \Delta U = \frac{mgR}{5} \qquad \left\{\because g = \frac{GM}{R^2}\right\}$$

Hence, the correct answer is (C).

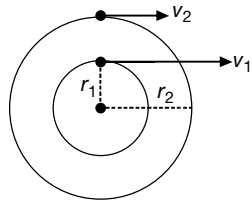
4. By Law of Conservation of Energy

$$\Delta U = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2}{5}gR}$$

Hence, the correct answer is (D).

5. $\dfrac{T_1^2}{T_2^2} = \dfrac{r_1^3}{r_2^3}$



$$\Rightarrow \quad \frac{1}{64} = \frac{r_1^3}{r_2^3}$$

$$\Rightarrow \quad r_2 = 4r_1$$

$$\Rightarrow \quad r_2 = 4\times10^4 \text{ km}$$

$$v_1 = \frac{2\pi r_1}{T_1} \text{ and } v_2 = \frac{2\pi r_2}{T_2}$$

$$\Rightarrow \quad v_1 = \frac{2\pi(10^4)}{1} \text{ kmh}^{-1}$$

and $v_2 = \dfrac{2\pi(4\times10^4)}{8} = \pi\times10^4 \text{ kmh}^{-1}$

$\Rightarrow$ Speed of S_2 relative to S_1 is v_{21}

$$v_{21} = -\pi\times10^4 \text{ kmh}^{-1}$$

Hence, the correct answer is (A).

6. $\omega = \dfrac{v_2 - v_1}{r_2 - r_1}$

Hence, the correct answer is (B).

7. $F = \begin{cases} \dfrac{GMm}{r^2} & , \quad r \geq R \\ 0 & , \quad r < R \end{cases}$

Hence, the correct answer is (A).

8. $F(r) = \begin{cases} \dfrac{GMm}{r^2} & , \quad r \geq R \\ \dfrac{4\pi G\rho rm}{3} & , \quad r < R \end{cases}$

where ρ is density of sphere

Hence, the correct answer is (C).

9. Given that $v_o = \dfrac{v_e}{2}$, where $v_e = \sqrt{\dfrac{2GM}{R}}$

Since $v_o = \sqrt{\dfrac{GM}{R+h}} = \dfrac{1}{2}\sqrt{\dfrac{2GM}{R}}$

$$\Rightarrow \quad R + h = 2R$$

$$\Rightarrow \quad h = R$$

Hence, the correct answer is (C).

10. Since the satellite is stopped suddenly, so its kinetic energy in this orbit becomes zero, however its gravitational potential energy in this orbit is

$$U = -\frac{GMm}{R+h} = -\frac{GMm}{2R}$$

Since $(U+K)_{\text{initial}} = (U+K)_{\text{final}}$

$$\Rightarrow \quad -\frac{GMm}{2R}+\frac{1}{2}m(0)^2 = -\frac{GMm}{R}+\frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}v^2 = \frac{GM}{2R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$$

Also, we know that $g = \frac{GM}{R^2}$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

Hence, the correct answer is (A).

11. $T = 2\pi\sqrt{\frac{R}{g}}$, substituting $g = \frac{GM}{R^2}$ we get,

$$T = 2\pi\sqrt{\frac{R^3}{GM}}$$

Hence, (A), (B) and (C) are correct.

12. Maximum speed is attained by the particle at centre. Applying Law of Conservation of Mechanical Energy (from surface to centre), we get

$$(U+K)_{\text{surface}} = (U+K)_{\text{centre}}$$

$$\Rightarrow \quad -\frac{GMm}{R}+\frac{1}{2}m(0)^2 = -\frac{3}{2}\frac{GMm}{R}+\frac{1}{2}mv^2$$

$$v = \sqrt{2\left(-\frac{GM}{R}+\frac{3GM}{2R}\right)} = \sqrt{\frac{GM}{R}}$$

Hence, the correct answer is (B).

13. $U = -\frac{GmM}{\sqrt{a^2+x^2}}$

Hence, the correct answer is (A).

14. Since $F = \frac{GMmx}{(a^2+x^2)^{3/2}}$

When $x = 0$, $F = 0$

Hence, the correct answer is (D).

15. For small x i.e., $x \quad a$, the force acting on the particle is

$$F \approx -\left(\frac{GMm}{a^3}\right)x$$

The negative sign showing that this force is directed towards the mean position (i.e. position where $F = 0$). Hence the particle will perform oscillations about the mean position i.e. the centre of the ring.

Hence, the correct answer is (B).

16. Given that $r = 3.6\times10^7$ m, $m = 5000$ kg, $v = 4000$ ms^{-1} and $\phi = 30°$

Angular momentum of satellite

$$L = mvr\sin\phi$$

$$\Rightarrow \quad L = 5000\times4000\times3.6\times10^7\times\sin30°$$

$$\Rightarrow \quad L = 3.6\times10^{14} \text{ Js}$$

Hence, the correct answer is (D).

17. Energy of the satellite is

$$E = K+U = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(5000)(4000)^2$$

$$\Rightarrow \quad K = 4\times10^{10} \text{ J}$$

$$\Rightarrow \quad U = -\frac{(6.67\times10^{-11})\times(5.97\times10^{24})\times5000}{3.6\times10^7}$$

$$\Rightarrow \quad U = -5.5\times10^{10} \text{ J}$$

$$\Rightarrow \quad E = K+U = -1.5\times10^{10} \text{ J}$$

Hence, the correct answer is (D).

18. Since $E = -\frac{GMm}{2a}$

So, semi-major axis a is given by

$$a = \frac{-GMm}{2E}$$

$$\Rightarrow \quad a = -\frac{6.67\times10^{-11}\times5.97\times10^{24}\times5000}{2\times(-1.5\times10^{10})}$$

$$\Rightarrow \quad a = 6.6\times10^7 \text{ m}$$

Hence, the correct answer is (A).

19. Eccentricity (e) of the orbit is given by

$$e = \left(1+\frac{2EL^2}{G^2M^2m^3}\right)$$

$$\Rightarrow \quad e = 1+\frac{2\times(-1.5\times10^{10})\times(3.6\times10^{14})^2}{(6.67\times10^{-11})^2\times(5.97\times10^{24})^2\times(5000)^3}$$

$$\Rightarrow \quad e = 0.804$$

Since, semi-minor axis b is given by

$$b = a\sqrt{1-e^2}$$

$$\Rightarrow \quad b = 6.6\times10^7\sqrt{1-(0.804)^2}$$

$$\Rightarrow \quad b = 3.92\times10^7 \text{ m}$$

Hence, the correct answer is (B).

20. Minimum distance $r_{\min}$ is

$$r_{\min} = a(1-e)$$

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$\Rightarrow \quad r_{\min} = 6.6\times10^7 \times (1-0.804)$

$\Rightarrow \quad r_{\min} = 1.29\times10^7$ m

Hence, the correct answer is (C).

21. Maximum distance $r_{\max}$ is

$$r_{\max} = a(1+e) = 6.6\times10^7\times(1+0.804)$$

$\Rightarrow \quad r_{\max} = 11.9\times10^7$ m

Hence, the correct answer is (C).

22. $F_1 = \dfrac{GMm}{(2R)^2} = \dfrac{GMm}{4R^2}$

Hence, the correct answer is (D).

23. Mass of complete sphere,

$$M = \frac{4}{3}\pi R^3\rho$$

Mass removed $M' = \dfrac{4}{3}\pi\left[\dfrac{R}{2}\right]^3\rho = \dfrac{1}{8}M$

Force due to hollow sphere F_2 at P equals force due to solid sphere at P minus force due to removed mass at P.

$$\Rightarrow \quad F_2 = \frac{GMm}{4R^2} - \frac{G(M/8)m}{(3R/2)^2}$$

$$\Rightarrow \quad F_2 = \frac{GMm}{R^2}\left[\frac{1}{4} - \frac{4}{9\times8}\right]$$

$$\Rightarrow \quad F_2 = \frac{GMm}{R^2}\left[\frac{1}{4} - \frac{1}{18}\right]$$

$$\Rightarrow \quad F_2 = \frac{GMm}{R^2}\left[\frac{9-2}{36}\right]$$

$$\Rightarrow \quad F_2 = \frac{7}{36}\frac{GMm}{R^2}$$

Hence, the correct answer is (D).

24. $\dfrac{F_1}{F_2} = \dfrac{GMm}{4R^2}\times\dfrac{36R^2}{7GMm} = \dfrac{9}{7}$

Hence, the correct answer is (A).

25. The system must revolve about the center of mass.
Hence, the correct answer is (B).

26. If we consider the origin at the heavier mass, then the center of mass is at a distance $\dfrac{\ell}{3}$ from the origin. So, the heavier star revolves in an orbit of radius $\ell/3$.
Hence, the correct answer is (C).

27. For m,

$$\frac{Gm(2m)}{\ell^2} = m\left(\frac{2\ell}{3}\right)\omega^2 \qquad \ldots(1)$$

For $2m$,

$$\frac{Gm(2m)}{\ell^2} = (2m)\left(\frac{\ell}{3}\right)\omega^2 \qquad \ldots(2)$$

Adding (1) and (2), we get

$$\frac{4Gm^2}{\ell^2} = \left(\frac{4m\ell}{3}\right)\omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3Gm}{\ell^3}}$$

Hence, the correct answer is (C).

28. $(\text{K.E.})_{\text{light}} = K_L = \dfrac{1}{2}\,\text{m}\left(\dfrac{2\ell\omega}{3}\right)^2 = \left(\dfrac{2}{9}m\ell\right)\omega^2$

$$(\text{K.E.})_{\text{heavy}} = K_H = \frac{1}{2}(2m)\left(\frac{\ell\omega}{3}\right)^2 = \frac{m\ell^2\omega^2}{9}$$

$$\Rightarrow \quad K_L + K_H = \frac{1}{3}m\ell^2\omega^2$$

Hence, the correct answer is (A).

29. Acceleration due to gravity near the surface of shell can be assumed to be uniform $(h \quad R)$.

$$g = \frac{G(2M)}{(2R)^2} = \frac{GM}{2R^2}$$

Since $h = \dfrac{1}{2}gt^2$

$$\Rightarrow \quad t = \sqrt{\frac{2h}{g}} = 2\sqrt{\frac{hR^2}{GM}}$$

Hence, the correct answer is (A).

30. $U_A = \sqrt{2gh} = \sqrt{2\left(\dfrac{Gm}{2R^2}\right)h} = \sqrt{\dfrac{Gmh}{R^2}}$

From A to B, field due to shell is zero, but field due to sphere is non-zero.

Hence, $t_{AB} < \dfrac{R}{V_A}$

$$\Rightarrow \quad t_{AB} < \frac{R^2}{\sqrt{GMh}}$$

Hence, the correct answer is (C).

31. Since $h \quad R$, so we have $K_A \approx 0$

Potential between A and B due to shell is constant. Applying the Law of Conservation of Energy, we get

$$K_A + U_A = K_B + U_B$$

$$\Rightarrow \quad K_B = U_A - U_B = m(V_A - V_B)$$

$$\Rightarrow \quad \frac{1}{2}mv_B^2 = m(V_A - V_B)$$

$\Rightarrow \quad v_B = \sqrt{2(V_A - V_B)}$

$\Rightarrow \quad v_B = \sqrt{2\left[-\frac{GM}{2R} + \frac{GM}{R}\right]}$

$\Rightarrow \quad v_B = \sqrt{\frac{GM}{R}}$

Hence, the correct answer is (D).

32-33. The correct answer is 32(C) and 33(B).

Combined solution to 32 and 33

If m be the mass of satellite, then by Law of Conservation of Angular Momentum, we have

$$mv_1(2R) = mv_2(4R)$$

$\Rightarrow \quad v_1 = 2v_2 \qquad \ldots(1)$

Also, by Law of Conservation of Mechanical Energy, we have

$$(U+K)_{2R} = (U+K)_{4R}$$

$\Rightarrow \quad -\frac{GmM}{2R} + \frac{1}{2}mv_1^2 = -\frac{GmM}{4R} + \frac{1}{2}mv_2^2$

$$\frac{1}{2}\left(v_1^2 - v_2^2\right) = \frac{GM}{R}\left(\frac{1}{2} - \frac{1}{4}\right)$$

$\Rightarrow \quad 4v_2^2 - v_2^2 = \frac{GM}{2R}$

$\Rightarrow \quad v_2 = \sqrt{\frac{GM}{6R}}$

Since, $v_1 = 2v_2$

$\Rightarrow \quad v_1 = \sqrt{\frac{2GM}{3R}}$

34. $\frac{mv_1^2}{r_1} = \frac{GmM}{(2R)^2}$

$\Rightarrow \quad r_1 = \frac{4R^2}{GM}v_1^2 = \left(\frac{4R^2}{GM}\right)\left(\frac{2GM}{3R}\right)$

$\Rightarrow \quad r_1 = \frac{8R}{3}$

Hence, the correct answer is (A).

35. Let M be the mass of the planet, m_1 be the mass of the moon 1, m_2 be the mass of moon 2 and m_3 be the mass of moon 3.

$$\frac{GM}{R_2^2} = \omega^2 R^2$$

$\Rightarrow \quad M = \frac{\omega^2 R_2^3}{G}$

$\Rightarrow \quad M = \left(\frac{2\pi}{3\times10^5}\right)^2 \times (9\times10^7)^3 \times \frac{1}{6.67\times10^{-11}}$

$\Rightarrow \quad M = 4\pi^2 \times 81\times10^{11} \times \frac{10^{11}}{6.67}$

$\Rightarrow \quad M = 4.8\times10^{24}$

Hence, the correct answer is (D).

36. $\frac{Gm_3}{R_3^2} = 0.2$

$\Rightarrow \quad \frac{6.67\times10^{-11}m_3}{(2\times10^5)^2} = 0.2$

$\Rightarrow \quad m_3 = \frac{0.2\times4\times10^{10}}{6.67\times10^{-11}}$

$\Rightarrow \quad m_3 \approx 0.12\times10^{21} = 1.2\times10^{20}$ kg

Hence, the correct answer is (C).

37. Centripetal Acceleration of Moon II is

$$a_C = \left(\frac{2\pi}{3\times10^5}\right)^2 \times 9\times10^7$$

$\Rightarrow \quad a_C = 4\pi^2 \times 0\times10^{-3}$

$\Rightarrow \quad a_C \approx 0.04\ \text{ms}^{-2}$

Hence, the correct answer is (B).

Matrix Match/Column Match Type Questions

1. A → (p, q, r); B → (r); C → (t); D → (p, q, r, s)
Conceptual

2. A → (q); B → (p); C → (p); D → (q)

$$E = \frac{Gm}{R^2} = \frac{G\left(\frac{4}{3}\pi R^3\right)\rho}{R^2}$$

$\Rightarrow \quad E \propto \rho R$

At surface, $V = -\frac{Gm}{R} = -\frac{G\left(\frac{4}{3}\pi R^3\rho\right)}{R}$

$\Rightarrow \quad V \propto \rho R^2$

$$V_{\text{centre}} = -\frac{3}{2}\frac{GM}{R}$$

At centre of a solid sphere field strength is zero.

3. A → (q); B → (r); C → (q); D → (p); E → (s)
At perihelion position planet is nearest to sun.

4. A → (p, q, r, s); B → (p, q, r, s); C → (q, r, s); D → (p, r)

$T = 2\pi\sqrt{\frac{r^3}{GM}}$, $v_0 = \sqrt{\frac{GM}{r}}$ and $E = -\frac{GMm}{2r}$

5. A → (p); B → (q); C → (q); D → (q)
Due to rotation of earth, variation in value of g is given by

$$g' = g - R\omega^2\cos^2\phi$$

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At pole $\phi = 90°$, so there is no effect of rotation of earth.

At equator g' will increase if ω decreases.

Further, T will increase with decrease in ω

From Kepler's Third Law $T^2 \propto r^3$, so r should also increase.

Since, $E = -\dfrac{GMm}{2r}$, so with increase in r, E also increases.

6. A → (r, t); B → (s); C → (p); D → (q)
Conceptual

7. A → (p); B → (q); C → (s); D → (t)

$$E = \frac{GM}{R^2},\ V = -\frac{GM}{R}$$

Gravitational field of the earth is actually the measure of acceleration due to gravity.

At height $h = R$:

$$E' = \frac{GM}{(R+h)^2} = \frac{GM}{4R^2} = \frac{E}{4}$$

$$V' = -\frac{GM}{2R}$$

i.e., E' decreases by a factor $\dfrac{1}{4}$ and V' increases by a factor of 2.

At depth $d = \dfrac{R}{2}$:

$$E' = E\left(\phi - \frac{d}{R}\right) = \frac{E}{2}$$

$$V' = -\frac{GM}{2R^3}\left(3R^2 - r^2\right)$$

$$V' = -\frac{GM}{2R^3}\left(3R^2 - \frac{R^2}{4}\right) = -\frac{11}{8}\frac{GM}{R}$$

i.e., E' decreases by a factor $\dfrac{1}{2}$ and V' also decreases by a factor $\dfrac{11}{8}$.

8. A → (q); B → (s); C → (p); D → (r)
Conceptual

9. A → (r, q); B → (s); C → (p); D → (r)
Conceptual

10. A → (s); B → (r); C → (p); D → (p)
Conceptual

11. A → (r); B → (q); C → (p); D → (r)
For planet revolving around sun in an elliptical orbit, we have from Kepler's Second Law

$$\frac{\Delta A}{\Delta t} = \frac{L}{2m}$$

$$\Rightarrow \quad \frac{\pi ab}{T} = \frac{L}{2m}$$

$$\Rightarrow \quad \frac{2\pi mab}{T} = L = \text{constant}$$

Integer/Numerical Answer Type Questions

1. Area that cannot be covered by the satellite moving in an orbit of radius r as shown in figure is

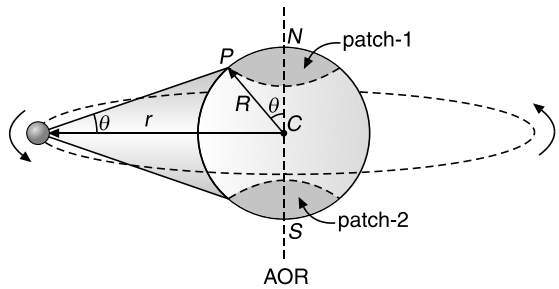
$$A = 4\pi R^2 - \frac{4\pi R^2\sqrt{r^2 - R^2}}{r}$$

$$\Rightarrow \quad A = 2\Omega R^2$$

where, $\Omega = 2\pi(1 - \cos\theta)$

$$\Rightarrow \quad \Omega = 2\pi\left(1 - \frac{\sqrt{r^2 - R^2}}{r}\right)$$

$$\Rightarrow \quad A = 4\pi R^2\left(1 - \frac{\sqrt{r^2 - R^2}}{r}\right)$$



According to the problem, $A = 0.25 \times 4\pi R^2$

$$\Rightarrow \quad 1 - \frac{\sqrt{r^2 - R^2}}{r} = 0.25$$

$$\Rightarrow \quad \frac{\sqrt{r^2 - R^2}}{r} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{r^2 - R^2}{r^2} = \frac{9}{16}$$

$$\Rightarrow \quad 16r^2 - 16R^2 = 9r^2$$

$$\Rightarrow \quad r = \frac{4}{\sqrt{7}}R = 1.515R$$

$$\Rightarrow \quad \frac{r}{R} = \frac{4}{\sqrt{7}} = 1.51$$

2. Since, $dW = \vec{F} \cdot d\vec{\ell}$

$$\Rightarrow \quad dW = m\vec{E}_g \cdot d\vec{\ell}$$

$$\Rightarrow \quad dW = m\left(2\hat{i} + 3\hat{j}\right) \cdot \left(\hat{i}dx + \hat{j}dy\right)$$

$$\Rightarrow \quad dW = m\left(2dx + 3dy\right) \qquad \ldots(1)$$

Since the particle is moved along the line

$$3y + 2x = 5$$

$$\Rightarrow \quad d\left(3y + 2x\right) = 0$$

$$\Rightarrow \quad 3dy + 2dx = 0$$

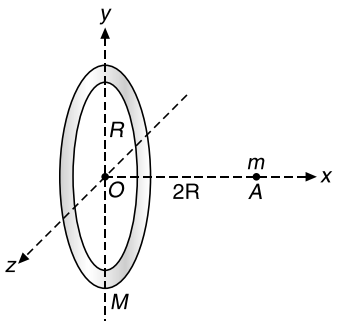
$$\Rightarrow \quad 3dy = -2dx$$

$\Rightarrow \quad dW = 0 \qquad \{\because \text{ of } (1)\}$

$\Rightarrow \quad W = 0$

3. Applying Law of Conservation of Energy, we get

$$(U+K)_A = (U+K)_O$$



$$U_A = -\frac{GMm}{\sqrt{R^2+4R^2}} = -\frac{GMm}{\sqrt{5R}}$$

$$K_A = 0$$

Finally, when m passes through O, we have

$$U_O = -\frac{GMm}{R} \text{ and } K_O = \frac{1}{2}mv^2$$

Since $(U+K)_A = (U+K)_O$

$$\Rightarrow \quad -\frac{GMm}{\sqrt{5}R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$\Rightarrow \quad v = \sqrt{\frac{2(\sqrt{5}-1)GM}{\sqrt{5}R}}$$

$$\Rightarrow \quad v = \left[2\left(1-\frac{1}{\sqrt{5}}\right)\frac{GM}{R}\right]^{\frac{1}{2}}$$

$$\Rightarrow \quad x = 2,\ y = \sqrt{5} \text{ and } z = 2$$

$$\Rightarrow \quad \frac{y^2}{xz} = \frac{(\sqrt{5})^2}{(2)(2)} = \frac{5}{4} = 1.25$$

4. By Law of Conservation of Energy, we have

$$(U+K)_{surface} = (U+K)_{at\ \infty}$$

$$\Rightarrow \quad -\frac{GmM}{R} + \frac{1}{2}mu^2 = 0 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad -\frac{2GM}{R} + u^2 = v^2$$

Since $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad -v_e^2 + u^2 = v^2$$

$$\Rightarrow \quad v^2 = -(11.2)^2 + (15)^2 = -125 + 225$$

$$\Rightarrow \quad v = 10 \text{ kms}^{-1}$$

5. By Law of Conservation of Angular Momentum, we have

$$m(v_0 \cos 60°)4R = mvR$$

$$\Rightarrow \quad \frac{v}{v_0} = 2$$

Applying Law of Conservation of Energy, we get

$$-\frac{GMm}{4R} + \frac{1}{2}mv_0^2 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}v^2 - \frac{1}{2}v_0^2 = \frac{3}{4}\frac{GM}{R}$$

$$\Rightarrow \quad v_0 = \frac{1}{\sqrt{2}}\sqrt{64\times10^6} = \frac{8000}{\sqrt{2}} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_0 = 4000\sqrt{2} \text{ ms}^{-1}$$

$$\Rightarrow \quad v_0 = 5656 \text{ ms}^{-1}$$

6. (a) Let x be the displacement of ring, then the displacement of the particle is $(3-x)$ m. Since, no external force is acting on the system and particles are initially at rest, so the centre of mass will not move. Hence,

$$(5.4\times10^9)x = (6\times10^8)(3-x)$$

$$\Rightarrow \quad x = 0.3 \text{ m} = 30 \text{ cm}$$

(b) By Law of Conservation of Linear Momentum, we have

$$0 = 5.4\times10^9 v_1 - 6\times10^8 v_2$$

$$\Rightarrow \quad v_2 = 9v_1 \qquad \ldots(1)$$

By Law of Conservation of Energy, we have

$$(U+K)_{\text{initial}} = (U+K)_{\text{final}}$$

$$\Rightarrow \quad -\frac{Gm_1m_2}{\sqrt{4^2+3^2}} + 0 = -\frac{Gm_1m_2}{4} + \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$\Rightarrow \quad Gm_1m_2\left(\frac{1}{4}-\frac{1}{5}\right) = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2(81v_1^2)$$

$$\Rightarrow \quad \frac{G(9m_2^2)}{20} = \frac{1}{2}(90m_2)v_1^2$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{Gm_2}{100}} = \sqrt{\frac{6.67\times10^{-11}\times6\times10^8}{100}}$$

$$\Rightarrow \quad v_1 = 0.02 \text{ ms}^{-1} = 2 \text{ cms}^{-1}$$

$$\Rightarrow \quad v_2 = 9v_1 = 18 \text{ cms}^{-1}$$

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7. Potential at origin is

$$V_0 = -\frac{2GM}{a}$$

Potential at point $(0,\ 0,\ 2\sqrt{3}a)$ is

$$V_P = -\frac{2GM}{\sqrt{a^2+12a^2}} = -\frac{2}{\sqrt{13}}\frac{GM}{a}$$

By Law of Conservation of Energy, we get

$$(U+K)_O = (U+K)_P$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{M}{2}\right)v^2 + \left(\frac{M}{2}\right)V_0 = \left(\frac{M}{2}\right)V_P$$

$$\Rightarrow \quad v = \sqrt{2(V_P - V_0)}$$

$$\Rightarrow \quad v = \sqrt{2\left(\frac{2GM}{a} - \frac{2}{\sqrt{13}}\frac{GM}{a}\right)}$$

$$\Rightarrow \quad v = 2\sqrt{\frac{GM}{a}\left(1-\frac{1}{\sqrt{13}}\right)}$$

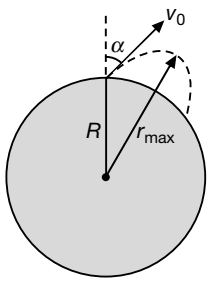
$$\Rightarrow \quad v = \sqrt{\frac{4GM}{a}\left(1-\frac{1}{\sqrt{13}}\right)}$$

$$\Rightarrow \quad \alpha = 4 \text{ and } \beta = 13$$

8. Let v be the speed of the projectile at highest point and r_{max} its distance from the centre of the earth. By Law of Conservation of Angular Momentum and Law of Conservation of Mechanical Energy, we have

$$mv_0 \sin\alpha = mvr_{max} \qquad \ldots(1)$$

$$\frac{1}{2}mv_0^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{r_{max}} \qquad \ldots(2)$$



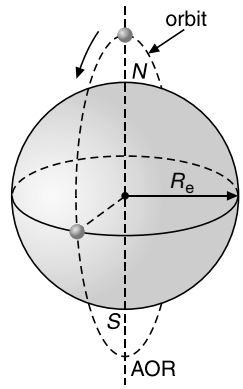
Solving these two equation, we get

$$r_{max} = \frac{3R}{2}$$

So, the maximum height is $h_{max} = r_{max} - R = \frac{R}{2}$

$$\Rightarrow \quad * = 2$$

9. A satellite which rotates with angular speed equal to earth's rotation has angular speed of revolution is given by



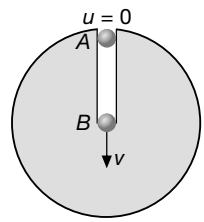
$$\omega = \frac{2\pi}{T}, \text{ where } T = 24 \text{ hr}$$

When satellite moves from a point above north pole to a point above equator, it traverses an angle $\frac{\pi}{2}$. So, this time taken is given by

$$T' = \frac{\pi/2}{(2\pi/T)} = \frac{T}{4} = 6 \text{ hr}$$

10. By Law of Conservation of Energy, we have

$$(U+K)_{surface} = (U+K)_{centre}$$



$$\Rightarrow \quad -\frac{GMm}{R} + \frac{1}{2}m(0)^2 = -\frac{3}{2}\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{GMm}{2R}$$

where m is the mass of the ball

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$$

Velocity of ball just after collision

$$v' = ev = \frac{1}{2}\sqrt{\frac{GM}{R}}$$

Let r be the distance from the centre upto the point P, where the ball reaches after collision. Then by Law of Conservation of Energy, we have

$$(U+K)_{centre} = (U+K)_P$$

where $U_P = -\frac{GMm}{2R^3}(3R^2 - r^2)$

$$\Rightarrow \quad -\frac{3}{2}\frac{GMm}{R}+\frac{1}{2}mv'^2=-\frac{GMm}{2R^3}(3R^2-r^2)+0$$

But $v'=\frac{1}{2}\sqrt{\frac{GM}{R}}$

$$\Rightarrow \quad -\frac{3}{2}\frac{GMm}{R}+\frac{1}{8}\frac{GMm}{R}=-\frac{GMm}{2R^3}(3R^2-r^2)$$

$$\Rightarrow \quad -\frac{11}{8}=-\left(\frac{3R^2-r^2}{2R^2}\right)$$

$$\Rightarrow \quad 22R^2=24R^2-8r^2$$

$$\Rightarrow \quad 8r^2=2R^2$$

$$\Rightarrow \quad r^2=\frac{R^2}{4}$$

$$\Rightarrow \quad r=\frac{R}{2}$$

So, the desired distance, is

$$s=R+\frac{R}{2}+\frac{R}{2}=2R$$

$$\Rightarrow \quad x=2$$

11. At a point P at a distance r from Moon, we have net gravitational field equal to zero

$$\Rightarrow \quad g=\frac{GM}{r^2}-\frac{G(81M)}{(D-r)^2}=0$$

$$\Rightarrow \quad D-r=9r$$

$$\Rightarrow \quad 10r=D$$

So, $r=\frac{D}{10}$ from the Moon and $\frac{9D}{10}$ from the earth.

$$\Rightarrow \quad x=9 \text{ and } y=10$$

$$\Rightarrow \quad y-x=1$$

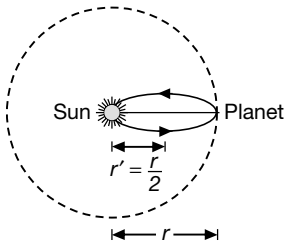
12. Consider an imaginary planet moving along a strongly extended flat ellipse, the extreme points of which are located on the planet's orbit and at the centre of the sun, the semi-major axis of the orbit of such a planet would apparently be half the semi-major axis of the planet's orbit. So, the time period of the imaginary planet T' according to Kepler's Law will be given by:

$$\left(\frac{T'}{T}\right)=\left(\frac{r'}{r}\right)^{3/2}$$

$$\Rightarrow \quad T'=T\left(\frac{1}{2}\right)^{3/2} \qquad \left\{\because\ r'=\frac{r}{2}\right\}$$

So, time taken by the planet to fall onto the sun is

$$t=\frac{T'}{2}=\frac{T}{2}\left(\frac{1}{2}\right)^{3/2}$$



$$\Rightarrow \quad t=\frac{\sqrt{2}}{8}T=\sqrt{\frac{2}{64}}T=\sqrt{\frac{1}{32}}=\frac{1}{\sqrt{2^5}}$$

$$\Rightarrow \quad x=5$$

13. For point mass at distance $r=3\ell$

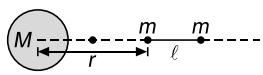
$$\frac{GMm}{(3\ell)^2}-\frac{Gm^2}{\ell^2}=ma \qquad \ldots(1)$$

For point mass at distance $r=4\ell$

$$\frac{GMm}{(4\ell)^2}+\frac{Gm^2}{\ell^2}=ma \qquad \ldots(2)$$

Equating the two equations, we have

$$\frac{GMm}{9\ell^2}-\frac{Gm^2}{\ell^2}=\frac{GMm}{16\ell^2}+\frac{Gm^2}{\ell^2}$$



$$\frac{7GMm}{144}=\frac{2Gm^2}{\ell^2}$$

$$m=\frac{7M}{288}$$

$$\Rightarrow \quad k=7$$

14. Since, $g_p=\frac{GM_p}{R_p^2}=\frac{4}{3}G\pi R_p\rho_p$

$$\Rightarrow \quad \frac{g_p}{g_e}=\frac{R_p\rho_p}{R_e\rho_e}$$

Also, $v_e=\sqrt{2gR}$

$$\Rightarrow \quad \frac{v_p}{v_e}=\sqrt{\frac{g_pR_p}{g_eR_e}}=\left(\frac{g_p}{g_e}\right)\sqrt{\frac{\rho_e}{\rho_p}}=\frac{\sqrt{12}}{11}\times\sqrt{\frac{3}{4}}$$

$$\Rightarrow \quad \frac{v_p}{v_e}=\frac{2\sqrt{3}}{11}\times\frac{\sqrt{3}}{2}=\frac{3}{11}$$

$$\Rightarrow \quad v_p=3 \text{ kms}^{-1}$$

15. The gravitational field due to cylinder at a distance x from its axis is $E_g=\frac{2G\lambda}{x}$

CHAPTER 4

$$\Rightarrow \quad m\left(\frac{2G\lambda}{x}\right)=\frac{mv^2}{x}$$

where λ is the linear mass density given by

$$\lambda=\rho(\pi R^2)$$

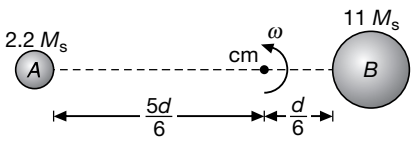
$$\Rightarrow \quad 2G\rho\pi R^2=v^2$$

$$\Rightarrow \quad v=R\sqrt{2G\rho\pi}$$

$$\Rightarrow \quad v=R(2G\rho\pi)^{\frac{1}{2}}$$

$$\Rightarrow \quad *=2$$

16.



$$\frac{\text{Total angular momentum about cm}}{\text{Angular momentum of B about cm}}=\frac{L}{L_B}$$

$$\Rightarrow \quad \frac{L}{L_B}=\frac{(2.2M_S)\left(\frac{5\omega d}{6}\right)\left(\frac{5d}{6}\right)+(11M_S)\left(\frac{\omega d}{6}\right)\left(\frac{d}{6}\right)}{(11M_S)\left(\frac{\omega d}{6}\right)\left(\frac{d}{6}\right)}$$

$$\Rightarrow \quad \frac{L}{L_B}=6$$

17. The time period T of the satellite is

$$T=2\pi\sqrt{\frac{r^3}{GM}}=2\pi\sqrt{\frac{r^3}{gR^2}}$$

$$\Rightarrow \quad T=2\sqrt{\frac{\pi^2r^3}{gR^2}}=2\sqrt{\frac{r^3}{R^2}}$$

where, $r=6400+1600=800$ km $=8000\times10^3$ m

and $R=6400\times10^3$ m

Substituting these values, we get

$$T=2\sqrt{\frac{(8000\times10^3)^3}{(6400\times10^3)^2}}=7071 \text{ s}$$

Further, orbital speed,

$$v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{gR^2}{r}}$$

$$\Rightarrow \quad v=\sqrt{\left(\frac{9.8}{8000\times10^3}\right)}\times(6400\times10^3)$$

$$\Rightarrow \quad v=7155 \text{ ms}^{-1}$$

Let t be the time interval between two successive moments at which the satellite is overhead to an observer at a fixed position on the equator. Since both satellite and earth are moving in the same sense (i.e. from west to east) with angular speed ω_S and ω_E respectively so, we can write the time of separation (t) as

$$t=\frac{2\pi}{\omega_S-\omega_E}$$

where $\omega_S=\frac{2\pi}{7071}$ and $\omega_E=\frac{2\pi}{86400}$

$$\Rightarrow \quad t=\frac{86400\times7071}{86400-7071}$$

$$\Rightarrow \quad t=\frac{611\times10^6}{79329}$$

$$\Rightarrow \quad t=7702 \text{ s}$$

18. Applying conservation of mechanical energy, we have

$$(U+K)_{\text{at }B}=(U+K)_{\text{at }A}$$

$$\Rightarrow \quad U_B+K_B=U_A+K_A$$

$$\Rightarrow \quad U_B+0=U_A+\frac{1}{2}mv_A^2$$

where $U_B=mV_B$ and $U_A=mV_A$, so we have

$$\frac{1}{2}mv_A^2=U_B-U_A=m(V_B-V_A)$$

$$\Rightarrow \quad v_A=\sqrt{2(V_B-V_A)} \qquad \ldots(1)$$

Potential at A

$$V_A=\begin{pmatrix}\text{Potential Due}\\ \text{to the Complete}\\ \text{Sphere at } A\end{pmatrix}-\begin{pmatrix}\text{Potential Due}\\ \text{to the Spherical}\\ \text{Cavity at } A\end{pmatrix}$$

$$V_A=-\frac{3}{2}\frac{GM}{R}-\left(-\frac{GM'}{r}\right)=\frac{GM'}{r}-\frac{3}{2}\frac{GM}{R}$$

where

$$M=\frac{4}{3}\pi R^3\rho,\ r=\frac{R}{2} \text{ and } M'=\frac{4}{3}\pi r^3\rho=\frac{\pi\rho R^3}{6}$$

Substituting the values, we get

$$V_A=\frac{G}{R}\left(\frac{\pi\rho R^3}{3}-2\pi\rho R^3\right)=-\frac{5}{3}\pi G\rho R^2$$

Potential at B

$$V_B=\begin{pmatrix}\text{Potential Due}\\ \text{to the Complete}\\ \text{Sphere at } B\end{pmatrix}-\begin{pmatrix}\text{Potential Due}\\ \text{to the Spherical}\\ \text{Cavity at } B\end{pmatrix}$$

$$\Rightarrow \quad V_B=-\frac{GM}{2R^3}(3R^2-r^2)-\left(-\frac{3}{2}\frac{GM'}{r}\right)$$

where

$$M=\frac{4}{3}\pi R^3\rho,\ r=\frac{R}{2} \text{ and } M'=\frac{4}{3}\pi r^3\rho=\frac{\pi\rho R^3}{6}$$

$$\Rightarrow \quad V_B = -\frac{11}{8}\frac{GM}{R} + \frac{3GM'}{R}$$

$$\Rightarrow \quad V_B = \frac{G}{R}\left(\frac{\pi\rho R^3}{2} - \frac{11\pi\rho R^3}{6}\right) = -\frac{4}{3}\pi G\rho R^2$$

$$\Rightarrow \quad V_B - V_A = \frac{1}{3}\pi G\rho R^2$$

So, from equation (1)

$$v = \sqrt{\frac{2}{3}\pi G\rho R^2}$$

Also, $v_e = \sqrt{\frac{2GM_{\text{net}}}{R}}$

where $M_{\text{net}} = M - M' = \frac{7}{6}\pi R^3\rho$

$$\Rightarrow \quad v_e = \sqrt{\frac{2G\left(\frac{7}{6}\pi R^3\right)\rho}{R}}$$

$$\Rightarrow \quad v_e = \sqrt{\frac{7}{3}\pi G\rho R^2}$$

$$\Rightarrow \quad \frac{v_e}{v} = \sqrt{\frac{7}{2}}$$

$$\Rightarrow \quad \frac{v_e^2}{v^2} = \frac{7}{2}$$

$$\Rightarrow \quad \frac{2v_e^2}{v^2} = 7$$

19. At height h, we have

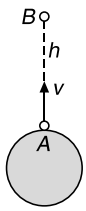
$$g_h = \frac{gR^2}{(R+h)^2} \qquad \ldots(1)$$

Given that $g_h = \frac{g}{9}$

Substituting in equation (1) we get,

$$\frac{1}{9} = \left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad h = 2R$$



Applying Law of Conservation of Energy, from A to B, we get

$$(U+K)_A = (U+K)_B$$

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0$$

Since $h = 2R$, so we have

$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{3R}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{2GM}{3R}$$

$$\Rightarrow \quad v = \sqrt{\frac{4GM}{3R}}$$

Since $v_e = \sqrt{\frac{2GM}{R}}$

Given that $v_e = v\sqrt{N}$

$$\Rightarrow \quad \sqrt{\frac{2GM}{R}} = \sqrt{\frac{4GM}{3R}}\sqrt{N}$$

$$\Rightarrow \quad N = \frac{3}{2} = 1.5$$

20. Let E be the gravitational field at x due to the complete sphere.
If E_1 be the field due to hole and E_2 be the field due to the remaining portion, then we have

$$E = E_1 + E_2$$

$$\Rightarrow \quad E_2 = E - E_1$$

$$\Rightarrow \quad E_2 = \frac{GM}{x^2} - \frac{Gm}{\left(x - \frac{R}{2}\right)^2} \qquad \ldots(1)$$

where, $M = \frac{4}{3}\pi R^3\rho_0$ and $m = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\rho_0$

Substituting the values in equation (1), we get

$$E_2 = -\left(\frac{\pi G\rho_0 R^3}{6}\right)\left[\frac{1}{\left(x - \frac{R}{2}\right)^2} - \frac{8}{x^2}\right]$$

$$\Rightarrow \quad E_2 = -\left(\frac{\pi G\rho_0 R^3}{6}\right)\left[\frac{1}{\left(2R - \frac{R}{2}\right)^2} - \frac{8}{(2R)^2}\right]$$

$$\Rightarrow \quad E_2 = -\frac{\pi G\rho_0 R^3}{6}\left(\frac{4}{9R^2} - \frac{2}{R^2}\right)$$

$$\Rightarrow \quad E_2 = -\frac{\pi G\rho_0 R}{6}\left(\frac{4-18}{9}\right)$$

$$\Rightarrow \quad E_2 = \frac{14}{54}\pi G\rho_0 R$$

$$\Rightarrow \quad E_2 = \left(\frac{7}{27}\right)\pi G\rho_0 R$$

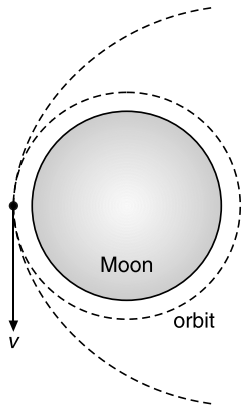
Since, $E = \left(\frac{a}{a+20}\right)\pi G\rho_0 R$

$$\Rightarrow \quad \frac{a}{a+20} = \frac{7}{27}$$

$$\Rightarrow \quad a = 7$$

CHAPTER 4

21. Figure shows the corresponding situation



Since the spaceship follows a parabolic trajectory tangential to the moon. When at the surface of moon it has speed equal to escape speed

$$v_e = \sqrt{\frac{2GM}{R}}$$

Now to transform it into a circular orbit, its speed should be decreased to orbital speed v_0 given by

$$v_0 = \sqrt{\frac{GM}{R}}$$

So, change in speed is

$$\Delta v = v_e - v_0$$

$$\Rightarrow \quad \Delta v = \left(\sqrt{2}-1\right)\sqrt{\frac{GM}{R}}$$

$$\Rightarrow \quad x = 2$$

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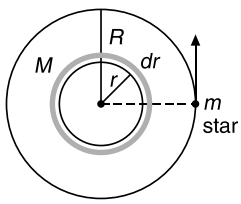
1. Since $dm = \rho dV$, where $dV = 4\pi r^2 dr$



$$\Rightarrow \quad dm = \left(\frac{k}{r}\right)\left(4\pi r^2 dr\right) = 4\pi k r dr$$

$$\Rightarrow \quad M = \int_0^R dm = \int_0^R 4\pi k r dr = 4\pi k \frac{r^2}{2}\Big|_0^R$$

$$\Rightarrow \quad M = 2\pi k\left(R^2 - 0\right) = 2\pi k R^2$$

For circular motion, gravitational force will provide the centripetal force so that

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \quad \frac{G\left(2\pi k R^2\right)m}{R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \quad v = \sqrt{2\pi G k R}$$

Time period T is given by

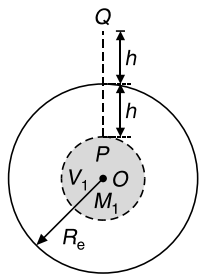
$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{2\pi G k R}} \propto \sqrt{R}$$

$$\Rightarrow \quad T^2 \propto R$$

Hence, the correct answer is (C).

2. Let M be the mass of earth, M_1 be the mass of shaded portion and R be the radius of earth.

At height h, $g_h = \dfrac{GM}{(R+h)^2}$



At depth h, we have

$$M_1 = \rho V_1 = \frac{M}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi(R-h)^3 = \frac{M(R-h)^3}{R^3}$$

Weight of body is same at P and Q, so

$$mg_P = mg_Q$$

$$\Rightarrow \quad g_P = g_Q$$

$$\Rightarrow \quad \frac{GM_1}{(R-h)^2} = \frac{GM}{(R+h)^2}$$

$$\Rightarrow \quad \frac{GM(R-h)^3}{(R-h)^2 R^3} = \frac{GM}{(R+h)^2}$$

$$\Rightarrow \quad (R-h)(R+h)^2 = R^3$$

$$\Rightarrow \quad R^3 - hR^2 - h^2R - h^3 + 2R^2h - 2Rh^2 = R^3$$

$$\Rightarrow \quad R^2 - Rh^2 - h^3 = 0$$

$$\Rightarrow \quad R^2 - Rh - h^2 = 0$$

$$\Rightarrow \quad h^2 + Rh - R^2 = 0$$

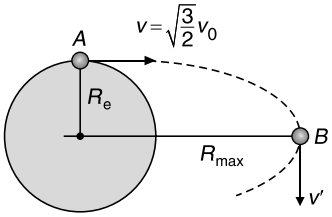
$$\Rightarrow \quad h = \frac{-R \pm \sqrt{R^2 + 4R^2}}{2}$$

$$\Rightarrow \quad h = \frac{-R + \sqrt{5}R}{2} = \left(\frac{\sqrt{5}-1}{2}\right)R$$

Hence, the correct answer is (A).

3. Since, $v_0 = \sqrt{\frac{GM}{R_e}}$

$$(U+K)_A = (U+K)_B$$



$$\Rightarrow \quad \frac{-GMm}{R_e} + \frac{1}{2}mv^2 = \frac{-GMm}{R_{max}} + \frac{1}{2}mv'^2 \qquad \ldots(1)$$

Also, $vR_e = v'R_{max}$ $\ldots(2)$

Solving equations (1) and (2), we get

$$R_{max} = 3R_e$$

Hence, the correct answer is (B).

4. According to Gauss Law for gravitation, we have

$$E(4\pi r^2) = \int \rho_0 4\pi r^2 dr$$

$$\Rightarrow \quad Er^2 = 4\pi G \int_0^r \rho_0 \left(1 - \frac{r^2}{R^2}\right) r^2 dr$$

$$\Rightarrow \quad |E| = E = 4\pi G\rho_0 \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$$

For E to be maximum, $\frac{dE}{dr} = 0$

$$\Rightarrow \quad r = \sqrt{\frac{5}{9}}R$$

Hence, the correct answer is (B).

5. Given that $E_G = \frac{Ax}{(x^2+a^2)^{3/2}}$ and $V_\infty = 0$

Since, $\int_{V_\infty}^{V_x} dV = -\int_\infty^x \vec{E}_G \cdot d\vec{x}$

$$\Rightarrow \quad V_x - V_\infty = -\int_\infty^x \frac{Ax}{(x^2+a^2)^{3/2}} dx$$

Substitute $x^2 + a^2 = z$ i.e., $2xdx = dz$

$$\Rightarrow \quad V_x - 0 = -\int_\infty^x \frac{Adz}{2(z)^{3/2}} = \left(\frac{A}{z^{1/2}}\right)\Big|_\infty^x = \left(\frac{A}{(x^2+a^2)^{1/2}}\right)\Big|_\infty^x$$

$$\Rightarrow \quad V_x = \frac{A}{(x^2+a^2)^{1/2}} - 0 = \frac{A}{(x^2+a^2)^{1/2}}$$

Hence, the correct answer is (A).

6. For orbit close to the planet, $R + h \approx R$

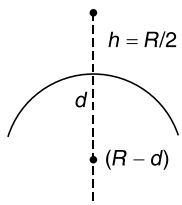
$$v_{orbit} = \sqrt{\frac{GM}{R}}$$

Since, $v_{escape} = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad \frac{v_{orbit}}{v_{escape}} = \frac{1}{\sqrt{2}}$$

Hence, the correct answer is (C).

7. Since, $g_1 = \frac{GM}{(R+h)^2} = \frac{GM}{(3R/2)^2}$ $\ldots(1)$



Also, $g_2 = \frac{GM(R-d)}{R^3}$ $\ldots(2)$

Given that, $g_1 = g_2$

$$\Rightarrow \quad \frac{GM}{(3R/2)^2} = \frac{GM(R-d)}{R^3}$$

$$\Rightarrow \quad \frac{4}{9} = \frac{(R-d)}{R}$$

$$\Rightarrow \quad 4R = 9R - 9d$$

$$\Rightarrow \quad 5R = 9d$$

$$\Rightarrow \quad \frac{d}{R} = \frac{5}{9}$$

Hence, the correct answer is (D).

8. At equator, we have $g_e = g - R\omega^2$

For $h \quad R$, we have

$$g_2 \approx g\left(1 - \frac{2h}{R}\right) = g - \frac{2gh}{R}$$

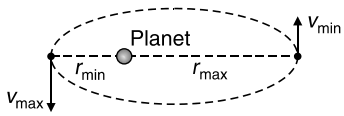
Since, $g_1 = g_2$

$\Rightarrow \quad R\omega^2 = \dfrac{2gh}{R}$

$\Rightarrow \quad h = \dfrac{R^2\omega^2}{2g}$

Hence, the correct answer is (D).

9. Applying conservation of angular momentum, we get



$r_{min}v_{max} = r_{max}v_{min}$ …(1)

Given that, $v_{min} = \dfrac{v_{max}}{6}$

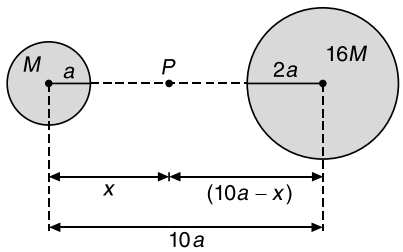
From equation (1), we get

$\dfrac{r_{min}}{r_{max}} = \dfrac{v_{min}}{v_{max}} = \dfrac{1}{6}$

Hence, the correct answer is (A).

10. The minimum speed will take the body to the neutral point P after which the bigger planet will automatically attract the body towards itself. So,

$$\frac{GM}{x^2} = \frac{G(16M)}{(10a-x)^2}$$



$\Rightarrow \quad \dfrac{1}{x} = \dfrac{4}{(10a-x)}$

$\Rightarrow \quad 4x = 10a - x$

$\Rightarrow \quad x = 2a$ …(1)

Applying law of conservation of mechanical energy, we get

$$-\frac{GMm}{8a} - \frac{G(16M)m}{2a} + KE = -\frac{GMm}{2a} - \frac{G(16M)m}{8a}$$

$\Rightarrow \quad KE = GMm\left(\dfrac{1}{8a} + \dfrac{16}{2a} - \dfrac{1}{2a} - \dfrac{16}{8a}\right)$

$\Rightarrow \quad KE = GMm\left(\dfrac{1+64-4-16}{8a}\right)$

$\Rightarrow \quad \dfrac{1}{2}mv^2 = GMm\left(\dfrac{45}{8a}\right)$

$\Rightarrow \quad v = \sqrt{\dfrac{90GM}{8a}} = \dfrac{3}{2}\sqrt{\dfrac{5GM}{a}}$

Hence, the correct answer is (B).

11. $W = 196 - mR\omega^2$

Hence, the correct answer is (D).

12. Gravitational field on the surface of a solid sphere

$I_g = \dfrac{GM}{R^2}$

From graph, $\dfrac{GM_1}{(1)^2} = 2$ and $\dfrac{GM_2}{(2)^2} = 3$

$\Rightarrow \quad \dfrac{M_1}{M_2} = \dfrac{1}{6}$

Hence, the correct answer is (D).

13. Initially, the body of mass m is moving in a circular orbit of radius R. So, it must be moving with orbital speed given by

$v_0 = \sqrt{\dfrac{GM}{R}}$

After collision, let the combined mass moves with speed v_1, then by conservation of momentum, we get

$$mv_0 + \left(\frac{m}{2}\right)\left(\frac{v_0}{2}\right) = \left(\frac{3m}{2}\right)v_1$$

$\Rightarrow \quad v_1 = \dfrac{5v_0}{6}$

Since after collision, the speed is not equal to orbital speed at the point. So, motion cannot be circular. Also, velocity will remain tangential, so it cannot fall vertically towards the planet. Their speed after collision is less than escape speed $\sqrt{2}v_0$, so they cannot escape gravitational field. Hence their motion will be elliptical around the planet.

Hence, the correct answer is (A).

14. The escape velocity is

$v_e = \sqrt{\dfrac{2GM}{R}}$

So, $v_A = \sqrt{\dfrac{2GM}{R}}$ and $v_B = \sqrt{\dfrac{2G(M/2)}{R/2}} = \sqrt{\dfrac{2GM}{R}}$

$\Rightarrow \quad \dfrac{v_A}{v_B} = 1 = \dfrac{n}{4}$

$\Rightarrow \quad n = 4$

Hence, the correct answer is (A).

15. Since, $U_1 + K_1 = U_2 + K_2$

$\Rightarrow \quad -\dfrac{GM_em}{10R} + \dfrac{1}{2}mv_0^2 = -\dfrac{GM_em}{R} + \dfrac{1}{2}mv^2$

$$\Rightarrow \quad \frac{9}{10}\left(\frac{GM_em}{R}\right)+\frac{1}{2}mv_0^2=\frac{1}{2}mv^2$$

$$\Rightarrow \quad \frac{9}{10}\left(\frac{1}{2}mv_e^2\right)+\frac{1}{2}mv_0^2=\frac{1}{2}mv^2$$

$$\Rightarrow \quad v^2=\frac{9}{10}v_e^2+v_0^2$$

$$\Rightarrow \quad v^2=\frac{9}{10}\times(11.2)^2+(12)^2$$

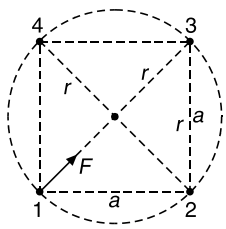
$$\Rightarrow \quad v^2=112.896+144$$

$$\Rightarrow \quad v=16.027\approx 16 \text{ kms}^{-1}$$

Hence, the correct answer is 16.

16. Net force on 1 acting towards the centre of circle is

$$F=\frac{GM^2}{a^2}(\sqrt{2})+\frac{GM^2}{2a^2}=\frac{Mv^2}{r}$$



$$\Rightarrow \quad \frac{Mv^2}{\left(\frac{a}{\sqrt{2}}\right)}=\frac{GM^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right) \qquad \left\{\because r=\frac{a}{\sqrt{2}}\right\}$$

$$\Rightarrow \quad v^2=\frac{GM}{a}\left(1+\frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow \quad v=\sqrt{\frac{GM}{a}\left(1+\frac{1}{2\sqrt{2}}\right)}=1.16\sqrt{\frac{GM}{a}}$$

Hence, the correct answer is (B).

17. Let m_0 be the mass of rocket, then

$$E=\frac{GM_em_0}{R_e} \text{ and } E'=\frac{GM_mm_0}{R_m}$$

Since $V_e=64V_m$

$$\rho R_e^3=64\rho R_m^3$$

$$\Rightarrow \quad R_e=4R_m$$

$$\Rightarrow \quad \frac{E'}{E}=\left(\frac{M_m}{M_e}\right)\left(\frac{R_e}{R_m}\right)=\left(\frac{1}{64}\right)(4)=\frac{1}{16}$$

$$\Rightarrow \quad E'=\frac{E}{16}$$

Hence, the correct answer is (C).

18. $E=\frac{GM}{(3a)^2}+\frac{2GM}{(3a)^2}$

$$\Rightarrow \quad E=\frac{GM}{3a^2}$$

Hence, the correct answer is (C).

19. $M=\int_0^R(4\pi r^2dr)\frac{K}{r^2}$

$$\Rightarrow \quad M=4\pi KR$$

$$\Rightarrow \quad G\left(\frac{4\pi KR}{R^2}\right)=\frac{v_0^2}{R}$$

$$\Rightarrow \quad v_0=\sqrt{4\pi GK}$$

Since $T=\frac{2\pi R}{v_0}=\frac{2\pi R}{\sqrt{4\pi GK}}$

$$\Rightarrow \quad \frac{T}{R}=\text{constant}$$

Hence, the correct answer is (A).

20. Given that $g_h=\frac{g}{2}$

Since $g_h=\frac{gR^2}{(R+h)^2}$

$$\Rightarrow \quad \frac{g}{2}=g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad \frac{R}{R+h}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad h=(\sqrt{2}-1)R$$

$$\Rightarrow \quad h=6400\times 0.414$$

$$\Rightarrow \quad h=2649.6 \text{ km}$$

$$\Rightarrow \quad h=2.6\times 10^6 \text{ m}$$

Hence, the correct answer is (D).

21. Since $T=\frac{2\pi r}{v_0}$ and $v_0=\sqrt{\frac{GM}{r}}$

$$\Rightarrow \quad T=2\pi r\sqrt{\frac{r}{GM}}=2\pi\sqrt{\frac{r^3}{GM}}$$

$$\Rightarrow \quad T=2\pi\sqrt{\frac{(202)^3\times 10^{12}}{6.67\times 10^{-11}\times 8\times 10^{22}}} \text{ s}$$

$$\Rightarrow \quad T=7812.2 \text{ s}$$

$$\Rightarrow \quad T \quad 2.17 \text{ hr}$$

So, number of revolutions is

$$N=\frac{24}{T}\approx 11$$

Hence, the correct answer is (C).

CHAPTER 4

22. Given that $\frac{g_e}{g_p}=\frac{9}{4}$

Since $g=\frac{GM}{R^2}$

$\Rightarrow \quad \frac{M_e\times R_p^2}{R_e^2\times M_p}=\frac{9}{4}$

$\Rightarrow \quad 9\times\left(\frac{R_p}{R_e}\right)^2=\frac{9}{4}$

$\Rightarrow \quad R_p=\frac{R_e}{2}=\frac{R}{2}$

Hence, the correct answer is (D).

23. Areal velocity is given by

$$\frac{\Delta A}{\Delta t}=\frac{L}{2m}$$

Hence, the correct answer is (C).

24. $E_1=U_f-U_i$

$\Rightarrow \quad E_1=\frac{GMm}{R}-\frac{GMm}{(R+h)}$

$\Rightarrow \quad E_1=\frac{GMmh}{R(R+h)}$

KE of satellite in this orbit is

$$KE=E_2=\frac{GMm}{2r}=\frac{GMm}{2(R+h)}$$

$\Rightarrow \quad E_2=\frac{1}{2}\frac{GMm}{(R+h)}$

Given that $E_1=E_2$

$\Rightarrow \quad \frac{h}{R}=\frac{1}{2}$

$\Rightarrow \quad h=\frac{R}{2}$

Hence, the correct answer is (A).

25. Since $U=-2\left(\frac{1}{2}mv^2\right)$

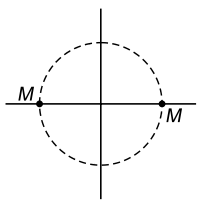
In order to escape, we have $U+K=0$

$\Rightarrow \quad K=mv^2$

Hence, the correct answer is (B).

26. $2R=d=2\times10^{11}$ m

$\Rightarrow \quad R=10^{11}$ m



$\Rightarrow$ Let v_0 be the minimum speed of meteorite at O, then by Law of Conservation of Energy, we have

$$\frac{1}{2}mv_0^2-\frac{2GMm}{R}=0$$

$\Rightarrow \quad v_0=\sqrt{\frac{4GM}{R}}=\sqrt{\frac{4\times6.67\times10^{-11}\times3\times10^{31}}{10^{11}}}$

$\Rightarrow \quad v_0=2.83\times10^5\ \text{ms}^{-1}\approx2.8\times10^5\ \text{ms}^{-1}$

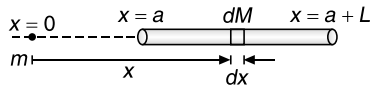
Hence, the correct answer is (A).

27. Since, $v_0=\sqrt{2gR}$, $v_e=\sqrt{2gR}$

$\Rightarrow \quad \Delta v=\sqrt{gR}\left(\sqrt{2}-1\right)$

Hence, the correct answer is (B).

28.



$$dF=\frac{GmdM}{x^2}, \text{ where } dM=\lambda dx$$

$\Rightarrow \quad dM=\left(A+Bx^2\right)dx$

So, $dF=-Gm\int_a^{L+a}\frac{\left(A+Bx^2\right)dx}{x^2}$

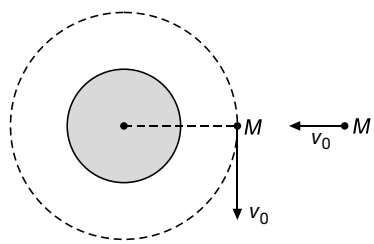
$$F=-Gm\left[-A\left(\frac{1}{L+a}-\frac{1}{a}\right)+BL\right]$$

$\Rightarrow \quad F=-Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

Hence, the correct answer is (C).

29. Orbital velocity v_0 is given by

$$v_0=\sqrt{\frac{GM}{R}}$$



After collision, we have

$$mv_0\left(-\hat{j}\right)+mv_0\left(-\hat{i}\right)=2m\vec{v}$$

$\Rightarrow \quad \vec{v}=-\frac{v_0}{2}\hat{i}-\frac{v_0}{2}\hat{j}$

$\Rightarrow \quad |\vec{v}|=\frac{v_0}{\sqrt{2}}=0.7v_0$

$\Rightarrow \quad v<v_0$

So, the path will be elliptical.

Hence, the correct answer is (C).

30. Since $\frac{v^2}{r}=\frac{GM}{r^2}$

$\Rightarrow \quad v^2=\frac{GM}{r}$

$\Rightarrow \quad \frac{T_A}{T_B}=\frac{\frac{1}{2}mv_A^2}{\frac{1}{2}2mv_B^2}=\frac{1}{2}\left(\frac{v_A}{v_B}\right)^2$

$\Rightarrow \quad \frac{T_A}{T_B}=\frac{1}{2}\frac{R_B}{R_A}=1$

Hence, the correct answer is (A).

31. Since $U=-\frac{k}{2r^2}$

Force acting on the particle is $F=-\frac{dU}{dr}=\frac{k}{r^3}$

This force provides necessary centripetal force, so

$\frac{mv^2}{r}=\frac{k}{r^3}$

$\Rightarrow \quad mv^2=\frac{k}{r^2}$

Kinetic energy of particle, $K=\frac{1}{2}mv^2=\frac{k}{2r^2}$

Total energy of the particle $=U+K=-\frac{k}{2r^2}+\frac{k}{2r^2}=0$

Hence, the correct answer is (C).

32. Since, central force is given by

$F_c\propto\frac{1}{R^n}$

$\Rightarrow \quad F_c=k\left(\frac{1}{R^n}\right)$

$\Rightarrow \quad m\omega^2R=k\frac{1}{R^n}$

$\Rightarrow \quad m\frac{(2\pi)^2}{T^2}=k\frac{1}{R^{n+1}}$

$\Rightarrow \quad T^2\propto R^{n+1}$

$\Rightarrow \quad T\propto R^{\frac{(n+1)}{2}}$

Hence, the correct answer is (C).

33. Initially, total energy is $E_i=-\frac{GMm}{2R}$

Final total energy is $E_f=-\frac{GM\left(\frac{m}{2}\right)}{2\left(\frac{R}{2}\right)}-\frac{GM\left(\frac{m}{2}\right)}{2\left(\frac{3R}{2}\right)}$

$\Rightarrow \quad E_f=-\frac{2GMm}{3R}$

Required difference in energies is $\Delta E=E_f-E_i$

$\Rightarrow \quad \Delta E=-\frac{GMm}{R}\left(\frac{2}{3}-\frac{1}{2}\right)=-\frac{GMm}{6R}$

Hence, the correct answer is (D).

34. $F_1=\frac{GM_eM_m}{r_1^2}$ and $F_2=\frac{GM_eM_s}{r_2^2}$

$\Rightarrow \quad \Delta F_1=-\frac{2GM_eM_m}{r_1^3}\Delta r_1$ and $\Delta F_2=-\frac{2GM_eM_s}{r_2^3}\Delta r_2$

$\Rightarrow \quad \frac{\Delta F_1}{\Delta F_2}=\frac{M_m\Delta r_1}{r_1^3}\frac{r_2^3}{M_s\Delta r_2}=\left(\frac{M_m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right)$

Given that

$\Delta r_1=\Delta r_2=2R_{\text{earth}}$, $M_m=8\times10^{22}$ kg

$M_s=2\times10^{30}$ kg, $r_1=0.4\times10^6$ km,

$r_2=150\times10^6$ km.

$\Rightarrow \quad \frac{\Delta F_1}{\Delta F_2}=2$

Hence, the correct answer is (A).

35. Effect of rotation of earth on acceleration due to gravity is given by $g'=g-R\omega^2\cos^2\phi$

where ϕ is the latitude angle. There will be no change in gravity at poles because $\phi=90^\circ$ at the poles and at all other points as ω increases, g' will decrease.

Hence, the correct answer is (C).

36. This force provides the centripetal force to the particle to move in a circular orbit.

$\Rightarrow \quad \frac{mv^2}{r}=\frac{16}{r}+r^3$

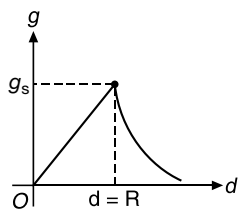
Kinetic energy, $K=\frac{1}{2}mv^2=\frac{1}{2}(16+r^4)$

$\Rightarrow \quad \frac{K_1}{K_2}=\frac{\frac{16+1}{2}}{\frac{16+256}{2}}=\frac{17}{272}=0.0625$

$\Rightarrow \quad \frac{K_1}{K_2} \quad 6\times10^{-2}$

Hence, the correct answer is (A).

37. Variation of g inside earth surface



For $d < R$, $g = \left(\frac{Gm}{R^2}\right)d$

For $d = R$, $g_s = \frac{Gm}{R^2}$

For $d > R$, $g = \frac{Gm}{d^2}$

Hence, the correct answer is (D).

38. Here, the weight of person on the equator is W. If the earth rotates about its axis, then weight is $\frac{3W}{4}$

Radius of the earth $= 6400$ km

The acceleration due to gravity at the equator is

$$g_e = g - R\omega^2$$

$$\Rightarrow \quad \frac{3}{4}g = g - R\omega^2$$

$$\Rightarrow \quad R\omega^2 = \frac{g}{4}$$

$$\Rightarrow \quad \omega = \sqrt{\frac{g}{4R}} = \sqrt{\frac{10}{4\times 6400\times 10^3}} = 0.625\times 10^{-3} \text{ rads}^{-1}$$

$$\Rightarrow \quad \omega \approx 0.63\times 10^{-3} \text{ rads}^{-1}$$

Hence, the correct answer is (A).

39. **The correct answer is (A).**

40. $v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

So, increase in velocity is

$$\Delta v = \sqrt{gR}\left(\sqrt{2}-1\right)$$

Hence, the correct answer is (A).

41. Let the area of the ellipse be A. According to Kepler's Second Law, areal velocity of a planet around the sun is constant, i.e., $\frac{dA}{dt} = \text{constant}$, so we have

$$\frac{t_1}{t_2} = \frac{\text{Area of } abcsa}{\text{Area of } adcsa} = \frac{\frac{A}{2}+\frac{A}{4}}{\frac{A}{2}-\frac{A}{4}} = \frac{\frac{3A}{4}}{\frac{A}{4}} = 3$$

$$\Rightarrow \quad t_1 = 3t_2$$

Hence, the correct answer is (C).

42. Gravitational pull on the astronaut $F_G = \frac{GMm}{(R+h)^2}$

Net force on the astronaut is zero.

Hence, the correct answer is (C).

43. $V_P = V_{\text{sphere at } P} - V_{\text{cavity at } P}$

Since, $V = -\frac{GM}{2R^3}\left(3R^2 - r^2\right)$

$$\Rightarrow \quad V_{\text{sphere at P}} = -\frac{GM}{2R^3}\left(3R^2 - \left(\frac{R}{2}\right)^2\right) = -\frac{11GM}{8R}$$

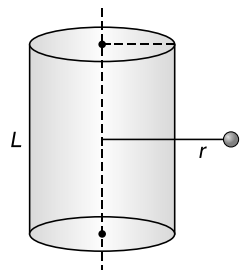
Now, $V_{\text{cavity at P}} = -\frac{3}{2}\frac{G\left(\frac{M}{8}\right)}{\left(\frac{R}{2}\right)} = -\frac{3GM}{8R}$

$$\Rightarrow \quad V_P = -\frac{11GM}{8R} - \left(-\frac{3GM}{8R}\right) = -\frac{GM}{R}$$

Hence, the correct answer is (B).

44. Centripetal force is provided by the gravitational force which is proportional to $\frac{1}{r}$. So, $F \propto \frac{1}{r}$

$$\Rightarrow \quad \frac{mv_0^2}{r} \propto \frac{1}{r}$$



$$\Rightarrow \quad v_0 = \text{constant}$$

Since $T = \frac{2\pi r}{v}$

$$\Rightarrow \quad T \propto r$$

Hence, the correct answer is (C).

45. Potential $V(r)$ due to a large planet of radius R is given by

$$V_0(r) = -\frac{GM}{r} \text{ for } r > R$$

$$V(r) = \frac{-GM}{R} \text{ for } r = R$$

$$V_{in} = -\frac{3}{2}\frac{GM}{R}\left(1 - \frac{r^2}{3R^2}\right) \text{ for } r < R$$

Hence, the correct answer is (B).

46. $F = F_{\text{gravitational force on M}} = \frac{Mv^2}{R}$

$$\Rightarrow \quad F = 2\left(\frac{GM^2}{\left(\sqrt{2}R\right)^2}\frac{1}{\sqrt{2}}\right) + \frac{GM^2}{(2R)^2} = \frac{Mv^2}{R}$$

$$\Rightarrow \quad \frac{GM^2}{\sqrt{2}R^2} + \frac{GM^2}{4R^2} = \frac{Mv^2}{R}$$

$$\Rightarrow \quad v = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$$

Hence, the correct answer is (D).

47. At surface, $E_i = -\dfrac{GMm}{R}$

In orbit, $E_f = -\dfrac{GMm}{2(3R)} = -\dfrac{GMm}{6R}$

$\Rightarrow$ Required energy is

$\Rightarrow \quad \Delta E = E_f - E_i = \dfrac{5GMm}{6R}$

Hence, the correct answer is (A).

48. Energy required to make the spaceship reach the free space is

$$\Delta E = \frac{GMm}{R}$$

Since $g = \dfrac{GM}{R^2}$

$\Rightarrow \quad \Delta E = gR^2 \times \dfrac{m}{R}$

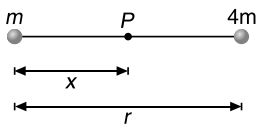
$\Rightarrow \quad \Delta E = mgR$

$\Rightarrow \quad \Delta E = 1000 \times 10 \times 6400 \times 10^3 = 64 \times 10^9$ J

$\Rightarrow \quad \Delta E = 6.4 \times 10^{10}$ J

Hence, the correct answer is (C).

49. Let x be the distance of the point P from the mass m where gravitational field is zero.



$\Rightarrow \quad \dfrac{Gm}{x^2} = \dfrac{G(4m)}{(r-x)^2}$

$\Rightarrow \quad \left(\dfrac{x}{(r-x)}\right)^2 = \dfrac{1}{4}$

$\Rightarrow \quad x = \dfrac{r}{3}$ ...(1)

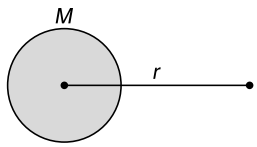
Gravitational potential at a point P i.e. at $x = \dfrac{r}{3}$ is

$$V = -\frac{Gm}{x} - \frac{G(4m)}{(r-x)} = -\frac{Gm}{\left(\frac{r}{3}\right)} - \frac{G(4m)}{\left(r - \frac{r}{3}\right)}$$

$\Rightarrow \quad V = -\dfrac{3Gm}{r} - \dfrac{3G(4m)}{2r} = -9\dfrac{Gm}{r}$

Hence, the correct answer is (D).

50. The acceleration due to gravity at a height h from the ground is given as $\dfrac{g}{9}$.



$$\frac{GM}{r^2} = \left(\frac{GM}{R^2}\right)\frac{1}{9}$$

$\Rightarrow \quad r = 3R$

The height above the ground is $2R$

Hence, the correct answer is (A).

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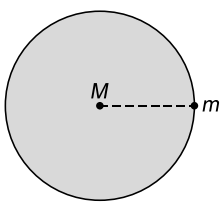
Single Correct Choice Type Problems

1. For a particle revolving in a circular orbit of radius r due to the gravitational attraction of inner cloud of mass M, we have

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$\Rightarrow \quad M = \dfrac{v^2 r}{G} = \dfrac{2mv^2 r}{2Gm}$

Since $K = \dfrac{1}{2}mv^2 = \text{constant}$



$\Rightarrow \quad mv^2 = 2K$

$\Rightarrow \quad M = \dfrac{2Kr}{Gm}$

$\Rightarrow \quad dM = \dfrac{2Kdr}{Gm}$ ...(1)

Also, we know that

$$dM = \rho(r)dV$$

$\Rightarrow \quad dM = \rho(r)4\pi r^2 dr$

So, equation (1) becomes,

$$\rho(r)4\pi r^2 dr = \frac{2Kdr}{Gm}$$

$\Rightarrow \quad \dfrac{\rho(r)}{m} = n(r) = \dfrac{K}{2\pi Gm^2 r^2}$

Hence, the correct answer is (B).

CHAPTER 4

2. Given that $v_e = 11.2 \text{ kms}^{-1} = \sqrt{\dfrac{2GM_e}{R_e}}$

By Law of Conservation of Energy, we have

$$K_i + U_i = K_f + U_f$$

$$\Rightarrow \frac{1}{2}mv_s^2 - \frac{GM_s m}{r} - \frac{GM_e m}{R_e} = 0 + 0$$

where, r is the distance of rocket from sun

$$\Rightarrow v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2GM_s}{r}}$$

Since, $M_s = 3\times10^5 M_e$ and $r = 2.5\times10^4 R_e$

$$\Rightarrow v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2G3\times10^5 M_e}{2.5\times10^4 R_e}}$$

$$\Rightarrow v_s = \sqrt{\frac{2GM_e}{R_e}\left(1 + \frac{3\times10^5}{2.5\times10^4}\right)}$$

$$\Rightarrow v_s = \sqrt{\frac{2GM_e}{R_e}\times13}$$

$$\Rightarrow v_s \quad 42 \text{ kms}^{-1}$$

Hence, the correct answer is (C).

3. Given, $R_{\text{planet}} = R = \dfrac{R_{\text{earth}}}{10}$

Since, density $\rho = \dfrac{M_{\text{earth}}}{\frac{4}{3}\pi R_{\text{earth}}^3}$

Also, $\rho = \dfrac{M_{\text{planet}}}{\frac{4}{3}\pi R_{\text{planet}}^3}$

$$\Rightarrow M_{\text{planet}} = \frac{M_{\text{earth}}}{10^3} = \frac{M_e}{1000}$$

Let the acceleration due to gravity at surface of planet and at the surface of earth be g_p and g_e respectively. Then

$$g_p = \frac{GM_{\text{planet}}}{R_{\text{planet}}^2} = \frac{GM_e(10)^2}{(10)^3 R_e^2} = \frac{GM_e}{10R_e^2}$$

$$\Rightarrow g_p = \frac{g_e}{10}$$

The value of g inside the planet at a distance x from centre of the planet is

$$g_{\text{inside}} = g_{\text{surface of planet}}\left(\frac{x}{R}\right) = g_p\left(\frac{x}{R}\right)$$

So, total force acting on wire is

$$F = \int_{\frac{4R}{5}}^{R} (\lambda dx) g_p \left(\frac{x}{R}\right)$$

$$\Rightarrow F = \frac{\lambda g_p}{R}\left(\frac{x^2}{2}\right)\Bigg|_{4R/5}^{R}$$

Substituting the given values, we get

$$F = 108 \text{ N}$$

Hence, the correct answer is (B).

4. In circular orbit of a satellite, potential energy U is

$$U = -2\times(\text{kinetic energy}) = -2\times\frac{1}{2}mv^2 = -mv^2$$

Just to escape from the gravitational pull, its total mechanical energy should be zero. Therefore, its kinetic energy should be $+mv^2$.

Hence, the correct answer is (B).

5. For annular disc, gravitational potential at the point P lying on the axis at a distance x from centre is

$$V_P = -\frac{2GM}{\left(R_2^2 - R_1^2\right)}\left(\sqrt{R_2^2 + x^2} - \sqrt{R_1^2 + x^2}\right)$$

where $R_2 = 4R$, $R_1 = 3R$ and $x = 4R$

$$\Rightarrow V_P = -\frac{2GM}{7R^2}\left(4\sqrt{2}R - 5R\right)$$

Also, $V_\infty = 0$

Since $W_{P\to\infty} = m_0\left(V_\infty - V_P\right)$, where $m_0 = 1$ unit

$$\Rightarrow W_{P\to\infty} = \frac{2GM}{7R}\left(4\sqrt{2} - 5\right)$$

Hence, the correct answer is (A).

6. For $r \le R$, $\dfrac{mv^2}{r} = \dfrac{GmM}{r^2}$ …(1)

Here, $M = \left(\dfrac{4}{3}\pi r^3\right)\rho_0$

Substituting in Equation (1), we get $v \propto r$

i.e. v-r graph is a straight line passing through origin.

For $r > R$, we have

$$\frac{mv^2}{r} = \frac{Gm\left(\frac{4}{3}\pi R^3\right)\rho_0}{r^2}$$

$$\Rightarrow v \propto \frac{1}{\sqrt{r}}$$

The corresponding v-r graph will be as shown in option (C).

Hence, the correct answer is (C).

7. In case of binary star system, angular velocity and hence, the time period of both the stars are equal.

Hence, the correct answer is (D).

8. Time period of a satellite very close to earth's surface is 84.6 min. Time period increases as the distance of the satellite from the surface of earth increases. So, time period of spy satellite orbiting a few 100 km above

the earth's surface should be slightly greater than 84.6 min . Therefore, the most appropriate option is (C) or 2 h

Hence, the correct answer is (C).

9. Since $T_1 = 2\pi\sqrt{\frac{\ell}{g}}$

$$T_2 = 2\pi\sqrt{\frac{\ell}{g_h}}$$

Now $g_h = \frac{gR^2}{(R+h)^2}$

$$\Rightarrow \quad g_h = \frac{g}{4} \qquad \{\because\ h = R\}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \sqrt{\frac{g}{g/4}} = 2$$

Hence, the correct answer is (D).

10. Force on satellite is always towards earth, therefore, acceleration of satellite S is always directed towards centre of the earth due to which net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout. Since, the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

Hence, the correct answer is (A).

11. From Kepler's Third Law, we have

$$T^2 \propto r^3$$

$$\Rightarrow \quad T \propto (r)^{3/2}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\Rightarrow \quad T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = (365)\left(\frac{1}{2}\right)^{3/2}$$

$$\Rightarrow \quad T_2 \approx 129 \text{ days}$$

Hence, the correct answer is (B).

12. $F \propto R^{-5/2}$

This gravitational force of attraction provides necessary centripetal force to planet to revolve about the massive star.

$$\Rightarrow \quad mR\omega^2 \propto R^{-5/2}$$

$$\Rightarrow \quad \omega^2 \propto R^{-7/2}$$

$$\Rightarrow \quad \frac{4\pi^2}{T^2} \propto R^{-7/2}$$

$$\Rightarrow \quad T^2 \propto R^{7/2}$$

Hence, the correct answer is (B).

13. $\Delta U = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$

$$\Rightarrow \quad \Delta U = -\frac{GMm}{2R} + \frac{GMm}{R} \qquad \{\because\ h = R\}$$

$$\Rightarrow \quad \Delta U = \frac{GMm}{2R} = \frac{1}{2}m\left(\frac{GM}{R^2}\right)R = \frac{mgR}{2}$$

Hence, the correct answer is (A).

14. Since, $g = \frac{GM}{R^2}$

$$\Rightarrow \quad g \propto \frac{1}{R^2}$$

$$\Rightarrow \quad \frac{\Delta g}{g} = -2\frac{\Delta R}{R}$$

So, g will increase, if R decreases

$$\Rightarrow \quad \frac{\Delta g}{g} = -2(-1\%) = 2\%$$

Hence, the correct answer is (C).

Multiple Correct Choice Type Problems

1. Gravitational field at a distance r due to mass $m\left(=\frac{4}{3}\pi r^3\rho\right)$ is

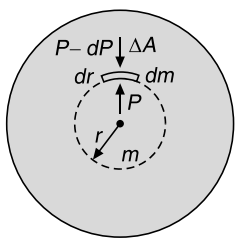
$$E = \frac{G\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = \frac{4G\rho\pi r}{3}$$

Consider a small element of width dr and area ΔA at a distance r from the centre. Pressure force on this element is due to the gravitational force on dm from m inwards towards the centre.

$$\Rightarrow \quad (dP)\Delta A = E(dm)$$

where $dm = (\Delta A)(dr)\rho$ and $m = \left(\frac{4}{3}\pi r^3\right)\rho$

$$\Rightarrow \quad -dP\Delta A = \left(\frac{4}{3}G\pi\rho r\right)(\rho\Delta A dr)$$



$$\Rightarrow \quad -\int_0^P dP = \int_R^r \left(\frac{4G\rho^2\pi}{3}\right)rdr$$

$$\Rightarrow \quad -P = \frac{4G\rho^2\pi}{3\times 2}(r^2 - R^2)$$

CHAPTER 4

$$\Rightarrow \quad P=\frac{2G\rho^2\pi}{3}(R^2-r^2)$$

$$\Rightarrow \quad P=k(R^2-r^2)\text{, where } k=\frac{2G\rho^2\pi}{3}=\text{constant}$$

For, $r=\frac{3R}{4}$, $P_1=k\left(R^2-\frac{9R^2}{16}\right)=k\left(\frac{7R^2}{16}\right)$

For, $r=\frac{2R}{3}$, $P_2=k\left(R^2-\frac{4R^2}{9}\right)=k\left(\frac{5R^2}{9}\right)$

$$\Rightarrow \quad \frac{P_1}{P_2}=\frac{63}{80}$$

For, $r=\frac{3R}{5}$, $P_3=k\left(R^2-\frac{9}{25}R^2\right)=k\left(\frac{16R^2}{25}\right)$

For, $r=\frac{2R}{5}$, $P_4=k\left(R^2-\frac{4R^2}{25}\right)=k\left(\frac{21R^2}{25}\right)$

$$\Rightarrow \quad \frac{P_3}{P_4}=\frac{16}{21}$$

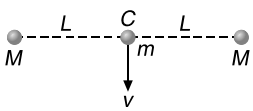
For $r=\frac{R}{2}$, $P_5=k\left(R^2-\frac{R^2}{4}\right)=k\left(\frac{3R^2}{4}\right)$

For $r=\frac{R}{3}$, $P_6=k\left(R^2-\frac{R^2}{9}\right)=k\left(\frac{8R^2}{9}\right)$

$$\Rightarrow \quad \frac{P_5}{P_6}=\frac{27}{32}$$

Hence, (B) and (C) are correct.

2.



Let v is the minimum velocity, then by Law of Conservation of Energy, we have

$$(U+K)_C=(U+K)_\infty$$

$$\Rightarrow \quad \left(-\frac{GMm}{L}\right)2+\frac{1}{2}mv^2=0+0$$

$$\Rightarrow \quad v=2\sqrt{\frac{GM}{L}}$$

Hence, (B) and (D) are correct.

3. $$v_{es}=\sqrt{\frac{2GM}{R}}=\sqrt{\frac{2G\left(\frac{4}{3}\pi R^3\right)\rho}{R}}=\sqrt{\frac{4G\rho}{3}}R$$

$$v_{es}\propto R$$

Surface area of P is $A=4\pi R_P^2$

Surface area of Q is $4A=4\pi R_Q^2$

$$\Rightarrow \quad R_Q=2R_P$$

Mass of R is $M_R=M_P+M_Q$

$$\Rightarrow \quad \left(\frac{4}{3}\pi R_R^3\right)\rho=\left(\frac{4}{3}\pi R_P^3\right)\rho+\left(\frac{4}{3}\pi R_Q^3\right)\rho$$

$$\Rightarrow \quad R_R^3=R_P^3+R_Q^3$$

$$\Rightarrow \quad R_R^3=9R_P^3$$

$$\Rightarrow \quad R_R=9^{1/3}R_P$$

$$\Rightarrow \quad R_R>R_Q>R_P$$

$$\Rightarrow \quad V_R>V_Q>V_P$$

Also, $\frac{V_R}{V_P}=9^{1/3}$ and $\frac{V_P}{V_Q}=\frac{1}{2}$

Hence, (B) and (D) are correct.

4. Gravitational field is the acceleration due to gravity.

So, $g=\begin{cases}\frac{GM}{r^2} & r\geq R \text{ (Outside)}\\ \frac{4}{3}\pi G\rho r & r<R \text{ (Inside)}\end{cases}$

$$\Rightarrow \quad \frac{F_1}{F_2}=\frac{r_2^2}{r_1^2}\text{(Outside)}$$

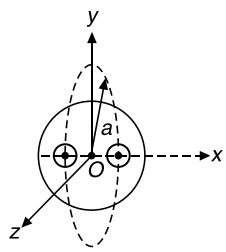
where $r_1>R$ and $r_2>R$

and $\frac{F_1}{F_2}=\frac{r_1}{r_2}$ (Inside)

where $r_1<R$ and $r_2<R$

Hence, (A) and (B) are correct.

5. The spherical cavities can be assumed to be negative masses placed symmetrically about origin O. So gravitational force due to this object at origin is zero.



The dotted circle is lying in the yz plane and is an equipotential surface (as we can see that two cavities are lying symmetrically on each side of the EPS (Equi-Potential Surface) so gravitational potential is same at all the points lying on the circle $y^2+z^2=a^2$ $\left(\text{where } a^2=\text{constant}\right)$.

Hence, (A), (C) and (D) are correct.

Reasoning Based Questions

1. Force acting on astronaut is utilised in providing necessary centripetal force, thus he feels weightlessness, as he is in a state of free fall.
Hence, the correct answer is (A).

Integer/Numerical Answer Type Questions

1. At height h, we have

$$g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2} \qquad \ldots(1)$$

Given that $g_h = \frac{g}{4}$

Substituting in equation (1) we get,

$$\frac{1}{4} = \left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad h = R$$

Applying Law of Conservation of Energy, from A to B, we get

$$(U+K)_A = (U+K)_B$$

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{R+h} + 0$$

Since $h = R$, so we have

$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{2R}$$

$$\Rightarrow \quad \frac{v^2}{2} = \frac{GM}{2R}$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$$

Since $v_e = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad v_e = \sqrt{2}v$$

$$\Rightarrow \quad N = 2$$

2. For point mass at distance $r = 3\ell$, we have

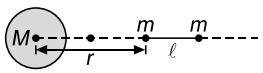
$$\frac{GMm}{(3\ell)^2} - \frac{Gm^2}{\ell^2} = ma \qquad \ldots(1)$$

For point mass at distance $r = 4\ell$, we have

$$\frac{GMm}{(4\ell)^2} + \frac{Gm^2}{\ell^2} = ma \qquad \ldots(2)$$

Equating these two equations, we get

$$\frac{GMm}{9\ell^2} - \frac{Gm^2}{\ell^2} = \frac{GMm}{16\ell^2} + \frac{Gm^2}{\ell^2}$$



$$\Rightarrow \quad \frac{7GMm}{144} = \frac{2Gm^2}{\ell^2}$$

$$\Rightarrow \quad m = \frac{7M}{288}$$

3. Since, $g_p = \frac{GM_p}{R_p^2} = \frac{4}{3}G\pi R_p\rho_p$

$$\Rightarrow \quad \frac{g_p}{g_e} = \frac{R_p\rho_p}{R_e\rho_e}$$

Also, $v_e = \sqrt{2gR}$

$$\Rightarrow \quad \frac{v_p}{v_e} = \sqrt{\frac{g_pR_p}{g_eR_e}} = \left(\frac{g_p}{g_e}\right)\sqrt{\frac{\rho_e}{\rho_p}} = \frac{\sqrt{6}}{11} \times \sqrt{\frac{3}{2}}$$

$$\Rightarrow \quad \frac{v_p}{v_e} = \frac{3}{11}$$

$$\Rightarrow \quad v_p = 3 \text{ kms}^{-1}$$